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Performance Evaluation of Developed Mathematical Models of Hot Air Balloon for Drone Application

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Abstract

There is a growing demand for flying drone with diverse capabilities for both military and civilian applications, successfully designing a hot air balloon necessitates an accurate model of its dynamics. In many of these applications, control of the balloons is important and requires proper mathematical modelling of the mechanical and thermal dynamics of the balloon. Although complex thermodynamic models have been developed, this research would address the use of simple heat transfer and performance relationship to develop a comprehensive hot-air balloon model for drone applications. The mathematical model comprises of a heat transfer model and a performance model, considering that the motion of the system is governed by heat transfer between the system and the surroundings. For the heat transfer, two models were developed (model 1 and model 2). Model 2 proved to be more accurate than Model 1 upon simulation. The simulation runs of the first and second model were compared using the simulation data inspector on Simulink to determine the difference between the signals graphically. The result shows that the second model (orange) is more responsive to heat loss than the first model (green).

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Keywords: balloon; dynamics; heat loss; model; simulation; simulink;

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1. Introduction

Hot-air balloons are lighter-than-air (LTA) aircrafts that contains heated air, and a gondola or wicker basket, which carries passengers and a source of heat. Researchers have become interested in them due to their inherent properties of high payload to weight ratio, long endurance capabilities and lower fuel consumption [1]. They are

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used for scientific experiments such as; space-flights, astronomical and telecommunications research, aerial surveying to provide valuable data to military, aircrafts applications and the like. They represent a platform that is unique and promising for many applications that involve an extended period of airborne presence [2]. The balloon is always unconditionally stable during flight because the weight of the balloon is always concentrated at the bottom, below the center of buoyancy. The control of the air balloon relies on the operator, if the operator wishes to descend, he could either discontinuing firing the burner, which leads to the hot air inside the envelope to cool naturally through heat exchange with the atmosphere or open a little opening at the top of the envelope, thus releasing some of the hot air, which reduces the buoyant force. On the other hand, when the air is heated by burning fuel, the balloon gains height [3]. The heated air in the envelope builds a pressure that is greater than the surrounding air as a result the balloon remains inflated. Usually, there is an opening at the bottom of the envelope through which the expanding hot air is allowed to escape, which prevents a large pressure gradient from developing. What this means is that the heated air pressure inside the balloon will end up being a little greater than the pressure in the cooler surrounding air. Therefore, the control input of a hot-air balloon is therefore unidirectional and switches between on and off states, precise positioning of the balloon is not required for most applications [3].

1.1. Model Based Design

Model based design is a virtual and mathematical process of developing control systems that are utilized in motion control, automotive, aerospace and industrial equipment applications [4]. It enables engineers to create virtual systems in parts, using continuous time and discrete time building blocks in order to determine whether the product will work together before it is manufactured or embedded code is written on a piece of hardware. Successful design of this system necessitates an accurate model of its dynamics. In many of these applications, control of the balloons is important and requires proper mathematical modelling of the mechanical and thermal dynamics of the balloon. The mathematical models of hot air balloons beyond simple buoyancy calculations have been largely driven by high-altitude balloon flights. Some of the early work on modelling was done by [3]. Their models took into account thermodynamic influences of solar and infrared radiation, as well as optical/infrared absorptivity and related irradiative properties of balloon films, which are important for long flight durations and day–night transitions. A model by [5] addressed the trajectory control problem for hot-air balloons such that they can reach a target location by controlling their altitude and riding the wind field judiciously. Hot air balloons have been of a particular interest to many noble people in the industry and academia, the extent of its literature is very vast, a few of which have been reviewed in this report as their findings establish the case for developing a model for drone applications. Some of the early literatures was done by [6], who developed a model for predicting the vertical motion of high altitude balloon systems in anticipation of increasing the use of balloons for transportation instruments and other scientific and commercial payloads [7], [8], [9]. Their models took into account, the influence of direct and reflected solar radiation and related radiative properties of balloon films, this was the starting point for subsequent researches that would be carried out. Stefan, 1979 developed a performance model that described the internal heat transfer [5]. The model was utilized for illustrating the effects of heat and the lift generated with respect to fuel consumption. Through his model, he was able to conclude that larger balloons are more economical than smaller balloons, black balloons offer better fuel economy when flying in sunshine, but inferior to an aluminized balloon without sunshine. [1] proposed a model for simulation the nonlinear dynamics of airships. The properties of aerodynamics and flight mechanics were incorporated into their model. The result was a simulated program developed and applied to analyze the control responses of airships.

2. Methodology

The envelope is assumed to be a sphere and the motion of the hot-air balloon depends on the heat transfer between the lifting gas inside and the ambient temperature. The mathematical model comprises of a heat transfer model and a performance model, considering that the motion of the system is governed by heat transfer between the system and the surroundings.

HEAT TRANSFER MODEL (Model 1)

The heat transfer model shows the heat transfer between the system and its surroundings

$$Q_{in} = mc_{air}\Delta T \ldots \tag{1}$$

$$\frac{\delta Qin}{\delta t} = mc_{air}\,(T_{heater} - T_g)$$

$$Q_{out} = Q_{cond} + Q_{rad} + Q_{conv\ldots} \tag{2}$$

$$\frac{\delta Qout}{\delta t} = \frac{\delta Qcond}{\delta t} + \frac{\delta Qrad}{\delta t} + \frac{\delta Qrad}{\delta t}$$

$$\frac{\delta Qout}{dt} = \frac{KA\Delta T}{D} + \varepsilon\sigma A\left(T_g^4 - T_{ambient}^4\right) + hA\left(T_g - T_{ambient}\right) \ldots \tag{3}$$

$$Net\ heat\ = Heat\ gain - Heat\ loss,\ \frac{\delta Qin}{\delta t} - \frac{\delta Qout}{\delta t} = \frac{mc\delta Tg}{\delta t}$$

$$\left[\frac{\delta Qin}{\delta t} - \frac{\delta Qout}{\delta t}\right]\frac{1}{mc} = \frac{\delta Tg}{\delta t} \ldots \tag{4}$$

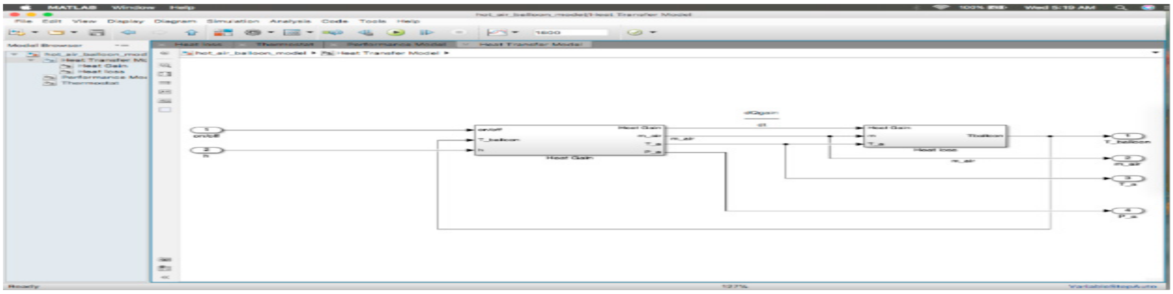

Fig. 1. A simulink build showing heat gain and heat loss blocks of model 1

MODEL 2

$$Q_{in} = mc_{air}\Delta T \ldots \tag{1}$$

$$\frac{\delta Qin}{\delta t} = mc_{air}\,(T_{heater} - T_g)$$

$$Q_{out} = Q_{iconv} + Q_{irad} + Q_{cond} + Q_{econv} + Q_{erad\ldots} \tag{2}$$

$$\frac{\delta Q_{out}}{\delta t} = h_i A(T_g - T_s)^{\frac{5}{4}} + \varepsilon_i\sigma A\left(T_g^4 - T_s^4\right) + \frac{KA(T_s - T_{ambient})}{D} + h_e A(T_s - T_{ambient})^{\frac{5}{4}}$$

$$+\varepsilon_e\sigma A\left(T_s^4 - T_{ambient}^4\right) \ldots \tag{3}$$

$$Net\ heat\ = Heat\ gain - Heat\ loss,\ \frac{\delta Q_{in}}{\delta t} - \frac{\delta Q_{out}}{\delta t} = \frac{mc_{film}\delta T_g}{\delta t}$$

$$\left(\frac{\delta Q_{in}}{\delta t} - \frac{\delta Q_{out}}{\delta t}\right)\frac{1}{mc_{film}} = \frac{\delta T_g}{\delta t} \ldots \tag{4}$$

$$\delta Q_{gain} = h_e A(T_a - T_s)^{\frac{5}{4}} + \varepsilon_e\sigma A\left(T_{ambient}^4 - T_s^4\right) = \frac{mc\delta T_s}{\delta t} \ldots$$

(5)

$$h_e(T_a - T_s)^{\frac{5}{4}} + \frac{\varepsilon_e\sigma A\left(T_{ambient}^4 - T_s^4\right)}{mc} = \frac{\delta T_s}{\delta t} \ldots \tag{6}$$

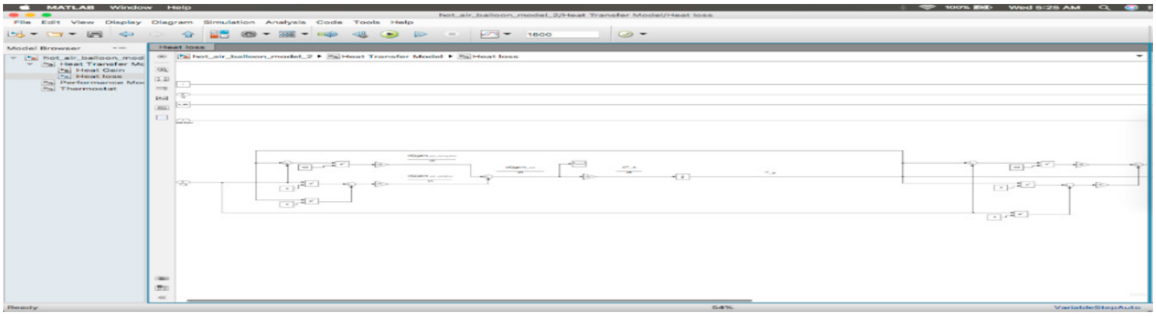

Fig. 2. A closer look at the added heat loss in model 2

PERFORMANCE MODEL

The performance model shows how the temperature generated from the heat transfer generates lift. The model was incorporated into both heat transfer models separately, which ends up generating two separate mathematical models due to the difference in the heat transfer models.

$$L = (\rho_a - \rho_g)V_g \ldots \tag{1}$$

$$L = V_g(\frac{\rho_a}{R_a T_a} - \frac{\rho_g}{R_g T_g} \ldots \tag{2}$$

$$R_a = R_g\,(\text{Assume same gas constant}), \rho_a = \rho_g\,(\text{Assume no pressure gradient})$$

$$L = V_{g\rho a}\left(1 - \frac{T_a}{T_g}\right)\ldots \tag{3}$$

$$\text{For neutral buoyancy}, L - G = 0, \Rightarrow L = G$$

$$\text{Substitute } L \text{ for } G \text{ to obtain the temperature required to maintain a gross weight } G \text{ at a steady altitude,}$$

$$\Rightarrow LV_{g\rho a} = 1 - \frac{T_a}{T_g}, \; T_g = \frac{T_a}{(1 - \frac{L}{V_{g\rho a}})}, \quad L = G$$

$$T_g = \frac{T_a}{(1 - \frac{G}{V_{g\rho a}})} \ldots \tag{4}$$

$$T_a = T_0 - 0.0065h, \rho_a = \rho_0\left(1 - 0.0065\,\frac{h}{T_0}\right) T_a = T_0 - 0.0065h, \rho_a = \rho_0\,(1 - 0.0065\,\frac{h}{T_0})^{4.2561}$$

$$\text{where}, T_0 = 288.15\ K \text{ and } \rho_0 = 1.225\,\frac{kg}{m^3}, \text{fromnewton'slaw of motion:}$$

$$m\ddot{x} + R\dot{x} + kx = F \ldots \tag{5}$$

$$\text{the system is assumed to be undamped and offers no form of resistance, therefore}$$

$$R = 0, \text{and } k = 0, \; \Rightarrow m\ddot{x} = F, \;\; m\frac{\delta u_x}{\delta t} = L - G, \; (V_{\rho g} + \frac{G}{g})\,\frac{\delta u_x}{\delta t} = L - G$$

$$\frac{\delta u_x}{\delta t} = \frac{1}{\left[V\rho_a\left(\frac{T_a}{T_g}\right) + \frac{G}{g}\right]}\left[Vg\rho_a\left(1 - \frac{T_a}{T_g}\right) - G\right]\ldots \tag{6}$$

$$\iint \frac{\delta u_x}{\delta t} = \chi(t) \ldots \tag{7}$$

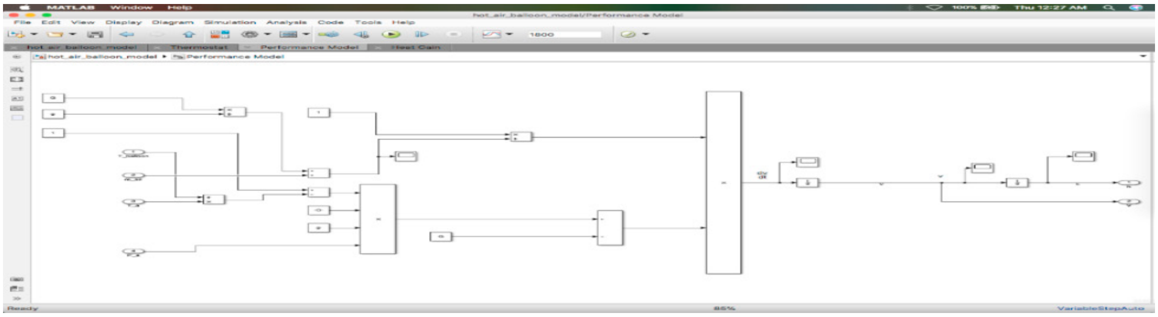

Fig. 3. A Simulink build showing the performance model

3. Result and Discussion

Having developed the mathematical model and build on Simulink, parameters were loaded on the model to determine if it works. The parameters loaded and result of the simulation of the first model is shown below.

Table 3.1 Model 1 Parameters

Variable	Value
C_{air}	1005.4 joule/kg K^{-1}
T_{heater}	373K
K	0.0265joule/mseconds1K^1

V	2200^3
D	0.0001 m
h	3.37 watt/m^2 K^{-1}
ε	0.87
σ	5.6703e-08
G	1500 N

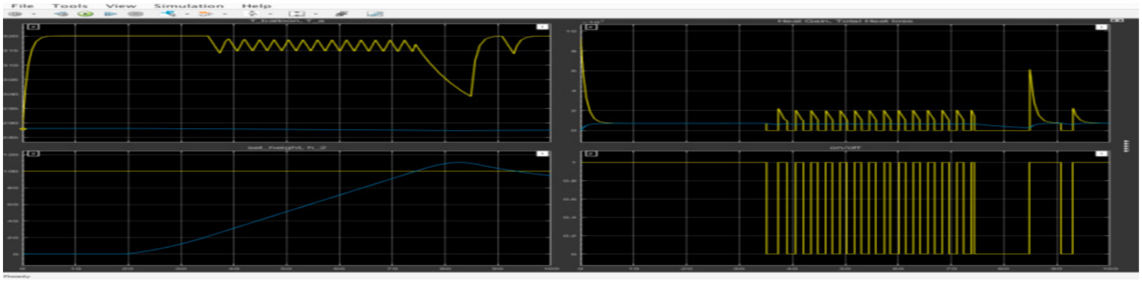

Fig. 4. A closer look at the altitude signal at the bottom left from simulation result of model 1

Various important signals are displayed on the oscilloscope to monitor the behavior of the system. Among these signals, the altitude signal at the bottom left of the scope shows that the envelope will lift off the ground. A closer look is as shown above. Parameters were loaded on the second model and simulation was run to determine if it works, the parameters loaded and results of the simulation are shown below.

Table 3.2 Model 2 Parameters

Variables	Values
C_{air}	1005.4 joule/kg K^{-1}
C_{film}	1700 joule/kg K^{-1}
T_{heater}	373K
h_i	16 watt/m^2 K^{-1}
h_e	3.37 att/m^2 K^{-1}
K	0. 265 les/m s^{-1} K-^{1}
D	0.0001 m
ε_i	0.85
ε_e	0.87
σ	5.6703e08watt/m^2K^4
V	2200 m^3
G	1500 N

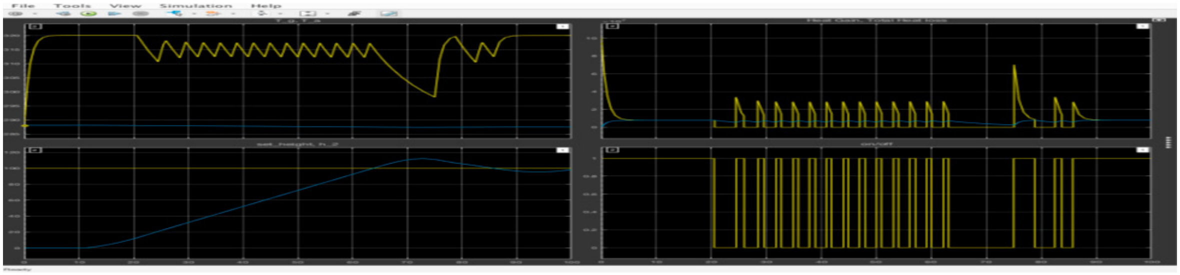

Fig. 5 Simulation results from model 2

Also, similar to the first model, various signals are displayed on the oscilloscope to monitor the behavior of the system. Among the signals, the altitude signal at the bottom left shows that the envelope will lift off the ground. A closer look at the signal is shown in the diagram above. The simulation runs of the first and second model are compared using the simulation data inspector on Simulink to determine the difference between the signals graphically. The results are shown below.

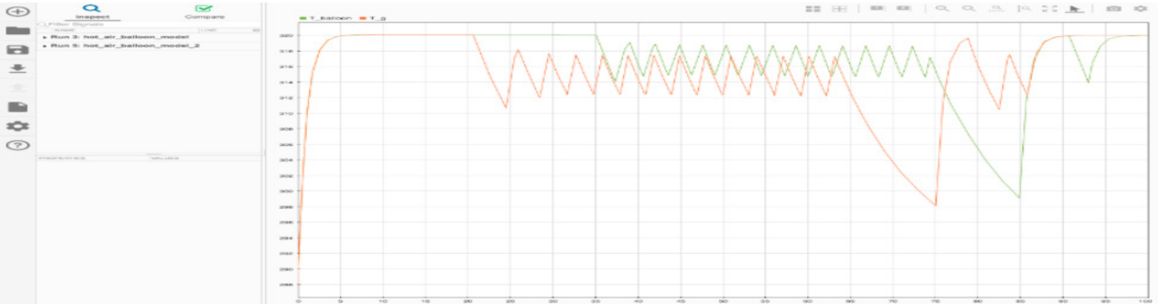

Fig. 6. The second model (orange) is more responsive to heat loss than the first model (green)

4. Conclusion

During the course of this research, two mathematical models for a hot air balloon were developed and scaled down, simulated and evaluated by varying parameters to determine optimal designs and important parameters. The models were simulated and evaluated by comparing their performance. The simulation runs of the first and second model were compared using the simulation data inspector on Simulink to determine the difference between the signals graphically. The result shows that the second model (orange) is more responsive to heat loss than the first model (green). Although the results of this concept are promising, there are certain drawback such as the system is assumed to be a lumped system therefore, the net heat chances or flow and balloon temperature chances were instantaneous. This is impractical considering that it takes time for heat to travel from the heating element to the balloon envelop, this can be observed in the sudden spikes of the compared heat gain and heat loss signals between the two models.

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