

1                   Autonomous Formation Flight using  
2                   Solar Radiation Pressure

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11   **Keywords**

12   CubeSat, Formation Flight, Autonomous Satellite Operations

13   **Abstract**

14   Autonomous formation flight enables new satellite missions for novel applications. The cost and limits of propulsion  
15   systems can be overcome if environmental resources are being benefitted of. Currently, atmospheric drag is used in  
16   low Earth orbit to this end. Solar radiation pressure, which is of similar order of magnitude as aerodynamic ram  
17   pressure, is however always neglected. We introduce this force and show that it can be exploited. We demonstrate  
18   through simulations that a formation geometry is established quicker if the solar radiation pressure is modeled.

19



1 Besides being a resource for orbital control, the correct  
 2 modeling of solar radiation pressure is also important  
 3 for applications where satellite position needs to be  
 4 maintained with high accuracy. This is the case, for  
 5 instance, for formations with satellite-satellite VHF  
 6 beam-forming techniques requiring close proximity of  
 7 satellites [21].

8 Since the solar radiation pressure force is of  
 9 appreciable amount, it also promises benefits for  
 10 controlling formations around celestial bodies such as  
 11 the Earth's moon, which is, however, not addressed  
 12 here.

## 13 2 Orbital Environment

14 CubeSats are exclusively used in LEO due to  
 15 conceptual and technological constraints such as the  
 16 utilization of Commercial-Off-The-Shelf (COTS)  
 17 components, power needed for space-to-Earth  
 18 communication, availability of a planetary magnetic  
 19 field for basic attitude control or reliable and free  
 20 availability of navigation information such as two-  
 21 line-elements (TLE). Notable exceptions are the  
 22 MarCO satellites[22], which performed a Mars fly-by.  
 23 However, these exceptional nanosatellites are not  
 24 typical CubeSats: neither in terms of technology nor  
 25 financial budget.

26 CubeSat are usually found on circular Earth orbits for  
 27 which the orbital speed  $v$  and the orbital period  $T$ ,  
 28 which depend solely on the radius  $r$  of the orbit, are  
 29 defined as the following:

$$v_{\infty} = \sqrt{\frac{\mu}{r}} \quad (1)$$

$$T = 2\pi \sqrt{\frac{r^3}{\mu}} \quad (2)$$

30 The symbol,  $\mu$ , is the standard gravitational parameter  
 31 of Earth.

### 32 2.1 Residual Atmosphere

33 In LEO, a highly rarefied yet appreciable atmosphere  
 34 exists, which is called thermosphere due to its  
 35 relatively high gas temperature. The high temperature  
 36 of this upper atmosphere layer is the consequence of  
 37 the absorption of solar ultra-violet (UV) radiation  
 38 causing heating. The UV radiation is highly dependent  
 39 on the solar cycle and therefore the heating of the  
 40 thermosphere, its temperature, its chemical  
 41 composition and its neutral density.

42 Neutral density causes aerodynamic drag which, over  
 43 long periods of time, leads to a decay of orbital altitude  
 44 of CubeSats and eventually their demise upon re-  
 45 entering denser atmospheric layers.

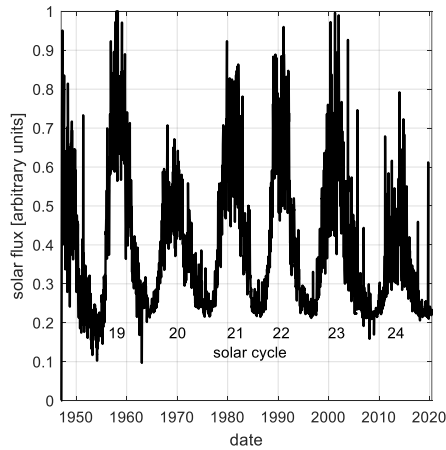
46 Orbital lifetime calculations of Qiao [23] enable the  
 47 formulation of a rule of thumb for CubeSats:

- 48 • at an altitude of 400 km, the orbital lifetime is  
 49 around one year. This is typically the minimum  
 50 lifetime for meaningful satellite operations
- 51 • at an altitude of 600 km, the orbital life time is  
 52 around 25 years. This is the maximum  
 53 permissible lifetime according to the IADC  
 54 Debris Mitigation Guidelines[24]

55 Therefore, a suitable altitude range for CubeSats is  
 56 400-600 km. In the following, the analyses focus on  
 57 altitudes from within that range.

58 Several atmospheric models exist. Here, we selected  
 59 the NRLMSISE-00 model [25] as it is publicly  
 60 available and provides the necessary information such  
 61 as temperature, chemical composition, neutral density  
 62 and variation with diurnal, annual and the repeating  
 63 11-year solar cycle. The solar cycle is shown in [Figure](#)  
 64 [2](#)[Figure-2](#) [26]. Moreover, the model resolves  
 65 variations with Earth's longitude and latitude.

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66  
 67 *Figure 2 solar activity, F10.7 cm solar radiation flux*  
 68 *(smoothened for clarity)[26].*

69 To derive atmospheric conditions for our subsequent  
 70 aerodynamic analyses, we investigate the variability of  
 71 the atmosphere during solar cycle 23, which occurred  
 72 between the years 1996 and 2007 (minimal to minimal

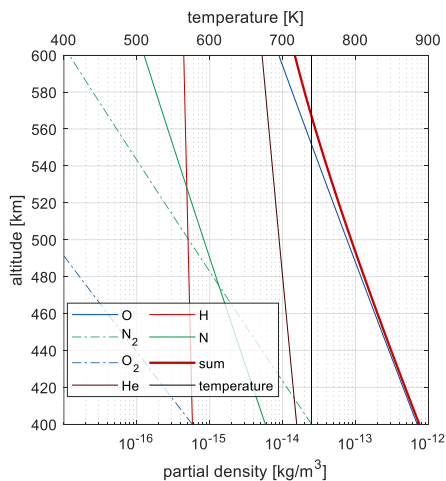
1 solar activity). The atmospheric conditions at the  
 2 beginning of this cycle will serve as reference for the  
 3 simulations in the following sections.

4 Figure 3 shows the chemical composition of the  
 5 atmosphere in the desired 400-600 km range as well as  
 6 the temperature according to the chosen model. It can  
 7 be seen that within the considered altitude range the  
 8 atmosphere mostly consists of atomic oxygen, which  
 9 decreases approximately exponentially with altitude.  
 10 The exact chemical processes leading to such models  
 11 are not fully understood and are thought to be  
 12 responsible for inaccurate neutral density and thus  
 13 drag predictions[27]. The temperature is constant over  
 14 altitude at around 740 K. The pressure exerted onto a  
 15 perpendicular surface is the change of momentum due  
 16 to impact of the atmospheric particles per unit surface.  
 17 If we assume the particles stick to the surface, the  
 18 aerodynamic pressure  $p_{aero}$  is:

$$p_{aero} = \rho_{\infty} v_{\infty}^2 = \rho_{\infty} \frac{\mu}{r} \quad (3)$$

19 Here,  $\rho_{\infty}$  is the atmospheric density and  $v_{\infty}$  is the  
 20 orbital velocity as per computed with Eq. (1)(+).

21

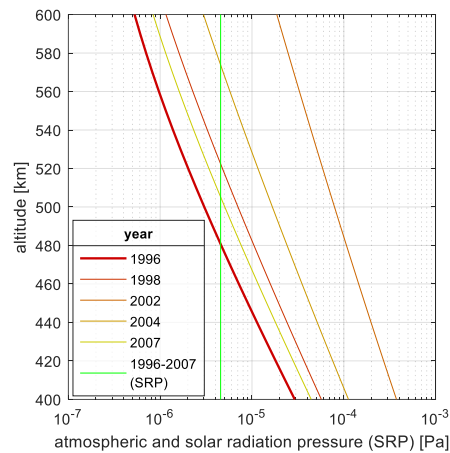


22

23 Figure 3 chemical composition of the low earth orbit  
 24 atmosphere (01:30 am, 01/01/1996 UTC).

25 For fully elastic impacts the change of momentum is  
 26 doubled and therefore also the pressure. The actual  
 27 impact will be a blend of fully elastic and fully sticking  
 28 gas-surface-interactions (GSI). It will be addressed in  
 29 the following sections.

30 Figure 4 shows the aerodynamic ram pressure  
 31 as a function of altitude and selected instants during  
 32 solar cycle 23 on the night side of the Earth at the  
 33 equator.



34

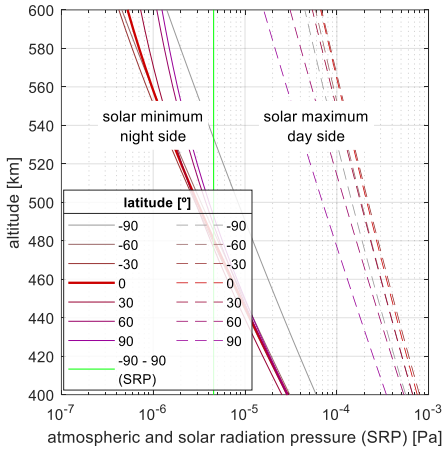
35 Figure 4 aerodynamic ram pressure and solar radiation  
 36 pressure for the solar cycle 23. At lat=0, lon=0, 1:30  
 37 am(night time).

38 According to the atmospheric model, the aerodynamic  
 39 pressure can change by almost two orders of  
 40 magnitude over the solar cycle. Thus, also the  
 41 available control forces change significantly over the  
 42 solar cycle.

43 Figure 5 shows the density's latitude  
 44 dependence for the year 1996 low-solar-activity  
 45 reference at night time and for the high solar activity  
 46 2004 during the day.

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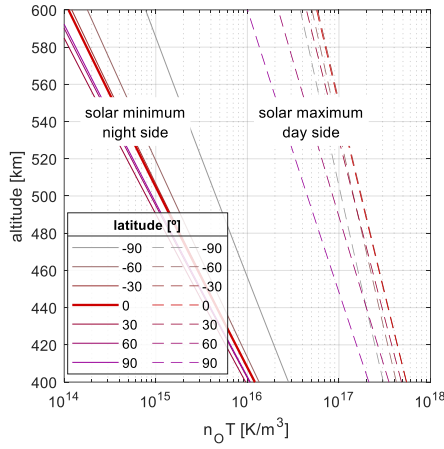
1 atmospheric and solar radiation pressure (SRP) [Pa]  
 2 Figure 5 latitude effect on atmospheric pressure (1996 – at  
 3 solar minimum, night side, 2002 day side)

4 From the figure, it can be inferred that the latitude  
 5 dependence is moderate—particularly for lower  
 6 altitudes and the night time. Comparing the 2002 night  
 7 time neutral densities shown in Figure 5 with  
 8 the 2002 night time neutral densities shown in Figure  
 9 4, it can be seen that daytime densities are  
 10 higher than night time densities as is expected for ideal  
 11 gasses heated by solar radiation during the day.

12 During the solar cycle minima and night time, the  
 13 atmosphere is less dense and therefore provides  
 14 smaller aerodynamic forces for satellite orbit control.  
 15 Thus, the low solar activity and night time atmospheric  
 16 conditions serve as a conservative values and are used  
 17 for our analyses. Control will be better for all other  
 18 conditions.

19 Ultimately, a control algorithm shall account for exact  
 20 atmospheric conditions.

21



22 parameter  $n_o T$  as function of latitude and instant  
 23 during solar cycle [ $K/m^3$ ]  
 24

25 Surface physics models used in the remainder of this  
 26 study depend on the product atomic oxygen partial  
 27 density and atmospheric temperature,  $n_o T$ . Its variation  
 28 over the solar cycle, latitude and altitude is shown in  
 29 Figure 6.

30 **2.2 Solar Radiation Pressure**

31 Whereas the solar cycle causes high variability of UV  
 32 radiation, which is responsible for the variability of the  
 33 upper atmosphere, the total solar irradiance is  
 34 counterintuitively largely constant [28].  
 35 Consequently, the solar radiation pressure is also  
 36 invariable over the solar cycle. Its magnitude is shown  
 37 for comparison purposes in Figure 4 and  
 38 Figure 5. In analogy to the aerodynamic  
 39 forces, the solar radiation pressure force depends on  
 40 change of momentum during impact of the particle,  
 41 which here is a photon. If it is fully absorbed, the  
 42 pressure reads:

$$p_{SRP} = \frac{G}{c} = \frac{(1,366 \pm 0.8) W/m^2}{300,000,000 m/s} = (4.55 \cdot 10^{-6} \pm 0.003) Pa \quad (4)$$

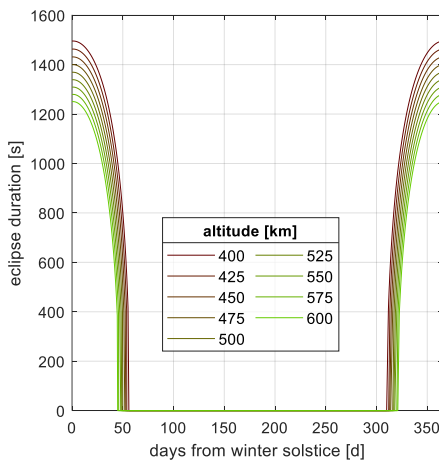
43 The variable  $G$  is the solar constant and  $c$  is the speed  
 44 of light. If the surface is fully reflective the momentum  
 45 change is doubled and hence also the solar radiation  
 46 pressure.

47 In low earth orbit, a spacecraft experiences periods of  
 48 eclipse during which solar radiation pressure is  
 49 unavailable. For sun-synchronous dusk-dawn orbits  
 50 (SSO, 6 am/pm local time of ascending/descending

1 node [LTAN/LTDN]) these periods are minimal. They  
 2 are computed and illustrated in Figure 7 for the two  
 3 extreme altitudes of those considered using Analytical  
 4 Graphics Inc.'s (AGI) Systems Tool Kit version 12.  
 5 The 23.5° day/night inclined terminator line can be  
 6 seen. The orbital plane is inclined by 97° for the 400  
 7 km altitude orbit and 97.7° for the 600km altitude  
 8 orbit. At the summer solstice both are inclined into the  
 9 same direction (Figure 7 right) preventing eclipse at  
 10 altitude. At the winter solstice however (Figure 7 left)  
 11 both inclination are opposite in direction. A spacecraft  
 12 in LEO is shadowed for some time.



24 Figure 7 orbital plane (400 and 600 km altitude) for a dusk/dawn orbit at winter solstice (left and center) and summer solstice



26  
 27 Figure 8 duration of eclipse for the considered altitude  
 28 range.

29 The second source of radiation from the Sun, i.e. solar  
 30 wind consisting of particles, is negligible and is  
 31 therefore not accounted for here [29].

32 Within this research, we further neglect gravitational  
 33 forces such as those by the J2 and J4 of the Earth and  
 34 disturbances caused by the Moon and the Sun. As  
 35 these forces are additional forces that can be exploited,

13 Figure 8 illustrates the duration of the eclipse  
 14 over the year and for the considered altitude range.  
 15 Eclipses are longer for higher altitudes. Accounting  
 16 for the longer orbital periods in higher altitude as per  
 17 Eq. (2), one can compute the percentage of eclipse.  
 18 It is highest at the winter solstice for the lower altitude  
 19 and amounts to 27%. In the following astrodynamics  
 20 analyses, we use this conservative value for our further  
 21 analyses. For flight software, the actual eclipse time  
 22 should be used for optimal formation flight control.  
 23

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36 our algorithm is hence conservative. If shown feasible,  
 37 our algorithms and underlying assumptions suffice to  
 38 solve the engineering problem. Future versions of our  
 39 algorithm will make use of these forces for optimal  
 40 formation flight control.

### 41 3 Surface-Physics

42 The forces exerted onto the satellites by the residual  
 43 atmosphere and the solar light depend on the surface's  
 44 properties. For both phenomena, the interactions are  
 45 analogous: momentum is transferred from the  
 46 impinging particle—either oxygen atom or a photon—  
 47 to the spacecraft. In either case, the momentum  
 48 transferred and thus the resultant pressure is dependent  
 49 on whether the impact is plastic, i.e. the particle  
 50 remains on the surface, or elastic, i.e. the particle is  
 51 bounced back. In the latter case, the momentum  
 52 transfer is doubled that of the former. The detailed  
 53 surface physics differ for both phenomena, which is  
 54 outlined below.

#### 55 3.1 Gas-Surface-Interaction

56 Particles impinging on a surface may be absorbed or  
 57 may immediately bounce back. In the case of  
 58 absorption, they may eventually desorb or react and  
 59 leave the surface as a molecule with released bond  
 60 energy [30]. However, this process is unlikely for the

1 highly-rarefied flow regime in LEO because of the  
2 scarcity of the reactants.

3 The actual gas-surface-interaction will be a blend of  
4 the immediate back-bouncing—also known as  
5 specular reflection—and adsorption-desorption  
6 process. This was first modelled by Maxwell [31]. The  
7 process depends on the surface-material-gas-particle  
8 bonding energy and the temperature of the material.

9 The amount of energy accommodated, i.e. transferred,  
10 on the surface can be modeled as in [32] using the  
11 energy accommodation coefficient  $\alpha$ :

$$\alpha = \frac{T - T_r}{T - T_w} \tag{5}$$

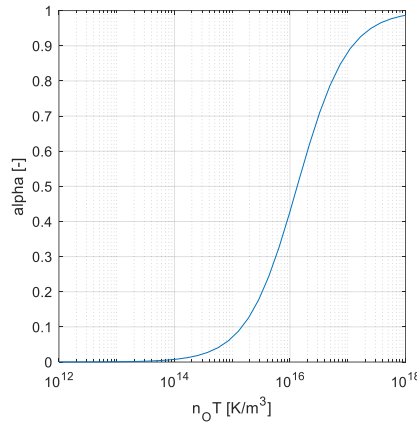
12 Here,  $T$ ,  $T_r$  and  $T_w$  are the temperatures of the  
13 impinging atoms, the reflected atom and the surface  
14 wall, respectively.  $\alpha$  becomes zero if reflected atoms  
15 have the same temperature as impinging atoms and are  
16 therefore not leaving while accommodating any  
17 energy on the surface. Conversely,  $\alpha$  becomes unity if  
18 reflected atoms depart at wall temperature and hence  
19 accommodate almost all energy at the surface. As we  
20 will see later, in the first case the aerodynamic forces  
21 are higher than in the second. Because higher forces  
22 are desirable for satellite control, materials with low  
23 energy accommodation are advantageous.

24 Pilinski [33] proposes the following semi-empirical  
25 model for  $\alpha$ :

$$\alpha = \frac{7.5 \cdot 10^{-17} n_o T}{1 - 7.5 \cdot 10^{-17} n_o T} \tag{6}$$

26 Here,  $n_o$  is number density of atmospheric atomic  
27 oxygen and  $T$  is the temperature of the incident atom.  
28 For the relevant range of the parameter  $n_o T$  as outlined  
29 in section 2.1 and shown in Figure 6,  $\alpha$  is shown in  
30 Figure 9.

31



32

33 Figure 9  $\alpha$  as function of  $n_o T$ .

34 For the solar minimum and night time, when the  
35 atmosphere is thinner, the parameter  $\alpha$  is very small or  
36 negligible. The decreased aerodynamic forces due to  
37 the thin atmosphere are partially offset by better  
38 aerodynamic performance due to a lower energy  
39 accommodation coefficient,  $\alpha$ .

### 40 3.2 Optical Surface Properties

41 The interaction of light, *i.e.* photons, with the surface  
42 of the spacecraft depend on its optical properties.  
43 There are four main mechanisms: absorption  $a$ ,  
44 diffusive and specular reflection  $r_D$ ,  $r_S$  and  
45 transmission  $t$ . As energy is conserved, their sum  
46 equals one, as in Eq. (7):

$$1 = a + r_D + r_S + t. \tag{7}$$

47 Solar panel covers are glassy and allow transmission  
48 of light. However, the underlying material is  
49 nontransparent. Hence, effectively, the transmissivity  
50 of solar cells is zero.

51 List [34] provides typical surface property values for  
52 common solar cells, which are tabulated in Table 1  
53 differentiated for beginning-of-life (BOL) and end-of-  
54 life properties.

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1 *Table 1 optical properties of solar cells* [34]

optical property	value
$a_{\text{BOL}}$	0.92
$a_{\text{EOL}}$	0.92
$\Gamma_{\text{D,BOL}}$	0.007
$\Gamma_{\text{D,EOL}}$	0.03
$\Gamma_{\text{S,BOL}}$	0.0727
$\Gamma_{\text{S,EOL}}$	0.05

2  
3 Solar radiation pressure forces are higher for higher  
4 reflectivities. Materials with higher reflectivities are  
5 therefore advantageous. However, satellite surfaces  
6 are typically entirely covered with solar cells to  
7 maximize power generation. To this end, absorption of  
8 the solar light is necessary which is a directly  
9 conflicting surface property requirement.

### 10 3.3 Solar-aerodynamic Forces

11 Sentman [35] provides equations to determine the  
12 aerodynamic force coefficients for the rarefied flight

29 *Table 2: aerodynamic coefficients for rarefied orbital flight regime..*

$C_D = \frac{P}{\sqrt{\pi}} + \cos \theta \left(1 + \frac{1}{2s_\infty^2}\right)Z + \frac{\cos \theta}{2} \frac{v_r}{v_\infty} (\cos \theta \sqrt{\pi}Z + P)$	[35]
$C_L = \sin \theta \frac{1}{2s_\infty^2}Z + \frac{\sin \theta}{2} \frac{v_r}{v_\infty} (\cos \theta \sqrt{\pi}Z + P)$	[35]
where	
$P = \frac{e^{-\cos^2 \theta s_\infty^2}}{s_\infty}$	[35]
$Z = 1 + \int_0^{\cos \theta s_\infty} e^{-y^2} dy$	[35]
$\frac{v_r}{v_\infty} = \sqrt{\frac{1}{2} \left(1 + \alpha \left(\frac{4RT_W}{v_\infty^2} - 1\right)\right)}$	[35]
$s_\infty = \frac{v_\infty}{\sqrt{2R/MT_i}}$	[36]

30

31 *Table 3 force induced by solar radiation pressure*

$\vec{F} = -p_{\text{SRP}}((1 - r_S)\vec{e}_{\text{Sun}} + 2\left(r_S \cos \theta + \frac{1}{3}r_D\right)\vec{e}_N) \cos \theta A$	[34]
--	------

32

## 33 4 Methods

### 34 4.1 Solar-Aerodynamic Formation Flight

35  
36 For autonomous formation flight, we implemented the  
37 Hill-Clohessy-Wiltshire (HCW) equations in the  
38 notations of Ivanov [37]. They model the local  
39 movement of the satellites relative to each other.

13 regime using the energy accommodation coefficient as  
14 input. They are repeated in [Table 2](#) for  
15 convenience. Here,  $\theta$  is the angle between the surface  
16 normal and the incoming particle direction,  $v_\infty$  is the  
17 speed of the impinging particle,  $\alpha$  is the energy  
18 accommodation coefficient as per Eq. (6).  $R$  is the  
19 ideal gas constant and  $T_i$  is atmospheric temperature.

20 Similarly, List [34] provides with the equations forces  
21 exerted on a surface due to solar radiation pressure,  
22 which are given in [Table 3](#),  $p_{\text{SRP}}$  solar-radiation  
23 pressure as per equation (4),  $r_D$  and  $r_S$  are optical  
24 surface properties as per Eq. (7) and [Table 1](#).  
25  $A$  is the surface area.

26 The outcome of these equations, *i.e.* the solar and  
27 aerodynamic force coefficients, is given in Figure 10  
28 and Figure 11.

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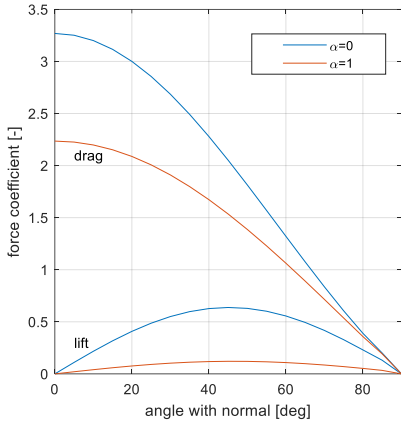
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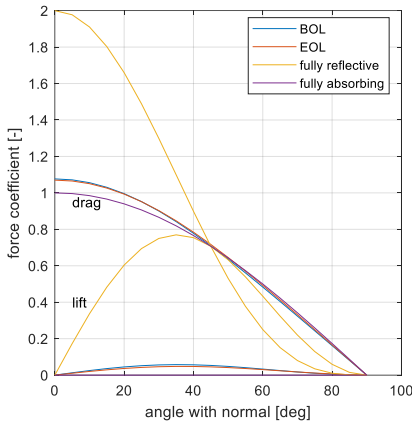
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1  
2 Figure 10 aerodynamic force coefficient for lift and drag vs.  
3  $\theta$ .

4 The HCW equations are a set of ordinary differential  
5 equations with a right hand side accounting for  
6 external forces, which are in our case those of  
7 aerodynamics and of solar radiation pressure.



8  
9 Figure 11 solar force coefficients for lift and drag vs.  $\theta$ .

10  
11 In contrast to the approaches of others, we  
12 implemented rotations around all three axes using  
13 Euler angles, i.e. roll, pitch and yaw. We apply an  
14 often employed convention using two extrinsic and  
15 one intrinsic rotation. The rotation axes are shown in  
16 Figure 1. For this research, permissible angles  
17 are multiples of 45°. In total four units of aerodynamic

18 surfaces are considered for a 1U CubeSat. Such  
19 surfaces consisting of a frame, solar cells and a spring-  
20 enabled deployment mechanisms are commercially  
21 available and frequently used for CubeSats. A  
22 conceptual drawing is given in Figure 1.   
23 **!Error! Reference source not found.**

#### 24 4.2 Control Law

25 We employ a classical Linear-Quadratic-Regulator  
26 (LQR) control algorithm following the example of  
27 Ivanov [37]. It controls at each time step all three axes.  
28 The regulator determines the control forces required  
29 for optimal control. Similarly to Ivanov, we employ a  
30 distributed formation flight control method:

- 31 • The maximum  $x$ -component of the three  
32 computed control forces (per satellite) is  
33 determined. The amount is subtracted from each  
34 satellite's  $x$ -component control force. Thus, the  
35 updated  $x$ -component control force is negative  
36 or zero. This shift is needed because drag can  
37 only provide forces opposite of the direction of  
38 flight (negative forces).
- 39 • For the directions perpendicular to the  $x$ -axis, i.e.  
40  $y$  and  $z$ , the average control force is computed  
41 and subtracted from the corresponding control  
42 force for each satellite. This allows minimizing  
43 the required control force for each satellite. It  
44 benefits the formation flight control as the  
45 available forces are typically smaller than the  
46 required control force.

47 The shift and averaging of required control forces  
48 requires the exchange of this information from one  
49 satellite to another through an inter-satellite  
50 communication link. To this end, the normally  
51 available telemetry and telecommand communication  
52 system can be used. By design, its range is at least  
53 3000 km as needed for LEO-ground communication.  
54 Thus, the range is vastly sufficient for the purposes  
55 considered here.

56 A routine computes the available solar-aerodynamic  
57 forces for the given 45° granularity of the Euler angles  
58 (see Figure 1). Restricting the permissible  
59 Euler angles prevents the controller from performing  
60 optimally and reduces stability because the required  
61 direction of the required control cannot usually not be  
62 achieved. However, this restriction also reduces the  
63 computational load, which is a scarce resource on a  
64 nanosatellite. This engineering choice has been seen  
65 favorable for our simulations. A detailed analysis will  
66 be made in our future research.

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1 We implemented the methods in MATLAB 2019 and  
 2 executed the code on a standard office laptop with  
 3 Intel I7 CPU in one computational thread. Run time  
 4 was about 3 hours per flight case (altitude, modelling)  
 5 as presented below.

## 6 5 Performance of Algorithm

### 7 5.1 Design Test Case

8 Orbital control for formation flight is required for four  
 9 situations:

- 10 1. Deployment, *i.e.* the establishment of the default  
 11 formation geometry from the initial  
 12 configuration after launch. Typically, the  
 13 satellites are in the same orbital plane and close  
 14 to each other in the beginning.
- 15 2. Reconfiguration, *i.e.* establishment of a  
 16 formation geometry from an existing formation  
 17 geometry.
- 18 3. Re-establishment of the formation geometry  
 19 from an arbitrary configuration; for instance,  
 20 after a formation flight control anomaly
- 21 4. Maintenance, *i.e.* the keeping of the formation  
 22 geometry in view of disturbances, control sensor  
 23 uncertainty and actuator imperfections. This  
 24 situation is the nominal one during which  
 25 routine operations for accomplishing the  
 26 mission objectives, for instance sensor use, is  
 27 carried out.

28 Table 4 CLUSTER mission parameters.

parameter	value
number satellites	3
distance satellite 1 – satellite 3	230 m
radius circular flight path middle satellite	50 m
angle of satellite 2's local orbital plane to main orbital plane	30°
duration of simulation	120 orbits
step size	3 s (constant)

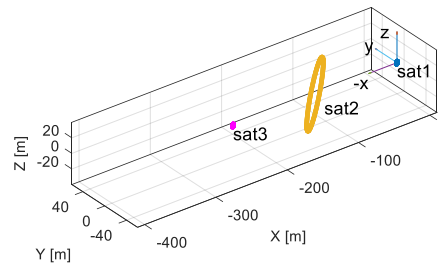
29

30 Within this research, we focus on the first and  
 31 demonstrate our algorithm with a mission scenario we  
 32 coined Cubesat of Luxembourg's University for Space  
 33 Technology and Earth Research (CLUSTER). It  
 34 consists of three 1U CubeSats. The choice was made  
 35 because, conventionally, a CubeSat deployer allows  
 36 the ejection of three CubeSat units. Our three satellites  
 37 would be simultaneously ejected with a very small  
 38 differential speed dictated by the required separation

39 springs [38] between the individual CubeSats. The  
 40 algorithm is capable to simulate an arbitrary number  
 41 of satellites.

42 The main parameters for this mission are listed in  
 43 Table 4. The demonstration formation  
 44 geometry consists of, in Local Horizontal Local  
 45 Vertical (LHLV) coordinates, two satellites flying at  
 46 constant distance and a third satellite circling an  
 47 intermediate position. The target formation geometry  
 48 is illustrated in Figure 12.

49



50

51 Figure 12 target formation geometry.

52 In Earth-centred coordinates, satellites 1 and 3 share  
 53 the same Kepler elements except for the true anomaly.  
 54 Satellite 2's eccentricity and right ascension node is  
 55 slightly different from those of the other two satellites  
 56 leading to the circulating motion in LHLV. The  
 57 formation features particular small satellite distances.  
 58 Applications of such small formations are numerous.  
 59 For instance, distributed sensors architectures requires  
 60 such geometries. The small-distance flight requires the  
 61 accounting of small disturbances such as aerodynamic  
 62 drag and solar radiation pressure. In this article, we  
 63 show how to not only integrate the disturbance but also  
 64 how to make use of it for formation flight control.

### 65 5.2 Results: Effect of Solar Radiation Pressure

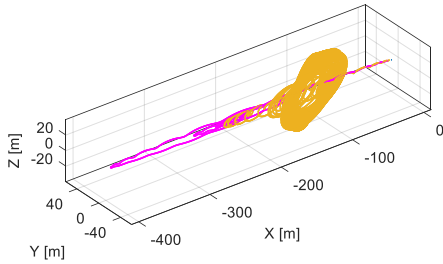
66 Figure 13 shows the trajectories of the three satellites  
 67 relative to each other over a period of 120 orbital  
 68 periods as computed by our algorithm that we coined  
 69 CosmosAlpha in the local 3D LHLV coordinate  
 70 system. All three satellites start at the origin. Satellite  
 71 3 moves toward the  $-x$  direction, overshoots its target  
 72 location and then returns to it at  $x=-230$  m. The  
 73 overshoot is due to the imperfect control algorithm;  
 74 however, its robustness is demonstrated by the  
 75 satellite's eventual return. Satellite 2 starts to move out  
 76

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1 of the z-x plane approaching the target relative circular  
2 movement with a period of one orbital period  
3 (compare the results of the solution obtained  
4 the control algorithm shown in [Figure](#)  
5 [13](#) [Figure 13](#) to the analytical solution in [Figure](#)  
6 [12](#)).



7  
8 *Figure 13 trajectories of satellite 2 (beige) and 3 (magenta)*  
9 *relative to satellite 1 computed with our algorithm using*  
10 *solar-aerodynamic forces at an altitude of 525 km.*

11 In the following, we focus on analyzing the trajectory  
12 of satellite 2 only. The satellite moves in the 3D space  
13 and is therefore particularly suitable to show the  
14 features of our algorithm. [Figure 14](#), shows the  
15 coordinates x, y and z (from top to bottom) over time  
16 of the position of the 2<sup>nd</sup> satellite for different altitudes  
17 in between the 400 km and 600 km limits. Comparing  
18 the computational cases in which only aerodynamic  
19 forces are modeled (dashed lines), it clearly can be  
20 seen that satellite 2 moves faster to its target trajectory  
21 at lower altitudes. This is expected since the denser  
22 atmosphere leads to higher aerodynamic forces  
23 available to the formation flight controller. [Figure](#)  
24 [14](#) [Figure 14](#), shows 120 orbits of the flight for the x-  
25 coordinate. For the y and z coordinate only a zoom to  
26 the first 20 orbits is shown for clarity of the display.

27 The solid lines show the trajectories of satellite 2 with  
28 modelled solar radiation pressure. Direction of the  
29 solar radiation and eclipse time correspond to a 6 am  
30 LTAN orbit and is therefore approximately in the  
31 direction of the -y axis. It can be seen that exploiting  
32 solar radiation pressure in addition to aerodynamic  
33 forces leads to a faster establishment of the formation  
34 geometry.

35 It can also be seen that at the upper limit of the 400-  
36 600 km range, the satellite does not reach the target  
37 trajectory. It is a consequence of very small available  
38 aerodynamic forces. Solar-aerodynamic formation  
39 flight would require larger control surfaces for  
40 formation flight.

### 41 5.3 Verification of the Effect of Eclipse

42 The simulations for which results are presented in the  
43 previous sections are conservative as they model the  
44 loss of solar radiation pressure due to eclipse for a  
45 fraction of the orbital duration. For the considered 6:00  
46 am LTAN orbit, eclipse only takes place around the  
47 winter solstice as illustrated in section [2.2.2](#), [Figure](#)  
48 [8](#). The effect of eclipse is investigated exemplarily  
49 here for one altitude, i.e. 525 km. [Figure 15](#) [Figure 15](#),  
50 shows the trajectory of the second satellite with  
51 modeled eclipse and without. As before, 120 orbits are  
52 shown for the x-coordinate, but only 20 orbits for the  
53 y and z coordinate for clarity. For comparison  
54 purposes, we also show the trajectory not accounting  
55 for solar radiation pressure. It can be seen that the  
56 accounting for eclipse results in a slightly shortened  
57 duration of the formation deployment. The results are  
58 in-line with the expectations as the eclipse period is  
59 relatively short.

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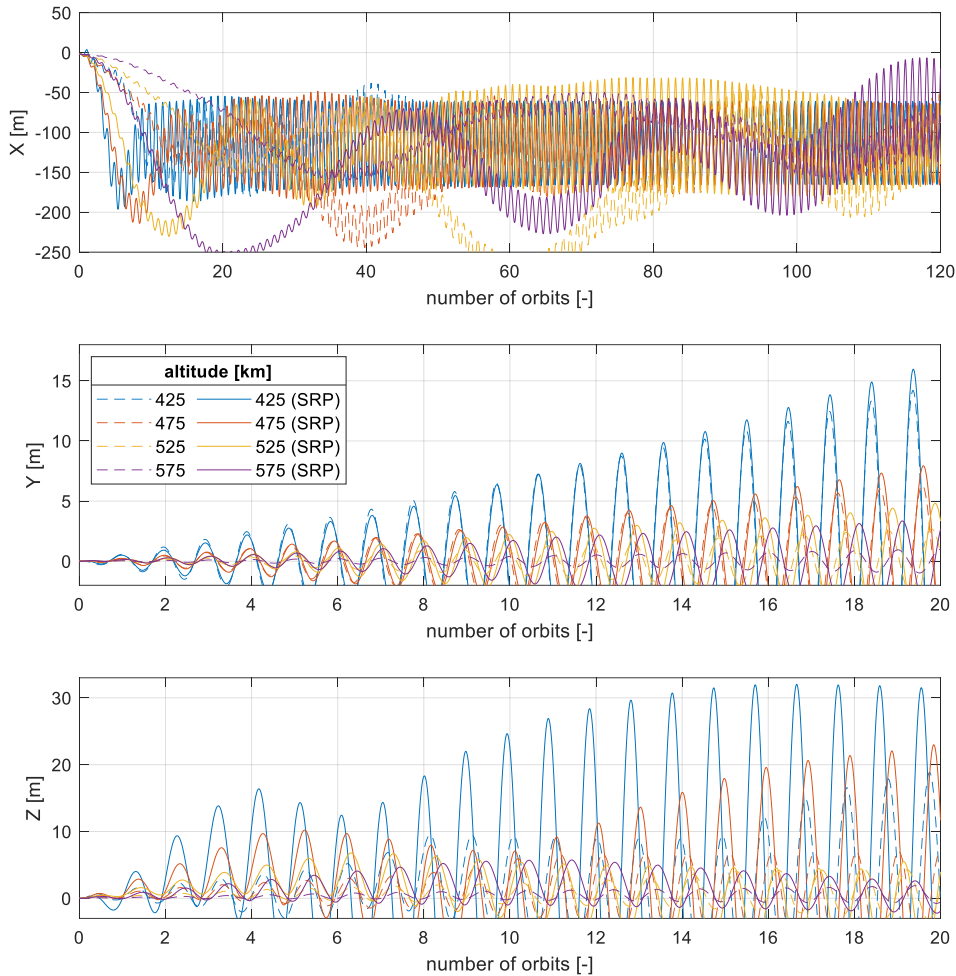
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1  
2 *Figure 14 formation deployment with eclipse, results for satellite 2.*

### 3 **6 Summary and Conclusions**

4 We report on our progress in developing the formation  
5 flight algorithm coined Cosmos in its version Alpha.  
6 We accounted for the first time for the presence and  
7 use of solar radiation pressure and show that the force  
8 is of significance compared to other disturbances in  
9 low earth orbit. We demonstrate that solar radiation  
10 pressure can be used to control the flight of satellites  
11 to establish a CubeSat formation in conjunction with  
12 aerodynamic forces. Emphasis was put on the use of

13 conservative assumptions for these forces and their  
14 duration. Additional forces available for formation  
15 flight control were also neglected. Hence, our  
16 algorithm is realistic. A future version accounting for  
17 more effects will be higher performing.

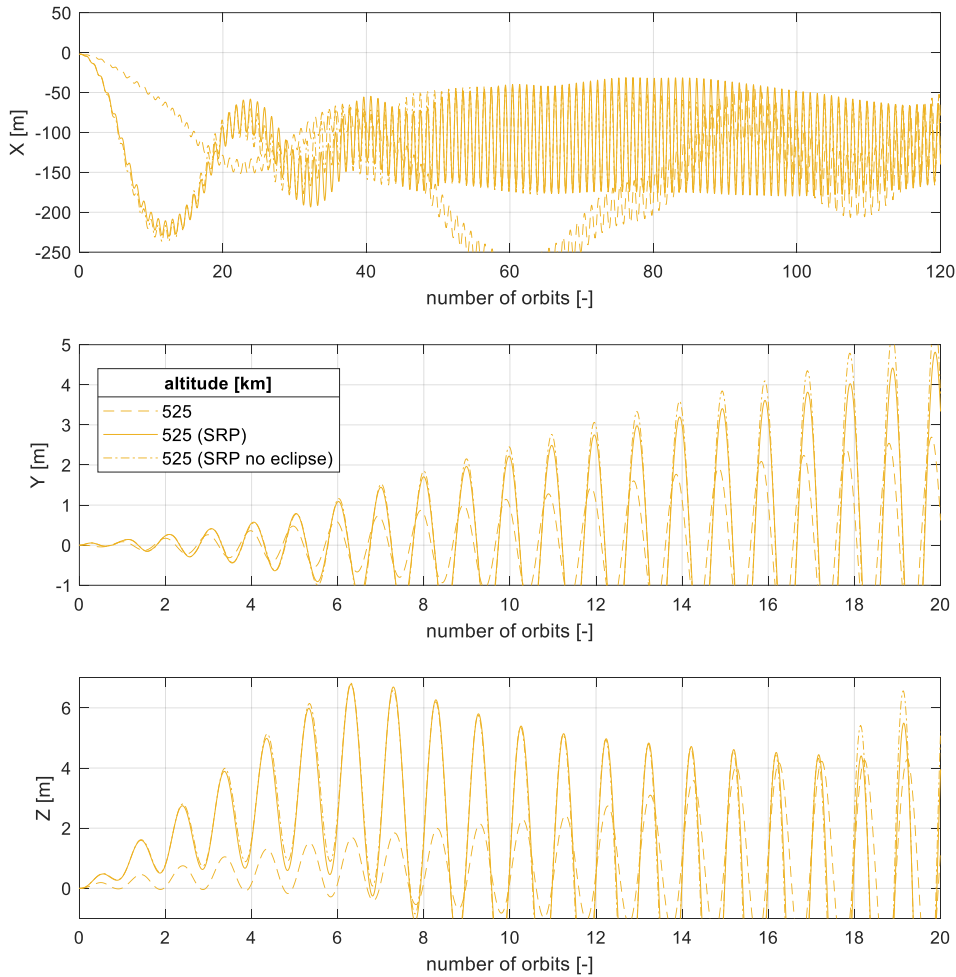
18 Our simulations show that the formation geometry is  
19 reached faster with modeling of the solar radiation  
20 pressure. Consequently, it can also be concluded that  
21 neglecting of the solar radiation pressure leads to  
22 suboptimal formation flight control.

## 7 Acknowledgments

5

2 The authors thank Ms. Fiona Singarayar-Byerley and  
3 Mr. Lawrence Byerley for the proofreading the  
4 manuscript.

6



7

8 *Figure 15 effect of eclipse on the duration of the deployment of the nanosatellite formation geometry.*

9

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