A Logical Analysis of Freedom of Thought

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Abstract

This paper studies the logical form and properties of one prominent category of epistemic rights: the freedom of thought and belief. We do so in the broadly Hohfeldian formalization of rights developed by [Markovich, 2020,Markovich, 2019], but extended with tools from doxastic logic. The resulting analysis reveals subtle differences in the way freedom of thought can be analyzed, and how these differences affect the logical properties of this doxastic right and the normative positions it incorporates.

Keywords: rights and duties, normative positions, epistemic rights, doxastic logic, action logic, multi-modal logic, conceptual analysis

1 Introduction

The freedom of thought and belief is one of the most fundamental and intimate human rights declared not only by the United Nations' Universal Declaration of Human Rights, but also by the European Convention of Human Rights and the vast majority of western constitutions. This paper studies freedom of thought and belief from a logical point of view.

We investigate freedom of thought as an epistemic right. According to a new approach in philosophy, an epistemic right is one that protects and governs the distribution and accessibility of epistemic goods [Watson, 2021]. For the development of a formal analysis within the theory of the normative (or Hohfeldian) positions (see Section 2), we assume that an epistemic right, in the narrow sense, is a right pertaining to a certain state of knowledge or belief of the right holder. [2] Next to the freedom of thought, such rights include someone's right to know—or to not know—her medical test's result, the citizens' right

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[2] In a broader sense, rights, where not the right-holder's, but the duty-bearer's epistemic state is concerned by the right, can also be considered as epistemic rights, such as the right to be forgotten, or the right to privacy, studied in e.g. [Aucher et al., 2011,Aucher et al., 2010,Cuppens and Demolombe, 1996].

to know the declaration of assets of members of parliament, the consumers'
right to not be misled by advertisements, or, for example, the right to truth:
the right, in the case of grave violations of human rights, of the victims and
their families or societies to have access to the truth of what happened. In this
paper, we focus on freedom of thought and belief, and investigate its content
and logical properties through an analysis using Hohfeldian conceptions.

While many systems at the intersection of epistemic logic, deontic logic, and
the logic of agency have been developed e.g. [Broersen, 2011], the notion of epis-
temic rights *as normative positions* has not yet been investigated in logic. The
deontic logic literature has been mostly concerned with epistemic obligation,
Åqvist's paradox [Åqvist, 1967,Hulstijn, 2008], and the theory of knowledge-
based obligations [Pacuit et al., 2006]. In their work on privacy policies, Aucher
et al. [Aucher et al., 2011,Aucher et al., 2010] investigated both the obligation
and the permission to know something, differentiating between obligatory and
permitted knowledge and obligatory and permitted messages.

The notion of epistemic rights *per se* is not completely new to the philo-
sophical literature, but so far it has been restricted to the right to believe when
discussing justification in epistemology, see for instance [Dretske, 2000], or fo-
cused on epistemic obligation [Feldman, 1988,Stapleford, 2012]. It is a recent
development that epistemic rights are discussed *as* a group of legal rights by
Watson [2018, 2019, 2020], a categorization with which we agree. Before, if
epistemic rights were discussed together with normative positions, it was al-
ways in comparison or contrast with them [Wenar, 2003,Altschul, 2021]. Wenar
even claims that the "epistemic (...) realms contain no claims, powers, or im-
munities" [Wenar, 2015]. This paper can be seen as challenging that view, by
showing that analyzing the epistemic rights using Hohfeldian categories yields
interesting insights.

The paper is structured as follows. In section 2, we briefly introduce the
theory of normative positions that we will use, and section 3 presents the details
of the language and semantics, including the doxastic operators. In section 4
we turn to freedom of thought proper, and section 5 concludes by pointing to
directions for future work. The contribution of this paper is conceptual, and the
mathematical observations we make are elementary. The proofs are thus omit-
ted, and we leave aside the study of the meta-logical properties (completeness,
decidability and tracktability) of the systems that we are using.

2   The Theory of Normative Positions

The theory of normative positions goes back to Hohfeld's typol-
ogy [Hohfeld, 1923] and its seminal formalizations in [Kanger, 1971]
and [Lindahl, 1994]. See [Makinson, 1986] and [Sergot, 2013] for a
critical assessment of this tradition. New formal approaches to Ho-
hfeldian rights have been developed and presented by several au-
thors, for instance [Sartor, 2005], [Gelati et al., 2004,Gelati et al., 2002],
[Governatori and Rotolo, 2008], and more recently in [Dong and Roy, 2017],
and [Markovich, 2020,Markovich, 2019]. The work in this paper builds on the

latter, but we change the semantics somewhat and extend it with tools from epistemic logic.

Hohfeld proposed to distinguish between four types of atomic right-positions (Claim, Privilege, Power, Immunity) and their correlative duty positions (Duty, No-claim, Liability, Disability). See Figure 1, taken from [Markovich, 2020]. The right-positions in the left square are claim-right and privilege. A claim-
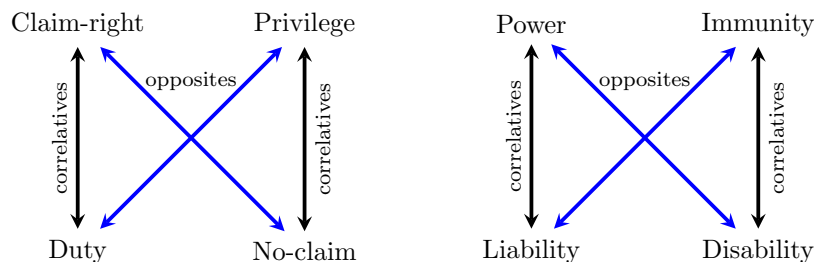
Fig. 1. The Hohfeldian atomic types of rights, and their correlative

right of an agent concerns the counter-party's actions. The counter-party has an obligation to do the certain thing, and this obligation is directed to the right-holder. Hohfeld calls this a duty, in the narrow sense. The seller's right against the buyer that the latter pays the purchase price, for instance, is a claim-right. The freedom or privilege[3] to do something, on the other hand, is understood as not being the subject of a claim-right coming from the counter-party. A land owner's right to use her own land refers to her privilege in the sense that the counter-party does not have the claim-right against her to refrain from that use.[4] Privilege can thus be seen as a directed version of the standard (weak) permission in deontic logic.

The normative positions in the right square capture the agent's ability to change an (other) agent's normative positions. For that reason, they have been called "higher order" or *capacitative* [Fitch, 1967]. They thus capture the norm-changing potential, or lack thereof, of an agent [Dong and Roy, 2017,Markovich, 2020]. A land owner, for instance, has a right—here a power—to sell her land and the other agent(s), for instance, the one who so far has rented a house on it, is (are) exposed, that is, liable (in the Hohfeldian sense), to this change: his relevant normative positions will change. The land owner has immunity, though, regarding her neighbor selling the land: the neighbor is unable to change the owner's normative positions concerning the land. This counts as a disability, meaning that he does not have a power to do that.

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[3] 'Freedom' is an often used alternative for 'privilege' in the literature dealing with Hohfeld.

[4] In this particular case the owner has, in fact, such a privilege against any counter-party. The owner's position is a so-called absolute position or a multital (vs paucital) right. Her property rights, thus this privilege of hers too, are to be considered against every other agent. [Simmonds, 2001] and [Markovich, 2020].

While it can be argued that the type of deontic actions—changing someone's normative positions—that are involved in the capacitative square is of a different kind than the actions that claim-rights and privileges concern [Jones and Sergot, 1996,Dong and Roy, 2017,Markovich, 2020], here we analyze them using a simple combination of alethic and agentive modalities. We do so because these are actually actions whose execution is *possible* if and only if the actor has the power to do so. This simplification allows us to focus on the formalization of freedom of thought while keeping our logical language and its interpretation relatively simple. In the Conclusion we briefly discuss the consequences of adopting a more dynamic modeling of actions for some of the results presented below.

The main characteristic of Hohfeldian theory is that the normative positions are inherently relational. Not only duties are directed, but also all the other positions: for instance, the dual[5] of a duty, a privilege (or freedom) is to be interpreted as being free from a given other agent's, the counterparty's claim-right. One can, however, also express the idea of an absolute duty in this relational framework. Hohfeld himself differentiated between paucital and multital positions: in the former we consider one given relation between two parties, while in the latter one agent is a party in a series (conjunction) of such relations [Simmonds, 2001,Markovich, 2020]. We will formalize freedom of thought as such a multital right: we have it against/with regard to everyone else.[6]

Hohfeld's theory identifies four atomic types of rights to resolve the terminological confusion arising from (over)using the word 'right' while meaning different concepts. Legal language, though, still uses the word 'right' or sometimes, as in our case, 'freedom', to refer to different positions, or, often, their complex combinations. As we argue below, this is indeed the case for freedom of thought. It consists of a combination of at least three atomic types: a multital privilege, a multital claim-right, and a multital immunity.

3   Language and Semantics

We analyze freedom of thought using a combination of standard deontic logic augmented with directed operators [Markovich, 2020], and epistemic/doxastic logic.

Definition 3.1 Let A be a finite set of agents and Φ a set of propositional letters. The language $\mathcal{L}$ is defined as follows:

$$p \in \Phi \mid \varphi \wedge \psi \mid \neg\varphi \mid \{E_a\varphi \mid O_{a \to b}\varphi \mid B_a\varphi\}_{a,b \in A} \mid \Box\varphi$$

$\mathcal{L}$ thus extends the propositional logic with four modalities. E_a is the agency modality and should be read as "agent a sees to it that...". $O_{a \to b}$

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[5] Or "opposite" in the less precise Hohfeldian term.

[6] As a matter of fact, *everyone* has it against everyone else (which would be a complete directed graph from the graph-theoretical point of view [Markovich, 2019]), but for the sake of simplicity, here we analyze *one's* freedom of thought.

is a directed obligation modality, and should be read as "agent a has a duty towards b that...". B_a, on the other hand, is a doxastic modality, to be read as "agent a believes that...". The $\Box$ modality is the universal, alethic modality "it is necessary that." All these modalities have duals: the weak permissions operator, i.e. $P_{a \to b}$..., which stands for $\neg O_{a \to b} \neg$...; $\langle B_a \rangle$... which stands for $\neg B_a \neg$...; and $\Diamond$..., which stands for $\neg \Box \neg$....

We make the following assumptions regarding the logical behavior of these modalities. We take the deontic modalities $O_{a \to b}$ to be normal modalities validating the D axiom, i.e. $O_{a \to b}\varphi \to P_{a \to b}\varphi$. So the deontic fragment of our language is standard deontic logic. For the agentive modalities E_a, we take them to be non-normal, validating only substitution under logical equivalence and the T axiom ($E_a\varphi \to \varphi$). As it turns out the logical behavior of freedom of thought will be strongly influenced by what additional assumptions are made about the logic of E_a, for instance that the agents always see to it that necessarily true formulas hold ($\Box\varphi \to E_a\varphi$), or that the operator is regular (if $\Box(\varphi \to \psi)$ then $E_a\varphi \to E_a\psi$). Observe that from the T axiom it also follows that no agent can see to it that a contradiction holds. The doxastic modalities B_a are assumed to be normal modalities validating D ($B_a\varphi \to \langle B_a \rangle\varphi$). We do not, in particular, assume that the belief modalities are either positively or negatively introspective.

Given these assumptions, the language $\mathcal{L}$ is interpreted over frames containing a neighborhood function for each E_a, a deontic ideality relation for each $O_{a \to b}$, and a doxastic accessibility relation for each B_a.

Definition 3.2 Let A be a finite set of agents. A frame $\mathfrak{F}$ is a tuple of the following form:
$$\mathfrak{F} = \langle W, \{f_a, R_a^B, R_{a,b}^O\}_{a,b \in A}\rangle$$

Here W is set of possible worlds. The function $f_a : W \to \wp\wp(W)$ is a neighborhood function such that, for all $w \in W$ and $X \in f_a(w)$, we have $w \in X$. Both $R_a^B \subseteq W^2$ and $R_{a,b}^O \subseteq W^2$ are serial, binary relations. A model $\mathcal{M}$ is a frame $\mathfrak{F}$ together with a valuation function $V : \Phi \to \wp(W)$.

With this in hand the truth conditions of formula of our language is defined in the standard way. We have only defined explicitly the case for the modalities.

Definition 3.3 Let $||\varphi|| = \{w : \mathcal{M}, w \models \varphi\}$. Then:

- $\mathcal{M}, w \models E_a\varphi \Leftrightarrow ||\varphi|| \in f_a(w)$

- $\mathcal{M}, w \models \mathbf{O}_{a \to b}\varphi \Leftrightarrow \forall w'(wR_{a,b}^O w' \Rightarrow \mathcal{M}, w' \models \varphi)$

- $\mathcal{M}, w \models \mathbf{B}_a\varphi \Leftrightarrow \forall w'(wR_a^B w' \Rightarrow \mathcal{M}, w' \models \varphi)$

- $\mathcal{M}, w \models \Box\varphi \Leftrightarrow \forall w', \mathcal{M}, w' \models \varphi$

When a formula of the form $\Box\varphi$ is true in a model we will say that φ is necessarily true, and similarly for $\Box\neg\varphi$ and "necessarily false". Otherwise we will say that φ is contingent in a model. Validity in models, frames, and classes thereof, are defined as usual.

Since we do not make any specific assumptions regarding the interaction between these modalities, the set of validities over our intended class of frames is completely axiomatized by all propositional tautologies, the logic ET for the agentive modality E_a, KD for $O_{a \to b}$ and B_a, S5 for $\Box$, and the standard inclusion axioms relating the universal modality $\Box$ to the other operators in the language.

4   Freedom of Thought

We are now ready to address freedom of thought. We first provide some legal foundation for our analysis, and then move to the formalization itself.

4.1   Legal Foundations

The United Nation's document, The Universal Declaration of Human Rights [7], as its name says, declares what are the human rights. [8]   Article 18 is about freedom of thought: "Everyone has the right to freedom of thought, conscience and religion; this right includes freedom to change his religion or belief (...)." Article 19 says: "Everyone has the right to freedom of opinion (...); this right includes freedom to hold opinions without interference (...)." [9] [10]   The United Nation's Office of the High Commissioner for Human Rights' (OHCHR) general Comment Nr. 22 interpreting Article 18 and 19 says: [11]

> The right to freedom of thought, conscience and religion (which includes the freedom to hold beliefs) in article 18.1 is far-reaching and profound; it encompasses freedom of thought on all matters, personal conviction and the commitment to religion or belief (...). [Article 18] does not permit any limitations whatsoever on the freedom of thought and conscience or on the freedom to have or adopt a religion or belief of one's choice. These freedoms are protected unconditionally, as is the right of everyone to hold opinions

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[7] The document was proclaimed by the United Nations General Assembly in Paris on 10 December 1948 (General Assembly resolution 217 A) as a common standard of achievements for all peoples and all nations. https://www.un.org/en/universal-declaration-human-rights/

[8] We do not go into the philosophical discussion on where these rights come from (natural law vs. legal positivism), nor on what political or legal legitimacy the UN has. We only analyze formally what the declared human rights' content and implications are based on the official interpretation. The reader unwilling to accept the Declaration as a legal source because of the legitimacy questions regarding international law can instead consider a national constitution's relevant article, the wording of which is very much alike the Declaration.

[9] While Article 19 is about freedom of opinion and its expression, we believe that its internal part—freedom of opinion—is to be discussed together with the internal part of freedom of thought referring to the same thing as far as the formalization is concerned.

[10] The freedom of thought, conscience and religion has an internal and an external realm: the internal concerns the beliefs without concerning their expression, while the external concerns the manifestation of beliefs, such as religious practices—just like the separation between freedom of opinion and its expression. We intentionally cite only the parts of the Declaration concerning the internal realm, as our current investigation is only concerned with this.

[11] General Comment No. 22: The right to freedom of thought, conscience and religion (Art. 18): . 30/07/93. CCPR/C/21/Rev.1/Add.4, General Comment No. 22. (General Comments) https://bit.ly/37T15Uc

without interference in article 19 (...) The Committee observes that the freedom to "have or to adopt" a religion or belief necessarily entails the freedom to choose a religion or belief, including the right to replace one's current religion or belief with another or to adopt atheistic views, as well as the right to retain one's religion or belief. Article 18 bars coercion that would impair the right to have or adopt a religion or belief, including the use of threat of physical force or penal sanctions to compel believers or non-believers to adhere to their religious beliefs and congregations, to recant their religion or belief or to convert.

In the terminology of normative positions, freedom of thought, as its name suggests, includes a freedom (or privilege), but a multital one. There is no duty of ours toward anyone regarding our beliefs' content. But this privilege in itself would be a rather weak position, so freedom of thought also involves protections, in two ways. On one hand, it means a claim-right against everyone else not to interfere with it. What interfering with practicing a freedom of thought would be is, of course, debatable. One might raise the question whether it is really possible at all, for example, to force someone to believe in something. The common reference in this regard is Orwell's dystopia, 1984, and its thought police and thought crime concepts. [12] Whether forcing someone to adopt or change a belief is possible in reality is rather a question that psychology or neuroscience could answer; we do not need to commit ourselves in this matter, we only need to represent that it is forbidden. A freedom and a joint claim-right against everyone else is a frequent combination: these are what are usually called civil liberties. [13] On the other hand, the Declaration, according to its official interpretation, is also ruling out the possibility of changing this freedom (for example, by a country introducing penal sanctions, that is, duties to accept some specific view). This is what Hohfeld identifies as immunity: the other party's inability to change our normative positions (for example, in this case, imposing a duty as to what to believe in).

4.2 Formalization

Three components have, thus, to be analyzed: the freedom, the claim-right, and the immunity components of freedom of thought. We look at them in turn. Throughout we assume that the right-holder is a given agent a, and that the right bears on a's doxastic attitudes towards a given proposition φ.

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[12] In [Hulstijn, 2008], when combining deontic and epistemic logics, the possible need for the—as Hulstijn refers to it—'freedom of thought' axiom $K_i\varphi \rightarrow PK_i$ is raised "to exclude the definition of 'thought crimes' in Orwell's 1984" (the author then recants this as it would go against the purpose of access control policies). We don't think knowledge would be a good description of thought, especially in the context of this freedom. We, therefore, will use a belief operator.

[13] In the reception of Hohfeld, it was raised that he missed identifying this kind of liberty as an atomic type of right, but as is shown in [Markovich, 2020], this combination of Hohfeldian notions expressing what a civil liberty is rejects these opinions: a civil liberty (that [Bentham, 1843] calls vested liberty, while [Wright, 1963] calls simply as 'right'), is not an atomic, but a compositional or molecular type of right.

4.2.1   Freedom

As observed in the previous section, freedom of thought consists partly in being, indeed, a freedom in the Hohfeldian sense. One has no duty towards anybody else not to believe φ:

$$\bigwedge_{b \in A} \neg O_{a \to b} \neg \mathbf{B}_a \varphi$$

For readability we will use the dual $P_{a \to b}...$ instead of $\neg O_{a \to b} \neg ...$:

$$\bigwedge_{b \in A} P_{a \to b} \mathbf{B}_a \varphi \qquad\qquad \text{(FoT-F-}B_a\varphi)$$

This formulation does not rule out that a is in fact under the obligation to believe φ, i.e. that she is not permitted not to believe it. This privilege is indeed consistent with an(y) agent b having a claim-right against a that she (a) believes φ. To rule that out one can instead require that a has a multital privilege to hold any of the three possible attitudes towards φ: belief, disbelief, or suspending judgment.

$$\bigwedge_{b \in A} (P_{a \to b} \mathbf{B}_a \varphi \wedge P_{a \to b} \mathbf{B}_a \neg \varphi \wedge P_{a \to b} (\langle \mathbf{B}_a \rangle \varphi \wedge \langle \mathbf{B}_a \rangle \neg \varphi)) \qquad \text{(FoT-F-}a\text{-}\varphi)$$

Semantically, this condition restricts the application of freedom of thought to propositions that are contingent in a particular model. [14] We have indeed assumed that the doxastic modalities B_a are normal and that beliefs are consistent, i.e. they validate the D axiom. Under these assumptions about the belief modalities, (FoT-F-a-φ) predicts that freedom of thought does not apply to necessary truths or necessary falsities.

This restriction to contingent formulas is a consequence of the idealizations that we have made regarding the belief operators. The standard logical models of belief assume that the logic of modalities is either KD45 or K4.3 [Fagin et al., 2003,Stalnaker, 2006,Pacuit, 2013]. Here the modalities B_a are weaker. Yet, they still represent the agents as consistent and logically omniscient. This has the direct consequence that agents do not have the freedom to, for instance, suspend judgment about necessary truth or necessary falsities. A number of solutions to the logical omniscience problem have been proposed (c.f. [Hawke et al., 2019] and references therein), but introducing them here would go beyond the scope of this paper. This is a question of the adequate model of belief, not primarily of the logical form of freedom of thought. Even if we were to decide to adopt a weaker doxastic logic, this would arguably not affect the logical form of (FoT-F-a-φ). What this would change is its logical behavior.

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[14] It furthermore imposes a richness condition on the set of states that are normatively ideal from the perspective of a towards b. This richness assumption appears less controversial than the restriction to the contingent formula that we discuss this the main text.

4.2.2 Claim-right

Freedom of thought also consists of a multital claim-right to refrain from interfering with us practicing our freedom, that is, forcing us to not hold certain beliefs. As with freedom, we will first consider this claim-right as bearing on simply believing φ, and consider later on the consequences of extending it simultaneously to disbelief and suspending judgment.

A first attempt at capturing this claim-right is in terms of the others' correlated duty to refrain:

$$\bigwedge_{b \in A} \mathbf{O}_{b \to a} \neg E_b \neg B_a \varphi \qquad \text{(FoT-C1)}$$

This first attempt is, perhaps, overly strong. It rules out any attempt to convince someone, or simply teaching or instructing. This is not what freedom of thought forbids. It is rather the *forceful* intervention into someone's beliefs. The idea that someone is forced or prevented, against her will, to hold or form certain beliefs is beyond the scope of the language and the models that we are working with. Something which *is*, however, within the expressive power of our language is the idea that interventions which not only result in a not believing something, but rather make this impossible, are forbidden.

$$\bigwedge_{b \in A} \mathbf{O}_{b \to a} \neg E_b \neg \Diamond B_a \varphi \qquad \text{(FoT-C-}a\text{-}\Diamond\text{)}$$

The logical behavior of this formalization of the claim-right turns out to be not completely satisfactory, and it depends heavily on the assumptions that one makes regarding the logic of E_a. Recall that the T axiom for E_a rules out that a does the impossible. On the other hand, since $\Box$ is the universal modality, we get $\mathcal{M}, w \models \Diamond B_a \varphi$ implies $\mathcal{M}, v \models \Diamond B_a \varphi$ for *any* v in W. In other words, $\mathcal{M}, w \models \Diamond B_a \varphi$ implies that $\neg \Diamond B_a \varphi$ is true nowhere in $\mathcal{M}$. But then it is also impossible for b to actively rule out that possibility, which in turn entails that a has a (trivial) claim-right against b regarding a's belief in φ.

Observation 1 *For any model $\mathcal{M}$ and state w, if $\mathcal{M}, w \models \Diamond B_a \varphi$, then for all v we have $\mathcal{M}, v \models \bigwedge_{b \in A} \mathbf{O}_{b \to a} \neg E_b \neg \Diamond B_a \varphi$.*

Perhaps surprisingly, the other direction of the implication is also valid, provided that one makes the additional assumption that in any state there is at least something trivial that a does. Recall that this additional assumption translates syntactically to $\Box \varphi \to E_a \varphi$, which semantically corresponds to the fact that W is an element of $f_a(w)$ for any a and w.[15] This is a property that some agency operators satisfy, notably any normal ones like the "Chellas stit" [Belnap et al., 2001]. We do not need full normality, though. It is sufficient that f_a "contains the unit" [Pacuit, 2017].

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[15] In standard neighborhood semantics without the universal modality the syntactic correspondent of this condition is $E_a \top$.

Observation 2 *For any model $\mathcal{M}$ where all f_a contain the unit, and state w, if $\mathcal{M}, w \models \bigwedge\limits_{b \in A} \mathbf{O}_{b \rightarrow a} \neg E_b \neg \Diamond B_a \varphi$ then $\mathcal{M}, w \models \Diamond B_a \varphi$.*

FoT-C-a-$\Diamond$ and $\Diamond B_a \varphi$ thus become equivalent when we assume that agents can always see to it that necessary truths hold. This is a form of deontic collapse: the claim-right component of freedom of thought, an "ought", collapses to a modal fact about a's belief, an "is". To the extent that one sees this as an undesirable consequence of this particular model, this can be used as yet another argument against assuming that $\Box \varphi \rightarrow E_a \varphi$. Classical agency operators, e.g. in [Kanger and Kanger, 1966] or the *dstit* and the *astit* [Belnap et al., 2001] also invalidate this principle. On the other hand, the culprit is not *only* the assumption that necessary truths are always (trivially) seen to it that. For one thing the direction from $B_a \varphi$ to FoT-C-a-$\Diamond$ follows just from assuming that agents are not doing impossible things, which is a plausible and in any case much more common assumption. Furthermore, the full equivalence follows rather from the *combination* of assuming $\Box \varphi \rightarrow E_a \varphi$ and the fact that $\Diamond B_a \varphi$, given a model, is never contingent. This suggests that FoT-C-a-$\Diamond$ might not be quite the right analysis of this claim-right.

As an alternative to (FoT-C-a-$\Diamond$) we could instead express the claim-right not as bearing on the sheer possibility of holding a particular belief, but instead on being forced to *adopt* a particular belief, here viewed as something that the agent actively does. [16] This would give the following:

$$\bigwedge_{b \in A} \mathbf{O}_{b \rightarrow a} \neg E_b \neg E_a B_a \varphi \qquad \qquad \text{(FoT-C-}a\text{-}E)$$

FoT-C-a-E is logically independent both from FoT-C1 and FoT-C-a-$\Diamond$. It furthermore avoids the ought-is collapse that we observed for the latter. As before, however, we gain some logical interactions between these different formalizations of the claim-right by assuming $\Box \varphi \rightarrow E_a \varphi$. Indeed, with that additional assumption FoT-C-a-E implies FoT-C-a-$\Diamond$:

Observation 3 *For any model $\mathcal{M}$ where all f_a contain the unit, and state w, if $\mathcal{M}, w \models \bigwedge\limits_{b \in A} \mathbf{O}_{b \rightarrow a} \neg E_b \neg E_a B_a \varphi$ then $\mathcal{M}, w \models \bigwedge\limits_{b \in A} \mathbf{O}_{b \rightarrow a} \neg E_b \neg \Diamond B_a \varphi$.*

The converse direction still fails, however, even when all f_a contains the unit. So even in this case FoT-C-a-E avoids the ought-is collapse.

As we did for the privilege to believe, the claim-right component of freedom of thought can of course be expanded to the three possible attitudes that an

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[16] Using an action operator in front of the belief operator to refer to some kind of agency regarding one's own belief, on the one hand, accords well with the phrasing of the OHCHR comment on the Declaration talking about to *have* or *adopt* a belief; and, on the other hand, has its epistemological foundations in the view of doxastic voluntarism [Chignell, 2018].

agent can hold with respect to a particular formula φ.

$$\bigwedge_{b \in A} \mathbf{O}_{b \to a}(\neg E_b \neg E_a B_a \varphi \wedge \neg E_b \neg E_a B_a \neg \varphi \wedge \neg E_b \neg E_a(\langle B_a \rangle \varphi \wedge \langle B_a \rangle \neg \varphi))$$

$$(\text{FoT-C-}a\text{-}\varphi)$$

As before, this expansion also restricts the claim-right to consistent formulas, even without the assumption that agents always see to it that necessary propositions hold. Indeed, if φ is necessary in a particular model then the second and the third conjuncts become false because $\neg \varphi$ becomes necessarily false. If φ is instead necessarily false, then it is the first and the third conjuncts that become false.

Another notable fact regarding FoT-C-a-φ, is that this formula is consistent even in the case that a and b are the same agent. So imposing the claim-right in that case boils down to saying that everyone has a duty towards herself not to force herself to hold a particular belief regarding φ. From the legal point of view, this is rather questionable. This conclusion follows from the fact that we do not make any assumptions regarding the iteration of agency operators, so it could of course be avoided by, for instance, assuming that refraining from refraining, i.e. $\neg E_a \neg E_a...$, is equivalent to doing $E_a...$—c.f. again the discussion in [Belnap et al., 2001]. On the other hand, this conclusion could be avoided without entering this substantial debate about doing and refraining, by simply restricting the multitality to all agents $b \neq a$.

4.2.3   Immunity

The last component of freedom of thought is the immunity it involves. As we observed above, freedom of thought is viewed as inalienable and indispensable. No legal statement nor legislative act could take that freedom away. Any act entailing the negation of freedom of thought would turn out to be an invalid law. In other words, such action is not possible, which translates into an Hohfeldian immunity.

$$\bigwedge_{b \in A} \neg \Diamond (E_b(\neg(\text{FoT-F-}a\text{-}\varphi)) \vee E_b(\neg(\text{FoT-C-}a\text{-}\varphi))) \qquad (\text{FoT-I-}a\text{-}\varphi)$$

Unlike our first formulation of the freedom and the claim-right constituents of freedom of thought, (FoT-I-a-φ) implicitly covers the three possible attitudes that a can take towards φ (believing, disbelieving, and suspending judgment).

This formulation of immunity behaves differently from its freedom and claim-right components when it comes to φ being necessarily true or necessarily false. Recall that if φ is necessarily true or necessarily false in a particular model then both the freedom and the claim-right become necessarily false. This is not the case for (FoT-I-a-φ). This formula is satisfiable even when φ is not contingent. If, however, we assume $\Box \varphi \to E_a \varphi$, then, as before, (FoT-I-a-φ) becomes always false when φ is not contingent.

There is one important aspect which is not covered, however, by (FoT-I-a-φ), namely that this immunity might apply recursively, so to speak,

to limit everyone's powers to change that very immunity. One could indeed interpret the wide-ranging character of freedom of thought in that sense, i.e. that it also protects itself. [17] The expressive power of our language is too weak to capture such cases of self-reference, so we limit ourselves to some general observations.

The self-referential character of this immunity could be captured by more expressive languages containing fixed-points operators, for instance the modal mu-calculus [Pratt, 1981,Kozen, 1983]. This language extends basic modal ones like ours with the smallest and largest fixpoint operators μ and ν, as well as propositional variables, here simply x. One can use these additional resources to capture the self-reference in the immunity as follows:

$$\mu x. \bigwedge_{b \in A} \neg \Diamond (E_b \neg (\text{FoT-F-}a\text{-}\varphi) \vee E_b \neg (\text{FoT-C-}a\text{-}\varphi) \vee E_b \neg x)$$

This formula should be interpreted as saying that it is impossible for any agent b to either negate the freedom, the claim-right, or *this very immunity*, expressed by the variable x, here bound by the operator μ. The scope of this operator indicates what the variable x implicitly refers to. So once unpacked, this self-reference through x contains another self-reference, which needs to be unpacked, and so on, creating an ascending hierarchy of higher-order immunities regarding lower-order ones.

Without going into the details of this potential fixed-point extension of our language, we can already observe that the variable x is in the scope of an even number of negations, which means that the formula itself semantically corresponds to a monotone operator, which in turns guarantees the existence of smallest and largest fixpoints. So this formulation would be recursive, but not viciously circular.

4.2.4 Logical properties of freedom of thought

We are now in position to put together the three components of freedom of thought, and highlight some of their logical properties. First, to recast, our proposal for the formalization of a's freedom of thought regarding proposition φ is the following.

$$(\text{FoT-F-}a\text{-}\varphi) \wedge (\text{FoT-C-}a\text{-}\varphi) \wedge (\text{FoT-I-}a\text{-}\varphi) \tag{FoT-a-φ}$$

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[17] While we intentionally refrained from becoming involved in the natural law–positive law debate, here we need to mention that considering the UN Declaration's article to be self-referential might greatly depend on what philosophical assumptions one has: according to a natural law approach, the whole Declaration can be considered as merely descriptive; it might be said, therefore, that the impossibility of changing or taking away freedom of thought doesn't come from this very article, it comes because of people's inability to intervene with what is there by nature (which is often referred to as the inalienability of human rights). This way, the point of regarding the immunity not to be self-referential is that whatever is written in the Declaration does not change people's immunity concerning any change in their human rights, with their freedom of thought among them. We leave the discussion of this approach and its logical consequences, though, to future work.

That is, we analyze freedom of thought as a conjunction of three different multital, complex normative positions. These constituting normative positions are themselves complex because they cover all three possible doxastic attitudes that a can hold towards φ.

This complex right can be reformulated as a conjunction of three corresponding statements of freedom of thought regarding believing φ, disbelieving φ, and suspending judgment. In other words, (FoT-a-φ) is equivalent to the conjunction of a freedom, a claim-right and an immunity regarding a believing φ, a freedom, a claim-right and an immunity regarding disbelieving φ, and similarly for suspending judgment. Indeed, the freedom part of this right is simply a conjunction of three freedoms, one for each doxastic attitude. One obtains a similar conjunction for the second part, the claim-right, by observing that these are conjunctions of normal obligation operators, which of course distribute over conjunctions. Finally, the immunity can be rewritten with a $\square$ operator scoping over a conjunction of negated agency statements, and this $\square$ is also a normal modality. This observation is important to the extent that some of the atypical logical behavior that we have observed in the previous sections, for instance regarding necessary truths or falsities, can be avoided, if one wishes, by formulating restricted versions of freedom of thought, applicable for instance only to one of the three possible doxastic attitudes.

The logical behavior of (FoT-a-φ) is otherwise very limited. Necessary truth can be substituted in its scope, but otherwise all three properties that constitute normal modalities fail for this formula.

Observation 4

(i) *Substitution under logical equivalence holds: $\square(\varphi \leftrightarrow \psi)$ and (FoT-a-φ) together imply (FoT-a-ψ).*

(ii) *Closure under Conjunction fails: (FoT-a-φ) and (FoT-a-ψ) together do not imply (FoT-a-$\varphi \wedge \psi$).*

(iii) *Regularity fails: $\square(\varphi \rightarrow \psi)$ and (FoT-a-φ) together do not imply (FoT-a-ψ)*

(iv) *Necessitation fails: φ being valid in our class of models does not entail (FoT-a-φ).*

5   Conclusion

We analyzed the logical structure and properties of freedom of thought. We argued, *contra* [Wenar, 2015], that there is a theoretical basis for viewing this right as a particular case of Hohfeldian positions, i.e. a combination of freedom, claim-right and immunity, and that these can be analyzed using a combination of deontic logic, doxastic logic, and logic of agency. We then proceeded to study the logical behavior of these rights, and showed how this behavior depends on making particular assumptions on its constitutive belief, obligation, and agency operators. This logical analysis has also allowed to show an ought-is collapse for one formalization of the claim-right, and highlighted the potential recursivity

of so-called inalienable immunities. This last point constitutes a natural next step to extend our analysis, both from a legal and a logical point of view.

One important modeling choice that should be revisited in future work is to capture the agency operator in a "static" way, following the tradition in the theory of normative positions [Kanger, 1971,Sergot, 2001] or in stit theory [Belnap et al., 2001], for instance. This choice turns out to be crucial in explaining the ought-is collapse for (FoT-C-a-$\diamond$). We conjecture that this collapse would have not occurred had we used dynamic modalities in the style of Dynamic Epistemic Logic [Pacuit, 2013], as we do in [Markovich and Roy, 2021] for the right to know, or update semantics [Klein and Marra, 2020], to study the effect of the agents' actions.

Another important point that we have not touched on is the relation between freedom of thought and so-called conscientious objections. The OHCHR comment writes:

> The Covenant does not explicitly refer to a right to conscientious objection, but the Committee believes that such a right can be derived from article 18, inasmuch as the obligation to use lethal force may seriously conflict with the freedom of conscience and the right to manifest one's religion or belief.

The possibility to decline an otherwise existing duty is a power in Hohfeldian terms. We already showed that a (multital) claim-right, a (multital) freedom and a (multital) immunity are all part of the freedom of thought, so having this power would mean that all the four atomic right positions are incorporated in this one human right. Aiming at the formal representation of this power's derivability from the freedom brings up some considerations that we haven't made, though. On the one hand, the OHCHR interpretation talks about a very special duty that can be refused: using lethal force, but there are other cases in the legal literature where the notion of conscientious objection comes up, for instance medical practitioners not providing certain treatments to their patients [Shanawani, 2016], so the crucial point in formalization should be the reason of a "serious conflict" with beliefs, which refers to a supposed possibility of incompatibility between given beliefs and given actions. This brings us—as the comment also refers to it—to the external realm of the freedom of thought that we have intentionally omitted from our investigation so far: the manifestation, the freedom of choosing one's actions accordingly. There the immunity, for instance, is not absolute: duties can be imposed regarding our actions even if these have something to do with the (otherwise) free manifestation of our beliefs. Also the power we discuss here is not absolute: while one could make up a religion declaring paying taxes incompatible with one's innermost beliefs, courts would hardly accept rejection of paying taxes counting as a conscientious objection. This, of course, can be interpreted also not as defeasibility of the power to reject actions being in serious conflict with our conscience, but a need to define the notions of conscience and serious conflict more precisely. We leave this investigation to later papers.

References

[Altschul, 2021] Altschul, J. (2021). Epistemic entitlement. *Internet Encyclopedia of Philosophy*.

[Aucher et al., 2011] Aucher, G., Boella, G., and Torre, L. (2011). A dynamic logic for privacy compliance. *Artificial Intelligence & Law*, 19(2/3):187 – 231.

[Aucher et al., 2010] Aucher, G., Boella, G., and Torre, L. V. D. (2010). Privacy policies with modal logic: the dynamic turn. In *Deontic Logic in Computer Science (DEON 2010)*, pages 196–213.

[Belnap et al., 2001] Belnap, N. D., Perloff, M., and Xu, M. (2001). *Facing the future: agents and choices in our indeterminist world*. Oxford University Press, Oxford.

[Bentham, 1843] Bentham, J. (1843). *The works of Jeremy Bentham*. W. Tait ; Simkin, Marshall, and Co., Edinburgh ; London.

[Broersen, 2011] Broersen, J. M. (2011). Deontic epistemic stit logic distinguishing modes of mens rea. *J. Appl. Log.*, 9(2):137–152.

[Chignell, 2018] Chignell, A. (2018). The Ethics of Belief. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2018 edition.

[Cuppens and Demolombe, 1996] Cuppens, F. and Demolombe, R. (1996). A deontic logic for reasoning about confidentiality. In Brown, M. A. and Carmo, J., editors, *Deontic Logic, Agency and Normative Systems, DEON '96: Third International Workshop on Deontic Logic in Computer Science, Sesimbra, Portugal, 11-13 January 1996*, Workshops in Computing, pages 66–79. Springer.

[Dong and Roy, 2017] Dong, H. and Roy, O. (2017). Dynamic logic of power and immunity. In *Logic, Rationality, and Interaction : 6th International Workshop, LORI 2017, Sapporo, Japan, September 11-14, 2017, Proceedings*, volume 10455 of *Lecture Notes in Computer Science*, pages 123–136. Springer, Berlin, Heidelberg.

[Dretske, 2000] Dretske, F. (2000). Entitlement: Epistemic rights without epistemic duties? *Philosophy and Phenomenological Research*, 60:591?606.

[Fagin et al., 2003] Fagin, R., Moses, Y., Halpern, J. Y., and Vardi, M. Y. (2003). *Reasoning about knowledge*. MIT press.

[Feldman, 1988] Feldman, R. (1988). Epistemic obligations. *Philosophical Perspectives*, 2:235–256.

[Fitch, 1967] Fitch, F. B. (1967). A Revision of Hohfeld's Theory of Legal Concepts. *Logique et Analyse*, 10(39/40):269–276.

[Gelati et al., 2002] Gelati, J., Governatori, G., Rotolo, A., and Sartor, G. (2002). Actions, institutions, powers. Preliminary notes. In *International Workshop on regulated Agent-Based Social Systems: Theories and Applications*, pages 131–147.

[Gelati et al., 2004] Gelati, J., Rotolo, A., Sartor, G., and Governatori, G. (2004). Normative autonomy and normative co-ordination: Declarative power, representation, and mandate. *Artificial Intelligence and Law*, 12(1):53–81.

[Governatori and Rotolo, 2008] Governatori, G. and Rotolo, A. (2008). A computational framework for institutional agency. *Artificial Intelligence and Law*, 16(1):25–52.

[Hawke et al., 2019] Hawke, P., Özgün, A., and Berto, F. (2019). The fundamental problem of logical omniscience. *Journal of Philosophical Logic*, pages 1–40.

[Hohfeld, 1923] Hohfeld, W. N. (1923). Fundamental legal conceptions applied in judicial reasoning. In Cook, W. W., editor, *Fundamental Legal Conceptions Applied in Judicial Reasoning and Other Legal Essays*, pages 23–64. New Haven : Yale University Press.

[Hulstijn, 2008] Hulstijn, J. (2008). Need to know: Questions and the paradox of epistemic obligation. In van der Meyden, R. and van der Torre, L. W. N., editors, *Deontic Logic in Computer Science, 9th International Conference, DEON 2008, Luxembourg, Luxembourg, July 15-18, 2008. Proceedings*, volume 5076 of *Lecture Notes in Computer Science*, pages 125–139. Springer.

[Jones and Sergot, 1996] Jones, A. and Sergot, M. (1996). A formal characterization of institutionalised power. *Journal of the IPGL*, 3:417–443.

[Kanger, 1971] Kanger, S. (1971). New foundations of ethical theory. In Hilpinen, R., editor, *Deontic Logic: Introductory and Systematic Readings*, pages 36–58. D. Reidel, Dordrecht.

[Kanger and Kanger, 1966] Kanger, S. and Kanger, H. (1966). Rights and parlamentalism. *Theoria*, 32:85–115.

[Klein and Marra, 2020] Klein, D. and Marra, A. (2020). From oughts to goals: A logic for enkrasia. *Studia Logica*, 108(1):85–128.

[Kozen, 1983] Kozen, D. (1983). Results on the propositional μ-calculus. *Theoretical computer science*, 27(3):333–354.

[Lindahl, 1994] Lindahl, L. (1994). Stig Kanger's Theory of Rights. In D. Prawitz, B. Skyrms, D. W., editor, *Logic, Methodology and Philosophy of Science IX*, pages 889–911. Elsevier Science Publsiher, New York.

[Makinson, 1986] Makinson, D. (1986). On the formal representation of rights relations: Remarks on the work of Stig Kanger and Lars Lindahl. *Journal of Philosophical Logic*, 15(4):403–425.

[Markovich, 2019] Markovich, R. (2019). Rights and Punishment: The Hohfeldian Theory's Applicability and Morals in Understanding Criminal Law. *IFCoLog Journal of Logics and their Applications*, 6(5):847–864.

[Markovich, 2020] Markovich, R. (2020). Understanding Hohfeld and Formalizing Legal Rights: the Hohfeldian Conceptions and Their Conditional Consequences. *Studia Logica*, 108.

[Markovich and Roy, 2021] Markovich, R. and Roy, O. (2021). Formalizing the Right to Know: Epistemic Rights as Normative Positions. accepted for LNGAI2021.

[Pacuit, 2013] Pacuit, E. (2013). Dynamic epistemic logic i: Modeling knowledge and belief. *Philosophy Compass*, 8(9):798–814.

[Pacuit, 2017] Pacuit, E. (2017). *Neighborhood semantics for modal logic*. Springer.

[Pacuit et al., 2006] Pacuit, E., Parikh, R., and Cogan, E. (2006). The logic of knowledge based obligation. *Synthese*, 149(2):311–341.

[Pratt, 1981] Pratt, V. R. (1981). A decidable mu-calculus: Preliminary report. In *22nd Annual Symposium on Foundations of Computer Science (sfcs 1981)*, pages 421–427. IEEE.

[Åqvist, 1967] Åqvist, L. (1967). Good samaritans, contrary-to-duty imperatives, and epistemic obligations. *Nous*, 1(4):361–379.

[Sartor, 2005] Sartor, G. (2005). *Legal Reasoning. A Treatise of Legal Philosophy and General Jurisprudence*. Springer.

[Sergot, 2001] Sergot, M. (2001). A computational theory of normative positions. 2(4):581–622.

[Sergot, 2013] Sergot, M. (2013). Normative Positions. In Gabbay, D., Horty, J., Parent, X., van der Meyden, R., and van der Torre, L., editors, *Handbook of Deontic Logic and Normative Systems*, pages 353–406. College Publications.

[Shanawani, 2016] Shanawani, H. (2016). The challenges of conscientious objection in health care. *Journal of Religion and Health*, 55:384–393.

[Simmonds, 2001] Simmonds, N. (2001). Introduction. In *Hohfeld: Fundamental legal conceptions as applied in judicial reasoning*, Classical Jurisprudence series. Ashgate, Aldershot, new ed. / edited by David Campbell and Philip Thomas. edition.

[Stalnaker, 2006] Stalnaker, R. (2006). On logics of knowledge and belief. *Philosophical studies*, 128(1):169–199.

[Stapleford, 2012] Stapleford, S. (2012). Epistemic duties and failure to understand one's evidence. *Principia: an international journal of epistemology*, 16(1):147–177.

[Watson, 2021] Watson, L. (2021). *The Right to Know: Epistemic Rights and Why We Need Them*. Routledge.

[Wenar, 2003] Wenar, L. (2003). Epistemic rights and legal rights. *Analysis*, 63(2):142–146.

[Wenar, 2015] Wenar, L. (2015). Rights. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2015 edition.

[Wright, 1963] Wright, G. H. v. G. H. (1963). *Norm and Action : a Logical Enquiry*. International library of philosophy and scientific method. Routledge and Kegan Paul, London.