LbCS navigation controllers of Twining Lagrangian swarm individuals

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***Abstract*—This paper presents stabilizing velocity controllers for the individuals of two Lagrangian swarms, which navigates from their initial configuration space to their final configuration space, ensuring intra and inter swarm individual collision avoidance. The motion of the individuals is based on Reynold's rules of separation, alignment, and cohesion. Using the three pillars (safety, shortest and smoothest path) of Lyapunov based control scheme (LbCS), the velocity controllers of the individuals of the two swarms are derived from multiple Lyapunov functions. The effectiveness of the controllers is validated through computer simulations.**

***Index Terms*—Swarm Robotics, Lyapunov function, Potential function, Collision Avoidance, Velocity Controllers**

I. Introduction

When it comes to accomplishing specific real-world tasks and solving problems that lack human efficiency, researchers have, from time to time, resorted to exploring nature for solutions. According to [1], a continuous surge from the mid-1980s to explore various behavior patterns existing in wildlife has resulted in 21st-century artificial intelligence. One such form of bio-inspired artificial intelligence emerging is swarm intelligence, which results from rigorous studies based on the behavior patterns of groups, or swarms, of social insects such as ants and bees. Swarm systems take their inspiration from societies of insects that can perform tasks that are beyond the capabilities of the individuals, resulting in the creation of swarm robotics to solve challenging and complex real-world problems such as optimization [2], and target search [3], [4].

Swarm robotics is a field of multi-robotics in which a large number of robots are coordinated in a distributed and decentralized way [5]. Many simple robots can perform complex tasks in a more efficient way than a single robot, giving robustness and flexibility to the group. Swarm intelligence demonstrates the viability of task accomplishment of multi-agents with no centralized control [6].

To date, robot swarms have been demonstrated to solve tasks such as coordinated movement [7], transportation of objects [8], self-assembly [9], and collective construction of structures [10]. Over the past decade, researchers have used two types of modeling approaches to comprehend swarming the *Eulerian* and the *Lagrangian* approaches. Comprehensive reviews of these approaches and their advantages and disadvantages can be found in [11] and [12]. While multi-robot systems have been studied extensively in swarm robotics [13]–[15], to the authors' knowledge, twining swarm systems are yet to be researched.

In this paper, two Lagrangian planar swarms are constructed using the Lyapunov-based Control Scheme (LbCS), building on the work done in [16]. The Lagrangian swarms are developed based on the hypothesis that swarming interplay between long-range attraction and short-range repulsion between the individuals in the swarms. Kumar et al. [17], [18] states that in the Lagrangian approach, the state (position, instantaneous velocity, and instantaneous acceleration) of each individual and its relationship with other individuals in the swarm is studied; it is an individual-based approach, in which spatial coordinates of the individual can influence the velocity and acceleration. Both swarms have a different number of individuals in their respective systems and will have two equilibrium points. The motion of the swarm individuals is based on Reynold's rules [19], which are (1) collision avoidance with neighbors, (2) matching velocity of the neighbors, and (3) staying close to the neighbors [20]. While approaching each other, the two swarms will have to avoid each other, therefore assuming the role of a moving obstacle. In this research, a case where there is no obstacle is considered. For individuals in the sensing range of the other individuals, attractive and repulsive potential functions that are part of a total potential function are formed using the artificial potential field technique [17], [21]. The decentralized velocity-based control laws are then derived from the total potential function, which gives rise to a gradient system. Furthermore, using a Lyapunov-like function [22]–[24], velocity-based controllers for each individual in both swarms are derived.

II. Lyapunov-based Control Scheme

LbCS is a method of an artificial potential field used widely in motion planning and the development of robotic models. Both nonlinear acceleration and velocity controllers can be derived using LbCS. LbCS function can be derived for target attraction and obstacles avoidance for many moveable members of the system in an environment. A variety of problems has been shown to be feasible and stable in [18], [25]–[32].

The major advantage of using LbCS is that the continuous controllers could be derived easily [23], [24]. However, LbCS has a possibility of algorithm singularities (local minima) as its disadvantage. One can read more on the basics of LbCS in [33]. Fig 1 and Fig 2 are created for a robot in a workspace

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with its initial position at $(10, 10)$. The robot avoids the obstacle with a radius of 20 to reach its target.

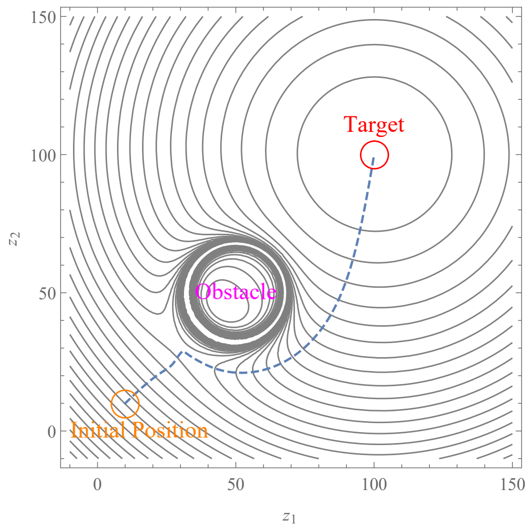

Fig. 1. Contour plot for the Lyapunov function.

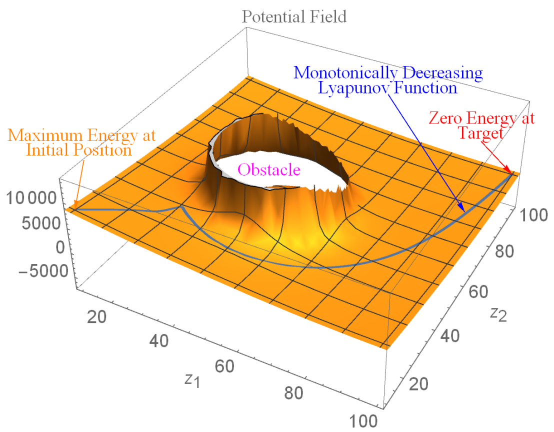

Fig. 2. 3D visualisation of the Lyapunov function.

III. A Two-Dimensional Swarm Model

At the most fundamental extend, consider two swarms, first of $m \in \mathbb{M}$ individuals of point masses and second of $n \in \mathbb{N}$ individuals of point masses. The first swarm is denoted by $k = 1$ and the second swarm is denoted by $k = 2$. The point masses will be used to create two swarm models in an environment, and its stability will be assessed. The independent parameters, which are the translational components (x, y), are used to characterize the individuals' positions in a 2D arrangement.

Let the position of the i^{th} individuals of the k^{th} swarm where $k \in \{1, 2\}$ and swarm position at time $t \geq 0$ be $(x_{ki}(t), y_{ki}(t))$, for all $i \in \{1, 2, 3, \dots, n\}$ and $1 \leq m \leq n$ with $(x_{ki}(t_0), y_{ki}(t_0)) = (x_{ki0}, y_{ki0})$ as the initial conditions.

The vector configuration then becomes $\mathbf{x}_i := (x_i, y_i) \in \mathbb{R}^2$ for the i^{th} individual and $\mathbf{x} := (\mathbf{x}_{11}, \mathbf{x}_{12}, \mathbf{x}_{13}, ..., \mathbf{x}_{1m}, \mathbf{x}_{21}, \mathbf{x}_{22}, \mathbf{x}_{23}, ..., \mathbf{x}_{2n}) \in \mathbb{R}^{2(m+n)}$ becomes the configuration vector for $m + n$ individuals with the initial conditions vector denoted by $\mathbf{x_0} := (\mathbf{x}_{k1}(0), \mathbf{x}_{k2}(0), \mathbf{x}_{k3}(0), ..., \mathbf{x}_{2n}(0)) \in \mathbb{R}^{2(m+n)}$.

Definition 3.1: The i^{th} individual of k^{th} swarm where $k \in \{1, 2\}$ is a point mass residing in a disk with center (x_{ki}, y_{ki}) and radius $r_{ki} > 0$. It is described as the set

$$B_{ki} := \{(z_1, z_2) \in \mathbb{R}^2 : (z_1 - x_{ki})^2 + (z_2 - y_{ki})^2 \leq r_{ki}^2\}. \quad (1)$$

The *centroid of the* k^{th} *swarm* is defined as

$$(x_{Ck}, y_{Ck}) := \left(\frac{1}{n_k} \sum_{i=1}^{n_k} x_i, \frac{1}{n_k} \sum_{i=1}^{n_k} y_i \right) \quad (2)$$

, where $n_1 = m$ and $n_2 = n$. The instantaneous velocities at $t \geq 0$ of the i^{th} individuals of two swarms is represented as $(v_{ki}(t), w_{ki}(t)) := (x'_{ki}(t), y'_{ki}(t))$. The system of first-order ODEs by the above notations for the i^{th} individuals of two swarms, assuming the initial condition at $t = t_0 \geq 0$ is given as:

$$x'_{ki}(t) = v_{ki}(t), \; y'_{ki}(t) = w_{ki}(t), \; x_{ki0} := x_{ki}(t_0), \\ y_{ki0} := y_{ki}(t_0). \quad (3)$$

We proceed by removal t from the equation for simplicity , let $\mathbf{x}_{ki} := (x_{ki}, y_{ki}) \in \mathbb{R}^2$ be our state vectors. Also let $\mathbf{x}_0 = \underbrace{(x_{k_{10}}, y_{k_{10}}, x_{k_{20}}, y_{k_{20}}, ..., x_{1_{m0}}, y_{1_{m0}}, x_{2_{n0}}, y_{2_{n0}})}_{\text{2(m+n) terms}}$ for $k \in \{1, 2\}$.

If the instantaneous velocity (v_{ki}, w_{ki}) has a state feedback law of the form, for $i \in \{1, 2, 3, \dots, n\}$,

$$(v_{ki}(t), w_{ki}(t)) = (-\mu_{ki} f_{ki}(\mathbf{x}(t)), -\varphi_{ki} g_{ki}(\mathbf{x}(t))),$$

for scalars $\mu_{ki}, \varphi_{ki} > 0$ and functions $f_{ki}(\mathbf{x}(t))$ and $g_{ki}(\mathbf{x}(t))$, to be constructed accordingly later, and if we define $\mathbf{g}_{ki}(\mathbf{x}) := (-\mu_{ki} f_{ki}(\mathbf{x}), -\varphi_{ki} g_{ki}(\mathbf{x})) \in \mathbb{R}^2$ and $\mathbf{G}(\mathbf{x}) := (\mathbf{g}_{11}(\mathbf{x}), \mathbf{g}_{12}(\mathbf{x}) ..., \mathbf{g}_{1m}(\mathbf{x}), \mathbf{g}_{21}(\mathbf{x}), \mathbf{g}_{22}(\mathbf{x}) ..., \mathbf{g}_{2n}(\mathbf{x})) \in \mathbb{R}^{2(m+n)}$, then the two swarm of $m+n$ individuals is

$$\dot{\mathbf{x}} = \mathbf{G}(\mathbf{x}), \quad \mathbf{x}(t_0) = \mathbf{x}_0. \quad (4)$$

An equilibrium point of the system if achieved, we will denote it by

$$\mathbf{x}_e := (\mathbf{x}_{11e}, \mathbf{x}_{12e}, ..., \mathbf{x}_{1me}, \mathbf{x}_{21e}, \mathbf{x}_{22e}, ..., \mathbf{x}_{2ne})$$

. Thus we have the following $2(m + n)$ equilibrium points

$$\mathbf{x}_e := (x_{11e}, y_{11e}, x_{12e}, y_{12e}, ..., x_{1me}, y_{1me}, \\ x_{21e}, y_{21e}, x_{22e}, y_{22e}, ..., x_{2ne}, y_{2ne}).$$

The stability of $\mathbf{x}_e$ will be examined by Lyapunov's Direct Method.

IV. Absence of Stationary Obstacles

Let us consider two swarms, first with $m \in \mathbb{M}$ individuals and second with $n \in \mathbb{N}$ individuals. Their configuration space is free of static obstacles. The components of a tentative Lyapunov function for this case will be considered in this section. The velocity controllers are then constructed for both of the swarm to ensure the stability of the system. Later, the velocity controllers are then used to simulate the two swarm model.

A. Lyapunov Function Components

It is considered that the two swarms of $m \in \mathbb{M}$, $1 \leq m \leq n$ and $n \in \mathbb{N}$ individuals to be identical. We let the total swarm individuals of swarm $\mathbb{M}$ be n_1 and the total swarm individuals of swarm $\mathbb{N}$ be n_2. The first swarm is represented by $k = 1$ and second swarm is represented by $k = 2$, hence $r_i = r_{ka}\ \forall\ ki \in \{1, 2, 3, \dots, n_k\}$.

1) Attraction to the Centroid: The attractive potential function which will enable the i^{th} individuals to be attracted towards the swarm centroid is proposed to be, for $i \in \{1, 2, 3, \dots, n_k\}$:

$$R_{ki}(\mathbf{x}) := \frac{1}{2}\left[(x_{ki} - x_C)^2 + (y_{ki} - y_C)^2\right]. \tag{5}$$

2) Target of the two Swarms Individuals: By assigning a target to the swarm's centroid, we will assign a target to the swarm of $n \in \mathbb{N}$ agents.

Definition 4.1: The target for the centroid of the individual swarm is a disk with center (a_k, b_k) and radius $r_{k\tau}$. It is described as the set

$$\tau_k = \{(z_1, z_2) \in \mathbb{R}^2 : (z_1 - a_k)^2 + (z_2 - b_k)^2 \leq r_{k\tau}^2\}. \tag{6}$$

To get the centroid attraction to these targets, we shall utilize target attraction functions, for the Lyapunov function to be presented, the function

$$T_k(\mathbf{x}) := \frac{1}{2}\left[(x_C - a_k)^2 + (y_C - b_k)^2\right]. \tag{7}$$

3) Inter-individual Collision Avoidance: To avoid inter-agent collision between the i^{th} and the j^{th} agent, $j \neq i$, $i, j \in \{1, 2, 3, \dots, n_k\}$, we will consider an obstacle avoidance function

$$Q_{kij}(\mathbf{x}) := \frac{1}{2}\left[(x_{ki} - x_{kj})^2 + (y_{ki} - y_{kj})^2 - (2r_a)^2\right]. \tag{8}$$

The Lyapunov function will also account for short-range repulsion between individuals.

4) Individual Collision Avoidance of Swarms: The swarm individuals of both swarms must avoid collision upon path intersection. The following avoidance function could be employed to achieve this.

$$W_{kij} = \frac{1}{2}\left[(x_{1,i} - x_{2,j})^2 + (y_{1,i} - y_{2,j})^2 - (r_{1a} + r_{2a})^2\right] \tag{9}$$

B. A Tentative Lyapunov Function

Let $\alpha_k > 0$, $\gamma_{kij} > 0$, $\beta_{kij} > 0$ and $\lambda_{kij} > 0$ be real numbers, and for $i, j \in \{1, 2, 3, \dots, n_k\}$, the tentative Lyapunov function suppressing t for system becomes (4),

$$L(\mathbf{x}) = \sum_{k=1}^{2} \alpha_k T_k(\mathbf{x}) + \sum_{k=1}^{2}\sum_{i=1}^{n_k} T_k(\mathbf{x}) \left(\sum_{\substack{j=1\\ i\neq j}}^{n_k} \frac{\beta_{k,i,j}}{Q(\mathbf{x})_{k,i,j}} + \sum_{j=1}^{p} \frac{\lambda_{k,i,j}}{W(\mathbf{x})} + \gamma_{k,i} R(\mathbf{x})\right), \tag{10}$$

where

$$p = \begin{cases} n_1 & if\ k = 2, \\ n_2 & if\ k = 1. \end{cases}$$

.

C. Velocity Conrollers

Alone the trajectory of the system (4) the derivative of $L(\mathbf{x})$ is given by,

$$\dot{L}(\mathbf{x}) = \sum_{k=1}^{2}\sum_{i=1}^{n_k} [f_{ki}(\mathbf{x}) v_{ki} + g_{ki}(\mathbf{x}) w_{ki}]$$

where

$$f_{ki}(\mathbf{x}) = \frac{\partial L(\mathbf{x})}{\partial x_{ki}}, \tag{11}$$

and

$$g_{ki}(\mathbf{x}) = \frac{\partial L(\mathbf{x})}{\partial y_{ki}}. \tag{12}$$

Let there be constants $\mu_{ki}, \varphi_{ki} > 0$ for $k \in \{1, 2\}$ then

$$v_{ki} = -\mu_{ki} f_{ik}(\mathbf{x}), \quad w_{ki} = -\varphi_{ki} g_{ki}(\mathbf{x}), \tag{13}$$

are our velocity controllers of system (4).

D. Stability Analysis

$L(\mathbf{x})$ is positive over the domain

$$D\big(L(\mathbf{x})\big) := \Big\{\mathbf{x} \in \mathbb{R}^{2(n_1+n_2)} : Q_{kij}(\mathbf{x}) > 0, W_{kij}(\mathbf{x}) > 0, \forall\ k = \{1, 2\}, i, j = \{1, 2, 3, \dots, n_k\}, i \neq j\Big\}.$$

Moreover, using (13)

$$\dot{L}(\mathbf{x}) = -\sum_{k=1}^{2}\sum_{i=1}^{n_k} \left[\frac{v_{ki}^2}{\mu_{ki}} + \frac{w_{ki}^2}{\varphi_{ki}}\right] \leq 0,$$

It is clear to see that $L(\mathbf{x_e}) = 0, L(\mathbf{x}) > 0 \forall \mathbf{x} \neq \mathbf{x_e}$ from equation (10) and $\dot{L}(\mathbf{x}) \leq 0$. Thus, we conclude that the system (4) is stable.

V. Simulation Work

The simulation is done in Wolfram MATHEMATICA 12.1 software. The initial configurations of the swarm individuals are randomly generated.

To deploy two swarms in an environment, firstly the swarm size is determined. In this example, each swarm contains 8 individuals. Convergence parameters of swarm 1 and swarm 2 are $\mu_{1,i} = \varphi_{1,i} = 0.001$ and $\mu_{2i} = \varphi_{2,i} = 0.001$, respectively. Similarly, the target cohesion parameter are $\alpha_{1i} = \alpha_{2i} = 50$, and inter-individual obstacle avoidance parameter are $\beta_{1,i,j} = \beta_{2,i,j} = 10$. Centroid cohesion parameter of swarms are $\gamma_{1,i} = \gamma_{2,i} = 0.1$. The individual collision avoidance parameter for the two swarms are $\lambda_{1ij} = \lambda_{2ij} = 10$. At $t = 0$ swarm individuals are stationed at their initial position. The red-colored individuals are members of swarm 1, and blue-colored individuals are members of swarm 2, which are shown in Fig 3. The targets of swarm 1 and swarm 2 are T_{S1} and T_{S2}, respectively. $L(\mathbf{x})$ monotonically decreases over time as shown in Fig 4. In Fig 5, the instantaneous translational velocities of individuals of each swarms is shown respect to time. The swarm individuals cluster around the centroid as time evolves, and then they move towards their target of the centroid as a well-spaced cohesive group ensuring collision avoidance is shown in Fig 6.

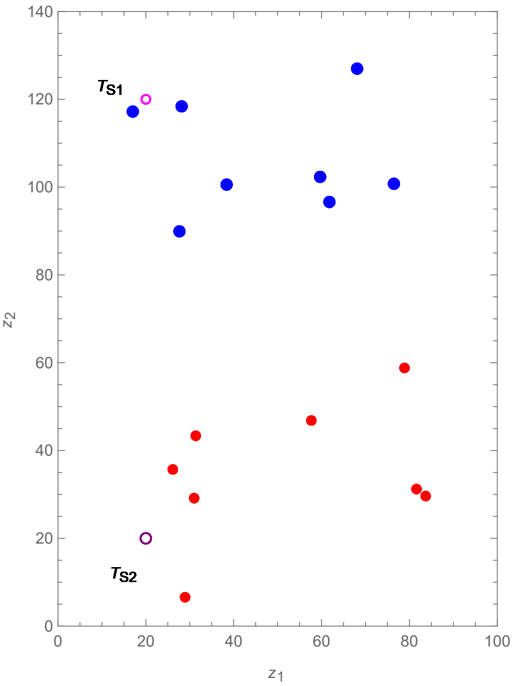

Fig. 3. Positions where the individuals are stationed at $t = 0$.

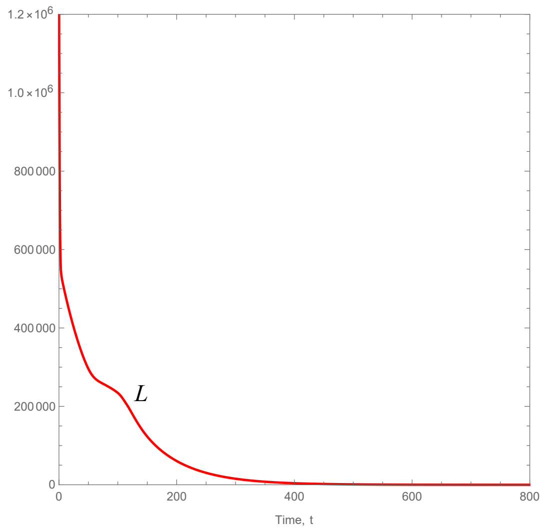

Fig. 4. The Lyapunov Function as time increases from $t = 0$ to $t = 800$.

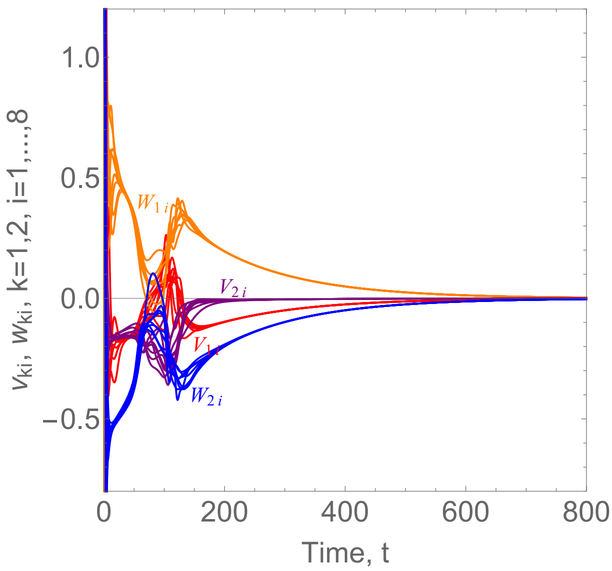

Fig. 5. Translational velocities of individuals of both swarms.

VI. Conclusion and Further Work

This paper presented stabilizing velocity controllers of two Lagrangian swarm individuals for navigation from their initial state to their equilibrium state. The controllers were derived from multiple Lyapunov function, a total potential, developed using LbCS. The controllers effectiveness were validated via simulations. In the future, this model could be generalized for $n \in \mathbb{N}$ swarms in an environment which is also cluttered with stationary obstacles.

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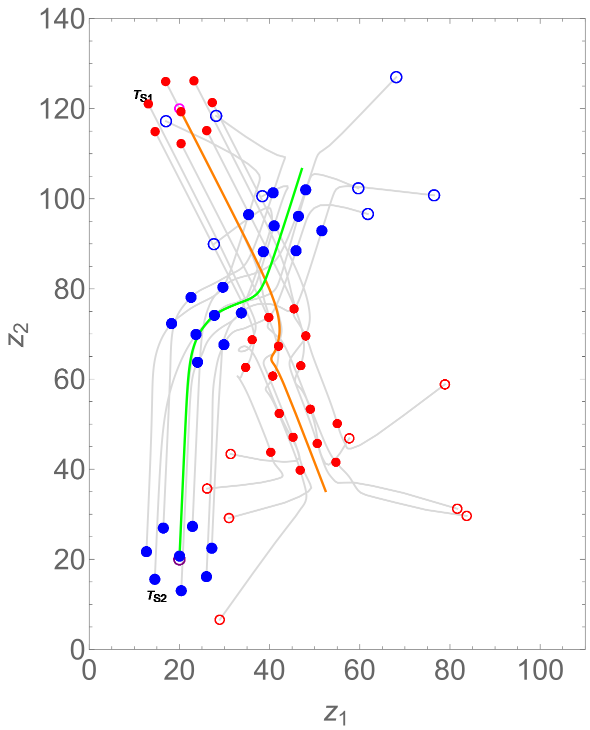

Fig. 6. The position of both swarms at $t = 22$, $t = 113$ and $t = 800$.

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