Success versus failure: Efficient heat devices in thermodynamics

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Classical equilibrium thermodynamics provides, in a general way, upper Carnot bounds for the performance of energy converters. Nevertheless, to suggest lower bounds is a much more subtle issue, especially when they are related to a definition of *convenience*. Here, this issue is investigated in a unified way for heat engines, refrigerators, and heat pumps. First, irreversibilities are weighted in the context of heat reservoir stability for irreversible engines by using the thermodynamic distance between minimum energy and maximum entropy steady states. Some stability coefficients can be related to a majorization process and the obtention of Pareto fronts, linking stability and optimization by means of efficiency and entropy due to correlations between system and reservoirs. Second, these findings are interpreted in a very simple context. A region where the heat device is efficient is defined in a general scheme and, below this zone, the heat device is inefficient in the sense that irreversibilities somehow dominate its behavior. These findings allow for a clearer understanding of the role played by some well-known figures of merit in the scope of finite-time and -size optimization. Comparison with experimental results is provided.

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I. INTRODUCTION

As noted by Seifert [1], “From its very beginnings, thermodynamics has fascinated scientists by posing deep conceptual issues that needed to be resolved for understanding and optimizing quite practical matters such as the design of heat engines.” Since the seminal work by Carnot [2] to the stochastic thermodynamics framework, spectacular experimental and theoretical advances have encouraged the thermodynamic coverage of heat devices. It includes systems ranging from macroscopic dimensions to nanoscales including biomolecular devices [3–8].

As a straightforward consequence, the thermodynamic optimization of heat devices is receiving nowadays great attention heightened by the contemporary growing importance of saving energy resources in any energy conversion process for heat engines (HE), refrigerators (RE), and heat pumps (HP) [9–11]. Common features of optimization criteria are their inherent dependence on the elected model in each case (classical, mesoscopic, or quantum) and that the objective function should be dependent on the stated optimization problem [12–14]. Accordingly, appropriate theoretical frameworks have been used and, in all of them, important efforts have been devoted to the derivation of specific trade-offs and upper bounds [15–19] to guide more efficient designs.

One of the main goals of this paper is to focus on the opposite side. It is shown that the interplay between reversibility and irreversibilities (losses) could be used to define a threshold in regards to the efficiency of energy conversion processes in the landscape of heat reservoir stability for irreversible engines. After all, the contact surface between the system and the reservoirs is irretrievably affected by heat transfers and the departure from equilibrium in reservoirs could offer valuable information regarding the heat engine performance. The resulting thresholds (*lower bounds*) of these singular states are obtained by leveling reversibility and losses. All the results are general and apply without resorting to any particular model or explicit optimization criterion based on usual trade-off functions. However, the above findings allow for a clearer interpretation of the meaning of some figures of merit used in studies of heat device optimization. In between these thresholds and the corresponding maximum Carnot values, a fairly good agreement with experimental results for both HE and RE is found. Lately, the role of stability in optimization processes has been addressed, first, with a compromise between fluctuations and operation regimes in small systems for cyclic and steady-state processes [15,16,20] and, later on, addressing a possible role of stability of operation regimes in an optimization mechanism, which could favor evolution and adaptation [21], with possible applications under more realistic energetic demands [22,23]. Recently, it has also been discussed that in a variety of natural phenomena the time evolution of nonequilibrium states follows entropy demands [24]. This, ultimately, should be related to the heat transport mechanisms between systems and reservoirs. Thus it is expected that some constraints should be imposed by heat reservoirs.

The paper is structured as follows. In Sec. II two stability coefficients that measure the nearness to equilibrium states for the reservoirs are proposed. In Sec. III these coefficients are linked to a departure from reversibility and the degree of irreversibility of an energy converter; this allows one to define a success region. A comparison with experimental results is presented. Finally, in Sec. IV some remarks and conclusions are outlined.

II. HEAT DEVICE PERFORMANCE AND RESERVOIR STABILITY

Some basic results are first collected. The simplest model for a HE assumes a cyclic operating system between two infinite heat baths (reservoirs) with low and high temperatures T_c and T_h, respectively. The efficiency is defined in terms of the absorbed heat $|Q_h|$ at T_h and the heat $|Q_c|$ delivered at T_c as $\alpha \equiv \eta = \frac{|W|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$, where $|W| = |Q_h| - |Q_c|$ is the net useful work output. η is bounded between $\eta_{\min} = 0$, when the work done is null, and the Carnot efficiency $\eta_c = 1 - \tau$ ($\tau \equiv \frac{T_c}{T_h}$). For an inverse cycle working as RE, the cooling heat $|Q_c|$ is extracted from the reservoir at T_c while some work $|W|$ is fed into the cyclic working system, which in turns delivers $|Q_h| = |W| + |Q_c|$ to the hot thermal reservoir at T_h. Now, $\alpha \equiv \epsilon = \frac{|Q_c|}{|W|} = \frac{|Q_c|}{|Q_h|-|Q_c|}$ is the coefficient of performance (COP). As for heat engines, ϵ is lower bounded by a COP value $\epsilon_{\min} = 0$ when no heat is extracted from the cold reservoir, i.e., $|Q_c| = 0$. The reversible COP is $\epsilon_c = \frac{T_c}{T_h - T_c} \equiv \frac{\tau}{1-\tau}$. For the inverse cycle working as a HP, the heating heat $|Q_h|$ is delivered to a reservoir at T_h, while some work $|W|$ is required in the cyclic working system, which in turns extracts some heat $|Q_c|$ from the cold thermal reservoir at T_c. The COP of the HP is $\alpha \equiv \nu = \frac{|Q_h|}{|W|} = \frac{|Q_h|}{|Q_h|-|Q_c|}$. Different from previous cases, now ν attains a minimum nonzero value $\nu_{\min} = 1$ because of the general relation between the COPs for a RE and a HP: $\nu = \frac{|Q_h|}{|W|} \equiv 1 + \frac{|Q_c|}{|W|} = 1 + \epsilon$. The Carnot COP is $\nu_c = \frac{T_h}{T_h - T_c} \equiv \frac{1}{1-\tau}$.

Reversible (infinite in many cases) heat sources need the assumption that relaxation times are sufficiently short with respect to the engine operation times. Although operation regimes of actual heat devices depend on internal variables of the working fluid, they lastly depend on the properties of the heat reservoirs. Energy transfers between both components (system+reservoirs) irretrievably have consequences on their internal modes and the entropy of the compound system. In actual conditions, there are indeed correlation dynamics between reservoirs and the system that can lead, ultimately, to instabilities of the heat reservoirs. These correlations have been properly addressed [25] considering that they are linked to an irreversible contribution to the entropy change of the system, $\Delta_i S_s(t)$. It represents a relative entropy between the actual state of the compound system and that in which the heat reservoirs are in thermal equilibrium with no correlations between the system and the reservoirs. As long as this quantity is different from zero, perturbations on the internal modes and internal reorganization take place in the heat reservoirs, departing them from true equilibrium. In this way, only reversible system-reservoir heat exchanges will exhibit a value $\Delta_i S_s(t) = 0$. This is directly related, in the case of one reservoir, to the maximum work theorem, $T\,\Delta_i S_s(t) = W(t) - \Delta F_s(t) \geqslant 0$, where $\Delta F_s(t)$ denotes the nonequilibrium free energy. The time parameter, t, accounts for the possible variations that could be produced due to the dynamics in the coupling between the system and the reservoirs, allowing one to address, for example, nonstationary cyclic heat engines.

On the other side, let us recall that stability criteria are defined in terms of states of minimum internal energy and maximum entropy [26]. Here it will be argued that configurations of minimum internal energy of the heat reservoirs, U, and maximum entropy of the heat reservoirs, S, are stable equilibrium states. By considering the coupled system (system plus reservoir) as closed, conservation of internal energy implies that changes in internal energy in the reservoirs stem only from heat exchanges, that is, $\Delta U = -Q_h(t) - Q_c(t)$, and entropy change of heat reservoirs is given by $\Delta S = -Q_c(t)/T_c - Q_h(t)/T_h$. By using the definition of α in each case it is obtained that

$$\Delta U = \begin{cases} -|Q_h(t)|\eta \leqslant 0, & \text{HE,} \\ \frac{|Q_c(t)|}{\epsilon} \geqslant 0, & \text{RE,} \\ \frac{|Q_c(t)|}{\nu - 1} \geqslant 0, & \text{HP,} \end{cases} \tag{1}$$

$$\Delta S = \begin{cases} \frac{|Q_h(t)|}{T_h}\frac{\eta_C - \eta}{1-\eta_C} \geqslant 0, & \text{HE,} \\ \frac{|Q_h(t)|}{T_h}\left(1 - \frac{\epsilon(1+\epsilon_C)}{\epsilon_C(1+\epsilon)}\right) \geqslant 0, & \text{RE,} \\ \frac{|Q_h(t)|}{T_h}\left(1 - \frac{\nu_C(\nu-1)}{\nu(\nu_C-1)}\right) \geqslant 0, & \text{HP.} \end{cases} \tag{2}$$

If the reference states are put to zero, then $\Delta U = U(t)$ and $\Delta S = S(t)$. Both $U(t)$ and $S(t)$ are monotonous functions of $\alpha \equiv (\eta, \epsilon, \nu)$. For these functions $U_{\min}(t)$ is achieved when the efficiencies are the highest (η_C, ϵ_C, and ν_C) carried out under quasistatic processes; meanwhile, $S_{\max}(t)$ corresponds to minimum efficiency [$\eta = W(t) = 0$, $\epsilon = Q_c(t) = 0$, and $\nu = 1$, $Q_c(t) = 0$ for HE, RE, and HP, respectively]. It will be of interest to establish how far or close is a given operation state from that of maximum entropy and minimum internal energy. For that reason let us define two parameters $E_U \leqslant 1$ and $E_S \leqslant 1$ given by [note that $U_{\min}(t)$ is negative in a HE]

$$E_U \equiv \begin{cases} \frac{U(t)}{U_{\min}(t)} = \frac{\eta}{\eta_C}, & \text{HE,} \\ \frac{U_{\min}(t)}{U(t)} = \frac{\epsilon}{\epsilon_C}, & \text{RE,} \\ \frac{U_{\min}(t)}{U(t)} = \frac{\nu-1}{\nu_C-1}, & \text{HP,} \end{cases} \tag{3}$$

$$E_S \equiv \frac{S(t)}{S_{\max}(t)} = \begin{cases} \frac{\eta_C - \eta}{\eta_C}, & \text{HE,} \\ 1 - \frac{\epsilon(1+\epsilon_C)}{\epsilon_C(1+\epsilon)}, & \text{RE,} \\ 1 - \frac{\nu_C(\nu-1)}{\nu(\nu_C-1)}, & \text{HP,} \end{cases} \tag{4}$$

which could be interpreted as a measure of the nearness between the two extremal situations $U_{\min}$ and $S_{\max}$ and the actual state of the heat reservoirs (characterized by α), under the assumption that the heat transferred (Q_h or Q_c) is the same.

$E_U(\alpha)$ and $E_S(\alpha)$ are shown in the upper panel of Fig. 1. The configurations where $E_U \to 1$ can only be achieved under reversible conditions. Thus $\Delta_i S_s(t) = 0$ and, therefore, reservoirs are in equilibrium with no correlations with the system. Another situation in which this occurs is at the opposite side, where $E_S \to 1$ since $W(t) = 0$ for HEs and all input power is transferred directly to the hot reservoir [$W(t) = |Q_h(t)|$] for REs and HPs. These two extremal situations are incompatible with real processes: the first one is useless and the other is linked to zero power output in HE and zero cooling power in RE. For the intermediate situations there exists a nonzero correlation dynamics between reservoirs and system.

Let us now introduce a distance between E_U and E_S by means of a p norm (p being a natural number) as

$$d_p(\alpha) \equiv \|E\|_p = (|E_U|^p + |E_S|^p)^{1/p}, \tag{5}$$

where $E = (E_U, E_S)$. A surprising result is that [except for the case $p = 1$ for HEs in which case $d_1(\alpha) = 1$], independently

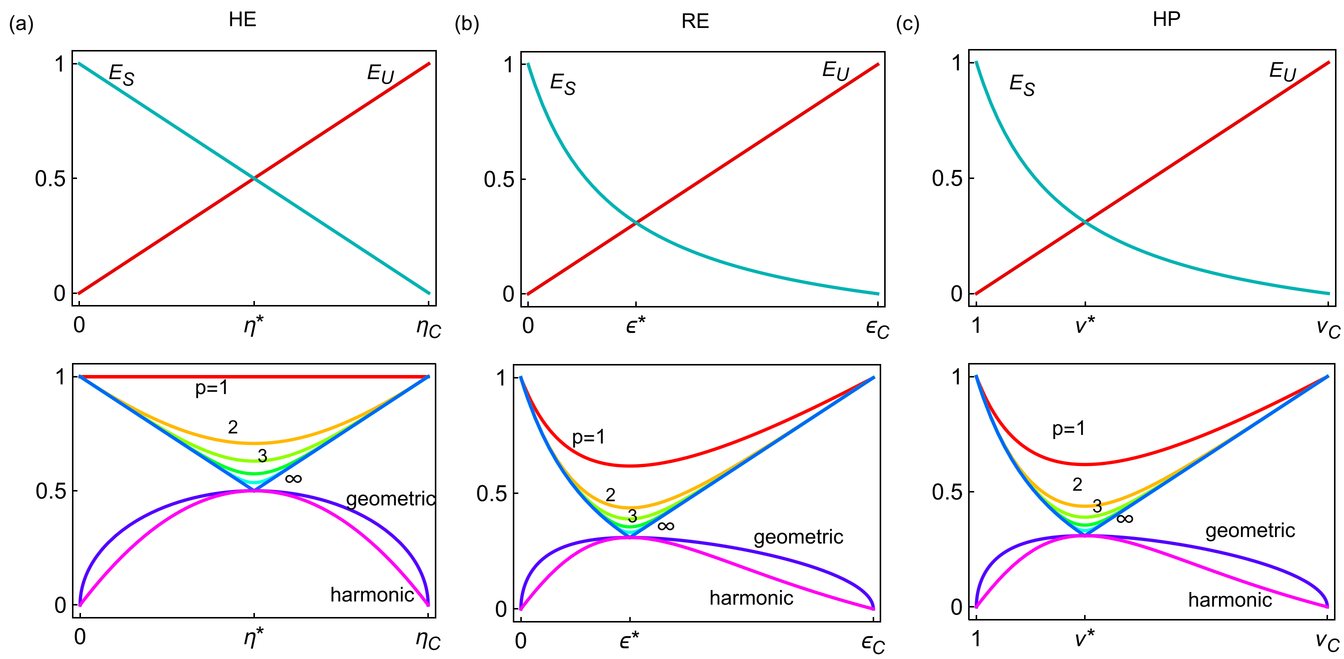

FIG. 1. In the first row E_U and E_S are plotted as functions of $\alpha \equiv (\eta, \epsilon, \nu)$ for the range $\alpha_{\min}$ to $\alpha_{\max} = \alpha_C$. Below each inset the p-norm $[d_p(\alpha)]$ for $p = \{1, 2, 3, 5, 10, \infty\}$. Note that all of them have their minimum value at α^*. The values $\tau = 0.4$ for the HE and $\tau = 0.8$ for the RE and the HP are used. The geometric and harmonic means are depicted, showing that their maximum values are also located at α^*.

from the chosen distance, Taxicab or Manhattan distance ($p = 1$), the Euclidean distance ($p = 2$), up to the infinite norm, $d_p(\alpha)$, is a concave function with one minimum at α^*, given by

$$\alpha^* = \begin{cases} \frac{\eta_C}{2}, & \text{HE}, \\ \sqrt{\epsilon_C + 1} - 1, & \text{RE}, \\ \sqrt{\nu_C}, & \text{HP}. \end{cases} \quad (6)$$

This point represents the closest that one state (characterized by α) can be from both extremal situations. For $\alpha > \alpha^*$ the heat reservoirs would be closer to $E_U = 1$.

Proving that for any p norm the same α^* is obtained relies on the fact that, for $p = 1$ at α^*, $E_U = E_S$ and $E_U' = -E_S'$ ($'$ refers to $d/d\alpha$). This guarantees that $d_p' = 0$ holds for any $p < \infty$. These two constraints also exhibit a similar result for the mean value of E_U and E_S. In either case, if both are considered as normalized quantities, in which case the geometric is the correct mean, or as weighted quantities, in which case the harmonic one should be used, both means exhibit their maximum value at α^* (see lower panel on Fig. 1). Since the nature of both means is that extremal values dominate, α^* is located at a position when both stabilities have the same influence over the reservoir state. All this strengthens the idea that α^* represents a threshold between two stable extremal configurations.

Intuition dictates that the natural evolution of energy converters will tend to favor optimum energetic performance. Thus, if stable points are attraction configurations, those that tend to a null performance are doomed to vanish (under the perspective of adaptation or evolution of energy converters) and those leaning to optimum efficiency will battle with a trade-off among reversibility, the actual needs for energy conversion and their resilience, linked to reservoir stability. This is a feature that will require further analysis although some results on the interplay between stability and energetic optimization have been published [21].

Note that, for HEs, $\alpha^* \equiv \frac{\eta_c}{2}$ is the first term of the power expansion of the Curzon-Ahlborn efficiency at maximum power in the finite-time thermodynamics framework [27], $\eta_{CA} = 1 - \sqrt{1 - \eta_c} \approx \frac{\eta_c}{2} + \frac{\eta_c^2}{8} + O(\eta_c^3)$. As such, it appears in the optimization of many different systems (see, for instance, Ref. [28]). This value was also obtained by Van den Broeck [29] in the frame of linear irreversible thermodynamics under strong coupling conditions between the heat and work fluxes. Concerning $\alpha^* \equiv \sqrt{\epsilon_C + 1} - 1$ for RE, it was first obtained in finite-time frameworks by Yan and Chen [30], taking as target function $\epsilon \dot{Q}_c$, where $\dot{Q}_c$ is the cooling power of the refrigerator. Later and independently it was derived by Velasco *et al.* [31] using a maximum per-unit-time COP and by Allahverdyan *et al.* [32] in the classical limit of a quantum model with two n-level systems interacting via a pulsed external field. It is also the extension of the symmetric low dissipation model for finite-time Carnot refrigerator devices as shown by de Tomás *et al.* [33].

III. SUCCESS IN ENERGY CONVERSION

Some physical insights can be derived from the above results. Consider a function $\Delta = R - I$ in such a way that condition $\Delta > 0$ ($R > I$) defines a success region where reversibilities (R) dominate irreversibilities (I) while condition $\Delta < 0$ ($R < I$) defines a failure region where irreversibilities (I) dominate reversibilities (R). The threshold is thus defined by the constraint $\Delta = 0$ ($R = I$). This constraint imposes a limit value for α, denoted as α^*, which delimits the success and failure regions in terms of the efficiency: the success one is then bounded by $\alpha^* < \alpha < \alpha_c$ while the failure one is bounded by $\alpha_{\min} < \alpha < \alpha^*$. The Δ function can be easily quantified in Carnot-like heat devices: reversibility can be taken into account by a dimensionless factor R, $0 \leqslant R \leqslant 1$, defined as

$$R = \frac{\alpha - \alpha_{\min}}{\alpha_c - \alpha_{\min}}, \tag{7}$$

while as a measure of irreversibility (losses) another dimensionless factor I, $0 \leqslant I \leqslant 1$, is introduced as

$$I = \frac{\Delta S_{\text{univ}}}{\Delta S_{\text{univ}}^{\max}}, \tag{8}$$

where $\Delta S_{\text{univ}} \geqslant 0$ stands for the non-negative entropy change of the thermodynamic universe (system and environment) according to the second law and $\Delta S_{\text{univ}}^{\max}$ denotes its maximum possible value. R, I, and Δ are straightforward to obtain for cyclic heat devices working between two infinite heat baths.[1] For a HE,

$$\Delta S_{\text{univ}} = -\frac{|Q_h|}{T_h} + \frac{|Q_c|}{T_c} = \frac{|Q_h|}{T_h}\frac{\eta_c - \eta}{1 - \eta_c} \geqslant 0, \tag{9}$$

with a maximum when no work is delivered, i.e., $\eta = 0$,

$$\Delta S_{\text{univ}}^{\max} = \frac{|Q_h|}{T_h}\frac{\eta_c}{1 - \eta_c}, \tag{10}$$

$$R = \frac{\eta}{\eta_c}, \quad I = \frac{\eta_c - \eta}{\eta_c} = 1 - \frac{\eta}{\eta_c}, \tag{11}$$

$$\Delta = \frac{\eta}{\eta_c} - \left(1 - \frac{\eta}{\eta_c}\right) = \frac{2\eta}{\eta_c} - 1. \tag{12}$$

It is direct that Δ becomes zero for $\eta^* = \frac{\eta_c}{2} \equiv \frac{1-\tau}{2}$.

In accordance with the comments above, this value should be considered as the minimum value of the efficiency of any thermodynamically successful HE device.

For the inverse cycle working as a refrigerator,

$$\Delta S_{\text{univ}} = \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c} = \frac{|Q_h|}{T_h}\left(1 - \frac{\epsilon}{\epsilon_c}\frac{1 + \epsilon_c}{1 + \epsilon}\right) \geqslant 0, \tag{13}$$

with a maximum value when $\epsilon = 0$,

$$\Delta S_{\text{univ}}^{\max} = \frac{|Q_h|}{T_h}. \tag{14}$$

Now, expressions for R, I, and Δ read as

$$R = \frac{\epsilon}{\epsilon_c}, \quad I = 1 - \frac{\epsilon}{\epsilon_c}\frac{1 + \epsilon_c}{1 + \epsilon}, \tag{15}$$

$$\Delta = \frac{\epsilon(2 + \epsilon + \epsilon_c)}{\epsilon_c(1 + \epsilon)} - 1, \tag{16}$$

where Δ becomes zero for $\epsilon \equiv \epsilon^* = \sqrt{1 + \epsilon_c} - 1 \equiv \frac{1}{\sqrt{1-\tau}} - 1$. As η^* for HE, ϵ^* determines the minimum COP of any thermodynamically successful RE.

[1] This constraint not only covers the case of reversible cycles, but also the case of finite systems with finite heat reservoirs where the entropy production of the system has a negative entropy contribution from the correlation between the system and the reservoir [25].

For the inverse cycle working as a HP device,

$$\Delta S_{\text{univ}} = \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c} = \frac{|Q_h|}{T_h}\left(1 - \frac{\nu_c}{\nu}\frac{\nu - 1}{\nu_c - 1}\right) \geqslant 0, \tag{17}$$

with a maximum value when $\nu = 1$,

$$\Delta S_{\text{univ}}^{\max} = \frac{|Q_h|}{T_h}, \tag{18}$$

$$R = \frac{\nu - 1}{\nu_c - 1}, \quad I = 1 - \frac{\nu_c}{\nu}\frac{\nu - 1}{\nu_c - 1}, \tag{19}$$

$$\Delta = \frac{(\nu - 1)(\nu + \nu_c)}{\nu(\nu_c - 1)} - 1, \tag{20}$$

which becomes zero for $\nu^* = \sqrt{\nu_c} = \frac{1}{\sqrt{1-\tau}} \equiv \epsilon^* + 1$. Again, this value fixes the minimum COP of any successful HP device.

Thresholds η^* and ϵ^* (ν^*) can be interpreted in the context of the work-entropy relations [11,34]. For a HE this relation reads as $|W| = |W_{\max}| - T_c\Delta S_{\text{univ}}$, where W denotes the work output, $|W_{\max}|$ is its maximum possible value, and $T_c\Delta S_{\text{univ}}$ is the so-called lost work as a consequence of the global irreversibilities (lost-work theorem). For a given heat input $|Q_h|$ it is easy to show that condition $R = I$ implies that ΔS_{univ} is one-half of its maximum, i.e., $|W| = |W|_{\max}/2$. A straightforward consequence is that half of the available work is lost. For a RE the work-entropy relation reads $|W| = |W|_{\min}| + T_h\Delta S_{\text{univ}}$, where now $|W|$ denotes the actual input work, $|W_{\min}|$ the minimum value, and $T_h\Delta S_{\text{univ}}$ the needed extra work because of irreversibilities. Direct calculations show that condition $R = I$ for a given cooling heat $|Q_c|$ implies that $|W| = (1 + \sqrt{1 + \epsilon_C})|W|_{\min}$, i.e., the required extra work (in addition to the minimum value) is $\sqrt{1 + \epsilon_C}|W_{\min}|$ with $\Delta S_{\text{univ}} = \frac{\Delta S_{\text{univ}}^{\max}}{1+\sqrt{1+\epsilon_C}}$.

IV. CONCLUDING REMARKS

A comparison of the obtained thresholds with some experimental results for a variety of power and refrigeration plants is presented; see Figs. 2 and 3. An agreement is observed in most cases and only a few points are slightly below the predicted success region. There is no physical inconsistency in the fact that some experimental values could attain values below the obtained lower bounds (as can be seen, for example, in Refs. [29,39–41]); this only shows that, according to the meaning of Δ, these plants should be qualified as thermodynamically inefficient heat devices. Let us recall that, in this work, while theoretical results have been obtained under strong assumptions by balancing reversibility and irreversibilities, experimental results account for quite different and more intricate arrangements where the design does not necessarily fit the theoretical balance of irreversibilities.

So far, no explicit optimization has been made on the usual thermodynamic figures of merit. However, an intuitive link between the threshold given by α^* and optimization can be justified and even some information can be obtained prior to any optimization.

The thermodynamic entropy and internal energy allows the definition of stability parameters, lastly depending on α. However, notice that E_U and E_S (R and I) for HEs conform to a discrete distribution E, with $\sum_i E_i = 1$ ($i = \{U, S\}$), while

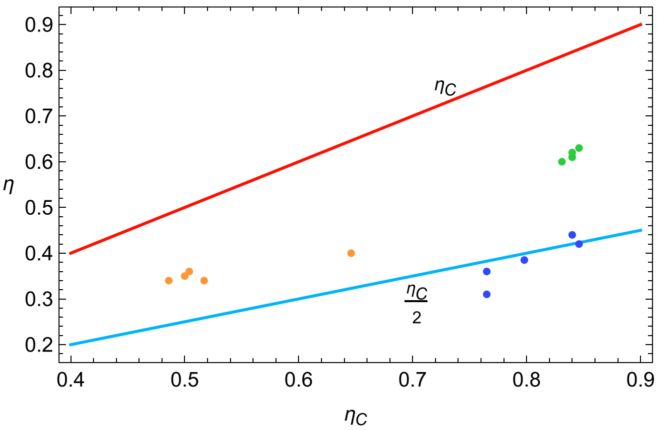

FIG. 2. Comparison between experimental results [11,35–37] [nuclear power plants with Rankine-like cycles (orange dots): gas-turbine Brayton-like (blue dots); gas-turbine Brayton-like in combined arrangements (green dots)]. Carnot value η_c (red solid line) and $\eta^* = \frac{\eta_c}{2}$ (blue solid line).

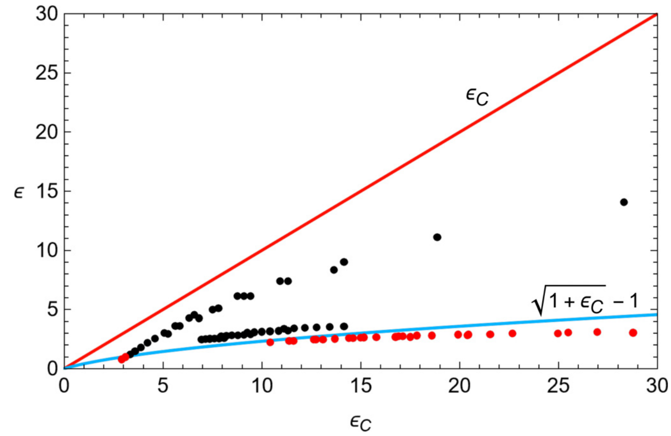

FIG. 3. Comparison among three sets of experimental results for RE [38] (points). Carnot value ϵ_c (red solid line) and $\epsilon^* = \sqrt{1+\epsilon_c} - 1$ (blue solid line).

for REs and HPs E_U and E_S conform to an incomplete (nonnormalized) distribution, in which case the escort distribution (by normalization) can be easily obtained. Then, the Renyi entropy, H, for this distribution can be related to the p norm as [42,43]

$$H_p(\alpha) = \frac{p}{1-p}\ln(\|E\|_p), \tag{21}$$

where in the case where $p \to 1$ the Shannon entropy is recovered. This would represent a kind of "second layer" entropy, in the sense that one obtains information entropy based on thermodynamic entropy. It can be shown that precisely at α^* the Renyi entropy exhibits its maximum. For $p > 0$ and $p \neq 1$ this function is Schur concave. This can be exploited to obtain information on optimization regardless of the fact that no information on the specific model has been provided as follows. The presence of a maximum on H_p and the Schur concave property imply the existence of a majorization on E_U and E_S, which preserves order in the entropy space [44,45]. In a first approach, both functions depend on α, which would be the natural candidate to define dominance. However, on a deeper level, a connection between the efficiency, α, and entropy change of the system, $\Delta_i S_s(t)$, could indicate that the latter could be used in the majorization of E_U and E_S. This would have two consequences: (a) the term $\Delta_i S_s(t)$, linking entropy production and correlations between system+reservoirs, could be analyzed under the light of optimization, which leads to the second point (b) regarding multiobjective optimization where majorization (through nondominated vectors) are key. For some models, it has been reported that the Pareto front exhibits an endorreversible behavior (commonly described in terms of α) [21]. Thus using α as the discriminant for majorization makes sense, strengthening the intuitive link between success, optimization, and stability.

An open issue would be to demonstrate that, in fact, $d_p(\alpha) = d_p(\Delta_i S_s(t))$. In such a case it would be obtained that at α^* the system-reservoir correlations would be the largest, which intuitively makes sense in HEs since $\eta_C/2$ is a kind of universal lower bound for the maximum power regime. The equivalent for REs and HPs is by no means evident, although ϵ^* corresponds to the lower bound of the maximum χ regime in REs for weakly dissipative Carnot refrigerator models [46].

To conclude, a simple, general, and unified derivation of thresholds (*lower bounds*) for the performance of cycling heat devices without resorting to any explicit optimization criterion and simply balancing reversibility and irreversibilities has been presented. They delimit by below a region where the heat devices perform successfully. Perhaps, according to the Seifert remark [1], the main qualitative conclusion to emphasize would be the capability of the conceptual issues involved in thermodynamics to delimit efficient design of heat devices, either manmade or those, biochemical in nature, evolving under natural resources availability. Indeed, optimal manmade designs or efficient adaptation to a natural environment can greatly surpass the obtained lower bounds.

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