“Like Flying Blind:” Instructors’ Stories About Teaching Undergraduate Mathematics During the Coronavirus Pandemic

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Abstract

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Efforts to improve learning outcomes in undergraduate mathematics are backed by calls for instructors to move away from persistent pedagogical norms, such as traditional "chalk-talk lectures" and timed closed-book exams. Although the movement towards active learning pedagogies and alternative assessments is gaining ground, uptake has been slow. But when traditional practices became virtually impossible to maintain during the coronavirus pandemic, many instructors quickly gave up old methods for new ones.

This qualitative study sheds light on how twenty-eight instructors of undergraduate mathematics experienced teaching through the coronavirus pandemic. It documents stories they told to describe their experiences and explores how they adjusted their teaching practices. A purposeful sample of instructors whose teaching experience spans 58.5 years participated in the study. They taught lower- and upper-division courses during the pandemic and came from colleges and universities across the United States. The primary data collection method was semi-structured interviews.

Four salient storylines emerged from data: (1) Remembering other times of change and disruption, (2) Pivoting to emergency online instruction, (3) "Like flying blind:" navigating a new normal, and (4) Coping. In addition, analysis of participants' stories revealed that they adjusted five dimensions of practice for teaching during the pandemic: (1) Representing Mathematical Content, (2) Choosing Tasks and Content, (3) Monitoring: "Leaning Over Their

Shoulders," (4) Building a Community of Mathematics Learners and Doers, (5) Assessing for Learning During the Pandemic.

The lessons we learn from this emergency will be "educative" for this crisis and the next. Moreover, the coronavirus pandemic also offers an opportunity to examine longstanding norms in undergraduate mathematics education. This study offers recommendations for practitioners, leadership, and further research.

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Dedication

To my parents in heaven,

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CHAPTER I: INTRODUCTION

> When everything changes...what we can construct, if we keep notes and survive, are hindsight accounts of the connectedness of things that seem to have happened: pieced-together patternings...showing how particular events and unique occasions, an encounter here, a development there, can be woven together with a variety of facts and a battery of interpretations to produce a sense of how things go, have been going, and are likely to go. (Geertz, 1995, pp. 2-3)

Background

Because the coronavirus spreads through human contact, many colleges canceled in-person classes for safety. According to the United Nations (2020), "by mid-April 2020, 94 percent of learners worldwide were affected by the pandemic, representing 1.58 billion children and youth, from pre-primary to higher education, in 200 countries" (p. 5). Some instructors had as little as one week to prepare for emergency remote instruction. Despite the abrupt shift amidst illness, grief, loss, caregiving responsibilities, and uncertainty, "instructional MacGyvers," as Hodges et al. (2020) referred to instructors, used various digital technologies to teach online during a global health crisis. This study seeks to shed light on the experiences of mathematics instructors who taught undergraduate students. First, it documents the stories they tell to describe their experiences. This first inquiry is broad in scope to allow stories that I may not have considered to emerge. As far as I know, before the coronavirus pandemic, no post-pandemic analyses of stories about teaching undergraduate mathematics during a pandemic exists. Second, this study explores those stories to understand how instructors adjusted their pedagogical practices for teaching during the pandemic.

The pandemic has disrupted persistent practices in undergraduate mathematics education. For example, the "chalk-talk lecture," which, according to Artemeva and Fox (2011), was "the central pedagogical genre of the undergraduate mathematics lecture classroom" (p. 345), did not translate well online. Likewise, closed-book exams, "the timed, unseen exam where students are

not allowed to access external material" (Iannone & Simpson, 2012, p. 1), became virtually impossible. So, instructors improvised. They found other ways to engage and assess students online. Moreover, the pandemic exacerbated and put a magnifying glass on inequities that existed in the mathematics community. Historically underrepresented students (e.g., Black students) in mathematics classrooms come from communities hardest hit by the virus and a failing economy (Horsford et al., 2021). Those same students came from under-resourced communities and were less likely to have access to the technology and infrastructure needed for learning online.

In sum, the pandemic exposed longstanding issues in undergraduate mathematics education—how it's taught and assessed, for example. Further, it revealed existing problems of access and representation. These are not new problems in undergraduate mathematics education; however, the pandemic worsened them and drove the point home. Professional societies responded by encouraging instructors to use this pandemic as an opportunity to examine persistent norms in undergraduate mathematics education. For example, The National Council of Science Museums Leadership in Mathematics Education (NCSM) and the National Council of Teacher of Mathematics (NCTM) point out, "we have the opportunity to be innovative and to think purposefully about addressing traditional/systemic structures, practices, and beliefs that have allowed inequities to persist" (2020, p. 15). Thus, the pandemic offers a chance to challenge the status quo.

These calls from professional societies to "build back better," a phrase borrowed from the United Nations Children's Fund (UNICEF), require sustainable changes. Since instructors are agents of change in education, knowledge about how they adjust to teaching during this unique era can inform and mobilize lasting change. And as Casey (1995) puts it, "what better way to

grapple with making sense of our rapidly changing world than through the study of stories" (p. 240). Clandinin and Connelly (2000) maintain that teachers' stories are rich units for analysis. Furthermore, what is often missing from research on instructors and instruction in undergraduate mathematics education, are stories told in their voices. The lessons we take away from the stories of undergraduate mathematics instructors can inform and instruct how instructors navigate the unpredictable future, especially because the pandemic's effects may be long-lasting.

Purpose of the Study

The purpose of this study is to shed light on the experiences of instructors who taught undergraduate mathematics during the coronavirus pandemic. The following questions guide this study:

1. How did instructors of undergraduate mathematics "story" their experiences teaching during the coronavirus pandemic?
2. How did instructors of undergraduate mathematics adjust their practices for teaching online during the pandemic?

Procedures for the Study

This qualitative study was conducted using narrative inquiry methodology. Nardi (2016) recommends narrative approaches for research in undergraduate mathematics education because they "bring the richness and vibrancy of storytelling into how data is [sic] collected and interpretations of it shared" (p. 361). Thus, the questions that guided this study were explored through narrative inquiry. Grounded theory strategies were also helpful for understanding the perspectives of many participants about an unprecedented experience.

Participants

Twenty-eight instructors of undergraduate mathematics participated in this study. Purposeful sampling (Patton, 2015) was used to recruit information-rich cases (Merriam & Tisdell, 2016), who taught during the coronavirus pandemic in 2020. Recruitment was done online and through personal connections. The flyer in Appendix A was posted on my Twitter account at the end of December 2020. I tagged professional mathematics societies and networks to bring attention to the flyer (e.g., Math Twitter Blog-o-sphere (@MTBoS), #iteachmath, @BlackinMath, @maanow, @amermathsoc). Friends in my network retweeted, liked, shared, and tagged other professional communities of mathematics instructors. I also posted to the Facebook group, "STEM Faculty Blundering Through Remote Teaching in A Pandemic," and MAA member communities (e.g., SIGMAA on RUME). Four participants, who I knew personally, were invited to participate via email—not via online platforms.

Data Collection

After participants agreed to participate in the study, they filled out an Informed Consent form shown in Appendix C. Participants provided background information (e.g., number of years of teaching experience) via the Qualtrics Pre-Interview Questionnaire in Appendix D. The information gathered from this questionnaire serves to familiarize readers with the participants. Appendix B is a snapshot of their profiles. The semi-structured, open-ended protocol in Appendix E guided the interview. Developing trust and rapport with participants was necessary for a conversational experience to feel comfortable sharing their stories. The interviews took place between the end of December 2020 and early February 2021. They lasted 45-96 minutes and were conducted via Zoom video conferencing to adhere to social distancing regulations. After each interview, I downloaded audio files and transcripts from the Zoom cloud storage and

stored them in my password-protected Google Drive. Participants were assigned pseudonyms. During the interviews, I took notes of specific utterances to explore further or emerging connections within and across participants' stories. In addition, participants shared pedagogical artifacts that they developed and used during the pandemic.

Data Analysis

After each interview, I wrote memos to keep track of my "thoughts, musings, speculations, and hunches" (Merriam & Tisdell, 2016, p. 200). Chapter III offers more detail about the data analysis process. Herewith, I briefly describe the procedures for addressing each research question.

To answer research question 1:

1. I used an inductive, grounded theory approach to understand how participants storied their experiences. "What stories did they choose to tell?" I asked myself as I read through each transcript, line by line. Next, I used NVivo software to manage the data and identify and code emergent themes that recur across participants' stories. Then, all relevant themes were categorized into storylines.
2. Dewey's work on the theory of experience, scholarship in undergraduate mathematics education, digital technology in mathematics education, education in emergencies, teacher stories in education research provided guidance and language for identifying and categorizing themes.
3. I wrote a narrative that captures the collective story of the participants and the zeitgeist of this unprecedented time in undergraduate mathematics education.

To answer the second research question, I used a procedure similar to that for research question 1; however, I examined the transcripts for pedagogical decisions they made to adjust for teaching

during the pandemic. Those were organized according to common dimensions of teaching practice.

In short, I followed Charmaz's (2014) advice to remain open to the data so that I was able "to discover subtle meanings and have new insights" (p. 137). Because of my position as an undergraduate mathematics instructor, this grounded approach helped minimize the influence of my experiences in coding the data.

Conclusion

This study gets at the heart of anthropologist's Geertz (1995) advice, which is that "when everything changes...what we can construct, if we keep notes and survive, are hindsight accounts of the connectedness of things that seem to have happened…to produce a sense of how things go, have been going, and are likely to go" (pp. 2-3). Moreover, it centers instructors' voices to shed light on how they experienced teaching and living through the coronavirus pandemic. In Chapter II, I outline the literature relevant for understanding this work. The research design and procedures are presented in Chapter III—the methodology section. Chapters IV and V present the study's findings, each describing the results for the first and second research questions, respectively. This dissertation closes with a discussion in Chapter VI, which includes a summary, conclusions, and recommendations.

CHAPTER II: LITERATURE REVIEW

Introduction

The purpose of this study is to shed light on the experiences of instructors who taught undergraduate mathematics during the coronavirus pandemic. In particular, it documents stories they told to describe their experiences and the pedagogical decisions they made to adjust for teaching during a pandemic. This chapter begins with a discussion of the concepts that frame this research. The following section describes the literature search. The remaining sections outline three bodies of literature relevant for understanding this study and its significance: (1) Perspectives in Undergraduate Mathematics Education, (2) Perspectives on Technology Use, (3) Education in Emergencies.

Conceptual Framework

As a novice researcher, the questions "What do you think is going on?" and "What do you think this is an example of?" (Casanave & Li, 2015, p. 108) helped identify the concepts that frame this study. The concepts germane to this work come from John Dewey's theory of experience and Donald Schön's work on reflective practice. Jean Clandinin and Michael Connelly's view that stories lived and told by people provide rich insights for understanding their experiences also inspires this work. The conceptual framework for this research has disciplinary roots in, at least, philosophy and anthropology, which has become increasingly popular due to "the emergence of modes of inquiry that are more qualitative and less quantitative in nature" (Silver & Herbst, 2007, p. 43). This section consists of four subsections: 1) Dewey's Theory of Experience, 2) Schön's Reflective Practice, 3) Clandinin and Connelly on Experience and Story, and 4) Concluding Thoughts on the Conceptual Framework.

Dewey's Theory of Experience

According to Dewey (1938), all experiences influence subsequent experiences—a principle he termed "continuity of experience." As he puts it, "wholly independent of desire or intent, every experience lives on in further experiences" (p. 16); however, not all experiences are educative. That experience may not lead to growth, deems some experiences mis-educative, whereas educative experiences "live fruitfully and creatively in subsequent experiences" (Dewey, 1938, p. 17). Another principle, interaction of internal and external factors, which form a "situation," comes into play when interpreting experience. The internal refers to "feelings, hopes, aesthetic reactions, and moral dispositions, [and the external refers to] the existential conditions, that is, the environment" (Clandinin & Connelly, 2000, p. 50). Moreover, as Rodgers (2020) explains:

> Interaction and continuity, the elements of experience, are the x and y axes of experience. Without interaction, learning is sterile and passive, never fundamentally changing the learner. Without continuity, learning is random and disconnected, building toward nothing either within the learner or in the world. (p. 847)

The idea that teachers' stories of experience can serve as moving forces that can be educative is rooted in these notions. Furthermore, so that an experience is educative, Dewey asserts that reflection is a necessary step. Schön (1983) agrees with Dewey. He states:

> Through reflection, [a practitioner] can surface and criticize the tacit understandings that have grown up around the repetitive experiences of a specialized practice and can make new sense of the situations of uncertainty or uniqueness which he may allow himself to experience. (p. 61)

For Dewey (1938), we learn and grow from reflecting on experiences, not experiences themselves. His theory of experience helps conceptualize what teachers do and say they do.

Schön's Reflective Practice

Building on Dewey's ideas about reflection, Schön (1983) emphasizes the importance of reflective practice for professional growth. When practitioners reflect on the strengths and

weaknesses of their actions, they may use that information to modify future practice. Reflective practice has been used widely in mathematics education for teacher development (e.g., Artzt et al., 2015). My Google Scholar search for "reflective practice" and "teacher education" returned 75,000 results. Yet, some scholars argue that researchers misuse the term. For example, Rodgers (2002) writes, "reflection has suffered from a loss of meaning. In becoming everything to everybody, it has lost its ability to be seen" (p. 843). Echoing Rodger's sentiment, Hébert (2015) asserts:

> The term is overused in teacher education programs to the point that it has lost meaning...Hence, we should be wary to assume that a student who has engaged in reflection has automatically enhanced his or her practice and is 'ipso facto a good teacher.' (p. 370)

But in Walshaw's (2010) view, "The ideas expressed in the literature on the reflective practitioner have considerable persuasive purchase in our attempts to understand effective practice" (p. 488). When teachers reveal their reflections, they provide a space for making sense of their pedagogical actions.

Clandinin and Connelly on Experience and Story

Clandinin and Connelly (2000) use Dewey's work on the nature of experience as a backdrop for their scholarship on experience and story. For them, "Dewey transforms a commonplace term, experience, in our educators' language into an inquiry term, and gives us a term that permits better understandings of educational life" (Clandinin & Connelly, 2000, p. 2). Thus, Clandinin and Connelly's work on narrative inquiry draws on Dewey's notions of continuity of experience, interaction, and situation. They position the researcher to ask questions, collect data, make meaning of that data, and write research "that addresses both personal and social issues by looking inward and outward, and addresses temporal issues by looking not only to the event but to its past and to its future" (Clandinin & Connelly, 2000, p. 50).

In addition to Dewey's influence, other scholars in the social sciences shaped Clandinin and Connelly's views of narrative inquiry. For example, Alasdair MacIntyre's (1981) work on narrative unity gave them a way to situate Dewey's notion of continuity into their conceptualization of narrative inquiry. Anthropologist Clifford Geertz (1995) acknowledges that narratives, including their narrators, are beneficial for studying change. Another anthropologist, Mary Catherine Bateson (1994), writes, "adaptation comes out of encounters with novelty that may seem chaotic. In trying to adapt, we may need to deviate from cherished values, behaving in ways we have barely glimpsed, seizing on fragmentary clues" (p. 8, as cited in Clandinin & Connelly, 2000, p. 7). This quotation from Bateson (1994) connects Dewey's notions of continuity and interaction with change. Clandinin and Connelly add that "improvisation and adaptation to change allow the past to be connected and to have continuity with the future...people improvise and adapt, that is, they learn" (p. 7). Bateson (1994) also contends that in the written text of the research, the researcher must be sure to use the active voice and the word "I." Thus, narrative inquiry has its own set of rules for constructing narrative texts. With its multidisciplinary, multilayered foundation, narrative inquiry has been "flourishing" as a qualitative methodological approach (Chase, 2005); however, Gottlieb and Lasser (2001), and others, criticized narrative inquiry for its reliance on first-person accounts of experience and researcher interpretation.

Often, skeptics question the veracity of stories that people tell about their experiences. Clandinin and Connelly, and other narrative methodologists, acknowledged criticisms of narrative inquiry and addressed some; others, they maintain, are difficult to ascertain. They also refer those skeptics to writers of memoirs and autobiographies for answers, citing "the

insolubility of sorting out distinctions of fact and fiction" (p. 179). Additionally, Gottlieb and Lasser (2001) acknowledge:

> The scientific method has much value and has improved our lives in numerous ways, but it has an inherent flaw. Just like the drunk who lost his keys looks for them under the lamppost because that is where the light is, so the scientific method can only manipulate or control variables with which we are already familiar. (p. 191)

Despite its criticisms, using narratives as a method for data collection and object of inquiry for understanding teachers' experiences is on the rise. In Carter's (1993) view, "this trend has been upsetting [only] to some who mourn the loss of quantitative precision, and, they would argue, scientific rigor" (p. 5). She adds:

> Teachers are not privileged authors who somehow have direct access to truth and the power to tell the whole story. Stories, including those told by teachers, are constructions that give a meaning to events and convey a particular sense of experience. They are not videotapes of either reality, thought, or motivation. Thus, we cannot escape the problems of veracity and fallibility in our work. (Carter, 1993, p. 8)

Other proponents of using stories to study teachers and teaching acknowledge the problems associated with using first-person accounts; however, they maintain that the benefits of this approach outweigh the concerns about truth. For example, Clandinin and Connelly (1987) note that "what is especially interesting about these studies is that one way or another, they purport to study 'the personal,' that is the what, why and wherefore of individual pedagogical action" (p. 487). Thus, as central elements in research, teachers' stories have been used to make meaning of challenging and complex issues. For example, Walker's (2014) collection of stories from 35 black mathematicians about their lived experiences proves that Black people have succeeded and continue to succeed at mathematics, despite counter-narratives. Nardi (2016) explored rapprochement between mathematicians and mathematics educators using a narrative approach. Elsewhere, scholars (e.g., Gholson, 2016; Jett, 2011; Kaasila, 2007) have used narrative inquiry to study students' postsecondary mathematics experiences; however, the possibilities for

studying instruction, using instructors' stories, in undergraduate mathematics are largely untapped.

What is a Story?

According to Jonassen and Hernandez-Serrano (2002), "stories are the oldest and most natural form of sense making. Stories are the 'means [by] which human beings give meaning to their experience of temporality and personal actions'" (p. 66). Those who use stories in education research define "story" in various ways. For example, Sfard and Prusak (2005) assert that people's identities are stories. "No, no mistake here," they clarify, "We did not say that identities were *finding their expression* in stories—we said they *were* stories" (Sfard and Prusak, 2005, p. 14; emphasis in original). In Carter's (1993) view, a story is a "telling or re-counting of a string of events" (p. 5). Three essential elements, according to Carter (1993), constitute a story:

> (a) a situation involving some predicament, conflict, or struggle; (b) an animate protagonist who engages in the situation for a purpose; and (c) a sequence with implied causality (i.e., a plot) during which the predicament is resolved in some fashion. (p.5)

Across the various interpretations of "story" and its structure, scholars agree that storytelling is a human activity. There is a teller, a listener, and humans use stories to make meaning of their worlds. Thus, Polkinghorne (1988) argues for "research strategies that can work with the narratives people use to understand the human world" (p. xi).

Stories in Research on Undergraduate Mathematics Education (RUME)

> Telling a story makes the moment live beyond the moment. Stories function to alter the ways we view mundane everyday events. Stories can, indeed, accomplish change. (Riessman, 2008, p. 63)

Teachers' stories as a vehicle for studying teachers and teaching practice are underutilized in RUME; however, scholars have used this medium for many years to analyze aspects of teachers' knowledge and conceptions (e.g., Connelly & Clandinin, 1990). Researchers in K-12 education have recognized that teachers' practices go beyond what they can be observed

doing in the classroom—the cognitions that drive those actions are very much a part of their pedagogical practice (Artzt et al., 2015; Fennema & Franke, 1992). Despite this recognition that "teaching is more than meets the eye," there is slow uptake of methodologies in RUME that examine instructors' pedagogical thoughts. In fact, "much of RUME has focused on learners, including conceptual or cognitive change, change in affect, change in discourse, and change in participation in classroom and mathematical practices" (Reinholz et al., 2020, p. 1). But Shavelson and Stern (1981) argue that examining teachers' pedagogical thoughts is essential to changing pedagogical practice. They maintain that "the need for research on teaching to examine teachers' intentions and the link between intentions and behavior, and not just behavior alone, has been justified on several grounds" (Shavelson & Stern, 1981, p. 455). Research in undergraduate mathematics has only a few examples of this kind of study (e.g., Leinhardt & Steele, 2005; Mesa et al., 2020; Speer & Wagner, 2009); however, more interview studies to explore the thoughts that drive college mathematics teachers' practice are needed (Speer et al., 2010). During interviews with the researcher, "the teacher could tell, explain, confirm, reflect and thus 'represent' her thoughts, judgments, decisions, and ideas in public words to the researcher, who could then in turn study and analyse them to make sense of that internal world," writes Freeman (1994, p. 81). Fluid, open-ended interviews allow for instructors' stories to emerge. Through these stories, researchers can begin to understand what undergraduate instructors can be observed doing in their classrooms. Riessman's (2008) claim, which I cited at the beginning of this section, that "stories can, indeed, accomplish change" (p. 63), makes a good argument for the place of instructors' stories in RUME.

In the search for studies wherein mathematics teachers' voices were central elements of the research, I found a limited number in RUME. For example, Mesa et al. (2020) interviewed

eight instructors to understand how they transitioned from lecture to inquiry-based instruction in their linear algebra courses. Fortune and Keene (2021) studied the impact of an online professional development workshop geared towards inquiry-oriented instruction on one professor's practice. They interviewed him before, during, and after the workshop to understand how he changed his practice from his perspective. Such papers were hard to find because RUME's scope is broad, with a wide array of terms representing the topics studied. Here, I make a tangential argument for a prescriptive way for authors to determine appropriate keywords for their manuscripts. In any case, a number of those stories in RUME focus on undergraduate mathematics learners' experiences. For example, Adiredja and Zandieh (2020) explored the experiences of women of color studying linear algebra. They used a counter-storying approach to interrogate a dominant narrative that women of color are unsuccessful in this course. Other scholars have used the counter-storying approach to oppose deficit views of African American students in mathematics. Jett (2019) used four African American males' stories to understand how they persisted through their undergraduate programs in mathematics and gained admission to graduate programs in the field.

According to Clandinin and Connelly (1998), whereas earlier research "focused on teacher skills, attitudes, characteristics, and methods, [they] sensed excitement throughout the research community when attention turned to teachers' thought processes" (p. 149). As such, teachers' storied experiences provide legitimate avenues to explore teachers' thinking, which are shaped by and shape their experiences. Buhagiar's (2018) thematic analysis of a veteran secondary mathematics teacher's story examines how he changed his pedagogical approach from traditional to inquiry based. Interview data reveal his motivations for embracing the change. A pre-service secondary mathematics teacher (PSSM) in Clarke's (2009) study was transitioning

from a “chalk-and-talk” teacher-centered approach to a technology-enhanced, learner-centered approach. The PSSM’s reflections during interviews revealed to Clarke (2009) the type of professional development he needed to make this pedagogical shift. In sum, my claim here is that stories deserve a place in research on undergraduate mathematics education. They have the potential to offer insights and advance research in the field. Lerman (2000), in his call for a social turn in mathematics education, proposes that the use of narratives in research is “fruitful.” Moreover, they put the instructor’s voices at the center of efforts to address persistent problems in undergraduate mathematics education. The current educational landscape, challenged by the coronavirus pandemic, lends itself to be studied through stories experienced and told.

Concluding Thoughts on the Conceptual Framework

Concepts from Dewey on the theory of experience, Schön’s reflective practice, and Clandinin and Connelly’s work on experience and story make up the conceptual framework for this research. Other scholars used similar combinations of these ideas as a conceptual lens for studying teachers’ experience (e.g., Drake, 2006; Fridley-Hereford, 2005; Trevino, 2006). Clandinin and Connelly (1998), whose work builds on Dewey’s theory of experience and Schön’s work on reflective practice, used this approach to understand teachers’ reform practices (e.g., Clandinin & Connelly, 1998). Research in undergraduate mathematics education could benefit from studying stories of experience to understand shifts in instructors’ pedagogical practices. In the next section, I describe the method for finding relevant literature to support this work.

Method for Conducting the Review

The search for the literature that grounds this study was conducted by searching library databases[1] (e.g., Education Research Complete) for keywords[2] (e.g., mathematics, undergraduate, teaching) or combinations of them (e.g., undergraduate mathematics teaching). The same keyword searches were used in Google Scholar. Other sources were found searching journals[3] that publish papers on undergraduate mathematics education (e.g., *PRIMUS, Journal for Research in Mathematics Education, International Journal of Undergraduate Mathematics Education*) and publications of professional societies (e.g., MAA Notes). In addition, I searched proceedings from the Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education (SIGMAA on RUME) annual conferences for topics relevant to this study. Finally, the reference lists of those articles were used to identify other relevant literature. The remainder of this review is organized into three main sections: 1) Perspectives Undergraduate Mathematics Education, and 3) Perspectives on Technology Use, 3) Education in Emergencies.

Perspectives in Undergraduate Mathematics Education

Research on undergraduate mathematics education is far from limited. To illustrate this point, consider that in 2001 the International Commission on Mathematical Instruction (ICMI) released a 599-page report, *Teaching and Learning of Mathematics at University Level*. Fifteen years later, it released *Research on Teaching and Learning at the Tertiary Level*—a smaller

[1] Other databases include ERIC, Education Full Text, and Academic Search Premier

[2] Other keywords include university, college, higher education, postsecondary, tertiary, pedagogy, instruction, education in emergency, digital technology, educational technology distance education, distance learning, online learning, remote education, narrative inquiry, teacher story, narrative study, teacher decision making, teacher thoughts, teacher cognition

[3] Other journals include *Journal of Mathematical Behavior*, *Educational Studies in Mathematics*, *Educational Psychologist*, *The College Mathematics Journal*, *College Teaching*, *The Mathematics Teacher*

volume that gives a sweeping review of research in the field. In addition, the SIGMAA on RUME hosted 21 conferences and published 23 proceedings. The most recent proceeding is 1313 pages long and "was organized around the following themes: results of current research, contemporary theoretical perspectives and research paradigms, and innovative methodologies and analytic approaches as they pertain to the study of undergraduate mathematics education" (Karunakaran et al., 2020, p. iii). Other significant bodies of work include Biza et al.'s (2016) *Research on Teaching and Learning Mathematics at the Tertiary Level*, Carlson and Rasmussen's (2008) *Making the Connection: Research and Teaching in Undergraduate Mathematics Education*, the four volumes of *Research in Collegiate Mathematics Education* published by the American Mathematical Association (AMS) and *The Teaching and Learning of College Mathematics: Current Status and Future Directions*.

Despite the considerable body of literature to support change in undergraduate mathematics education, some issues remain persistent. I address those issues in broad strokes: 1) What Should Be Taught in Undergraduate Mathematics Education, 2) How Undergraduate Mathematics is Taught, and 3) Who Has Access.

What Should Be Taught in Undergraduate Mathematics Education

According to Transforming Post-Secondary Education in Mathematics (TPSE Math), the goal of undergraduate mathematics education should be to "enable any student, regardless of his or her chosen program of study, to develop the mathematical knowledge and skills necessary for productive engagement in society and in the workplace" (Saxe & Brady, 2015, p. v). Social, political, and economic contexts motivate this vision. For example, consider the post-Sputnik curriculum "undergirded by the desire to restructure education in ways that created a much more mathematically and scientifically literate populace" (Schoenfeld, 2016, p. 505). Reformists

maintain that programs can achieve this goal with curricula centered around students' conceptual understanding. Yet, many years after Sputnik, undergraduate mathematics curricula remain primarily traditional despite evidence of change. Furthermore, deficiencies in mathematics education persist. The Mathematical Association of America (MAA), and other professional mathematical societies, maintain that this "status quo is unacceptable" (Saxe & Brady, 2015, p. 1). Thus, they ask that programs modernize curricula to improve learning outcomes and meet the needs of the twenty-first century.

Traditional versus Modern Curricula

The traditional curriculum is characterized as "a collection of pre-established truths and procedures that learners must assimilate" (Laursen & Rasmussen, 2019, p. 360). Those unhappy with the traditional curriculum argue that it does not motivate students to think and create; it encourages students to parrot subject matter (Schoenfeld, 1980), which, according to Schoenfeld (1980) and others, is counterproductive for preparing students for the workforce. And as the National Research Council ([NRC], 1991) points out, "rarely do workers or researchers confront mathematical problems requiring primarily calculation. Authentic problems are often ambiguous, admitting many forms and several answers" (p. 12). Still, those who defend the traditional curriculum argue "that the understanding that reform demands can only come after a solid grounding in calculation skills; that the problem lies elsewhere, in students' work habits and attitude to learning" (McCallum, 2000, p. 13). Whereas the modern curriculum promotes conceptual understanding and problem solving, the traditional curriculum involves memorization and repetition. Moreover, modernists want the curriculum to illustrate mathematics as an accessible, creative, and exploratory human endeavor (Rasmussen & Kwon, 2007). Unlike traditional mathematics, reform mathematics engages students in using technology and solving

interdisciplinary applications. The traditional design is also characterized by a mile-wide, inch-deep curriculum. But according to the NRC (1991), "The size of undergraduate mathematics by itself creates tremendous inertia which impedes reform" (p. 2). Thus, reformists (e.g., calculus reformists) have called for deep explorations of fewer topics (Bressoud, 2019).

Professional mathematical associations published volumes geared towards revamping the undergraduate mathematics curriculum. *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025* highlights recommendations from seven such publications. The authors found that adopting a reform-based curriculum is the "most productive approach to preparing the next generation of citizens literate in science, technology, engineering, and mathematics" (Saxe & Brady, 2015, p. 7). The *Committee on the Undergraduate Program in Mathematics' (CUPM) Curriculum Guide 2004* offers six overarching recommendations, which include:

> (1) [incorporating] activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. (p. 1)
> (2) Employ[ing] a broad range of examples and applications to motivate and illustrate the material...Promote awareness of connections to other subjects (both in and out of the mathematical sciences) and strengthen each student's ability to apply the course material to these subjects. (p. 1)
> (3) [incorporating] activities that will help all students progress in learning to use technology appropriately and effectively as a tool for solving problems, [and] as an aid to understanding mathematical ideas. (p. 2)

While the *Curriculum Guide 2004* makes the recommendations without a guide for implementing them, the newest version, *Curriculum Guide 2015*, suggests how departments may implement these recommendations "while maintaining the essential components of the traditional mathematics major" (p, 1). So innovative curricula are on the rise. Yet, the traditional curriculum continues to dominate undergraduate mathematics education. Reinholz et al. (2020) argue that shifts in departmental cultures are necessary for shifts in pedagogical practices.

The Calculus Reform Movement

The research and literature on teaching and learning calculus, since the calculus reform was born in 1986, is extensive. Based on my review, the following are some goals of the reform: fewer topics, taught deeply; use technology; emphasize conceptual understanding; use multiple representations of mathematics other than symbolic (e.g., graphical, descriptive); use real-life contexts; engage students in mathematical discussions and explorations (Bressoud, 2019; Dubinsky, 1994; Mesa & White, 2021). Schoenfeld's (1995) "A Brief Biography of Calculus Reform," as the title suggests, offers an account of the movement's life. He writes, "I was present at the birth, and I can tell you that the success of the newborn was hardly assured" (p. 1). Yet, he acknowledges that change is evident, for example, in the uptake of technology. Bressoud (2019) agrees with Schoenfeld (1995). He writes, "It is important to recognize that the Calculus Reform effort was not a failure. It made a real difference as can be seen by comparing textbooks of the 1980s and today" (Bressoud, 2019, p. 1). But he also claims that "those who worked at the forefront of the Calculus Reform movement had a vision that has not been realized" (p. 1). Mesa and White (2021) observed 66 Calculus I lessons taught across 18 institutions. Some reform aspects were evident in the lessons they observed (e.g., representing mathematics in ways other than symbolic); however, the "low presence of the reformed practices during the main space, the lecture, suggests that the calculus curriculum, as present in textbooks and enacted by instructors in these classrooms, it is not as reformed as reformers would have hoped" (Mesa & White, 2021, p. 15). So, more than three decades after the calculus reform movement was born, the efforts have not failed; however, uptake is slow. Professional societies continue to call for a calculus curriculum, more broadly, an undergraduate mathematics curriculum that incorporates more

technology, more conceptual understanding and less mechanical skills, a balance between applications and theory, and mathematical representations that are not entirely symbolic.

Introductory Mathematics Courses (Lower-Division Courses)

In *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*, Cohen (1995) defines introductory college mathematics, developmental, or lower-division courses as:

> Mathematics taught at [below calculus] in two-year colleges and in the lower division of four-year colleges and universities…[which] include college algebra, trigonometry, introductory statistics, finite mathematics, and precalculus, as well as courses characterized as developmental mathematics. Noncalculus-based mathematics courses for technical or occupational programs and mathematics courses for elementary teachers and those for liberal arts majors are also considered part of introductory college mathematics. (p. ix)

According to Cohen (1995), these courses serve students "who intend to study calculus but enter college unprepared to do so" (p. ix). Some programs offer courses as remediation. Introductory mathematics courses are also "gateways" to natural and social sciences careers. Chen and Simone (2016) reports that a striking number of freshmen (i.e., 68% at the two-year level, and 40% at the four-year level) enroll in developmental courses; however, "fewer than half of students enrolled in developmental courses at two-year institutions completed these courses and only 59 percent of four-years students completed them" (Rutschow, 2019, p. 1). Thus, lower-division courses are fraught with low-completion rates, high failure rates, and gatekeeping issues, particularly for traditionally underrepresented students in mathematics (Martin et al., 2010).

Upper-Division Courses

Traditionally, the upper-division undergraduate mathematics curriculum centers on abstract algebra, analysis, complex variables, and linear algebra. Over the years, these courses, considered difficult for students, have received growing attention since the calculus reform

movement began: linear algebra (e.g., Bagley & Rabin, 2016; Selinski et al., 2014), abstract algebra (e.g., Karaali & Yih, 2020; Larsen et al., 2013) and proofs (e.g., Mejia-Ramos et al., 2012; Selden & Selden, 2003). They prepare students for advanced study in mathematics, say, in graduate school. According to Quarfoot and Rabin (2019), "at the college level, the ability to understand and construct proofs is essential for students to transition from computationally-oriented calculus sequences to more theoretically oriented upper-division mathematics courses" (p. 1005). As a result, many mathematics departments have included standalone courses on proof in the first two years of college. In addition, the Committee on Undergraduate Programs in Mathematics ([CUPM], 2015) also recommends that the undergraduate curriculum provide opportunities for students to engage in mathematical research.

Despite the growing body of work on undergraduate mathematics curricula (i.e., what should be taught), sustainable change depends heavily on how instructors facilitate learning (i.e., how undergraduate mathematics should be taught). "Many people are coming to the point of view that what needs to be changed is not the content, but the pedagogy—not what we teach, but how we teach it," Dubinsky (1994) writes. "This is true, not only for calculus," he adds, "but throughout undergraduate mathematics" (p. 1).

How Undergraduate Mathematics is Taught

Nearly 30 years ago, Dubinsky (1994) pointed out that change in undergraduate mathematics education requires not only change in the curricula but a change in pedagogy. Since then, the scholarship on undergraduate mathematics education grew in breadth to cover pedagogical issues. Professional societies and scholars wrote guidelines for revising pedagogical practice (e.g., MAA's *Guide to Evidence-Based Instructional Practices in Undergraduate Mathematics*; *A Source Book for College Mathematics Teaching*). While evidence shows that

some instructors are revamping their teaching practices, traditional lecture remains the most common method of teaching (Reinholz et al., 2020; Viirman, 2021). Related to the question about how undergraduate mathematics should be taught is how instructors assess that students are learning what they should. Timed closed-book exams are also a longstanding traditional practice, commonly used to assess students' learning in undergraduate mathematics education (Iannone & Simpson, 2012). People unhappy with these traditional methods (i.e., traditional lectures) argue that they are ineffective (Freeman et al., 2014). With evidence that non-traditional methods do a better job at fostering and measuring learning, programs are called upon to revise how they teach and assess.

Dubinsky (1994) emphasizes that revising pedagogy must be done "in conjunction with research into what it means for a student to learn a mathematical concept" (p. 1). Accordingly, Dubinsky and McDonald (2001) developed the Actions, Processes, Objectives, and Schemas (APOS) Theory. Rooted in Piaget's constructivism, the APOS perspective became a theoretical underpinning for studies in undergraduate mathematics education in the late 1980s. According to Inglis (2015), "APOS theorists were among the first to pay serious attention to how students come to understand mathematical concepts, and to think carefully about how this should inform pedagogic interventions" (p. 414). Those unfamiliar with this construct may see Inglis (2015, pp. 413-414) for an illustrative example. The overarching tenet is that students can construct their understanding. Others have built on this work (e.g., Arnon et al., 2013), and this theory continues to foreground many studies on undergraduate mathematics pedagogy (e.g., Salgado & Trigueros, 2015). The APOS theory is important to mention here because it represents the perspectives of many scholars who study undergraduate mathematics education. This theory brought with it the view that "students were no longer black boxes connecting stimuli to responses—they had minds

whose cognitive processes could be explored" (Selden & Selden, 1993, p. 433). Many of the student-centered, research-based instructional methods in undergraduate mathematics education were conceptualized using APOS as a theoretical lens—among those are active learning methods and alternative methods of assessments.

Active-Learning Methods

Bressoud et al. (2015) point out that students may not learn during traditional lectures because they "have difficulty identifying the most important aspects of what they are seeing and hearing. Either they try to record everything, creating notes that are of little use, or they focus on what they imagine to be important, the template solutions" (p. 4). Advocates for active-learning methods promise that when done well, "in a setting where they can be directed and encouraged by the instructor" (Bressoud et al., 2015, p. 20), active-learning methods allow students to participate in the learning process. Further, the Conference Board of The Mathematical Sciences ([CBMS]; 2016) calls on the mathematics community to ensure active learning opportunities for students.

There is no one agreed-upon definition of active learning. The CBMS (2016) offers one definition; however, Braun et al. (2017) find that definition to be broad. A broad definition, they argue, "increases the risk of faculty, administrators, and other stakeholders 'talking past' one another, as much is left to the imagination regarding what actually happens with such methods" (p. 124). So, there is no established definition, but there are different interpretations of active learning. Prince (2004) offers the following interpretation:

> Active learning is generally defined as any instructional method that engages students in the learning process. In short, active learning requires students to do meaningful learning activities and think about what they are doing. While this definition could include traditional activities such as homework, in practice, active learning refers to activities that are introduced into the classroom. The core elements of active learning are student activity and engagement in the learning process. Active learning is often contrasted to the

> traditional lecture, where students passively receive information from the instructor. (p. 223)

Active learning could be as simple as incorporating short activities scattered throughout the lecture, where students work with pairs (Prince, 2004). On the other hand, active learning could be sophisticated as engaging in inquiry-based learning (IBL). According to Kogan and Laursen (2014), IBL:

> Engage[s] students in exploring mathematical problems, proposing, and testing conjectures, developing proofs or solutions, and explaining their ideas. As students learn new concepts through argumentation, they also come to see mathematics as a creative human endeavor to which they can contribute...Consistent with current socio-constructivist views of learning, IBL methods emphasize individual knowledge construction supported by peer social interactions. (p. 184)

Regardless of the definition or interpretation, at its core, active learning is student-centered and rooted in constructivist theory. Professional societies maintain that such student-centered strategies "address the learning needs of increasingly diverse student cohorts, strengthen interest and persistence in mathematics and science, and address systemic inequities" (Reinholz et al., 2020, p.2).

The evidence that active learning strategies are successful is compelling. For example, Freeman et al. (2014) conducted a meta-analysis of 225 studies that compared traditional lecture versus active learning using students' outcomes in science, technology, engineering, and mathematics. They found that "on average, students in traditional lecture courses are 1.5 times more likely to fail than students in courses with active learning" (Freeman et al., 2014, p. 8411). Theobald et al.'s (2020) meta-analysis shows that traditionally underrepresented students in mathematics classrooms perform better in active learning environments. Yet, despite what we know about the effectiveness of active-based instruction, lecturing continues to be the dominant pedagogical approach (Bennett, 2020). Despite a growing body of advocates for active-learning pedagogy, "skeptical faculty regard active learning as another in a long line of educational fads"

(Prince, 2004, p. 233). Moreover, Bennett (2021) posits that instructors resist active learning pedagogies for several reasons, including a lack of institutional support. Instructors are not the only skeptics—students are, too. Deslauriers et al. (2019) found that students, who are "poor judges" of their learning, perceive that they learn better in traditional lectures. They explain that "the cognitive effort involved in [active learning] may make students frustrated and painfully aware of their lack of understanding, in contrast with fluent lectures that may serve to confirm students' inaccurately inflated perceptions of their own abilities" (Deslauriers et al., 2019, p. 19255). Braun et al. (2017) agree that students may be biased against active learning, so they resist for many reasons; however, they advise that even the most resistant students eventually appreciate active learning when they understand its value.

Traditional versus Alternative Forms of Assessments

Advocates for alternative methods (e.g., open-book tests) argue that such traditional methods poorly assess deep learning. Instead, they demonstrate what students can memorize (Feller, 1994). Contrary to "Instructors' report that their exams require students to demonstrate and apply understandings," Tallman et al.'s (2016) analysis of 150 Calculus I exams across the US revealed that "a very high percentage of exam items focus on skills and methods for carrying out computations, while a very low percentage of items prompt students to explain their thinking" (p. 131). Moreover, those unhappy with closed-book tests argue that they are unrealistic. Smith (2001) shares an excerpt from his syllabus:

> The 'real world' is … a very large collection of 'open books.' Success does not require memorizing the contents of those 'books' but rather understanding how to use available resources in an intelligent way. Closed-book tests do little to measure such understanding, so all activities in this course–including tests–will be open-book. (p. 169)

Besides emphasizing memorization, closed-book tests are high stakes and cause students to feel anxious (Feller, 1994). On the other hand, evidence suggests that students experience lower

anxiety levels when taking open-book exams (e.g., Gharib et al., 2012). Taken together, these results motivate calls for instructors to adopt innovative modes of assessment. Thus, volumes such as *Assessment Practices in Undergraduate Mathematics* provide dozens of alternatives for measuring and informing learning (e.g., project-based learning, oral exams).

Still, in undergraduate mathematics education, the timed, closed-book test is the common assessment method (Iannone, 2020; Iannone & Simpson, 2012). Furthermore, contrary to the broader perspective, some findings suggest that students prefer closed-book tests. For example, students in Iannone and Simpson's (2015) study found closed-book exams fairer and better evaluators of their competencies. To the best of my knowledge, the reasons instructors preserve traditional assessment modes are not widely explored in the literature on undergraduate mathematics education. But Gold et al. (1999) suggest that some faculty resist alternative forms of assessment because they "fear that it will be used to push us to lower our expectations of our students, to lower our 'standards'" (p. 9). Houston (2001) argues that lack of knowledge about innovative assessment methods, the purpose of assessments, and tendencies to teach how they were taught are why instructors maintain traditional ways of assessing learning.

Changing How Undergraduate Mathematics Is Taught

With all these innovations to improve the teaching of undergraduate mathematics and research to support them, instituting widespread change in undergraduate mathematics is difficult (Logue, 2016); however, "We know teaching practice has changed and continues to change because we have read about it...we have seen it, and we have made changes ourselves" (Tocci et al., 2019, p. vii). Fortune and Keene (2021) affirm that scholars are working to confront and support changing traditional practices and cultures that remain persistent. In addition, they found that faculty collaborations help support instructional change.

While some argue that lecturing at the chalkboard is a thing of the past, some contend that this relic is an essential feature of the mathematics classroom. Selden and Selden (1993) acknowledged the changes in what they called "a largely unsuspecting mathematics community" (p. 432) and prophesied that lectures would play a diminished role in instruction. But nearly two decades later, Artemeva and Fox (2011) found that the majority of 50 mathematics instructors in seven countries "accounted for their use of chalk talk in their lectures as a means of providing their students with an experience of the processes of mathematics" (p. 356); however, Thurston (2004) criticizes lectures citing that they are "often dysfunctional" (p. 165; as mentioned in Lew et al., 2016). In *A Common Vision for Undergraduate Mathematical Sciences Programs In 2025*, the MAA (2015) urges instructors to adopt pedagogical strategies that are more engaging for students, even in a traditional lecture setting.

But lectures and other traditional instructional activities may be sticking around, not only because individual faculty are holding onto them. For example, Dancy and Henderson (2010) surveyed over 700 physics faculty in the United States about their instructional practices. They found that faculty who held progressive views about teaching continue to use traditional methods because they "teach in environments that do not support innovation—the chairs are bolted down, large numbers of students have expectations for traditional instruction, or their colleagues do not use innovative instructional strategies" (as cited in National Research Council [NRC], 2011, p. 61). Johnson et al. (2018) also suggest that lectures persist largely due to instructors' personal beliefs, but also in part to institutional cultures. Whatever the reason for persistent practices, research is needed to understand how instructors position themselves to change long-standing pedagogies that research shows to be ineffective in undergraduate mathematics education.

Who Has Access

Research in undergraduate mathematics education widened to include not only what is taught and how it is taught, but social and political issues that affect who has access and how they experience mathematics (Lerman, 2000). Studies on teaching and learning undergraduate mathematics that include perspectives on gender, race, culture, identity, class, economic status, and the intersectionality of those constructs are on the rise. Adiredja and Andrews-Larson (2017) put forth a well-informed synthesis of the literature on the "sociopolitical turn," a phrase coined by Gutiérrez (2013), in undergraduate mathematics education. They argue that postsecondary mathematics education needs research "that explicitly focuses on efforts to understand and mitigate systemic differences in ways that people experience and are afforded educational opportunities, particularly differences that privilege one group over another" (p. 241). Without partaking in the "gap-gazing fetish" (Gutierrez, 2008), the preceding quotation from Adiredja and Andrew-Larson (2017) leads me to think of a quotation from Ladson-Billings (1997). She asks:

> But some people do well in mathematics in our society. Why? Certainly, individual differences exist that cannot be easily generalized to explain mathematical abilities. However, statistically, we can see whole-group patterns that may suggest some tendencies. White middle-class male students typically do well in mathematics, as do some groups of students of Asian descent. Is there anything about the culture of mathematics that is compatible with White middle-class male students' culture and experiences? Is there anything about White middle-class male students' culture that makes it compatible with mathematics as it is taught in our schools? (p. 699)

Scholars dedicated to this work documented the inequities among racial and other groups in the mathematics classroom (Borum & Walker, 2012; Collins et al., 2020). They also report how traditionally underrepresented students navigate the undergraduate mathematics pipeline (Jett, 2013; Joseph et al., 2017; McGee & Martin, 2011). Martin (2009) provides a critical analysis of how race has been treated in mathematics education research, policy, and practice—a paper that

over 400 works have cited. In 2003, the *Journal for Research in Mathematics Education* published a "Special Equity Issue" with articles that address identity and power. The search string, "race AND equity AND undergraduate mathematics education," in Google Scholar returned over 56,000 results—this illustrates the breadth of the sociopolitical perspective in the field of undergraduate mathematics education. With this "sociopolitical turn," social theories became pillars for studying undergraduate mathematics education. Adiredja and Andrews-Larson (2017) define the "the sociopolitical turn" as a "growing body of work in mathematics education…[which] highlight[s] the interrelatedness of knowledge, power, identity, and ways that these are situated within and influenced by broader social discourses" (p. 446). While cognitive domains of learning remain important for research, critical race theory and feminist theory, for example, became integral parts of research on undergraduate mathematics education.

With eyes on the social and political, researchers in mathematics education brought the human element to the fore. Goffney et al. (2018) emphasize that "unfortunately, for many students, mathematics classrooms are experienced almost exclusively as windows. We need research-based illustrations of teachers' and researchers' initiatives that promote forms of what I refer to as rehumanizing mathematics" (p. 1; original in italics). In "Rehumanizing mathematics for Black, Indigenous, and Latinx Students," they describe what counts as rehumanizing mathematics; I cannot discuss them here because of space limitations. But consider these questions put forth to the mathematics community by Su (2020a) in *Mathematics for Human Flourishing*:

> If you teach, how do you affirm your students' dignity as creative human beings in the way that they do mathematics? What inequities do you notice in mathematical spaces? Who is harmed by those inequities? Think deeper than the obvious answers. What actions can you take to address feelings of not belonging in math communities? How can you honor each person you meet as a dignified mathematical thinker? Who are the forgotten

> among you, mathematically speaking? Whom will you love, whom will you read differently? (pp. 235-236)

These questions resonate with ongoing calls to make mathematics accessible to all students; to see mathematics as a human activity that all students can do and learn.

Schoenfeld (2016) offers an eloquent chronology of mathematics education research. He alludes to the challenges that remain, provides readers an opportunity to look back, see how far mathematics education has come, and get a glimpse into the future of mathematics education at all levels. Reinholz et al. (2020) point out that "professional societies have been actively encouraging mathematics departments to increase their use of student-centered instructional strategies to address the learning needs of increasingly diverse student cohorts, strengthen interest and persistence in mathematics and science, and address systemic inequities" (p. 2). As agents of change, instructors and their voices should be positioned as subjects of inquiry in studies seeking to confront these norms.

Perspectives on Technology Use

As LaRose and Talbert (2016) put it, "we live in a world in where technology is ubiquitous and omnipresent, where computers masquerade as phones, and everyone is always online" (p. 505). Yet, digital technologies and research around how instructors use them in their undergraduate mathematics classrooms have received little empirical attention. In 2021, only two (i.e., Lockwood & Mørken, 2021; Sümmermann et al., 2021) of the 25 articles featured in the *International Journal of Research on Undergraduate Mathematics Education* focus on technology use. In 2020, only one (i.e., Lockwood & De Chenne, 2020) of the 21 focused on technology. The lack of focus could be because teaching with digital technology in tertiary classrooms "tends to be far more fragmented and small scale in many cases" (Clark-Wilson et al., 2020, p. 1224). But teaching remotely during the coronavirus pandemic relied heavily on

technology. Even those who resisted for many years were forced to use digital technology to keep teaching from a distance. Thus, this discussion does not paint technology as bad or "as panaceas to cure educational ills" (Cuban, 1986, p. 6) but provides context for this research. I organize the following discussion into two themes relevant to this study: (1) Technologies Used for Undergraduate Mathematics Instruction, (2) Online Learning and Instruction in Mathematics Education.

Technologies Used for Undergraduate Mathematics Instruction

In this paper, technologies refer to both digital technology and educational technologies—digital technologies include technologies that were not necessarily designed for educational purposes (e.g., video streaming). Clark-Wilson et al. (2020) synthesize the research on teaching mathematics with technology. Though their focus was on secondary schools, they found that teachers generally use technologies in four ways: (1) to support their work (e.g., grading), (2) to do and represent mathematics, (3) "as a support for connecting, organising in communities, communicating, and sharing materials," and (4) "to support students' more independent work" (pp. 1225-1226). The literature in undergraduate mathematics education shows that postsecondary faculty use technology in similar ways, though at a smaller scale. Broadly speaking, people either view technology as a way to make things better or, conversely, as making things worse. To capture this dualism, I discuss not only the affordances of incorporating technologies in the mathematics classroom but the drawbacks, too. Additionally, instructors are "somewhat neglected players in research considering the relations between digital technologies and mathematics education" (Hoyles & Lagrange, 2009, p. 8). Thus, I also attend to how, from their perspective, instructors use technologies to support their work.

Teaching with Technologies in Undergraduate Mathematics Classrooms

Pittard (2013) affirms that a host of emerging technologies "provide the opportunity to deliver truly excellent learning and teaching as never before" (p. 116). Some examples of promising technological advances for teaching and learning include teachers' use of virtual worlds and simulations, game-based learning, personal and mobile devices, display technologies and interfaces, and online and blended learning, to name a few. In addition, research shows that technology use in mathematics classrooms can positively impact student achievement, motivation, and attitude (Higgins et al., 2019). Thus, the enthusiasm to incorporate technologies in undergraduate mathematics classrooms is growing. But Bressoud et al. (2015) found that in over 300 Calculus I programs in colleges across the United States, technologies have no effect or negative effect on students' attitudes about mathematics. They write, "it is not certain if the use of technology makes for a more successful learning and teaching experience" (p. 38). Moreover, Clark-Wilson et al. (2020) point out that previous research at the tertiary level hardly addresses how instructors use technologies for teaching.

Nonetheless, a search for relevant literature showed that instructors employ technology for teaching in undergraduate mathematics classrooms in substantive ways. For example, Saxe and Brady (2015) point out that:

> flipped and blended classes, classroom management systems (e.g., Blackboard, Moodle), web-based interactive systems (e.g., WeBWorK), adaptive learning platforms, massive open online courses (MOOCs), and other distance learning platforms are examples of ways in which faculty are using technology to improve student learning. (p. 29)

The following year, *PRIMUS* published special Issue *26*(6) focused on how technologies (e.g., online homework and classroom response systems) influence teaching and learning undergraduate mathematics. For example, Lunsford and Pendergrass (2016) describe their experiences using online homework systems in their face-to-face statistics and calculus courses.

They define an online homework system as "any computer system that delivers homework assignments to students and provides them immediate graded feedback on their responses" (p. 533; e.g., WebAssign). According to the authors, online homework systems improve student learning and engagement. From teachers' perspective, such systems also alleviate the tedium of grading and providing feedback on written homework assignments. But Clark-Wilson et al. (2020) refer to online homework systems as generally "commercial and industry-driven…with limited (or no) involvement of the academic research community of mathematics educators" (p. 1226). *PRIMUS* Issue 26(6) also features Latulippe's (2016) reflections on how incorporating clicker technology created a safe classroom environment where students felt safe to participate.

Few studies have been conducted to understand how instructors employ technology in undergraduate mathematics classrooms. Yet, examples in the literature show how instructors teach with technologies such as computer algebra systems, graphing calculators, spreadsheets, online applets, course management systems, and Massive Open Online Courses (Pollatsek et al., 2015). Pollatsek et al. (2015) include examples of how instructors use these tools for exploration, computation, communication, assessment, and motivation. For many years, instructors and students practiced mathematics as a two-dimensional inscription activity in undergraduate mathematics classrooms. But adaptive computer algebra systems (e.g., Mathematica) and dynamic geometry environments (e.g., GeoGebra) make it possible for them to engage with mathematics in three-dimensional spaces. With the increased attention to active learning pedagogies, technologies to support inquiry-based instruction are on the rise (e.g., flipped classrooms). Generally, in flipped classrooms, instructors ask students to engage with new ideas before class by reading, watching videos, and using technology (Pollatsek et al., 2015). Then class time is used for deeper exploration and interaction with the content (Setrin et al., 2021).

According to Kerrigan and Prendergast (2021), “the flipped approach has been very successful in undergraduate STEM courses with multiple studies reporting increases in student achievement” (p. 2). They found that implementing the flipped model in a precalculus course with about 100 first-year engineering students dropped failure rates and improved students’ attitudes towards learning. Still, the flipped model comes with limitations and challenges. For example, this model is time consuming for instructors. Additionally, it relies heavily on student discipline since the onus is on students to prepare for class (Akçayır & Akçayır, 2018).

Professional societies strongly encourage instructors to use various tools for enhancing instruction. Consider the MAA’s (2004) claim that:

> Technology can promote students’ exploration of and experimentation with mathematical ideas. For example, students can be encouraged to ask “what if?” questions, to posit conjectures, to verify or refute them, and to use technology to investigate, revise, and refine their predictions. (p. 24)

Despite such affordances, the chalkboard lecture remains a mainstay in the undergraduate mathematics classroom. Chalk talk, as Artemeva and Fox (2011) call it, is often regarded by instructors as “t*he* way to teach mathematics” (p. 366; emphasis in original). Thus, many instructors employ simple forms of technology such as chalk and the blackboard. Additionally, Hoyles and Lagrange (2009) point out that “most digital technologies do not make explicit how they work or how they can be used in mathematics education” (p. 3). But instructors have written about incorporating digital technologies not created for educational purposes in their undergraduate mathematics classrooms. Recently, Öçal et al. (2021) described how students used a smartphone application, which uses a Global Positioning System (GPS), to measure the height of a flagpole. However, they, too, found that using digital tools such as smartphones, not originally designed as an educational tool, “can be perceived by educators as too complex.

Therefore, the use of such technology in mathematics teaching can be limited" (p. 1015). For this reason and others, teaching with technology has some downsides.

Although technology enthusiasts have argued that incorporating technology into lectures can improve teaching and learning, skeptics argue that technology presents "drawbacks" (Selden & Selden, 1993). One supporting argument for this claim is that technology can lead students to develop misconceptions. For example, Zbiek et al. (2006) point out that graphing calculators have "limitations in representing continuous phenomena with discrete structures and finite precision numerical computations" (p. 1175). While Artemeva and Fox (2011) assert that instructors' decision to practice chalk talk lectures should not prevent them from using advanced technologies, participants in their study argue that "technology demands semiotic reduction and distortion…[which] undermine the complex and subtle narrative of 'doing mathematics' (pp. 357-358). Moreover, incorporating certain technologies (e.g., presentation software) reduces the spontaneity and pacing control chalk talk lectures offer. Because of these drawbacks, instructors often resist integrating technologies into their pedagogical practice. Research is needed to explore how instructors of undergraduate mathematics teach with those technologies for the affordances they provide and how they minimize drawbacks.

Emerging Needs Insofar as Technology in Undergraduate Mathematics Education

A wide variety of technological tools are available for teaching and learning mathematics. Using technologies in the classroom to enhance learning outcomes is well studied and documented in the literature. Moreover, Clark-Wilson et al. (2020) draw our attention to the issue that:

> Globally, it is the large EdTech companies that are generating big data sets that are ripe for the development of learning analytics, dashboards, and artificially intelligent algorithms that enhance or personalize learners' and teachers' experiences for a range of purposes. However, traditional research designs to evaluate the educational effectiveness

> for such systems become problematic when learners' and experiences within such systems become "unique", as are the experiences of their teachers and lecturers. (p. 1238)

So, what are some emerging needs for technologies and research around their use? According to Beatty and Geiger (2009), there is a need for "digital technologies [to] enhance the learning and teaching of mathematics towards social aspects of acquiring knowledge" (p. 252). Yet, today the research on teaching and learning mathematics with technologies continues to have a primarily constructivist focus. But with the call for active and engaging pedagogies, technologies that foster collaboration amongst instructors and learners, for example, are needed more than ever. More research is required to bolster and support this movement to shift from paper-and-pencil tests. Sangwin et al. (2009) investigate the use of computer-aided assessments to support teachers in grading, scoring, and providing automatic feedback on students' tasks. They found that while computer-aided assessments save teachers time, they are geared towards grading traditional tests. Madison (1999) underlines the MAA call for non-traditional assessments. Thus, instructors need sophisticated tools to evaluate innovative modes of assessments such as inquiry-based and project-based assessments.

Besides emerging needs for technological tools that support learners and instructors, there are emerging needs for research. For example, research to explore how technology use in classrooms favors some learners and instructors over others is needed. For instance, scholars found that the use of technology may close gender gaps in achievement, favoring males (Forgasz et al., 2010; Hoyes, 1998). Leonard et al. (2019) frame technology use in classrooms as "a racialized experience because of the digital divide, which negatively impacts the poor and communities of color" (p. 102). There is a need for research that looks at technology from sociopolitical and sociocultural lenses (e.g., critical race theory, culturally responsive pedagogy). Few researchers have looked into what technologies instructors use for inclusive education, how,

and whether such technologies address existing inequities. For example, many instructors have begun to adopt open educational resources ([OER]; e.g., textbooks), generally available for free on the Internet, in their classrooms. Ryan and Nawalaniec (2021) replaced an expensive linear algebra textbook with online modules and an open-access textbook in one course. They found that learners in an OER section of a linear algebra course performed as well as or better than students in two traditional sections.

Instructors and Technology

Often, instructors are viewed as resisting technology in their mathematics classrooms. For example, Cuban (1986) said that "the teacher has been singled out as inflexibly resistant to 'modern' technology, stubbornly engaging in a closed-door policy toward using new mechanical and automated instructional aids" (p. 2). According to Tauson and Stannard (2018), instructors resist utilizing technologies available to them for many reasons. In particular, they assert, mathematicians are cautiously uncomfortable with using technology in their classrooms. Gordon (2000) explains that "some may fear that the technology will replace mathematics as the primary focus of the course" (p. 82), and others may not have given thought to the benefits of using technology. But Habib and Deshotel (2018) interviewed engineering faculty to understand the challenges of adopting innovative technology. Faculty in their study acknowledged the benefits for student learning; however, they reported challenges such as a steep learning curve (e.g., for writing codes) and time constraints as factors that hindered their adoption of the technology. Another explanation for instructors' resistance to using technologies in their classrooms is that they do not receive enough training and institutional support.

Dusick (1998) found that factors other than training and support affected instructors' willingness to use technology. For example, cognitive factors such as anxiety about technology

controlled their decisions. Another is the educator's espoused beliefs about the place of technology in teaching and learning. As Clark-Wilson et al. (2020) point out, some instructors may be less likely to use technologies in their classrooms because they believe "by-hand fluency" is necessary. Moreover, objections to using technology "are often related to teachers' opinions of the value of technology integration, and an aversion to changes in practice, not dissimilar from any adult working in an environment that is changing" (Tauson & Stannard, 2018, p. 16). Besides, innovations in technology are growing faster than instructors and researchers can keep pace (Schoenfeld, 2016). Reasons that instructors use or resist technologies are documented in the literature.; however, there is a relative lack of scholarship on innovations geared towards teacher usability. There is a need to explore technological innovations that offer practical benefits to teachers. For example, teachers spend a great deal of their non-teaching time grading students' work and providing guided feedback. Thus, technologies for reducing tedium and time spent grading and research around their use would be beneficial.

Pittard (2013) affirms that "it is more important to remember that the first aim in adopting and developing any technology is that it is there for learning" (p. 111). Touting the benefits of learning gains assumes that teachers make decisions about their instructional activities of choice solely based on student outcomes. But Cuban (1986) considers the ideal technological tool for teaching as one that increases productivity and allows students to acquire more information with less effort from the teacher. Moreover, Sinclair and Yerushalmy (2016) assert that research on technology should attend to both learning and tool use theories. Neuropsychologists Osiurak et al. (2010) maintain that humans use tools for what they afford. Artigue (2009) said that research on the affordances of technology for mathematics learning has shifted from the cognitive benefits for students to inquiry about teachers' implementation of

technology. Today the focus continues to be void of teachers' perspectives (Clark-Wilson et al., 2020). Technological advances that attend to student learning outcomes are well studied and documented. Developing technology for usability and efficiency from the instructors' perspectives deserves attention. I propose that disseminating research implications for technology needs to include instructors' first-hand nuanced accounts of their experience using technology.

Online Learning and Instruction in Mathematics Education

In the literature on distance education, traditional in-person instruction is referred to as face-to-face instruction (FTF). Hybrid refers to a combination of in-person and online components, whereas online implies no in-person components. More recently, the hyflex mode allows students to decide on an ongoing basis whether they would like to meet in person, attend the online session (e.g., via Zoom), or participate asynchronously. For this purpose of the following discussion, online refers to virtual modalities of the delivery methods. Online could be synchronous or asynchronous. Synchronous classes have regularly scheduled meetings. Asynchronous classes do not have scheduled meetings; however, instructors may meet with students during virtual office hours to provide support.

Ellis (2000) wrote that "the movement to distance learning has been limited because no college, Internet company, or publisher has learned how to make it profitable" (p. 64). Fourteen years later, according to Shukla et al. (2014), the demand for distance education grew to accommodate the increased number of non-traditional students attending college. Recently, Kanwal (2020) wrote that "online education has become a common feature of university level courses" (p. 1). Though few, there are papers written about online learning and instruction in undergraduate mathematics education (e.g., Engelbrecht & Harding, 2005a, 2005b). Here I

describe one such study by Kanwal (2020), who explored how calculus students interacted with an online calculus class. The instructor recorded himself giving a lecture and writing explanations, which students watched asynchronously. He then posted the accompanying notes on the class' learning management system. The instructor assigned homework and administered tests on Pearson's MyMathLab. For a group project, the instructor asked students to use a computer algebra system, Maxima, to write and program a set of questions on integration. The instructor held in-person and online office hours. Kanwal (2020) found that the students in this online environment did not engage with mathematical tasks as expected. This finding resonates with Ellis's (2000) explanation that the effectiveness of online programs is limited by students' lack of motivation, "time management skills, and persistence to work through such materials alone" (p. 65). Moreover, Shukla et al. (2014) found that students in FTF classes had a higher success rate than students in other delivery methods.

Besides the limitations of students' interaction with the online environment, online instruction could be limiting for instructors for various reasons. For example, Gordon (2000) points out that mathematics, compared to other disciplines, is especially challenging to teach online because it limits the ease and accuracy of symbolic representation. This may be because of the belief that "mathematics is inherently representational—[that] no significant mathematical activity is possible without symbols (broadly understood to include such things are graphs)" (Selden & Selden, 1993, p. 438). Other than issues of representation, Saxe and Brady (2015) report that traditional standards and academic honesty are challenging to ensure online.

During emergencies, where access to computers and the Internet are available, teaching and learning online is the most common modality for teaching and learning to continue. Hodges

et al. (2020) argue that there is a difference between traditional online instruction and emergency remote teaching (ERT), which I refer to as pandemic pedagogy in this study. They explain:

> Emergency remote teaching (ERT) is a temporary shift of instructional delivery to an alternate delivery mode due to crisis circumstances. It involves the use of fully remote teaching solutions for instruction or education that would otherwise be delivered face-to-face or as blended or hybrid courses, and that will return to that format once the crisis or emergency has abated. The primary objective in these circumstances is not to re-create a robust educational ecosystem but rather to provide temporary access to instruction and instructional supports in a manner that is quick to set up and is reliably available during an emergency or crisis. (p. 7)

This study assumes that teaching during the coronavirus pandemic calls for emergency remote teaching and expects that the teaching is not online instruction at its best. A state of emergency called for instructors without training and experience in online teaching to adapt so that students could keep learning. Although online learning has been put in place to accommodate mostly non-traditional students, "the move to distance learning has been an opportunity to expand flexible learning modalities, setting the stage for a sustained shift towards more online learning in this sub-sector in the future" (United Nations, 2020, p. 13).

Education in Emergencies

In addition to the literature on undergraduate mathematics education and technology use, scholarship on education in emergencies is relevant for understanding this work. Next to necessities such as food, shelter, and health care, education is considered the "fourth pillar" of humanitarian effort (Kagawa, 2005; Milton, 2017). While the word emergency implies a temporary relief situation, the impact of the disruption caused can be long-lasting. The field of education in emergencies emerged in the mid-1990s from the realization that education often became inaccessible in places affected by armed conflict and natural disasters (Burde et al., 2017).

In the literature on education in emergency, emergencies tend to be categorized as either natural (e.g., floods, earthquake) or human-made (e.g., war, genocide). Much of the scholarship in this area focuses on emergencies in armed conflict. As Burde et al. (2017) point out, "to date, both scholars and practitioners have focused more on conflict settings, and as a result, there are insufficient data from natural disaster settings" (p. 621). Education during global health crises is far less represented in the literature. This gap may be because armed conflict occurs more frequently than global health crises. But other communicable diseases required similar responses to the coronavirus pandemic. For example, Santos and Novelli (2017) report, "As Ebola was spread through human contact, schools were identified as hotspots, 'locations of serious risk of transmission'...and therefore closed as a public health security measure" (p. 10). Three years later, these words are identical to what you might see in a newspaper article about schools during the coronavirus pandemic; except, health experts dubbed this pandemic the worst health crisis in the last 100 years. Further research is needed to understand how school systems continue to educate when communicable diseases disrupt the ability to teach and learn in a physical space on such a global scale.

During emergencies, humanitarian efforts are often geared towards primary education in times of crisis. The focus ensures that children learn reading, writing, and numeracy skills (Pigozzi, 1999). To this end, research in this relatively new field tends to explore issues of primary education. Unfortunately, there is a dearth of research that focuses on postsecondary education in areas affected by conflict and natural disasters, so much so that Milton (2017) refers to higher education as the "neglected sector" in emergency education conversations.

The remainder of this section discusses four cross-cutting themes in the literature in education in emergencies: 1) Emergency Pedagogy, 2) Access and Equity During Emergencies,

3) Distance Education and Educational Technology During Emergencies, and 4) Education in Emergencies: Opportunities for "Building Back Better."

Emergency Pedagogy

Much of the research on education in emergencies stem from an urgent need to understand what works in those situations. Thus, several efforts have been devoted to documenting the best teaching practices. For example, at a refugee school in Johannesburg, Pausigere's (2011) ethnographic study revealed that a "Direct Instructional Model" was the best approach for teaching mathematics. He explains:

> This model is teacher directed but in a positive manner that ensures student engagement and is mostly used to teach in difficult circumstances with limited space and resources...The teacher under this approach is expected to carefully structure every skill and concept, yet ensuring student engagement through the use of task-orientated approaches. (n.p.)

Adams and Engelmann (1996) found this kind of direct instruction effective. As McAleavy and Gordon (2020) suggest, "Some advocates of student-centred pedagogy might consider that the promotion of teaching presence and direct instruction were manifestations of poor teacher-centred pedagogy and a recipe for student passivity and disengagement" (p. 9). But when Duffin et al. (2020) compared direct instruction with inquiry-based pedagogy, they found no difference in their effect on students' motivation and achievement. Instead, social support and community building had a more significant impact on student learning outcomes (Duffin et al., 2020). Thus, it seems plausible that instructors should combine direct instruction with social and emotional components in their emergency practice. Burns (2020) recommends direct instruction as a suitable instructional strategy for teaching online during the pandemic. She also points out that community-building efforts foster "camaraderie, collegiality, and sense of community will increase learner engagement in and persistence with the course" (para. 11).

Moreover, during emergencies, when schools close, teachers are confronted with assessing students' learning from a distance. According to the United Nations Educational, Scientific, and Cultural Organization ([UNESCO]; 2020), schools around the world responded by "organizing exams with special arrangements, cancellation, postponement/rescheduling, going on-line (on-line assessment), and introducing alternative approaches to exams and validation of learning" (p. 3). But the United Nations (2020) point out that alternative solutions are not necessarily equitable. Moreover, some students (e.g., students with disabilities, students without reliable Internet access) may be disadvantaged.

Access and Equity During Emergencies

One finding of research on education in emergencies is that crises create new problems but exacerbate old ones. For example, "because established, historical gender bias and because social norms and actions break down during emergencies, girls are usually at particular risk" (Pigozzi, 1999, p. 15). Silwal (2016) found that girls dropped out at a higher rate than boys in a conflict-affected area in Nepal. In general, other scholars of education in emergencies report a similar finding. The United Nations (2020) reported that girls and other marginalized groups (e.g., children with disabilities) often do not benefit from remote instruction. In Liberia, for example, during the Ebola outbreak, Santos and Novelly (2017) report that schools were closed, often with no distance learning put in place. Consequently, "the negative outcomes of prolonged closures disproportionately impact displaced children. This situation is especially precarious for girls, most at risk of permanently dropping out" (United Nations, 2020, p. 14).

Distance Education and Educational Technology During Emergencies

Several modalities make it possible to keep teaching during emergencies worldwide: audio and radio, video and television, paper-based learning, mobile phone, and online teaching

and learning (United Nations, 2020). The choice of which modality to use depends on the nature of the emergency and the resources available. For example, in Haiti, where natural disasters often destroy school infrastructure, students received instruction via Nokia smartphones, whose batteries could be powered by the sun (Carlson, 2013).

Tauson and Stannard (2018) reviewed the literature on educational technology (EdTech) to investigate what works in emergencies. They present a robust synthesis of findings, all of which I cannot discuss here. One take-away is that EdTech does not fix all problems during an emergency—it is not a "silver bullet." The success of EdTech relies on communal efforts and teaching practices that are aligned with theories of learning. They contend that "EdTech is a tool that needs to be constructed with the principles of pedagogy in mind, such as active learning, engagement, and content that hooks. EdTech should support cognition and not only present content" (Tauson & Stannard, 2018, p. 61).

Much of the work on the use of EdTech in emergencies focuses on crisis and natural disaster emergencies. The coronavirus pandemic affected many people all at once; it was challenging to ensure that every teacher and every student had all the technology resources available to continue teaching and learning. The pandemic presents a new avenue for research on digital technology for teaching during a health crisis when physical gatherings are unsafe.

Education in Emergencies: Opportunities for "Building Back Better"

I borrow the phrase "building back better" from the United Nations International Children's Emergency Fund (UNICEF) to describe a principal tenet of education in emergencies. One of the guiding principles of the Inter-agency Network for Education in Emergencies (INEE) is that "crises which destabilize education can be approached not only as urgent situations of immediate need but also as opportunities for positive change" (inee.org). This motto resonates

with the United Nation's (2020) call for educators to implement existing solutions for learning problems that existed before the pandemic. Moreover, Pigozzi (1999) advises that interventions in emergencies can serve to rebuild and strengthen education systems. In building back better, the goal is to strive for flexibility, equity, and inclusion (United Nations, 2020).

Educators and professional societies have subscribed to this tenet of "building back better" and have begun to identify how the pandemic may offer opportunities to fix some old problems in undergraduate mathematics education. For example, Cleary and Levy (2020) titled their op-ed in a recent Mathematical Association of American publication, *MAA Focus*, "How Some Pandemic Necessities Could Improve Teaching and Learning." They ask, "How can we use this momentum to continue moving toward more effective teaching?" They call on their colleagues to use this opportunity "to learn and grow with an eye toward creating a more effective, and more equitable, learning environment. It will be hard work, but resources to help are widely available" (p. 7). According to Bressoud (2020), members of the Conference Board of the Mathematical Sciences met to discuss the implications for moving forward. The plan was to address questions such as:

> How do we ensure that those from under-resourced or otherwise challenged communities continue to have access to quality education? How do we continue to work toward the vision of a classroom in which all learners are actively engaged in constructing their understanding of mathematics? How can we (or even should we) maintain our long-established habits for how we do assessment? What are optimal means for authentic assessment, both online and in other environments? What is being, can be, and should be done to support our professionals who must operate in this new world? What are the technologies with which our members need facility? What are the promising technologies that need to be more widely distributed? Where are the greatest technological needs?

Anecdotal evidence begins to answer these questions; this study adds empirical evidence.

In "No Going Back, Only Going Digital," readers of the *Notices of the American Mathematical Society* (Volume 67, Number 8) will see that Wawrzyniak (2020) makes a case that mathematics instructors should maintain the digital presence that they created for teaching

during the pandemic. In addition, she believes that students should have online access to the course material and resources after the pandemic. Other survey reports show how undergraduate mathematics instructors changed their pedagogy to adjust. For example, Dumbaugh and McCallum (2020) describe a professor who reports success with "the seemingly antiquated tactic of oral exams with fresh purpose" (p. 61) in place of paper-and-pencil tests. There are also reports of increased flexibility and humanness in the mathematics community (Clark, 2020). Research to understand this shifting landscape is needed.

Conclusion

This literature review began with a discussion of the theories that are foundational to this research. Dewey's theory of experience, Schön's work on reflective practice, and Jean Clandinin and Michael Connelly's narrative inquiry provide a framework for understanding this study. Then, I described the method for finding relevant literature. This work is situated in three bodies of work: undergraduate mathematics education, technology use in mathematics education, and education in emergencies. My literature review presents a need for studies that put instructors' experiences at the center of inquiry into undergraduate mathematics pedagogy. Moreover, stories of how instructors adjust for teaching during an emergency are missing from the literature on education emergencies—the postsecondary lens is missing. Furthermore, the coronavirus pandemic provides a remarkable opportunity to explore persistent problems in undergraduate mathematics education. This body of work operates from this perspective. In the next chapter, I describe the research methodology.

CHAPTER III: METHODOLOGY

This chapter outlines the following: a brief rationale for the methodological approach, the recruitment process, a description of the participants, procedures for the study, and limitations of the study. This study used a combined qualitative methodological approach. Features of narrative inquiry and constructivist grounded theory were used to address the following research questions:

1. How did instructors of undergraduate mathematics "story" their experiences teaching during the coronavirus pandemic?
2. How did instructors of undergraduate mathematics adjust their practices for teaching during the pandemic?

Methodological Approach

According to Weaver-Hightower (2018), qualitative research allows us to examine "unusual events that disrupt or alter lives" (p. 16). Moreover, Nardi (2016) recommends narrative approaches for research in undergraduate mathematics education because they "bring the richness and vibrancy of storytelling into how data is [sic] collected and interpretations of it shared" (p. 361). Thus, the research questions that frame this study will be best answered through a qualitative methodology to capture the stories of the instructors who taught during an unprecedented time in history. Features of narrative inquiry and grounded theory inform the research design for this study. It is customary for researchers to choose one approach; however, Lal et al. (2012) found that narrative inquiry and grounded theory "can be potential allies in a qualitative study given that they are theoretically commensurable and methodologically complementary" (p. 14). The two methodologies have overlapping features (e.g., they focus on experience from the participants' perspectives), but they differ in their underlying philosophies,

purposes, and procedures. The theoretical underpinnings for narrative inquiry are at the core of this study; however, grounded theorists provided practical strategies for interviewing participants and analyzing the data. This study was designed using insights from narrative methodologists, Clandinin and Connelly (2000) and Riessman (2008). Grounded theory strategies were also helpful for understanding the perspectives of many participants about an unprecedented experience.

Narrative Inquiry

Storytelling is a natural way for humans to share and understand their experiences. One section of the literature review attends to the notion of story. As Clandinin and Connelly (2000) put it, "The answer to the question, Why narrative? Is, because experience" (p. 50). As a methodological approach, it explores the stories lived and told by the participants. Other scholars have used narrative inquiry to investigate and make meaning of mathematics educators' experiences (e.g., Kaasila, 2007; Larnell, 2016; Martin, 2009; Salmon-Nembhard, 2015; Walker, 2014). Riessman (1993) suggests that stories can mobilize progressive and lasting change. Therefore, to understand and represent a fascinating time in the history of undergraduate mathematics education, I rely on a key feature of the narrative approach: collecting and analyzing stories lived and told by instructors who taught undergraduate mathematics during the pandemic. Connelly and Clandinin (1994) note that:

> People live stories, and in the telling of these stories, reaffirm them, modify them, and create new ones. Stories lived and told educate the self and others, including the young and those such as researchers who are new to their communities. (as cited in Clandinin & Connelly, 2000, p. xxvi)

Stories of teaching during the pandemic can inform how we move forward in the unpredictable future.

In a narrative study, the research often focuses not only on what participants say but also on how they say it (Clandinin & Connelly, 2000). Freeman (1994) distinguishes the how (i.e., presentation) from the what (i.e., representation). I believe that there is value in studying how instructors talk about their experiences; however, this is outside of the scope of this project. The focus here is the representation of their stories. Narrative methodologists who are not concerned with the presentation of what participants say use thematic analysis. As such, I was drawn to Riessman's (2008) practical steps for thematic analysis, which focuses on the content of the stories. While individual stories were essential for this study, I present a collective narrative.

Recruitment and Participants

Twenty-eight participants were recruited for this study through criterion-based, purposeful sampling methods. According to Patton (2015):

> The logic and power of qualitative purposeful sampling derive from the emphasis on in-depth understanding of specific cases: information-rich cases. Information-rich cases are those from which one can learn a great deal about issues of central importance to the purpose of the inquiry. (p. 53, italics in original)

To participate in the study, volunteers had to be instructors who taught undergraduate mathematics since the coronavirus pandemic hit in March 2020. In telling their stories, the participants moved back and forth in time to compare experiences in ways that were helpful to this study. For example, they compared how they changed their practices to adapt when the pandemic first hit two semesters later when they had some experience teaching online and more time to prepare.

The flyer in Appendix A was posted on Twitter at the end of December 2020. My followers and their followers liked, retweeted, and shared the post. This call for participants was far-reaching and visible to a broad audience. Tagging professional communities and popular networks (e.g., Math Twitter Blogosphere #MTBoS, @globalmathdept, #iteachmath,

@BlackinMath, @maanow, @amermathsoc) using their Twitter handles and hashtags helped bring attention to my outreach for volunteers. About one month after posting the flyer on Twitter, my call for participants had 15,488 impressions (i.e., the number of people who saw the tweet) and 611 engagements (i.e., the number of people who interacted with the tweet). I also posted the flyer to a Facebook group (e.g., STEM Faculty Blundering Through Remote Teaching During a Pandemic), my LinkedIn profile, MAA Connect, and SIGMAA on RUME online discussion boards.

Over 50% of the participants in this study were recruited from Twitter. Early in the interview phase, I noticed that a large subset of that group had teaching experience under 20 years and had progressive teaching ideologies. For example, many of them talked about using alternative grading strategies (e.g., mastery-based grading). Therefore, to have a sample that reflects variation in perspectives, I invited participants with more than twenty years of teaching experience who had not adopted such progressive instructional methods. The table in Appendix B presents a snapshot of participants' profiles (e.g., teaching position, college level, state, number of years of teaching experience, level of experience with teaching online, and courses they taught in Spring and Fall 2020).

In a nutshell, participants had undergraduate teaching experience from a few months to 58.5 years. They taught a variety of lower-division (e.g., college algebra) to upper-division (e.g., linear algebra) mathematics. Many of them began their teaching journey as tutors or graduate teaching assistants and taught online for the first time during the pandemic. While participants do not represent all undergraduate mathematics instructors, I suspect their experiences teaching during the pandemic were not atypical of any other undergraduate mathematics instructor. All participants were assigned a pseudonym.

Data Collection

The data collected for this study consisted of interview transcripts, pedagogical artifacts, and researcher memos. After participants agreed to participate in the study, they completed the Qualtrics Consent Form in Appendix C and the Pre-Interview Questionnaire in Appendix D. When participants completed those forms, we scheduled interviews at days and times convenient to them between late December 2020 and early February 2021, during the winter break. The timing also allowed participants to reflect on the last year and speak about their plans for the new year. As participants told stories about their experiences teaching during the pandemic, both internal and external factors interacted to place those experiences on a continuum (i.e., in the past, present, and future; Clandinin & Connelly, 2000; Dewey, 1935).

Open-ended and story-generating interview questions allowed participants' stories and thoughts to emerge (Charmaz, 2014). I also drew from Artzt et al.'s (2015) *Becoming a Reflective Mathematics Teacher* to design questions that asked participants to look back and reflect on their pedagogical decisions. Questions also prompted participants to look ahead and share plans for the future. A few of the participants commented that the open-ended style of the interview was not easy for "the mathematician." But as Zinsser (1976) puts it, "somewhere in every drab institution are men and women who have a fierce attachment to what they are doing and are rich repositories of lore, [because all of the participants] had anecdotes that were unique both in substance and in the manner of telling" (p. 101). Semi-structured interviews allowed for flexibility to ask follow-up questions. The interviews lasted between 45 to 96 minutes. The version of the interview guide in Appendix E is a refined iteration of the IRB-approved version. After conducting a few interviews, I realized that some questions served better as probes when needed. Other questions were reworded for clarity based on feedback from participants. The last

question on the interview guide invited participants to explore aspects of their stories that I had not asked about previously. In addition to question design, building a rapport with participants during the interview was important to allow stories to emerge. I introduced myself as an insider who taught undergraduate mathematics for 13 years and taught during the pandemic. I approached the interview from a non-evaluative and non-judgmental stance.

All interviews, except for one, took place on Zoom for safety. One participant was interviewed on the phone. One benefit of Zoom and phone interviews is that participants from various U.S. locations could be interviewed without traveling. Muting my microphone allowed participants to share their stories without interruption from my end. I used Zoom's automatic transcription feature. This way, I did not have to transcribe all the interviews from scratch; however, I verified and edited in tandem with the Zoom playback feature to ensure accuracy. The phone interview was recorded using a hand-held recorder and transcribed manually.

Participants could upload a pedagogical artifact in portable document format on the pre-interview questionnaire. Kim (2016) posits that artifacts provide a way for researchers to reveal more about participants' stories in ways that their words may not. The term pedagogical artifact "refers to a material artifact used as a resource for teaching and learning" (Chazal, 2015, p. 147). Not all participants shared a document; however, for those who did, during the interview, I asked them to discuss its purpose and their experience using it. Participants generally shared instructional activities they created for students' individual or group work or their syllabi. For example, one instructor shared his teaching philosophy, and another shared his student evaluations. Some participants demonstrated how they used interactive applets or websites for instruction during the interview. I wrote notes about those artifacts that the Qualtrics questionnaire was not designed to collect. I downloaded the pre-interview questionnaire data

from Qualtrics, the audio recording, and transcript files from Zoom—these were organized by folders in password-protected, cloud-based storage.

During each interview, I took notes for various reasons (e.g., something a participant said that I wanted to explore in the literature) in sections of a notebook created for each participant. After each interview, I wrote memos about my reactions to participants' stories and emerging themes as a form of early data analysis. A separate notebook kept my methodological musings. Charmaz (2014) suggests that this kind of memo writing:

> Prompts you to analyze your data and codes early in the research process...memos catch your thoughts, capture the comparisons and connections you make, and crystalize questions and directions for you to pursue. Memo-writing creates an interactive space for conversing with yourself about your data, codes, ideas, and hunches. (p. 162)

Together these notebooks contained researcher memos that were useful for advanced data analysis.

Data Analysis

In this study, participant narratives, as told during semi-structured interviews, are the primary unit of analysis. Throughout the data collection phase, I took notes about emerging themes across participants' stories. After the data collection period ended, I read each transcript to familiarize myself with the participants' stories. The goal was to understand participants' stories without preconceived themes. While reading, I highlighted fragments of text that had meaning for the study's purpose and wrote memos of my initial impressions. I asked, "What is this expression an example of?" (Ryan & Bernard, 2003, p. 87) to help me identify codes. As I read each transcript, patterns emerged across the participants' stories. As I read the subsequent transcripts, I kept those patterns in mind—some data supported these developing categories. Others offered different points of view. I revisited the notes I wrote during the interviews for each participant to access the thoughts and impressions I had at that time and wrote further

memos about those, too. I wrote notes about the pedagogical artifacts participants shared that supported my preliminary impressions of the transcript data. Saldaña (2013) defines a code as a chunk of data that "symbolically assigns a summative, salient, essence-capturing, and/or evocative attribute for a portion of language-based or visual data" (p. 3). Some of the words and phrases that participants used to capture their experience in unique ways became codes—these are referred to as in vivo codes. Table 1 shows the initial codebook, based on initial impressions, organized by research questions.

Table 1

Initial Codebook

Research Question 1	Research Question 1	Research Question 2	Research Question 2
(not) feeling supported	Flying blind	alternative forms of assessment	monitoring- looking over their shoulders
attending to issues of access	keeping it simple	assessing what students know	open book open notes take home exams
ballooning class sizes	learning from colleagues	building a community of learners	orchestrating participation
choosing a mode	living at work	building relationships	providing out-of-class support
comparing lower division vs upper division	looking ahead	choosing tasks	representing content
comparing spring 2020 with subsequent semesters	offering flexibility	collaborating	shaping classroom mathematical discourse
considering student lives	pivoting online	connecting with students	standards based grading
coping	receiving training	engaging in mathematical activity	verbal and written communication
COVID Stories	remembering other times of disruption	establishing norms	
dealing with academic (dis)honesty	shepherding	fostering mathematical identities	
devising a plan	showing compassion	grading	
doing more work	surviving	Holy grail	
doing what we did before	teaching students in different time zones	interacting	
extending grace	trying to survive	interpreting what students know	
	using technology		

Charmaz (2014) advises coding "with words that reflect action... [because that] reduces tendencies to code for types of people...assigning types to people casts them with static labels" (pp. 116-117). The stories that participants shared with me are a small part of who they are as

people and represent a short, albeit significant, period in their lives; therefore, following Charmaz's advice, I was careful not to pigeonhole the participants. Weaver-Hightower (2018) advises choosing episodes to represent participant stories. He writes:

> Themes, by their nature, imply commonality and agreement. Take care not to suggest to readers that your themes indicate uniformity of participants' thoughts...Even amidst themes, clearly convey when disagreements, schisms, and tensions divide participants. Outliers can hold just as much importance in qualitative research as those who fit the themes. (p. 109)

To that end, in the finding's sections, you will notice that some of the stories are not representative of all participants' thoughts and experiences; however, they present an alternate view or are unique and unexpected but relevant to the study.

Data files (i.e., transcripts and pedagogical artifacts) were uploaded in NVivo, a computer-assisted qualitative data analysis software (CAQDAS), to manage the large data set. In the initial coding process, I assigned nodes to the chunks of data I identified manually. I remained open to the data, got rid of some codes, edited others, and wrote new ones in this phase. In the next phase, the codes were categorized (i.e.., focused coding; Charmaz, 2014). This process "requires decisions about which initial codes make the most analytic sense to categorize your data incisively and completely. It also can involve coding your initial codes" (Charmaz, 2014, p. 138). Focused coding also included making comparisons between codes and identifying patterns across cases using the constant comparative method (Charmaz, 2014). Throughout the coding process, I examined the stories to identify the pedagogical decisions they made to adjust to teaching during a pandemic and their rationales for those decisions. Ultimately, I wrote about experiences that may "live fruitfully and creatively in subsequent experiences" (Dewey, 1938, p. 17).

Trustworthiness and Ethical Considerations

To ensure the trustworthiness of this study, in addition to receiving approval from the Institutional Review Board, rigor was incorporated in the data collection and analysis process. According to Merriam and Tisdell (2016), "these qualities are essential because, in all research, we have to trust that the study was carried out with integrity and that it involves the ethical stance of the researcher" (p. 260). Multiple data sources (e.g., interview transcripts, pedagogical artifacts, researcher memos) were used to triangulate the data. Since my connection to the study could directly affect how I interpreted the data, I wrote and reflected on my reactions to the data. I reviewed those critically to identify any possibility of bias. I discussed the findings with colleagues and committee members to check for bias. In addition, a knowledgeable colleague coded some of the raw data to check for consistency. A detailed description of the procedures for conducting the study is provided. In the report of the study, I included rich, thick descriptions and included direct quotations "to contextualize the study such that readers will be able to determine the extent to which their situations match the research context, and, hence, whether findings can be transferred" (Merriam & Tisdell, 2015, p. 259). While my intention is generalizability, the results of this study may inform practice in undergraduate mathematics education for the near and distant future.

Limitations of the Methodology

I acknowledge that there are limitations in this research design, some of which are inherent in a qualitative research methodology. Subjectivity and bias are limiting factors in qualitative research. While being a part of the same community as my participants helped build rapport during the interviews, my interpretation of their stories was influenced by thirteen years of experience teaching undergraduate mathematics, my beliefs about the field, and my

experience teaching during the pandemic. During the data analysis process, I remained open to and reflexive about what participants shared throughout the data collection and analysis process. I was mindful of my interpretations while keeping their perspectives at the forefront. No one experience was identical to mine or the other, so each story was uniquely important to the collective story. Participants' language is used throughout the writing of this study's report.

Another inherent feature of qualitative research that relies heavily on interview data, when there is no observation involved, is that participants may choose to tell a story that reflects an ideology instead of how they feel or what they experienced. Consider Cohen's (1990) Mrs. Oublier, a teacher whose espoused beliefs about reformed teaching practices did not match her actual teaching practices. Atkinson and Silverman (1997) assert that "interviews consist of retrospective narratives. What people say may not be what they do, have done, and would do in the future. Interviews are performances that research participants give for particular purposes" (as cited in Charmaz, 2014, p. 78). Whether my participants were like Mrs. Oublier was not a concern of mine because I trusted that they were each sharing a perspective about an unprecedented event that I should not ignore for the sake of truth. Charmaz and Belgrave (2012), in support of interview research, point out that "interviews can also give research participants a space, time—and human connection—to reflect on these events anew and to clarify meaning and actions while providing rich data that spark analytic insights" (as cited in Charmaz, 2014, p. 80). Notwithstanding my agreement with Charmaz and Belgrave (2012) and my openness to the stories that participants share, features of this research methodology (e.g., triangulation of the data, constant comparative method) addressed the possibility of this limitation.

Conclusion

In summary, this qualitative inquiry was designed to explore the experiences of undergraduate mathematics instructors who taught during the pandemic. Twenty-eight undergraduate mathematics instructors were interviewed using a semi-structured, open-ended protocol. Interviews were the primary data source and lasted between 45-96 minutes. I collected pedagogical artifacts from participants who were willing to share them. Emergent themes and categories were identified. I then compared these ideas across cases (i.e., cross-case analysis) to look for similarities and differences among participant thoughts. Issues of trustworthiness and ethical considerations were addressed in the research methodology. Next, I organize the findings of the study across two chapters—one per research question: Chapter IV reports how instructors of undergraduate mathematics in this study "storied" their experiences teaching during the coronavirus pandemic; Chapter V presents, based on their first-hand accounts, how they adjusted their practices for teaching during the pandemic.

CHAPTER IV: FINDINGS I

The purpose of this study was to shed light on the experiences of instructors of undergraduate mathematics who taught during the coronavirus pandemic. This chapter presents results for the first research question: How did instructors of undergraduate mathematics "story" their experiences teaching during the coronavirus pandemic? But what is a story? For answers to this question, Carter (1993) points us to works in literary studies (e.g., Miller, 1990; Scholes, 1982). Simply put, a story is "telling or re-counting of a string of events" (Scholes, 1982, p. 59). And why "story?" According to Carter (1993):

> Stories capture, more than scores or mathematical formulae ever can, the richness and indeterminacy of our experiences as teachers and the complexity of our understandings...Readers, in turn, seek coherence and causal connections among these incidents and conventions as they construct for themselves, often retrospectively, the meaning or theme of the story. (pp. 6, 8)

So "story" because stories give us a way to represent and understand experience (Clandinin & Connelly, 2000). When teachers tell stories for research purposes, they offer a window to their thoughts and practices. In Noddings' (1991) words, "stories have the power to direct and change our lives" (p. 157). Moreover, stories retold offer possibilities for "new directions and new ways of doing things" (Clandinin & Connelly, 2000, p. 189). Stories about teaching during the pandemic can "produce a sense of how things go, have been going, and are likely to go" (Geertz, 1995, p. 3). Four salient storylines emerged from the data:

1. Remembering Other Times of Change and Disruption
2. Pivoting to Emergency Online Instruction
3. "Like Flying Blind": Navigating a New Normal
4. Coping

Writing about narrative inquiries is "personal, familiar, perhaps 'up-close,' highly readable, friendly, and applied for a broad audience" (Creswell & Poth, 2018, p. 232); I strive to

achieve this form of writing in this report. There are ample quotations throughout to bring participants' voices into the study (Creswell & Poth, 2018) and avoid distorting their stories' contexts. Each participant shared unique stories, and I attempted to distill the essence of multiple storied accounts where possible. I say where possible because not all individual stories represent uniformity across participants' experiences. These stories offer a glimpse into the participants' realities when a global health crisis altered their ability to teach "normally." All participant names are pseudonyms. Appendix B provides a snapshot of participants' profiles.

Remembering Other Times of Change and Disruption

These days, "unprecedented" is a popular word in our conversations—we use it to define the year 2020 because many have never experienced these circumstances before, all in one year to boot. A global pandemic, a national fight for racial justice, and a contentious presidential election are among the reasons that 2020 was a year like no other. "Upended" is another popular word—many use it to describe the state of education. Participants in this study also uttered these words (i.e., unprecedented and upended) to describe their worlds since the coronavirus pandemic hit the United States in March 2020; however, one participant, Aryeh, did not use those words in telling his story. Instead, he described the move to distance learning brought on by the pandemic as a "radical change." Aryeh's 58½-year-teaching career—the longest for the 28 participants in this study—began when there was a demand for science teachers in response to the Soviet Union's launch of Sputnik in 1957. He recounted:

> There was great demand for math teachers then because of the Sputnik situation with Russia sending the spaceship into the sky. That was an implication to the United States that we were behind in the scientific race, which meant behind in the arms race because science was equal to arms back then. So, if you were ahead in science, that probably meant that you were ahead in arms. And so, the country became very panicky, and it was giving out all sorts of fellowships and scholarships to people to go to grad school and study science.

Since Sputnik, Aryeh said that he has "seen a lot of changes" in undergraduate mathematics education—none like the disruption brought on by COVID-19. But they were significant enough to hold a space in Aryeh's memory and resurface when he storied his experiences about teaching undergraduate mathematics during the coronavirus pandemic. Inadvertently, perhaps, Aryeh connected this disruption to other times of change he witnessed in the past by recounting remembered events.

From Aryeh's perspective, changes in K-12 mathematics education in the 1980s and the open admission policy of the 1970s resulted in "radical change" in undergraduate mathematics education. "It is my belief that the new curriculum which has been in place since the 80s in the high schools is what has led to this incredible increase in remediation," he said. "Students come into college not knowing proofs, not even having a concept of what a proof is," he added, because "what they did in the 80s was, they reduced emphasis on proof. They gutted geometry." Additionally, according to Aryeh, the "highly controversial" open enrollment policy instituted in public colleges in the 1960s and 1970s is also responsible for what he referred to as "lowering of math standards." He explained, "with open enrollment, we did away with the usual standards, and we started letting in people who were who previously would not qualify for college." From Aryeh's view, these changes affected and continue to influence undergraduate mathematics curricula. "We had no remedial classes back in the 60s…calculus was the lowest level math course at the time…Over the years, we had this incredible increase in remedial classes," he said. Aryeh argued that with change in the high school mathematics curriculum and open enrollment, an increasing number of students came to college unprepared to take calculus.

Initially, Aryeh's account of these events seemed unrelated to the pandemic teaching experience. But after careful analysis, similar themes, such as "reduced emphasis" on content

and “lowering standards,” emerged across participants’ stories. For example, many instructors in this study said they covered less content in their emergency online classrooms. I connected Aryeh’s comment that students “have not been learning much at all” to other concerns about lowering learning standards and rigor during the pandemic. “I often worry about this,” one participant Pierre said. “Are people teaching up to the same level? Are students being held to the same standards? Are students still learning the same amount of material?” he added. But, I wondered, is covering more material the same as having high standards and the same as maintaining rigor? Culver et al. (2021) note that “there are two primary approaches to defining academic rigor: course work-load demands and expectations for course learning in the form of cognitive challenge” (p. 1141). But teaching and learning while living through a public health crisis required a lot of cognitive bandwidth. As a result, instructors said teaching during an emergency forced them to cover less material and focus on the “stuff” that mattered most. Nonetheless, a few participants worried that students may not have received enough preparation for a subsequent course. So, they predicted an increased need for remediation as the pandemic recedes.

“Black Lives Matter” was a famous word trio in 2020. People chanted these words at protests nationwide. George Floyd, a Black man, lost his life at the knee of a White police officer. Subsequent killings of other Black people sparked outrage. During our interview, Aryeh told a similar story. “You know about that incident? The Kent State situation?” he asked. “No,” I told him. “Look at it on YouTube,” he suggested. Then, he offered this version of what happened:

> This is what happened. It was the spring of 69 or 70, one or the other. The kids were rioting at Kent State University. I guess the president of the college or somebody called in the national guard to Kent State University to quell the riots. Students were rioting. They couldn’t have classes on campus because there was too much rioting going on. So,

> the national guard came on campus, and the students were going crazy there. They were throwing things at the national guard, maybe smoke bombs; it was just little things, like spitballs. They're throwing rocks and things like that at the national guard. The national guard guys lost control and started firing shots. They ended up killing four students. I think it was four. They killed at least one student. There is an iconic picture of this girl—probably a college student; she might have been a high school student. Kids from high schools were getting involved in it also—kneeling next to this guy who'd been shot by the national guard. So once this happened, chaos broke out across the country on many campuses...they did away with finals. I think this is what happened. You couldn't give final exams. I think you were told to make up a grade based on what tests we had given up until that point. Because I don't think there were any more classes after that point.

This is a story of a time, fifty years before the pandemic when students protested President Nixon's decision to invade Cambodia, which also disrupted teaching and learning; however, Aryeh admitted that in comparison, "our disruption is a little bit different; quite unique, I must say." By "our disruption," he meant the pandemic and the abrupt shift to emergency remote instruction. And this time was "a little bit different" because technology provided a way for Aryeh and others to keep teaching, albeit from a distance. Aryeh explained that he taught his class via email communication.

Aryeh did not mention the Black Lives Matter movement, the protests of summer 2020, or the killing of George Floyd. But his recount of the Kent State riots and the killing of Jeffrey Miller—the student whose body lies on the ground in the iconic picture Aryeh mentioned—reminded me of the protests that followed chilling images of Mr. Floyd's lifeless body at the knee of a police officer. Besides, in 2020 people were not protesting war, but the country was grappling with a contentious presidential election. Amidst this social and political unrest, teaching and learning operated online. This is not to say that classrooms were immune to the effects of these events. Wesley shared:

> When George Floyd got killed, I had this feeling, 'Why are we doing this? Why are we learning about the brachistochrone when people are in the streets demanding justice?' So, I took that to my class. Because it was the only thing, I could think to do. What is the purpose of us doing this thing when all of this other stuff is also happening? How can we

> approach the rest of this class in a way that feels meaningful when civil rights two is happening right outside our door?

Other instructors explained that teaching in 2020 forced them to consider how social and political events may affect students' ability to engage in the mathematics classroom. Irina said that she and her colleagues in her department discussed racial justice in ways they did not before. "If we don't respond to this in our classrooms, it feels like we are not sending a message that we think is an accurate representation of mathematics," she said. "So, we kind of sat down and said, 'How do we address this in our classrooms?'"

In sum, Aryeh's recollections illustrate Schoenfeld's (2016) view that "What takes place in schools and classrooms is intimately connected to what happens in society at large… Politics, policy, and the social environment of education all shape what happens in mathematics classrooms" (p. 497). In telling his story of experience, he recalled other events that impacted instruction or that "radically" changed the trajectory of undergraduate mathematics education. Much like the events Aryeh remembered, the effects of the pandemic will have curricula and sociopolitical implications for education.

Pivoting to Emergency Online Instruction

Each participant's story included elements of "pivoting online." Here, I highlight four common storylines from the collective experience of pivoting to online instruction: (1) receiving the news, (2) transporting practices, (3) (not) feeling supported, and (4) learning from and collaborating with colleagues.

Receiving the News

Participants described the moment in March 2020, around spring break, when they heard about pivoting to emergency online instruction. After that, teaching as they knew it changed indefinitely. Chaya recounted:

> I remember I was giving a class, and [someone] from the math office came in and said, this was March 11th last year, "This is the last day of your classes. We're closing for two weeks for online education because of the virus." By the time I got to the math office, they said no, we will be closing for the whole semester. So, naturally, I thought we'd be back in the fall, but I had no idea how far-reaching this pandemic was going to be.

The coronavirus outbreak prompted campuses to cancel in-person classes. At the time, some schools were on or about to start their spring break. The news was not surprising because they were aware of national efforts to slow the spread of COVID-19. But participants said they felt uncertain about what was ahead. Many of them had not taught online before. Based on the results of the Pre-Interview Survey, 24 out of 28 participants had little to no experience teaching online. In addition, they had to move their courses online abruptly and swiftly. Caelum recalled:

> We were on spring break when all of this happened. Then we heard we were going to switch to online. We were telling the students not to come back or that they have another week off. Then, when that week is over, we needed to have a plan. So, it's like, wait, what? In that week, I had to figure out what to do, and it was hard. When you're planning a course, it's a lot that goes into it. It's not just what am I going to teach; it's how I'm going to teach it. What do I want them to do? What is the timing of everything? And so, to have to do all of that in a week and in a format that I wasn't used to was really difficult.

Under normal circumstances, an online course takes six to nine months to develop (Hodges et al., 2020). But, like Caelum, most participants said they had as little as one week to prepare their emergency online courses. Wesley received the news on the same day as Chaya. He told me:

> So, March 11th hit. We were in the middle of spring break. We had just gotten the spring break when everything shut down. Immediately, our president said: 'We're going to extend spring break for one more week. When we come back, we will be online entirely for at least two weeks, and then we'll reevaluate at that time.' We ended up being online for the rest of the term. And so, of course, the immediate response was, how do I take all of this stuff online? Specifically, how do I take this active learning stuff that I am doing, which is all about students working together in small groups on specific math problems? How do I do that remotely?

Overall, feelings of determination accompanied those feelings of uncertainty. In response to the news of shifting online for safety, instructors swung into action. They scrambled to learn all they

could, in a short time, about digital technologies available to move their face-to-face practices to an online format.

Before this pandemic, no blueprint exists for suddenly moving courses online during a public health crisis. But, upon receiving the news, some participants found comfort in their existing knowledge and practices. Before going on spring break, Sofía anticipated the switch to online. She shared:

> Prior to going on spring break, the last time I saw my students in person, I mentioned to [my students] that I have experience teaching online. So, if it comes to that and we have to move to that, I hope you know that I would be comfortable in that area. And I'll try to make it as comfortable and as best an experience as I can for them.

Like Sofía, Caelum found resolve in practices she had already adopted before the pandemic. She explained:

> In the spring, I was piloting a flipped version of calc two. Students would watch some videos before class; they'd have a couple of problems they had to do. They'd come to class, and we'd work through stuff together. There was already a pretty good online infrastructure because they had this stuff. They had to do it before class. So, when we switched online, I had a little bit easier time, I think, than some of my colleagues because that infrastructure was already there.

Caelum referred to the instructional strategy (i.e., flipped classroom) she adopted before the pandemic hit in March 2020. The flipped classroom is a newish instructional model, about 15 years old. It is a student-centered, active-learning strategy wherein students interact with new content before class and then explore it further during class (Pollatsek et al., 2015). Caelum and others expressed that news of pivoting online found them more prepared than others. They had already adopted reform pedagogies (e.g., flipped classroom model, project-based learning) that transferred easier online compared to traditional practices.

Along with the news that they had to move their courses online in a matter of days, participants said they received messages which eased their nerves and made them feel hopeful. A

few participants explained that Barrett-Fox's (2020) "Please Do a Bad Job of Putting Your Courses Online" set the tone for their response to the frenzy. Barrett-Fox (2020) wrote:

> For my colleagues who are now being instructed to put some or all of the remainder of their semester online, now is a time to do a poor job of it. You are NOT building an online class. You are NOT teaching students who can be expected to be ready to learn online. And, most importantly, your class is NOT the highest priority of their OR your life right now. Release yourself from high expectations right now because that's the best way to help your students learn. (para. 1)

She follows this advice with a list of dos and don'ts for instructors to consider when the pandemic first hit in March. One participant, Annika, said she received an email from the dean of faculty at her school echoed Barrett-Fox's sentiments:

> We got an email from our dean of faculty last spring. He shared an article I think it was in Chronicles of Higher Ed that said the expectation is not that we're going to become these great online teachers. This is not a permanent thing, and so we have to do the best we can. But also have some balance with what reality is. Those aren't the exact words he used, but I just remember reading that email towards the beginning of the pandemic and taking a deep breath.

The article Annika referred to may have been Barrett-Fox's (2020) blog post because it went viral and cited several columnists who wrote for the *Chronicle of Higher Education*. "Do-the-best-you-can" tones reverberated across campuses, according to participants' stories. Participants who received such messages, along with news of the pivot, said they felt a bit more confident to step into unpredictable and uncharted waters.

Transporting Practices

According to some faculty, certain practices were difficult to transport from the face-to-face classroom to the online space. For example, Zachary revealed that once the pandemic hit, he "felt like his hands [were] tied" insofar as implementing embodiment in his courses (i.e., using bodily movements to gesture and think about mathematics). Others found it challenging to engage students in active learning online; therefore, they chose a more traditional lecture approach online for convenience. In Irina's face-to-face classroom, she said that students were

more willing to engage in active learning, whereas online, they preferred less interaction. When she asked students to work in groups in Zoom, she found that they were quiet and mostly unproductive. Her students wanted her to be always present and preferred direct instruction. "Even though I would like for them to be doing more active work, we know all the research that supports that learning," she admits, "it wasn't as much as I would normally like in a classroom." "But I think it gave students at least this feeling of support that is sort of implied in a normal semester, just by being in a classroom with [their] instructor." Irina said she ended up lecturing but found ways to encourage student participation. What Irina describes here resembles the pedagogical approach some scholars (e.g., Pausigere, 2011) recommend for teaching during emergencies (i.e., direct instruction). Direct instruction is a teacher-centered method. But Mendenhall et al. (2015) suggest that despite evidence-based opportunities student-centered approaches offer, they may be difficult to implement during emergency conditions for various reasons (e.g., lack of resources). In this study, many instructors said that some of their students did not have access to a quiet place to engage actively in discussions. So, learning from home, in some cases, hindered student-centered activity.

Sofía shared a similar sentiment about transporting active learning to the online classroom:

> I'm also very big on active learning. When you're in person, that's very easy to do because you can put students in groups, you can have students collaborating, they're in the same room. That was one of the things I struggled with; how do I continue to do active learning type things in an online format? Even though I know the technology is there, it was just kind of trying to brainstorm and think about how am I going to do all of this? So again, I knew that the technology existed, but I was really just trying to think of how to do all that stuff.

Both Irina and Sofía taught synchronous classes (i.e., classes with live sessions at a pre-scheduled time). However, Annika, who taught an asynchronous, junior-level, discrete

mathematics class of preservice teachers, had a different experience with active learning online. She offered the following explanation:

> I think because they were sophomore junior-level teachers who had been in our program for a little while. I had some of them in classes before, and I had had them in person before, right. So, they were used to doing problem solving in groups and having to share their thinking, and they were used to some of the expectations that I would have around that. So, I felt like I could leverage that to push a little bit further in terms of what I could expect for active learning in an asynchronous environment.

In comparison, about her calculus class, she said:

> I don't see what pieces of that I could take to a 35-student calculus one course being taught asynchronously nicely. Because there, the expectations of students who typically take calculus one are very different, I think, from someone in a teacher preparation program. So, I think I was able to push a little bit of the active learning there [in the discrete mathematics course] in a way that I wasn't in calculus one.

Other instructors who taught a mix of lower- and upper-division courses shared a similar experience. This finding suggests that instructors who taught upper-division classes had more success transporting reform-based methods (e.g., active learning) online.

Traditional practices were also challenging to transport online. Chalk talk—a traditional pedagogical tool where the instructor paced back and forth at the front of the classroom while writing lecture notes on the chalkboard—became virtually impossible online. Artemeva and Fox (2011) found that chalk talk is the most pervasive pedagogical genre across undergraduate mathematics classrooms. So many instructors felt disoriented in the online space. Bruce explained:

> The challenge was not so much the teaching of the material. I didn't have a setup where I could just immediately write something down, and people could see it easily. So, I had to prepare things in advance. So, the spontaneity of—there's a style of teaching, I don't know if you know—called the improv style, where anytime a student asks a question, you take the improv response, which is yes, rather than no. And so that that's a little harder to do when you don't have an easy way to put things on the board.

Thus, some of them attempted to transport this practice online. "When we were told to all be online, I went down to my basement, and I set up. Luckily, I had a big whiteboard," Luis said;

however, he quickly realized that it was challenging to replicate traditional chalk-talk lectures. So, he and others used tablets, electronic whiteboards, and presentation tools to keep some semblance of chalk talk online. But the writing space was limited compared to a wall of blackboards. Stiles (2000) identified using technology to "[mirror] traditional didactic practices" online as one of the "most serious errors in the educational design of web-based courses" (as cited in Engelbrecht & Harding, 2005a, p. 256). But many instructors transported practices in ways that were convenient to them and based on the technologies they had available. Liam explained:

> I probably don't quite have an adequate computer system; that was my first dilemma. Because ideally, and I see a lot of people have this now, I should have an electronic whiteboard of some sort…You can't really do mathematics by using the touchpad and the built-in whiteboard. With some of these systems, you can't write clearly enough…So the first thing I did was use the mathematical typing program LaTeX to create much more detailed notes so that I could post these notes in advance. Then, ask students to read them and then go over them, allowing for questions as we went along. Then I jumped over to my little whiteboard to add some more information as these questions came up or something occurred to me as I was teaching. And I think that was kind of the best that could be done under those circumstances.

So, in many cases, instructors used what they had available and prepared supplementary materials to accompany the lecture.

I found that instructors' ability and willingness to transport practices from their face-to-face classroom to the online classroom had much to do with several factors—not limited to but including—convenience, the lack of physicality in the virtual space, the mode of instruction (e.g., synchronous, or asynchronous), the level of the course (e.g., lower, or upper-division mathematics), and class size. The comfort level with the technology was another factor—I will explore this in an upcoming section on using technology. Finally, whether faculty felt supported

or not, which I discuss next, also affected their decisions about practices they could transport from the face-to-face classroom to the online classroom.

(Not) Feeling Supported

Support had different meanings amongst participants. When I asked, "who or what was most helpful to you during this time?" participants mostly said, colleagues—I will say more about this in the next section. Some said their partners or spouses were supportive. Azra shared:

> The most helpful has been my husband. He's also a math professor, and so he was doing similar things all along…I was recording things weekly while he had been done with his recordings over the course of the summer. Since he was done over the summer, he could take on a lot of slack at home because we have two children doing distance learning. He's also great in terms of technical support.

Other instructors who taught from home with children who learned from home or had other caregiving responsibilities said their spouses were supportive. Others said their spouses offered pedagogical advice. For example, Darren said his wife is an English teacher who taught online before. So, while she was not teaching mathematics, she offered pedagogical advice engaging his students online. Using discussion boards was one thing he said his wife recommended. "She helped me to stay alive," he exclaimed. Zachary, whose wife is also a teacher, said, "We talked about teaching a lot in the house." He did not share the details of their conversations around teaching, but he admitted that they were helpful. Thus, I found that some spouses became workplace colleagues.

Aside from stories of familial support, a few participants mentioned that their students were helpful. For example, Chaya credited her students with helping her navigate the online platform:

> The first thing they taught me about was how to share screen…Another student told me that I can get an online calculator and that went great. I could share my notes, and I have the online calculator! Students were asking: Can we see your notes? Can we have a copy? Can you record the lesson? But how do I get it to them? So, one student said, "I got this."

> And he taught me how I could set up Dr. [Chaya's] YouTube math channel. All my notes can be there; the students can get them…So it's synchronous, but if they wanted it to be asynchronous, they could have all my notes and listen to me talk them through it at their pace. So, the most helpful have been my computer-savvy students.

Connecting and communicating with students online was challenging. But, Chaya added, "those students who care enough to keep the camera on who asked me questions and participate in class, I feel I get to know them." A few instructors said seeing their students' faces and hearing their voices alleviated some of the awkwardness of teaching online.

Other than colleagues, family, and students, only a few participants told stories of feeling supported by their leadership. Most of them expressed disappointment with the top-down guidance issued by their administration to keep teaching in person. Consider that not all colleges and universities moved online, and some partially. As a result, some participants felt that their colleges jeopardized their lives by requiring them to teach in person. Alexa's university remained open. But since she was immunocompromised, she requested to teach from home. "Awful" was how she described the process of obtaining permission to teach from home. "It's like sage on the stage is the official pedagogy of the university," the reason Alexa gave for why her leadership wanted to keep teaching and learning in person. Another participant, Amie, said she felt betrayed by her administration for requiring her to teach in person.

> [They] said we have to be in person. And my president didn't fight back on that even when we had a huge outbreak. So, I feel betrayed, and I think that is a really common thing among faculty and staff at my school though, we feel betrayed...The main thing that characterizes my experience was frustration with the requirements put on me by the administration. The fact that I had to be in class every day and was not allowed to do one day a week online.

Only a few participants said they were required to teach in person. But of those who did, none of them felt safe being in the face-to-face classroom, even with masks on.

Participants expressed other frustrations with their leadership besides safety concerns about teaching in person. The impact of the pandemic (e.g., low enrollment) caused many part-

time faculty to lose their jobs. As a result, some instructors picked up the slack in the fall semester. Liam shared his frustration:

> The workload was far and above what it should be, with very little attempt [from the administration] to offer some sort of compensation. I'm not talking about being paid more. But I'm talking about the fact that as a tenure-line faculty member, contractually, part of my responsibilities is to carry on my research activity. At the same time, the kind of ballooning of demands to do online teaching very much ate into that. Now, I could have seen a simple-minded answer might have been, 'in better circumstances, you can expect to see such a reduction in your teaching load as compensation for the extra work you're doing now.' That type of response from the university administration would have been much more encouraging. But instead of saying that, what I see from university administration is that class sizes are about 50% larger.

Participants said that ballooned class sizes were overwhelming, especially online. In addition, as Liam pointed out, some instructors spoke about how picking up more teaching responsibilities meant they could not focus on research obligations.

In contrast, other participants, such as Annika, had a different experience from Alexa and Amie, whose campuses continued to offer in-person instruction. "No one was forcing us to be on campus to teach in person. So even though they're hurting by not having students being on campus," she said, "I think those kinds of institutional support have been helpful." Carly also felt supported by her department:

> My [Chair] is the most wonderful supportive boss... [She] made sure that we have a scheduled meeting every week. Occasionally, she has to cancel them, but we have a time set aside when we all get together, and she shares information, gets feedback from us, and allows us to just kind of vent a little bit. So that's really been, I think, what makes us, makes me feel the most comfortable with the situation, supported, and able to hash out any kind of problems that we're having.

Other participants also shared that frequent department meetings created a sense of camaraderie amongst colleagues—a feeling that they were "in this together." Annika, who teaches in mathematics and science and mathematics education departments, said she felt more supported by the education department. She compared the meeting cultures of the two departments:

> I will say my experience in the two different departments has been quite different. In the science and math ed department, we're kind of going through this as a team. We still have bi-weekly department meetings...In the math department, our first department meeting, I think we had one at the beginning of fall. Our next one now, it's going to be a few months. Those are pretty short, and I don't really see a lot of my colleagues. Other than that, I hear doors open, and occasionally we see each other in the hallway. But that's about it. So, the field is very different. You asked about support, so I guess what I'm trying to say is I think I'm lucky that I'm in this second department. There, it feels like I have a lot of support.

It turns out that frequent meetings were helpful to most participants. Yet others reported that they gained nothing worthwhile from often meeting with colleagues. For example, Kevin said that department meetings were spent "going off the rails" about the best ways to prevent cheating. Those meetings, to him, were not helpful and often created a lack of community. At the bi-weekly meetings hosted by Nellie's department, they shared pedagogical ideas:

> This didn't happen in the spring, but this past fall, we had every-other-week meetings just in our department. We would just talk about pedagogy and what was going on in class, and they had good ideas. The idea to have the group homework came from one of my colleagues. And I sort of took that and then made it my own.

So, the substance of those meetings mattered, too. Instructors said frequent meetings where they could voice their concerns, felt supported, and walked away with valuable solutions were most helpful. This finding confirms McAllister et al.'s (2020) finding that "regular, face-to-face meetings have proved essential for presenting ideas, talking through disagreements, problem-solving difficulties, and building a consensus" (p. 670). But scheduling regular meetings could be tricky. During the pandemic, video-conferencing technology made it convenient for departments to gather more than before. Reinholz et al. (2020) call for studies that explore change in mathematics departments. Examining the substance of frequent department meetings and whether they influence change could be beneficial for research in undergraduate mathematics education. Other participants told stories about collaborations with course coordinators in their departments. "The courses that I am teaching that are very large are

coordinated courses," Irina said, "so most of those decisions are made by someone else and kind of handed to me." But although Irina, an early-career lecturer, did not have full autonomy to make decisions about her course, she had a close working relationship with her coordinator. She added:

> It was definitely interesting starting a new job in the middle of all of this. I'm the assistant coordinator for a course. But thankfully, I've worked very closely with the head coordinator and then some of the other leadership people I've met with frequently. So that's made it easier to transition because then I have an excuse to meet and talk to people in the department.

Other instructors who worked with course coordinators did not have the same experience as Irina. At the end of every interview, I asked participants, "is there a question I should have asked you but did not?" Therese responded:

> Has teaching during a pandemic heightened any tensions between you and your colleagues? Because some people are going to be making these big moves. For example, I'm making all these flexibility moves. What happens when you're in a department where they're like, 'no, we have our reputation to uphold. Only 12% of this class we pass, or no one's going to take us seriously as an institution.' I work somewhere where any teaching idea I say out loud everyone's like, let's try it for a semester or two. Then if it's bad after two semesters, generally, they're like, we need to put this away for right now. But I feel like I have friends who are teaching during the pandemic, who are trying to lean towards being more flexible, and course coordinators are just like, 'no, we refuse, everything has to be as hard as it was before, if not harder. Also, what disagreements are they having, maybe on proctoring software, for example?

So, while Therese felt encouraged by her program to make shifts in her practice, her questions are important to consider. It seems that course coordinators and department chairs had the power to support instructors as they adopted new practices for pivoting online. But in some cases, course coordinators opposed offering flexibility to students for the sake of maintaining rigor and standards. In some cases, the department's culture either supported or undermined change.

Institutions offered training for faculty to move their classes online in a hurry, primarily through centers for teaching and learning. But some instructors said they did not receive the kind

of just-in-time training they felt they needed to go online quickly. “The college was not that helpful,” Chaya remarked, “the stuff that the college gives me are bells and whistles that I don’t care about.” She added:

> They want to teach me how to make a clever banner on Google Classroom. I don’t want that. So, my heading says math [###] lesson number three, that’s fine. I don’t need more than that. Obviously, if I’m teaching elementary school, I have to find another way. The students are happy if I can just explain it to them. They want to learn the material. The ones who want to learn, I’m here to help them. But I’m more interested in how to teach the content online. My students were the ones who taught me how to annotate. I picked it all up from them. [My chair] said to me, ‘you should give a video now explaining the stuff that we really need to know, like the screen share and making your cursor larger and stuff like that.

So, for Chaya and others, the form should follow the function insofar as training. Instructors wanted practical tools and tips specific to the discipline that they could immediately put into practice. Another participant, Pierre, learned how to use Canvas at his university. But felt that it was “developed more with the aims and intentions of humanities faculty in mind.” Chaya, Pierre, and others thought their training was not tailored for mathematics instructors. But others reported better experience with the instructional support provided by their university. Kevin, for example, found it particularly helpful that the instructional support staff member at his college was a high school mathematics teacher. Kevin’s point of view resonates with others that pedagogical content knowledge was essential for supporting them with teaching online.

Instructors who felt insecure in their positions also felt insecure to try new ideas for teaching, in general, but also for teaching online. Irina shared this perspective:

> One of the things I have been thinking about a lot, partly because I just went through the whole job market thing, is that we have all these different positions of people who teach these undergrad math courses, and some are much more secure than others. I am lucky enough that even though my position is not a tenure track, it is full-time; it is permanent in the sense that there’s no ‘in three years, you’re out,’ like a postdoc or something. I’m at a department that despite being, quote-unquote, just lecturers, we are very much part of the faculty. There’s a culture of ‘we are all part of this faculty.’ That’s not the case in every institution. The security, the culture, I think, has a really big influence on

> instructors' ability to respond to the craziness. If you don't feel secure in your position, it's hard to try something new or different or nontraditional despite the fact that we're in a time that needs nontraditional. Nothing is traditional; that's out the window.

As an early-career instructor, Therese expressed a similar concern. She said, "I was very concerned. I'm starting out and having all of these ideas. But I think that the experience of teaching could be so much worse if you're now adding in these extra emotions." Therese referred to the "emotions" of not feeling secure enough to make certain decisions in the classroom. This account from Sofía speaks to this sentiment:

> In terms of leniency with deadlines, something I was afraid to do was allow students to make up work. Maybe I was afraid to be lenient because, again, I'm tied to what the department wants me to do. I'm an employee of theirs, and I'm not tenured, so I can't just go and do whatever I feel like doing. I'm teaching a coordinated course, which means that other professors were sort of all supposed to be kind of doing the same thing, same time, type of deal. So, I was sometimes afraid to be lenient, you know, for the reasons I mentioned before. Also, just because they're going to be like, well, why are your students doing so well? It's because you gave them extra time or because you took late homework.

But the pandemic gave Sofía a chance to go against inflexible department cultures to build some leniency into her course:

> These are things that I've wanted to do anyway. We now have a space to do this because there are legit concerns with students who just can't keep up and are having trouble, and we need to be there to be able to support them and help them along.

Others spoke about using the pandemic as an opportunity to implement practices their department would have otherwise discouraged.

Learning from and Collaborating with Colleagues

In Mesa and White's (2021) view, professional learning communities wherein faculty can discuss teaching and share experiences "can help faculty engage in the practices advocated by the reform while maintaining their professional autonomy over classroom work" (p. 16). Eighteen out of the twenty-eight participants in this study identified colleagues as most helpful for making the transition. This storyline illustrates how colleagues leaned on each other amidst

the pandemic. “The support of my coworkers is probably the number one thing because I feel so refreshed,” said Carly, “after we all get together and share what’s going on and to know that I’m not alone.” While only a few participants told stories of supportive institutions, most of them credited their colleagues for their ability to pivot to online instruction. According to participants, sharing these experiences with colleagues brought them closer. They offered practical advice and found solutions to common problems together. In Wesley’s words:

> There’s a sense of solidarity that we were all suddenly thrown into the same boat. Of course, it’s not the same, but this is a reductionist way of saying we were all suddenly faced with the same challenge. We were all suddenly trying to solve the same kinds of problems like there was a real sense of community and a real sense of working together to address these challenges that was really helpful and really inspiring.

David said he feared using technology. “I’m still very traditional when it comes to technology which is just a nice way of saying I’m a technophobe,” he told me. So, without his colleagues, he “probably just would have fallen back into a normal routine that was comfortable, which would not have allowed [him] to use any of those more sophisticated resources and technologies,” said David. Liam also appreciated the support from colleagues in his department:

> The positive note was that I found my colleagues to be extremely helpful. There was a session about getting online organized by the math department and run by our fellow faculty members. It was to the point, by people who understood the subject that we teach; therefore, they could focus their responses on what was meaningful. I’m finding that people who allegedly are experts on the methods of remote learning are not particularly knowledgeable about the specific issues in my field. Often their suggestions are completely inappropriate. Whereas the information that came from my colleagues was generally quite good. They were being helpful in the sense that they could tolerate my bizarre questions. Everything was welcomed, and it made it much easier to communicate. So, I think we already see models couched by expertise and willingness to hear lots of points of view is a much better model for helping people do new things.

Liam’s point is far-reaching. His account here has implications for professional learning helpful for instructors to do new things in improved ways.

In addition to colleagues within their departments, participants looked to the global community of mathematics teachers. As Katie put it, “the whole global community of math

teachers has been helpful." Twitter, Facebook, and other social media platforms provided a space for collaboration. Leon said one of the things he liked about Twitter is that he can access information that colleagues have thought about carefully and deeply in only 280 characters. Therese confirms that "Math Twitter" was an extension of her departmental colleagues:

> I like having new ideas. I like being on math Twitter because I feel like I just pop out like, hey, how do you feel about this thing? And everyone who's interested in teaching will give me an opinion, and lots of times have good suggestions. Well, sorry, sometimes they'll be like [Therese], this is a bad idea, but I need people to tell me about that. I'm in a small department, so there's not as much opinion for me to find. So, the Internet is such a good resource pre-pandemic and especially now steering mid-pandemic.

A few participants received support from networks they had membership in before the pandemic. For example, Mark told me about the Academy of Inquiry-Based Learning, a professional development community where he received resources to support his transition to online instruction. Participants also credited newspaper (e.g., *Chronicle of Higher Education, Inside Higher Ed*) columnists and online bloggers for providing useful advice.

Experts in online teaching and educational technology (i.e., scholars and those with experience with online teaching) were "Samaritans" to the mathematics community; they were quick to help and offer advice. For example, Jenae Cohn and Beth Seltzer are instructional technologists at Stanford who put together "Teaching Effectively During Times of Disruption." A few participants referenced Cohn and Seltzer's document during our interview as particularly helpful for transitioning to online instruction. Participants also cited professional societies (e.g., MAA, AMS, NCTM) as resourceful because they developed websites with loads of information for teaching online. They also organized webinars and presentations where participants huddled with colleagues near and far to learn and offer advice for pivoting online. The sentiment that professional societies were resourceful was shared by many participants and reflected by Sofía, who said:

> There were all of a sudden, you know, all of these webinars and workshops that were being put together by people from all over the math community, from the AMS and MAA and NCTM and other places as well. And I just tried to take as much of it in as I possibly could. NCTM, National Council Teachers of Math[ematics], had a hundred days of professional development, all free webinars, and you didn't have to be a member. Now you can watch all these things for free. These are people who are seasoned experts, who are willing to share. Then you've got all these people willing and ready to listen. Also, I feel like we've seen a lot of that happening. So that's definitely been the most helpful to me.

Sofía and others took advantage of the many professional community gatherings available because they did not have to travel—they could access valuable resources for free from home. Thus, the pandemic afforded many instructors easy-and-free access to different forms of professional learning opportunities.

"Like Flying Blind": Navigating a New Normal

"It's just like flying blind," said Luca, "it's kind of like when pilots and captains navigate only by instruments, right, there is a big storm. There are no visuals. That's what I feel." According to dictionary.com, the idiom 'flying blind' "dates from World War II, when it was used by pilots who could not see the horizon and therefore had to rely on instruments." In many ways, this idiom represents how participants navigated a new normal. Technology helped for sure. But technology could not solve all the unimaginable circumstances that came with moving online during an emergency. Instructors told stories of unique situations they experienced and shared how they improvised and adjusted accordingly—I capture these narratives in the following sub storylines: choosing a delivery mode, stories about using technologies, dealing with academic (dis)honesty, enacting care, and shepherding.

Choosing a Delivery Mode

Instructors were deluged with information about the advantages and disadvantages of synchronous, asynchronous, hybrid, and hyflex modes instruction. Synchronous instruction

required instructors and students to meet online regularly via a specified video-conferencing platform (e.g., Zoom). Asynchronous instruction also occurs online, except there are no specified real-time interactions between the instructor and students. Hybrid is a mixture of in-person meetings and online activities. Hyflex (i.e., a combination of hybrid and flexible) is hybrid with the option for students to attend in person, online, or both. Participants in this study chose a delivery mode based on several factors and modified their initial choices as needed. Consider these reflections from Amie and Irina, who decided to deliver their instruction asynchronously:

> I tried to figure out, you know, what's the most equitable thing I can do to keep the course going. I was reading all these articles about don't have synchronous meetings, don't require them to have their videos on. I was just like, oh my gosh, it's going to be so hard. How am I going to make it a good semester? Basically, I turned it entirely asynchronous. (Amie)

> I decided to teach them asynchronously. There were no designated class meeting times because I knew that students were spread across the world. So, time zones were all over the place. Some of the students were trying to do virtual internships, some of them were helping with family businesses. There were all kinds of weird, unpredictable things. So, students really appreciated that they had full control over their time as long as they got things done by the deadlines, they could choose when it made the most sense for them. It was a lot of work for me because then my schedule was everyone's schedule. And making sure resources were up early enough that they could access them. (Irina)

Katie, who chose the asynchronous mode in the Spring, decided to switch to synchronous in the fall:

> I tried asynchronous classes in the spring. When we suddenly went online, our students were all across the country, and my class met at 8 am Eastern time, which is like 5 am for many of my students. So, we went asynchronously. I just recorded a lecture and had things that I asked the students to do. This was in a class where I had already set a precedent of daily quizzes. So, there was like a lecture, take a quiz, a lecture, take a quiz, which I thought wasn't terrible. But I don't think it was nearly as effective as having the synchronous class meetings. I didn't fully appreciate it until I had to do synchronous in the fall and realized that even online, synchronicity was really important. (Katie)

Irina and Katie mentioned teaching students in other time zones, and so did other participants. Unfortunately, time zones and locations interfered with how students interacted with their instructors and classmates.

Geographical location also affected connectivity, and hence instructors' decisions about which mode of instruction worked for them and their students. Pierre offered the following account:

> We were prompted to use asynchronous delivery, especially because [of our location]. We're very rural, so we can't rely, even on faculty, to have good Internet connections from their homes, let alone students. One of my colleagues teaches asynchronously precisely because you can't count on the network working from her home because she's way out in the country or something, even more so with students.

In some cases, their colleges or universities choose hybrid to keep some in-person components or a hyflex mode to give students the flexibility to choose. Chloe taught using the hyflex option but said that students hardly attended class in person. Other than location, participants described different experiences with delivery modes for students in lower vs. upper-division courses. Based on the interview data, synchronous instruction worked better for lower-division courses (e.g., algebra, precalculus).

Regardless of the delivery mode, instructors needed a learning management system ([LMS]; i.e., a course site) to organize course information and materials (e.g., BlackBoard, Google Classroom, Canvas). Those who did not use a course site in the past started using one; all participants except Aryeh used an LMS. Aryeh delivered his calculus lecture notes asynchronously via email. I asked Aryeh about his decision not to use a video-conferencing platform or an LMS. "I just figured that this would be as good as I could do," he said. "I didn't get into the Zoom. I just didn't. I think it worked out pretty well. I'm not saying it was better than if I had used Zoom. But I think it was okay." From Aryeh's and all other participants' perspectives, none of these modes could replace the traditional delivery method, within the classroom walls, in person. But they all stepped out of their comfort zone, used the information technology available, and did what they could to keep teaching.

Stories About Using Technologies

"I'm thinking to myself, what if this happened 20 years ago?" one participant, Luis, wondered. In an earlier section of this chapter, I retold Aryeh's story about 50 years before the coronavirus pandemic hit in March 2020. In the spring of 1970, campuses across the country shut down when protests against President Nixon's decision to invade Cambodia turned violent. "You couldn't give final exams," Aryeh said. "I think you were told to make up a grade based on what tests we had given up until that point." Now, 50 years later, instructors taught and assessed from a distance when the pandemic forced campuses to cancel in-person classes. "I think technology has been a saving grace for education as a whole," said Luis, who learned how to use the computer algebra system, Maple, for teaching online. Overall, participants in this study shared Luis' sentiment that technology saved education during the pandemic.

Using technologies to enhance undergraduate mathematics lessons is a reform feature; however, undergraduate instructors use technologies minimally in their classrooms (Mesa & White, 2021). Undoubtedly, the pandemic forced even the most technology-resistant instructor to use some form of technology, whether basic or sophisticated, or so it seems. From scanning documents to sending notes to their students to writing code in LaTeX to prepare lecture notes, every instructor in this study said they tried new technologies or gained new technological skills. This section captures how they experienced using technologies while navigating emergency online instruction. It also presents reasons instructors shared for deciding which technologies to use or not use. In the next chapter, I discuss the technologies they used and how they used them in substantive ways for practices germane to teaching mathematics. Also, Appendix F provides a list of technologies that participants said they used.

Instructors were awash in technology for teaching online. Yet, deciding which ones to use, in a hurry, under difficult circumstances, was overwhelming. Sofía offered this account:

> I did get overwhelmed, though, because honestly, sometimes I want to do this, and I know this software exists, but here's like ten other ones that also do the same thing. So sometimes it's also overwhelming because how do I know which one's good? I don't know which one's going to do what I want it to do.

Thus, instructors had many concerns that impacted their use of technologies during the pandemic. "It has to be free," Sofía said. Another participant, Irina, explained:

> One tool that has made a world of difference, that I used a little bit in the fall and will be using extensively in the spring, is Desmos classroom activities. Desmos is this free online graphing calculator. It is a great resource, and it's free for students.

Instructors reported that many of their students experienced financial instability amidst the pandemic, so they opted for technologies that cost little or nothing to students. Because, according to one participant, Leon, asking students to pay even as little as $20 for software "kind of adds insult to injury." Instructors worried that asking students to purchase tools could amplify the pandemic's financial burden and obscure their access to mathematics. As such, many instructors shifted towards open educational resources, which, according to Ryan and Nawalaniec (2021), saves money for students and has a positive impact on learning outcomes.

Another concern was whether everyone had the resources to stream lessons virtually. Whether they and their students had a strong Internet connection and working devices influenced instructors' pedagogical decisions. Yosef explained:

> One of the reasons I didn't want to have proctoring for a test, or even a really rigorously-timed test, is that I know that students may not have the same sort of equipment I do. There certainly are stories about students driving to the McDonald's parking lot and using the Wi-Fi there to go to their classes because they don't have reliable Wi-Fi at home. Some students didn't have a laptop; they just had a phone...they didn't have the full experience.

While some colleges and universities gave students devices, some participants argued that they were not functional (e.g., Chromebook). In addition, participants who taught synchronously said

they had latency issues and sometimes could not stream the classes effectively because of poor Internet connection. Carly taught in a rural town and shared that the infrastructure was not in place for high-speed Internet. And some instructors, because of geographical location, also had trouble with Internet access.

Aside from access issues, participants said their students' fluency with technology was another hurdle to overcome. Carly added:

> There's an awful lot of students who don't feel comfortable with technology. I explained to a student yesterday what the tabs mean on a browser. There are students who have just like, they've only used a Chromebook. They've never used a laptop before. They've never had to take a picture and upload it into our learning management system.

There was agreement across participants' stories that while students have the technical fluency to operate their smartphones, many either did not have access to or the fluency to engage with the course content online. The faculty also commented on how their comfort level affected their use of technology. Although most instructors said they learned new technologies on the fly, they were worried about whether they knew enough to troubleshoot problems that arose. For example, Luis said about using Maple for the first time, "The risky thing was, for me, I had not used Maple in this fashion before," he said, "so it was kind of like this might work, or I can just completely mess this up." Yet, most participants and their students stepped out of that zone to keep teaching and learning online. They discarded technologies that did not meet their pedagogical goals and students' needs and found new ones in the process. Speaking on behalf of his colleagues, Luis said, "I think now, all of us have become more aware of the technological tools out there." Another participant, Katie, said, "I picked up a bunch of different tools that I was able to use to intentionally design my course to be online." In the end, instructors preferred technologies accessible for their students and themselves regardless of their comfort levels and fluencies with technology. According to Sofía, it "has to be easy for the students to use."

All participants, except one, used a video-conferencing platform to keep teaching; Aryeh communicated with his students via email only. Most participants used Zoom because, as Luis pointed out, "Zoom was crafted for this. You can share your screen for teaching and presenting the material." In some cases, the administration decided which platforms instructors could use. In addition, many of them had never used these platforms before the pandemic. As a result, the learning curve was steep. There was a lot to learn in a short time: how to use the mute and video buttons, send students into breakout rooms, create polls, manage cascading texts in chatboxes, record lectures, and much more. "I'm still very traditional when it comes to technology," David said, "which is just a nice way of saying I'm a technophobe, and the idea of giving lectures over Zoom and using breakout rooms—it was scary." Others had similar reactions. In addition, a few participants were concerned that using such web-based technology exposed their and students' personal information. For example, Liam said he was uncomfortable using video-conferencing platforms (e.g., Zoom). He explained:

> I thought that there could be several issues in terms of information that's collected by these online platforms and safety issues connected with online. Pirate phishing activity and so on...There was a big expose and the Wall Street Journal about how Zoom was not really safe as a platform. Because there was an uproar, Zoom responded. I think clearly if we look at the economics of these stocks of these companies, they're doing very well, and we should expect them. To be more giving in terms of what they're taking from us. I could see that these people should be paying us for the information they're collecting from our students. I think that's not happening; instead, we're being exploited.

In addition to video-conferencing platforms, proctoring software was "intrusive" and invasive of students' privacy from the perspective of some participants. And so, despite many options on the market for lockdown browsers that keep students from straying to other websites while taking a test, most participants did not use online surveillance for proctoring exams.

Nonetheless, the video conferencing platforms allowed instructors to keep some semblance of the face-to-face experience (e.g., present lectures, pose and respond to questions,

students work collaboratively). Yet, some features of the face-to-face experience were lost. “You lose a lot, I think, of the interpersonal connections moving from in-person class to Zoom,” said David. “So that is definitely a drawback of Zoom over face-to-face. I don’t know how to recreate the face-to-face experience,” he added. In addition, not seeing their students’ faces on screen made it more complicated. Despite this downside, many instructors said they were flexible with camera policies. They told stories about students whose living conditions made it uncomfortable for them to turn on their cameras. Leon explained:

> I always wanted to err on the side of who knows what’s happening at their house. Who knows where they are? What if they don’t want to share their situation? I would much prefer to make these people feel comfortable than I feel comfortable. Why don’t they make a mode where students can turn on their cameras, but only the instructor can see? That would be great. That would be transformative. Students don’t want some random stranger looking in their room.

Interestingly, Sofia used Big Blue Botton, an open resource platform. She said, “the camera feed was set to only come to me. So, if students decided to turn their cameras on, only I could see it.” No other instructor said they had access to such a feature. Most instructors in this study ditched camera-on policies and found workarounds. For example, Zachary told me:

> I got over the whole camera thing. That was important. I had to get over it. And it took me about halfway through the semester last semester to realize that students can be perfectly engaged without showing their faces…There is something to be said about seeing faces. It makes us feel like we’re almost in a normal situation. But for me, it’s engagement. So, are you answering the chat questions? Are you doing the poll? Are you participating in the breakout rooms? Are you putting things on the artifacts, on the slides, on whatever you have to fill in? To me, that is much more important than you showing your face in the meeting.

In the end, while video conferencing diminished the classroom experience in ways, it afforded opportunities to engage students in new and innovative ways.

Not all instructors taught to black screens. According to instructors’ stories, those who taught students in upper-level courses, or students in learning communities, seemed to have more luck seeing their students’ faces. Those students had taken classes with the instructors before,

with each other, and were friends, in some cases. When I heard Yosef say that his students turned on their cameras and engaged with each other online, I asked him, "how did you get your students to turn on their cameras and be engaged?" Yosef replied, "I never asked them to turn on the camera. I think it was, probably for some or most of them, the way they could see their friends. This was a cohort. They have taken many classes together." Then he told me this heart-warming story: "On the last day of class, and this was really on some sort of pre-arranged signal, the students in the class held up handwritten notes thanking me." Yosef's experience was unique. But those who managed to forge interactions online, similar to in-person interactions (e.g., students responding to questions in chorus), said they remained flexible and constantly reminded students about the benefits of on-screen interactions.

Instructors used the learning management systems ([LMS]; e.g., Blackboard, Canvas, Google Classroom) to house their emergency online courses. Whether they taught synchronous, asynchronous, hybrid, or hyflex courses, many instructors learned to use one just in time for pivoting online. Even those who used one before the pandemic said they learned new features in the wake of emergency remote instruction. Through the LMS, instructors organized materials for students; students turned in their assignments; instructors gave students feedback on their work; instructors communicated with students and vice versa; students sought help and helped each other on discussion boards. Other aspects of their work, instructors said, they could not carry out effectively with the built-in features of platforms. For example, when Canvas grading did not work, they used Gradescope instead. Some who used Blackboard said it was not suitable for organizing course materials. Katie explained, "Blackboard made it very hard for my students to find things. So, they couldn't keep track of what was assigned and where were they supposed to submit it." She switched to Google Classroom. Not every instructor did away with aspects of

their school's chosen LMS that were not working, either because they were required to use it or preferred to keep using it. Sofia explained that there were latency issues because Big Blue Button was a web-based, standalone application. But when her department suggested a switch to Zoom, she kept using Big Blue Botton. "I became very comfortable and proficient in Big Blue Button," she said. Also, for convenience, she did not want her students to create another account.

During the pandemic, the choice was not whether to use technologies or not. It was a matter of which ones and how. Despite challenges and concerns, participants said they gained new tools. In fact, some participants admitted that they plan to take technological allowances they stumbled upon (e.g., online office hours) back to the face-to-face classroom when they return. Overall, they used technology that made the transition easier, that everyone could access for free and equitably, and technology that instructors and students had the technical fluency to use. For the most part, they chose to use technology that did not compromise students' privacy.

Dealing with Academic (Dis)honesty

"Online, it's easier to cheat. So, people who would never do it in a classroom are more likely to do it online," said Caelum. "So, no doubt there's been an increase in cheating," Aryeh noted. "And that's made me uncomfortable because I can't walk around the room looking at students and seeing if they have a little jib sheet nearby," he added. Dealing with cheating was a common storyline in this study. For example, Chaya told a story from her experience:

> You're not going to find that [question] in the test bank because I just made that up. What happened is that my printer has a slight defect. I can buy a new printer or get a Q tip and clean it, but it produces a thin line down the middle. I didn't know about Chegg…but my granddaughter was the one who told me she said, 'Grandma, I saw your exam. You just have to Google it. And your question is there. There's an exam with the fine line down the middle. So, it's your exam. It's nobody else's. It's not a coincidence. Half an hour into the exam, it's already out there. People are giving the answers…I can't tell you how upset I was. I worked so hard [on the problem].

A few other instructors complained about Chegg, a homework help website where students could pay expert tutors to provide solutions in little or no time. But, as Caelum put it, "just like in an English paper, you can tell if somebody's not writing in their voice." Hence, according to some instructors, they detected cheating easily and repeatedly.

A few instructors admitted that the more they tried to maintain traditional aspects of testing, the more students cheated. So, in response, they gave take-home, open-book exams. In addition, some instructors said they gave students more time to take the exam (e.g., a weekend), and others said they wrote more than one version of the test. For example, Yosef explained:

> I wrote out the exams. There are usually a couple of questions on exams over computations. I was able to make small differences in them so that no two people got exactly the same test. It took a little bit of time to personalize it. And I never had someone correctly answering one of the questions they have not been assigned. So that was one of my checks that there wasn't any cheating going on. Then, I'd give it out, say on Thursday, cancel class on Friday, and then the test would be due Saturday at midnight. They were not timed; they were open-book, open-notes. The only thing I asked them to do was to take it in one sitting and not talk to anybody. I said it was okay to go to Moodle to find notes, but I didn't want them to search on the Internet. And there's an element of trust in there.

Instructors also wrote novel, open-ended questions; as Pierre puts it, they "asked more thinking questions and problem-solving questions." These kinds of questions would make it harder for students to cheat, or so they thought. But students still went to companies like Chegg. At any rate, instructors did what they could to reduce students' temptation to cheat, recognizing that it was impossible to eradicate cheating. For example, a few of them gave alternative forms of assessment (e.g., oral exam) and reduced the emphasis on grades, which I will discuss in the next chapter. Some instructors said they deemphasized the need to get correct answers. David explained:

> I made it very clear from the beginning that, to me, it's never important if they get the right answer. To me, what matters is that their work is shown clearly, and I can understand it. And if you know you make some tiny mistake along the way, that's minus

> one point out of 10 points or something. So, I don't know if that sort of decreased the temptation to go to a place like Chegg and just get answers to the problems.

In addition, some instructors responded tried to prevent the urge to cheat by lowering the stakes. "When I hear my colleagues talk about, worrying about cheating, what I think is, well, there's an impetus to cheat because each of the assessments is so high stakes," said Leon. "If you see your student in class every day, and every day they have a quiz or an assessment, are they really going to hire somebody to take each assessment, each week, each one of them?" he asked. Leon and others argued that students were more likely to cheat with fewer high-stakes assessments. Instead of forcing traditional testing practices into the emergency online environment, Mark said that he took a different approach:

> Instead of spending hours complaining about cheating, I started thinking about what the real goals were. I guess what's really helpful is giving flexibility so that students can use other resources and try to write better problems and remove; what I've heard are the things that will drive people to want to cheat.

Like Mark, other instructors recognized that increased stress levels brought on by the pandemic, for example, drove students to cheat. As a result, they focused not on stopping cheating but on facilitating better assessments.

Meanwhile, some instructors shared details about how they and colleagues spent time and energy focusing on the question, "how to stop cheating?" According to Mark:

> That was just such a big topic in my department in the spring. All these people were so upset. Oh, so many hours were taken up by people complaining about their exams being found on Chegg. And how everything is kind of falling apart because everyone is finding their exams on Chegg…And not really any talk about changing how we're assessing students. There were always conversations about tracking IP addresses and a process for reporting people and class action lawsuits against the Chegg company.

No one in this study said they tracked IP addresses or sued Chegg. Several lockdown browsers (e.g., Respondus) were available, but instructors in this study did not use them for various reasons. "It's possible that somebody cheated," Yosef said, "but I think the cost of that was less

than the cost would have been if I used Proctorio or one of these very intrusive exam things." Many instructors shared this sentiment: lockdown browsers and other proctoring technologies invaded students' privacy. Others used what Luis called "Zoom proctoring." He explained, "I tried my best to replicate a standard exam day. So, what I did was I told them beforehand they have to have cameras on, and I told them that I want you to present your working space so I can see where you're writing." Kevin, who noted he tried to focus on why students were cheating rather than preventing, said:

> In my department, we never went to proctorial services—the most elaborate solution I saw was an instructor whose students were on the webcam on their computer. They had Zoom on their phones, going on the side, going over the entire desk so he can see what their hands were doing. It was quite elaborate.

Even when instructors worked hard to secure their tests, students found workarounds. As a result, some instructors remained skeptical of online instruction because it undermines their ability to administer tests in traditional ways. "Teaching on Zoom would be fine," Chaya said, "fine if it wasn't for the cheating."

Only a few instructors in this study said they reported incidents of academic misconduct to the administration. Most instructors who caught students cheating said they handled it internally because, as Arthur explained, "The way the institution approaches this has been difficult to navigate." Arthur's institution reacted to complaints about academic misconduct by telling instructors to use a lockdown browser. But he chose not to use one. "I'm not here to punish students, to chase them down, to surveil them by all means," he added; however, he expressed that instances of cheating should not be overlooked:

> I have produced a good number of misconduct reports, and it makes me sad to think about it...The way the institution approaches this has been difficult to navigate. I have heard from other people at other institutions that they have had similar sorts of concerns. That's one thing that I think has been very difficult; threading the needle between appropriate levels of oversight, authenticating student work, enforcing policies that are

> potentially going to result in consequences for students, while at the same time remembering that there are people on the other end of the zoom call, these people are engaging in misconduct, some small number of them. They are desperate to pass a class while working two jobs and taking care of their family, or they might just be someone who doesn't care about their education at all and is committing misconduct out of apathy. Both of those things should be addressed, if we can, before misconduct, and if we can't, then after it's found. I think that the pandemic has made that very difficult. (Arthur)

Other instructors were concerned about how reporting students who cheated may affect them in the long run. Thus, they promoted a culture of academic integrity and stressed its importance for scholarly work. As Luca put it:

> I talked for weeks about academic honesty; I like to frame it in the positive, not in the negative; it's good. Why? The purpose of being in college is to become a better professional is not only about learning math, and you want to learn how to be ethical in your profession.

Based on what participants said during the interviews, I suspect instructors were at various points along the continuum of upholding academic integrity and rigor and lowering expectations.

Instructors described how they felt when they caught their students cheating; I found that acts of cheating evoked strong emotional reactions in faculty. For example, Caelum said that she felt hurt. "After all I've done for [them], [they] decide to cheat," she exclaimed. Another participant, Chaya, said, "Cheating was so rampant, I couldn't sleep." From Liam's point of view:

> It's so annoying because it's disrespectful. You really feel like you've gone out of your way to be more in tune with your students and make an effort for the things they need. You want them to step up in the same way. You certainly don't want this feeling that they're putting one over on you. You don't want to feel like you're the fool. I think that's why it boils over so much.

Like Liam, other participants reported a feeling of betrayal when students cheated. Amie shared her sentiment: "I felt hurt that they would do this during the pandemic. This is a crisis, people are dying, and they're cheating on their pre-calc exam. It made so much more work for me." Other participants used emotive phrases to describe how they experienced cheating. "I was routinely

catching cheaters," Pierre said, "I find it very discouraging." He described the moment he found his questions on Chegg as "one of the most depressing moments" in his teaching career. Mark used "frustrated" and "violated" to describe how he felt when students cheated on a take-home exam. Noddings (2003) describes teaching as relational. Undoubtedly, the spike in academic misconduct upset instructors because trust, an essential component of any relationship, was repeatedly broken in the online classroom.

Enacting Care

Nel Noddings' work on "ethics of care" in education inspired the name of this storyline. She writes:

> My contention is, first, that we should want more from our educational efforts than adequate academic achievement and, second, that we will not achieve even that meager success unless our children believe that they themselves are cared for. (1995, pp. 675-676)

Liam's reflection echoes Noddings' (1995):

> I think our students do better because we do care about them. If you're tuned in, in a human way, you want your students to succeed. You realize that they have different needs. You want to address it at the level that works for each individual student that takes a lot of work. Sometimes a lot of thought...And yet, we won't give it up, despite what we say in our rhetoric. It's a responsibility.

Living during a pandemic was difficult for faculty and students alike. One participant expressed it in this way:

> So, there's one angle that is it's like a crazy lot more work. Another angle is, my students, I think, they're going through a lot too. I'm going through a lot as a parent of young kids. They are dealing with their own familial issues, their own struggles, in terms of needing to work, wanting to find safe work. Also, issues of not having a quiet space in the home to do their work, having their own parents expecting them to do other household support, take their time away from their studies. So, I've been really cognizant of that, and I think it's just hard for everybody; I think everybody's having a hard time. (Annika)

Moreover, students who are traditionally underrepresented in mathematics and come from economically disadvantaged backgrounds were most affected by the pandemic. A few instructors

told me that students with disabilities struggled, too, because suddenly, the accommodations (e.g., additional time for testing) they received in person were challenging to provide online.

According to participants, they had a newfound awareness of students' lives. For example, Leon said:

> I think I have a better sense of my students' lives than I did in the past. I know I'm privileged. I know my students have had issues or had to deal with something. And I've always tried to be empathetic. I think you really start to see how hard their lives really are...we have students who crawl over broken glass just to get their education. That's rather inspiring.

Like Leon, the pandemic opened Nellie's eyes to her students' experiences that may have affected their ability to engage before and during the pandemic.

> I mean, it's made me think of more about my students' home lives, how that impacts their learning and how that impacts the physical space in which they're studying. And then you know all the other things that they're dealing with in their lives that might impact their time or their attention. So that's probably the biggest thing, just having more space in my mind to consider all of those impacts.

Instructors had a clearer view of students' living conditions because the pandemic exacerbated and shed light on a fraction of their realities. For example, Zachary described one of his students' realities:

> That's a really sobering thing to think about—That students want to take a class, they want to be engaged in the class, but they literally cannot get in the door because of things that are not their fault. Add to that the economic piece of the puzzle where students' entire work lives have changed as a result of this. I have this student who more or less came to class every day from his UPS truck driving his route. That's what he was doing. And it took, thankfully, the journals; otherwise, I wouldn't have known because he was very shy about it. He finally said in one journal, by the way, I'm driving my UPS truck during class. That's why I don't participate. Wow.

Zachary and others were surprised to learn about students' circumstances that directly disrupt their classroom engagement.

Another participant, Luca, explained that he grew concerned about students asked his students to "Please fill this survey to provide information about your resources available for this

remote course. This information will be very valuable for preparation and adaptation of the course structure." He asked questions such as, "What kind of devices do you have available for this course?" Then on a "Wellness Survey" shared with me, Luca asked students at the start of every lecture, "Did you have breakfast? Did you sleep well? How do you feel right now? Are you prepared for today's class?" Part of caring for his students meant that he checked in with them every day in this way. Luca said he became "a little bit more aware of disparities" among his students based on their survey responses. The "Wellness Survey" reminded me of what Annika said during her interview about caring for her students. "I don't know if you've heard the phrase in Ed," she said, "there's this phrase you can't Bloom [as in Bloom's Taxonomy] until you Maslow. You have to do the Maslow Hierarchy of Needs before you can really engage." McLeod (2018) describes Maslow's Hierarchy of Needs as:

> a motivational theory in psychology comprising a five-tier model of human needs, often depicted as hierarchical levels within a pyramid. Needs lower down in the hierarchy must be satisfied before individuals can attend to needs higher up. From the bottom of the hierarchy upwards, the needs are physiological, safety, love and belonging, esteem, and self actualization. (n. p.)

Luca's surveys illustrate the "Maslow before you bloom" ideology. For him and others, students' wellbeing and access to the necessary tools were important for learning mathematics.

Enacting care meant that instructors verbally communicated it to students and made it part of their pedagogy. Yosef explained:

> I repeatedly reminded them that I knew that they were taking class under difficult conditions. I was open to flexibility based on things that were going on in their lives, although that didn't happen very much. I had one student who did get COVID and was quarantining in a room for a couple of weeks. I forgave a homework, which is kind of a minor thing.

Extending flexibility was a common way for participants to care for students in their emergency classrooms. For example, Chaya told me about a student who lost her father due to COVID. She allowed the student to complete the Spring course during the summer:

> Her father was in a refrigerated truck. They couldn't bury him. They couldn't do anything. But at least she wound up getting the A+, and she's that much closer to being a high school teacher…she's one semester closer to making a living wage to help her mother with the kids. She's the oldest, and someone's got to help. It's hard when you don't have money and you're in college.

Leon also gave students incomplete grades. "I've given a number of incompletes at the end of the semester." He added:

> I feel like I put a lot of students on my shoulders and tried to drag them across the finish line. This past semester, we had at least four or five students who got the coronavirus. I have students whose parents got the coronavirus, and the students were in charge of taking care of family members; taking care of may be the family business because the parents couldn't do that. There's just so much. I wanted to give the flexibility to these students to be able to prosper and thrive... I want to give my students as many chances as possible.

During the interviews, I asked, "How has this experience, teaching during the pandemic, changed you as a teacher, if at all?" Many of them said they became more compassionate. The following quotations express this sentiment:

> I might have even more compassion for students because we've just all been through this really difficult time. And I can even talk about the social upheaval that happened last year that had so many people so upset. I've always felt for my students, but I just feel like I'm even more sensitive to the emotional and social upheaval and how difficult it can be. Lots of times, when people are coming to community college, they've been on this downward roller coaster, and they're trying to get on the way up. (Carly)

> Being accommodating, being lenient, and taking into consideration the unique situations that each student might be living in while they're taking my course…I think that, if anything, that's one of the big things that has come out of all of this. We really have to consider what our students are dealing with at home, in their personal lives, in general, to be able to now show up to class, focus, and perform. I think it's definitely something for me. I mean, yes, I was aware of these things. These are things that I always thought about, but I think it's coming to the forefront, given our move to online instruction. I'm excited, and I'm happy that it has because I feel like there were instructors who maybe never thought about these things. And now it's like we're all thinking about it. Hopefully, this is something that improves education in general as we move forward. (Sofía)

> In some ways, it's made me a little more compassionate to people's situations. I've been forced to think more about what they have at home. What's their Internet like, things like that, which aren't normally that big a deal. I think about that when I'm setting up my classes—the way I communicate with them. Instead of just assuming that they're lazy and don't want to work, a lot of times, it is actually the case, but not always. (Caelum)

I offer these excerpts to illustrate how instructors shifted towards more compassionate pedagogy during the pandemic. Participants incorporated care in their pedagogy by extending grace, compassion, and flexibility.

Shepherding

Shepherds tend to and protect their sheep. They go out of their way to find lost sheep. In a biblical parable, according to Matthew 18:12-13 (NIV):

> If a man owns a hundred sheep, and one of them wanders away, will he not leave the ninety-nine on the hills and go to look for the one that wandered off? And if he finds it, truly I tell you, he is happier about that one sheep than about the ninety-nine that did not wander off.

This parable, related to the theme of care, came to mind when instructors told stories about "lost students" (i.e., students who became disengaged or stopped coming to class for any number of reasons). This anecdote from Wesley illustrates this finding:

> I had a student who I loved, and I worked very hard to help him, but he was my problem child. I don't know. He was, what even to say, I think that he really, really, really wanted to do well and didn't have any idea how to do that. I tried really hard to coach him: 'Here's how you do well,' and just got nowhere. I don't know. I'm still mystified. I really wish that I could have done better by this dude. The instant, he didn't know what was happening, full stop. He would disengage from the work of the group and have these really strong emotional reactions. I don't know how to deal with that. I did whatever I could, but I'm not a trained counselor. I can talk till the cows come home about mindset and belief, and 'it's okay to fail,' productive failure, learn from your mistakes, blah, blah. But the emotional reaction was so strong that it just shut off all those.

Azra had a similar experience to Wesley with a student who she was unable to reach after he stopped showing up for class:

> One student was lost. We lost one student, which feels pretty terrible because when we were in class, that student was really bright, brilliant, and contributed so much. I'm not only saying they were smart, but I'm just saying they actually contributed and participated. Then right before the pandemic, that student got sick—I don't know what type of sickness, and they just never came back for the semester. It was very difficult. I tried to reach out. I tried to connect.

Then, later in the interview, when I asked Azra to reflect on something she did that she was not satisfied with, she mentioned that student again:

> I am very frustrated and upset about losing that student in the spring. I don't know what to say. I emailed the advisor. I emailed the student. Nothing. That sometimes happens on campus, too, where a student disappears, and I don't know what to do about that. I still haven't figured out how to deal with it. I don't know how to solve that problem.

During our interview, I asked Irina to talk about a moment that stands out in her memory about teaching during the pandemic. She, too, shared a story about a particular student:

> I had a student in the spring—this was when we started in person and then suddenly, in an emergency, we had to switch—who was very active in class; always there and on time, working hard and asking good questions, and was definitely on track to get an A. After we switched, he was still definitely logging in, but he communicated that he was having a hard time focusing. There were a couple different days where he was like, hey, I'm not gonna be able to make it to class today because my family has their own business. They had to lay off people, and I'm going in to help. So that obviously made it a challenge for him. Sometimes he didn't know until he woke up that morning. It was, oh, I have to go help with this business now, I have to literally leave right now. I can't make it to class later. I could tell that he was having a hard time. He stopped turning in as much homework. So, I reached out to him and kind of had a conversation. He was definitely going through some mental health challenges, as many students were. We kind of talked, and I pointed out some resources. I mean, I'm glad that I had enough of a connection with him that I could notice this difference and reach out to him. We had enough flexibility built in—if he had to miss a day for this emergency, he could...But that was one thing that I think stuck with me a lot; I need to make sure I know my students and have that connection to be able to reach out to them.

Irina's anecdote exemplifies the importance of building relationships with students, "sheep" in this analogy, metaphorically speaking. These stories represent an aspect of teaching that is not often discussed in the literature on mathematics education—tending to not only the students who are always present and ready to learn but those who are struggling and lost.

Coping

Towards the end of every interview, I asked participants, "is there a question that I should have asked you but did not?" "I feel like I did talk a lot about my teaching," Annika answered, "but I can talk more about some of the family things that I was dealing with on the side while

trying to navigate teaching online." Her response reminded me of the book, *Mathematicians Are People, Too: Stories from the Lives of Great Mathematicians*. Like students, they, too, dealt with angst and life-altering situations during the pandemic. Finding ways to cope with new pedagogical demands during a health crisis was a human element that emerged from participant stories.

In general, participants felt that teaching during the pandemic was a lot of work. "I didn't have time for anything else," said Caelum, "grading these things took forever, writing the assessments took forever, planning the sessions took forever...it was exhausting." Luis and others told me stories about the increased number of emails they received and students' expectations to respond at all hours a day. To deal with the cognitive overload, instructors had various coping strategies. "By coping," Pearlin and Schooler (1978), "refer to the things that people do to avoid being harmed by life strains. At the very heart of this concept is the fundamental assumption that people are actively responsive to forces that impinge upon them" (p. 2). Participants described how they responded to the stress of teaching and living during a pandemic.

A few participants said that music broke the ice in their online classrooms. Nellie said, "I started all my classes with music. And I invited my students to bring their music." This was her way, she said, for students to bring themselves to class and have a human experience. I asked her, "did the students comment on what it was like to have music at the start of class?" She responded, "So the first question is like what three things did you value most...nobody mentioned music." She searched her evaluation document for the word music and said she found "no instances" of music. I thought that was interesting because I wonder whether all students, all personalities appreciated music. Another instructor played music to relieve stress at the start of class. "I will also have music playing just something seemingly insignificant, but I feel like it

kind of helped ease into the beginning of class. If they're scared about math, okay, but you're listening to music. Maybe you forget that you're scared" (Sofia). Therese said that feedback from her students suggested that playing music before class had a positive effect on their pandemic learning experience.

Another intention behind the music was to create a welcoming environment—other instructors told me how they tried to do the same. Creating a welcoming environment for their students was one way to help their students cope. A few of them said they made space for students to escape the world outside of the virtual classroom. By escape, they meant a focus on doing mathematics. At the same time, others allowed space for students to talk about things other than mathematics. Luca recounted:

> I guess I was trying to do something that I'm personally not great at. I've been trying to improve talking about things besides math in my class. For some reason, sometimes I feel guilty if I don't talk 100% of the time in class about math. Then again, I think that I'm satisfied. Maybe you can phrase it as 'make it more human.' I think that we definitely, overall, need to make math more human. And I think that I took a couple of steps in that direction.

Another way to cope with the cognitive demand of teaching during a pandemic, some instructors said, was to "keep things simple." Aryeh described his plan for going online:

> This is what I basically figured. Back in March this year, when we were told school was closing. You gotta go over within one week. You gotta go over to the distance teaching. Alright, so I came up with my plan. And this was my plan for me and for the students. Simplicity.

Instructors could choose from various options for teaching online; however, surviving a pandemic was stressful enough, so some of the instructors chose low-technology, asynchronous options.

In the section on enacting care, I shared that participants extended flexibility to their students. But to cope with the demands of teaching online and surviving a pandemic, participants talked about giving themselves grace and flexibility. For example, Leon reduced the number of

projects in his course. Katie found a faster but meaningful way to grade; instead of using a point system, she used a binary system. She said, "binary grading is brilliant. It's pretty quick," because it is either "they don't understand this piece yet, they need to study it again or yeah, okay, I think they got it." Therese said giving herself a break was one way to deal with the frustration she felt:

> Keyword frustration, all-around frustration, tiredness grading on my laptop. My eyeballs hurt all the time grading on my tablet. My eyeballs still hurt because it's the screen. It's way harder to separate work from home stuff because, like, I'm working at home...I just took email notifications off of my phone around Christmas because I suddenly couldn't take it anymore. It's changed my life. I didn't check my email for 12 hours. It was a wild time for me.

Carly changed the deadlines for when assignments were due to reduce her workload on the weekends:

> Instead of our deadlines being Sunday nights, we made the deadlines Thursday nights to make our weekends less unpleasant. Because when students come on Sunday morning, 'oh no, I have all this to do and I don't really understand,' I'm like, 'don't mess up my Sunday. Let me have some time with my family. Mess up my Thursday.'

Making videos for instructional purposes, according to participants, took a lot of time. As a result, some of them told me that they decided to stop along the way and use videos available on the Internet instead.

As they reflect, a few participants realized that they did not do much to care for themselves. As a result, a work-life balance was missing in their day-to-day operations. "I feel like I should have done a little bit more to take care of myself," said Liam, who planned to make that a priority in the following semester. I did not intend to ask participants about self care, but a few mentioned it as a coping mechanism. They explained how they shouldered the emotional labor of emergency pedagogy. Many participants told stories of coping by doing whatever was necessary to preserve wellbeing and mental health.

"There's been some silver linings as far as I'm concerned," said Jessie. Despite the challenges of the pandemic, Luis admitted, "it afforded me the opportunity to try new things." According to Sofía, the pandemic is "a unique opportunity to rethink education, a time to experiment." She encouraged other instructors to try new ideas: "Maybe you've wanted to do project-based learning, and you couldn't, or you felt like you couldn't do it when you were in person." On the one hand, this positive outlook on the pandemic emphasizes Winston Churchill's call to "never let a good crisis go to waste." On the other hand, instructors said framing the pandemic as an opportunity was a helpful way to cope. I believe those instructors underwent a sort of cognitive reframing. According to Robson Jr. and Troutman-Jordan (2014):

> Reframing has been defined as changing the conceptual viewpoint in relation to which a situation is experienced…Revisiting and reconstructing one's view of an experience imbues it with a different, usually more positive, meaning in the individual's mind…Specifically, *cognitive* framing involves changing the way people see things and trying to find alternative ways of perceiving ideas, events, or situations. (p. 56)

Instructors could not change the impact of the pandemic. But some of them coped with this new normal by searching for silver linings.

Conclusion

The findings in this chapter answer the first research question: How did instructors of undergraduate mathematics "story" their experiences teaching during the coronavirus pandemic? Four major storylines emerged from participants' narrative accounts of teaching during the pandemic: remembering other times of change and disruption, pivoting to emergency online instruction, "like flying blind:" navigating a new normal, and coping.

Aryeh is a professor with 58.5 years of teaching experience, the most of all the participants. During our interview, he told stories about other times in history when events caused change and disruption in undergraduate mathematics education. From his recollection, Sputnik, the Kent State shooting, changes in K-12 curricula, and open enrollment all impacted

teaching and learning at the undergraduate level. Yet, the sudden shift to remote instruction because of the coronavirus pandemic was like no other disruption he, nor any of the other participants, had experienced in their teaching career.

Participants painted a picture of what it was like to receive the news in the spring semester. Some had the opportunity to teach remotely, whereas others had to teach in person or do a combination. Then they told me about decisions they made for pivoting online. Some reverted to traditional methods when tried-and-true practices (e.g., active learning) did not transfer to the online space. Support from their leadership was essential for helping them to make the transition. Based on my analysis, caring departments where participants had options and spaces to be heard frequently were most supportive for the transition to online instruction. Although some participants had supportive departments, most of them credited their colleagues across the global community as most helpful.

None of the participants I interviewed taught during a global health emergency before. Moreover, only a few of them had experience teaching online. Hence, this experience was "like flying blind," as Luca put it. Choosing a delivery mode and navigating the new technology-driven space was challenging for both instructors and their students. Academic integrity was also a much-talked-about issue. Instructors expressed a variety of emotions aroused by dealing with cheating. For the most part, they did not want to punish students but hoped they would learn what it meant to do scholarly work. The pandemic called on them to consider students' lives when making decisions. Because students faced unprecedented challenges, instructors offered flexibility and empathy. They tried to reach out to lost students—a practice I termed shepherding. Overall, a theme of care permeated participants' stories as they told stories of navigating a new normal.

Many instructors accepted that what they were doing during the pandemic would not be perfect and found ways to cope with teaching during a crisis. The workload was intense, so they found ways to find a work-life balance. Whether by taking a break from emails or finding effective ways to grade assessments, participants found ways to maintain their well-being. Their classrooms provided space for their students to escape or confront societal issues. Participants' response to the emergency was to triage (i.e., to do what was most important), offer grace to themselves and their students, and look for silver linings where possible.

I started this chapter by citing the purpose of this study: to shed light on the experiences of undergraduate mathematics instructors who taught during the coronavirus pandemic. In sum, their stories reminded me of this quotation:

> We learned that the sector can turn on a dime when we need to. We learned that when all of the oars are rowing in the same direction, we rise to the challenge and adapt to changes that one seemed improbable. We embraced discomfort together. We may even learn that some remote learning could be desirable for what it affords students. If we hold on to these learnings beyond a crisis, we will be less fragile in an already uncertain and changing context. (Lederman, 2020a, n.p.)

Participants' narratives made it possible to understand what it was like to teach during an unprecedented time in their lives.

CHAPTER V: FINDINGS II

This chapter presents the findings for the second research question: How did instructors of undergraduate mathematics adjust their practices for teaching during the pandemic? Teaching is more than meets the eye. So, where possible, I discuss evidence of factors participants said influenced their practices. The findings in this chapter are organized according to five dimensions of teaching practice prevalent in the data:

1. (Re)presenting Mathematical Content
2. Choosing Tasks and Content
3. Monitoring: "Leaning Over Their Shoulders"
4. Building a Community of Mathematics Learners and Doers
5. Assessing for Learning During the Pandemic

(Re)presenting Mathematical Content

According to Artzt and Armour-Thomas (2002), a teacher's "modes of representation are the forms for representing mathematical concepts or principles externally through the use of verbal or written words, diagrams, manipulatives, computers, or calculators" (p. 12). In the undergraduate mathematics classroom, lecturing via "chalk talk" is the traditional, most commonly used mode of representation. Chalk talk, "a pervasive pedagogical genre," as defined by Artemeva and Fox (2011), is "writing out a mathematical narrative on the board while talking aloud" (p. 345). Instructors were concerned with how to take this traditional practice to the online space. "For the math teachers, it's just so different because you know we always ask about something to write on," said one participant, Darren, "which is different, I guess, for all the other professors. They really weren't as concerned about, 'how can I show material?'" As one participant in the Artemeva and Fox (2011) study puts it, "you need to show…what you're

thinking; you need to show the process" (356). Moreover, undergraduate mathematics instructors write on the board "as a means of introducing students to the disciplinary thinking, practices, and procedures of "doing mathematics" (Artemeva & Fox, 2011, p. 356). The pandemic disrupted this disciplinary culture, especially for instructors who ran a traditional undergraduate mathematics classroom for many years before the pandemic. It became virtually impossible for instructors to write on chalkboard walls while talking through mathematical ideas.

Nonetheless, instructors searched for alternative ways to present mathematics to their students from a distance. Mathematical representations, according to Goldin (2014):

> are visible or tangible productions—such as diagrams, number lines, graphs, arrangements of concrete objects or manipulatives, physical models, mathematical expressions, formulas, and equations, or depictions on the screen of a computer or calculator—that encode, stand for, or embody mathematical ideas or relationships...they are external to the individual who produced them and accessible to others for observation, discussion, interpretation, and/or manipulation. (p. 409)

Instructors were concerned about how they would articulate and show the mathematics in the way Goldin (2014) described it so that students could access their thoughts and processes. Participant Luis explained:

> When we were told to all be online, I went down to my basement and set up. Luckily, I had a big whiteboard. So, I'm thinking to myself, I wonder how people try to adapt without having a whiteboard? Like how? What if you didn't have a whiteboard? What if you didn't have markers? What if you didn't have all this stuff? How would you have adapted? I had a whiteboard. I was using Google Meet with the whiteboard in the back. Then I realized after two weeks of doing it, this is not where it's supposed to be. Students were telling me, hey, move it to the left, to the right, the glare. I don't know what you're doing. Can you zoom out? Zoom in? I'm like, okay, I'm trying my best. I'm recording my lectures. I'm doing an hour, an hour, and 15 minutes to an hour and a half, just writing stuff down. Then I realized if I continue this way, I'm doing them a disservice.

So, Luis searched for an alternative for writing on a physical whiteboard. "I watched YouTube videos on how to present in mathematics," he said. Instead of writing on the physical whiteboard, he wrote on his iPad's note-taking application, which he connected to Zoom, while he talked

through mathematical ideas. From his perspective, this was better because he could save the notes for students. After all, space on the physical whiteboard is limited. Also, instructors erase what they have written on the board to make space for more notes. So generally, before the pandemic, instructors kept no permanent record of the notes they wrote on the board. During the pandemic, Luis also used Maple, a computer algebra system:

> I started to use Maple to demonstrate without drawing those squiggly lines. Okay, so we're looking at this curve, and what Maple does really nicely is that it creates the curve for you. It also shades in the region for you, and it generates the solid for you. It generates the disks for you. So, then I'm able to show them in real time.

For Luis, Maple enhanced this lesson on volumes of solid revolutions because students could visualize revolutions as a three-dimensional activity. According to him, "if this pandemic wouldn't have happened, and if this virtual learning wouldn't have occurred, I would have been doing the same thing." By "doing the same thing," Luis was referring to representing mathematics as a two-dimensional activity

Other instructors used a physical whiteboard during the pandemic. For example, Liam said he used one because teaching mathematics requires a place to write clearly. He explained:

> I probably don't quite have an adequate computer system; that was my first dilemma. Because, ideally, and I see a lot of people have this now, I should have an electronic whiteboard of some sort. I think the only functional ones are the ones that are using an app and Apple stylus on the whiteboard, and then you can use it effectively. You can't really do mathematics using the touchpad and the built-in whiteboard on some of these systems. You can't write clearly enough...So the first thing I did was use the mathematical typing program LaTex to create much more detailed notes so that I could post these notes in advance. Ask students to read them. Then, go over them, allowing for questions as we went along. Then, jumping over to my little whiteboard soI could add some more information as these questions came up, or something occurred to me as I was teaching. And I think that was kind of the best that could be done under those circumstances.

Though he did not refer to it as such, Liam and other instructors facilitated a sort of flipped model. He asked his students to interact with his mathematical ideas via LaTex-typed notes before class. According to Kerrigan and Prendergrast (2021), the flipped approach leaves in-class

time for student engagement and collaboration. But I suspect that providing notes ahead of time was more about convenience than taking this innovative approach. Moreover, it appears that Liam, and others, provided handouts with notes to circumvent the lack of physical writing space online. If students had the notes ahead of the lesson, then instructors could write less on screen but still communicate the ideas captured in those notes:

> Normally, my notes would be what I wrote on the board. It's probably the same amount of detail, but usually, they have to get it from the board instead of having it written in a LaTex document...But the fact that I could communicate that information in pages of notes and that the students could process it was a pleasant surprise.

Yet, Liam, and others, missed the old model for what it offered (e.g., generating dialogue while writing mathematical ideas on the board). Although "communicating the full amount of information [in pages of notes], worked out better than [he] expected," Liam said, "there's an issue with that in some way:"

> I might have maybe a half page of notes, which becomes the one or two-hour lecture because you're responding to what people say and the body language of whether they're getting it or not, and even ideas that occur to you. And I think that model, in a way, is much more important for students to watch you think in the classroom. People have this kind of maybe uninformed view of mathematics, that some of us, the gifted few, are born being able to do it. The rest of us, Oh, 'I can't do math,' as if that's an acceptable answer. And the idea that it's not a list of rote formulas, but rather, mathematics is a conversation. They need to see you doing that in the classroom as you're kind of thinking yourself along the way.

For Liam and others, presenting mathematics using chalk is complex; it is more than just writing out notes on the board for students. As Artemeva and Fox (2011) put it, chalk talk lectures allow instructors to introduce students "to the disciplinary thinking, practices, and procedures of 'doing mathematics'" (p. 356). Moreover, from their perspective, their ability to be spontaneous was diminished online.

Others spoke to this notion of spontaneity. "Usually, after a while, you get to the point where you could sort of improvise a lecture," said Yosef, "almost like a musician, you have a

few themes you want to make, and then how you get from one to the next might vary from one class to the other." He added:

> I was mentally picturing the class and then typing down what I was going to say. That led to about 14 pages of notes per class which took four or five hours to type up...Then, I would talk over them, usually about 30 to 35 minutes, and I would send out the recording the night before.

Chaya also wrote and shared lecture notes with her students before class then talked through the notes during class. She explained:

> My husband studies Jewish texts. He has a program called Daf Yomi, where every day they do one page a day, and it can cover the entire Gemara in seven and a half years. So, I watched what he did. He had the text in front of him, and he would then go online, and somebody would talk him through it. I said, 'I'm going to steal that idea.' So I wrote out my notes, not like I was writing on the board but in a very reader-friendly way. Not every word that I was going to say. But it was more than when I would write in my notes for in-person classes. I wrote a lot down. It took me a long time that first semester teaching online. I mean, it still takes a long time. I wrote everything down, and my plan was that I was going to send those notes to the students before each class. Now they could print them out, or they could have them in front of them on their computer, and they could watch or listen to the class on the phone.

It was important to Chaya, and others, that students could access their mathematical explanations through visual aids (i.e., notes). Typically, in the regular classroom, like Liam, Yosef, and Chaya explained, what instructors wrote on the board was less than what they prepared in a page or less of notes. But during the pandemic, they spent a considerable amount of time preparing detailed scripts for their lectures. In some cases, instructors added videos, either videos they recorded themselves or from other sources, to accompany notes.

Generally, instructors think about not only where and how they will share mathematical ideas with students but also how students will keep a record of those ideas. In other words, where and how do students write notes? According to Liljedahl (2020):

> Mathematics teaching, since the inception of public education, has largely been built on the idea of synchronous activity—students write the same notes at the same time, they do the same questions at the same time...From a teacher's perspective, this is an efficient strategy that, on the surface, allows us to transmit large amounts of content to groups of

20-30 students at the same time. (p. 145)

In the face-to-face classroom, instructors expect students to be present and copy notes from the board. But according to instructors in this study, the pandemic gravely affected students' ability to attend synchronous sessions. Thus, in addition to providing lecture notes, instructors recorded the lecture. This way, students could access the mathematical ideas and discussions about them asynchronously. Participants shared various reasons for providing written notes in addition to the video recordings, a practice they did not do before. From Chaya's point of view:

> I'm aware that not everybody can afford a fancy computer. A lot of students are watching my lessons on their phones. It's not a big screen, so it's important that they have the notes, print them out, and then they can look at them. They can hear me on the screen or hear me on the phone, but you've got to realize that not everybody is seeing it the same way while I am presenting.

Whereas in person, all students in attendance could access and copy the notes. Therefore, providing notes and recordings of the lectures made it possible for students to access the materials asynchronously. A few participants planned to continue this practice when they returned to the face-to-face classroom. In fact, Yosef said that he regrets not providing notes to his classes before the pandemic because "it does seem to improve matters in improving the learning," he said. "I'm not sure I would have had the time to do it," he added, if not for his reduced teaching load. Then Chaya said that if she continued this practice of providing handwritten notes, then she would give them to students after class. "If I were to give them the notes in advance in a regular class, they wouldn't come to class," she said.

Some participants who gave students the notes before class said it was beneficial for classroom discussions. While others shared that students did not read the notes; therefore, they ended up lecturing as if students had not seen it before. Chaya continued to wonder about the best way to present the content to her students, but not all at once (e.g., pages of notes before

class). She explained:

> I'm thinking of a way to do formulation problems in linear programming. Paper is cheap. So, if I have a problem, I don't want them to see the whole formulation. I'm going to show them 1a, which is just the objective function, and then 1b, I'm going to add in a constraint. I'm going to do it in a series, you know, like if you flick it like a cartoon when you flip the book, and you see it. So that hopefully they won't see everything at one time. They can look ahead and see what it is, but I'm trying to always think, what's the best way I can do it if it's a complicated theorem. How can you break it up into little pieces?

Chaya did not try this approach, but she considered it for the following semester. I found that she and other participants were concerned about breaking up the material into "little" pieces. Because to them, that pedagogical move was necessary for presenting the content to students. Talking students through the content step by step is an element of "chalk talk" that participants took online.

It turns out that representing mathematical content online felt unsatisfying for most instructors who preferred the traditional lecture style. They contend that teaching online diminished their flexibility to think aloud while talking through and writing mathematical ideas. Still, they found ways to maintain some conventions of the chalk talk lecture online (e.g., verbalizing what they wrote, teacher-guided instruction). But not everyone used a version of "chalk talk" for representing mathematics online. Participants, especially those who had moved away from this pedagogical genre before the pandemic, described other ways they shared and communicated mathematical ideas in their online classrooms. For example, Wesley shared a Google Jamboard activity he created to convey the theorem that every convergent sequence is a bounded sequence. "What you do is you give people all of the correct code. But then it's out of order. They don't have to write the code. They just have to arrange the code," he said. Wesley typed detailed lines of the theorem, then posted screenshots of each line on Google Jamboard. "And I also threw in some stuff that was wrong. Some statements that shouldn't be there. So, let's see which one is true, extraneous, or incorrect," he added. These incorrect statements were

common mistakes that students make in writing the proof. He allowed students to work in groups to organize the elements of the proof in the correct order and then remove the incorrect lines. Wesley's choice to replace writing each line of the proof as he talked through it could be a solution to Chaya's question, "if it's a complicated theorem. How can you break it up into little pieces?" In addition to Jamboard, participants used web-based calculators, such as Desmos. For example, Irina said she used Desmos online to explore functions in her college algebra course. In this way, they represent mathematical ideas, and students can interact with them.

One participant, Carly, admitted that she "went back to lecture practice methods online because it's just so much easier." But a few instructors in this study modified how they represented content in various ways. They added elements not part of their usual repertoire (e.g., Maple simulations). Asynchronous activity that resembles a flipped model was another way they adjusted for the emergency online format. For example, they asked students to read detailed lecture notes before class and bring questions to the discussion. According to participants, the adjustments meant more time preparing complementary material for lectures. Although a few participants said they embraced the affordances of presenting content online (e.g., recorded lectures and notes for students), many were concerned presenting mathematical ideas online came with a cost. Most of the participants in the study, showing material and improvising as musicians do, were lost in the online format. It seems that they would agree with participants in the Artemeva and Fox (2011) study who argue that chalk talk is central to teaching mathematics.

Choosing Tasks and Content

The tasks instructors choose to represent mathematics become the conduits to connect students to the mathematical ideas. According to Artzt and Armour-Thomas (2002):

> Tasks provide opportunities for learners to connect their knowledge to new information and to build on their knowledge and interest through active engagement in meaningful

> problem solving. According to the general literature, to foster student involvement, tasks must be motivational, at an appropriate level of difficulty, and sequenced in a meaningful way so that different representations are available to help students clarify and connect their ideas. (p. 12)

When I asked Caelum to describe how this experience (i.e., teaching during the pandemic) changed her as a teacher, she said:

> It's really made me focus on what is the most important thing that I'm trying to get them to understand...I hadn't really ever stopped to think about what is the most important thing that we're learning in this class. It's really made me refocus. I sort of feel like I've rethought what it means to be a mathematician. It's not about being able to do calculations. I mean, you should be able to do the calculations. Sure. But that's not it. What I'm really trying to give them is the ability to think logically, and calculus is just the vehicle for doing that, especially in today's world where people aren't thinking. So, I feel like that's really what I'm giving them—using those tools to get a better understanding of what calculus means. It's not how to do it but what it means, how it describes our world.

Caelum said that because of this new realization, the tasks she chose for her online classroom allowed students to engage actively with the content.

Other participants also said the pandemic forced them to focus on meaningful tasks, which according to Abell et al. (2018):

> provide opportunities for students to develop deeper mathematical meaning for ideas, model and apply their knowledge to new situations, make connections across representations and ideas, and engage in higher-level reasoning where students discuss assumptions, general reasoning strategies, and conclusions. (p. 30)

Moreover, research in undergraduate mathematics suggests that instructors should choose tasks appropriate for the learning environment (e.g., face-to-face vs. online; Abell et al., 2018). In some cases, they focused on conceptual ideas rather than procedural ones. Katie explained:

> I find that mathematics is a very small little corner of human endeavor. And that perspective makes me ask, what do my students as people need? They're not like a little calculus factory. Calculus is what? A percentage of their whole life? And so, I think of myself as developing my students as people and not as teaching them calculus. So, then I think it's that broader perspective that makes me keep asking why am I teaching you to do the determinant of a four-by-four matrix by hand? That is such a waste of human effort. A computer can do that way better than a person is ever going to do it. It just doesn't give you any insight. I mean, yeah, some things you should do by hand because

> they help you understand how things work that way. But that is not an example for almost any of my students in anything that they'll ever do. So, this year, I just refuse to teach it.

Instead, Katie's calculus students used Excel to find the determinant, only when they needed it to solve problems. They built a Markowitz optimal portfolio, wherein if "the covariance is negative, you can get a less risky portfolio than either of the individual stocks," Katie said, "so I had them work all of that cool thing out as an optimization problem."

The events of 2020 (e.g., the presidential election) provided unique opportunities for instructors to bring data and real-life scenarios into the online classroom. As Darren put it, "2020 was the year for statistics teachers because we had so many real, applicable situations to talk about." Carly also said, "because it was the election, we talked about gerrymandering, talked about apportionment, we talked about ways to tally votes like plurality." Social distancing restrictions resulted in long lines to get into stores. Chaya brought this problem to her probability and statistics course: "People are social distancing outside of Trader Joe's, one customer every six feet. They're entering according to a Poisson Process at a certain rate. What is the probability that the 11^{th} customer online will enter before an hour now?" This task was timely and relevant. So too, other instructors planned to design tasks from pandemic vignettes; however, some did not because they felt that students might not have wanted to engage with the topic. After all, the pandemic was wreaking havoc on lives. So, Leon, who taught a mathematical modeling class in the spring, and then in then in the fall semester, said:

> I had done three projects [in the spring], and this time [in the fall], I only did two projects. I got rid of a whole project. Some students were excited about the pandemic project. But some of them just didn't want to do that, which I quite understand. I'm going to be teaching again this spring. I think some of the students are just going to have a complete coronavirus overload, and they won't want it. They won't want to do a section on that. Some people find it really useful. And some people will probably be a bit traumatized by it.

Since the events of 2020 had impacted students negatively, some instructors avoided teaching

mathematics through those topics.

Students could use computational (e.g., Wolfram Alpha) and search engines (e.g., Google) to find solutions to computational and procedural problems in the online classroom. Consequently, the pandemic pushed some instructors to choose tasks aligned with reform goals for the undergraduate mathematics curriculum. They chose tasks that required students to use technology, emphasized conceptual understanding, provided opportunities for students to use representations of mathematics other than symbolic, situated in real-life contexts, and engaged students in mathematical discussions and explorations. In addition, because surviving a global health crisis was taxing enough, some instructors said they kept simplicity in mind and "reduced some of the complexity" (Leon). Thus, not only did instructors choose tasks appropriate for the environment (Abell et al., 2018), they also chose tasks sensitive to the time.

Monitoring: "Leaning Over Their Shoulders"

As a form of ongoing assessment, monitoring provides the instructor with necessary in-the-moment feedback about students' understanding. In turn, they use that feedback to regulate their instruction (Artzt & Armour-Thomas, 2002) and make decisions "in the moment that serve the individuals and the collective" (Franke et al., 2007, p. 228). But teaching online diminished instructors' ability to observe students' work and listen to their utterances by walking around the classroom. Many of the instructors in this study said they missed the ability to "lean over their students' shoulders." As David puts it:

> There's no replacement for leaning over the shoulder of a group and putting your face right in the middle of their faces and looking them in the eye and telling them they've got it. You're going to be great. You've got this. Okay, let's settle down.

Like David, other participants missed the ability to monitor their students' work in person. Many instructors said their students attended class off-screen (i.e., with their videos off) to make matters worse. As Yosef expressed:

> It's different not seeing people. There is an energy and a class you can read from students, facial expressions, and posture. Whether what you're saying is going over and out and making adjustments immediately. That is lost...You have to guess what's working, what's not, and you can't always cast very well.

According to participants, in general, it was hard to "cast very well" in their remote classrooms. This kind of ongoing assessment of students' thinking, vis-à-vis what they say, do, ask, or express through facial expressions and body language, was virtually impossible to replicate.

Participants monitored their students' thinking and activity in the online format using breakout rooms, online interactive and collaborative spaces, polling features, and chatboxes.

Breakout Rooms

Group work was difficult to implement online. In most cases, breakout rooms made it possible for students to collaborate and instructors to monitor student discussions. "Breakout rooms" is a Zoom feature that enables the meeting host to put participants into smaller groups. Other platforms had similar capabilities (e.g., Blackboard breakout groups). Most participants said they struggled to foster productive group work online because breakout rooms were ineffective for various reasons. For example, instructors who used them said that the rooms were often silent—students kept their cameras and microphones off. Another reason breakout rooms did not work well for some participants was that it was difficult to monitor all groups at once, which is different from the face-to-face experience where instructors could cast their eyes over the whole class. As David pointed out:

> Before the pandemic, I would literally have students get in small circles. I would present something on the chalkboard and then give them the problems to work on in class. I walked around from group to group and just checked on them, answered questions, and asked my own questions.

David and others said they visited breakout rooms, one at a time, to have some semblance of monitoring students while they worked in groups. But it was not the same as in the face-to-face classroom. Visiting one room at a time was not the same as looking over the entire classroom.

Wesley shared:

> There's no way for me to sort of wander around the classroom and sort of snoop on what everybody is doing, spot the student who needs some gentle encouragement, and nudge them a little bit. You can't do it. I've seen some workarounds. A Twitter friend of mine has figured out a way to do what he calls a Zoom breakout room panopticon.

Salamone (2020) shared a detailed, seemingly complex process for creating a Zoom breakout room panopticon. "Here is the workaround that I've used to make this happen," he said. "The basic idea is this: You join your own Zoom meeting multiple times, all from the same computer, and then place one of "'you' into each breakout room." (para. 5). Instructors could monitor and interact with all breakout rooms at once. Based on what they said in their interviews, none of the participants created panopticons; however, visiting breakout rooms one at a time was the best they could do to monitor students while they worked in groups.

Online Interactive Whiteboards and Collaborative Spaces

Online interactive whiteboards and collaborative spaces (e.g., Desmos' interactive interface, Miro boards, Google Docs, and Google Jamboard) also provided a way for students to collaborate, and instructors also used them to monitor their students' work. For example, Irina used Desmos and referred to it as "one tool that made a world of difference." She explained:

> Desmos is this free online graphing calculator. I mean, that itself is a great resource. It's free for students. It's actually better, I think, than your little handheld ones, you might get. But they have separate classroom activities that are a mix between the Desmos graphing calculators and PowerPoint slides and response systems. So, it has this great teacher interface where you can see where all the students are at and what they're submitting as their answers. And you can kind of watch all of that in real time. It sort of gives you that in a normal classroom, students are working and look over their shoulder and see what they're working on or just eavesdrop on the conversation in a way that you can't really do virtually as well. And so that interface gives you that. You're literally looking over all of their shoulders at the same time...I love it. I would not teach an online math course without it.

Caelum, like Irina, used a dynamic platform for monitoring students' work. Miro is an online whiteboard collaboration tool. Caelum said she could access students' activity on various boards

from one vantage point. Other instructors said their students worked collaboratively on Google Jamboards and Google Documents in their Zoom breakout rooms. In this way, they could observe students' progress and join their breakout rooms to support and move students along as needed. Unlike Desmos and Miro, instructors could not see various groups' works at once using Jamboards and Google Docs. So, they created separate links to each group's Jamboard or Google document. By doing so, they accessed the board or document via the link and the students' discussions via the Zoom breakout rooms. But several instructors who tried Google Docs and Jamboards to monitor student collaboration said they did not work well. Irina said:

> I tried to do group work at the beginning of the semester. I had Google Docs that they collaborated on in groups. So, I could watch all of the Google Docs in my different tabs and kind of poke into the breakout rooms. But students weren't responding to that very well.

Eventually, Irina said she stopped using breakout rooms. Instead, she "switched and started doing more keeping everyone in the same room and having more sort of instructor-led time but doing like clicker questions with the Zoom polling." In the online environment, digital interactive whiteboards and collaborative spaces provided a way for some instructors to monitor the students' activity during the lesson. At the same time, others chose to get feedback from students using polls.

Polling Features

Polling features were another source of ongoing feedback. "I learned about Poll Everywhere," Katie said, "That was one of my ways to get some insight into what the students were thinking and doing when I can't look at their paper. I really want to look over their shoulder and see what they've written down." For some participants, inviting all students to choose an answer gave them just-in-time feedback that they could use to regulate the flow of the lesson.

Irina shared the following reflection about using Zoom polls. First, she thought she could

use this feature in place of "think, pair, share," a collaborative learning strategy used in classrooms. But she explained why she used the polling feature without the "pair" aspect:

> I'd get the feedback. I started originally showing them the results first and then sending them into breakout rooms for two or three minutes to talk. When they'd come back, I'd poll them again. I realized that in the second poll, they were all choosing whichever was the most popular one in the first round. If I didn't show them the results in between going to the breakout rooms, the result stayed the same. Which left me feeling like I don't think they're talking much in the breakout rooms. I don't think they're getting much out of that. So instead, I kind of skipped the pair part.

Despite her view that "the pairing part was hit or miss," Irina felt that the polling feature allowed students to think about an idea and share their thoughts with the whole group.

Luca used the polling feature to do check-in activities at the beginning of class. Instructors usually ask, when they are in person, "how is everyone doing?" at the beginning of class, with hardly any responses. Luca felt that asking this question using the poll feature yielded informative responses from students. Those who utilized the polling feature of Zoom, like Irina, shared that they would like to find a way to do it in the classroom. The polling feature mimics classroom clickers that allow instructors to view students' responses to their questions. Instead of getting responses from a few students, a polling feature, or classroom clickers, could collect and display every student's response.

Chatboxes

Students and instructors could type a message to the group in the online chatboxes. In addition, they could send a message to each other, if allowed, or the instructor, privately. According to instructors' stories, students were more likely to communicate via chat rather than turn on their cameras or microphones. "Sometimes students would unmute and talk and ask questions. It was the same few who would unmute and speak," Alissa recalled, "but in the chatbox, I got a lot more." So, instructors leveraged this space to monitor and engage students. As Sofía pointed out:

> The chat was always super active, which for me meant, okay, I have to have eyes everywhere. I have to monitor the conversation and be part of the conversation, but at the same time, I'm sharing my whiteboard screen. I have to know what's going on. Students are private messaging me because maybe they want to tell me something that they don't want the rest of the class to see in the chat.

Like Sofía, others shared that their students used the chatbox to answer or ask questions privately. But Sofía felt like she had to "have eyes everywhere" to monitor the chat; however, Luca said his teaching assistant moderated the chat.

Instructors used the chatbox to elicit responses from students—not the sort of chorus response typical in the classroom, but responses from each student themself. The chat feature also allowed instructors to keep track of students participating in the class discussion. Then, they can encourage others to contribute. A few instructors told me about a strategy that, according to Darren, is called "chat blast," which is a mixture of providing wait time (i.e., time for students to think) and polling, with either multiple responses or open-ended questions. Hollingsworth and Johnson (2020) describe chat blast as a strategy "where you ask students to type a message into the chat but wait to hit enter. You wait enough time for everyone to think of an answer, and then instruct everyone to hit enter at the same time" (para. 11). Nellie seemed pleased with the feedback she gets from using chat blasts. "I feel like it gets students to say something," she said. "It gives me an idea of the array of ideas that students are having." Luca said he used questioning in tandem with the chat feature to keep students' attention and break up the monotony of the lecture:

> One of the challenges was how to bring their attention back every five to ten minutes. I tried to ask them a lot of questions during the session. Can you hear me? Is everyone okay? Do you agree with this equation that I just wrote? Can you guys please check my numbers? Can somebody tell me what this number is? So, a lot of questions, little questions like that, that they could reply to on the chat.

Other asked students to respond to questions in the chat to review material from a previous lesson or check for understanding throughout the lessons.

While most participants shared positive experiences using the chat feature, others shared concerns. For example, some instructors said it was challenging to sift meaningful discourse from the rolling text. Liam preferred when students used microphones to communicate:

> If they're going to start typing in the chat with four-by-four matrices and determinants, it's not going to happen. So, I was more interested in really how they would articulate the subject, and the fact that they weren't using their microphones was a problem for me.

Not being able to represent mathematics was a limitation of the chat feature. Another issue Wesley pointed out was that "people just won't type stuff because they didn't want to be wrong." I mentioned earlier that Sofía said her students sent private messages. It could be that, as some of Wesley's students, they were shy about contributing publicly.

Katie shared that she would like to have a chat feature in the face-to-face classroom for what it afforded in the online space. "I don't know how to take this back, but the chat feature on our Google Meets allows students to ask me a question without interrupting me, and I think they like that opportunity, and I want them to have that," she said. Darren shared a similar sentiment: "I don't know if I'll have the technology, but I think that's one thing I will try to kind of bring that chat blast into my classroom," he said. During our interview, he brainstormed about what that may look like:

> We don't have clickers. I'm just trying to find a way where I am asking the question. I'm going to count down, and maybe I'll have some flashcards. A, B, C, D, or some words or something, and expect everybody to give me an answer.

Like Darren, other instructors hoped to find a way to simulate the chat experience in the face-to-face classroom. The chatbox allowed instructors to monitor student thinking and understanding in ways they did not before. It also removed barriers to participating for some students. Particularly in the Spring 2020 semester, instructors said that students who had not contributed to in-person discussions became active in the chatbox discussions.

Although the distance made it impossible to "lean over students' shoulders" and read the

expressions on their faces, participants found other ways to monitor students' progress. Overall, participants kept track of their students' learning during synchronous sessions via breakout rooms, polls, chatboxes, and interactive whiteboards to adjust to the online space. "I don't think it was as good as it used to be, but we certainly tried," David concluded. I believe participants would uniformly agree with this sentiment.

Building a Community of Mathematics Learners and Doers

Franke et al.'s (2007) features of classroom practice for building a community of mathematics doers and learners frame the findings in this section: "shaping classroom mathematical discourse," "establishing norms for doing and learning mathematics," and "building relationships for doing and learning mathematics." Participants described how they nudged, encouraged, and invited students to participate in their learning experience. In addition, they told stories of how they tried to foster intellectual communities of learners and doers of mathematics during the pandemic.

Shaping Classroom Mathematical Discourse

According to Franke et al. (2007), students develop mathematical understanding when they have opportunities "to present problem solutions, make conjectures, talk about a variety of mathematical representations, explain their solution processes, prove why solutions work, and make explicit generalizations" (p. 230). Therefore, an essential aspect of teachers' practice is to provide those opportunities through discourse. Artzt et al. (2002) describe discourse as "the verbal exchange among members of the community in the classroom, both teachers and students" (p. 16). Facilitating discourse was difficult online for various reasons revealed in participant stories. The instructors recounted how they adjusted, in the pandemic classroom, to encourage students to participate and develop their mathematical knowledge and identities

(Franke et al., 2007). I present the findings for shaping classroom discourse in two categories: (1) interacting and (2) questioning and eliciting responses.

Interacting

Across participants' stories, it is evident that teaching from a distance made it difficult for instructors to interact with students and for students to interact with their peers. Participants expressed that teaching mostly names, not faces, on black screens made matters worse. Even those participants who said that before the pandemic, they were moving away from teacher-centered instruction and incorporating more interactive learning strategies had difficulty doing so in the online space. Irina, who used active-learning methods before the pandemic, said that she ended up using instructor-led methods in her pandemic classroom because students felt more supported in those situations:

> I would like them to be doing more active work. We know all the research that supports that learning. But I'm glad I paid attention to the emotional side of it by saying I support you. I'm glad students felt like I was there in class with them, helping them to learn this material and figuring out how to make it active in that moment.

In addition, Engelbrecht and Harding (2005b) claim that instructors are skeptical about online learning because it "is neither personal nor interactive and is consequently claimed to be inferior to traditional teaching strategies" (p. 267). They emphasize that "this is especially true for mathematics that comes with a century-long tradition of verbal teaching" (p. 267). Thus, the structure of their classroom interactions online resembled the traditional lecture-style classroom. But not all traditional practices, according to participants, translated well online.

As Irina puts it, "the face-to-face interaction doesn't flow as well on Zoom, and it's harder to make connections happen"; however, she and others used the tools available to them to foster interactions. In a previous section, I discussed how instructors used breakout rooms and chatboxes to monitor students' progress; instructors also used breakout rooms and chatboxes to

interact with students. Nellie described how she used the chat feature:

> Sometimes I also look at the responses in chat. Sometimes I summarize in my voice and say things like, 'I'm seeing a lot of people say two-thirds. We have a couple of other answers. Let's think together about which ones could work and which ones don't.' Sometimes, especially if a person has their camera on, if they put something in chat that I want to hear more about, I'll say, hey, you have your camera on. I saw your response in the chat. Can you tell us a little bit more about that?

In this way, Nellie invited students to explain their responses verbally. Instructors said that while most students did not turn on their cameras nor use their microphones, the interactions in the chat were more vibrant than they had been in the face-to-face classroom.

A few instructors used breakout rooms so that students could interact with each other and do mathematics; however, they had mixed reviews about student interactions in the breakout rooms. Some said they had better experiences when each student had a role in the group (e.g., note-taker, presenter, the person who shares the screen, etc.) and told students to present their work when they returned to the main room reported better experiences. On the other hand, some instructors said their students rebelled against group work in breakout rooms because, in some cases, they were unfamiliar with their classmates. Several instructors pointed out that in the Spring semester, interactions were less awkward because students had already formed relationships before the pandemic hit; however, instructors said their students did not necessarily know each other in the Fall semester. As one participant in the Engelbrecht and Harding (2005b) said, "when you have a small group of what are essentially strangers trying to work together without any common sense of association, it might provide with a very negative experience" (p. 267). As a result, students did not collaborate effectively in breakout rooms. "I don't think they're talking much in the breakout rooms. I don't think they're getting much out of that," Irina said.

Some instructors said that students refused to interact in breakout rooms because they

preferred to work with the instructor present at all times. Irina explained:

> Breakout rooms did not work well, I think, partly because I couldn't get to every room every day. So, they would come to class and hardly see me. I think students felt they needed to have their instructor there…I think it gave students at least this feeling of support that is sort of implied in a normal semester, just by being in a classroom with an instructor. But right now, with breakout rooms, they can't see me.

Irina ditched breakout rooms. She added:

> So, I switched and started keeping everyone in the same room. I was having more sort of instructor-led time. I asked clicker-type questions with the Zoom polling. I'd give them the question. They'd work through it. I would tell them the correct answer was B or whatever. And I say, does anyone who got that answer want to share how you solved it? Then someone would raise their hand, and they would unmute, and they would explain how they did it. And I would write while they explained. So, they still got to participate and share ideas. It wasn't as much as I would normally like in a classroom, but they still got some sort of class interaction.

Interactions in other participants' classes resembled what Irina described—the instructor lectured and posed questions to the whole class. From their perspective, it was challenging to generate mathematical conversations amongst students. Yet, a few instructors said they moved past supplying the correct answers. Instead, they encouraged students to explain their mathematical ideas with their cameras on or using their microphones. If not, then students typed responses that they could in the chatbox.

Familiarity was a determinant of the nature of discourse and interactions in the online space. According to participant stories, students who belonged to a cohort before the pandemic were more likely to turn on their cameras and microphones and publicly express their mathematical thoughts. For example, Yosef said most of his students turned on their cameras. His students were student teachers. He explained: "I never asked them to turn on the camera. I think it was probably for most of them the only way they could see their friends. This was a cohort that had taken many classes together." From their perspective, interactions in the Spring 2020 semester were better because instructors interacted with students in person for the first few

weeks of the semester. In the Fall 2020 semester, instructors met students for the first time online. As per Sofía, "the spring semester was half in person. I felt like that helped because I already had a relationship with the students. They knew me. I knew them. That we were able to continue that kind of community that we had online." Irina also noticed how interactions in the Spring semester differed from those in the fall:

> In the Spring, I had the benefit of the fact that I had already spent months in a classroom with these students working with them actively. I knew them all by name. I knew their personalities and what they needed. And in the Fall semester, since we've all been online, I haven't been able to get as much of that. I haven't been able to connect with them and in quite the same way. There are so many students that I know I've had in the last two semesters that I wouldn't recognize them on the street. I mean, they don't have their camera on. I don't see them. And finding a way to have a face-to-face conversation with them has been challenging. If we're in person, I can kind of walk around the room while they're doing group work and even just say one word to them. But that doesn't flow as well in Zoom. It's harder to make those connections happen.

Overall, the distance between the instructors and their students highlights the need for personal interactions in the mathematics classroom. For example, consider this reflection from Yosef:

> If the university is going to survive, we have to have features that are not available on YouTube. And what that really means is interaction between students and faculty. And I'll try to see if I can incorporate more humane interactions or find a way to make students comfortable with asking questions because a lot of them aren't. One of the things that I've read is that a surprisingly large impediment is that the students don't know how to address the instructor. I didn't give them instructions [for how to address me], but maybe in the spring, I'll start giving instructions.

Other instructors reflected on reasons for lack of interaction in the undergraduate mathematics classroom and how they may address those issues. For example, Luca emphasized that instructors "have to be more intentional about [their] interactions. Because [they] don't have the luxury of randomness anymore."

A few participants also reflected on how their practices may encourage one group of students to participate and possibly discourage another. For example, David discussed how instructors' actions might favor boys over girls. In his reflection, he also shared:

> For the first time, I had to ask myself, what if all the things I do in class have these unintended, very negative consequences. Like, the way I talk about what it means to be good in math. Am I more patient with the smart kids than I am with the not smart? Like I just suddenly was filled with all kinds of anxieties. Because nothing scares me more than imagining that I'm doing something bad and completely unaware of it. What if my blind spots could be hurting people is a terrible fear of mine.

Like David, the distance gave other participants time and space to reflect on how what they do in their classrooms may affect the quality of the interaction. Zachary told me about a shift in his mindset, from what he noted as equal to equitable instruction:

> I was of the mindset I'm just going to kind of let things go, give everybody the same opportunities, and that's fair. That's what I would think. But I've come to realize in this remote environment that there is no reason why I can't be reaching out to individual students that I notice are having trouble or not engaging.

For Zachary, students had legitimate reasons for not interacting or engaging with the course. This realization caused him to be deliberate in his interactions with students.

Though it was challenging to facilitate student-teacher and student-student interactions in the virtual classroom, instructors said they found ways to adapt. They also reflected on ways their practices may or may not have fostered participation from all students.

Questioning and Eliciting Responses

Questions and responses drive the discourse in the mathematics classroom. They also offer a way for instructors to investigate learning that is taking place. As Wiliam (2007) noted, "the shortest feedback loops are those involved in the day-to-day classroom practices of teachers, where teachers adjust their teaching in light of students' responses to questions or other prompts in real time" (p. 1063). In the online space, instructors posed questions and elicited responses in a variety of ways: narrow questions (e.g., yes, no, or multiple choice) that students could respond to via a poll or the chat feature, or open-ended questions that required students to offer more thoughtful responses via their microphones. Most instructors allowed students to volunteer responses; however, a few instructors said they elicited responses by calling students by name.

Liam explained:

> One of the techniques I use is to call on people in class. And that means they're put on the spot, in a sense. Some people would say, maybe that's unfair. Also, I serve a population where a lot of the math people are going to become teachers. Some of their job will be the issue of responding when somebody asks something that maybe you don't know offhand. So, I feel like it, in a way, is a positive thing too. But in that sense, I'm out of step, maybe with some of the thinking in my current issues about how one teaches in the classroom.

Like Liam, Katie said she continued her practice of cold calling students, noting that she tended to call on her students more when their cameras were off to pull them into the conversation. "I would be in the middle of the derivation and say, you know, Jones, what do you think the next step should be after I do this piece? I cold called students, and then they would talk, and they would talk to each other." Not every instructor used cold calling. From Luca's experience: "People were a little bit more shy, or maybe again they're freshmen. That's another thing. Right. I mean, it's a completely new experience. Sometimes just the name calling could be a little bit intimidating." So, most instructors in this study allowed students to volunteer answers publicly or respond anonymously. Discussion boards on their campus' Learning Management Systems (e.g., Blackboard) hosted streams of conversations; instructors and students could also ask and respond asynchronously to questions.

Students could ask questions in the chat, use the "raise hand" feature, or turn on their microphones to ask. But asking questions and waiting for students to respond, with no utterances and facial expressions to assess, was challenging in the virtual space; however, when students asked questions in the chat, their peers were more likely to respond than when they asked out loud. Students were also less likely to raise their hands or interrupt the flow of the lesson to ask a question. Some instructors incentivized student participation by giving them participation points to contribute to the mathematical discourse. Still, instructors said not many students, as they hoped, took advantage of points towards their grades.

Overall, the online classroom dialogue lacked spontaneity from the instructors' perspectives. For example, consider this reflection from Liam about conversations in his virtual classroom:

> Mathematics as a conversation. [Students] need to see you doing math in the classroom and thinking yourself along the way. So that little bit of spontaneity was diminished. It wasn't totally lost [in the online classroom] because I became more and more skillful about jumping between my notes and the whiteboard and what students had to say. It took a while to get enough people, not by any means the whole class, but I would say five or six out of a class of 35 sometimes who were willing to engage in that way, ask an open-ended question, admit that they didn't know something, and ask for help, about that. Or I just tried to push the topic a little bit more for more information. It just took a while to establish a relationship where that became possible more than it would have taken in person. So, I'm working much harder to create this stuff, and they're probably uncomfortable. They don't have the body language of me. Yes, I'm calling on them, but I'm not angry. I'm asking them, 'what did you think about it?' and giving them a little bit of an opening that's lost in electronic communication.

Amid muted microphones and dark screens, instructors found new ways to generate mathematical conversations amongst students. Like Liam, many of them found that the quality of discourse diminished in the virtual space. But a few instructors, while they agree that the mathematical discourse is not the same as in person, admitted that they found more productive ways for them and students to interact with mathematical content and pose and respond to questions.

Artemeva and Fox (2011) identified *board choreography* and *discursive signaling as* typical elements of traditional chalk lectures that instructors believe impact classroom discourse. By board choreography, they mean not only what instructors write on the board, but their gestures and movements in front of the board in the classroom:

> Not only is the text "moving" on the board, the teachers are moving in space as well, using pointing gestures…to indicate relationships between parts of the chalk talk narrative on the board, signaling points, highlighting key issues, referring to problem sets and textbook chapters that are not necessarily physically present in the classroom, and strategically positioning themselves physically in relation to the text written on the board or to the class as they speak. (p. 360)

Discursive signaling, according to Artemeva and Fox (2011), "not only signposts the process of "doing mathematics"—mathematical logic—on the board but also signals staging points in the lecture, creates spaces for reinforcement, questioning, interaction, and so forth" (p. 362). Instructors rely on feedback and visual cues from students to inform their blackboard choreography and discursive signaling. But online, especially because many instructors could not see students' faces, these features central to most instructors' pedagogical practice were diminished. By Yosef's account:

> It's different not seeing people in class. There is an energy in class you can read from students. Facial expressions and posture, whether what you're saying is going over or not, and make adjustments immediately. And that is lost when you teach where you're recording the class in advance because you have to guess what's working, what's not, and you can't always guess very well.

The lack of face-to-face interactions negatively impacts instructors' ability to ask questions and elicit responses. As a result, the effect on discourse was enough that most instructors look forward to the eventual return to in-person instruction.

Establishing Norms for Doing and Learning Mathematics

An essential element of teachers' practice is constructing norms that support learning in the mathematics classroom. Norms, as defined by Wood (1998), are the "interlocking networks of obligations and expectations that exist for both the teacher and students [that] influence the regularities by which students and teacher interact and create opportunities for communication to occur between the participants" (p. 170 as cited in Franke et al., 2007, pp. 237-238). Teaching online during a pandemic meant that instructors had to cultivate new norms for engagement. Some participants felt the limits of their compassion were tested because they were sometimes unsure whether what students revealed about their lives was legitimate. However, participants said their newfound awareness of students' realities caused them to put flexible norms in place. Here I highlight the top three norms or rules participants discussed: encouraged-but-not-enforced

camera policies, forgiving absenteeism, extending deadlines. A few participants shared their versions of a "COVID-19 syllabus," where they communicated revised norms and expectations—I discuss this briefly at the end of this section.

Encouraged-But-Not-Enforced Camera Policies

Every instructor interviewed for this study was concerned about students' reluctance to turn on their cameras and use their microphones—it was a controversial topic. Nonetheless, participants said they encouraged students to appear on the screen. None of them told me they mandated students to appear on the screen during synchronous sessions. Wesley offered the following reflection:

> I wish I would have had a specific conversation on day one about cameras. Here are all the reasons that I will never require you to have your camera on. Here are all the reasons that I will always ask you to have your camera on. What do you think about this? Let us discuss. Not just give them the choice but give them the ownership of the choice and help them think about their choice. I think that would have made a big difference.

Like Wesley, other instructors acknowledged students' agency in showing their faces on screen. For them, the benefits of students' on-camera presence (e.g., engagement) did not outweigh the discomfort of the alternative. Zachary explained how he addressed the elephant in the room with his students:

> I got over the whole camera thing. That was important. I had to get over it. And it took me about halfway through the semester last semester to realize that students can be perfectly engaged without showing their faces. So why am I going to give them a hard time? I can't force them to come on. So, this semester, I took that lesson to heart. I did a Zoom poll. And I said, something like, my camera will be, and then they fill in the blank: on, always off, sometimes on, sometimes off, on when I'm having a good hair day, things like that, silly things. Then we had a conversation about it. And ultimately, I told them, "I'm not going to push you. I'm going to encourage you to put your camera on, just because there is something to be said about seeing faces. It makes us feel like we're almost in a normal situation."

More than seeing faces, instructors suspected that cameras turned on were indicative of engagement, and cameras off meant that students were not participating. Zachary accepted

alternative ways for students to engage. He maintained:

> For me, it's engagement. Are you answering the chat questions? Are you doing the poll? Are you participating in the breakout rooms, and are you putting things on the slides, on whatever you have to fill in. That, to me, is much more important than showing your face in a meeting.

Though instructors found other ways for students to engage, they reflected on the participation lost in the virtual space and longed to see their students' faces and non-verbal cues.

Usually, some instructors institute rules governing how students participate in traditional college mathematics classrooms. For others, participation is spontaneous and voluntary. Usually, there are no requirements for where students sit in the physical classroom. But in the virtual classroom, everyone has the same seat; there is no front or back of the room. Still, participants admitted that engaging students was challenging both in person and online. Students choose to participate or not for various reasons. During our interviews, a few participants made a connection between students who may have kept their cameras off and those who may have sat quietly at the back of the physical classroom. Zachary shared an interesting viewpoint. "We have students that are quiet and introverted but incredibly engaged," he said. "But unfortunately, they don't fit the model of what we've perceived as the stereotypically engaged student." Teaching in the online environment forced Zachary to consider students' personalities and reconsider norms for engagement.

Instructors shared other reasons for their encouraged-but-not-enforced norm for cameras. Issues of access and equity made it difficult to institute a camera-on policy. Not every student had access to a camera; some joined synchronous sessions via their smartphones. For other students, low-Internet bandwidth made it challenging to use their web cameras. In some cases, instructors shared that privacy laws made it illegal to require students to turn on their cameras. Participants said that they were uncomfortable with surveilling students in their private spaces.

According to Leon:

> I always wanted to err on the side of who knows what's happening at their house. Who knows where they are? What if they don't want to share their situation? I would much prefer to make these people feel comfortable than me feeling comfortable.

Instructors were concerned that some students worked extra hours and picked up new jobs during the pandemic, which meant working during class time. One instructor told me about a student attending class from a delivery truck. For reasons outside of students' control, instructors said they had no choice but to deal with the discomfort of teaching black screens.

Overall, from the participants' perspective, students having their cameras on could improve learning and engagement. Nonetheless, instructors in this study did not enforce camera-on policies nor penalize students for not turning on their cameras. In the end, participants said they found other ways, some acceptingly and others begrudgingly, for students to engage in mathematical activity with or without cameras.

Forgiving Absenteeism

Generally, instructors have explicit rules for attendance. For others, the rule is unspoken. Because traditional teaching and learning rely heavily on synchronous activity, many of the instructors in this study expressed frustration with high rates of absenteeism in their online classroom; however, they responded by adjusting norms and expectations for student attendance. A few instructors gravitated towards the asynchronous option so that there was zero expectation for attendance during uncertain times. "I tried to figure out what's the most equitable thing I can do to keep the course going. So, I was reading all these articles about don't have synchronous meetings...so I turned it entirely asynchronous," Amie recalled.

For those who taught synchronously, the most common reaction among participants was to relax attendance policies and not to penalize students for missing class. "I'm happier about the expectation surrounding attendance," Alissa remarked. She explained:

> In some classes, I used to worry about attendance...So in my intro remedial class, attendance used to be 5% of the grade. No more. In my syllabus now, it says, I'm not going to determine what's an excused absence right now. There's no merit at all in me saying that's good enough to be excused...as far as grading it but also just like the norms around it.

A few other instructors dropped their attendance policies because the effects of the pandemic made it difficult to hold students accountable for absences. For example, students who moved back home in different time zones found it difficult to attend synchronous sessions. But with the shift to online, courses have features that support students who missed a class in ways they had not before. "Everything's online; the recordings are posted," Alissa said. When her students were absent, she pointed them to course materials and recorded lectures, then offered out-of-class support. As a result, understanding students' situations became even more critical during the pandemic. In his reflection, Yosef told me that he revised his thoughts on students' absences:

> There were 19 people in the class. I usually had 12 or 13 when we're in the synchronous class. There were some who are never in the synchronous class; then there are some who are in and out. And the challenge is just if somebody isn't there, the reason they're not there is because they're just not a dutiful student, or are there other things going on in their lives that are messing them up.

In the end, instructors in this study had flexible policies for attendance but maintained the expectation that students should show up for synchronous sessions.

Extending Deadlines

According to participants, issues with missed deadlines and requests for extensions increased during the pandemic. But generally, they offered more flexibility with deadlines and got rid of late penalties. From Therese's perspective, students "need a little just a little pressure, just a little bit to turn stuff in." So, she offered two deadlines: "the real deadline," she said, "and when that one has passed, it will then show them the late deadline, so no one's like, oh, I actually have an extra week to turn this in." Although accepting late work was inconvenient, like with absenteeism, instructors understood various reasons students could be submitting late work.

Some instructors felt that the limits of their graciousness with extending deadlines were tested. They were concerned about deception but were willing to be flexible considering the unprecedented times. Luis admits that it was hard to "gauge being too strict or lenient" when it came to accepting missed work. "We still have to have a certain level standard," he maintained, but felt that it was "hard to enforce when everyone is going through a pandemic."

Classroom norms are not unidirectional. Students also have expectations for how instructors will participate in the classroom community. For example, Irina said the following about her students:

> The students were actually pretty forgiving. When I was like, hey, like I'm going through this too, it's a pandemic, for me, too. I'm doing my best. I care about you. They totally responded to that, which was really great. They were forgiving when things didn't quite get out on time and that I would push the deadline back.

For both instructors and their students, hard-deadline policies were counterintuitive during a time when everyone needed the grace to cope.

In closing this section on norms in the pandemic classroom, I share this quotation from participant Zachary which supports the perspectives of most instructors in this study regarding deadlines and late penalties:

> I tried to adapt what I had previously with respect to late policies. I found that I was making exceptions left and right, more than ever before, and not caring, because we're in the middle of a pandemic. The country was dissolving. Does it really matter that you're turning in math homework two days late? Am I really going to deduct 4% of your grade? Do I really care about that 4%? No. This semester I abandoned all late penalties. Students asked me that the first day or two, do we get penalized? I said no, but you can't do certain things without us having a quick little conversation, and then I'll open things up again if they're closed. It seems annoying and pointless to have those points deducted...You meet students where they are. You want them to do this stuff. I tell them that's really the important thing...The deadlines are for me. I don't want you to feel buried by all the stuff, and I want to be able to give you timely feedback on these things. It doesn't help me if I don't know that you don't know this week one material when it's week 8. That ship has sailed. Everything builds, unfortunately.

For these instructors, deadlines and late penalties are norms put in place to manage learning,

assessment, and feedback. Negotiation and flexibility of these policies were necessary for pandemic pedagogy.

The "COVID-19 Syllabus"

I asked participants to share a pedagogical artifact they created for teaching during the pandemic. A few of them shared a syllabus, a COVID-19 version. As an example, Zachary pointed out, "the word 'Zoom' did not exist in my syllabus, before Spring 20, before Fall 20, which is funny to think about." So, participants revised their syllabus to adjust to the new situation—to provide guidance for distance learning and updated course requirements. What stood out to me were the warm and humane tones communicated across those syllabi, which connect to the ideas of caring pedagogy illustrated in the previous chapter. This is not to say that expressions of grace, compassion, and flexibility did not exist before the pandemic on undergraduate mathematics syllabi. But, from some participants' perspectives, these elements of care have become more prominent in their pedagogy. For example, in her syllabus, Therese acknowledged the unusualness of the pandemic and invited her students to reach out for support. This excerpt is from her syllabus:

> This is not a normal semester. If you are in distress for any reason, do not hesitate to reach out to me. We can discuss solutions that may allow you to keep progressing in the course, but that also work within the contexts of your current life.

Alissa removed the grade for attendance on her syllabus. This was her attempt at less stringent requirements:

> I used to worry about attendance and due dates and stuff. In my intro remedial class, attendance used to be 5% of the grade, no more in my syllabus. Now it says, 'I'm not going to determine what's an excused absence right now.' There's no merit at all in me saying that's good enough to be excused. That's not for me to decide. I felt like people's health history is like an open book now. Like, oh well, you can only work from home if you have a serious medical condition. Well, I am so pleased to reveal to you that I have a serious medical condition. Would you like to know the particulars? It's not okay for

> people to have to say, my grandmother has COVID, or my child has a cold, like none of that.

Course syllabi often include rules for engagement and attendance. Some participants shared what they called "COVID statements" on their syllabi. These were acknowledgments of the current state of the world and how the associated challenges. But mainly, they addressed revised attendance policies. Arthur described his COVID statement:

> It's right under the syllabus. It's just called the COVID statement, and that is basically a list of, what I wouldn't call, policies but sort of. I guess it's like you have a teaching statement or research statement and a covid statement. It's my statement, mostly for students, explaining we are in unprecedented, complicated, and confusing circumstances. These are our priorities; this is what's important. When you approach this class, it is essential that you approach it with those priorities in mind: health, safety, wellbeing. Learning and grades, maybe that's your priority list...The most important thing right now is not attending class. You don't have to attend every class, even though it says attendance is mandatory. What that means is I want everyone to be here every day, not that I will fail you if you skip class because I'm mad at you. A lot of students don't understand that distinction.

A few other instructors shared Arthur's sentiment to include such statements, written in response to the pandemic, on post-pandemic syllabi. "I can certainly distill the important parts of that into some kind of document or section of a syllabus in the future," he said. Looking ahead, Amie questioned whether "there are good reasons to allow students to not come to class if they don't want to." Then, she cited J. Luke Wood's work which supports compulsory attendance. According to Amie, Wood's research found students "won't go to class if it's not mandatory." She also found that failure rates in her class were higher when she did not require attendance. So, during the pandemic, the syllabus states that attendance is required, but she "tried to be lenient about it and flexible if people had valid reasons not to come."

Building Relationships for Doing and Learning Mathematics

Franke et al. (2007) remind us that "teaching is relational" (p. 227). In their view, an important feature of pedagogical practice is "developing relationships with students and the class

in a way that supports opportunities for participation in the classroom's mathematical work" (p. 230). Nellie reflected on this idea: "I think, you know, us going remote has made me realize how relationship-focused learning really is." Participants in this study described an array of experiences that I coded as 'building relationships.' From Franke et al.'s (2007) perspective, these include: understanding students' lived experiences; considering personalities, identities, and mathematical engagement; building connections, getting to know students, and allowing students to get to know each other.

Understanding Students' Lived Experiences

The pandemic disrupted students' lives, particularly those underrepresented in mathematics classrooms. However, based on their stories, instructors gained an acute awareness of students' lived experiences with the shift to emergency remote instruction. When "pedagogical practices are brought into line with the lived experiences of students, student performance is positively influenced" (Franke et al., 2007, p. 244). From this perspective, most participants said that now more than ever, they consider how students' realities impact their interactions in the mathematics classroom.

In reflecting on how the pandemic changed him as a teacher, Mark said, "I'm taking the context of students' lives into mind when I'm working with them. It's something I was doing before, but I'm just really being more conscious of it now." In fact, other participants reported similar shifts in their attention to students' lives. For them, it became clearer that external socioeconomic factors impacted students' abilities to engage in the classroom, particularly because the pandemic exacerbated existing inequities.

At the end of the interviews, I asked participants, "Is there a question that I should have asked you but did not?" David pondered:

> I'm super curious about is whether or not mathematicians in particular, but educators more broadly, are open to the idea of talking about the things that are happening in the socio-ecological model or making conscious efforts to recognize the great variety of environments that our students are trying to learn in.

The Black Lives Matter movement and the sociopolitical events of Summer 2020, David said, made it hard to ignore the experiences of his students of color. Meanwhile, Amie said that she was conscious that these sociopolitical issues affected her students' lives but did not feel comfortable acknowledging them in her classroom. Amie's experience resonates with Franke et al.'s (2007) claim that "bringing the lived experiences of students into the mathematics classroom is both challenging and controversial" (p. 244). But other instructors said they curated spaces in their virtual classrooms for students to discuss how they were experiencing their worlds. Some participants said they learned more about students' lives and experiences than they knew before through students' journals entries and surveys.

Considering Personalities, Identities, and Mathematical Engagement

Students' social identities (e.g., ethnicity, gender, race, and socioeconomic status) influence their engagement with mathematics. As a result, some participants attended to these issues in their online classrooms to improve participation and community building. For example, consider this account from Mark, who explained how he attended to gender equity for participation:

> To build community and really have a way to encourage participation, I made index cards with all the students' names...I tried to rotate through them. I found that it really worked nicely because the students knew I had them, but they really wouldn't see them. I pull names from the index card to make sure I get to everyone at some point. Then I realized, oh, you know what, there's less women, so they're not coming up as much, so let me just have two stacks.

Similarly, David said he considered the relationship between gender and participation when creating groups online:

> There's also a wide range of personalities, and some, mostly men, are very aggressive

> and want to dominate group discussions. They might make condescending, you know, faces or comments if someone in their group isn't what they perceive to be up to snuff. Not exclusively men, but I'm afraid we do have the lion's share of bad interpersonal skills. Anyway, I have to be very careful and conscientious about how I crafted these groups because I don't want them to be negative experiences.

Attention to the structure of groups in the online classroom was helpful for fruitful collaborations amongst students.

Equally important, participants discussed how students' personalities came into play in the pandemic classroom. A few participants in this study became aware of how students' personalities may afford them an experience in the mathematics classroom different from others. For example, they noted that students, who were less likely to engage in person, were more likely to be active in the online chatbox. While there has been a spotlight on mathematics anxiety in the research, participants considered that some personality traits might cause students to suffer from this anxiety more than others. Therese thought timed exams might be challenging for students with, as she put it, "any kind of anxiety."

Nevertheless, some participants recognized their role in developing students' mathematical identities. Both students' individual (i.e., personality) and membership (e.g., gender) identities come together to create inclusive classrooms (Langer-Osuna & Edmonde, 2017). In this way, some instructors in this study described approaches (e.g., tasks they chose, classroom norms, etc.) they took so that students could see themselves as competent mathematics learners and doers. Irina told me about such an activity she did during the pandemic:

> My students read "Living Proof," which is a free eBook from the Mathematics Association of America. It's stories that mathematicians are telling. I said you can pick any two stories you want and reflect on it. I gave them some different prompts like, what's similar to you? What about the story? What do you find surprising? And I did highlight particular stories I would recommend you particularly pay attention to...I kept seeing these responses from my African American students who read stories of African American mathematicians and really felt like they saw themselves in these mathematicians. I want them to see that. Realizing how much of a difference that made, I'm doing this every semester. These students need to see this, and I can only do so much.

> I am one person with one identity, and they need to see all of this. So, every class is going to be required to do this if I can find a way to shove it in there somewhere, for sure.

Irina mentioned her identity. Interestingly, not many, but a few other instructors, talked about their personalities and identities in conversation about building their students' mathematics identity. For example, "I'm not a very power-oriented person," Yosef said, "and maybe some of that reduces the stress that students have in the class." As emphasized by Langer-Osuna and Esmonde (2017), "students' and teachers' identity work is organized around relationships of power and those relationships of power are connected to a sense of belonging to mathematics" (p. 645).

For other instructors, fostering mathematical identity was not a particular activity, but adjustments were made in the interactions they facilitated in their classroom space. In sum, these findings contribute to the scholarship on the connection between mathematical identity and individual or membership identities. In addition, they show that teaching online during the pandemic encouraged instructors to adopt approaches for fostering students' mathematical identities now and in the future.

Building Connections: Getting to Know Students and Allowing Students to Get to Know Each Other

Ladson-Billings (2009) asserts that if students and instructors know more about each other, students develop "a greater commitment to learning because of their commitment to their teacher" (p. 136). Earlier I described how participants in this study learned more about their students' lived experiences, identities, and personalities during the pandemic. This section is about opportunities for teachers and students to connect. According to David, it was necessary "to develop personal connections with [his] students." He said, "I think that was always a big component of my teaching, but it felt this year particularly important because it was harder to do.

I felt like I had to be more conscious and conscientious about it." So too, other participants reflected on the need for personal connections during these uncertain times and planned to keep doing so post-pandemic. The following quotation from Irina also illustrates this finding. "I am working to connect with students a little bit more actively and finding ways to do that more actively...it's something I will keep doing...even when we're in person, that effort is still going to be important," she said. Irina explained that connecting with her students made it easier for her to communicate with them. About a student with irregular attendance, who confessed that he was struggling with online learning, Irina said, "I'm glad that I had enough of a connection with him that I could notice this difference and reach out to him." On the other hand, other instructors said they could not connect with some students, mainly because it was harder to communicate online. Kevin explained:

> The one thing that I'm trying to figure out is a way to communicate to the students better. It's their lack of communication with me, not just in the course, but just in general. In person, if a student needs to talk to you, they just walk up to the front of the room and be like, hey, I need to talk, or they show up to your office. But online, my students weren't checking their email daily. A lot of my students, I'd say, hey, shoot me an email with what's going on so that I can communicate, and then they just wouldn't.

Zachary asked his students to write weekly journals. He said he was able to communicate with students because the journals gave him a way to respond to what they were experiencing. He admitted:

> I always try to get to know my students. But this whole experience makes me realize it was always if they came to me and they talked to me, I got to know them...But it was never me necessarily asking them in. Maybe it was me trying to be respectful of their privacy, and they don't want to tell me their story...But maybe giving them more chances to tell their story upfront, other than just in a journal entry, is good and fine...You know, if a student comes to me to talk, it's usually very serious. I'd like to know what's happening before it's very serious. Not to say I could prevent the serious thing, but at least then I know this person at their better moments, not when they're in crisis. I know it seems like an obvious thing, but it's easy to forget when you have 80 students, a hundred students. Can you actually really get to know every one of those students in that way? But it's worth a try.

Because of ballooned class sizes, other participants said it was difficult to connect with large groups of students.

Zachary noted that collecting journal entries during the pandemic was his attempt to connect with students. “I built individual relationships with the students through the journaling and other things and office hours, whatever else, but I don’t feel like I built that community where they really trust each other,” he said. When students feel connected with each other, “they’re okay making mistakes in front of each other” (Zachary). According to the instructors, connections were easier to maintain in the Spring semester than in the fall because students already knew each other.

Despite the difficulty in making those connections happen in the fall semester, participants described other strategies for fostering connections amongst students in the online space. Instructors used discussion boards, Slack, Campuswire, and other communication platforms to connect students outside the online classroom. Some instructors said they encouraged students to form study groups. Azra said she was intentional about building community in her course. Her students responded to video prompts on Flipgrid, and she required each student to respond to two of their classmates’ posts. “They actually watched each other’s videos, and they made comments, and in the end, I think that connected them outside of the class,” Azra told me. She added that her students in an advanced mathematics course “had to meet together outside the class hours to do their homework together. So that kind of also forced them to make connections. Maybe that could be the trick that they were forced to...Having sort of semi-formal reasons for students to meet outside of class meaning, but with class purposes, I think that that might have helped” (Azra).

I began this section with Ladson-Billings’ (2009) assertion that students commit to

learning when they feel committed to their teachers. I end with a quotation from Franke et al. (2007) that encourages educators and scholars to:

> push beyond seeing teachers' work as eliciting students' thinking in mathematics and consider what it means for teachers and students to get to know one another in ways that lead to different opportunities for participation in mathematics for teachers and students. (p.244)

In general, participants' stories revealed growing attention to personal connections in the mathematics classroom. Though it was difficult to do remotely, instructors in this study gained a heightened awareness of its importance. In addition, they used their knowledge of students to adjust for building community in the pandemic classroom.

Perhaps not seeing their students' faces and teaching to black screens caused instructors to develop a keen sense that community building is as important as planning and delivering instruction. But ultimately, the themes discussed in this section, "shaping classroom mathematical discourse," "establishing norms for doing and learning mathematics," and "building relationships for doing and learning mathematics," were necessary ingredients for building a community of mathematics learners and doers in the online space.

Assessing for Learning During the Pandemic

As the traditional ways of assessing students (i.e., proctored, timed, closed-book tests, quizzes, exams) became virtually impossible, participants were disoriented in the online space. According to Luis:

> The assessments were another challenge. How do I give a test? It was challenging for me. How am I supposed to give a test? Am I supposed to make everybody turn on their cameras? What if they don't have webcams? Not everybody has a webcam. So, monitoring and keeping people accountable, that's hard to do when you're far away and when you're physically in two different places.

In response to the shift to the remote environment, instructors shared that they struggled to assess learning in meaningful ways. Still, assessment in undergraduate mathematics education, based on

this study's findings, is a world divided.

Some instructors looked for ways to continue traditional assessment methods online and planned to return to those practices when they go back to the face-to-face classroom. On the other hand, a few instructors, forced by the pandemic to reconceptualize the purpose of assessments, said they would consider the new methods in their post-pandemic classrooms. Others said they moved away from traditional means of assessment before the pandemic, and teaching during the pandemic affirmed their reasons for doing so. In any case, some participants used this opportunity to rethink the purpose and methods of assessments. In contrast, others maintained that the 'old way' is the most authentic way to evaluate learning. But Sofía concluded that:

> Some things that maybe we were afraid to do before in education, I feel like now you might have to, or we have more of an opportunity to be able to do it. And so, the same goes with assessments. I mean, alternative forms of assessment. So not the same old, same old. I mean, we weren't getting stuff that was useful anyway, so why not rethink the way we're assessing students and thinking about the purpose of assessments.

Despite their take on assessing students' learning—its purposes, the best ways to do it, how to grade it, and what happens after—doing it online was challenging for all participants in this study.

It was time-consuming for all, and they struggled to determine the best ways to assess learning online with integrity. Some participants said that differences in students' access to the Internet and technology, and their varying levels of literacy for technology, drove their decisions about assessment. For example, timed assessments were problematic because not all students had a strong Internet connection. In addition, because students could not turn in physical papers in the pandemic classroom, instructors required them to scan and upload documents for some tests. But students' varying levels of literacy for technology made it challenging for them to submit written solutions electronically.

As an adjustment, some instructors shifted from written explanations to multiple-choice or short-answer assessments in online education platforms (e.g., WebAssign, WebWork, etc.). Thus, they shifted a lot of the old ways into new formats that were difficult for them to monitor. A few instructors who considered lockdown browsers and other monitoring systems admitted that not only did lockdown browsers and camera surveillance have technical hitches, but they were also inequitable. For example, in Pierre's department, they "were strongly encouraged not to do proctored exams, in part because of the Internet issue. Also, in part, because the proctor would require money. And our students aren't rich." In some cases, instructors said students would be required to pay for the proctorial services.

Indeed, cheating was a significant theme across participant stories. As Aryeh put it, "cheating was enormous." Academic dishonesty motivated shifts in modes and purposes of assessments. As Nellie put it:

> I think we're all doing assessment differently now, and I think that a lot of the things people are coming up with are just so much more useful, so much more reasonable, more interesting. I do think that as terrible as a pandemic has been in many different areas, it has opened up the possibility for positive change.

With this in mind, the focus of this section is to highlight how participants assessed learning online in response to teaching online during the pandemic: frequent, low-stakes assessment; open-book, open-notes, take-home, untimed exams; the holy grail: writing non-Googleable, non-Cheggable questions; alternative forms of assessment; grading assessments and giving feedback.

Frequent, Low-Stakes Assessments

"I think I should have went to the low stakes model many years ago," David said, "I know that our colleagues in education have been talking about this for some time. It's just some of us are a little slow to follow." David and others moved away from two or three tests and administered smaller, low-stakes assessments frequently in their online classrooms. For example,

David explained:

> I got rid of high-stakes assessments and replaced them with more low-stakes assessments. So specifically, we had seven midterm exams, essentially one every other week. We did not have a cumulative final, and we did not have a big midterm. We had seven small midterms instead of what I used to do, which was a single midterm and a final.

David found this change valuable and said, "I think I will do those again." Other participants agree that replacing traditional midterms and cumulative final exams with smaller, frequent assessments is worth keeping.

Those who made this adjustment had different motivations for increasing the number of assessments and lowering the impact of each one on students' grades. "How do you prevent students from cheating?" Kevin asked, "To me, the answer is, do it differently. I am not doing exams. I'm only doing smaller quizzes scattered throughout the term. Make them more low stakes." Some participants viewed these smaller, more frequent assignments as less formal assessment methods; therefore, they required no surveillance and deemphasized grades.

Preventing cheating was not the only motivator for increasing the number of assessments and making each one weigh less. Supporting learning from a distance brought a need for increased feedback. "I tried to give more feedback to try to reduce the distance," Yosef explained. For Sofía, more assessments were necessary so that students could "know how they're doing in the course. They need to know what they need to fix or change or improve on before the next exam." Similarly, Katie said that frequent quizzes allowed her to "look over students' shoulders" remotely:

> I didn't do a lot of high stakes enormous exams. Instead, I was doing quizzes every day or once a week...And I feel like my students really benefited. It was like saying I couldn't look at their paper and offer them immediate help with what was going wrong. But if a lot of people struggle with something on the quiz, then on the next Monday I would explain to them. Here's the thing that I saw a lot of people do. Here's why that wouldn't work; here's what you should do instead. The students remarked that that was one of the most helpful things of the class for them. So maybe normally, I would do it over their shoulder as they're learning the concept. Instead, I did it after they turned in their quiz.

Frequent feedback from students' quizzes gave Katie information for subsequent lessons. Wiliam (2007) underlines the importance of assessment as an "interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it" (p. 1054). What instructors did with the feedback from these assessments varied across participants. For example, some allowed students to revise and resubmit their work. On WebAssign, an online educational platform, Caelum gave her students unlimited attempts to respond to quiz questions. In this way, they could use automated feedback to revise their answers.

I asked participants to tell me about a practice they were reluctant to try in the past. Chaya said giving frequent assessments is something she never did before but tried during the pandemic:

> One student wrote that 'giving a quiz every third lesson doesn't let us fall behind. It's wonderful. We can't put your course on hold. We have to stay on top of it.' It was a lot of grading, but it's for the benefit of the students. The more quizzes, the more they have to stay on top of it. Just giving a midterm and a final is a cop out because they're not going to work. I never gave a lot of quizzes, but I have to...It helps the students.

By "stay on top of it," Chaya implied that students remained diligent about and engaged with the content.

Despite these benefits, grading the increased number of assessments was time-consuming and arduous for Chaya and other instructors. As a result, some instructors used online learning systems (e.g., WebAssign) and Google Forms to administer quick assessments and alleviate grading time commitment. This way, grading was immediate, and feedback was generated automatically. Although David was not satisfied with the feedback students got from automated assessments:

> In the beginning, you're like, oh my god, that's amazing. I can't imagine never having to grade homework. Again, that's awesome. And it is super convenient, but I do feel like

students don't get feedback the same way they would if you're grading their homework. I will discuss the grading and feedback in more detail later in this section; however, suffice it to say that frequent, low-stakes assessments, despite the tedium, were common adjustments to assessing learning online.

Open-Book, Open-Notes, Take-Home, Untimed Exams

Closed-book exams, a routine in undergraduate mathematics education, abruptly came to a halt with the emergency shift to online instruction. Instructors, some begrudgingly, allowed students to consult their textbooks and notes during exams. I use the word begrudgingly here because most of the instructors in this study favored the more traditional option; for them, closed-book tests ensure the most rigor. Also, I used the word allowed here loosely because instructors had no control over whether students used notes or not—the distance made it difficult to monitor and regulate students' behavior. "I can't walk around the room looking at students and seeing if they have a little jib sheet nearby," Aryeh said, "the fact is it has to be open-book open-notes." So, in the end, all assessments were open-book, open-notes, despite some instructors' preference for closed-book exams.

Instructors found it hard to trust their students and the validity of open-book, open-notes, take-home tests. First, some students not only looked for answers on the Internet but went to companies like Chegg for homework solutions. As a result, instructors were concerned that students submitted work that did not represent their knowledge. Second, some instructors argued that open-book, open-notes assessments do not prepare students for on-the-spot recall and retrieval of information, which Liam argues is important. For Liam, closed-book tests assess students' ability to think, and problem solve in novel situations where there are no materials at their fingertips—what Liam calls "stand and deliver":

That you have to be able to stand and deliver sometimes, it's not unreasonable to test

> people. That the information is distorted is a given. But you try to overcome that with the law of large numbers. You have to have enough different avenues that you're measuring between homework and tasks and conversation in the classroom. So that you hope to smooth out what are the imbalances in testing.

The essence of Liam's argument is that closed-book tests may not offer adequate feedback about students' knowledge; however, for him, testing how students think on their feet is necessary. "If mathematics is done correctly, it will teach you to think. It shouldn't be all memorizing, but a little is good," Chaya commented.

Some participants argue that memorization is also necessary for acquiring and communicating deeper knowledge. In Alyssa's view: "Mathematics is a language, and you can't communicate if you don't know the vocabulary of the language...There are some vocabulary ideas that you have at your disposal just as communication tools." Chaya, who said she would not allow students to use their textbooks or notes when they return in person, shares a similar sentiment: "I don't want open book exams because I feel there are certain basic things that they should know that will help them. They can't look up everything," she added.

While some participants were not thrilled about open-book, open-notes, take-home exams, others welcomed the change. For instance, Yosef, who has more than 40 years of teaching experience, admitted:

> It struck me over the years that the way we test math does not reflect the way people use math in the real world; nobody, to my knowledge, has ever said I need to know the antiderivative of the secant stat. People who use math use notes; they talk to their friends. There's not a rush to get the answer. And I guess I was exploiting the opportunity of the virus to do tests the way I thought tests should have been done all along.

As a sort of counterargument to Yosef's claim, Liam had the following view:

> The standard thing you hear is, in the real world, people work in teams. So, the idea that you ask other people to help you is actually a valuable skill; I think that's a false reading of the situation. Yes, people work in teams. But you're expected to know something, not expected to have the team do the work for you. That doesn't last very long.

Again, he argued that students should have formal assessments without access to supporting

materials, whether from books, notes, or friends.

As far as timed exams are concerned, instructors also had opposing views. Some instructors administered and proctored tests during a synchronous class session in the way Luis explained:

> I tried my best to replicate a standard exam day. What I did was I told them beforehand that they have to have cameras on. I told them that I want you to present your working space so I can see where you're writing. I told them that I will post like it during our scheduled time slot. So, the moment our class starts, if it's at 8:45, I would be on Zoom as well. I posted the exam as a PDF on Google Classroom when the time struck at 8:45 am. They would be able to see the PDF, and then once they have the PDF in front of them, they can even print it out if they want. They would write on a separate piece of paper, and when they're done, they can upload into Google Classroom before the end of class.

Luis did not encourage open-book, open-notes; however, he admitted that students might have found ways to access materials he could not see via his Zoom camera. The reasons participants gave for administering exams this way centered around surveillance and ensuring academic integrity. But Caelum, who timed exams in the Spring 2020 semester, admitted that she chose not to do so in the Fall 2020 semester because it was stressful for students, particularly in the online space. Students who had Internet connection issues while taking the exam, for example, panicked about not submitting their scanned exam on time. Some instructors made their exams like traditional take-home tests to account for inequities in students' access to technology (e.g., a printer at home and reliable Internet access). They allowed their students to take the exam outside of the scheduled class time. In some cases, instructors gave their students 72 hours to submit the exam online. Yosef explained:

> I'd give it out, say on Thursday, cancel class on Friday. And then the test would be due Saturday at midnight. And they were not timed; they were open books, open notes. The only thing I asked them to do was to take it in one sitting and not talk to anybody.

Similarly, Amie also offered open-book, open-notes exams but discouraged students from consulting with each other. "But then what ended up happening with the exams is that the

students worked together, which was not allowed," she told me, "You know, it's open-book, open-notes, not open collaboration."

Instructors said that in addition to notes, they allowed students to use their calculators during online exams. Irina, for example, gave her students access to this resource. She explained:

> I kind of realized that for a lot of the questions—even though you certainly could solve it without a graphing calculator, as mathematicians, we probably would solve it without a graphing calculator—There was this level of comfort, the students felt, let me just grab it and see what's going on. And I don't blame them. If I'm exploring a new function as a mathematician, I look at the graph to see what's happening. But the graphing calculators are not cheap. And since we moved our exams online, we said, let's just give them access to the Desmos graphing calculator during the exam. And students loved having that access. I don't know how we would figure out how to do that with a paper exam or if we might try to do an online exam still; it's not clear. But that universal access to this resource, that even if technically they don't need it, still gives them comfort in what is otherwise a very stressful scenario. I'm hoping to push for that and figure out a way to make it happen when we go back to regular in-person teaching.

This account from Irina represents a unanimous sentiment across participants' stories: in response to a stressful and challenging time, they adjusted their assessment practices (e.g., allow students to use resources such as the graphing calculator). I end this section with a quotation from Annika's story, who said she changed how she gave exams because she "was trying to make it as easy as possible for everybody." But, she added, "considering all the craziness that's been going on, I feel like my motto the whole time has been like we're still in a pandemic."

I mentioned earlier that assessment is a world divided. So, too, is the world of open-book, open-notes, untimed exams versus closed-book, closed notes, timed exams; however, the shift to emergency online instruction forced participants to assess differently—some reluctantly, others with openness.

The Holy Grail: Writing Non-Googleable, Non-Chegg-able Questions

"I just tried to change everything, so it just wasn't Googleable and Chegg-able," Darren emphasized as he told me about the kinds of questions he wrote for his online exams. Not only

were most instructors' exams open-book and open-notes, but the nature of the online environment also gave students open access to the Internet. As a result, some instructors said they wrote questions differently from when they taught in person—questions that a Google search or a Chegg request may not fully answer. Their goal was to stop students from cheating or at least reduce their ability to find solutions online. Caelum told me that she "threw out all of the memorization stuff, all of that got offloaded to homework." She explained, "the Internet can do calculus computations, Wolfram Alpha can do it, you can go to Chegg, or somebody else can do it for you." Katie referred to such questions as "pattern-parroting questions." She added:

> We need exams that recognize that people are going to have the whole Internet at their fingertips...we can ask them things that are realistic instead of asking them things that Wolfram Alpha [a computational engine] will print out for them on the screen anytime.

As an adjustment to the open-book, open access-to-the-Internet environment, some instructors found that "non-Googleable, non-Chegg-able" questions better assess students' understanding. For example, during our interview, Caelum, who said she "got better at writing the assessment questions," shared an exam she gave her students:

> This was an example of one of my assessments. You can see, for this one, in particular, this was one where they had to understand the different properties of the definite integral in order to answer the question. So, this isn't something that they could put into a search engine and get an answer, for they have to know that.

Although she asked conceptual questions in upper-level courses in the past, she admitted she had not done it in calculus before the pandemic. Caelum compared the feedback she received from the assessments she gave before and during the pandemic:

> It was so clear what they didn't understand, and I honestly don't think I ever got that on one of my in-person exams. If I gave them some sort of computation to do, I didn't really understand what part of that they didn't get. But for these, it was pretty clear.

Like Caelum, other instructors acknowledged that they wrote more conceptual questions in the online classes. They asked students to explain their thinking, in some cases, to stop students from

cheating.

But these kinds of conceptual and novel questions were not necessarily easier for students to answer. “I can make the problem, and it turns out to be too hard,” Liam acknowledged. As Katie pointed out:

> Students have learned for so many years, from whatever other math classes they had before, that what’s important is getting this right answer, the same answer that’s in the back of the book as fast as possible. Solving questions where there isn’t an obvious direction to go is difficult for students. Maybe that’s too highfalutin; maybe not everybody needs to be able to solve questions that have never been answered before. But I feel like that’s the kind of brain we want to develop. Maybe I’m wrong about that.

Caelum shared that she came to this realization before the pandemic. But for a few other instructors, the realization came with the shift to online instruction during the pandemic. Sofía urged other instructors to seize this opportunity to rethink their assessments:

> This is a time to experiment...why not rethink the way we’re assessing students by thinking about the purpose of assessments? When different types of assessments are appropriate...even though it’s a scary time. I think it's also a good time.

Sofía’s point about rethinking the purpose of assessments resonates with one observation I made while analyzing participant stories. Participants’ decisions for how they assessed learning before and during the pandemic—the questions they asked, their modes of assessment—depended heavily on views about what it means to learn and do mathematics.

Another significant aspect of writing “non-Googleable, non-Chegg-able” questions, I found, is that some instructors collaborated with colleagues to do this work. Katie admitted that writing those questions was not an easy feat:

> It’s a holy grail. We wish we had cool exam questions, just buckets, and books full of them that are meaning-focused. They’re questions that require the student to understand what’s going on, questions that inspire them to want to figure out the answer, questions that have a cool AHA in the moment.

Caelum agrees. Because it was challenging to write thoughtful questions from scratch, she worked with a colleague:

> One of my colleagues helped me make these exam questions that we both used. We never used them in exactly the same way. It was like, here's an idea, here's a thought. We'd even share the actual questions we made and then use them completely differently.

A few other participants worked with colleagues to develop a 'holy grail' of questions or at least close to it. They sought questions that decreased the chances of academic dishonesty. In turn, they created questions that diagnosed students' needs and monitored their progress (Steen, 1999). Moreover, discussing and sharing ideas with colleagues about the purposes of assessments was helpful during this isolating time.

Alternative Forms of Assessments

Most of the instructors in this study continued with the traditional style of assessing students by giving quizzes and exams in their online classrooms; however, some instructors moved away from this culture with the shift to remote instruction. They told me about the other methods of assessments they used to reduce the stress of testing during what was already a difficult time; to reduce the emphasis on grades; and to determine whether students were learning, what was going wrong, and what to do about it (Wiliam, 2007). Oral exams, verbal and written communication were the most common alternative assessments participants used in their virtual environments.

Oral Exams

According to a few participants who used them, oral exams allowed them to probe and guide students in real time; however, they made it clear that having small classes made utilizing this mode of assessment possible. Irina explained:

> I did oral exams for my classes. They were smaller classes, so that helped, maybe 20 students or so. For the midterms, they would have a standard written, take-home exam. They turn it in, and then they would have to meet with me afterward. We would talk about what they turned in and explain why they made the choices they made. What if this number was different? How would that change how you would solve it? Things like that. I loved it for so many reasons. I don't think I would have tried it if we weren't online. I think that was the thing that really pushed me to say I gotta do something different with

> the way we do testing. Especially with these were four courses that were asynchronous, so we didn't otherwise have a scheduled meeting time unless they came to office hours or things like that. So, this meant that every student had to see my face and have this conversation with me a couple times a semester. That helped a little bit with keeping track of everyone and connecting with them. I would do it again in a heartbeat with an online class or not. It just needs to be smaller classes to be manageable to have that many meetings.

From Irina's account, giving oral exams in her Introductions to Proofs course extended the assessment beyond what students wrote on their papers. She could have a conversation with her students about their mathematical thinking, meanwhile building connections with them. Recall that I asked participants to share a pedagogical artifact that represents their pandemic teaching experience. Irina walked me through the guide to her final exam, an oral exam, which she said was inspired by a Twitter post from Francis Su. To illustrate the nature of the reflective questions she asked students during the oral exam, consider the following question from Su's (2020) "7 Exam Questions for a Pandemic (or any other time):"

> What mathematical ideas are you curious to know more about as a result of taking this class? Give one example of a question about the material that you'd like to explore further and describe why this is an interesting question to you. (para. 14)

In her reflection, Irina said that her students enjoyed this format: "They had a lot of fun...I really enjoyed seeing them sort of flourish." Although, she admitted that she was able to do that sort of final exam because she had already assessed the course content before the semester ended.

Leon did not use oral exams but planned to use them in his calculus course after he observed one of his colleagues administering a group oral exam:

> Another thing that I think I'm going to use when I teach multivariable calculus next semester is something I saw [a colleague] do when she did an oral exam. So, you get them in groups of four or five people. Then, you have them each explain their answers for these questions. The thing I like about that is that you're really seeing the people talk about math in a way that conveys their understanding. It's intimidating the first time they do it until they realize that you're not there to jump on them; you just want to see that they've learned. An oral exam can be more of a discussion than a scary exam. It'll also allow you to ensure that the people know what they're talking about and haven't just either copied from somebody else. They can actually take the ideas and put them into

> words which is really what you'd like somebody who understands some mathematical concepts to be able to do.

Like Leon, Nellie got the idea of students' collaborating on assessments from a colleague. Except, she implemented the idea and shared her experience during our interview. Her students did the oral finals together, and each took turns answering questions from the group homework assignments. "I thought that was a good way to sort of motivate students to work together," she said, and students "can utilize each other as resources." Annika cited "Collaborative Oral Take-Home Exams," from *Assessment Practices in Undergraduate Mathematics*, as helpful for implementing group oral exams. She, too, did it with a class of 20 students. Though she was reluctant to try this mode of assessment before, she said that she plans to take it back to the face-to-face classroom.

Those who considered, but did not try oral exams, explained that they were reluctant for a few reasons. For example, Leon pointed out that oral exams can intimidate students. In addition, instructors were also concerned that students would collaborate dishonestly (i.e., for group orals). But Nellie, who was also reluctant in the past, explained:

> I think that oral exams can be completely intimidating, and that's not my goal. I don't want to frighten the students. I like the fact that it was sort of interactive. If students get stuck, I can sort of give them hints; therefore, potentially see a broader understanding or a deeper understanding depending on what exactly happened in that interaction. So, I liked that because I feel like sometimes, in written exams, students just get stuck and then quit. What has prevented me from doing it before is some of it is certainly chugging along. This is working fine; let's just keep going doing this thing. I'm not sure oral exams would have really even occurred to me before because I think the students would have found it really intimidating. So, it's a mixture of not wanting to frighten the students, but probably more so just let's just keep doing this thing we've always done that's working fine.

Nellie's point about inertia, that is, "let's just keep doing this thing we've always done that's working fine," was another reason participants shared for not using oral exams.

Verbal and Written Communication

"One of the things I added to the course this year, as an explicit component of their grade,

was based on communication skills," David said. "I had never put anything in their grading before that wasn't some sort of math assessment." So, he and a few other participants in this study assessed students' ability to communicate mathematics. David explained:

> I had an assessment that was about communication skills. We practiced it throughout the whole semester when I would go into their breakout rooms. I always make sure that one person was presenting the problem and they're talking their way through it. They are the presenter, and that was their role. We talked about what it means to be a good presenter, and I wrote down some bullet points. So, they practice this communication in breakout rooms all semester. At the end of the semester, everybody had to give a five-minute presentation. I gave everybody a problem. Their final assignment for the year was to present that problem to the whole class, and they were graded on knowledge of the subject, articulation, and time management, or something like this. Basically, the first two were, how well did you know the problem and how well did you articulate it...I was pretty satisfied overall with how it went. I feel like it was a useful addition to the class.

Like David, other instructors asked students to explain their work, either verbally or in written form, so that they could assess how students represent their mathematical ideas, not as an oral exam, but more informally. David pointed out:

> If you just give them a bunch of derivative and integral problems, they can knock them out real easily. But to be able to explain the calculations is much harder for them. They can mimic somebody's manipulations without thinking about what they're doing. But to get them to explain that is not a skill they've ever practiced before.

Participants' students communicated their mathematical thoughts and engagement in various ways: during class sessions, Flipgrid videos, and journal reflections.

David explained above that his students practiced explaining their ideas in small groups in Zoom breakout rooms and then presented the whole group at the end of the semester, whereas Leon's students posted videos of their presentations on Flipgrid. "I kind of don't want to go back to in-class presentations," Leon told me during our interview. Then he shared a recording of one excited students' presentation. Leon explained why, for him, Flipgrid videos might replace in-person presentations:

> Because it's an online presentation, I can let it be whatever length it needs to be and not be restricted by how many students there are in the class or how many presentations there

> need to be. It allows the students to redo it until they're happy with it. So, if they realized that they didn't do a good job the first time around, they could redo it. It also lets them reply to each other's things, so there's support. They can say things like 'I like your project idea and topic idea.' So, these students are giving nice feedback. There's a lot of good stuff in this Flipgrid for the presentations. I want to use that again.

Like Leon, others who used Flipgrid for student presentations found it convenient; they did not use class time for presentations. Students presented their ideas asynchronously; their peers provided feedback asynchronously, too. The instructors did not ask students to watch all the presentations, as they would do when presentations occur in person. Instead, Leon assigned each student five videos to watch and give feedback. He asked that they watch five other presentations. "So, the reason why I feel like that works well is it means that you don't have to watch too many presentations," he explained, "but you get feedback from five other people." VoiceThread was another way for students to submit recordings of their mathematical explanations.

Some participants used journals to access students' reflections about their engagement and progress in the course. In those journals, participants said students communicated experiences happening in their lives, but they also used them to reflect on their learning processes. Nellie explained:

> I decided to just have my students write little journal entries like four or five or six times during the semester. I just wanted the students to sort of look at their own learning process. I have them write after every chapter that we had been going through in our text. What was your favorite part of this chapter? Why did you like it? Where did you struggle the most? So, the questions were both content-focused but also sort of analyzing one's own learning and the progress that's being made. I was trying to get an idea of what students were thinking about these things. It was a way to stay connected with them on an individual basis when I could no longer see any of them in person. I think the students really took it seriously. It was not an arduous task but, but it was a way for them to just take five minutes and think about it, and also as a way for me to read it and then respond and reaffirm, 'yes, I've noticed this as well, or oh I'm surprised you say that because I think that I was really proud when you did blankety blank.' I mean, this sort of feedback. It helped me feel more connected to my students that helped me feel like I could at the very least try to help the students that were not necessarily noticing their progress to see something.

This quotation from Nellie captures the essence of what other participants shared regarding their reasons for asking students to respond to journal prompts.

During the interviews, I asked participants to tell me about a practice they used during the pandemic that they were reluctant to try in the past. “Journals were a thing for classes that I’ve been reluctant to try,” Therese said. “Because this is the first time I’m not like ‘oh my God, the job market, it’s going to ruin my evaluations.’” As an aside, Therese and other early-career faculty in this study shared similar concerns about trying new approaches in their classrooms. I asked Therese why she thought assigning journals could ruin her evaluations. She explained, “having students write journals about their math classes is sort of unconventional because you bring writing into the math class.” She added, “but journals tell me how they have been engaging with the material or something...The students who benefit from it benefit a lot because they feel like they’ve got this direct line of communication to me because I answer their journal entries.” They typed them in Google Docs; this way, it was easy to refer to the previous weeks’ reflections. When I asked her what practices she tried during the pandemic that she would take back to the face-to-face classroom, she said, “journals are coming in some format.” Other participants who assigned journals for the first time during the pandemic expressed a similar sentiment.

Grading Assessments and Giving Feedback

Earlier I mentioned that most participants in this study increased the number of assessments in their online classrooms. While this was meant to reduce the emphasis on grades, they said they spent more time grading those assignments as a result. Alissa explained that she was “protecting both their time and [her] time by only doing meaningful tasks.” She said, “If I’m spending way more time giving students feedback than they’re spending on the assignment, it’s

not a good assignment." So, Alissa modified her tasks to save time on grading. Automatic grading was another way for participants to reduce the time spent grading. For example, Luca and his team of teaching assistants assigned activities "that usually are graded automatically in Canvas. If they submit something, they will get a point." He added, "I tried to make a lot of the grading automatic as possible because otherwise I'll be just grading night and day."

Not only was grading time-consuming, but participants said that doing it electronically was challenging. So, they needed easy-to-use tools that make grading less-time consuming and afford them a smooth feedback process. In the pandemic classroom, most participants, in place of collecting papers, asked students to upload their assignments to the course hub (e.g., Google Classroom). Then, they graded pictures or portable document files (PDFs) of student work on their touch-screen devices. But a few of them used grading software. "We used a tool called Gradescope that I really like," Luca said. "It is very useful and makes the grading very simple." Leon and other instructors who used Gradescope had similar reviews. Leon explained why he stopped using Google Classroom for grading purposes:

> In the spring, students uploaded their assignments to Google Classroom. That was a mess in terms of getting things. How many clicks do you have to do to grade an assessment and then give them feedback? It was just so many clicks. So, I had my ear to the ground to hear for a different option. There's this thing called Gradescope. And what Gradescope lets the students do is upload pictures of their work. What I liked is that the grading was just so much more smooth in terms of giving them a score, clicking the next button, then it showed up on the screen. You don't have to flip the pages. You don't have to click on ten different things. You click one button; it goes to the next one.

Kevin did not use Gradescope because, like a few instructors in this study, grading in his course was done automatically in an online education platform, Pearson's MyMathLab; however, he admitted, "in the ideal world, that's not how I would actually grade things." Thinking about what he will do differently in the Spring 2021 semester, Kevin said, "I'm going to use Gradescope. Some people I know use that for their own grading, and they said it's much faster. So, I'm going

to experiment with that and see if it works."

A few participants implemented new grading practices and explained that adjusting due to the demands of the new remote environment provided them with substantive feedback for improving learning. "The way I do feedback now is definitely as a result of the pandemic," said Darren, adding that now he tries to garner more information about students' thinking. "I'm really trying to see what they're thinking and try to give them feedback as if I'm in their minds," he said. Insofar as learning-focused feedback, many of the participants referred to standards-based grading, or mastery-based grading, during our interviews. Although these are newer formats in undergraduate mathematics education, I was not surprised that participants spoke about them—there are advocates everywhere. According to Spencer (2012), "Standards-based grading derives from the idea that teachers ought to have clearly defined academic goals for their students, be able to determine if they've met them, and then communicate that to students" (p. 5). Students use the feedback to revise and resubmit their work. The goal is to reach proficiency, or mastery, of the course standards or objectives.

A few of the participants who spoke about standards-based grading during the interviews adopted this method for the first time during the pandemic. Others used these practices before the pandemic but doing them in the new learning environment was not without challenges; they either stopped or modified their practices. The rest considered it for their new remote classes but decided not to implement it for various reasons. "I didn't use mastery-based grading this quarter because I didn't want to figure that out, for the first time, while teaching online," Annika explained. Kevin, who said that "students really benefit from opportunities to revise their work," also said that he did not take the standards-based approach because "it was too much during the pandemic, my classes are so large, it became difficult to manage the retakes." According to

Leon, “the biggest downside for standards-based grading is just the amount of record keeping and the number of reassessments that have to happen.” As a response to the demands of the pandemic, he said he reduced the number of standards students needed to master; however, he said, “I don’t feel like I compromised at all in terms of whether this student understood the concept or not.” Reducing the number of specific standards was one way to adjust to the new normal. Amie managed retakes by retesting old standards on new exams. Also, students who were close to mastery could email their work instead of scheduling a retake meeting. The standards-based system minimizes the emphasis on points and letter grades. So, the binary option (i.e., pass or no pass) was helpful for grading during the pandemic because grading those assessments took less time. As Leon put it, students either understood a concept or not.

Katie, a staunch advocate for the standards-based grading, brought it from her face-to-face classroom to her remote classes. “It was excellent,” she said. I asked her, “how did that work in this environment, standard-based grading?” I close this section on assessment with her response because it sums up the viewpoints of many participants in this study, whether they used standards-based grading or not, who adopted new practices for assessing learning amidst the pandemic:

> I think it’s particularly well-suited for this environment because I didn’t do a lot of high-stakes enormous exams. Instead, I was doing quizzes every day or once a week, and it covered one or two of the standards. So, I feel like my students were less tempted to cheat because they knew they could do it again. I feel like my students really benefited. I couldn’t look at their paper and offer them immediate help with what was going wrong. But if a lot of people struggle with something on the quiz, then on the next Monday I would explain to them: here’s the thing that I saw a lot of people do; here’s why that wouldn’t work; here’s what you should do instead. The students remarked that was one of the most helpful things of the class for them. Maybe normally, I would do it over their shoulder as they’re learning the concept. Instead, I did it after they turned in their quiz. Then they took another quiz, and it wasn’t too late for them. It was essentially the only time that I looked closely at the work they did. I don’t mind grading two or three quizzes on the same concept from that student because the grading is quick, I find with standards-based grading. I’m not worrying about, is that minus six points out of 15, or did I give the

> other student minus eight points for that mistake? It's either they don't understand this piece yet, they need to study it again, or I think they got it…I'm also looking closely to see what am I going to teach next class to clear up the problems for a lot of these students. It focuses me on the right things. So instead of worrying about how many points something should be worth, I'm worrying about, did my students learn what I was trying to teach them or not, or do they need to go back over it?

This account from Katie resonates with Wiliam's (2007) assessment for learning. According to Black et al. (2004), assessment for learning is "any assessment for which the first priority in its design and practice is to serve the purpose of promoting pupils' learning" (p. 8), as cited in Wiliam, 2007, p. 1062). "What is important is not the intent behind the assessment, but the function it actually serves," Wiliam (2007, p. 1062) says. Some of the instructors who adjusted their assessment practices during the pandemic said they would bring these new practices into the classroom because they were more consistent with their goals for teaching and learning.

Conclusion

I began this study with the research question: How did instructors of undergraduate mathematics adapt their practices for teaching during the pandemic? In the end, I changed it to: How did instructors of undergraduate mathematics adjust their practices for teaching during the pandemic? The findings presented in this section answer the latter question. To me, adapt has a more permanent ring to it, whereas adjust represents a for-the-moment change. The truth is the subtle difference in word choice represents my impression that the evidence herewith inconclusively supports permanent change. This is not to say that participants may not keep the changes they made during the pandemic, post-pandemic, and beyond in the face-to-face classroom; however, it is too early to tell. With that said, all participants in this adjusted their practices due to the pandemic—the sudden shift to the remote classroom made it virtually impossible to keep teaching normally.

The quintessential image of a mathematics professor lecturing while writing notes on the

blackboard was no more in the remote classroom. There was no blackboard; there was no "chalk talk" (Artemeva & Fox, 2011) as we remember it. Instead, instructors found new ways to represent mathematical ideas through the computer screen. The tasks they choose to connect learners to these ideas also changed. In some cases, they made changes to maintain academic integrity. Still, in other cases, instructors said they seized the opportunity to explore the mathematics of current events (i.e., pandemic, the election, and fight for social justice). Instead of "looking over their shoulders," instructors used technologies available to monitor students' learning progress.

Building community and making connections in the classroom were particularly challenging; interacting with black squares and names instead of faces and voices made it worse. But instructors found ways to connect with students and have them collaborate. They also responded to the things happening in their students' lives by incorporating flexibility and compassion into their pedagogy to adjust to the pandemic and social unrest in the country.

High-stakes, timed, proctored, and closed-book exams were challenging to implement with integrity in the remote environment. So, instructors found alternative ways to assess students' learning and evaluate their progress. For example, frequent, low-stakes, take-home, and open-book tests, with opportunities for revision, replaced traditional assessment methods. For some participants, the purpose of assessments shifted; the focus was more on learning than on scores and grades. Moreover, others said the demands of online teaching caused them to strip the course down to its most important objectives. As Annika put it, "being forced into this way of doing things has caused me to slow down and think about my priorities and what matters." And for every instructor, their learning goals and students' wellbeing took precedence over letter grades.

In the end, despite the challenges brought about by the pandemic, the instructors in this study made essential adjustments to their practices. They rolled with the punches and made the choices they made relied heavily on their teaching beliefs and proficiencies. During the interviews, I asked each instructor to talk about something they did in their remote classrooms that they are satisfied with—none of them said 'nothing.' They made at least one adjustment to their teaching practices that worked well. In some cases, they adjusted in ways they intend to take them back when they return to the face-to-face classroom. To answer this research question, "How did instructors of undergraduate mathematics adjust their practices for teaching during the pandemic?" I find, in general, they used knowledge of their students, focusing on the human element, and considered what it means to learn and do mathematics to adjust for the pivot to online instruction.

CHAPTER VI: SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

> None of us are to be found in sets of tasks or lists of attributes; we can be known only in the unfolding of our unique stories within the context of everyday events. (Paley, 1990, p. xxi).

This chapter summarizes the study and important conclusions drawn from the data presented in the previous chapters. Next, it provides a discussion of the study's limitations and delimitations, followed by recommendations for practice and further research.

Summary of the Study

The goal of research in undergraduate mathematics education is to improve teaching and learning; therefore, studying change is a necessary aspect of the field (Reinholz et al., 2020). The coronavirus pandemic provides an opportunity to study how instructors adjusted when practices that persisted for decades became virtually impossible. Two primary objectives initiated this inquiry. First, this study recognizes the humanity of the instructors living and teaching through a global health crisis. Thus, it captures how 28 instructors of undergraduate mathematics storied their experiences. Second, it explores how they adjusted their practices for teaching during this era. This work operates under the conceptual assumptions that experiences can be educative (Dewey, 1938), that reflecting on practice can lead to change (Schön, 1983), and that teachers' stories are rich units for analysis in education research (Clandinin & Connelly, 2000).

In this study, story is "both the method and the object of inquiry" (Carter, 1993, p. 5). The first research question for this study asks: How did instructors of undergraduate mathematics "story" their experiences teaching during the coronavirus pandemic? I focused on common stories across participants' accounts of their experiences to address this research question. There are stories about remembered events that connect the past to this disruptive present, pivoting to new ways of instruction when the coronavirus pandemic hit in March 2020, navigating that new

normal, and finding ways to cope. The second research question asks: How did instructors of undergraduate mathematics adjust their practices for teaching during the pandemic? This research question was addressed by analyzing their stories to reveal how practices changed, especially because some traditional methods (e.g., closed-book exams) became virtually impossible during the pandemic. The findings show that they adjusted how they represented mathematical content online, chose tasks and content, monitored students' progress, built a community for doing and learning mathematics online, and assessed student learning. The instructors in this study used increased awareness of students' lives and their conceptions of what it means to do and learn mathematics as compasses for adjusting to teaching during the coronavirus pandemic. It is unknown whether they will maintain those changes over time in the post-pandemic classroom.

Conclusions

This section presents conclusions about the findings in Chapters IV and V about participants' experiences teaching during the pandemic. Then, I relate these findings to three bodies of literature that inform this study: Perspectives in Undergraduate Mathematics Education, Perspectives on Technology Use, and Education in Emergencies. Finally, I discuss how these stories of experience teaching during the coronavirus pandemic either substantiate or add to previous findings.

Perspectives in Undergraduate Mathematics Education

Here I discuss the findings of the study in the context of the literature reviewed for this study, which includes three themes in undergraduate mathematics education research: 1) What Should Be Taught in Undergraduate Mathematics Education, 2) How It Should Be Taught, and 3) Who Has Access. In addition, a fourth subsection, Socio-ecological Perspectives in

Undergraduate Mathematics Education, emerged from my data analysis; however, to the best of my knowledge, it is not widely discussed in the literature on undergraduate mathematics education.

What Should Be Taught

For simplicity's sake, the principles of the calculus reform guide this discussion on what should be taught. Some goals of the reform are fewer topics, explored deeply, emphasizing "graphical representations, verbal descriptions and communication skills, as well as projects" (Bressoud, 2019, p. x). Incorporating technology and realistic applications are other aspects of reform curricula. It turns out that, as a matter of coincidence, some instructors' course goals during the pandemic were consistent with some of these goals. For example, many instructors reduced the number of topics they covered in response to the pandemic. Instructors were disoriented by the loss of the chalk and blackboard, but some quickly adapted and used technology (e.g., Desmos, Maple) to supplement instruction with dynamic graphical representations. Other instructors explained that they chose tasks to discourage students from cheating. In doing so, they assigned more meaningful tasks and connected mathematics to the events of 2020 (e.g., the presidential elections, the pandemic, the fight for racial justice). More findings in Chapter V illustrate how representations, content, and assessments during the pandemic resemble the vision for the calculus reform; my goal is not to repeat them here. But I posit that the pandemic presented, at the very least, an opportunity for instructors to try new things. Despite some instructors' vow to take some of these pandemic course goals to the post-pandemic classroom, there is no evidence in this study to predict the sustainability of those adjustments.

How Undergraduate Mathematics Should Be Taught

Much of the scholarship that calls for change in how instructors teach undergraduate mathematics is rooted in constructivist learning theory (e.g., active learning, inquiry-based learning). The overarching idea is that students can construct their mathematical understandings. As Dubinsky (1994) argues, each student "must construct knowledge for her—or himself and the role of the teacher is not to explain mathematics in a classroom, but to induce students to construct it in their minds" (p. 2). As a result, pedagogies supporting active student engagement and collaboration are rising. Still, instructor-led lectures, or "chalk talk lectures," pervade undergraduate mathematics classrooms (Artemeva & Fox, 2011).

Instructors' stories reveal that active learning pedagogies also were difficult to enact in the online space for several reasons. One reason is that some instructors lacked fluency with technology for fostering engagement and collaboration online. A second reason, I suspect, is that some instructors held on to espoused beliefs that traditional methods (e.g., chalk talk lecture) are most effective. Another reason could stem from the mode of instructional delivery—asynchronous instruction diminished engagement in some cases. According to instructors in this study, a fourth reason was that not all students could actively and equitably participate in the online learning process. Fifth, instructors said that students expressed frustration with active learning methods such as group work, confirming Deslauriers et al.'s (2019) finding that students perceive traditional lecture methods as more effective. But teaching and learning in active and engaging environments require cognitive effort, both on the instructors' and students' part, which was probably demanding during a pandemic. So, for the most part, instructors enacted some aspects of traditional chalk-talk lectures online (e.g., instructor-led discussions). Even those who

adopted active learning strategies before the pandemic reverted to instructor-led lectures, but with opportunities for engagement and discussion.

Instructors who were skilled in active methods and cultivated a culture for it in their pre-pandemic classrooms found workarounds for engagement and collaboration in the remote classroom (e.g., breakout rooms, Google Docs); however, they, too, found it challenging. So, when the traditional lecture became virtually impossible, active learning in its purest form was difficult to enact. As a result, instructors found themselves somewhere in the middle and facilitated engaging lectures. Even the staunchest chalk talk lecturers were pushed to foster engagement. They posed questions throughout the lesson and invited students to engage with the mathematical ideas by publicly explaining their thoughts. This result supports previous findings that the Direct Instruction Model is suitable for emergencies (Pausigere, 2011). Although teacher-directed, this model "is mostly used to teach in difficult circumstances with limited space and resources" (Pausigere, 2011, p. 4).

Moreover, instructors "carefully structure every skill and concept, yet ensuring student engagement through the use of task-orientated approaches" (Pausigere, 2011, p. 4). Direct instruction is not well-regarded by all scholars because they characterize it as teacher-centered (Shukla et al., 2014). But those who support this kind of instruction argue that it allows instructors to teach skills for fluency (Slocum & Rolf, 2021). Also, instructors stated that students, especially first-year students, needed the support, scaffolding, and connectedness of direct instruction under challenging circumstances. What I refer to as engaging lectures, Burn and Mesa (2017) describe as interactive lectures, which they characterize as "a mix of lecture and opportunities for students to engage in the course content (e.g., fielding questions, working problems in class)" (p. 24). Instructors also found that this method was practical for teaching

large sections of students. As an aside, many instructors taught larger than usual class sizes during the pandemic.

Closed-book, timed, and proctored exams, another dominant practice in undergraduate mathematics education, became virtually impossible online. I discuss how instructors adjusted for giving exams online in Chapter V, which aligned methods scholars have suggested for many years (e.g., MAA publications *Assessment Practices in Undergraduate Mathematics* and *Supporting Assessment in Undergraduate Mathematics*). For many instructors of undergraduate mathematics, how to assess student learning online was a major concern, probably more than how to teach online. And despite a considerable amount of guidance for interesting ways to assess learning in mathematics, instructors searched for ways to maintain aspects of traditional testing online (e.g., using proctoring software). The Conference Board of Mathematical Sciences asked: "How can we (or even should we) maintain our long-established habits for how we do assessment? What are optimal means for authentic assessment, both online and in other environments?" (Bressoud, 2020). This study shows that the answers to these questions depend on who you ask. For example, those who argued that students should memorize information were more likely to express that closed-book exams were optimal for authentic assessment. In contrast, those who argued that students should not memorize procedural information, but access only when needed for conceptual processes, were more willing to try open-book, untimed, unproctored exams.

So far, I have discussed two components of instructors' work: instruction and assessment. Scholars often study these aspects with a cognitive lens. But pedagogies rooted in constructivist theories alone are insufficient to inform pedagogical practice. The narratives in this study demonstrate that other perspectives (e.g., social, emotional) are important for informing how

instructors teach mathematics in the undergraduate mathematics classroom. For years, education researchers have moved beyond the cognitive to study how social, political, and cultural issues impact student learning (Adiredja & Andrews-Larson, 2017; Cobb, 1994; Gutiérrez, 2013; Ladson-Billings, 1997; McGee & Martin, 2011). Amid the global health crisis, a national fight for racial justice, the presidential elections, and economic instability, instructors shifted their thinking to consider these perspectives. As Gutiérrez (2013) maintains, "This shift in thinking lays the foundation for sociopolitical perspectives to inform education, to imagine new possibilities for relationships between people, mathematics, and the globe" (p. 37). Instructors' response to students' sociocultural, socioeconomic, and sociopolitical pressures was instrumental for building a community of mathematics doers and learners during the pandemic.

In the literature on undergraduate mathematics education, perspectives on caring pedagogy are hardly explored. Bartell (2011) makes a case for developing caring relationships in the mathematics classroom, but there is room for research that explores this perspective in the undergraduate mathematics classroom. Features of instructors' pandemic pedagogy were the care, compassion, and grace that guided their actions. Most instructors said that they became more compassionate during the pandemic. This is not to say that instructors were indifferent before the pandemic, but the events of 2020 awakened compassion in their pedagogy. For instance, they made stringent classroom policies flexible to accommodate students' varying circumstances. This finding was substantiated in themes such as "enacting care" and "shepherding." For example, some instructors made a concerted effort to connect with students struggling to meet the course requirements and who missed several classes at a time. The pandemic made it clear that students struggle because of situations out of their control. I anticipate that these humane adjustments will make their way to the post-pandemic classroom,

more so than any other new pedagogical tools or strategies instructors gained. Moreover, Kirkman et al. (2021) found that "Faculty have become more sympathetic to the problems of students and colleagues, which may help inclusiveness and response to underrepresented groups" (p. 10).

Furthermore, the pandemic human connections directly link to learning. When the distance made it difficult to connect with students on a human level, instructors struggled to foster engagement. Franke et al. (2007) call on teachers to push beyond building relationships around mathematical thinking. They write, "relationships based on getting to know students' mathematical thinking alone are limited" (p. 243). Research in undergraduate mathematics education has focused on what should be taught and how it should be taught without much attention to how instructor-student interpersonal connections impact learning. But, based on the findings of this study and the awareness instructors gained during the pandemic, I argue that the interpersonal relationships and rapport that instructors build with their students may influence learning in the same way, if not more, than what they teach or how they teach. The findings from Glazier (2016) support this conclusion regarding the importance of building rapport. Her study's qualitative and quantitative data show that instructors' accessibility and connection with students contribute to their success. She writes:

> Students face many challenges in pursuing their degrees and, most of the time, instructors cannot do much to help students with those challenges. Rapport building provides an instructor-driven method to improve online student retention—one that appears to be especially effective at helping our most at-risk students. Rapport building leads to significant improvements in student success, without additional budget requests, policy revisions, or any committee meetings at all. (p. 14)

The findings of this study are consistent with these results. Participants in this study used rapport-building strategies to connect with their students online, which included: reaching out to students who missed class or struggling (i.e., shepherding); showing care by offering flexibility,

learning about students' lives; making space in their classrooms for students to speak about issues, happening outside of the mathematics classroom, that affect them, and so on.

There is considerable literature on group identity (e.g., gender, race), the intersectionality of those identities, and students' mathematics engagement (e.g., Bowe et al., 2017; Cosby, 2020; Leyva, 2017; Martin, 2012). Another noteworthy result that emerged from the data was a perspective that personalities also affect mathematical engagement. For example, one participant, Zachary, explained that teaching online during the pandemic caused him to consider how students with introverted personalities participate in the mathematics classroom. On the one hand, students may have turned off their cameras as a way to sit in the back of the classroom, except remain unseen. On the other hand, features of the online classroom (e.g., the chatbox) provided them opportunities to participate in ways they may not have in the face-to-face classroom. Researchers have studied how students' individual personalities impact their academic performance. For example, Kalutskaya et al. (2015) reviewed the literature on shyness in the educational context. Hopefully, this result creates a space for conversations about how different personality traits impact engagement in the undergraduate mathematics classroom.

I suppose that teaching through a computer, to black screens, caused many instructors to bemoan interactions in the remote classroom. Bauersfeld (1980) claimed that "researchers in mathematics education have not spent much time on these dimensions of human interaction" (p. 24). Yet, forty years after Bauersfeld's claim, the meme in Figure 1 circulated when the pandemic first hit, and classes were canceled.

Figure 1

Meme from Warwick Math Memes Facebook page

This image implies that human interactions, necessary for the virus to spread, do not occur in the mathematics classroom. But instructors in this study confirmed what researchers have agreed on for many years—that the nature of classroom interactions "afford and constrain what is learned, how it is learned, and which students learn it" (Franke et al., 2007, p. 237). Even instructors who were not keen on interactive lectures before the pandemic found themselves searching for ways to generate human interaction online. While in the face-to-face classroom, it is not unusual for students to be unresponsive in class; the long moments of silence were unwieldy online. Interestingly, this experience taught some instructors that wait time is important. A few admitted that when they learned to characterize the long silence as opportunities for students to think, they, though few, responded thoughtfully. In the end, most found workarounds (e.g., chat feature, polling, breakout rooms, one-on-one or group video conferences as office hours). Especially for instructors who taught asynchronously, online office hours were conducive to fostering instructor-student interactions.

So, what does this bode for the future of undergraduate mathematics instruction? It is too early to tell. But I presume that "sage on the stage" will return in the post-pandemic classroom because most instructors seem to identify with this style; it is embedded in the culture. Yet, optimistically, instructors will continue to provide opportunities throughout their lectures for students to interact with mathematics and each other. During the pandemic, they prioritized

humanity for their students and themselves. Hopefully, instructors will not forget the ways they coped and adjusted their practices for pandemic teaching because "principles of emergency education are not very different from good practice in any education situation" (Sinclair, 2002, p. 30).

Who Has Access

The last ten years have witnessed growth in research that addresses the inequities in undergraduate mathematics education. Dr. Ladson-Billings' (1997) question, "But some people do well in mathematics in our society. Why?" (p. 699), is still relevant for today's mathematics education research. This study confirms previous findings that the culture of mathematics does not position students from all backgrounds to access the domain equitably. As Martin (2019) points out, "Black, Latinx, Indigenous, women, and poor students, have experienced long histories of underrepresentation in mathematics" (p. 460). Those same groups of students were disproportionately affected by the pandemic.

Surprisingly, many instructors said they became more aware of students' situations that affect their ability to learn mathematics. They used this new awareness of students' realities to inform their pandemic pedagogy. This finding suggests that instructors' knowledge of students' lives for teaching—a construct that I did not come across in my review of the literature on undergraduate mathematics education—is helpful for making pedagogical decisions. This assertion is like other findings in the literature on mathematics education (e.g., knowledge of students' conceptions; Ball et al., 2008; Shulman, 1986) and is in line with principles of culturally responsive pedagogy (Gay, 2018). As Franke et al. (2007) put it, "when the classroom environment and pedagogical practice are brought into line with the lived experiences of students, student performance is positively influenced" (p. 244). When instructors use their

knowledge of students' lived experiences to inform instruction, only then, I argue, can they begin to address issues of access and equity in their classrooms. The saying "you can't teach us if you don't know us and care about us," from Ukpokodu's (2016) book title, sums up this assertion well.

Socio-ecological Perspectives in Undergraduate Mathematics Education

"Are you familiar with the term socio-ecological model?" one participant, David, asked during our interview. "It's the idea that education does not happen in a vacuum," he explained. Instructors who taught in 2020 have begun to consider how individual, interpersonal, community, and societal factors influence mathematics classrooms if they have not done so before the pandemic. But how do those factors manifest themselves in undergraduate mathematics classrooms? Do undergraduate mathematics instructors function along social and political ideologies, or do they maintain a neutral stance? Earlier studies have explored these topics (e.g., Martin et al., 2010). This study offers a glimpse into how instructors can demonstrate their views that mathematics does not operate in a vacuum. For instance, a few instructors said they acknowledged sociopolitical issues in class instead of sweeping them under the rug. The storied experiences narrated in this study suggest that, during the pandemic, many of the instructors removed boundaries for what appears to be "mathematical" and "nonmathematical" (Shah, 2017). Undergraduate mathematics classrooms are very much a part of and not separate from the socio-ecological model in which they sit. As Gutiérrez (2013) puts it, "the effects of a global society...are present in the learners who arrive in our mathematics classrooms every day" (p. 37). A few participants said they made space in their classrooms to have social discourse about George Floyd's death because they recognized that it affected students' ability to engage.

This finding also underlines the point that scholars in education have made for a long time: classroom practices and curricula couched in real-world contexts provide students with authentic and engaging learning experiences (Schoenfeld, 2016). When the online space made it difficult to engage students, some instructors brought the outside world into the classroom through the tasks and activities. Based on their accounts, I conclude that instructors who leveraged the pandemic, the social movement, and the elections to bring meaningful contexts into the mathematics classroom provided engaging experiences for their students online. As mentioned in the literature review, political and economic contexts affect change on a macro level (e.g., post-Sputnik curriculum). But based on the findings of this study, I conclude that instructors have the power to bring the external into their everyday practice to affect change on a micro level.

In addition to bringing "nonmath" issues to classroom discussions and content, instructors used their new awareness of students' lived experiences to make pedagogical decisions. The pandemic spotlighted systemic, economic, and racial inequities that minoritized students experience. For example, there were stories of students who did not have a quiet place to study or access to high-speed Internet or students who worked to supplement family income. These issues existed before the pandemic, but the pandemic magnified them. Overall, this finding is consistent with principles of culturally relevant pedagogy, which according to Franke et al. (2007):

> Helps students develop sociopolitical consciousness, a sense of agency, and positive social and cultural identities while also supporting them to "read" the world using mathematics, developing students' mathematical power, and changing their disposition towards mathematics. (p. 247)

I opened this section by stating that the findings of this study confirm that the undergraduate mathematics classroom is not immune to happenings on the outside. In closing, consider this

reflection from participant Irina: “Math isn’t objective. If we don’t respond to this in our classrooms, it feels like we are not sending a message that we think is an accurate representation of mathematics.” This quotation represents the view of a few participants who said they have become aware of how different social and political constructs turn students away from mathematics. But unlike before the pandemic, they are willing to attend to these issues in their classrooms.

Perspectives on Technology Use

Scholars of educational technology have been trying to convert those who resisted for many years. Although the list of proven benefits is long, the number of instructors who use digital technologies in their undergraduate mathematics classroom is far less than those who do not use technology. But when the coronavirus pandemic made it unsafe to teach in person, technology made it possible to teach from a distance. Thus, the decisions were no longer about whether or not to use technology. Instead, the decisions had to be about *which* technologies to incorporate. Still, several factors influenced those decisions. This study finds that instructors made decisions about which technologies to use based on their facilities, the training available to them, the equipment they had access to, or their overall beliefs about how technology can enhance or diminish the learner’s experience and teaching practice. A similar conclusion was reached by Tauson and Stannard (2018). On the one hand, we see one instructor in this study who used emails as a medium for instruction. On the other hand, we also see instructors who tried dynamic software for the first time in the pandemic classrooms, who vow to never return to “chalk talk” lecture presentations. While it would seem that the pandemic will be an impetus for more technology use in the future, I suspect that convenience, more than anything else, pushed instructors to try new technologies. Whether simple or sophisticated, instructors had no choice

but to use some type of technology to keep teaching during this public health emergency. So, there is no sufficient evidence to suggest that instructors continue to use the technologies in the wake of the pandemic.

Skeptics have argued that technology has presented many drawbacks in the mathematics classroom for a long time. But during the pandemic, instructors experienced some affordances of technologies aligned with reform goals for undergraduate mathematics education. Some of these goals include emphasizing "the importance of graphical representations, verbal descriptions and communication skills, as well as projects and deep explorations of selected topics" (Bressoud, 2019, para. 3). For example, one participant in this study, who learned how to use Maple during the pandemic, touted its benefits for representations in calculus. Before the pandemic, his students visualized two-dimensional drawings of solids. When this instructor switched to using Maple during the pandemic, he demonstrated how the functions rotate to form the solids in real time.

Moreover, some instructors who were reluctant to assign projects and presentations stumbled upon new ways for students to communicate their mathematical ideas. For instance, instructors asked their students to record and post voice notes and video clips of their presentations using applications such as Voice Thread and Flipgrid; this was beneficial in a few ways. For example, they saved in-class time, and there was more flexibility with presentation length. Also, students could redo the presentation until they were satisfied. Doing it this way also removes the pressure of presenting in front of the class. Appendix F organizes technologies instructors used according to what they afforded. In Chapters IV and V, I discussed how instructors used different technologies to support learning and teaching in their virtual classrooms.

While some argue that "the first aim in adopting and developing any technology is that it is there for learning" (Pittard, 2013, p. 111), participants' stories of experience confirm Cuban's (1986) claim. He asserts that the best technology not only enhances learning but increases productivity with less effort from the teacher. With the increased workload of the pandemic, instructors sought tools for efficiency. According to Gordon (2000), mathematicians are cautiously uncomfortable with using technology, citing that some may not have thought about the benefits of using technology. But instructors who taught during the pandemic were forced to consider a confluence of technological tools. Their feedback captured in this study adds to a robust body of literature. What is more, the stories in this study offer a wide range of perspectives on using technology from technophobes, technophiles, and recent converts.

This study also has implications for getting instructors to try new technologies. Many of the instructors in this study said they adopted technologies that their colleagues used and recommended. For example, one participant, Liam, described a workshop hosted by colleagues in his department: "It was to the point, by people who understood the subject that we teach; therefore, they could focus their responses on what was meaningful," he said. There is evidence to support professional development models wherein instructors share their ideas and experiences with their colleagues.

In the last few years, much more information on emerging technological needs and the relevant tools in undergraduate education has become available. With the push for active and collaborative learning came technologies that allow students to engage with mathematics in meaningful ways. Several new tools support instructors in their non-teaching work (e.g., grading and providing feedback). From the results of this study, we see that these new technologies made the transition to remote instruction feasible. As it turns out, some instructors appreciate the

affordances of technology in the online classroom and wish to have those same affordances when they return to the face-to-face classroom. Polling features and chatboxes, for example, allowed students to engage in ways they did not before. “I don’t know if I’ll have the technology,” one participant said, “but I think that’s one thing I will try to kind of bring, that chat blast into my classroom.” In sum, this study’s findings answer other questions posed by the Conference Board of Mathematical Sciences (CBMS): “What are the technologies with which our members need facility? What are the promising technologies that need to be more widely distributed? Where are the greatest technological needs?” (Bressoud, 2020). Further investigation will be needed to explore how instructors can bring these technologies they stumbled upon online to the face-to-face classroom.

Scholars warn that emergency remote teaching should not be confused with teaching and learning online during regular times (Hodges et al., 2020). Still, the results of this study confirm earlier findings that students struggled to engage with mathematics content asynchronously (Kanwal, 2020). This finding leads me to conclude that synchronous models are probably best suited for undergraduate mathematics education when face-to-face instruction is not possible. Based on a CBMS survey, Kirkman et al. (2021) reported that online synchronous was the most common instructional method used by 2-year and 4-year programs across the U. S. Moreover, instructors in this study also shared that synchronicity was necessary. Also, when I compared stories from instructors who taught lower-division courses (i.e., first and second-year courses) with those who taught higher-division courses, I found that synchronous classes worked better for students enrolled in lower-division courses. For instance, one participant who taught a calculus course (i.e., lower-division) and an abstract algebra course (i.e., upper-division) said her abstract algebra students did better with asynchronous activity. The asynchronous model

"requires of students to make their own decisions on when and where to do what, to reflect upon the materials and their responses and permit students to work at their own pace" (Engelbrecht & Harding, 2005b, p. 266). Upper-division students can be assumed to have the academic maturity needed for student-directed learning. Also, students in their first two years probably require pacing and guidance from their instructors during synchronous sessions.

Although instructors' swift transition to online learning prompted institutions to want to offer more classes online, many instructors in this study remain antagonistic to teaching mathematics online. Instead, they look forward to the eventual return in the face-to-face classroom for reasons outlined in the findings' chapters. Their rationales fall in line with Gordon's (2000) position that mathematics is challenging to teach online because it limits the ease and accuracy of symbolic representation. In addition, they found teaching online to be limiting in several other ways. For example, the ability to improvise was diminished. Also, the nonverbal cues from students that instructors use to pace and guide their instruction were lost in the online space. Moreover, instructors find it challenging to assess learning virtually. Thus, while teaching online during the pandemic presumably moved the needle in a positive direction for using technology in the undergraduate mathematics classroom, I cannot predict the same for teaching undergraduate mathematics online. A few instructors plan to use more technology in the eventual return to the classroom; however, most of them look forward to being in the physical classroom because this experience has not convinced them that teaching mathematics online could be as effective as being in person. At the same time, results of a recent survey show that "a majority of departments believe that more faculty are interested in teaching online courses" (Kirkman et al., 2021, p. 6), but only a few instructors in this study said they would consider teaching online in the future.

Education in Emergencies

Previous work on emergency education has been limited to primary education in crisis-ridden situations (e.g., armed conflict) and natural disasters. As a result, the research on undergraduate education in emergencies is limited. Moreover, "No post-pandemic analyses exist to inform us on how best to support faculty" (McCollum, 2020, para. 7). This study explores two gaps in the literature on education in emergencies: education in emergencies at the undergraduate level and education during a global health crisis. The findings also confirm previous results. For example, I mentioned earlier that participants in this study resorted to teaching in ways Pausigere (2011) found effective in emergencies. This approach has been used in refugee contexts and other situations where students learn under challenging circumstances.

There is not enough evidence in this study's results to affirm earlier findings that girls were particularly at risk; however, the findings of this study confirm reports that minoritized groups are negatively affected during emergencies (Sinclair, 2001). Those traditionally underrepresented in undergraduate mathematics classrooms mainly were those who picked up extra hours of work to supplement lost family income, those who contracted the virus or had to care for family members who contracted the virus, and others for whom societal ills were exacerbated during the pandemic. In addition, Horsford et al. (2021) found that "COVID-19 and systemic racism had a disproportionate and traumatic impact on Black students, families, and communities" (p. 4).

One of the first responses to the pandemic was to provide students with devices; however, Tauson and Stannard (2018) report that providing students with hardware was not sufficient for learning during an emergency—many other factors came into play. For example, whether or not students had access to a home environment conducive to learning. Another factor

was whether the necessary infrastructure (e.g., Internet connectivity) was in place. It is important to note minoritized groups of students not having all the resources they need for learning is not a new problem; this is an old problem that the pandemic made worse. The stories presented in Chapters IV and V describe pedagogical practices that instructors used during the pandemic to support students. Since "principles of emergency education are not very different from good practice in any education situation" (Sinclair, 2002, p. 30), the findings of this study provide lessons for addressing these long-standing issues in mathematics education.

A common perspective on emergencies is that they provide opportunities to "build back better." Chapter II highlighted some recent publications from the undergraduate mathematics education community that support this view. We also see this agenda reflected across the narratives reported in Chapters IV and V. Instructors identified pandemic practices that they believe will improve teaching and learning in their post-pandemic classrooms. Even those who were not pleased with teaching online admitted that they gained new tools that may enhance their teaching in the future. This finding is consistent with the belief that "crises which destabilize education can be approached not only as urgent situations of immediate need but also as opportunities for positive change" (inee.org). The findings of this study on undergraduate mathematics education during an emergency do not provide "silver bullet" answers. Still, they provide insights useful for exploring them further and "building back better."

Limitations and Delimitations

The limitations of this study stem from the methodology, the chosen recruitment methods, and others that are inherent to qualitative research. I discuss these in the methodology section. In short, I describe how I minimized subjectivity, especially because I also taught undergraduate mathematics during the pandemic. As with all research, there is no way to

eliminate bias, but I was careful to monitor and minimize their influence in this study (Merriam & Tisdell, 2016). For instance, I discussed the data and my interpretations with knowledgeable colleagues. One concern about narrative studies is that participants' stories may not match what they do. But I attempted to address this concern by collecting artifacts from participants and using the constant comparison method (Charmaz, 2014). But verifying participants' stories was not a focus of this study. Because, as Charmaz and Belgrave (2012) point out, "interviews can also give research participants a space, time—and human connection—to reflect on these events anew and to clarify meaning and actions while providing rich data that spark analytic insights" (as cited in Charmaz, 2014, p. 80). I conducted the interviews during the winter break for convenience, so observations to support interview data were not an option.

Most participants were members of online communities and seemed to have similar interests in progressive pedagogies (e.g., inquiry-based learning). I put out a call for participants to Math Twitter Blog-o-sphere (@MTBoS), and there are several assumptions that one could probably make about MTBoS followers. For example, the instructors may be adept with technology. Moreover, on a website that supports this community, its members describe themselves as instructors who all have "a passion for our craft, and a desire to get a little bit better each year" (Mathtwitterblogosphere.com). This implies that many of the participants in this study actively seek ways to improve their practice. So, the results of this study that point to the pandemic as an opportunity to change may be skewed by selection bias. I considered this limitation during the recruitment process. I tried to minimize this bias by inviting instructors who taught using traditional methods (e.g., lecturing) and were not members of such online communities.

As I consider the limitations of this study, I am aware that there are also delimitations. Weaver-Hightower (2018) suggests explaining "why the study you didn't do might be interesting but that it wasn't what you wanted to accomplish" (p. 33). I realize now that collecting demographic data (e.g., race, gender) could have resulted in some interesting results. However, this was a missed opportunity to explore stories that emerged during the interviews. For example, a Black woman's approach to teaching during the pandemic, in the aftermath of George Floyd's death, may be different from a White man's approach. Instructors who were women seemed to have more caregiving responsibilities than their male counterparts. But these are avenues that I could not explore because I did not collect demographic data. When I designed the pre-interview survey, I did not ask questions about demographics because that was not the focus of this study. Thus, further investigation is recommended on how different groups of people experienced teaching undergraduate mathematics during the pandemic.

Recommendations for Research

The purpose of this study was to shed light on the experiences of instructors who taught undergraduate mathematics during the coronavirus pandemic. It could serve as a base for future studies in undergraduate mathematics education. Several opportunities for future research have been identified.

Data for this study were collected nine to ten months after the pandemic hit in March 2020. Future work may explore faculty experiences after one year and beyond. In this way, instructors would have had more time to reflect on their experiences. In addition, some of them may have returned to the face-to-face classroom despite the ongoing coronavirus pandemic. It will be interesting to learn how instructors continue to adjust for teaching during this new

normal. Moreover, the postsecondary lens is missing in research on education in emergencies. Post-pandemic analyses will inform this and the next global health crisis.

This study collected stories from instructors to understand their experience teaching during the pandemic. Future studies should explore the student experience. Earlier studies about learning online involve students who chose distance education. Some questions worth asking are: How about the students who had no choice and made the transition during a global emergency? Do they perceive the online classroom to be conducive to learning mathematics? How does that compare with their prior in-class experience and the eventual return to in-person learning? How do they perceive their instructors as accommodating to their various circumstances? There are many avenues to explore from students' perspectives because their stories are important for improving learning online, in emergencies, and in general.

During our interviews, instructors often referred to practices unique to the culture of teaching undergraduate mathematics (e.g., lecturing). So, I wonder, is there such an undergraduate mathematics teacher identity? Brown and McNamara (2011) emphasize that "there are no identities as such. There are just identifications with particular ways of making sense of the world that shape that person's sense of his self [sic] and his actions" (p. 27). Also, consider Sfard and Prusak's (2005) position that equates a person's stories with their identities: "No mistake here," they wrote, "we said they were stories" (original in italics; p. 14). Thus, I recommend Sfard and Prusak's narrative identity framework to explore undergraduate mathematics instructors' identities through the stories that they tell about their practices. What actions do they identify with? How do those identifications impact how they adjusted for teaching during the pandemic? Have their identities changed? In general, how do their identities change?

An interesting avenue to explore for future research would be disparities in experiences teaching during the pandemic amongst various groups. A few women wondered, during our interviews, whether other men shared their experiences. The instances were not sufficient for me to generate a finding; however, in my view, these are important issues for future research. For example, two women told me that they taught from home alongside their husbands, who shared caregiving responsibilities for their children. Their children interrupted them and their husbands alike. Except, students complained about the interruptions on their end-of-term evaluations, but not on their husbands'. Another woman shared that her students demanded more flexibility from her than they did from her male counterparts. Two adjunct instructors mentioned that job security was more a concern for them than their full-time colleagues. Thus, it seems worthwhile to examine the experiences of marginalized groups of faculty (e.g., women, Black women, part-time faculty).

In future work, investigating how different student personalities engage in the mathematics classroom might prove important for practice. For example, we see that some students who chose not to show their faces or speak on their mics participated in the chat. I wonder how providing students with various ways to participate in the class will improve engagement and belongingness in the undergraduate mathematics classroom?

This study operates from the lens that experiences can be educational (Dewey, 1938), that reflective practice can be transformative (Schön, 1983), and that stories are rich units for research analysis (Clandinin & Connelly, 2000). Therefore, I recommend that researchers in undergraduate mathematics education incorporate ideas from Dewey, Schön, Clandinin, and Connelly into their research methods. Moreover, interview studies that put undergraduate mathematics instructors' stories of experience at the center of inquiries can help understand the

work they do. I agree with Freeman (1994), during interviews with the researcher, "the teacher could tell, explain, confirm, reflect and thus 'represent' her thoughts, judgments, decisions, and ideas in public words to the researcher, who could then in turn study and analyse them to make sense of that internal world" (p. 81). Much of the research in undergraduate mathematics education rely on metrics (e.g., student performance) to support reform practices. But such data provide a small glimpse into what actually takes place in undergraduate mathematics classrooms. Qualitative research hinged on instructors' reflections is needed.

Instructors continued to show up for their students despite dealing with the impact of the pandemic on their own lives. Surviving a life-threatening disease, caring for small children and sick family members, and coping with loss while empathizing with students required increased emotional labor. Emotional labor, write de Ruiter et al. (2021), "refers to managing emotions and emotional expressions to be consistent with the expectations about appropriate emotional expression that exist[s] within an occupation (p. 2). Participants in this study described how they managed their emotional labor by practicing self-care routines. But I wonder how instructors negotiate emotional labor in the wake of the pandemic in their face-to-face classrooms; specifically, how they respond and manage their emotions as they tend to individual student needs. Mahoney et al. (2011) point out that "Though academics are the primary proponents of emotional labor, few studies examine college professors" (p. 407). Further research is needed to explore this phenomenon in undergraduate mathematics education. The results may inform how faculty interactions and relationships with students.

Finally, I recommend that research in undergraduate mathematics education, as Franke et al. (2007) put it:

> push beyond seeing teachers' work as eliciting students' thinking in mathematics and consider what it means for teachers and students to get to know one another in ways that

> lead to different opportunities for participation in mathematics for teachers and students. (p. 244)

The pandemic has made it clear that interpersonal connections and rapport directly influence student outcomes. Based on their accounts, instructors had more success engaging students in the Spring 2020 semester compared to the Fall 2020 semester. One reason they offered was that they had already got to know and built relationships with their students before the pandemic hit.

Recommendations for Practice

Instructors should "understand that the usefulness and appropriateness of [these] prescriptions for practice...in relation to the specific circumstances of practice in their own setting" (Erickson, 1986, p. 153, as cited in Merriam & Tisdell, 2015, p. 269). Thus, I make these recommendations for practice based on the findings of this study.

I agree with Clark (2020) when she writes, "I hope we can all notice larger lessons of flexibility, resilience, and humanness, and I hope these lessons follow you to infinity and beyond" (p. 1144)—this is one of the most important lessons I want people to take away from this study. The pandemic uncovered and exacerbated student struggles. Thus, instructors became acutely aware of sociopolitical, economic, racial stressors and anxieties that students grapple with outside of the classroom that may affect how they engage in the classroom. Many participants in this study described how they were pushed to adjust their practices and norms to support those students. Since "principles of emergency education are not very different from good practice in any education situation" (Sinclair, 2002, p. 30), I recommend that instructors continue to practice such pedagogies of care in their pandemic classrooms and beyond.

Furthermore, I think that teaching online during the pandemic taught us that human connections and interactions are vital for learning to take place in classrooms. When it became harder to connect with students online, it became harder to teach them. So, I recommend that

instructors find ways to connect with their students on a human level. Intentional rapport-building was helpful for classroom engagement during the pandemic. I hope that instructors will continue to humanize their classrooms in this way. Some rapport-building strategies include reaching out to students who missed class or struggling (i.e., shepherding); showing care by offering flexibility, learning about students' lives (e.g., through journal entries); making space in their classrooms for students to speak about issues happening outside of the mathematics classroom, that affect them, and so on.

Amidst the pandemic, instructors covered less content—opting to teach only what was important to cope with the pressures of the pandemic; however, covering less content aligns with the calculus reform's goal for "leaner" curricula and may be worth keeping in the post-pandemic classroom. Under normal circumstances, instructors could expect that covering less material could leave time for deeper exploration. This recommendation relies on a widespread agreement because expectations for what students learn should be uniform across the curriculum.

Another recommendation that follows from the results of this study is online office hours. Instructors in this study said that more students showed up for meetings online than they did in person. Therefore, I encourage practitioners to use online office hours to interact with students outside of the classroom. Instead of listing a room number, students can click on a link that directs them to a virtual room. Online office hours allow for flexible meeting times as well.

Practitioners should reconsider norms for student participation and engagement. They should reflect on ways that traditional norms for participation and engagement exclude certain groups of students. Bologna (2020) explains that students may not engage in the online classroom for various reasons. For example, they may feel shy and self-conscious. I posit that students do not participate in the face-to-face classroom for those same reasons. Instructors

should make efforts to understand what is preventing students from participating and engaging in their classrooms. Moreover, consider strategies for eliciting participation online, where it was more difficult to do compared to in-person. For example, instructors may brainstorm how to bring features of the chatbox to the in-person learning experience.

Online communities (e.g., MAA Connect, Math Twitter Blog-o-Sphere) offered instructors support and many resources for making the quick transition to remote teaching. Thus, I suggest that instructors join such communities, where they can expect to gain tools for improving their craft. In addition, membership in some communities (e.g., on Twitter) is free, and instructors may find valuable information for professional development. In less than 300 characters, instructors can access pragmatic classroom practices translated from empirical research.

Connecting with students online was difficult. But instructors found some ways to build relationships and maintain connections—a few of them are worth keeping (e.g., check-ins with students before and after class). Moreover, as one participant, Caelum, encourages, "Instead of just assuming that they're lazy and don't want to work," find out what situations outside of class that may affect students' ability to engage. A common way for instructors to learn about students' lives was through student journal entries.

The mathematics classroom does not operate in a vacuum. When possible, instructors should help students use mathematics to interpret and understand their worlds. So more than contrived applications, students need to engage with the mathematics occurring in their worlds. In other words, prioritize connections over applications. Professional societies curated resources for instructors interested in teaching the mathematics of COVID-19. In light of "humanizing" the mathematics classroom, I recommend Raygoza et al.'s (2020) "Humanizing Online Teaching,"

which "is not centered on the technical aspects of online teaching but rather pedagogical practices that promote care for the whole student and class collective" (p. 1). One salient result of this research is that care is a necessary feature of practice. Therefore, I recommend Callwood's (2020) dissertation, *Developing Educators' Capacity for Natural and Ethical Caring: A Mixed-Methods Study,* for practitioners interested in the pedagogy of care.

I wonder whether all instructors must become purists in active learning pedagogies. For instructors who prefer to lecture, I recommend incorporating some of the features of interactive pedagogy. There are examples of this type of practice in the literature. Interested instructors may consult Burn and Mesa's (2017) "Not Your Grandma's Lecture: Interactive Lecture in Calculus I in the CSPCC Two-Year Cases." They highlight common features of an interactive lecture: some time spent lecturing, incorporating technology, discussing homework problems, and students working on problems in class. More importantly, interactive lectures have features "that attend to active learning, student engagement with mathematics, and student-instructor interactions that promote relationship building, opportunities for feedback, and validation strategies" (Burn & Mesa, p. 28).

Recommendations for Leadership

The pandemic upended the status quo. Thus, the findings of this study about teaching during the pandemic have implications for how leadership may implement and sustain change. Instructors in this study implemented ideas that they learned from mostly their colleagues, in some cases, practices they were unwilling to try before (e.g., using interactive technology to engage students). Therefore, I recommend professional development opportunities in undergraduate mathematics where colleagues share ideas and experiences to translate and

disseminate evidence-based practices. These can be cheap and hassle-free monthly meetings where faculty members take turns sharing “what works.”

Moreover, new models for professional development should be couched in reflective practice. Participants in this study said that some questions during the interview prompted them to reflect on their practices. For example, could you talk about something you did that you are really satisfied with? Could you talk about something you did that you are not satisfied with? What do you plan to do differently this spring semester? In some cases, their reflections revealed how they might change aspects of their practice.

Department culture can either support or undermine instructors’ ability or willingness to adopt new practices. For example, participants in this study who belonged to departments where faculty often gather to talk about teaching, where different points of view were heard and respected, said that was instrumental for pivoting online. Thus, regular department meetings are encouraged. Additionally, institutions may explore how such cultures help promote organizational change under normal circumstances. Departments should also foster a sense of belonging and respect for non-tenure-track faculty and course assistants, who often feel like their hands are tied when it comes to trying new approaches. Students may not always be comfortable with new approaches, even when they work well. Thus, instructors may be hesitant to try innovative methods for fear that doing so may affect how students rate them on end-of-term evaluations. Reformed department cultures should support and reward instructors’ attempts to try new and innovative pedagogies.

Centers for teaching and learning were instrumental in assisting instructors with the shift to online instruction during the pandemic. So, they could be helpful for disseminating evidence-based teaching practices and training instructors for implementing those practices; however, as

participants pointed out in this study, instructional technologists with expertise in the content and pedagogy of the content are most helpful.

Virtual workshops were accessible and hassle-free during the pandemic. Institutions and professional societies may find ways to continue to offer workshops online. They are also cheaper to run, in that there are no travel, venue, and food costs. More diverse bodies of instructors could access those opportunities for growth.

Leadership should also explore aspects of pandemic teaching worth keeping and provide instructors with the necessary resources. For example, many instructors found their classes were more interactive through online chats and interactive, collaborative spaces. Thus, faculty need technology that will allow them to bring these effective parts of online learning to the in-person classroom.

Finally, institutions should approach teaching and performance evaluations and student ratings of instructors with caution, especially for marginalized groups of faculty (e.g., women, Black faculty, part-time faculty). Under challenging circumstances, instructors did what they could to keep teaching online while surviving a global health crisis.

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Appendix A

Recruitment Flyer

Appendix B

Participant Profiles

Table 2
Participant Profiles

	Pseudonym	Title	Level	State	Teaching Experience (in yrs.)	Experience teaching online	Spr.'20 Course(s)	Fall '20 Course(s)
1	Alexa	Clinical Professor	Four year	Massachusetts	More than 20	None at all	Discrete Mathematics (1 section), Linear Algebra (2 sections)	Discrete Mathematics (2 sections), Math Explorations (1 section)
2	Alissa	Associate Professor	Four year	Texas	11-20	A moderate amount	Introduction to Mathematics for Elementary Teachers, Concepts and Applications (for pre-service teachers at elementary/middle grades), linear algebra	Introduction to Mathematics for Elementary Teachers, Seminar for Mathematics Majors, Functions and Modeling (for pre-service secondary teachers)
3	Amie	Assistant Professor	Four year	Georgia	11-20	None at all	Precalculus	Precalculus, Abstract Algebra
4	Annika	Assistant Professor	Four year	Washington	6-10	None at all	None	Introductory (Differential) Calculus, Discrete Models for Middle-Level Mathematics Teachers
5	Arthur	Assistant Professor	Four year	Tennessee	6-10	A little	Statistics (x2), Introduction to Mathematical Computing, Introduction to Complex Analysis	Statistics (x4), Introduction to Combinatorics and Graph Theory, Mathematics Seminar
6	Aryeh	Professor	Four year	New York	More than 20	None at all	Multivariable Calculus, Advanced Calculus, Actuarial Math. II	Intro. to Probability and Math. Statistics, Advanced Calculus, Actuarial Math. I
7	Azra	Professor	Four year	California	11-20	A little	Math 1: Math, Philosophy, and the Real World	Math 174: Abstract Algebra II: Representation Theory

	Pseudonym	Title	Level	State	Teaching Experience (in yrs.)	Experience teaching online	Spr.'20 Course(s)	Fall '20 Course(s)
8	Caelum	Assistant Professor	Four year	Virginia	11-20	None at all	Elementary Differential Equations	Calculus 1
9	Carly	Professor	Four year	Massachusetts	6-10	None at all	Intro Stats, Special Topics in QR	Calculus 1, Intro Stats, World of Patterns, Teaching Elementary Mathematics
10	Chaya	Associate Professor	Four year	New York	More than 20	None at all	Introduction to Probability & Statistics, Mathematical Methods in Statistics, Linear Programming & Game Theory	Introduction to Probability & Statistics, Mathematical Methods in Statistics
11	Darren	Part-time or adjunct	Four year	Pennsylvania	1-5	A little	Analysis and Interpretation of Statistical Data in Education	None
12	David	Professor	Four year	Pennsylvania	More than 20	None at all	None	honors calculus
13	Irina	Lecturer	Four year	Kentucky	1-5	A moderate amount	Calculus 2	College Algebra
14	Jessie	Instructor	Two year	Delaware	11-20	A little	Math for Health Sciences	Math for Health Sciences
15	Katie	Professor	Four year	Maryland	11-20	A little	Linear Optimization, Mathematical Methods for Economics	Dynamic and Stochastic Modeling, Mathematical Methods for Economics
16	Kevin	Lecturer	Four year	Colorado	6-10	None at all	None	Pre-Algebra, College Algebra
17	Leon	Professor	Four year	New York	11-20	None at all	Mathematical Models, Integral Calculus	Mathematical Computing, Mathematical Design

	Pseudonym	Title	Level	State	Teaching Experience (in yrs.)	Experience teaching online	Spr.'20 Course(s)	Fall '20 Course(s)
18	Liam	Professor	Four year	New York	More than 20	None at all	Linear Algebra, Abstract Algebra,	Linear Algebra, Abstract Algebra, Number Theory
19	Luca	Assistant Professor	Four year	California	6-10	A little	Linear Algebra	Linear Algebra, Precalculus
20	Luis	Part-time or adjunct	Four year	New York	6-10	None at all	Calculus I, College Algebra for Precalculus	Calculus II, Calculus with Applications I
21	Mark	Professor	Four year	California	11-20	None at all	Abstract Algebra I, Calculus	Abstract Algebra II (two sections)
22	Nellie	Professor	Four year	California	11-20	None at all	Calculus 2	Calculus with Elementary Functions II
23	Pierre	Professor	Four year	Mississippi	More than 20	None at all	Calculus I with Analytical Geometry, Modern Algebra	Calculus I with Analytical Geometry, Modern Algebra
24	Sofia	Part-time or adjunct	Four year	New Jersey	6-10	A moderate amount	Topics in Mathematics for the Liberal Arts; Methods of Teaching Secondary Mathematics	Topics in Mathematics for the Liberal Arts; Curriculum & Instruction - Mathematics
25	Therese	Assistant Professor	Four year	Texas	1-5	None at all	None	Calculus I, Topology
26	Wesley	Professor	Four year	Utah	6-10	A little	Functions Modeling Change, Introduction to Statistics, Multivariate Calculus	Advanced Calculus, Introduction to Statistics, Multivariate Calculus
27	Yosef	Professor	Four year	Illinois	More than 20	None at all	Intro. to Math. Research	Intro. to Abstract Algebra
28	Zachary	Associate Professor	Two year	Illinois	11-20	None at all	Math for Elementary Teachers 1, Math for Elementary Teachers 2,	Math for Elementary Teachers 1, Math for Elementary Teachers 2,

Appendix C

Informed Consent

Protocol Title: Pandemic Stories: Emerging Pedagogies and Perspectives in Undergraduate Mathematics Education
Principal Researcher: Anisha Clarke, Doctoral Candidate,
718-737-2947, apc2137@tc.columbia.edu

Q1 NAME_____

Q2 Principal Researcher: Anisha Clarke, Doctoral Candidate, 718-737-2947, apc2137@tc.columbia.edu

Q3 INTRODUCTION You are invited to participate in this research study, "Pandemic Stories: Emerging Pedagogies and Philosophies in Undergraduate Mathematics Education." You may qualify to take part in this research study because you are an undergraduate mathematics teacher teaching during the pandemic. Approximately twenty people will participate in this study, and it will take 2 hours of your time to complete over the course of two days.

Q4 WHY IS THIS STUDY BEING DONE? This study is being done to examine the stories that undergraduate mathematics educators tell about their experiences teaching during the coronavirus pandemic.

Q5 WHAT WILL I BE ASKED TO DO IF I AGREE TO TAKE PART IN THIS STUDY? If you decide to participate, the primary researcher will ask you to complete a questionnaire, ask you to share a pedagogical artifact (e.g., syllabus, an activity you developed for online learning) in portable document format (PDF), and interview you.

Prior to the interview, you will provide background information (e.g., years of teaching experience) on a Qualtrics questionnaire. During the individual interview, you will be asked to describe your experience teaching mathematics during the pandemic. You will be asked to discuss a pedagogical artifact (i.e., the purpose for creating it and your experience with using it) that you will share with the researcher. The interview will be audio-recorded and auto transcribed. After the audio transcript has been reviewed for accuracy, the audio recording will be deleted. If you do not wish to be audio-recorded, you will not be able to participate. You may opt to turn off your camera so that only your voice will be recorded.

The interview will take approximately ninety minutes. You will be given a false name in order to keep your identity confidential.

For your safety during the pandemic, the interviews will be conducted via Zoom online platform at a date and time that is convenient to you. During the interview, please sit in a quiet space, away from non-participants.

Q6 WHAT POSSIBLE RISKS OR DISCOMFORTS CAN I EXPECT FROM TAKING PART IN THIS STUDY? This is a minimal risk study, which means the harms or discomforts that you may experience are not greater than you would ordinarily encounter while talking to a colleague about your experience teaching remotely during the pandemic. However, there are some risks to consider. You might feel embarrassed to discuss problems that you experienced teaching during the pandemic. You do not have to answer any questions or share anything you do not want to talk about. You can stop participating in the study at any time without penalty. Your information will be kept confidential.

The primary researcher is taking precautions to keep your information confidential and prevent anyone from discovering or guessing your identity, such as using a pseudonym instead of your name and keeping all information on a password-protected computer and in a password-protected Google Drive folder.

Q7 WHAT POSSIBLE BENEFITS CAN I EXPECT FROM TAKING PART IN THIS STUDY? There is no direct benefit to you for participating in this study. Participation may benefit the field of undergraduate mathematics education research.

Q8 WILL I BE PAID FOR BEING IN THIS STUDY? You will not be paid to participate. There are no costs to you for taking part in this study.

Q9 WHEN IS THE STUDY OVER? CAN I LEAVE THE STUDY BEFORE IT ENDS? Your participation in the study is over when you have completed the pre-interview questionnaire, the individual interview and emailed your pedagogical artifact(s) in portable document format (PDF). However, you can leave the study at any time even if you have not finished.

Q10 PROTECTION OF YOUR CONFIDENTIALITY. Any electronic or digital information (including audio (and video) recordings) will be stored on a computer that is password protected. What is on the audio recording will be written down, and the audio recording will then be destroyed. There will be no record matching your real name with your pseudonym.

For quality assurance, the study team, the study sponsor (grant agency), and/or members of the Teachers College Institutional Review Board (IRB) may review the data collected from you as part of this study. Otherwise, all information obtained from your participation in this study will be held strictly confidential and will be disclosed only with your permission or as required by U.S. or State law.

Q11 HOW WILL THE RESULTS BE USED? The results of this study may be published in journals and presented at academic conferences. Your identity will be removed from any data you provide before publication or use for educational purposes. Your name or any identifying information about you will not be published. This study is being conducted as part of the dissertation of the primary researcher.

Q12 CONSENT FOR AUDIO AND OR VIDEO RECORDING Audio recording is part of this research study. You can choose whether to give permission to be recorded. If you decide that you don't wish to be video recorded, then you may turn off your video during the Zoom interview. If you do not wish to be audio recorded, then you will not be able to participate in this research study.

Q13 By checking the "I agree" box and typing your name, you are electronically signing this consent form to give permission to be recorded. You affirm that an electronic signature has the same effect as a written signature. To agree: Check the "I agree" box and click NEXT to participate in the study. If you do not wish to be recorded, then close out of this browser window.

o I agree

Q14 WHO MAY VIEW MY PARTICIPATION IN THIS STUDY

If you consent to allow written materials viewed at an educational setting or at a conference outside of Teachers College, Columbia University, then please check "I agree." If not, then check "I do not agree."

o I agree (1)

o I do not agree (2)

Q15 OPTIONAL CONSENT FOR FUTURE CONTACT

The primary researcher may wish to contact you in the future. If you give permission for future contact, then please check "I agree." If you do not give permission for the researcher to contact you in the future for other research opportunities, then please check "I do not agree."

o I agree

o I do not agree

Q16 PARTICIPANT'S RIGHTS

I have read the Informed Consent Form and have been offered the opportunity to discuss the form with the researcher.

I have had ample opportunity to ask questions about the purposes, procedures, risks, and benefits regarding this research study.

I understand that my participation is voluntary. I may refuse to participate or withdraw participation at any time without penalty.

The researcher may withdraw me from the research at their professional discretion.

If during the course of the study, significant new information that has been developed becomes available, which may relate to my willingness to continue my participation, the researcher will provide this information to me.

Any information derived from the research study that personally identifies me will not be voluntarily released or disclosed without my separate consent, except as specifically required by law.

Identifiers may be removed from the data. De-identified data may be used for future research studies or distributed to another researcher for future research without additional informed consent from you (the research participant or the research participant's representative).

I should receive a copy of the Informed Consent Form document.

By checking the "I agree" box and typing your name, you are electronically signing this content form to participate in this study. You affirm that an electronic signature has the same effect as a written signature. You also affirm that an electronic signature has the same effect as a written signature. You also confirm that you are 18 years or older and an undergraduate mathematics instructor who transitioned from face-to-face instruction to remote teaching during the pandemic.

To agree: Check the "I agree" box and click NEXT to participate in the study. If you do not wish to participate in this study, then simply close out of this browser window.

o I agree (1) _____

Appendix D

Pre-Interview Questionnaire

Dear Participant,

I am pleased that you are interested in sharing your pandemic teaching story with me. Please take a few minutes to respond to the questions below. Your responses will help me to learn a little about you before meeting for an interview. Thank you!

1. What is your name?
2. Please share your contact information.
3. Which of the following best describes your teaching position?
    - Professor
    - Associate Professor
    - Assistant Professor
    - Lecturer
    - Instructor
    - Part-time or adjunct
    - Other faculty position
4. What level of undergraduate mathematics do you teach?
    - Two-year community college
    - Four-year college or university
5. In which state is your college/university located?
    - My college/university is not located in the United States (1) ... Wyoming (53)
6. How many years have you taught undergraduate mathematics?
    - Less than 1 year
    - 1-5 years
    - 6-10 years
    - 11-20 years
    - More than 20 years
7. Before the pandemic, how much experience did you have with teaching online?
    - A great deal
    - A lot
    - A moderate amount
    - A little
    - None at all
8. What format of instruction did you use in the Spring 2020 semester?
    - Asynchronous: students and instructors do not meet at a set time
    - Synchronous: students and instructors meet at a set time
    - Hybrid: a combination of in-person and online instruction

    - In person
    - I did not teach in the Spring 2020 semester
9. What format of instruction did you use in the Fall 2020 semester?
    - Asynchronous: students and instructors do not meet at a set time
    - Synchronous: students and instructors meet at a set time
    - Hybrid: a combination of in-person and online instruction
    - In person
    - I did not teach in the Spring 2020 semester
10. What is the name of the course(s) you taught in the Spring 2020 semester? If you did not teach in the Spring 2020 semester, then write N/A.
11. What is the name of the course(s) that you are teaching in the Fall 2020 semester? If you did not teach in the Fall 2020 semester, then write N/A.
12. Is there anything that you created, and used in your virtual classroom after the switch to remote instruction that you can share with me? (Examples: syllabus, exam, lesson plan, PowerPoint slides, instructions for a group activity, non-traditional assessment).
    - Yes
    - No
13. Upload documents here in portable document format (PDF).

Appendix E

Interview Guide

Principal Investigator: Anisha P. Clarke

Semi-structured Interview Guide

Hello. [Nice to meet you virtually]. How are you doing?
[I will share a little about my background. I have a bachelor's degree in applied mathematics and a master's degree in mathematics education. I have been teaching mathematics to undergraduate students for 13 years. This year I taught precalculus and algebra].
Thank you for agreeing to participate in my dissertative research. It means a lot to me to have your support.

The purpose of this study is to understand, through the narratives of undergraduate mathematics educators, like yourself, who taught in 2020 ~~during the coronavirus pandemic~~, how things are going and are likely to go in undergraduate mathematics education. Often narratives are written about teachers, void of their voices.

I expect our conversation to last about 60-90 minutes.

I will record this interview for data analysis. If you choose to be audio-recorded, I will notify you when the audio-recorder is started and stopped. If you do not want to be audio-recorded, you will not participate in this study. If you do not want to be video recorded, then you may turn off your camera. You may choose to withdraw from this interview at any time without penalty. You will be assigned a pseudonym.

I will mute my microphone when you speak to reduce background noises and avoid interrupting you, but please know that I am actively listening.
Do I have your permission to record?

1. Please get us started by talking about your mathematics teaching journey. You can frame that story however you choose to.

2. I would like you to paint a full picture of what it was like to teach mathematics this past year.
    a. PROBE (if needed) As you look back, is there a particular moment or memory that stands out to you?
    b. PROBE (if needed) Do you remember/recall anything else?
    c. PROBE (if needed) Could you tell me about any challenges you encountered?

3. Who/What has been most helpful to you during this time? How?
    a. PROBE (if needed): What kinds of support for teaching online did you receive, and from who/where?

4. Is there a helpful tip/resource that you received, or that you figured out on your own, for teaching mathematics during the pandemic that you can share?

5. Could you talk about something you did that you are really satisfied with?

6. Could you talk about something you did that you are not satisfied with?
    a. PROBE (if needed): Looking back, is there anything you wish you had done differently?

7. What do you plan to do differently this spring semester?

8. What practices have you tried for the first time during the pandemic (other than teaching online) that you have been reluctant to try in the past, if any?

9. Of the things that you tried for the first time during the pandemic, is there anything that you plan to take back to the face-to-face classroom when you return?

10. IF PARTICIPANT SHARED AN ARTIFACT
    [You shared a (pedagogical artifact) with me. Please tell me about it.
    a. What was its purpose?
    b. What was your experience using it?
    c. You student's experience with using it]

11. How has this experience (teaching during the pandemic) changed you as a teacher, if at all?

12. Is there any question I should have asked you but did not?
    a. PROBE (if needed). Is there anything else you think I should know to understand better your experience with teaching mathematics during the pandemic?

Thank you for sharing your story with me! I appreciate your time.
<STOP RECORDING>

Appendix F

Technologies Participants Used for Teaching During the Pandemic

Assessment Delivery & Online Educational Platforms

WebAssign
WebWork
Poll Everywhere
Zoom Polling
Pearson My Math Lab

Collaboration

Google Jamboard
Piazza
Google Docs

Presentation

Beamer
Voice Thread
Google Docs
PowerPoint Slides
Flipgrid
Tablets
Wireless stylus pen (e.g., Apple pencil)

Grading

Grade Scope
Google Classroom
Blackboard

Document Preparation

Spreadsheets
Latex
Overleaf

Course Hub/Course Management System/Learning Management Systems

Moodle
Google Classroom
Canvas
Blackboard

Videoconferencing

Zoom
Google Meet
Blackboard Collaborate
WebEx
Big Blue Button

Modeling and Simulation

Computer Algebra Systems (e.g., Maple)
Desmos

Video-Sharing Platform

YouTube
Flipgrid

Communication Platform

Slack
Campus Wire
Campus Everywhere