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Cache Code Math September Unit: Repeats in Mathematics and Programming

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Repeats in Mathematics and Programming

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Fifth-Grade Math Mini-Lessons

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Slide Deck to Support Teaching

Math Routine #1: What Makes This Equation True?
Go Math! Lesson: This mini-lesson could fit with any Chapter 1 lesson, but especially 1.1 or 1.2

Use a Think-Pair-Share format for each equation: Think - silent individual think time; Pair - share their solution and why they think it is that solution with their partner; and Share - share solutions and mathematical thinking as a whole class.

The equations will likely not be difficult for students to solve, and hence, the emphasis should be on explaining *why* a number makes the equation true. The first two equations will be a review of considering multiplication as repeated addition. You can encourage responses about groups being repeated and the explanation that multiplication is a simpler way to express repeated addition. The next two equations will be more challenging for students to explain *why* a number makes the equation true, and they will likely rely on the associative property of multiplication (though they may not name the associative property).

A key goal for this routine is to help students distinguish between repeated addends and repeated factors. An equation like 6x4 refers to repeated groups of 4 or repeated groups of 6, whereas 4x4x4x4x4x4 is exponential (4^6 or four to the sixth power).

I'm going to show some equations and ask you what number will make the equation true. First, I'll give you individual think time. When you think you know what number goes in the blank, give me a silent thumbs up. Then, I'll have you share your solution and why you think that is the correct solution with a partner. Finally, we'll share our thinking with each other as a whole class. Ready?
(Show each equation and conduct a Think-Pair-Share. For each equation, ask *What makes this equation true?* and *Explain how you know it makes it true*.)

1. 1 x ___ = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1
   Think: *What makes this equation true?*
   Pair and Share: *Explain how you know it makes it true.*

   1 x _**10**__ = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 **(Possible responses: there are ten 1s; 1 is repeated ten times)**

2. 4 + 4 + 4 + 4 + ___ + ____ = 6 x 4
   Think: *What makes this equation true?*
   Pair and Share: *Explain how you know it makes it true.*

4 + 4 + 4 + 4 + _**4**__ + __**4**__ = 6 x 4 **(Possible responses: there are six groups of 4; 4 is repeated six times)**

3. 2 x 2 x 2 x 2 = __ x 2 x 2
   Think: *What makes this equation true?*
   Pair and Share: *Explain how you know it makes it true.*

   2 x 2 x 2 x 2 = _**4**_ x 2 x 2 **(Possible responses: 2x2 is 4 so 4 makes the equation true because the blank needs to equal 2x2; both sides need to equal 16 so 4 goes in the blank)**

4. 2 x 2 x 2 x 2 = __ x 2
   Think: *What makes this equation true?*
   Pair and Share: *Explain how you know it makes it true.*

   2 x 2 x 2 x 2 = _**8**_ x 2 (**Possible responses: 2x2x2 is 8; both sides need to equal 16 so 8 goes in the blank to make 16; 8 is the same as 2x2x2 so we can write 8 instead of 2x2x2; students might incorrectly respond with 6 because they skip count the 2s: 2, 4, 6 or incorrectly respond with 4 because there are four 2s**)

Discuss: *How are these equations (#3 and #4) different from our first two equations? How are they similar?* **(They are repeated factors whereas the first two equations had repeated addition.)**
Discuss: *What do you notice about the equations? What do you notice about numbers being repeated in the equations?*

Teacher statement after discussion:
Say: *"Just like multiplication equations let us efficiently write repeated addition, tomorrow we will learn an efficient way to write repeated multiplication. Today you used what you know about multiplication and about combining factors to make the sentences true, and tomorrow I'll show you another way you can write repeated multiplication. Also, in the computer lab, you'll learn to use the "repeat" blocks to make code easier to write. As you work in math and the computer lab, be thinking about how we find shortcuts when things repeat."*
Math Routine #2: Is This Equation True or False?

Go Math! Lesson: This mini-lesson could fit with any Chapter 1 lesson, but especially 1.2 or 1.3

Use a Think-Pair-Share format for each equation: Think - silent individual think time; Pair - share their solution and why they think it is that solution with their partner; and Share - share solutions and mathematical thinking as a whole class.

The goal for this number sense routine is to build on the previous routine by focusing on repeated multiplication and to explore how to more efficiently write equations like 4x4x4 (as 4^3 instead of 4x3). A secondary focus is to revisit the difference between repeated addition (multiplication) and repeated factors (exponents).

I'm going to show some equations and ask you if they are true or false. First, I'll give you individual think time. When you think you know if it is true or false and why, give me a silent thumbs up. Then, I'll have you share your solution and why you think that is the correct solution with a partner. Finally, we'll share our thinking with each other as a whole class. Ready?
(Show each equation and conduct a Think-Pair-Share.)
Show each equation and conduct a Think-Pair-Share. For each equation, ask *Is the equation true or false?* and *How do you know?*)

1. True or False? 2 x 2 x 2 = 2 x 4
   Think: *Is the equation true or false?*
   Pair and Share: *How do you know it is true/false?*
   True because the 4 replaces the 2x2; true because 2x2 is the same as 4; both sides equal 8 so it is true; ask Bev and math teachers for other possible responses.

2. True or False? 4 x 4 = 2 x 2 x 2 x 2
   Think: *Is the equation true or false?*
   Pair and Share: *How do you know it is true/false?*
   True because 2x2 is the same as 4; true because both sides equal 16.

3. True or False? 4 x 3 = 4 x 4 x 4
   Think: *Is the equation true or false?*
   Pair and Share: *How do you know it is true/false?*
   False because one side equals 12 and one side is bigger than 16; False, but someone might think it means 3 groups of 4, but that is 12 and 4x4x4 is not 12.

Discuss: *What can we do to make the equation in #3 true?*
To make it true, it should be 4 x 3 = 4 + 4 + 4 or 4 x 3 = 3 + 3 + 3 + 3 or 4 x 3 = 3 x 4 or 16 x 4 = 4 x 4 x 4 or 4 x 3 = 12. (Students likely will not come up with the exponent notation: 4^3 = 4 x 4 x 4. Offer it as another equation that would make this equation true and foreshadow lesson 1.4 on Exponents. SAY: Yes, these are all ways to make the equation true. Another way is 4^3 = 4 x 4 x 4. You say this "four to the third power" and it means that the factor 4 is repeated 3 times.)

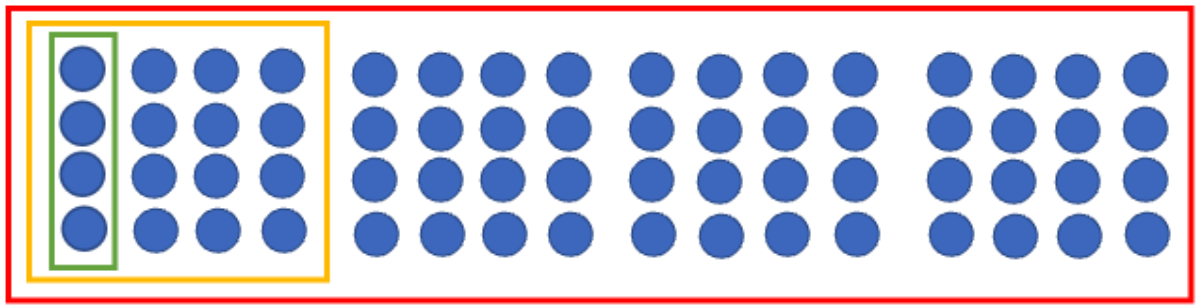
4. How are these equations (4 x 3 = 4 + 4 + 4 and 4^3 = 4 x 4 x 4) similar and how are they different?
   One equation is about repeated addends and the other equation is about repeated factors. One is repeated addition and one is repeated multiplication. The equation 4x3 is read as "four groups of 3," but 4+4+4 is also correct because of the commutative property of multiplication. The equation is still balanced.

Discuss:
What did you notice about the equations today?
What did you learn from comparing these equations and hearing each other's ideas?

Teacher Statement After Discussion:
Say: *"Just like multiplication equations let us efficiently write repeated addition, using exponents is an efficient way to write repeated multiplication. To make 4 x 3 = 4 x 4 x 4 true, one way is to use the exponent notation 4^3 so that the equation reads: 4^3= 4 x 4 x 4 ."*
Say: *"If we represent 4 x 3 with an image of dots, what does it look like?"*

Four groups of three cats = 4x3 = 12

Say: "If we represent 4^3 with dots, what does it look like?"



$4^1 = 4$
$4^2 = 4x4 = 16$
$4^3 = 4x4x4 = 64$

Say: *"Also, in the computer lab, you'll learn to use the "repeat" blocks to make code easier to write. As you work in math and the computer lab, be thinking about how we find shortcuts when things repeat."*

Math Lesson Adaptation #3: Efficient Exponents
Go Math! Lesson: This mini-lesson could fit with any Chapter 1 lesson, but especially with Lesson 1.4 or after 1.4. Coordinate with the Computer Lab Specialist and teach this after the students' Week 3 lesson on the Repeat Loop Block.

In this mini-lesson, we use students' learning from the Computer Lab coding lessons (Week 3 in the Computer Lab) to show exponential notation visualizations. We compare what exponential growth (repeated multiplication) looks like with what multiplication (repeated addition) looks like. Students will practice writing exponential notation, expanded form, and word form of exponents using the Scratch coding visuals.

Say: *"Remember that multiplication is a way to write repeated addition more efficiently. For example, 3 + 3 + 3 + 3 + 3 is five groups of 3, which can be represented as 5 x 3. Today, let's explore ways to write repeated multiplication more efficiently. We'll learn to more efficiently write equations like 3 x 3 x 3 x 3 x 3 in exponent form."*

Say: "*In a number in exponent form, which number is used as the repeated factor? What number tells how many times to repeat a factor? Let's talk about six to the fourth power:* **6^4***"* **(For example, in 6^4, 6 is the repeated factor, and the exponent tells us to repeat it 4 times.)**

Say: *"Now we are going to revisit your computer lab Scratch activity, but this time write what we see in mathematics notation. We'll use exponent form and word form to do it. I'll also ask you to write expanded form so we know what the exponent form is representing."* (Students need white boards)

- *"Seven to the second power or seven squared (7^2) is 7x7. Let's see it in Scratch."*
    - Project the Scratch activity Repeated Multiplication from *Computer Lab Activity #3* on the board and enter 7 as base and 2 as exponent and watch the visualization. (Click on the green flag to run the program, not the code itself.)
    - *"Now, write in exponent form on your white board. Write in word form on your white board. Write it in expanded form on your white board."* **(Exponent form: 7^2, Word form: seven to the second power *or* seven squared, Expanded form: $7^2 = 7 \times 7$)**

- "*Four to the third power or four cubed (4^3) is 4x4x4. Let's see it in Scratch."*
    - Project the Scratch activity Repeated Multiplication from *Computer Lab Activity #3* on the board and enter 4 as base and 3 as exponent and watch the visualization.

- *"Now on your white board write exponent form, expanded form, and word form for four to the third power."* **(Exponent form: 4^3, Word form: four to the third power, Expanded form: 4^3 =4 x 4 x 4)**

- *Let's look at (or look back at) problem 1 on page 24 in your book*:

1. 10×10

Exponent form: _____

Word form: _____

_____

  - *What number is our repeated factor (base)?* **(10)** *How many times will we repeat it (exponent)?* **(2)**
  - *Let's see it in Scratch* (project the Scratch activity on the board and enter 10 as base and 2 as exponent and watch the visualization).
- Repeat the same process with problems 2 and 3 (and any other problems in this Go Math lesson). Let students direct you on what numbers to enter into Scratch. With each problem, ask:
  - *What is our repeated factor (base)? How many times will we repeat it (exponent)*?

2. $10 \times 10 \times 10 \times 10$ **3.** 10^2

Discussion:
What did you notice?
What patterns did you see?
Can you make some predictions about what numbers look like when our repeated factor (base) is 10? Have you seen any shortcuts?

Teacher Statement After Discussion:
Say: *"We know that using exponents is an efficient way to write repeated multiplication, and today we saw a special application when the repeated factor is 10. We found a shortcut that helps us predict the number of zeros our answer will have, depending on the exponent on the base of 10. (For example, 10^6 will have 6 zeros, and 10^{12} will have 12 zeros.) We used Scratch to visualize how numbers grow when we apply an exponent, and the repeat loop block helps us shortcut the repeated multiplication."*

Math Lesson Adaptation #4: Visualizing Repeating Factors
Go Math! Lesson: This mini-lesson could fit with any Chapter 1 lesson, but after Lesson 1.4. Coordinate with the Computer Lab Specialist and teach this after the students complete all the Computer Lab Activities. In this mini-lesson, you will use three of the Computer Lab Scratch Activities: Scratch Repeated Addition (from Activity 2), Scratch Repeated Multiplication (from Activity 3), and Scratch Repeated addition_mult (from Activity 4: Comparison of growth by multiplication and exponents and Activity 5: Writing your own code for multiplication and exponent).

In this mini-lesson, we use students' learning from the Computer Lab coding lessons (Weeks 3 and 4 in the Computer Lab) to show exponential notation visualizations. We compare what exponential growth looks like with what multiplication (repeated addition) looks like. Students will revisit problem 17 (Yolanda problem) in *Go Math!* Lesson 1.4 to further justify (using the Scratch visualization) what they wrote about her mistake.

Say: *"What did you learn during coding in the Computer Lab this week (or last week)?"* **(using the repeat block; seeing exponential growth versus amounts being multiplied)**
Say: *"What does the repeat loop block do?"* **(example: if you need to move forward 4 spaces, you can use the repeat block to instruct 4 times rather than use 4 forward blocks)**
Say: *"What do you remember about this Scratch activity in the computer lab?"* (show the Scratch Repeated Multiplication activity from Activity 3: Visualizing exponential growth and try out various base and exponent numbers like 2^4, 2^8, 3^2, 3^3, 3^4, 3^5)

- *What did you notice? What did you learn?*
- *What kinds of equations did your Scratch sprite show you?*

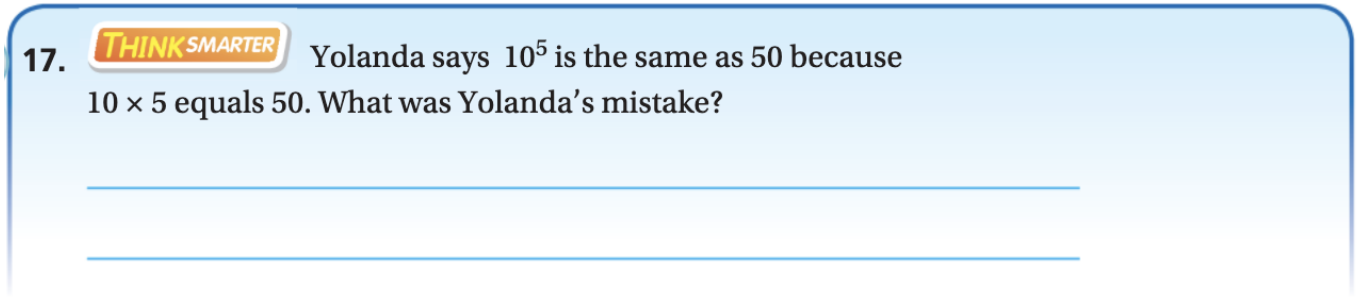
Project the Scratch activity Repeated Addition from *Computer Lab Activity #2* on the board and enter 6 as factor1 and 3 as factor2 and watch the visualization. (Students will see 3 groups of 6 to make 18)
Discuss: *What did you see? How do we represent what we saw mathematically?* (Three groups of 6. We can represent the visualization as 6 + 6 + 6 or 3 x 6 or 6 x 3. This could also be 3 + 3 + 3 + 3 + 3 + 3 or six groups of 3 because of the commutative property.)

Project the Scratch Repeated Multiplication from *Computer Lab Activity #3* on the board and enter 6 as base and 3 as exponent and watch the visualization.

Discuss: *What did you see? How do we represent what we saw mathematically?* (We saw 6=6, then 6 x 6=36, then 6 x 6 x 6=216. We can represent the visualization as 6^3 which is six to the third power or 6 cubed.)
Move back and forth between 6x3 (Scratch Repeated Addition) and 6^3 (Scratch Repeated Multiplication) and **Discuss**: *How are these visuals similar and different? How are the equations we used to represent them similar and different?*

Project Scratch Repeated Addition and Multiplication and enter 2 as First_number and 5 as Second_number to show 5x2 and 2^5 here with the gray and colorful cats computing the different equations side-by-side. Do it again with 4x3 and 4^3 and **Discuss:** *How do you know that 4 to the third power is not equal to 12 and 4 x 3 = 12?*

Say: *Based on what we just looked at with 2^5 and 4^3, let's look at problem 17 on page 26. How do those relate to this problem?*


17. THINK SMARTER Yolanda says 10^5 is the same as 50 because 10×5 equals 50. What was Yolanda's mistake?

"What number is the base? What number is the exponent? How did we write this on p. 26?
What would we see if we ran the program? How would the repeated multiplication cat (10^5) be different from the repeated addition cat (10x5)?"
(You could enter 10^5 on Scratch, but it will take a REALLY long time.)

Teacher Statement After Discussion:

Say: *"Just like multiplication equations let us efficiently write repeated addition, using exponents is an efficient way to write repeated multiplication. Today we saw how repeated addition looks much different than repeated factors or exponential growth. Like the loop block in the computer lab, exponent notations is a shortcut way to write repeated multiplication. Using the repeat loop block in Scratch and code.org is a shortcut way to instruct your program to repeat an action."*