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A critical analysis of jet-induced scour formulas

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Abstract: *Many studies investigate erosive processes occurring in non-cohesive granular materials downstream of grade-control structures, dam spillways, headcuts, and other hydraulic structures. Because of the complexity of the scour mechanism, the analysis of the scour phenomenon caused by plunging jets is generally conducted by using physical models and particularly for specific structure geometries. In this regard, many researchers proposed empirical approaches to estimate the main scour lengths, but their contributions are limited to tested conditions and cannot be generalized. This lack of generality has been (partially) overcome by other, more recent, approaches, that are either semi-theoretical or fully theoretical. Previous works assessed the predictive capability of the most well-known empirical relationships but did not present a comparative analysis between empirical and (semi-)theoretical relationships. The aim of this paper is to contribute to fill this gap of knowledge. Namely, we present an experimental validation of the most popular relationships using a large database. In addition, we compare the predictive capability of some (semi-)theoretical relationships with that of the best known empirical formulas. In doing so, we provide interesting insights into the different approaches, highlighting their limits in assessing the main scour features. Overall, this paper provides a critical and updated analysis of different approaches for scour problems caused by plunging jets.*

Keywords: *hydraulic structures, jet scour, non-cohesive sediment, scour mechanism.*

1. INTRODUCTION

Erosive processes occurring downstream of hydraulic structures may represent a threat to their stability. Consequently, many studies investigated scour processes in granular and rocky beds, providing tools to estimate the main features of the scour hole. Because of the three-phasic nature of the scour mechanism, the analysis of the phenomenon caused by plunging jets is generally conducted by using physical models and is limited to specific structural configurations and geometries. To this end, for scour in non-cohesive granular beds, many empirical equations were developed. Among others, Kotoulas (1967) proposed an empirical formula for scour depth downstream of a free overfall jet. Mason and Arumugam (1985) and Mason (1989) developed an equation to predict the scour depth associated with a variety of structures ranging from free overfalls to tunnel outlets, whereas D'Agostino (1994) analyzed the scour downstream of a sharp-crested weir. Nevertheless, these formulas are often dimensionally not correct and only valid in the tested range of parameters. Moreover, their range of application is frequently not defined (Pagliara et al. 2004). This lack of generality has been (partially) overcome by other, more recent approaches, that are either semi-theoretical or fully theoretical. In particular, the former are generally based on the jet diffusion theory in the turbulent cauldron, whereas the latter are based on first principles.

Previous works examined the predictive capability of the most well-known empirical relationships. Among others, Whittaker and Schleiss (1984) presented a compilation of the existing methods to estimate the features of jet-induced scour and compared their accuracy with prototype data. Pagliara et al. (2004) assessed the performance of the main empirical approaches by contrasting their predicting capability with a large experimental database derived from different authors. More recently, Castillo and Carrillo (2017) compared the performance of several methods to predict the scour downstream of a ski jump, including (semi-)empirical formulas and CFD simulations. Nevertheless, none of the mentioned studies presents a comparative analysis of empirical and (semi)theoretical relationships pertaining to 3D jet-induced scour processes.

Therefore, the aim of this paper is to contribute to fill this gap of knowledge. To this end, we tested the predictive capability of well-known (semi-)empirical formulas and compare their performance with that of the theoretical approach proposed by Bombardelli et al. (2018), based on the phenomenological theory of turbulence (PTT). The paper is organized as follows. We first describe the experimental setups adopted by Pagliara et al. (2008a) and (2008b) and Bombardelli et al. (2018), as we employed experimental datasets derived from these studies. Then, we critically discuss the approaches proposed by Mason and Arumugam (1985), Mason (1989), Bormann and Julien (1991), Hoffmans (2009), and Bombardelli et al. (2018). Finally, we provide interesting observations on the predicting capability of the mentioned methods, by highlighting differences and similitudes.

2. EXPERIMENTAL SETUP

Datasets used in this investigation comprise more than 100 experimental tests conducted at the Hydraulics Laboratory of the University of Pisa in collaboration with ETH Zurich (Switzerland). Experiments were conducted in two different rectangular channels, with a width B_c. Figure 1 shows a diagram of the experimental apparatus, along with the main hydraulic and geometric parameters involved, i.e., water discharge Q, water depth above the original bed D, diameter of the jet D_{jet}, inclination of the jet α, maximum scour depth Δ and length L of the scour hole, longitudinal and vertical coordinates x_i and z_i. Table 1 summarizes the range of hydraulic and geometric parameters pertaining to the selected datasets. Tests involved different types of uniform granular materials, with d_{50} indicating the median diameter of the sediment bed. The jet was generated by a circular pipe which could be placed either at the center of the channel (*full-model* arrangement) or close to the channel glassed wall (*half-model* arrangement, see Pagliara et al. 2008b for details). Pagliara et al. (2008a) adopted a *full-model* configuration; conversely, tests of Pagliara et al. (2008b) and Bombardelli et al. (2018) were conducted with a *half-model* configuration. All tests considered herein relate to the 3D scour configuration as defined by Pagliara et al. (2008a), i.e., $B_m/B_c < 3$, with B_m as the maximum width of the scour hole.

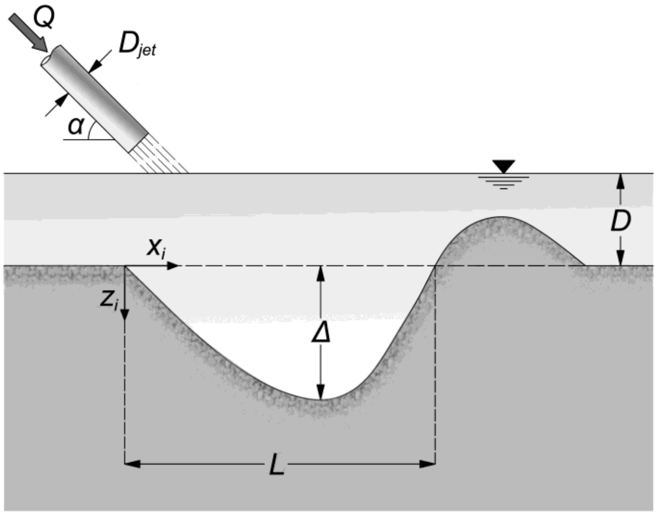

Figure 1 - Definition sketch of jet-induced scour along with the main geometric and hydraulic parameters.

It is worth mentioning that, under constant discharge, the scour hole evolves until reaching the *dynamic* equilibrium configuration. When the jet action ceases, the amount of material kept in suspension/rotation deposits in the scour hole resulting in the *static* equilibrium configuration. Therefore, the maximum *static* scour depth Δ_s might considerably differ from its *dynamic* counterpart (i.e., $\Delta_s < \Delta$).

Table 1 Summary of employed datasets

Reference	B_c (m)	Q (l/s)	D_{jet} (mm)	D (m)	d_{50} (mm)	α (deg)
Pagliara et al. (2008a)	0.80	0.51-8.89	27.0	0.015-0.289	10.3-11.5	45°, 60°
Pagliara et al. (2008b)	0.50	0.70-5.50	21.7-35.0	0.155-0.315	1.2	45°, 60°
Bombardelli et al. (2018)	0.80	2.38-3.45	16.0-51.0	0.027-0.189	7.45	90°

3. SELECTED APPROACHES

In the following, we report and discuss the methodologies listed in Table 2. In Table 3, we provide a summary of the scour formulas considered in this paper, along with the values of the parameters and the range of hydraulic and geometric conditions for which they were calibrated/validated. Note that the variables listed in Table 3 are defined in the next section.

Table 2 Scour formulas considered in the present paper.

Author(s)	Approach	Analyzed structure(s)
Mason and Arumugam (1985)	empirical	free overfalls, low level outlets, tunnel outlets, flip buckets
Mason (1989)	empirical	rectangular jet
Bormann and Julien (1991)	semi-theoretical	rectangular grade control structure
Hoffmans (2009)	semi-theoretical	rectangular jet
Bombardelli et al. (2018)	theoretical	plunging jet

Table 3 Scour formulas by different authors.

Author(s) and formula		Parameters	Tested conditions
Mason and Arumugam (1985) $D+\mathit{\Delta}_s=K_1 q^x h^y D^{0.15} g^{-0.30} d^{-0.10}$	model	K_1 = 3.27 x = 0.60 y = 0.05	$0.07<D+\mathit{\Delta}_s<1.18$ m $0.325<h<2.150$ m $0.015<q<0.420$ m²/s $0.001<d_{50}<0.028$ m $25°<\alpha<85°$
	prototype	$K_1 = (6.42-3.10h^{0.10})$ $x = (0.60-300^{-1}h)$ $y = (0.15+200^{-1}h)$	$6.70<D+\mathit{\Delta}_s<90.0$ m $15.82<h<109.0$ m $2.36<q<220.0$ m²/s $d_{50}=0.25$ m $20°<\alpha<72°$
Mason (1989) $D+\mathit{\Delta}_s=$ $3.39q^{0.60}(1+\beta)^{0.30}D^{0.16}g^{-0.30}d^{-0.06}$		β (Ervine, 1976)	$0.33<h<2.00$ m $\alpha=45°$
Bormann and Julien (1991) $D_p+\mathit{\Delta}=K_2 q^{0.6} U g^{-0.8} {d_{90}}^{-0.4} \sin\beta'$		$K_2 = C_d^2\left[\frac{\gamma\sin\varphi}{\sin(\varphi+\theta)B(\gamma_s-\gamma)}\right]^{0.8}$ B (Neill, 1968) C_d (Beltaos and Rajaratnam, 1973) $q = UY_0$	$0.10<\mathit{\Delta}<1.4$ m $0.05<D_p<0.38$ m $0.3<q<2.5$ m²/s $1.58<d_{90}<1.71$ mm $18°<\lambda<90°$
Hoffmans (2009) $D+\mathit{\Delta}=c_{2V} q^{1/2} U^{1/2} (\sin\alpha)^{1/2} g^{-1/2}$	$d_{90}<0.0125$ m	$c_{2V} = 20 d_{90}^{-1/3} g^{-1/9} \nu^{2/9} \left(\frac{\rho_s-\rho}{\rho}\right)^{-1/9}$	$0.03<h<40$ m $0.004<q<275$ m²/s $0.1<d_{90}<100$ mm
	$d_{90}\geq 0.0125$ m	$c_{2V} = 2.9$	
Bombardelli et al. (2018) $D+\mathit{\Delta}=K_3\left(\frac{\rho}{\rho_s-\rho}\right)^{3/5}(Qh)^{2/5} d_{50}^{-2/5} g^{-1/5}$		K_3 = 0.50	$0.04<D+\mathit{\Delta}_s<0.67$ m $1.2<d_{50}<11.5$ mm $45°<\alpha<90°$

3.1. Mason and Arumugam (1985) and Mason (1989)

Mason and Arumugam (1985) considered a set of model and prototype scour data pertaining to different structures, including free overfalls, low level outlets, spillway flip buckets, and tunnel outlets. Their datasets included 47 model tests, for which both cohesive and non-cohesive substrates were adopted. The prototype datasets consisted of 26 scour data. The range of hydraulic and geometric conditions adopted in model and prototype tests, respectively, is summarized in Table 3, where h indicates the head drop, and q is the unit discharge. Mason and Arumugam (1985) validated a wide number of existing scour formulas against their datasets, allowing them to present the following expression to estimate the *static* scour depth:

$$D + \Delta_s = K_1 q^x h^y D^{0.15} g^{-0.30} d^{-0.10} \tag{1}$$

where g is the gravitational acceleration, and the coefficient K_1 and exponents x and y were calibrated using experimental data. Different values of K_1, x and y were proposed for model data and prototype data, as indicated in Table 3. It is worth noting that in both cases the expression is dimensionally incorrect and K_1 is not a coefficient and depends on data used for calibration.

Mason (1989) further extended Mason and Arumugam's work by examining the effect of air entrainment on scour features. In so doing, he conducted experiments with a rectangular jet for α = 45°, under a controlled air/water ratio β. Air was provided by means of a low-pressure fan and was injected under the water outlet through a control valve (see Mason,1989 for further details). This methodology allowed the air to be entrained in the water jet in small bubbles, leading to the following empirical equation:

$$D + \Delta_s = 3.39 q^{0.60} (1+\beta)^{0.30} D^{0.16} g^{-0.30} d^{-0.06} \tag{2}$$

In Eq. (2), β was calculated according to Ervine (1976). Overall, Eq. (2) has the same analytical structure of Eq. (1), with the air entrainment term replacing h. Mason (1989) suggested that Eq. (2) is valid for both models and prototypes.

3.2. Bormann and Julien (1991)

Bormann and Julien (1991) adopted a semi-theoretical approach to predict the scour downstream of a grade control structure. They considered a grade control structure with a sloping downstream face (inclination λ with respect to the horizontal) and drop height D_p. Their approach is based on jet diffusion theory (Beltaos and Rajaratnam, 1973). They assumed that the equilibrium condition is achieved when the shear stress acting on the granular bed equals the critical shear stress. In doing so, they adopted some empirically validated assumptions. Therefore, the proposed approach cannot be considered fully theoretical. Bormann and Julien (1991) derived the following equation to estimate the equilibrium scour depth:

$$D_p + \Delta = K_2 q^{0.6} U g^{-0.8} {d_{90}}^{-0.4} \sin\beta' \tag{3}$$

where the unit discharge q is calculated as $q = UY_0$, with Y_0 indicating the jet thickness and U the average velocity of the jet, d_{90} is the material size for which 90% is finer, β' is the inclination of the diffused jet with respect to the horizontal, depending on Y_0, λ, D_p, g, U, Y_t (downstream water level). The expression of K_2 is reported in Table 3, with C_d indicating the jet diffusion coefficient that depends on the inlet conditions (Beltaos and Rajaratnam, 1973), γ_s and γ the specific weights of sediment and water, respectively, φ is the submerged angle of repose of the sediment, θ is the inclination of the downstream face of the scour hole, and B is a local friction coefficient (Neill, 1968). Bormann and Julien (1991) validated their formula using large-scale experiments carried out in an outdoor channel 0.91m-wide, 27.4m-long, and 3.5m-deep. Ranges of hydraulic and geometric conditions of tests are summarized in Table 3.

3.3. Hoffmans (2009)

Hoffmans (2009) applied the linear momentum equation to a selected control volume, by assuming that the horizontal component of the resultant can be expressed using the equation of Forcheimer. Namely, he proposed the following equation:

$$D + \Delta = c_{2V} q^{1/2} U^{1/2} (\sin\alpha)^{1/2} g^{-1/2} \tag{4}$$

where the value of c_{2V} is reported in Table 3, with v indicating the kinematic viscosity of the fluid. Note that the value of c_{2V} was calibrated using a large set of plunging jet data collected by various researchers. The dataset encompassed a wide range of hydraulic conditions and different geometrical configurations, including overfalls and grade control structures (see Table 3). Based on dimensional arguments and experimental data, Hoffmans (2009) argued that the coefficient c_{2V} is almost constant for $d_{90} \geq 0.0125$m; conversely, for $d_{90} < 0.0125$m, c_{2V} is a function of the densimetric particle number and granulometric characteristics of the bed and an ad-hoc empirical expression is proposed to estimate it.

3.4. Bombardelli et al. (2018)

Following the methodologies developed by Gioia and Bombardelli (2005) and Bombardelli and Gioia (2006), Bombardelli et al. (2018) analyzed the jet-induced scour process for the 3D case. In so doing, they considered a jet of water entering a pool of depth D from a height h. By applying the phenomenological theory of turbulence and energetic considerations to the resulting turbulent cauldron, they derived a fully theoretical equation to estimate $D+\Delta$:

$$D + \Delta = K_3 \left(\frac{\rho}{\rho_s - \rho}\right)^{3/5} (Qh)^{2/5} d_{50}^{-2/5} g^{-1/5} \tag{5}$$

where K_3 is a multiplicative constant set equal to 0.50. The exponents of each variable were theoretically derived. Therefore, this formula is dimensionally correct. Note that the theoretical values of the exponents were compared with those of the best-known empirical formulas, revealing a considerable agreement. Most importantly, the approach of Bombardelli et al. (2018) provides unprecedented insight into the physics of the interaction between sediment and turbulent flow. It is worth mentioning that Bombardelli et al. (2018) also proposed an equation for the 2D case [with the same structure as Eq. (5)] for which specific exponents and the multiplicative constant were determined.

4. RESULTS AND DISCUSSION

All the empirical and semi-theoretical equations adopted herein involve the unit discharge q. This variable can be easily defined (and computed) for jets originating from drop structures. But what does q represent for circular jet-driven scour processes? To answer this question, we tested the previous approaches assuming $q = Q^*/B_c$ and $q = Q^*/D^*$, with $Q^* = Q$ and $D^* = D_{jet}$ for tests conducted in *full-model* configuration, whereas $Q^* = 2Q$ and $D^* = 2^{0.5}D_{jet}$ for tests conducted with the *half-model* configuration. Note that D^* is the equivalent jet diameter, allowing to obtain the same jet velocity U for *full* and *half-model* arrangements.

In the formula of Mason and Arumugam (1985) we assumed $h = U^2/(2g)$. Note that Eq. (1) provides similar results regardless of the coefficients adopted. Therefore, the following estimations were done using the coefficients calibrated for models. Figure 2 contrasts measured (subscript *meas*) and calculated (subscript *calc*) values obtained assuming $q = Q^*/B_c$ (Fig. 2a) and $q = Q^*/D^*$ (Fig. 2b). In both cases, the estimations provided by Mason and Arumugam's formula exhibit a similar deviation from the perfect agreement line. However, for $q = Q^*/B_c$ experimental values are significantly underestimated, whereas the opposite occurs for $q = Q^*/D^*$ and results are in agreement with the findings of Pagliara et al. (2004) (see Figures 2a and 3a of that study). From a practical point of view, the second approach is preferable, as it can be assumed to include a safety coefficient. Overall, such differences could depend on the geometrical configuration of the tests analyzed by Mason and Arumugam (1985), involving different structures like flip buckets and tunnel outlets. However, as the resulting equilibrium morphology is essentially 3D, it is believed that the assumption $q = Q^*/D^*$ is more consistent with the physics of the jet-scour processes.

It is worth remarking that Mason and Arumugam (1985) and Mason (1989) formulas were calibrated using *static* scour data, whereas data of Pagliara et al. (2008a) and (2008b) and Bombardelli et al. (2018) refer to the *dynamic* equilibrium configuration. Consequently, an underestimation of the data should be expected. Nevertheless, Pagliara et al. (2004) argued that the performance of Eq. (1) does not vary considerably considering the *static* and *dynamic* scenarios. The inclusion of the air entrainment effect does not alter the overall predicting capability of the approach proposed by Mason (1989). In fact, the average deviation of predicted values of $D+\Delta$ using Eqs. (1) and (2) is less than 10% for $q = Q^*/B_c$, and less than 2% for $q = Q^*/D^*$. This slight difference may be due to the fact that experimental tests adopted herein were conducted under black water conditions.

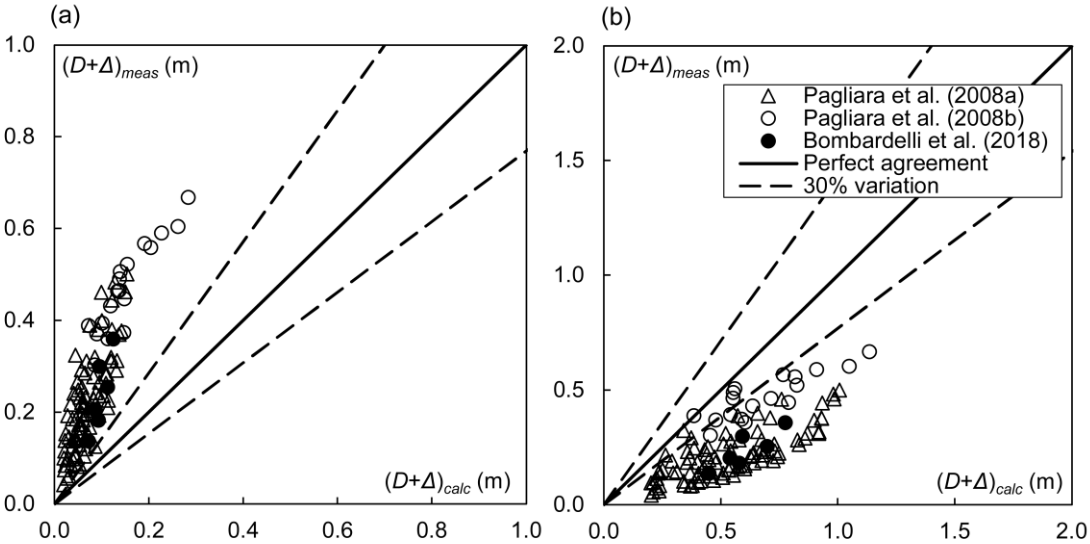

Figure 2 – Comparison of measured and calculated (using Eq. 1) values of the variable $D+\Delta$ for (a) $q=Q^*/B_c$ and (b) $q=Q^*/D^*$.

Likewise, Eq. (3) was tested assuming $D_p = D$; $U = (2gh)^{0.5}$, $\beta' = \alpha$, $\varphi = 25°$, $C_d = 1.8$, $\theta = \alpha$, and $B = 2.0$. Figures 3a and b show the results obtained for $q = Q^*/B_c$ and $q = Q^*/D^*$, respectively. It is worth mentioning that we also tested the performance of Eq. (3) assuming $q = UY_0$ (as suggested by the authors for grade-control structures), with $Y_0 = D^*$. In this case, we obtain $q = 4Q/(\pi D^*) = 1.27Q/D^*$. But considering that the exponent of q is equal to 0.6 in Eq. (3), the calculated values of $D+\Delta$ (not reported herein) are consistent with those shown in Fig. 3b. Figure 3 indicates that the predicting capability of Eq. (3) improves for Q^*/B_c, providing reasonable results (36% of data are within a ±30% deviation from the perfect agreement line). Overall, this approach tends to underestimate the diffusion length for jet-driven scour processes. Conversely, Eq. (3) systematically overestimates experimental data for $q = Q^*/D^*$. This occurrence should not be a surprise, as Bormann and Julien (1991) considered 2D structures and, consequently, assumed that the flow features in the downstream pool are essentially two-dimensional. This is in stark contrast with the physics of an impinging jet originating from a dam spillway. However, when the resulting equilibrium morphology becomes less three-dimensional, scour processes may exhibit some similitudes. Therefore, in such cases, B_c appears to be a more adequate reference length. Finally, the agreement between data and predictions (with $q = Q^*/B_c$) seems to improve with $D+\Delta$, i.e., with the increase of the diffusion length, for which, generally, scour features are more two-dimensional.

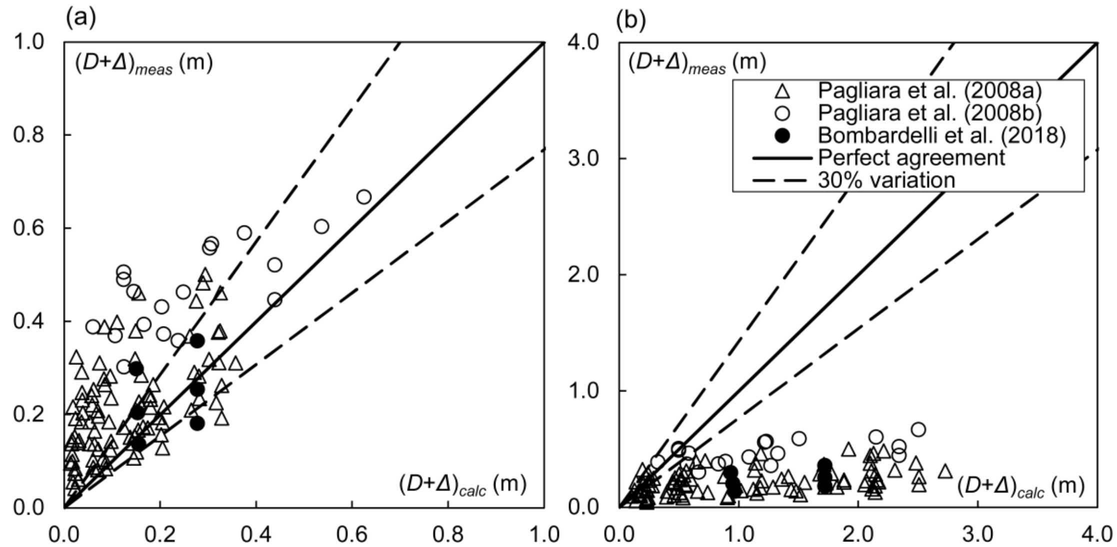

Figure 3 – Comparison of measured and calculated (using Eq. 3) values of the variable $D+\Delta$ for (a) $q=Q^*/B_c$ and (b) $q=Q^*/D^*$.

As regards the approach of Hoffmans (2008), we assumed $U = (2gh)^{0.5}$ and $v = 10^{-6}$ m²/s. Figures 4a and b show the comparison between measured and predicted values of $D+\Delta$ using $q = Q^*/B_c$ and $q = Q^*/D^*$, respectively. Results are consistent with those obtained using Eq. (3) although, overall, Eq. (4) performs better for both cases. Once again, this occurrence should be expected provided that Hoffmans (2008) applied the linear momentum equation to a control volume per unit width. More specifically, the performance of Eq. (4) is reasonably acceptable for $q = Q^*/B_c$ (the deviation of 42% of predicted data is less than 30%), although data are underestimated by 40% on average. Therefore, considerations similar to the previous case apply also to this approach.

Finally, we contrasted measured values of the variable $D+\Delta$ against those computed by using Eq. (5), for which we assumed $h = U^2/(2g)$ (Fig. 5). The PTT-based approach does not account for the effect of the jet inclination. Nevertheless, it should be mentioned that Pagliara et al. (2008a) and Hoffmans (1998) argued that it is not prominent for oblique jets with $\alpha > 60°$. Overall, it was found that the theoretical approach performs better than any of those analyzed in this paper. Regardless of the simplifications adopted to develop Eq. (5), the PTT-based approach has the advantage to be independent of the tested conditions and, in principle, is not affected by scale effects.

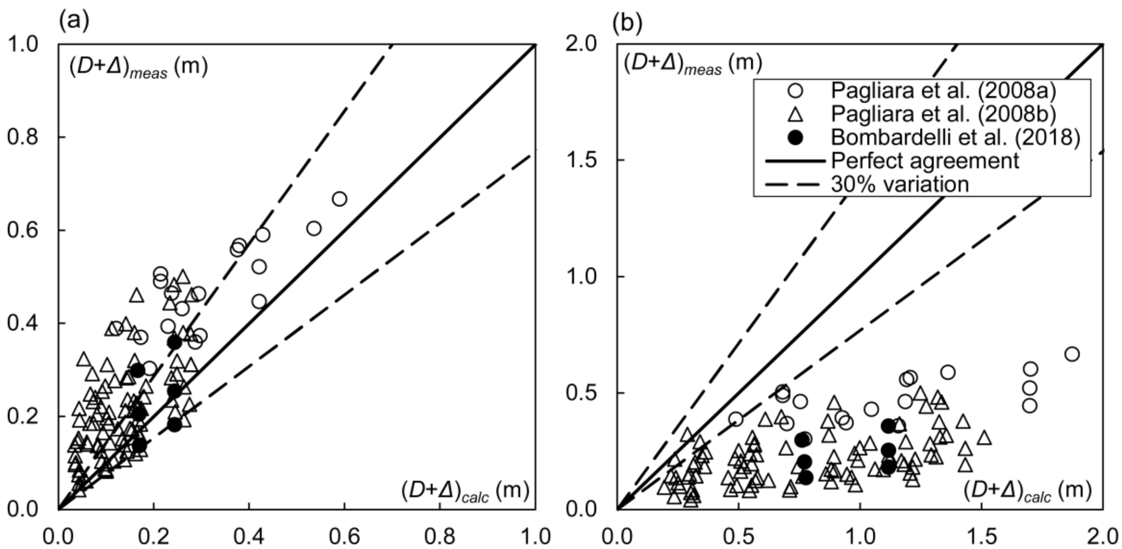

Figure 4 – Comparison of measured and calculated (using Eq. 4) values of the variable $D+\Delta$ for (a) $q=Q^*/B_c$ and (b) $q=Q^*/D^*$.

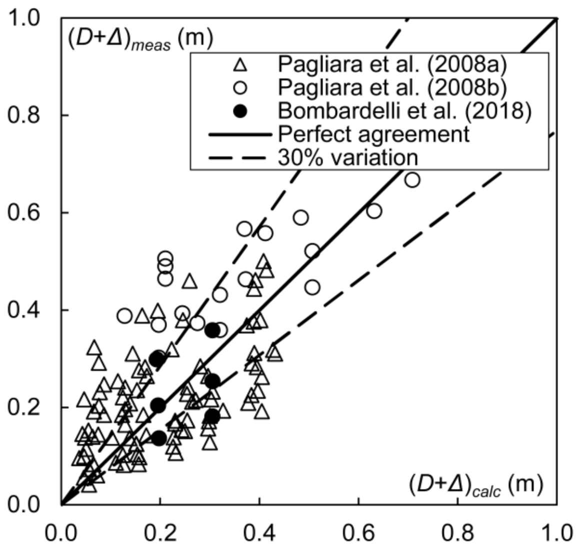

Figure 5 – Comparison of measured and calculated (using Eq. 5) values of the variable $D+\Delta$.

5. CONCLUSION

In this paper, we analyzed the performance of different approaches to estimate the scour depth caused by circular plunging jets. We presented a selection of empirical, semi-theoretical, and theoretical formulas and discussed their applicability to the case of a 3D equilibrium configuration. To this end, interesting observations were provided pertaining to the estimation of the unit discharge present in most of the analyzed approaches. In particular, we pointed out that different estimations of the unit discharge may lead to discordant results, and we indicated the most reasonable estimation method for each formula. In doing so, we highlighted differences and similarities characterizing the selected equations. By using a large dataset pertaining to the 3D equilibrium scour depth, we also corroborated the limits of applicability of both empirical and semi-empirical formulas. Conversely, we showed that the fully theoretical approach proposed by Bombardelli et al. (2018) provides reliable results regardless of the tested range of parameters.

6. ACKNOWLEDGMENTS

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