Engineering Structures 252 (2022) 113485

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Optimization framework for cost and carbon emission of timber floor elements

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ARTICLE INFO

Keywords:
Cost optimization
Carbon emission reduction
Timber floor
Eurocode 5

ABSTRACT

Long-span timber floor elements increase the adaptability of a building and they exhibit a significant market potential. High cost of the floor elements is a challenge, and the timber sector is under substantial pressure to find more economical solutions without weakening otherwise favourable environmental performance. The range of technical timber-based materials and components, structural typologies, overlays and ceiling systems represent an immense solution space when searching for a competitive design for a specific building application. Finding the optimum solution requires a computational procedure. In this study a recent development for the accounting of manufacturing resources for timber elements is utilized to build an optimization framework for cost and ECO2 minimisation of timber floor elements finalized at the factory gate. The design of the element is formulated as a discrete optimization problem which is solved by a mixed-integer sequential linearization procedure. Various material combinations and constraint combinations are treated. The optimization framework provides a tool for rapid design exploration that can be used in timber floor design situations. The results of the calculations carried out in this study provide insight on the general trends of optimum floor elements. The optimization model is used to analyse the characteristics of the optimum designs, and a comparison between the current and the proposed method for the second generation of Eurocode 5 is chosen as a vehicle for demonstrating achievable implications.

1. Introduction

The built environment is significantly contributing to the climate change today and represents therefore a substantial opportunity for mitigating it tomorrow. The role of the construction sector must increasingly be addressed as a measure to decelerate global warming [1]. Currently this sector is strongly identified with negative climatic impact, accounting for 36% of the global energy use and an associated 39% of the carbon dioxide emissions [2]. Even as 85% of the buildings we will inhabit in 2050 are already built [3], the construction sector is expected to erect some 230 billion square metres of new construction over the next 40 years [2]. The challenge is substantial, and the greenhouse gas (GHG) emissions related to the construction sector are likely to be doubled by 2050 [4]. The last three decades the GHG emissions from the construction sector have increased with 55% and are currently one of the three fastest growing sources [5].

It is a general understanding that the widely agreed emission reduction targets [6] cannot be met without appropriate actions in the construction sector. A recent study on material efficiency for reducing GHG in the construction sector [6] has examined various strategies such as more intensive use of materials, lifetime extension of buildings, lightweight design, and reuse of building components.

Another possibility is to develop new products that meet the imposed technical requirements while simultaneously being economically competitive with reduced GHG emissions. Such elements exhibit a substantial market potential, and the timber sector is endeavouring to gain market shares for commercial building applications. Long-span timber floors is required for this sector, and an associated span of minimum 7.2 m would be required to allow both basement parking space grid and an adaptable commercial building plan layout [7]. However, the competitivity of timber flooring systems for this segment is low, and the potential advantages in carbon emissions must be accompanied with suitable costs. It has been shown that the cost of timber floor elements can be nearly twice the cost of a comparable concrete hollow-core element [8], and the additional challenges of acoustics and

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https://doi.org/10.1016/j.engstruct.2021.113485
Received 2 March 2021; Received in revised form 30 September 2021; Accepted 23 October 2021
Available online 29 November 2021

Nomenclature

a_i	Position of neutral axes [m]
a_{RMS}	Root mean square acceleration due to human induced vibration [m/s^2]
B	System width of the flooring system defined as 1.5 times L [m]
b_i	Effective width of flanges [m]
B-LVLQ	Beech LVL type Q
CC	Constraint combination
C_ξ	Cost as calculated by the IDABC method [€/m^2]
D_L, D_T	The apparent stiffness (D) of flooring system is the bending stiffness of a section divided by the extent of the section longitudinally (D_L) and transversally (D_T) [Nm^2/m]
ECO2	Embodied carbon emissions [kgCO2eq]
$ECO2_\xi$	Embodied carbon emissions as calculated by the IDABC method [kgCO2eq/m^2]
EI_L, EI_T	Longitudinal and transversal bending stiffness (EI) longitudinally (EI_L) and transversally (EI_T) [Nm^2]
EI_T,midsection	Transversal bending stiffness of midsection [Nm^2]
F	Vertical force imposed by walking person [N]
f(**x**)	Relevant responses used in design
f_1	First natural frequency (fundamental frequency) [Hz]
FPL	Floor Performance Levels
f_w	Walking frequency [Hz]
g(x)	Self weight of the floor element [N/m]
GA_L	Shear stiffness in longitudinal direction. Only longitudinal members will in practice contribute to the shear capacity from bending [Nm^2/m^2]
GHG	Greenhouse gas
GL	Glulam
h_1	Top flange height [mm]
h_2	Joist (core) height [mm]
h_3	Bottom flange height [mm]
HB HLA1 (HB)	Construction plates of high density fibre board
HbtmFlg	Available material height for bottom flange [mm]
HC	Hu and Chui empirical serviceability parameter for dynamic response [-]
h_{CHS}	Depth of base floor element [mm]
Hjst	Available material height for joists [mm]
HtopFlg	Available material heights for top flange [mm]
IDABC	Item-Driven Activity-Based Consumption
k	Winkler foundation stiffness [N/m^2]
$k_{e,2}$	Frequency multiplier representing the transverse floor stiffness [DL]
K_{imp}	Higher modes multiplier for transient floor response [DL]
K_{sfd}	Constant in prediction of shear force deformations [DL]
L	Span length [L]
LC_ξ	Part of cost associated with labor as calculated by the IDABC method [€/m^2]
l_m	Mean modal impulse [Ns]
$L_{midsection}$	The length of the mid-section defined as half the span length of the floor element [m]
LVL	Laminated Veneer Lumber
L_{wink}	Length of Winkler foundation equal to the effective length of the transverse midsection [m]
m	Mass (kg) of floor per unit area (m^2)
M*	Modal mass [kg]
MISLP	Mixed-integer sequential linearization procedure
numStructTrns	The number of structural sections transversally [DL]
OSB 3 (OSB)	Construction plates of oriented strand board
p	Unit point load [N]
q	Distributed load in [N/m^2]
Q	Point load [N]
S-LVLQ	Spruce LVL type Q
T_ξ	Duration as calculated by the IDABC method [s/m^2]
V_{RMS}	Root mean square velocity due to human induced vibration [m(s]
$\boldsymbol{V}_\xi$	Expenditure vector as calculated by the IDABC method
w	Equivalent beam deflection at 1 kN point load [m]
w_1	Edge joist width [mm]
w_2	Field joist width [mm]
Wedge	Available material width for edge joists [mm]
Wfield	Available material width for field joists [mm]
w_{fin}	Deformation from permanent and imposed loads calculated as equivalent beam [m]
w_{mod}	Module width of floor element [m]
w_{sys}	System deformations due to self-load [m]
w_{wink}	Estimated two-way deflection due to 1 kN point load [m]
x	Vector of design variables [mm]
α	Fourier coefficient [–]
γ	Composite effect [DL]
ζ	Modal damping ratio [%]
η	Scaling factor for K_{imp} [DL]
ξ	An item subject to an activity in calculation of product expenditures of the IDABC method
$V_{product}$	The total expenditure of the product as calculated by the IDABC method

serviceability performance are causing the construction sector to be reluctant to accept timber floor elements widely [9]. For timber to become an attractive building material in this market, innovative, competitive and industrialized concepts with high technical qualities and minimal economic risks and investments need to be developed, documented and made readily available.

Timber flooring systems for long-span applications are normally glued thin flange elements with stiffeners and joists constituting the core. The number of joists and stiffeners, the internal added weight and insulation, and the dimensions of all members result in numerous potential combinations to be examined. This number increases drastically when the range of wood products and types of bonding are considered. When outfitting such as overlays and ceiling system is addressed, the number of combinations increases further. And finally, when support and load conditions and serviceability performance levels are regarded, the solution space is immense. With these many parameters, finding a competitive design may not be manageable by manual exploration, and the solution space can in practice only confidently be investigated when assessed computationally.

Timber structures have been optimized for greater material efficiency in [10,11], with the conclusion that the required amount of material in a construction can be substantially reduced, but the study does not reflect the resources of manufacturing nor the environmental impact from reduced potential reuse. Incorporating the total manufacturing cost and the environmental impact of the floor element in optimization are identified as main issues for the present work. For steel structures, cost optimization has been widely employed in the literature. In [12] a cost centre approach is used which resembles the cost accounting method used in the present work. The minimum cost designs of steel floors are obtained in [13], taking into account the cost of material, labour, equipment, overhead and including profit as well. In [14] the cost objective of composite floors is based on simple summation of costs of accrued material and manufacturing processes. The minimum cost is investigated in terms of how a change in steel price would affect the different structural principles that the composite floor is based on.

In a study by Mahn et al [15] optimization of wooden floors is

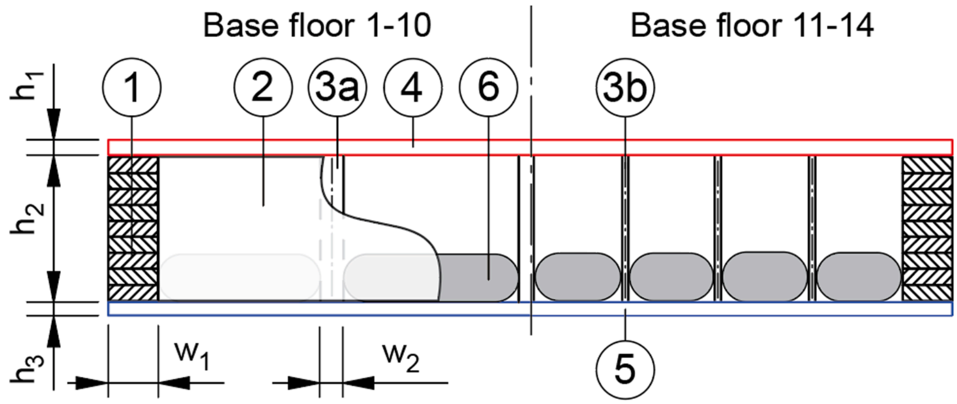

Fig. 1. Cross-section of base floors including design variables of the optimization problem.

Table 1
Material composition and due combinations.

Base floor	Edge joist and beam	Num fldJst	Field joist	Flanges	Possible combinations			
					Case 1–14	Case 15–28	Case 29–42	Case 43–56
1	GL30c	3	GL28c	S-LVLQ	44,496	129,024	48,384	129,024
2	GL30c	3	S-LVLS	S-LVLQ	32,256	86,016	32,256	86,016
3	GL30c	3	B-LVLS	S-LVLQ	40,320	107,520	40,320	107,520
4	GL30c	3	GL28c	B-LVLQ	42,336	98,784	42,336	98,784
5	GL30c	3	S-LVLS	B-LVLQ	28,224	65,856	28,224	65,856
6	GL30c	3	B-LVLS	B-LVLQ	35,280	82,320	35,280	82,320
7	S-LVLS	3	S-LVLS	S-LVLQ	9216	24,576	9216	24,576
8	B-LVLS	3	B-LVLS	S-LVLQ	31,200	83,200	31,200	83,200
9	S-LVLS	3	S-LVLS	B-LVLQ	8064	18,816	8064	18,816
10	B-LVLS	3	B-LVLS	B-LVLQ	27,300	63,700	27,300	63,700
11	GL30c	7	HB HLA1	S-LVLQ	24,192	64,512	24,192	64,512
12	GL30c	7	HB HLA1	B-LVLQ	21,168	49,392	21,168	49,392
13	GL30c	7	OSB 3	S-LVLQ	16,128	43,008	16,128	43,008
14	GL30c	7	OSB 3	B-LVLQ	14,112	32,928	14,112	32,928
Number of combinations					374,292	949,652	378,180	949,652

conducted in terms of acoustic performance, and in the context of increasing market impact of timber floors. The conclusions of the study are in line with the general concern of a low market share of timber floors. However, no further findings in the study offer support to the present work. Acoustic performance of timber floors is studied in [16], where a comparable hollow-core timber floor is parametrically described and optimized for sound insulation. Here it is reported that the various parameters could not simultaneously be minimised, leading to the definition of a compromise. A probabilistic robustness analysis based on the Pareto front of two significant parameters was performed to find the optimum compromise in [16].

Optimization is useful also when there are conflicting criteria, and when different objectives cause disagreeing designs. This is also the case for timber floor elements. Then, the methods of multiobjective optimization can be employed, for example, to consider cost management such as in [17], where three conflicting criteria (target costing, value engineering and quality function deployment) are integrated in a single-objective optimization to balance cost, functionality and customer satisfaction of a product.

One of the challenges of long-span timber floors has been uncertainties in vibration performance. Unless idealised support conditions and simple floor element construction, the assessment may require numerical analyses. However, the method as proposed for the second generation of Eurocode 5 [18,19] is based on research efforts over the last 30 years, resulting in a new and rigorous analytical calculation procedure. Currently this proposal is included in the Final draft of the second generation of Eurocode 5 and expected subject to formal vote. In the present work we have chosen to adopt this proposal as a vehicle for demonstrating the optimization framework, and compare the performance to the current common analytical method of assessing serviceability in Norway [20,21].

In this study, the cost and ECO2 minimisation of a novel timber floor element is presented, and the design approach is formulated as an optimization problem that is solved by an appropriate method. The manufacturing cost and ECO2 of the element are taken as objective functions, and they are evaluated by the parametric accounting method of resources in the manufacturing of timber elements, developed in [22]. This workflow is in accordance with the conclusions of Forintek and the Canadian Wood Council [23] stating that a precise manufacturing cost accounting in combination with an optimization workflow can offer an efficient solution for the development of competitive timber floor elements.

A mixed-integer sequential linearization procedure is employed to solve the formulated discrete optimization problem. Various material combinations and constraint combinations are treated. The optimization model is used to perform a parametric study for alternating span of the element. The results of optimization are used to analyse the characteristics of the optimum floor element designs.

The objective of the present work is to assess the potential of timber floor elements suitable for adaptable building applications. The optimization framework provides a tool for rapid design exploration that can be used in timber floor design situations. Moreover, the results of the calculations carried out in this study provide insight on the general trends of optimum floor elements.

The paper is organised as follows. In Section 2, the timber floor element is described in detail, including the cost and ECO2 evaluation. The treated optimization problem is presented in Section 3, followed by a computational study in Section 4. The implications of the results are

Table 2
Allowable dimension values.

Allowable dimensions		Material	Allowable values [mm]
Abbreviation	Description		
HtopFlg	Height of top flange	S-LVLQ	33,39,45,51,57,63,69,75
		B-LVLQ	20,30,40,50,60,70,80
Hjst	Height of joists	GL	90,115,135,180,225,270,315,360,405,450,495,540,585,630
		S-LVLS	200,220,240,300,360,400
		B-LVLS	120,160,200,240,280,320,360,400,440,480,520,560,600
HbtmFlg	Height of bottom flange	S-LVLS	(33,39,45,51,57,) 63,69,75
		B-LVLS	(20,30,40,50,) 60,70,80
Wedge	Width of edge joists	GL	36,48,66,73,90,115,140,165,190,215,140,260
		S-LVLS	27,33,39,45,51,57,63,75
		B-LVLS	40,50,60,80,100,120,160,200,240,280
Wfield	Width of field joists	GL	36,48,66,73,90,115,140,165,190,215,140,260
		S-LVLS	27,33,39,45,51,57,63,75
		B-LVLS	40,50,60,80,100,120,160,200,240,280
		HB	7,8,9,10,11,12
		OSB	12,15,18,22

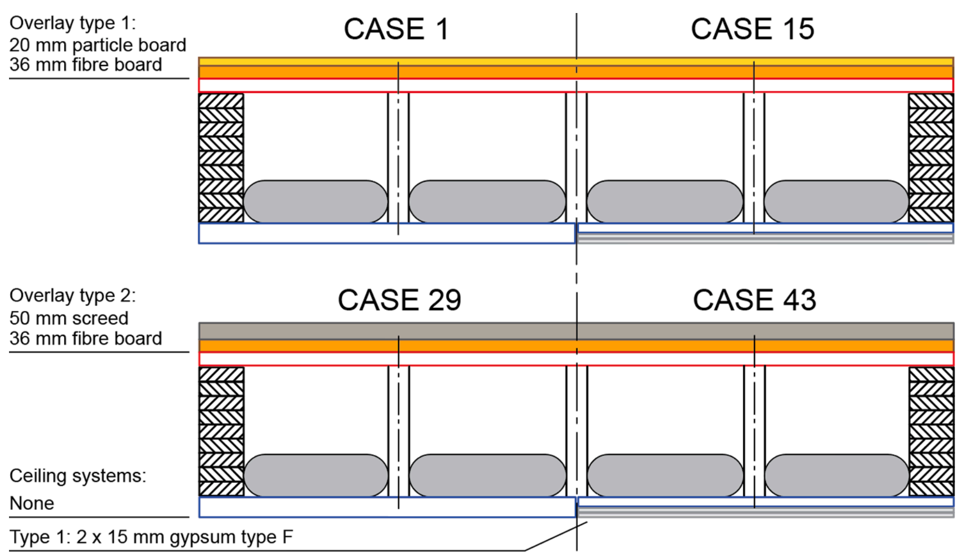

Fig. 2. Outfitting of base floor.

discussed in Section 5. Finally, conclusions of the research are drawn in Section 6.

2. Timber floor element

2.1. Primary structure

In the Nordic countries, timber flooring systems for long-span applications are typically constructed from a continuous top flange in Laminated Veneer Lumber (LVL) with joists in glulam (GL) each with a separate bottom flange. This design requires stiffeners between the joists to provide transverse bending stiffness. The stiffeners are laborious and the separate bottom flanges require the flooring system to have an additional fire resistance design. Of due reasons, in addition to the possibility of filling the space between the joists with a heavy mass, suggested a continuous bottom flange. A simply supported timber floor element constituting a closed hollow section as shown in Fig. 1 is consequently studied. By varying material combinations and the number of joists, fourteen base floors are defined. The base floor designs are created from an edging frame of joists ① and interconnecting transverse beams ②. Three or seven field joists ③ are fitted between the transverse beams positioned with equal centre to centre distances between all longitudinal members. In the cavities 100 kg/m^2 of gravel ⑥ is deposited to achieve acceptable acoustic performances. The continuous flanges are structurally glued on top ④ and bottom ⑤ of the frame. The design variables for the optimization problem as described in Section 3 are the dimensions shown in Fig. 1.

2.2. Material composition

The goal of the optimization is to explore the potential of changing material of edge frame members ① ②, flanges ④ ⑤ and field members ③. The materials are altered according to Table 1 to define the base floor designs. Base floor 1 is referred to as the reference floor. Glulam (GL) type GL30c and GL28c is according to [24]. LVL in spruce (S-LVL) and beech (B-LVL) is according to [25]. Two variants of LVL are used: LVLS has unidirectional fibre orientation, while LVLQ has a 20% of the fibres in crosswise direction. Construction plates in quality HB HLA1 (HB) and OSB3 (OSB) are used for field members [26]. In these cases, the number of field members is seven.

The various components of the floor element are available only in given dimensions. The standard delivery formats constitute the discrete values given in Table 2.

The bottom flange is intended either to be exposed to fire or covered by two layers of 15 mm gypsum type F. Rules for structural fire design [27] with guidance from [28,29] and chapter 11 in the Norwegian technical requirements for construction works [30] are used to calculate the required thickness. Hazard class 4 and fire class 3 are used,

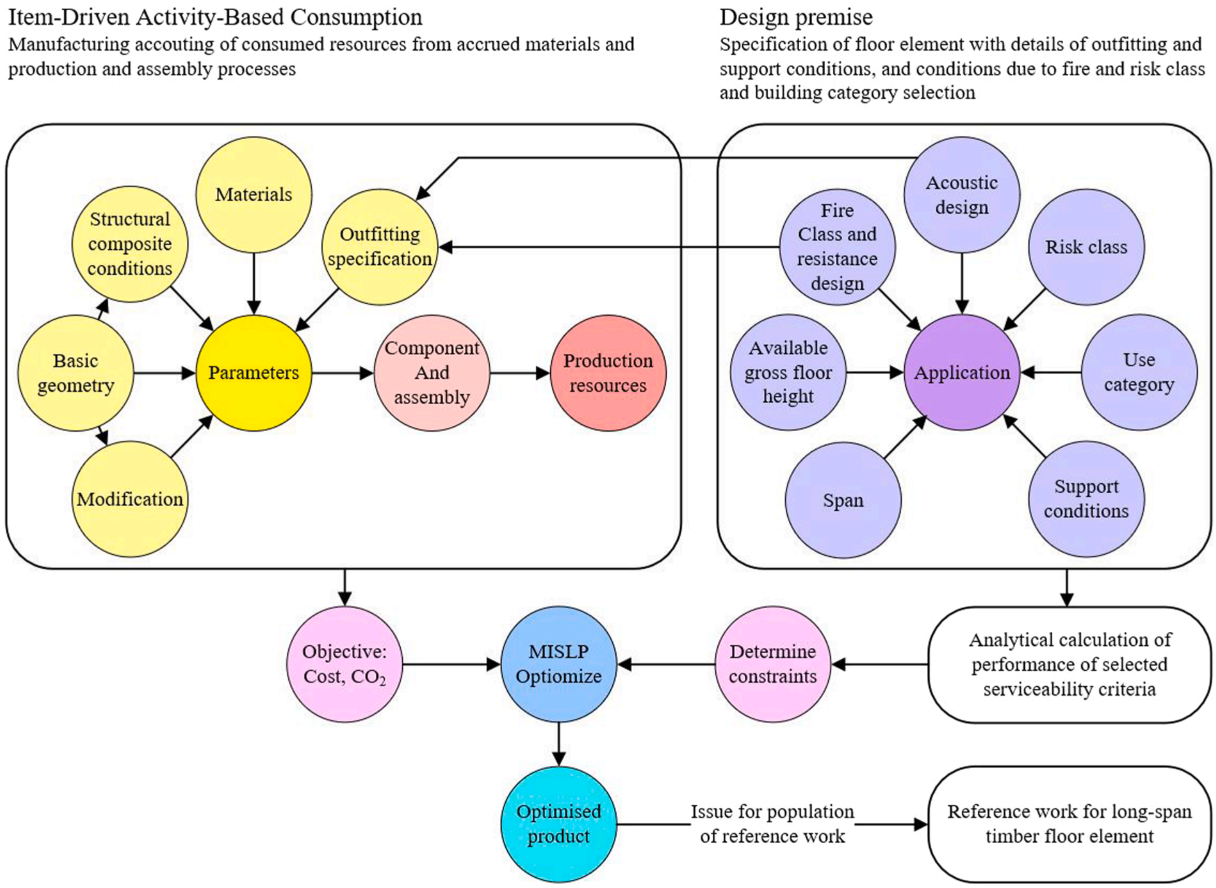

Fig. 3. Calculation framework for optimization.

presupposing a complete fire scenario of 90 min. The design philosophy is that the floor element shall have the capacity to withstand actions of accidental limit state without the bottom flange present. The minimum thickness of the bottom flange material is then calculated from the charring rate of the material for the fire scenario. Both spruce LVLQ and beech LVLQ have charring rate of 0.65 mm/min leading to a minimum thickness of 59 mm for exposed bottom flange (leaving the dimensions in brackets for H_{btmFlg} in Table 2 out), or 19.5 mm when two layers of gypsum type F is used as ceiling system. Restraining internal mass from fire exposure is not considered.

2.3. Design properties configuration

The base floors are fitted with a combination of non-structural overlay and ceiling system to acceptably estimate as built conditions. The ceiling system is designed to withstand fire exposure either as exposed bottom flange or covered by two layers of gypsum type F. The overlay is either type 1 or type 2 as indicated in Fig. 2. This results in four combinations of outfitting of the base floor designs 1 to 14, generating cases 1 to 56:

- Case 1 – 14: Base floor designs with overlay type 1 and exposed ceiling
- Case 15 – 28: Base floor designs with overlay type 1 and ceiling type 1
- Case 29 – 42: Base floor designs with overlay type 2 and exposed ceiling
- Case 43 – 56: Base floor designs with overlay type 2 and ceiling type 1

In Fig. 2 the associated cases of base floor 1 are shown.

For the optimization a constant module width (w_{mod}) of 2.4 m is used, and the design limit state is serviceability. Modifying support conditions, material specification, cavity mass, or thickness of edge beams will alter the optimization problem.

2.4. Economical and ecological performance

Cost and embodied carbon emissions are taken as objective functions in this study, and they are evaluated using a manufacturing expenditure accounting procedure developed in [22]. This method is called Item-Driven Activity-Based Consumption (IDABC). The method generates a parametric link between product specification and the expenditures in the manufacturing of a timber element. Expenditures cover manufacturing activities and accrued materials and it is presented as four indicators of competitiveness.

IDABC resembles the much used Time-driven Activity-Based Costing (TDABC) [37] in how the manufacturing line is modelled as resources combined to perform required activities. However, where the TDABC uses predetermined duration of activities to calculate costing, the IDABC method utilize information stored in the items subject to manufacture to calculate durations. For any item the activity requests a specific quantity based on predetermined SI unit associated with the activity, which in turn is used to calculate activity duration. Based on the duration of the activity and the definition of the activity and the underlying resources, manufacturing resources are determined.

Manufacturing of a floor element is typically divided into two parts: i) the making of components; and ii) the process of assembly. This separation is also seen in IDABC where components are made from direct material and then assembled to a final product. As can be seen in [22], an expenditure vector is generated for all items at every activity the item is subject to during the manufacturing.

Table 3
Constraint combinations and levels.

	Current Eurocode 5[1)]	Second generation of Eurocode 5	
		Resonant floor response	Transient floor response
CC	1	2a	2b
$h_{CHS,max}[m]$	1 or changing from 0.8 to 0.3 in Pareto-analysis		
$f_{1,min}[Hz]$	10	4.5	8
$f_{1,max}[Hz]$	–	8	–
$w_{1kN}[mm]$	1.3	$\{0.25 \quad 0.25 \quad 0.5 \quad 0.8 \quad 1.2 \quad 1.6\}^{1)}$	
Dynamic	$HC_{min} = 1[-]$	$a_{rms,max} = 0.005 \cdot R^{2)} \left[\frac{m}{s^2}\right]$	$v_{rms,max} = 0.0001 \cdot R^{2)} \left[\frac{m}{s}\right]$
$w_{fin,max}[m]$	$\frac{L}{200}$		

1) According to National Annex for Norway and SINTEF Building Research Design Guides [21]
2,3) Array for Floor Performance Levels (FPL) 1 to 6
3) Response factor levels [19]:$R(FPL) = \{4 \quad 8 \quad 12 \quad 20 \quad 30 \quad 40\}$

The expenditure vector V_ξ (Equation 3–1) comprise four quantities. This is the duration T_ξ [s], cost C_ξ [€], the part of cost associated with labor LC_ξ [€], and the ECO2 [kgCO$_2$eq], where ξ represent an item subject to an activity.

$$V_\xi = \{ T_\xi \quad C_\xi \quad LC_\xi \quad ECO2_\xi \} \qquad [s \quad € \quad € \quad kgCO_2eq] \tag{3.1}$$

The total expenditure of the product is the accumulated expenditures for body level and assembly level activities. See Equation 3–2.

$$V_{product} = \sum_{i=0}^{numBody} \sum_{j=0}^{numAct} V_{i,j} + \sum_{k=0}^{numAsmbly} \sum_{l=0}^{numAct} V_{k,l} \tag{3.2}$$

Specification of factory resources and activities are as defined in [22]. The conditions for applying the various activities, specification of fasteners, as well as principles of defining sections and material assignment likewise.

The cost objective is thus the total cost of the product finalized at the factory gates as offered by the wood component manufacturer $C_{product}$. The embodied carbon emissions of the product $ECO2_{product}$ have the same boundary conditions, normally referred to as cradle-to-gate, or A1 to A3 in the Environmental Product Declaration (EPD). The definition of resources and activities associated with the manufacturing of a timber

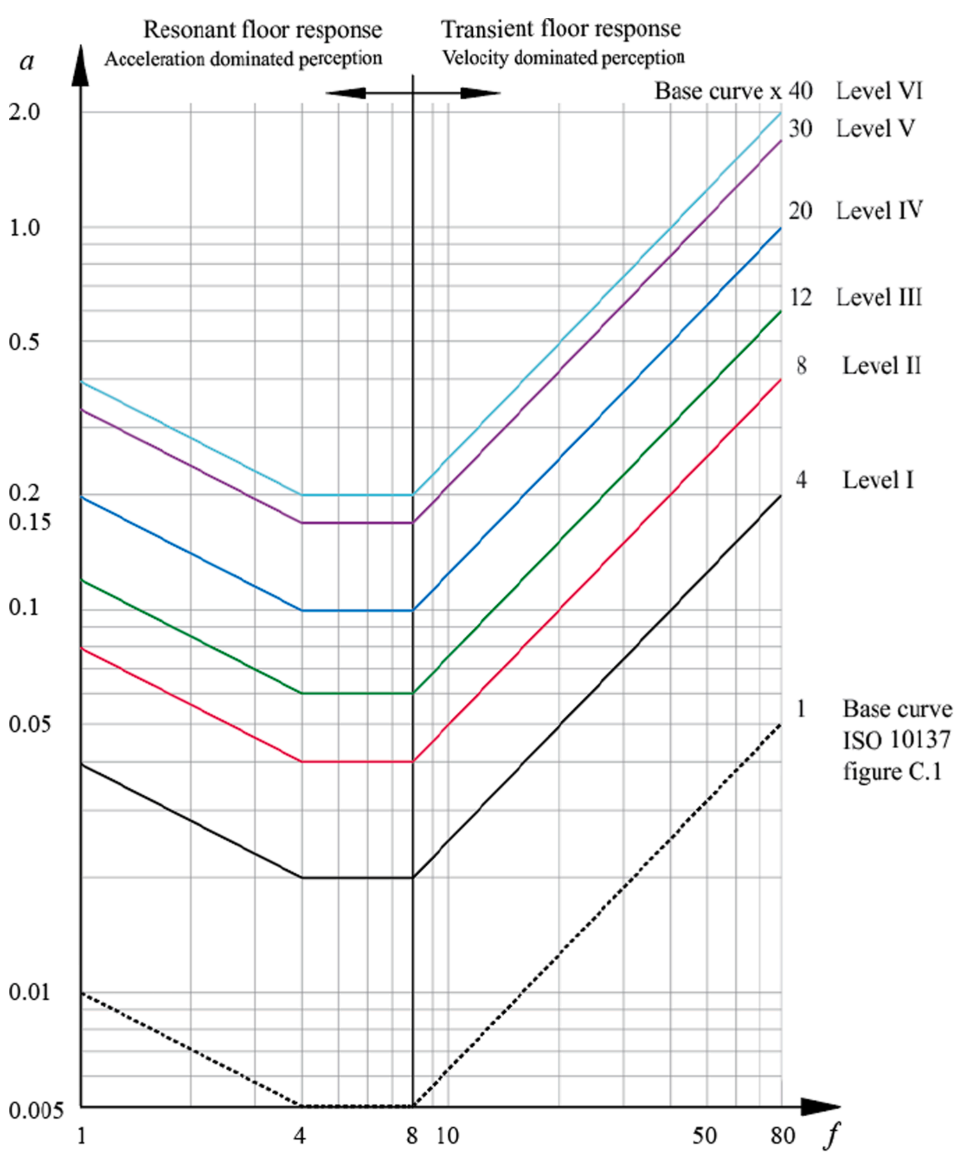

Fig. 4. Floor performance levels with respect to the ISO baseline curve.

element will change from one manufacturer to another, and the definition used in the present work is given in [22]. Unit cost and embodied carbon emissions of direct material are given in Appendix B.

3. Optimization

3.1. Framework

The optimisation framework consists of three modules as shown in Fig. 3: i) Design premise; ii) Item-Driven Activity-Based Consumption; and iii) Optimization. The background for the first two modules was described in Section 2. Their output is the cost and ECO2, and the constraint function values. This information is input to the optimization module (MISLP Optimise in Fig. 3). The output of the optimization module is the optimized product. In this Section, the details of optimization are provided.

All modules are parametric and they have been implemented using the principles of Object Oriented Programming in Python [38].

3.2. Problem formulation

3.2.1. Design variables

The optimization problem consists of an objective function that is to be minimised with respect to chosen design variables subject to given constraints. The structural responses used as constraint or objective functions are written as functions of the design variables. Therefore, the relevant responses used in design are written here in the form $f(\mathbf{x})$, where $\mathbf{x}$ is the vector of design variables. This vector consists of five dimensions of the cross-section (see Fig. 1 for the definition of symbols):

$$\boldsymbol{x} = \{ h_1 \quad h_2 \quad h_3 \quad w_1 \quad w_2 \} \quad [mm] \tag{4.1}$$

The design variables are discrete such that $h_1 \in H_{topFlg}$, $h_2 \in H_{jst}$, $h_3 \in H_{btmFlg}$, $w_1 \in W_{edge}$, and $w_2 \in W_{field}$. The corresponding allowable values are given in Table 2.

3.2.2. Constraint combinations and levels

As stated in the introduction the selected methods of serviceability constraints are adapted as a vehicle to demonstrate the optimization framework, and the selection is only based on their relation to the Eurocode. The constraints are arranged in three constraint combinations (CC) each with levels as stated in Table 3. Combination 1 is the current common practice for floor element design in Norway. This is based on the Ohlsson method of the current Eurocode [20], but where the Hu & Chui term rather than the unit impulse velocity is used [34]. Combination 2a and 2b is the method proposed for the second generation of Eurocode 5 [19]. The approach relates responses to human perception levels in terms of root mean square acceleration levels of the ISO baseline curve [35]. Acceleration levels dominate the human perception between 4 and 8 Hz. The ISO baseline curve level is constant in this frequency at $a_{RMS} = 0.005$ m/s^2. For human induced vibration, this frequency range is associated with a resonant floor design because the step frequency and the associated four first harmonics may coincide with the first natural frequency of the floor element.

Due to the ratio of stiffness and mass, long-span timber floor elements typically have a first natural frequency above 8 Hz. Above 8 Hz the ISO baseline curve is not constant (see Fig. 4). By integrating the baseline curve from 8 Hz, the corresponding velocity is constant at $v_{RMS} = 0.0001$ m/s [18]. This new constant is used as reference for floor performance levels above 8 Hz. For floor elements with first natural frequency above 8 Hz the floor response will be transient when subject to human induced vibration.

Additionally, element depth (h_{CHS}) is included as a constraint because of the financial importance the parameter has for tall timber building projects. The element depth is either a maximum value or increasingly constrained in a Pareto analysis to see the corresponding effect on cost and ECO2.

The equations for calculating the constraints are given in Appendix A.

3.2.3. Problem statement

The optimum design problem of the floor element can now be written as:

$$\begin{aligned} &\min \ f(\boldsymbol{x}) \\ &such \ \ that: \\ &\qquad f_{1,min} \leq \mathrm{f}_1(\mathbf{x}) \\ &\qquad f_{1,min} \leq \mathrm{f}_1(\mathbf{x}) \\ &\qquad f_{1,max} > \mathrm{f}_1(\mathbf{x}), \ \text{for CC2a only} \\ &\qquad HC_{min} \leq HC(\mathbf{x}), \ for \ CC1 \ only \\ &\qquad a_{rms,max} \geq a_{rms}(\mathbf{x}), \ for \ CC2a \ only \\ &\qquad v_{rms,max} \geq v_{rms}(\boldsymbol{x}), \ for \ CC2b \ only \\ &\qquad h_{CHS,min} < \mathrm{h}_{CHS}(\mathbf{x}) \\ &\qquad w_{1kN} \geq \mathrm{w}_{wink}(\mathbf{x}) \\ &\qquad w_{maxFin} \geq w_{fin}(\boldsymbol{x}) \\ &\qquad h_1 \in H_{topFlg} \\ &\qquad h_2 \in H_{jst} \\ &\qquad h_3 \in H_{btmFlg} \\ &\qquad w_1 \in W_{edge} \\ &\qquad w_2 \in W_{field} \end{aligned} \tag{4.2}$$

Where f(**x**) is cost C or embodied carbon emissions ECO2 of the floor element per area (m^2) as derived from Section 2. Note that some of the constraints will be removed depending on the constraint combination. The constants appearing on the left-hand side in the constraints are taken from Table 3 for each constraint combination.

The problem as stated in Equation 4–2 is a discrete nonlinear optimization problem consisting of five design variables and five or six constraints depending on the constraint combination. The problem is small-scale, and the objective and constraint functions are evaluated effortlessly through analytical expressions. For a given structural setup (span, materials, etc.), the problem may be solved by a brute force approach, where all combinations of design variable values are enumerated. This is performed for all cases to locate the cost and ECO2 minima as well as the computational effort required, but to rationally expedite the design space exploration, the problem of Equation 4–2 is solved by a suitable optimization method. As the design variables correspond to cross-sectional dimensions, they can be relaxed and treated as continuous variables during optimization. Moreover, the functions of the optimization problem are continuously differentiable. This allows the use of gradient-based optimization methods.

3.3. Mixed-Integer sequential linearization procedure

The optimization method employed in this study is based on solving a sequence of linear mixed-integer optimization problems. This method is a discrete extension of the well-known sequential linear programming (SLP) approach [39]. At each iteration point, the nonlinear functions are approximated by their linearization. The design variables are treated as continuous variables when solving the linearization. Discrete values can be enforced by introducing binary variables as follows.

Let × be a discrete variable with the allowable values $X = \{\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_d\}$. Then, introduce binary variables, $y_j \in \{0, 1\}, j = 1, 2, \cdots, d$. The variable × can be forced to have one of its allowable values by adding the following linear constraints to the optimization problem:

$$x = \sum_{j=1}^{d} \hat{x}_j y_j \tag{4.3}$$

$$\sum_{j=1}^{d} y_j = 1 \tag{4.4}$$

The latter equation ensures that exactly one binary variable takes the value 1, whereas the former equation sets the discrete value

Table 4
Indicators of accuracy of the optimization.

Deviation from global cost and ECO2 minimum	Current Eurocode 5 (with Hu and Chui)		Second generation of Eurocode 5			
			Resonant response		Transient response	
	Cost	ECO2	Cost	ECO2	Cost	ECO2
Mean error	3.81%	4.65%	1.41%	1.00%	1.83%	2.25%
Standard deviation of error	7.78%	8.73%	1.58%	1.18%	2.40%	3.49%

Table 5
Comparison to reference floor elements.

Floor element property			Base floor 1		Base floor 11	
			Reference	Optimum	Reference	Optimum
h_1 [mm]			45	33	45	39
h_2 [mm]			405	450	405	450
h_3 [mm]			63	63	63	63
w_1 [mm]			140 [1)]	140 [1)]	140 [1)]	140 [1)]
w_2 [mm]			66	48	8	7
Cost [€/m^2]			137.58	130.40	145.74	144.75
ECO2 [kgCO_2eq/m^2]			21.71	20.10	26.48	26.03
f_1 [Hz]	10	$\leq$	10.16	10.17	9.67	10.06
w_{1kN} [mm]	1.3	>	0.198	0.20	0.21	0.19
HC [-]	1	$\leq$	1.266	1.247	1.073	1.248
w_{maxFin} [mm]	45	$\geq$	16.79	16.31	18.02	16.82

[1)] Edge joist minimum width constrained to 140 mm

corresponding to the non-zero binary variable for *x*.

Each discrete variable is supplemented with its own binary variables and constraints of Equation 4–3 and Equation 4–4. During optimization, the discrete variables can be treated as continuous variables. Note that also the binary variables can be relaxed, so methods employing relaxation of discrete variables can be applied.

Consider the following optimization problem

$$\begin{aligned} &\min_{x} f(\boldsymbol{x}) \\ \text{such that}\quad &g_i(\boldsymbol{x}) \leq 0 \ \ , i = 1, 2, \cdots, m \\ &\boldsymbol{Ax} \leq \boldsymbol{b} \\ &\boldsymbol{Cx} = \boldsymbol{d} \end{aligned} \tag{4.5}$$

where g_i are nonlinear and continuously differentiable functions, and the matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ as well as the vectors b and $\boldsymbol{d}$ are constants. The vector of design variables,**x**, includes both continuous and discrete variables.

In one iteration of the *mixed-integer sequential linearization procedure* (MISLP), the original optimization problem is linearized at the current iteration point, $\mathbf{x}^k$:

$$\min_{x} f\left(\boldsymbol{x}^k\right) + \nabla f\left(\boldsymbol{x}^k\right)^T\left(\boldsymbol{x} - \boldsymbol{x}^k\right) \tag{4.6}$$

$$\text{such that}\quad g_i\left(\boldsymbol{x}^k\right) + \nabla g_i\left(\boldsymbol{x}^k\right)^T\left(\boldsymbol{x} - \boldsymbol{x}^k\right) \leq 0 \ \ , i = 1, 2, \cdots, m$$

$$\boldsymbol{Ax} \leq \boldsymbol{b}$$

$$\boldsymbol{Cx} = \boldsymbol{d}$$

The problem of Equation 4–6 is a mixed-integer linear optimization problem (MILP), which can be solved, for example, by the branch-and-cut method that is implemented in various optimization software packages. Even with the binary variables, this linearized problem can be considered small-scale for the timber floor optimization problem of Equation 4–2.

It is well-known that the SLP as well as the MISLP method may not converge in its basic form. The method can be stabilised by introducing so-called *move limits* that restrict the feasible set of the linearized problem. The move limits are written as additional bound constraints for the design variables. the move limits can be expressed as a portion of the total range of the variable, or in terms of local allowable change, say 15% of the current value. In any case, the move limits can be written as

$$\underline{\Delta}_i^k \leq x_i - x_i^k \leq \overline{\Delta}_i^k \tag{4.7}$$

where $\underline{\Delta}_i^k$ and $\overline{\Delta}_i^k$are the prescribed bounds. In this study, the bounds

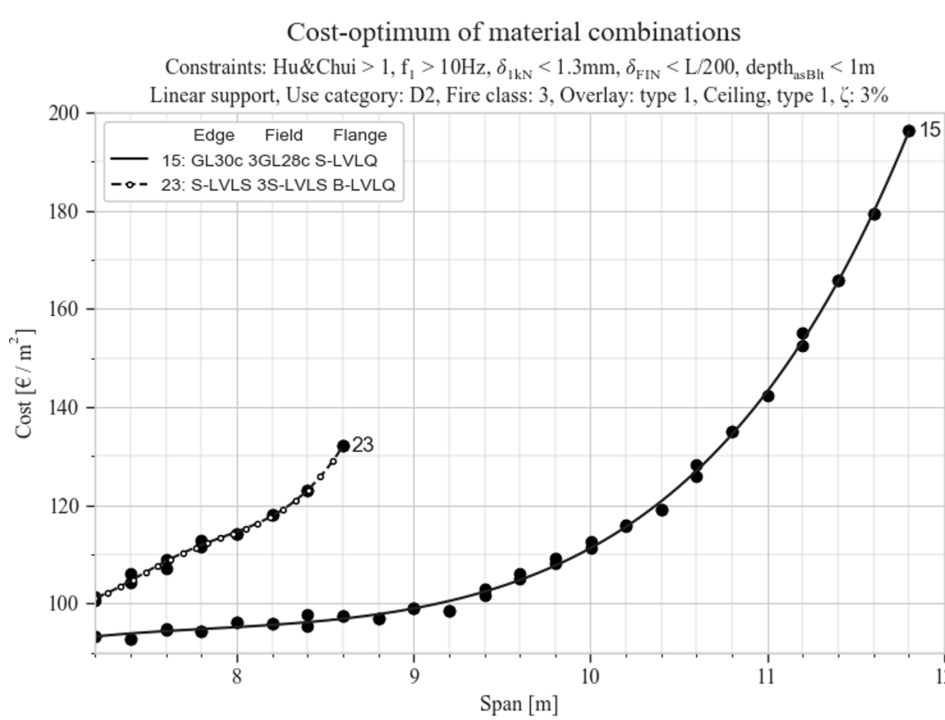

Fig. 5. Optimum data points and polynomial fitted curve for cases 15 and 23.

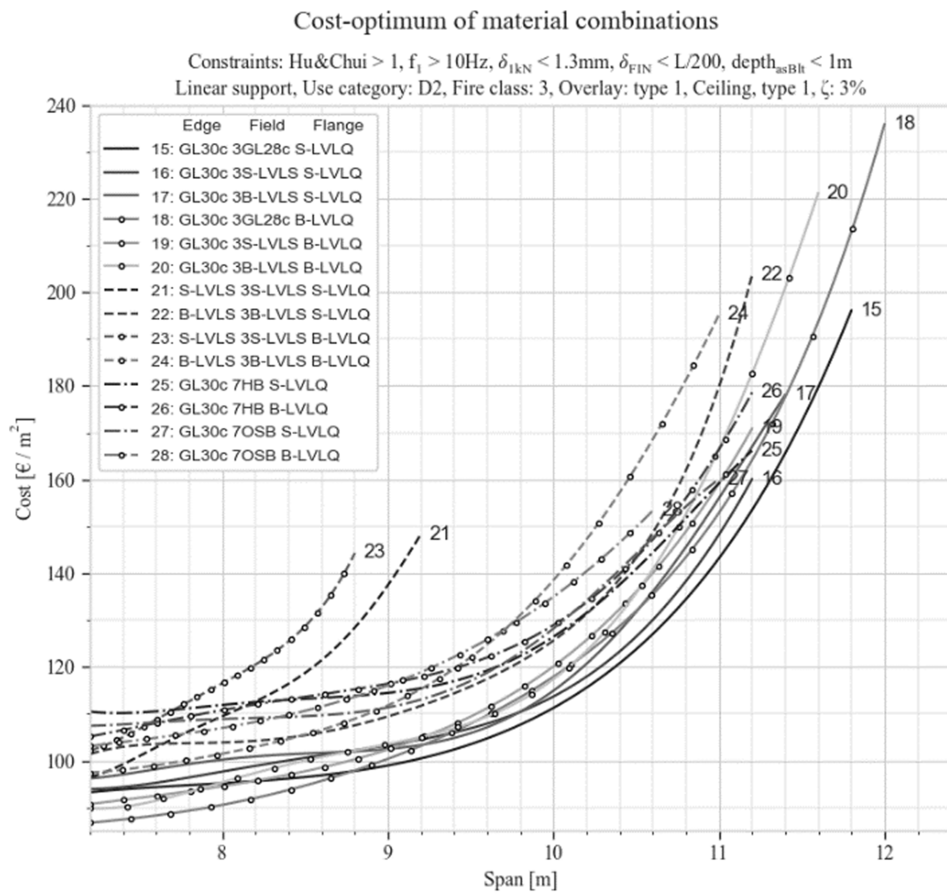

Fig. 6. Cost-optimum of material combinations for cases 15 through 28.

are related to the range of variable values, i.e.

$$\underline{\Delta}_i^k = C_1(\overline{x}_i - \underline{x}_i)$$

$$\overline{\Delta}_i^k = C_2(\overline{x}_i - \underline{x}_i) \tag{4.8}$$

where C_1 and C_2 are constants. In this study, the initial values $C_1 = 0.5$ and $C_2 = 0.5$ were used. Over the iterations, these constants are updated by the following rule

$$C_i \leftarrow (1-\gamma)C_i \tag{4.9}$$

where $\gamma = 0.001$ was used in this study.

For the application of the MISLP method on the timber floor optimization problem, binary variables are introduced as described in Equation 4–3 and Equation 4–4 which are the only linear constraints. Note that the binary variables appear only in the linear equality constraints that do not need linearization. For binary variables, no move limits were prescribed, but they were allowed to change from 0 to 1, or vice versa, when solving the linearized problem.

If the linearized problem is feasible, its optimum is a design, where all discrete variables attain an allowable discrete value. This feature is enabled by the binary variables. Without the binary variables, the linearized problem will likely provide a design, where the design variables non-allowable values, which means that solution process is not as efficient.

The design provided by the linearized problem may not necessarily satisfy the original nonlinear constraints. Moreover, the move limits may restrict the feasible set of the linearization such that even if its solution satisfies the original nonlinear constraints, the design may not be optimal for the original problem. Consequently, the linearization is performed sequentially until the obtained design does not change more than by a given tolerance.

For the MISLP method to begin, an initial design is required. In this study, the initial design is based on engineering judgement. The initial design for base floors 1 to 10 (three joists as field members) and 11 to 14 (seven webs as field members) is based on the cross-section of two floor element designs built and tested and used as a reference floor throughout the project that the present work is a part of. Base floor 1 and 11 have the same dimensions as the reference floors, while the initial design for similar base floors is adjusted by shifting the dimension up to the nearest matching dimension of available standard formats of associated materials.

It should be highlighted that the MISLP method is not guaranteed to find the optimum solution (local or global) of the nonlinear discrete timber floor design optimisation problem of Equation 4–2. The method was chosen in this study due to its simplicity and its ability to directly find discrete design through the use of the binary variables. As can be seen in the analysis presented in Section 4, the MISLP method works well for the optimisation problem at hand. Because optimisation is not the sole focal point of this study, no further methods were explored as MISLP provided satisfactory results.

As for the modules of objective and constraint, the modelling of the optimization problem is performed in Python [38], and the Google AI OR-Tools for Python [40] are used to solve the MILP sub-problem.

4. Results

The performance of the MISLP optimization technique was evaluated by comparing the design obtained by MISLP to the global minimum found by manual exploration of the solution space in all 56 cases. Both cost minimum and ECO2 minimum were compared. As seen from Table 1 the sum of possible combinations of the base floors is $9.5 \cdot 10^5$.

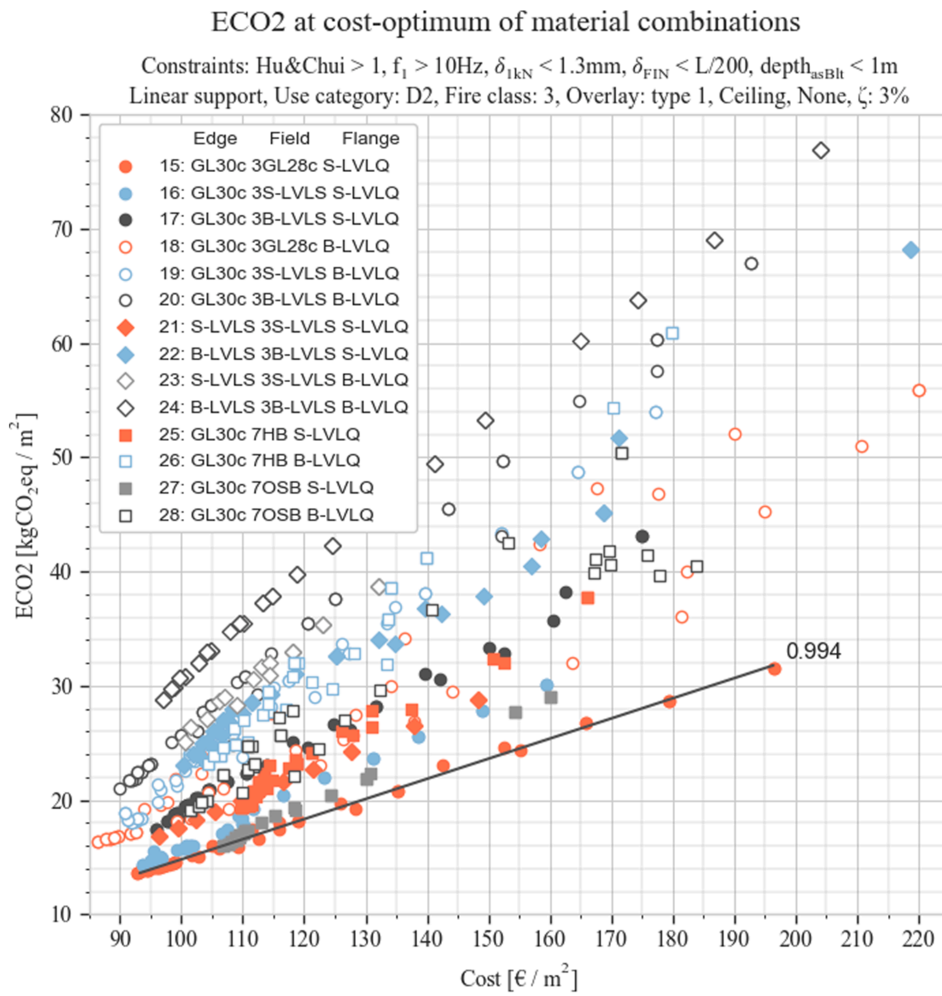

Fig. 7. ECO2 at cost-optimum of material combinations for cases 15 through 28. Each marker represents a single material combination for a certain span.

These combinations were run with four different outfitting giving a total of $2.65 \cdot 10^6$ combinations. The computational effort for performing this exploration is demanding. Currently a contemporary desktop computer (Intel(R) Core(TM) i7-8700 CPU at 3.20 GHz with 64 GB RAM) was calculating 2.65 runs per second, requiring 395 hrs to find the optimum solutions for these cases, or on average of 600 min per case. In comparison the average duration of the optimization approach was less than two seconds per case.

For the comparative study 9 m span was used and Floor Performance Level 4 was used when assessing the floor in accordance with the proposal for the second generation of Eurocode 5. The error was calculated as the ratio of the minimum found by the optimization method to the associated global minimum found by manual exploration. In Table 4 the statistical indicators of accuracy of the optimization method is presented. It can be concluded that in general, the MISLP approach performs very well, considering its simplicity and low computational time. The results from the manual exploration and due comparison to the MISLP optimum is given in Appendix C.

Optimum designs are also compared to the two reference floor elements (base floor 1 and 11). The reference floor element is an efficient design previously developed in the research programme financing the present work. For the reference floor elements, the width of the edge joists is constrained to a minimum width of 140 mm to allocate space for treaded rods. This constraint is therefore also used in the optimum case in the comparison presented in Table 5. The minimum width constraint of edge joists is not used elsewhere in the present work and is a special case to compare cost and ECO2 with the reference floor elements only. The optimum of cost and ECO2 produce the same solution for the variant of base floor 1 and 11 with minimum edge joist constraint.

For a floor element where the transverse deflection is not a negligible contribution to overall deflection, the Winkler method is enhancing the precision and provide good estimates for the two-way deflection. On average the analytical deflection of the floor element calculated as an equivalent beam produce 60% of the deformation computed numerically [41], whilst the Winkler theory of elastic foundation is evaluating the deflection 10% above the deformation computed numerically. For the application in the present paper the Winkler method both provide increased accuracy and estimates to conservative side for the deflection.

Because of the discrete design variables, the optimization will produce stepwise results. To increase the readability of the results and to better see trends, the results are plotted as a polynomial fit of degree 5 of the data points. In Fig. 5 two different cases are plotted to see the difference between the optimum solution and the fitted curve. For all cases a similar fit is seen, only with slight variations due to the steps of which dimensions for the design variables is offered.

5. Discussion

5.1. Principal findings

Fourteen base floors each with four different systems of ceiling and overlays were optimized and compared to manual exploration of global minimums of cost and ECO2. The optimization exhibits 1) compatible interaction with modules for calculation of objectives and constraints, 2) handling of changing composition of material and outfitting, 3) handling discrete design variables, 4) high and even level of accuracy, 5)

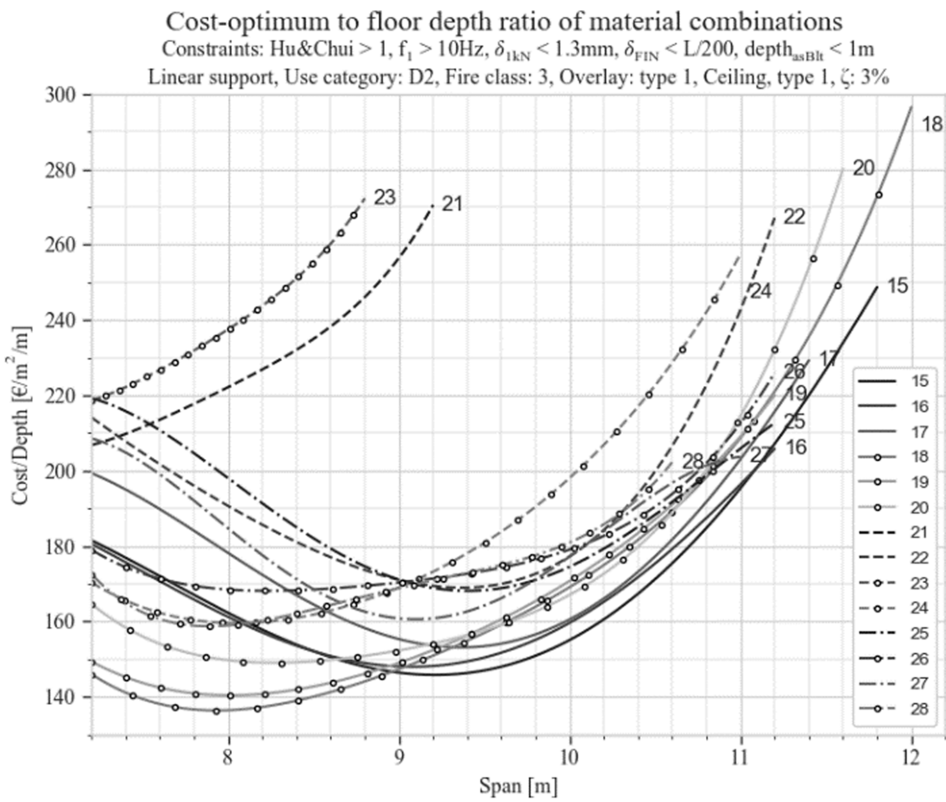

Fig. 8. Cost-optimum to floor depth ratio of material combinations for cases 15 through 28.

high convergence rate and consequently small calculation time.

To demonstrate the implications the method may offer the industry, the optimization framework is applied to produce cost-optimum designs for spans associated with adaptable buildings. The span ranges from 7.2 m to 12.6 m in steps of 0.2 m. Both the current common method of serviceability [20,21] and the method proposed for the second generation of Eurocode 5 [18,19] are used, and comparison between the methods is also presented.

5.2. *Implications*

5.2.1. *Cost optimum of base floor designs*

Current common method based on Hu and Chui [34] is used to generate cost-optimum for base floors with overlay type 1 and ceiling type 1. Ceiling type 1 implies that the minimum thickness of the bottom flange in practice is controlled by serviceability as opposed to fire resistance. The general trend shows a low gradient for cost for spans up to about 10 m. For greater spans, the cost increases more strongly. See Fig. 6. This is associated with available edge joist heights. Flange volumes are cost-drivers, and the cost-optimum solution will increase the height of joists unless constrained. Both case 15 and 18 with glulam frame and spruce and beech LVL respectively, perform well both with respect to span and cost. Floor elements with beech LVL (case 18) are slightly more expensive than spruce LVL (case 15) but offer marginally longer spans. Only limited spans are found for cases 21 and 23 due to the limited height offered for spruce LVLS. Only certain material combinations offer span towards 12 m.

5.2.2. *Correlation of cost and ECO2*

For the same cases as in 6.2.1, the embodied carbon emissions (ECO2) are plotted with respect to span in Fig. 7. Both cost and ECO2 are strongly linked to the volume of accrued materials, hence the strong correlation. A regression of case 15 shows that a linear model explains 99.4% (R^2) of the variance of the dependent variable, and that cost-optimum for most cases produce a well performing ECO2-design. As for section 6.2.1, case 15 demonstrates good performance. On the other hand, a larger variance in designs based on beech LVL can be seen. This is because the available thicknesses of spruce LVL which is offered at 6 mm steps, whilst the step is 10 mm for beech LVL. (See case 18 in Fig. 7).

5.2.3. *Cost to depth ratio of cost-optimum designs*

The ratio of cost to floor depth is presented in Fig. 8. Both parameters are indicators of competitiveness. By consulting Fig. 6, cost is increasing slowly until the available standard dimensions for the frame no longer offers increasing heights. By consulting Fig. 9, the depth of the floor is also seen steadily to increase, contributing the negative gradient of the ratio. As can be shown, as increased flange thickness is dominating the design, the gradient is turning positive.

5.2.4. *Comparison of serviceability methods*

To further examine effect of floor depth at cost-optimum, the current common method of Hu and Chui is compared to the new method proposed for the second generation of Eurocode for case 15 (See Fig. 9). The dashed lines represent resonant floor element design ($4.5 \leq f_1[Hz] < 8$), continuous lines represent transient floor element designs ($f_1[Hz] \geq 8$), and the dash-dot line representing the current common method. Concerning the new method for the Eurocode, resonant and transient floor designs both produce the same design for performance level 4 to 6. For resonant floor design, the best performance level found is three at a minimum span of 9 m satisfying the maximum fundamental frequency (f_1). The results suggest that lower building depths may be found for resonant floor designs with respect to both the current common method and transient floor design. The findings suggest that floor element designs with a resource-efficient solution to lowering fundamental frequency (as increased internal mass), may offer a potential for LSTFE with low building heights.

5.2.5. *Cost to depth trade-off*

The proposal for method of calculating serviceability in the second generation of Eurocode 5 [18,19] provides flexibility in floor design by

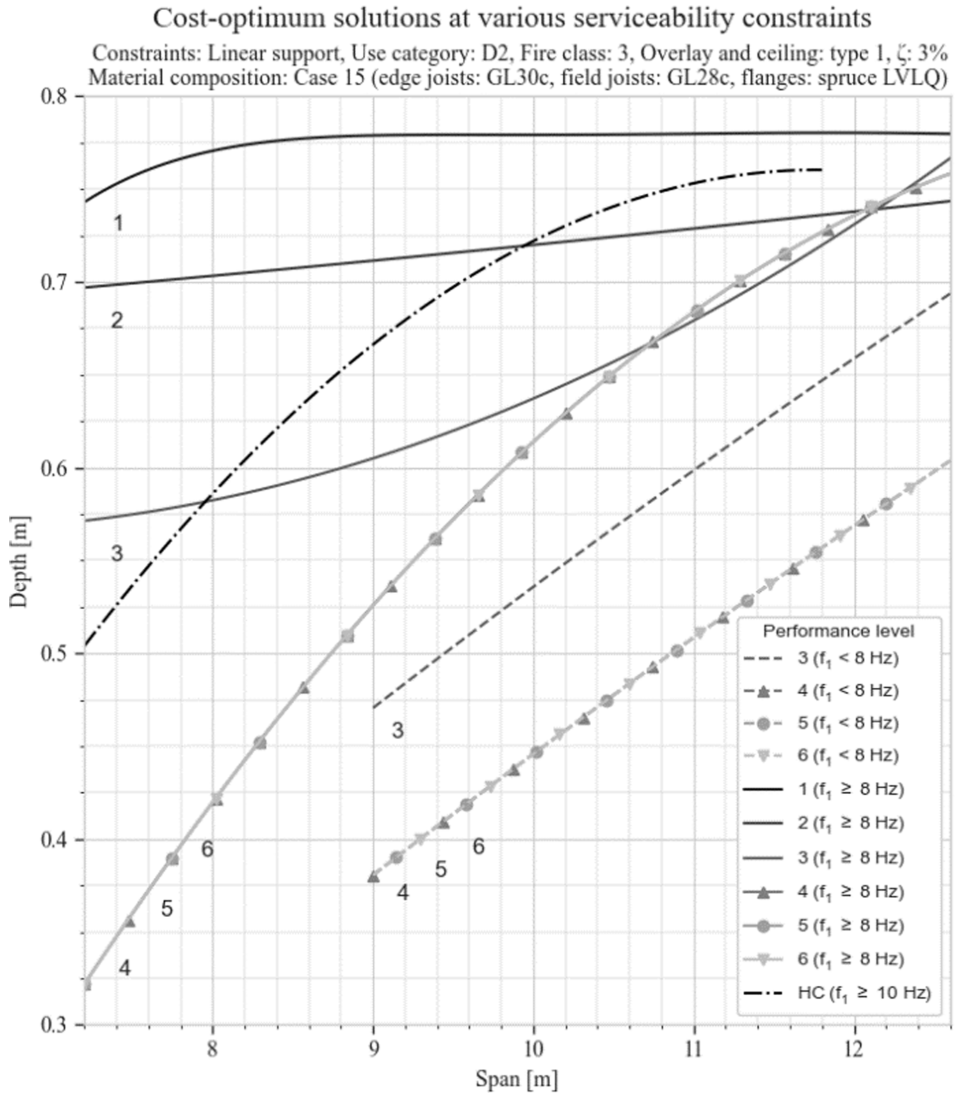

Fig. 9. Cost-optimum solutions at various serviceability constraints.

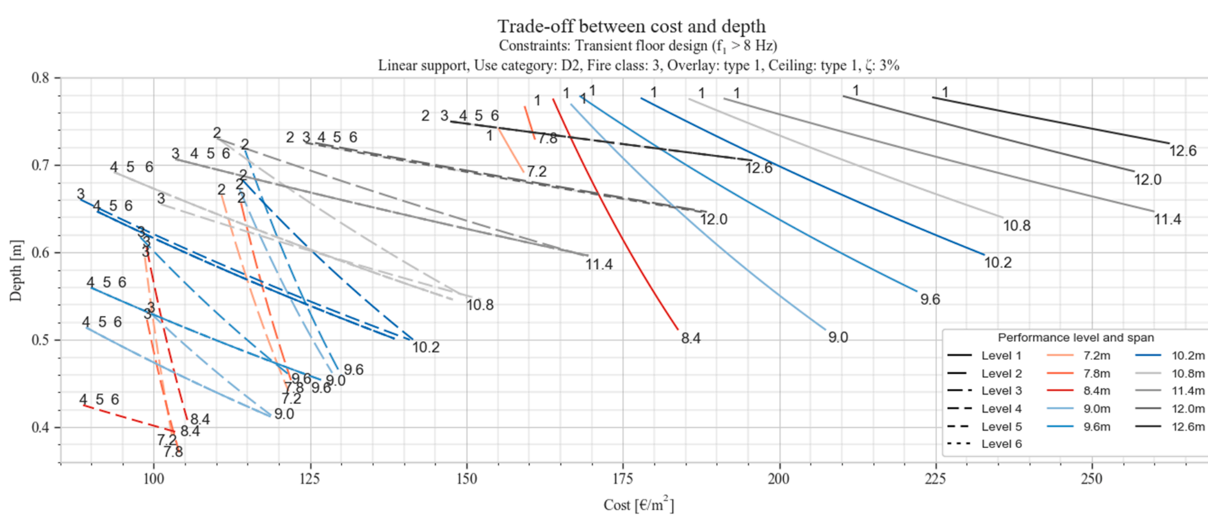

Fig. 10. Trade-off between cost and depth.

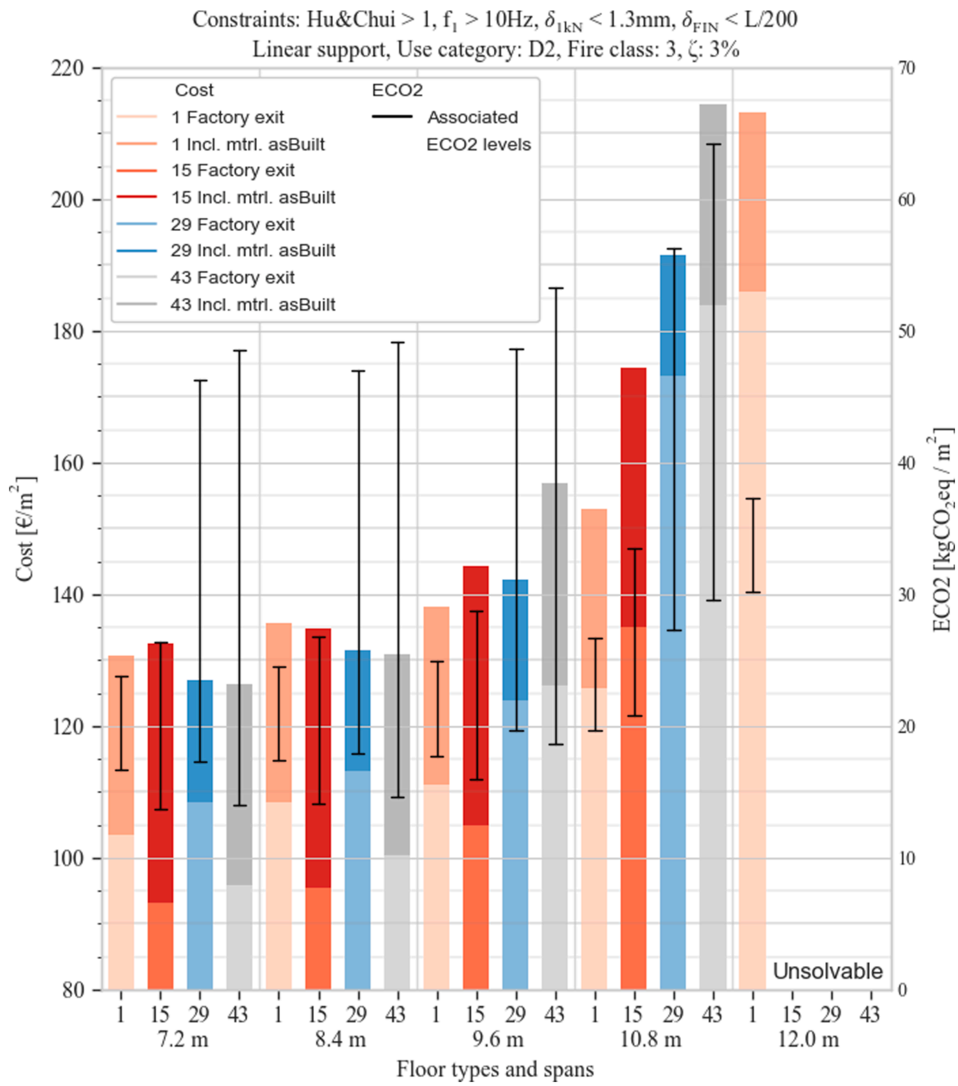

Fig. 11. Projected levels of cost and ECO2 due to additional materials (base floor 1 with four different outfitting).

employing the ISO baseline curve [35]. The method offers flexibility in calculations by either satisfying acceleration or velocity criterion and addressing required human perception levels for a specific building project through performance levels. The performance levels also serve as a convenient parameter to consider diverse socio-cultural attitude and expectations in the national annex. In the present work the performance levels proposed for Norway are used [19]. When applying a given performance level for a floor design, the cost-optimum will produce a floor depth. However, because floor depth also is a cost indicator for a building project, the interaction between floor depth and floor element cost is interesting. Consequently, the present work has applied the optimization framework for a trade-off analysis in the sense of multi-criteria optimization between depth and cost. In Fig. 10 the cost-optimum designs are presented for performance levels 1 to 6 for transient floor design, as floor depth is constrained from 0.8 m to 0.3 m in steps of 2 mm. As can be seen, depending on the performance level, the floor element cost increase as the floor depth is constrained. Generally, the ratio of cost to depth is increasing with increasing span. For performance level 1 at 8.4 m, reducing depth from 0.75 m to 0.5 m is associated with a 10% cost, whilst at 11.4 m reducing depth by 0.1 m is associated by an additional cost of 25%.

5.2.6. Cost and ECO2 as built

The cost and ECO2 used as objective function in the optimization framework cover manufacturing activities and accrued materials until the floor element is ready for transport at the factory gates. Associated as-built levels of cost and ECO2 requires further calculations of cost and ECO2 of activities including transport, installation and completion on-site. This is not covered in the present work, but projected levels of materials specified for as-built are presented in this section. In Fig. 11 cost and ECO2 for the floor element at factory gate are presented as bottom bars and bottom horizontal line. Cost and ECO2 of material added to the floor on site is presented as the top bar for the cost, and as the black line with top horizontal line for ECO2.

In this chart base floor 1 is used with four different outfitting. See Fig. 2 for a reminder of cases 1, 15, 29 and 43. As expected, the 50 mm screed contributes considerably to the ECO2 (cases 29 and 43), as well as the 2 × 15 mm gypsum type F (see case 1 compared to case 15). The latter may argue the case of using timber rather than gypsum in the fire resistance design when this is an option due to the ECO2 benefits of avoiding gypsum. The observed cost-optimum benefits from a heavy non-structural plate is caused by the effect the increased mass has on the serviceability criteria. However, the excessive cost is gained when adding the cost of the screed. The additional cost of installation will further increase this cost. For 7.2 m case 1 and 43 is explained by specific numbers:

The cost of the floor element at factory gate is 104 and 96 €/m^2 for cases respectively. When including required material for completion on site the costs are risen to 130 and 126 €/m^2. The weight of the screed of case 43 causes the serviceability constraint to be accepted with less structural timber than for the lighter floor element of case 1.

For the ECO2 the competitive figures are different. The increased structural timber of case 1 results in ECO2 of 17 $kgCO_2eq/m^2$ whilst case 43 contains 14 $kgCO_2eq/m^2$ as delivered from factory. The ECO2 figures as installed are respectively 23.5 and 48.5 $kgCO_2eq/m^2$.

5.3. Future research

Effect of transverse stiffeners are not included in the study, nor is the effect of changes of the edge beams. The effect of changes in these members will not be properly be reflected in an analytical assessment of the floor element performance. The optimization algorithm must therefore be implemented in a workflow where a numerical representation of the floor element is calculating serviceability performance, preferably using probabilistic methods of load modelling to lower the computational time to a level feasible of producing data for a reference work [42].

6. Conclusions

The optimization workflow implemented in this study provides a seamless dataflow between the designer and manufacturer (cost data). MISLP proved to be efficient and reliable and detect solutions close to the global optimum. The MISLP optimization method demonstrates adequate properties and performances required to be run directly from a server to generate immediate designs based on parameters collected from the user interface. The ability to reliably and efficiently explore the solution space in a rapidly growing market of novel engineered wood products opens a range of implications briefly demonstrated in section 6.2.

- Mean error and standard deviation between the global optimum and the solution obtained by MISLP are significantly larger for constraint combinations based on Hu and Chui than with acceleration and velocity. This is associated with the fact that the Hu and Chui constraint are composed of two other constraints.
- With respect to the 600 min analysis time per case for manual exploration of the solution space, the optimization approach took less than two seconds per case. The analysis duration may be sufficiently fast for an online reference work.
- Predetermination of floor element designs in conventional charts is challenging due to the six Floor Performance Levels (FPL). This is an argument for an online reference work.
- Glulam as joists is outperforming alternatives. Glulam has a competitive combination of cost, ECO2, stiffness and standard format range. The combination of glulam and spruce-LVL-Q in flanges performs generally best. Glulam in combination with beech LVL-Q slightly increase span, but at a high cost (2% increase in span at 20% increase in cost).
- Flange-driven performance increase is expensive, and it is increasingly dominating after 9.4 m for most base floor designs.
- Minimum cost and ECO2 correlate well as both are related to accrued material volume.
- The distribution of optimum design is generally responding well for FPL 1 to FPL 3, whilst similar designs is typically found for FPL 4 to FPL 6. This is linked to the activation of final deflection constraint.
- Pareto-analysis of the trade-off between cost and depth yields the cost increase as floor depth is constrained. Generally, the ratio of cost to depth is increasing with increasing span.
- The cost and ECO2 of the floor element as built deviate significantly from quantities as manufactured, depending on the design strategy for fire resistance and overlay.

The timber sector is under substantial pressure to find competitive solutions for an increasing demand for long-span floor elements suitable to adaptable and sustainable buildings. Due to the findings of the present work, a huge potential for the manufacturers in the successful adaption to algorithm aided design may be realistic, given that the infrastructure of the suppliers and production line can cope with the indeterminacy.

The combined investments in the modules of the presented workflow may offer the required computational foundation for a ready reference, thus assist in commercialisation of long-span timber floor elements suitable for adaptable building applications. All codes in the optimization workflow are based on open source which may simplify in producing public available results.

The present work is a contribution in the endeavour of industrialising timber manufacturing and in the establishment of parametric framework covering the value chain from design to manufacturing [43–45].

Note that changing cost and properties of materials, in addition to the manufacturing cost will influence the optimum design. The optimum design may therefore change between manufacturers.

CRediT authorship contribution statement

Sveinung Nesheim: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Writing – original draft, Writing – review & editing, Visualization. **Kristo Mela:** Methodology, Software, Validation, Investigation, Writing – original draft, Writing – review & editing. **Kjell Arne Malo:** Writing – review & editing, Supervision, Project administration, Funding acquisition. **Nathalie Labonnote:** Writing – review & editing, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This work is part of the four-year project “Woodsol – Wood frame solutions for free space design in urban buildings” realized through the research grant from The Research Council of Norway (254699/E50). The support is gratefully acknowledged. The authors would also like to acknowledge the contribution by Jaakko Huusko (formerly Tampere University) for the support on implementation of the computer code.

Appendix A. Design specification and equations

Appendix A.1. Bending stiffness

Longitudinal bending stiffness EI_L (Equation A–1) and governing transversal bending stiffness $EI_{T,midsection}$ (Equation A–7) is calculated with simple linear elasticity as stated in Eurocode 5 Rules for buildings [20] section 7.3.3, with effective width of flanges b_i and position of neutral axes calculated accordingly. The factor for composite effect (γ) is defined at a constant 1.0.

For EI_L numStructLng is the number of structural sections in the longitudinal direction. The floor element is divided into a set of longitudinal sections equal to the number of longitudinal members between the top and bottom flanges. The structural capacity is the calculated for each section and summed:

$$EI_L(\boldsymbol{x}) = \sum_{i=1}^{numStructLng} E_{tf,L}\left(I_{tf,L,i}(\boldsymbol{x}) + \gamma_{tf}A_{tf,ef,L,i}(\boldsymbol{x})a^2_{tf,L,i}(\boldsymbol{x})\right) + E_{j,L,i}\left(I_{j,L,i}(\boldsymbol{x}) + \gamma_j A_{j,L,i}(\boldsymbol{x})a^2_{j,L,i}(\boldsymbol{x})\right) + E_{bf,L}\left(I_{bf,L,i}(\boldsymbol{x}) + \gamma_{bf}A_{bf,ef,L,i}(\boldsymbol{x})a^2_{bf,L,i}(\boldsymbol{x})\right) \quad \text{(A1)}$$

Where the second moment of area I(**x**):

$$I_{tf,L,i}(\boldsymbol{x}) = \frac{1}{12}b_{tf,ef,L,i}\cdot\boldsymbol{x}[1]^3 = \frac{1}{12}b_{tf,ef,L,i}\cdot h_1{}^3 \quad \text{(A2)}$$

and the effective area A(**x**):

$$A_{tf,ef,L,i}(\boldsymbol{x}) = b_{tf,ef,L,i}\cdot\boldsymbol{x}[1] = b_{tf,ef,L,i}\cdot h_1 \quad \text{(A3)}$$

and neutral axis a(**x**) according to NS-EN 1995-1-1 Appendix B: B.2:

$$a_{j,L,i}(\boldsymbol{x}) = \frac{\gamma_{tf}\cdot E_{tf,L}\cdot A_{tf,ef,L,i}\cdot(h_1+h_2) - \gamma_{bf}\cdot E_{bf,L}\cdot A_{bf,ef,L,i}\cdot(h_2+h_3)}{2\cdot\left(\gamma_{tf}\cdot E_{tf,L}\cdot A_{tf,ef,L,i} + \gamma_j\cdot E_{j,L,i}\cdot A_{j,L,i} + \gamma_{bf}\cdot E_{bf,L}\cdot A_{bf,ef,L,i}\right)} \quad \text{(A4)}$$

$$A_{tf,L,i}(\boldsymbol{x}) = -a_{j,L,i}(\boldsymbol{x}) + \frac{h_1+h_2}{2} \quad \text{(A5)}$$

$$A_{bf,L,i}(\boldsymbol{x}) = a_{j,L,i}(\boldsymbol{x}) + \frac{h_2+h_3}{2} \quad \text{(A6)}$$

Effective width of flanges are chosen according to NS-EN 1995-1-1 section 9.1.2 (see Table 9.1 and Fig. 9.2)

For $EI_{T,midsection}$ numStructTrns is the number of structural sections in the transverse direction. There are one transverse section at each edge of the floor element, and one at the midsection of the floor element. The bending stiffness of the transverse edge sections are calculated similarly to the longitudinal edge sections only with the Young's modulus changed accordingly, whilst a simplification is used for the transverse midsection. For flanges with no shear capacity (without joist), the affected gamma factor is set to zero ($\gamma_{tf} = 0$ and $\gamma_{bf} = 0$) and the bending stiffness is reduced to the following form:

$$EI_{T,midsection}(\boldsymbol{x}) = E_{tf,T}I_{tf,T,i}(\boldsymbol{x}) + E_{bf,T}I_{bf,T,i}(\boldsymbol{x}) \quad \text{(A7)}$$

Shear stiffness of core only as this in practice contributes with the entire the shear capacity from bending:

$$GA_L(\boldsymbol{x}) = \sum_{i=1}^{numStructLng} G_{12,i}A_{L,i} = \sum_{i=1}^{numStructLng} G_{12,i}w_ih_2 \quad \text{(A8)}$$

The self weight of the floor element is given by g(**x**).

Appendix A.2 Element depth

The element depth is the sum of the layer thicknesses of the primary structure:

$$h_{CHS}(\boldsymbol{x}) = h_1 + h_2 + h_3 \quad \text{(A9)}$$

Appendix A.3 Fundamental frequency

The fundamental frequency (f_1) is calculated according to [19] section 9.3.4 as follows:

$$f_1(\boldsymbol{x}) = k_{e,2}(\boldsymbol{x})\frac{18}{\sqrt{w_{sys}(\boldsymbol{x})}} \quad \text{(A10)}$$

System deformations due to self-load, $w_{sys}(\mathbf{x})$, are calculated as:

$$w_{sys}(\boldsymbol{x}) = \frac{5\cdot g(\boldsymbol{x})\cdot L^4}{384\cdot EI_L(\boldsymbol{x})} + \frac{g(\boldsymbol{x})\cdot L^2}{8\cdot GA_L(\boldsymbol{x})} \quad \text{(A11)}$$

The frequency multiplier $k_{e,2}$ is calculated to reflect the effect of the transverse floor stiffness as reproduced in Equation A-12. For the present work the system width of the flooring system (*B)* is defined at a constant 1.5*L*, where *L* is the span.

$$k_{e,2}(\boldsymbol{x}) = \sqrt{1 + \left(\frac{L}{B}\right)^4\frac{D_T(\boldsymbol{x})}{D_L(\boldsymbol{x})}} \quad \text{(A12)}$$

The apparent stiffness (*D*) of flooring system is the bending stiffness of a section divided by the extent of the section [Nm^2/m]. The apparent bending stiffness longitudinal (D_L) and transversally (D_T) in given in and

$$D_L(\boldsymbol{x}) = \frac{EI_L(\boldsymbol{x})}{w_{mod}} \quad \text{(A13)}$$

$$D_T(\boldsymbol{x}) = \frac{EI_{T,midSection}(\boldsymbol{x})}{L_{midSection}} \quad \text{(A14)}$$

$L_{midSection}$ is the length of the mid-section in direction of span (L). In the present work where the floor element has only one compartment in the longitudinal direction, and where transverse beams only are located at the end of the floor element, the length of the mid-section is defined at half the span length of the floor element.

Appendix A.4 Unit load deflection

Mid span deflection due to a unit point load is used to assess serviceability in the methods of serviceability applied herein. The calculated deflection is strongly influenced by the support conditions and the analytical representation of the deflection. The present work applied the Winkler theorem for describing beams on elastic foundation [31,32] in order to improve the representation of two-way deflection from unit point load at midspan ().

$$p = \frac{d^4 w(\boldsymbol{x})}{dx^4} + k \cdot w(\boldsymbol{x}) \tag{A15}$$

This is done by equating a fictitious Winkler foundation [31,32] to the uniform deformation w(**x**) caused by the floor element acting as an equivalent beam ().

$$w(\boldsymbol{x}) = \frac{p \cdot L^3}{48 \cdot EI_L(\boldsymbol{x})} + K_{sfd} \frac{p \cdot L}{4 \cdot GA_L(\boldsymbol{x})} \tag{A16}$$

- p: unit point load 1 kN
- K_{sfd}: constant in prediction of shear force deformations. For rectangular section, $K_{sfd} = 1.2$ [33]
- GA_L: Shear stiffness in longitudinal direction. Only longitudinal members will in practice contribute to the shear capacity from bending, i.e. edge- ① and field joists ③ in Fig. 1.

By using the effective length of the transverse midsection of the floor element as the length of the foundation (L_{wink}), the Winkler foundation stiffness (k) can be expressed as:

$$k(\boldsymbol{x}) = \frac{\frac{p}{L_{wink}(\boldsymbol{x})}}{w(\boldsymbol{x})} \tag{A17}$$

Finally, the deflection constraint due to unit point load is calculated by determining the maximum deflection of the transversal cross section of the floor resting on the elastic foundation as:

$$w_{wink}(\boldsymbol{x}) = \frac{\beta(\boldsymbol{x}) \cdot p}{2 \cdot k(\boldsymbol{x})} \cdot \frac{2 + cosh\beta(\boldsymbol{x})x + cos\beta(\boldsymbol{x})x}{sinh\beta(\boldsymbol{x})x + sin\beta(\boldsymbol{x})x} \tag{A18}$$

Where

$$\beta(\boldsymbol{x}) = \sqrt[4]{\frac{k(\boldsymbol{x})}{4 \cdot EI_{T,midSection}(\boldsymbol{x})}} \tag{A19}$$

Appendix A.5. Dynamic response

Appendix A.5.1 Hu and Chui parameter

Hu and Chui (HC) parameter is applied on the following form:

$$HC(\boldsymbol{x}) = \frac{\left(\frac{f_1(\boldsymbol{x})}{18.7}\right)^{2.27}}{w_{wink}(\boldsymbol{x})} \tag{A20}$$

Appendix A.5.2 RMS acceleration

Resonant floor response (4.5 ≤ f_1 [Hz] < 8) assessed by acceleration:

$$a_{rms}(\boldsymbol{x}) = \frac{\alpha(\boldsymbol{x}) \cdot F}{7 \cdot \zeta \cdot M^*(\boldsymbol{x})} \tag{A21}$$

α: Fourier coefficient $\alpha = e^{-0.4 \cdot f_1}$
F: Vertical force imposed by walking person (700 N)
ζ: Modal damping ratio of 3%
M*: Modal mass $M^* = \frac{mLB}{4}$
m: Mass (kg) of floor per unit area (m^2)

Appendix A.5.3. RMS velocity

Transient floor response assessed by velocity ($f_1[Hz] \geq 8$).

$$v_{rms}(\boldsymbol{x}) = K_{imp}(\boldsymbol{x})\cdot\frac{0.7\cdot I_m(\mathbf{x})}{M^*(\boldsymbol{x})+70}(0.65-0.01\cdot f_1(\mathbf{x}))(1.22-11\cdot\zeta)\cdot\eta(\mathbf{x}) \tag{A22}$$

K_{imp}: Higher modes multiplier for transient floor response $K_{imp} = max\begin{cases}0.48\left(\frac{B}{L}\right)\left(\frac{D_L}{D_T}\right)^{0.25}\\ 1\end{cases}$

l_m: Mean modal impulse $l_m = \frac{42\cdot f_w^{1.43}}{f_1^{1.3}}$

f_w: Walking frequency (2 Hz)

$$\eta = \begin{cases}1.52-0.55\cdot K_{imp} 1.0 \leq K_{imp} \leq 1.5\\ 0.69 \quad otherwise\end{cases}$$

Appendix A.6 Final deformation

Deformation from permanent and imposed loads calculated as:

$$w_{fin}(\mathbf{x}) \leq w_{maxFin} \tag{A23}$$

For imposed loading category of use D2 (Areas in department stores) as defined in Eurocode 1 Actions on structure [36] is used. This is used for indeterminacy as the level is covering all categories. D2 states distributed load: q = 5000 N/m^2 and point load: Q = 7000 N. The deformation is calculated as equivalent beam.

Appendix B. Material supply cost and carbon emissions

Tables B1-B2

Table B1
Direct material supply cost and ECO2.

Material	Cost [€ per m^3]	ECO2 [g CO_2 per kg]
Adhesive	2965 [46]	1000 [46]
Beech LVL	635 [46]	364.9 [46]
Fasteners	See Table B-2	
Fibre board 36 mm	275 [8]	243 [8]
Glulam	510 [46]	144.2 [47]
Gravel 8/16	150 [46]	3 [48]
Gypsum type F	400 [8]	200.8 [49]
HB HLA 1	1000 approx. from [8]	661 [8]
OSB 3	520 [8]	208 [8]
Particle board P6/22	785 [8]	409 [8]
Screed	170 [8]	1355.9 [48]
Spruce LVL	595 [46]	254.9 [50]

Table B2
Fastener specification used to calculate fastener vector of body or assembly.

Fastened member	Type 1)	dia [mm]	Row-Dir 2)	multiplier 3)	D/A 4)	Value	unit-Cost [€]	unit-CO_2 5) [kgCO_2 eq]	Len 6) [m]	Type
Top flange	S	5	1	fldJstNum + 2	D	0.3	0.05	None	0.1	Partial tread flange head
Edge beam	S	8	3	2	D	0.05	0.1	None	None	Double threaded fastener
Field joist	S	8	3	2	D	0.05	0.1	None	None	
Trns. stiffener	S	6.3	3	2	D	0.05	0.1	None	None	NA
Bottom flange	S	5	1	fldJstNum + 2	D	0.3	0.1	None	0.1	Partial tread flange head
Overlay	N	4	1	fldJstNum + 2	D	0.1	0.01	None	None	NA
Ceiling	S	3	1	fldJstNum + 2	D	0.2	0.01	None	None	NA
Floor element	S	8	2	2	D	0.1	0.1	None	0.2	Partial tread flange head

1) N: nail, or S: screw. If screw and: Ø > 8 mm or machinability > 1 or density of material > 650 kg/m^3; predrilling is performed
2) Global direction of row of fasteners in body in question
3) Multiplier of the row as defined in rowing direction
4) Calculate the proceeding Value as distance between fasteners (D) or total amount of fasteners (A) along one row
5) None or a number. If None a value is calculated based on the volume using unitMassCO2eq of steel.
6) Length of fastener or None. If None length is calculated as twice the plate thickness or beam width (aspect ratio parsed as condition)

Appendix C. Dataset for quantifying accuracy of the optimization

Tables C1-C6

Table C1
Global cost optimum compared to MISLP for current common method (Hu and Chui).

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	cost	error
GL30c	GL28c	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_0_1	0.033	0.495	0.063	0.036	0.036	10.37	0.23	15.85	1.78	9.88	1.13	109.57	17.50	109.61	0.04%
GL30c	S-LVLS	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_1_2	0.033	0.495	0.063	0.036	0.039	10.35	0.23	15.70	1.77	9.81	1.13	111.51	19.21	111.42	−0.08%
GL30c	B-LVLS	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_2_3	0.033	0.495	0.063	0.036	0.04	10.69	0.22	14.35	1.50	9.15	1.29	115.02	22.57	114.47	−0.48%
GL30c	GL28c	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_3_4	0.02	0.585	0.06	0.036	0.036	10.61	0.22	14.48	1.52	9.77	1.24	110.02	26.92	107.34	−2.44%
GL30c	S-LVLS	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_4_5	0.02	0.54	0.06	0.036	0.045	10.27	0.24	15.19	1.71	9.84	1.09	111.89	29.01	112.03	0.13%
GL30c	B-LVLS	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_5_6	0.02	0.54	0.06	0.036	0.04	10.37	0.23	14.64	1.61	9.55	1.14	113.34	32.15	110.43	−2.57%
S-LVLS	S-LVLS	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_6_7	0.045	0.4	0.069	0.027	0.075	10.06	0.22	15.68	1.86	8.38	1.12	127.62	24.27	123.60	−3.15%
B-LVLS	B-LVLS	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_7_8	0.033	0.48	0.063	0.04	0.04	10.33	0.22	15.19	1.67	9.14	1.19	118.97	28.64	118.72	−0.21%
S-LVLS	S-LVLS	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_8_9	0.04	0.4	0.07	0.027	0.075	10.01	0.21	14.82	1.72	7.93	1.17	131.68	38.76	137.64	4.53%
B-LVLS	B-LVLS	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_9_10	0.02	0.56	0.06	0.04	0.04	10.59	0.21	13.78	1.40	9.01	1.32	119.98	39.86	116.94	−2.53%
GL30c	HB	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_10_11	0.033	0.495	0.063	0.036	0.007	10.35	0.23	16.01	1.80	9.71	1.15	124.35	23.61	124.18	−0.14%
GL30c	HB	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_11_12	0.02	0.54	0.06	0.036	0.007	10.11	0.23	16.14	1.87	10.15	1.05	122.32	33.23	122.06	−0.21%
GL30c	OSB	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_12_13	0.033	0.495	0.063	0.036	0.012	10.14	0.23	16.60	1.94	9.95	1.07	121.63	19.88	121.36	−0.22%
GL30c	OSB	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_13_14	0.02	0.585	0.06	0.036	0.012	10.55	0.22	14.66	1.54	9.74	1.26	121.71	29.60	119.11	−2.14%
GL30c	GL28c	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_0_15	0.033	0.63	0.033	0.036	0.036	10.96	0.27	13.39	1.29	12.34	1.10	99.08	14.47	98.92	−0.16%
GL30c	S-LVLS	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_1_16	0.033	0.63	0.033	0.036	0.033	10.64	0.28	14.13	1.46	12.84	1.00	101.13	16.05	100.71	−0.42%
GL30c	B-LVLS	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_2_17	0.033	0.585	0.033	0.036	0.04	10.64	0.28	13.69	1.41	12.13	1.00	102.69	20.17	102.39	−0.29%
GL30c	GL28c	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_3_18	0.03	0.63	0.03	0.036	0.036	10.99	0.28	12.89	1.22	12.84	1.09	99.69	21.88	97.59	−2.11%
GL30c	S-LVLS	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_4_19	0.03	0.63	0.03	0.036	0.039	10.98	0.27	12.72	1.20	12.72	1.10	102.15	24.05	101.33	−0.80%
GL30c	B-LVLS	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_5_20	0.02	0.63	0.04	0.036	0.04	10.78	0.26	12.85	1.26	11.51	1.11	106.05	28.32	102.49	−3.36%
S-LVLS	S-LVLS	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_6_21	0.063	0.4	0.075	0.027	0.075	10.09	0.17	13.93	1.55	6.63	1.43	141.90	27.39	137.88	−2.83%
B-LVLS	B-LVLS	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_7_22	0.033	0.6	0.033	0.04	0.04	10.74	0.26	13.17	1.28	11.53	1.11	109.31	28.32	108.87	−0.40%
S-LVLS	S-LVLS	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_8_23	0.06	0.4	0.07	0.027	0.075	10.02	0.17	13.33	1.45	6.57	1.42	144.48	44.09		
B-LVLS	B-LVLS	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_9_24	0.03	0.6	0.03	0.04	0.04	10.79	0.26	12.68	1.20	12.00	1.09	109.94	35.73	109.48	−0.42%
GL30c	HB	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_10_25	0.033	0.63	0.033	0.036	0.007	10.74	0.27	14.03	1.41	12.39	1.06	113.67	22.06	113.28	−0.34%
GL30c	HB	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_11_26	0.03	0.63	0.03	0.036	0.007	10.86	0.27	13.30	1.28	12.77	1.07	114.31	29.47	113.91	−0.35%
GL30c	OSB	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_12_27	0.033	0.63	0.033	0.036	0.015	10.74	0.26	13.79	1.38	12.14	1.08	113.43	17.99	110.12	−2.92%
GL30c	OSB	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_13_28	0.03	0.63	0.03	0.036	0.012	10.63	0.28	13.79	1.39	13.08	1.00	111.22	24.72	110.73	−0.44%
GL30c	GL28c	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_0_29	0.033	0.63	0.063	0.036	0.036	10.27	0.17	12.33	1.23	7.51	1.48	116.87	18.38	116.72	−0.13%
GL30c	S-LVLS	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_1_30	0.033	0.63	0.063	0.036	0.039	10.27	0.17	12.20	1.21	7.45	1.49	119.33	20.55	118.50	−0.70%
GL30c	B-LVLS	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_2_31	0.033	0.585	0.063	0.036	0.04	10.11	0.18	12.42	1.27	7.29	1.40	120.49	24.07	120.18	−0.26%
GL30c	GL28c	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_3_32	0.03	0.63	0.06	0.036	0.036	10.28	0.17	11.88	1.15	7.56	1.49	118.84	29.87	138.80	16.80%
GL30c	S-LVLS	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_4_33	0.02	0.63	0.06	0.036	0.051	10.02	0.18	12.43	1.28	7.59	1.32	120.28	30.64	114.44	−4.86%
GL30c	B-LVLS	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_5_34	0.03	0.585	0.06	0.036	0.04	10.16	0.18	11.92	1.18	7.32	1.42	122.46	35.57	118.43	−3.29%
S-LVLS	S-LVLS	S-LVLQ	Type2	None	10	NA	1.3	45	HC																
B-LVLS	B-LVLS	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_7_36	0.033	0.6	0.063	0.04	0.04	10.23	0.16	11.98	1.16	7.00	1.55	127.11	32.22	126.65	−0.36%
S-LVLS	S-LVLS	B-LVLQ	Type2	None	10	NA	1.3	45	HC																
B-LVLS	B-LVLS	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_9_38	0.03	0.6	0.06	0.04	0.04	10.28	0.16	11.50	1.08	7.03	1.58	129.08	43.72	126.54	−1.97%
GL30c	HB	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_10_39	0.033	0.63	0.063	0.036	0.007	10.23	0.17	12.49	1.25	7.37	1.50	131.38	25.96	131.00	−0.29%
GL30c	HB	B-LVLQ	Type2	None	10	NA	1.3	45	HC															132.37	
GL30c	OSB	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_12_41	0.033	0.63	0.063	0.036	0.015	10.29	0.17	12.20	1.20	7.20	1.56	131.17	21.89	127.86	−2.52%
GL30c	OSB	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_13_42	0.03	0.63	0.06	0.036	0.012	10.07	0.17	12.40	1.25	7.58	1.43	130.32	32.71	129.84	−0.37%
GL30c	GL28c	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_0_43	0.039	0.63	0.039	0.036	0.048	10.00	0.21	12.41	1.27	8.80	1.15	110.93	16.63	110.68	−0.23%
GL30c	S-LVLS	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_1_44	0.033	0.63	0.039	0.036	0.063	10.15	0.21	11.85	1.17	8.91	1.17	115.58	19.91	153.76	33.03%
GL30c	B-LVLS	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_2_45	0.033	0.63	0.039	0.036	0.05	10.07	0.21	11.96	1.20	8.92	1.15	113.58	23.75	109.58	−3.52%
GL30c	GL28c	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_3_46	0.03	0.63	0.04	0.036	0.048	10.01	0.22	12.13	1.22	9.18	1.12	110.80	25.14	110.34	−0.42%

(*continued on next page*)

Table C1 (*continued*)

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg		system	min	max	w_{1kN} [mm]	wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	cost	error
GL30c	S-LVLS	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_4_47	0.03	0.63	0.03	0.036	0.063	10.04	0.24	11.96	1.20	10.21	1.02	112.63	26.54	123.18	9.37%
GL30c	B-LVLS	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_5_48	0.03	0.63	0.03	0.036	0.06	10.25	0.23	11.25	1.07	9.72	1.12	115.48	32.48	110.19	−4.58%
S-LVLS	S-LVLS	S-LVLQ	Type2	1	10	NA	1.3	45	HC																
B-LVLS	B-LVLS	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_7_50	0.033	0.6	0.045	0.04	0.06	10.03	0.19	11.77	1.15	7.92	1.27	125.43	33.84	121.27	−3.32%
S-LVLS	S-LVLS	B-LVLQ	Type2	1	10	NA	1.3	45	HC																
B-LVLS	B-LVLS	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_9_52	0.03	0.6	0.04	0.04	0.06	10.09	0.20	11.42	1.09	8.39	1.24	125.32	42.36	167.88	33.96%
GL30c	HB	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_10_53	0.045	0.63	0.045	0.036	0.008	10.06	0.18	12.27	1.23	7.80	1.35	129.64	26.24	127.45	−1.69%
GL30c	HB	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_11_54	0.04	0.63	0.04	0.036	0.008	10.09	0.19	11.91	1.16	8.28	1.30	128.80	35.86	126.61	−1.70%
GL30c	OSB	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_12_55	0.039	0.63	0.051	0.036	0.018	10.09	0.18	12.02	1.19	7.50	1.41	130.46	21.79	127.10	−2.58%
GL30c	OSB	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_13_56	0.04	0.63	0.04	0.036	0.018	10.19	0.18	11.52	1.10	8.02	1.37	129.61	31.40	169.55	30.82%

Table C2
Global cost optimum compared to MISLP for second generation of Eurocode 5 (Resonant response).

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	Wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	cost	error
GL30c	GL28c	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_0_1	0.033	0.27	0.063	0.036	0.036	6.48	0.49	41.99	8.92	15.47	0.19	97.39	16.05		−100.00%
GL30c	S-LVLS	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_1_2	0.033	0.27	0.063	0.036	0.039	6.47	0.48	41.71	8.86	15.40	0.19	98.45	16.98		−100.00%
GL30c	B-LVLS	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_2_3	0.033	0.27	0.063	0.036	0.04	6.62	0.46	39.24	8.21	14.72	0.20	101.34	18.82		−100.00%
GL30c	GL28c	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_3_4	0.02	0.315	0.06	0.036	0.036	6.27	0.51	42.94	9.11	16.13	0.17	95.39	25.18		−100.00%
GL30c	S-LVLS	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_4_5	0.02	0.315	0.06	0.036	0.039	6.32	0.50	41.91	8.85	15.91	0.17	96.63	26.26		−100.00%
GL30c	B-LVLS	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_5_6	0.02	0.315	0.06	0.036	0.04	6.51	0.48	38.85	8.06	15.12	0.19	99.65	28.40		−100.00%
S-LVLS	S-LVLS	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_6_7	0.033	0.3	0.063	0.027	0.027	6.66	0.47	39.60	8.26	15.46	0.20	100.79	17.64	100.88	0.09%
B-LVLS	B-LVLS	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_7_8	0.033	0.28	0.063	0.04	0.04	6.77	0.43	37.11	7.49	14.09	0.23	105.38	22.68		−100.00%
S-LVLS	S-LVLS	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_8_9	0.02	0.3	0.06	0.027	0.045	6.17	0.53	43.73	9.36	16.03	0.15	97.97	27.35	96.71	−1.29%
B-LVLS	B-LVLS	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_9_10	0.02	0.28	0.06	0.04	0.05	6.04	0.52	44.67	9.47	15.68	0.15	103.03	32.44		−100.00%
GL30c	HB	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_10_11	0.033	0.27	0.063	0.036	0.007	6.48	0.48	42.06	8.90	15.28	0.19	112.10	19.68		−100.00%
GL30c	HB	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_11_12	0.02	0.315	0.06	0.036	0.007	6.55	0.47	39.54	8.16	15.43	0.20	110.04	29.30		−100.00%
GL30c	OSB	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_12_13	0.033	0.27	0.063	0.036	0.015	6.50	0.47	41.48	8.75	15.07	0.19	105.06	17.42	109.93	4.64%
GL30c	OSB	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_13_14	0.02	0.315	0.06	0.036	0.018	6.64	0.45	37.90	7.73	14.89	0.21	105.00	27.01		−100.00%
GL30c	GL28c	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_0_15	0.033	0.315	0.033	0.036	0.036	6.13	0.69	44.53	9.52	21.53	0.11	82.01	12.44	82.31	0.37%
GL30c	S-LVLS	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_1_16	0.033	0.315	0.033	0.036	0.039	6.14	0.69	44.10	9.42	21.39	0.12	83.27	13.53	84.15	1.06%
GL30c	B-LVLS	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_2_17	0.033	0.315	0.033	0.036	0.04	6.26	0.66	41.69	8.81	20.53	0.13	86.26	15.67	86.36	0.12%
GL30c	GL28c	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_3_18	0.02	0.36	0.03	0.036	0.036	6.08	0.80	44.52	9.47	25.44	0.10	78.70	17.47		−100.00%
GL30c	S-LVLS	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_4_19	0.02	0.45	0.02	0.036	0.039	6.77	0.79	35.99	7.22	28.18	0.13	78.98	16.94	77.20	−2.25%
GL30c	B-LVLS	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_5_20	0.02	0.45	0.02	0.036	0.04	6.97	0.75	33.26	6.52	26.72	0.14	82.38	20.00		−100.00%

(*continued on next page*)

Table C2 (*continued*)

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	Wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	cost	error
S-LVLS	S-LVLS	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_6_21	0.033	0.36	0.033	0.027	0.027	6.54	0.63	38.99	8.04	20.72	0.15	85.93	14.41		−100.00%
B-LVLS	B-LVLS	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_7_22	0.033	0.32	0.033	0.04	0.04	6.30	0.63	40.68	8.38	19.92	0.13	90.29	19.97	90.30	0.01%
S-LVLS	S-LVLS	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_8_23	0.02	0.36	0.03	0.039	0.039	6.12	0.78	43.50	9.16	24.96	0.10	81.51	20.31	81.59	0.10%
B-LVLS	B-LVLS	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_9_24	0.02	0.44	0.02	0.04	0.04	6.76	0.75	34.91	6.80	26.75	0.13	86.34	25.63		−100.00%
GL30c	HB	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_10_25	0.033	0.315	0.033	0.036	0.008	6.12	0.68	44.77	9.54	21.25	0.12	97.62	17.10	96.76	−0.88%
GL30c	HB	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_11_26	0.02	0.45	0.02	0.036	0.007	7.12	0.72	33.11	6.36	27.20	0.16	91.77	21.00	91.00	−0.84%
GL30c	OSB	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_12_27	0.033	0.315	0.033	0.036	0.018	6.13	0.67	44.10	9.36	20.92	0.12	91.62	14.28	95.75	4.51%
GL30c	OSB	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_13_28	0.02	0.405	0.02	0.036	0.022	6.57	0.80	37.95	7.72	28.24	0.12	87.21	17.86	88.94	1.98%
GL30c	GL28c	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_0_29	0.033	0.315	0.063	0.036	0.036	5.94	0.40	37.83	7.28	12.38	0.18	99.81	16.34	99.74	−0.07%
GL30c	S-LVLS	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_1_30	0.033	0.315	0.063	0.036	0.027	5.63	0.42	42.14	8.25	13.33	0.15	100.60	16.82	100.69	0.09%
GL30c	B-LVLS	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_2_31	0.033	0.27	0.063	0.036	0.05	5.53	0.45	42.87	8.41	12.66	0.14	103.35	19.71	101.50	−1.79%
GL30c	GL28c	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_3_32	0.02	0.315	0.06	0.036	0.048	5.39	0.48	44.76	8.71	13.60	0.12	97.81	25.48	95.48	−2.38%
GL30c	S-LVLS	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_4_33	0.02	0.36	0.06	0.036	0.027	5.46	0.45	43.84	8.50	14.04	0.14	98.43	26.01	97.52	−0.92%
GL30c	B-LVLS	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_5_34	0.02	0.315	0.06	0.036	0.04	5.38	0.48	44.55	8.66	13.53	0.12	99.65	28.40	99.73	0.08%
S-LVLS	S-LVLS	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_6_35	0.033	0.3	0.063	0.027	0.039	5.71	0.44	40.71	7.94	12.74	0.15	101.14	18.22		−100.00%
B-LVLS	B-LVLS	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_7_36	0.033	0.28	0.063	0.04	0.04	5.55	0.43	42.64	8.25	12.63	0.15	105.38	22.68	105.46	0.08%
S-LVLS	S-LVLS	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_8_37	0.02	0.36	0.06	0.027	0.027	5.43	0.47	44.16	8.57	14.03	0.13	99.30	27.14	99.27	−0.03%
B-LVLS	B-LVLS	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_9_38	0.02	0.32	0.06	0.04	0.04	5.43	0.45	43.38	8.28	13.24	0.13	103.67	32.71	103.68	0.01%
GL30c	HB	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_10_39	0.033	0.315	0.063	0.036	0.007	5.93	0.40	37.99	7.29	12.23	0.19	114.46	20.46	114.54	0.07%
GL30c	HB	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_11_40	0.02	0.315	0.06	0.036	0.008	5.43	0.46	44.43	8.62	13.61	0.13	110.96	29.83	110.12	−0.76%
GL30c	OSB	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_12_41	0.039	0.27	0.063	0.036	0.015	5.53	0.43	43.35	8.47	12.59	0.15	108.62	18.20	112.12	3.22%
GL30c	OSB	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_13_42	0.02	0.315	0.06	0.036	0.018	5.46	0.45	43.50	8.40	13.36	0.14	105.00	27.01	109.12	3.92%
GL30c	GL28c	S-LVLQ	Type2	1	4.5	8	1.2	45	Arms	0_0_43	0.033	0.36	0.033	0.036	0.036	5.65	0.58	40.56	7.78	17.43	0.11	84.46	12.73	84.69	0.27%
GL30c	S-LVLS	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_1_44	0.033	0.36	0.033	0.036	0.027	5.40	0.61	44.59	8.64	18.58	0.10	85.05	13.28	85.08	0.04%
GL30c	B-LVLS	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_2_45	0.033	0.36	0.033	0.036	0.04	5.82	0.55	37.62	7.12	16.54	0.13	89.02	16.42	88.62	−0.45%
GL30c	GL28c	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_3_46	0.02	0.45	0.02	0.048	0.036	5.54	0.77	42.28	8.09	25.73	0.08	79.48	15.67	74.72	−5.99%
GL30c	S-LVLS	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_4_47	0.02	0.45	0.02	0.036	0.039	5.59	0.79	41.29	7.92	25.31	0.08	78.98	16.94	76.76	−2.81%
GL30c	B-LVLS	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_5_48	0.02	0.45	0.02	0.036	0.04	5.78	0.75	38.10	7.22	24.01	0.09	82.38	20.00	79.59	−3.39%
S-LVLS	S-LVLS	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_6_49	0.033	0.36	0.033	0.027	0.027	5.38	0.63	44.75	8.68	18.58	0.09	85.93	14.41		−100.00%
B-LVLS	B-LVLS	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_7_50	0.033	0.32	0.033	0.04	0.05	5.34	0.61	44.27	8.43	17.35	0.10	92.66	21.02	92.59	−0.08%
S-LVLS	S-LVLS	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_8_51	0.02	0.4	0.03	0.027	0.039	5.56	0.70	41.23	7.88	20.66	0.09	83.27	20.37		−100.00%
B-LVLS	B-LVLS	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_9_52	0.02	0.44	0.02	0.04	0.04	5.62	0.75	39.96	7.47	24.23	0.09	86.34	25.63	83.48	−3.31%
GL30c	HB	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_10_53	0.033	0.36	0.033	0.036	0.007	5.58	0.58	41.70	8.00	17.47	0.11	99.05	17.35	99.06	0.01%
GL30c	HB	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_11_54	0.02	0.45	0.02	0.036	0.007	5.85	0.72	38.02	7.19	24.50	0.10	91.77	21.00	91.00	−0.84%
GL30c	OSB	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_12_55	0.033	0.36	0.033	0.036	0.018	5.66	0.56	40.18	7.65	16.93	0.12	94.26	14.73	96.52	2.40%
GL30c	OSB	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_13_56	0.02	0.405	0.02	0.036	0.022	5.43	0.80	43.51	8.35	25.40	0.08	87.21	17.86	89.34	2.44%

Table C3
Global cost optimum compared to MISLP for second generation of Eurocode 5 (Transient response).

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	cost	error
GL30c	GL28c	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_0_1	0.033	0.36	0.063	0.036	0.036	8.09	0.34	26.52	4.57	12.57	0.44	102.26	16.63	102.50	0.23%
GL30c	S-LVLS	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_1_2	0.033	0.36	0.063	0.036	0.039	8.09	0.34	26.31	4.54	12.49	0.44	103.68	17.87	104.32	0.62%
GL30c	B-LVLS	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_2_3	0.033	0.36	0.063	0.036	0.04	8.31	0.33	24.44	4.07	11.83	0.49	106.82	20.32	106.84	0.02%
GL30c	GL28c	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_3_4	0.02	0.45	0.06	0.036	0.036	8.50	0.32	22.97	3.63	12.14	0.53	102.71	26.05	100.23	−2.41%
GL30c	S-LVLS	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_4_5	0.02	0.405	0.06	0.036	0.045	8.03	0.35	25.45	4.34	12.56	0.42	103.52	27.55	102.26	−1.22%
GL30c	B-LVLS	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_5_6	0.02	0.405	0.06	0.036	0.04	8.09	0.34	24.68	4.17	12.26	0.44	105.13	29.90	105.01	−0.11%
S-LVLS	S-LVLS	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_6_7	0.033	0.36	0.063	0.027	0.039	8.06	0.35	26.41	4.58	12.50	0.42	104.55	19.00	105.58	0.99%
B-LVLS	B-LVLS	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_7_8	0.033	0.36	0.063	0.04	0.04	8.24	0.32	24.55	4.02	11.58	0.49	110.81	25.06	110.75	−0.05%
S-LVLS	S-LVLS	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_8_9	0.02	0.4	0.06	0.027	0.051	8.07	0.35	24.94	4.23	12.30	0.42	105.90	29.19	103.81	−1.97%
B-LVLS	B-LVLS	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_9_10	0.02	0.44	0.06	0.04	0.04	8.62	0.29	21.31	3.20	11.18	0.59	111.83	36.28	108.98	−2.55%
GL30c	HB	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_10_11	0.033	0.36	0.063	0.036	0.007	8.09	0.34	26.69	4.58	12.39	0.44	116.80	21.25	116.82	0.02%
GL30c	HB	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_11_12	0.02	0.405	0.06	0.036	0.007	8.02	0.34	26.08	4.44	12.75	0.43	114.75	30.87	114.70	−0.04%
GL30c	OSB	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_12_13	0.033	0.36	0.063	0.036	0.018	8.21	0.32	25.42	4.27	11.88	0.48	112.07	18.63	114.30	1.99%
GL30c	OSB	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_13_14	0.02	0.405	0.06	0.036	0.022	8.22	0.32	24.03	3.94	11.89	0.49	112.74	28.51	114.39	1.46%
GL30c	GL28c	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_0_15	0.033	0.45	0.033	0.036	0.036	8.28	0.43	23.98	3.91	16.26	0.37	89.33	13.31	89.44	0.12%
GL30c	S-LVLS	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_1_16	0.033	0.45	0.033	0.036	0.039	8.29	0.42	23.67	3.85	16.12	0.37	91.10	14.86	91.25	0.16%
GL30c	B-LVLS	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_2_17	0.033	0.45	0.033	0.036	0.04	8.51	0.40	21.97	3.45	15.25	0.42	94.49	17.92	94.38	−0.12%
GL30c	GL28c	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_3_18	0.02	0.495	0.03	0.036	0.036	8.10	0.50	24.65	4.10	19.45	0.30	86.02	18.35	85.84	−0.21%
GL30c	S-LVLS	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_4_19	0.02	0.495	0.03	0.036	0.039	8.16	0.49	24.03	3.96	19.15	0.31	87.95	20.05	85.90	−2.33%
GL30c	B-LVLS	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_5_20	0.02	0.495	0.03	0.036	0.04	8.41	0.46	22.08	3.49	18.04	0.35	91.48	23.41	88.62	−3.13%
S-LVLS	S-LVLS	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_6_21	0.033	0.4	0.039	0.027	0.057	8.03	0.43	24.62	4.17	14.85	0.34	97.74	17.63	95.81	−1.97%
B-LVLS	B-LVLS	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_7_22	0.033	0.44	0.033	0.04	0.04	8.27	0.40	23.00	3.64	15.20	0.39	98.45	23.55	98.27	−0.18%
S-LVLS	S-LVLS	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_8_23	0.03	0.4	0.04	0.027	0.051	8.09	0.41	23.34	3.83	14.44	0.36	99.51	26.52	103.81	4.32%
B-LVLS	B-LVLS	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_9_24	0.02	0.48	0.03	0.04	0.04	8.11	0.47	23.53	3.78	18.15	0.32	95.42	29.48	94.10	−1.38%
GL30c	HB	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_10_25	0.033	0.45	0.033	0.036	0.007	8.16	0.43	24.84	4.11	16.31	0.36	103.77	18.92	103.64	−0.13%
GL30c	HB	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_11_26	0.02	0.495	0.03	0.036	0.007	8.26	0.47	23.86	3.85	19.00	0.33	100.94	24.45	100.74	−0.20%
GL30c	OSB	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_12_27	0.033	0.45	0.033	0.036	0.012	8.03	0.43	25.52	4.30	16.58	0.34	101.64	15.53	101.41	−0.23%
GL30c	OSB	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_13_28	0.02	0.495	0.03	0.036	0.012	8.13	0.48	24.49	4.03	19.30	0.32	98.16	20.72	97.85	−0.32%
GL30c	GL28c	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_0_29	0.033	0.495	0.063	0.036	0.036	8.50	0.23	18.21	2.54	8.97	0.72	109.57	17.50	109.61	0.04%
GL30c	S-LVLS	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_1_30	0.033	0.45	0.063	0.036	0.045	8.08	0.25	19.99	2.99	9.21	0.59	110.74	19.20	109.43	−1.18%
GL30c	B-LVLS	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_2_31	0.033	0.45	0.063	0.036	0.04	8.18	0.25	19.31	2.83	8.96	0.62	112.29	21.82	112.17	−0.11%
GL30c	GL28c	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_3_32	0.02	0.54	0.06	0.036	0.036	8.20	0.25	19.10	2.76	9.51	0.62	107.58	26.63	104.97	−2.43%
GL30c	S-LVLS	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_4_33	0.02	0.54	0.06	0.036	0.039	8.28	0.24	18.59	2.64	9.37	0.64	109.69	28.49	107.20	−2.27%
GL30c	B-LVLS	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_5_34	0.02	0.495	0.06	0.036	0.04	8.02	0.26	19.61	2.90	9.37	0.57	110.60	31.40	110.43	−0.15%
S-LVLS	S-LVLS	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_6_35	0.039	0.4	0.063	0.027	0.075	8.06	0.25	19.62	2.92	8.59	0.59	120.48	22.71	116.83	−3.03%
B-LVLS	B-LVLS	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_7_36	0.033	0.48	0.063	0.04	0.04	8.56	0.22	17.40	2.34	8.37	0.77	118.97	28.64	116.06	−2.45%
S-LVLS	S-LVLS	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_8_37	0.04	0.4	0.06	0.027	0.063	8.10	0.25	18.81	2.71	8.55	0.61	121.98	35.30	126.42	3.64%
B-LVLS	B-LVLS	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_9_38	0.02	0.52	0.06	0.04	0.04	8.32	0.23	18.00	2.47	8.85	0.69	117.26	38.67	116.94	−0.27%
GL30c	HB	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_10_39	0.033	0.495	0.063	0.036	0.007	8.47	0.23	18.40	2.57	8.83	0.73	124.35	23.61	123.19	−0.93%
GL30c	HB	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_11_40	0.02	0.54	0.06	0.036	0.007	8.34	0.23	18.52	2.60	9.27	0.68	122.32	33.23	122.06	−0.21%
GL30c	OSB	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_12_41	0.033	0.495	0.063	0.036	0.012	8.30	0.23	19.08	2.74	9.04	0.68	121.63	19.88	121.36	−0.22%
GL30c	OSB	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_13_42	0.02	0.54	0.06	0.036	0.012	8.19	0.24	19.16	2.75	9.47	0.64	119.47	29.16	119.11	−0.30%
GL30c	GL28c	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_0_43	0.033	0.585	0.033	0.036	0.036	8.55	0.30	17.43	2.36	11.99	0.57	96.65	14.18	94.18	−2.56%
GL30c	S-LVLS	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_1_44	0.033	0.54	0.033	0.036	0.039	8.02	0.33	19.72	2.91	12.64	0.44	96.32	15.76	98.68	2.45%
GL30c	B-LVLS	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_2_45	0.033	0.54	0.033	0.036	0.04	8.31	0.31	18.08	2.54	11.87	0.51	99.97	19.42	99.72	−0.25%
GL30c	GL28c	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_3_46	0.02	0.63	0.02	0.036	0.048	8.08	0.45	19.42	2.84	17.76	0.33	91.70	17.15	86.60	−5.56%
GL30c	S-LVLS	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_4_47	0.02	0.63	0.02	0.036	0.051	8.19	0.44	18.70	2.68	17.38	0.35	94.77	19.98	88.62	−6.49%
GL30c	B-LVLS	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_5_48	0.02	0.63	0.02	0.036	0.04	8.05	0.44	19.23	2.81	17.57	0.33	93.33	23.00	90.28	−3.27%
S-LVLS	S-LVLS	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_6_49	0.045	0.4	0.069	0.027	0.075	8.03	0.22	18.60	2.66	7.44	0.67	127.62	24.27	123.60	−3.15%
B-LVLS	B-LVLS	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_7_50	0.033	0.56	0.033	0.04	0.04	8.51	0.28	17.02	2.25	11.31	0.59	106.60	27.13	103.56	−2.85%
S-LVLS	S-LVLS	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_8_51	0.05	0.4	0.05	0.027	0.075	8.01	0.24	18.23	2.57	8.23	0.61	125.28	36.09	131.24	4.76%

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Table C3 (*continued*)

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	cost	error
B-LVLS	B-LVLS	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_9_52	0.02	0.6	0.03	0.04	0.04	8.35	0.33	17.52	2.36	13.57	0.48	103.56	33.06	96.74	−6.59%
GL30c	HB	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_10_53	0.033	0.585	0.033	0.036	0.007	8.38	0.30	18.22	2.53	12.06	0.54	111.32	21.28	111.00	−0.29%
GL30c	HB	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_11_54	0.02	0.63	0.02	0.036	0.009	8.02	0.43	19.85	2.91	18.03	0.34	105.28	26.27	103.07	−2.10%
GL30c	OSB	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_12_55	0.033	0.585	0.063	0.036	0.012	9.02	0.19	15.19	1.85	7.83	1.01	126.11	20.77	107.94	−14.41%
GL30c	OSB	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_13_56	0.02	0.63	0.03	0.036	0.012	8.26	0.34	18.50	2.59	14.52	0.46	104.89	22.06		−100.00%

Table C4
Global ECO2 optimum compared to MISLP for current common method (Hu and Chui).

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	Wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	CO2	error
GL30c	GL28c	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_0_1	0.033	0.495	0.063	0.036	0.036	10.37	0.23	15.85	1.78	9.88	1.13	109.57	17.50	17.55	0.25%
GL30c	S-LVLS	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_1_2	0.033	0.54	0.063	0.036	0.027	10.32	0.22	15.98	1.81	10.29	1.15	111.85	18.62	18.62	−0.01%
GL30c	B-LVLS	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_2_3	0.033	0.495	0.063	0.036	0.04	10.69	0.22	14.35	1.50	9.15	1.29	115.02	22.57	22.11	−2.02%
GL30c	GL28c	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_3_4	0.02	0.585	0.06	0.036	0.036	10.61	0.22	14.48	1.52	9.77	1.24	110.02	26.92	26.63	−1.08%
GL30c	S-LVLS	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_4_5	0.02	0.585	0.06	0.036	0.033	10.30	0.23	15.29	1.71	10.16	1.13	112.07	28.38	28.28	−0.36%
GL30c	B-LVLS	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_5_6	0.02	0.54	0.06	0.036	0.04	10.37	0.23	14.64	1.61	9.55	1.14	113.34	32.15	31.41	−2.31%
S-LVLS	S-LVLS	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_6_7	0.051	0.4	0.069	0.027	0.063	10.05	0.21	15.74	1.86	8.30	1.14	127.88	24.26	23.38	−3.65%
B-LVLS	B-LVLS	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_7_8	0.033	0.48	0.063	0.04	0.04	10.33	0.22	15.19	1.67	9.14	1.19	118.97	28.64	28.01	−2.20%
S-LVLS	S-LVLS	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_8_9	0.04	0.4	0.07	0.027	0.075	10.01	0.21	14.82	1.72	7.93	1.17	131.68	38.76	41.31	6.57%
B-LVLS	B-LVLS	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_9_10	0.02	0.56	0.06	0.04	0.04	10.59	0.21	13.78	1.40	9.01	1.32	119.98	39.86	38.40	−3.68%
GL30c	HB	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_10_11	0.033	0.495	0.063	0.036	0.007	10.35	0.23	16.01	1.80	9.71	1.15	124.35	23.61	23.61	0.03%
GL30c	HB	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_11_12	0.02	0.54	0.06	0.036	0.007	10.11	0.23	16.14	1.87	10.15	1.05	122.32	33.23	33.23	0.00%
GL30c	OSB	S-LVLQ	Type1	None	10	NA	1.3	45	HC	0_12_13	0.033	0.495	0.063	0.036	0.012	10.14	0.23	16.60	1.94	9.95	1.07	121.63	19.88	19.87	−0.04%
GL30c	OSB	B-LVLQ	Type1	None	10	NA	1.3	45	HC	0_13_14	0.02	0.585	0.06	0.036	0.012	10.55	0.22	14.66	1.54	9.74	1.26	121.71	29.60	29.14	−1.55%
GL30c	GL28c	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_0_15	0.033	0.63	0.033	0.036	0.036	10.96	0.27	13.39	1.29	12.34	1.10	99.08	14.47	14.50	0.15%
GL30c	S-LVLS	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_1_16	0.033	0.63	0.033	0.036	0.033	10.64	0.28	14.13	1.46	12.84	1.00	101.13	16.05	16.02	−0.16%
GL30c	B-LVLS	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_2_17	0.033	0.585	0.033	0.036	0.04	10.64	0.28	13.69	1.41	12.13	1.00	102.69	20.17	20.16	−0.04%
GL30c	GL28c	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_3_18	0.02	0.63	0.02	0.036	0.115	11.65	0.33	10.85	0.89	14.67	1.04	118.58	20.65	19.21	−6.97%
GL30c	S-LVLS	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_4_19	0.03	0.63	0.03	0.048	0.033	10.68	0.27	13.58	1.35	13.28	1.04	104.89	23.84	22.60	−5.21%
GL30c	B-LVLS	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_5_20	0.02	0.63	0.03	0.09	0.04	10.62	0.27	13.48	1.32	13.94	1.02	113.83	27.41	26.03	−5.02%

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Table C4 (*continued*)

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	Wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	CO2	error
S-LVLS	S-LVLS	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_6_21	0.069	0.4	0.075	0.027	0.063	10.00	0.17	14.21	1.60	6.60	1.41	142.16	27.38	26.50	−3.24%
B-LVLS	B-LVLS	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_7_22	0.033	0.56	0.039	0.04	0.04	10.36	0.26	14.16	1.49	11.14	1.02	110.15	27.91	27.61	−1.08%
S-LVLS	S-LVLS	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_8_23	0.06	0.4	0.07	0.027	0.075	10.02	0.17	13.33	1.45	6.57	1.42	144.48	44.09		
B-LVLS	B-LVLS	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_9_24	0.03	0.6	0.03	0.04	0.04	10.79	0.26	12.68	1.20	12.00	1.09	109.94	35.73	35.40	−0.92%
GL30c	HB	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_10_25	0.033	0.63	0.033	0.036	0.007	10.74	0.27	14.03	1.41	12.39	1.06	113.67	22.06	22.05	−0.07%
GL30c	HB	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_11_26	0.03	0.63	0.03	0.036	0.007	10.86	0.27	13.30	1.28	12.77	1.07	114.31	29.47	29.45	−0.05%
GL30c	OSB	S-LVLQ	Type1	1	10	NA	1.3	45	HC	0_12_27	0.033	0.63	0.033	0.048	0.012	10.54	0.26	14.48	1.49	12.59	1.04	113.74	17.70	17.28	−2.37%
GL30c	OSB	B-LVLQ	Type1	1	10	NA	1.3	45	HC	0_13_28	0.03	0.63	0.03	0.036	0.012	10.63	0.28	13.79	1.39	13.08	1.00	111.22	24.72	24.69	−0.12%
GL30c	GL28c	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_0_29	0.033	0.63	0.063	0.036	0.036	10.27	0.17	12.33	1.23	7.51	1.48	116.87	18.38	18.40	0.11%
GL30c	S-LVLS	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_1_30	0.033	0.63	0.063	0.036	0.039	10.27	0.17	12.20	1.21	7.45	1.49	119.33	20.55	19.92	−3.05%
GL30c	B-LVLS	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_2_31	0.033	0.585	0.063	0.036	0.04	10.11	0.18	12.42	1.27	7.29	1.40	120.49	24.07	24.06	−0.04%
GL30c	GL28c	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_3_32	0.02	0.63	0.06	0.048	0.048	10.00	0.18	12.61	1.29	7.65	1.36	120.34	28.19	32.41	14.95%
GL30c	S-LVLS	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_4_33	0.02	0.63	0.06	0.036	0.051	10.02	0.18	12.43	1.28	7.59	1.32	120.28	30.64	29.35	−4.22%
GL30c	B-LVLS	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_5_34	0.02	0.63	0.06	0.073	0.04	10.00	0.16	12.38	1.24	7.52	1.48	128.52	34.85	33.63	−3.49%
S-LVLS	S-LVLS	S-LVLQ	Type2	None	10	NA	1.3	45	HC																
B-LVLS	B-LVLS	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_7_36	0.033	0.6	0.063	0.04	0.04	10.23	0.16	11.98	1.16	7.00	1.55	127.11	32.22	31.51	−2.22%
S-LVLS	S-LVLS	B-LVLQ	Type2	None	10	NA	1.3	45	HC																
B-LVLS	B-LVLS	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_9_38	0.03	0.6	0.06	0.04	0.04	10.28	0.16	11.50	1.08	7.03	1.58	129.08	43.72	42.67	−2.40%
GL30c	HB	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_10_39	0.033	0.63	0.063	0.036	0.007	10.23	0.17	12.49	1.25	7.37	1.50	131.38	25.96	25.95	−0.04%
GL30c	HB	B-LVLQ	Type2	None	10	NA	1.3	45	HC															37.45	
GL30c	OSB	S-LVLQ	Type2	None	10	NA	1.3	45	HC	0_12_41	0.033	0.63	0.063	0.048	0.012	10.01	0.17	12.98	1.34	7.54	1.46	131.49	21.60	21.18	−1.94%
GL30c	OSB	B-LVLQ	Type2	None	10	NA	1.3	45	HC	0_13_42	0.03	0.63	0.06	0.036	0.012	10.07	0.17	12.40	1.25	7.58	1.43	130.32	32.71	32.68	−0.09%
GL30c	GL28c	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_0_43	0.033	0.63	0.033	0.036	0.066	10.10	0.23	12.15	1.22	9.63	1.07	111.14	16.01	15.99	−0.12%
GL30c	S-LVLS	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_1_44	0.033	0.63	0.039	0.048	0.057	10.00	0.21	12.28	1.25	9.11	1.15	116.16	19.69	25.46	29.32%
GL30c	B-LVLS	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_2_45	0.033	0.63	0.051	0.036	0.04	10.03	0.19	12.08	1.22	7.92	1.28	116.10	23.26	22.46	−3.43%
GL30c	GL28c	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_3_46	0.02	0.63	0.03	0.073	0.09	10.10	0.24	11.84	1.15	10.95	1.01	124.58	23.16	21.87	−5.57%
GL30c	S-LVLS	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_4_47	0.03	0.63	0.03	0.036	0.063	10.04	0.24	11.96	1.20	10.21	1.02	112.63	26.54	26.26	−1.05%
GL30c	B-LVLS	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_5_48	0.03	0.63	0.03	0.066	0.05	10.00	0.22	11.95	1.18	10.10	1.09	118.53	31.35	30.35	−3.20%
S-LVLS	S-LVLS	S-LVLQ	Type2	1	10	NA	1.3	45	HC																
B-LVLS	B-LVLS	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_7_50	0.039	0.6	0.063	0.04	0.04	10.02	0.16	11.83	1.15	6.58	1.56	130.67	33.00	32.27	−2.20%
S-LVLS	S-LVLS	B-LVLQ	Type2	1	10	NA	1.3	45	HC																
B-LVLS	B-LVLS	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_9_52	0.03	0.6	0.04	0.04	0.06	10.09	0.20	11.42	1.09	8.39	1.24	125.32	42.36	60.85	43.66%
GL30c	HB	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_10_53	0.045	0.63	0.051	0.036	0.007	10.08	0.17	12.18	1.21	7.47	1.43	131.38	25.96	25.17	−3.06%
GL30c	HB	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_11_54	0.04	0.63	0.04	0.036	0.008	10.09	0.19	11.91	1.16	8.28	1.30	128.80	35.86	46.16	28.73%
GL30c	OSB	S-LVLQ	Type2	1	10	NA	1.3	45	HC	0_12_55	0.039	0.63	0.051	0.036	0.018	10.09	0.18	12.02	1.19	7.50	1.41	130.46	21.79	21.07	−3.28%
GL30c	OSB	B-LVLQ	Type2	1	10	NA	1.3	45	HC	0_13_56	0.04	0.63	0.04	0.036	0.018	10.19	0.18	11.52	1.10	8.02	1.37	129.61	31.40	39.92	27.13%

Table C5
Global ECO2 optimum compared to MISLP for second generation of Eurocode 5 (Resonant response).

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	Wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	CO2	error
GL30c	GL28c	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_0_1	0.033	0.27	0.063	0.036	0.036	6.48	0.49	41.99	8.92	15.47	0.19	97.39	16.05		
GL30c	S-LVLS	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_1_2	0.033	0.27	0.063	0.036	0.033	6.35	0.50	43.65	9.38	15.89	0.17	99.46	16.72		
GL30c	B-LVLS	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_2_3	0.033	0.27	0.063	0.036	0.04	6.62	0.46	39.24	8.21	14.72	0.20	101.34	18.82		
GL30c	GL28c	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_3_4	0.02	0.315	0.06	0.036	0.036	6.27	0.51	42.94	9.11	16.13	0.17	95.39	25.18		
GL30c	S-LVLS	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_4_5	0.02	0.315	0.06	0.036	0.033	6.16	0.52	44.45	9.52	16.51	0.16	97.47	25.96		
GL30c	B-LVLS	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_5_6	0.02	0.315	0.06	0.036	0.04	6.51	0.48	38.85	8.06	15.12	0.19	99.65	28.40		
S-LVLS	S-LVLS	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_6_7	0.033	0.3	0.063	0.027	0.027	6.66	0.47	39.60	8.26	15.46	0.20	100.79	17.64	17.62	−0.10%
B-LVLS	B-LVLS	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_7_8	0.039	0.24	0.063	0.04	0.04	6.23	0.48	43.81	9.31	14.89	0.17	106.22	22.26		
S-LVLS	S-LVLS	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_8_9	0.02	0.36	0.06	0.027	0.027	6.61	0.47	38.46	7.90	15.66	0.20	99.30	27.14	27.03	−0.42%
B-LVLS	B-LVLS	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_9_10	0.02	0.28	0.06	0.04	0.05	6.04	0.52	44.67	9.47	15.68	0.15	103.03	32.44		
GL30c	HB	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_10_11	0.033	0.27	0.063	0.036	0.007	6.48	0.48	42.06	8.90	15.28	0.19	112.10	19.68		
GL30c	HB	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_11_12	0.02	0.315	0.06	0.036	0.007	6.55	0.47	39.54	8.16	15.43	0.20	110.04	29.30		
GL30c	OSB	S-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_12_13	0.033	0.27	0.063	0.036	0.015	6.50	0.47	41.48	8.75	15.07	0.19	105.06	17.42	17.68	1.50%
GL30c	OSB	B-LVLQ	Type1	None	4.5	8	0.8	45	Arms	0_13_14	0.02	0.315	0.06	0.036	0.012	6.46	0.47	40.49	8.42	15.65	0.19	107.71	26.93		
GL30c	GL28c	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_0_15	0.033	0.315	0.033	0.036	0.036	6.13	0.69	44.53	9.52	21.53	0.11	82.01	12.44	12.51	0.60%
GL30c	S-LVLS	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_1_16	0.033	0.36	0.033	0.036	0.027	6.56	0.61	38.85	7.98	20.72	0.15	85.05	13.28	13.26	−0.15%
GL30c	B-LVLS	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_2_17	0.033	0.315	0.033	0.036	0.04	6.26	0.66	41.69	8.81	20.53	0.13	86.26	15.67	15.71	0.25%
GL30c	GL28c	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_3_18	0.02	0.45	0.02	0.048	0.036	6.73	0.77	36.83	7.36	28.58	0.13	79.48	15.67		
GL30c	S-LVLS	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_4_19	0.02	0.495	0.02	0.036	0.027	6.95	0.74	34.50	6.77	28.30	0.14	79.75	16.44	15.99	−2.73%
GL30c	B-LVLS	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_5_20	0.02	0.45	0.02	0.036	0.04	6.97	0.75	33.26	6.52	26.72	0.14	82.38	20.00		
S-LVLS	S-LVLS	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_6_21	0.033	0.36	0.033	0.027	0.027	6.54	0.63	38.99	8.04	20.72	0.15	85.93	14.41		
B-LVLS	B-LVLS	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_7_22	0.033	0.32	0.033	0.04	0.04	6.30	0.63	40.68	8.38	19.92	0.13	90.29	19.97	19.84	−0.67%
S-LVLS	S-LVLS	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_8_23	0.02	0.4	0.03	0.027	0.027	6.34	0.75	40.77	8.48	24.87	0.11	82.14	19.60	19.46	−0.71%
B-LVLS	B-LVLS	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_9_24	0.02	0.44	0.02	0.04	0.04	6.76	0.75	34.91	6.80	26.75	0.13	86.34	25.63		
GL30c	HB	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_10_25	0.033	0.315	0.033	0.036	0.008	6.12	0.68	44.77	9.54	21.25	0.12	97.62	17.10	16.60	−2.88%
GL30c	HB	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_11_26	0.02	0.405	0.02	0.066	0.007	6.51	0.78	39.87	8.04	29.48	0.12	94.41	20.85	20.49	−1.72%
GL30c	OSB	S-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_12_27	0.033	0.315	0.033	0.036	0.018	6.13	0.67	44.10	9.36	20.92	0.12	91.62	14.28	14.55	1.91%
GL30c	OSB	B-LVLQ	Type1	1	4.5	8	0.8	45	Arms	0_13_28	0.02	0.45	0.02	0.036	0.012	7.03	0.73	33.82	6.56	27.54	0.15	89.58	17.61	17.43	−1.06%
GL30c	GL28c	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_0_29	0.033	0.315	0.063	0.036	0.036	5.94	0.40	37.83	7.28	12.38	0.18	99.81	16.34	16.39	0.28%
GL30c	S-LVLS	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_1_30	0.033	0.315	0.063	0.036	0.027	5.63	0.42	42.14	8.25	13.33	0.15	100.60	16.82	16.85	0.23%
GL30c	B-LVLS	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_2_31	0.033	0.27	0.063	0.066	0.04	5.43	0.44	44.84	8.76	13.01	0.14	104.81	19.25	18.87	−1.99%
GL30c	GL28c	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_3_32	0.02	0.36	0.06	0.036	0.036	5.78	0.42	39.05	7.46	13.03	0.16	97.83	25.47	25.21	−0.99%
GL30c	S-LVLS	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_4_33	0.02	0.36	0.06	0.036	0.027	5.46	0.45	43.84	8.50	14.04	0.14	98.43	26.01	25.99	−0.08%
GL30c	B-LVLS	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_5_34	0.02	0.315	0.06	0.036	0.04	5.38	0.48	44.55	8.66	13.53	0.12	99.65	28.40	28.44	0.13%
S-LVLS	S-LVLS	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_6_35	0.033	0.3	0.063	0.027	0.033	5.58	0.45	42.76	8.39	13.17	0.14	102.05	17.93		
B-LVLS	B-LVLS	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_7_36	0.033	0.28	0.063	0.04	0.04	5.55	0.43	42.64	8.25	12.63	0.15	105.38	22.68	22.57	−0.46%
S-LVLS	S-LVLS	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_8_37	0.02	0.36	0.06	0.027	0.027	5.43	0.47	44.16	8.57	14.03	0.13	99.30	27.14	27.10	−0.14%
B-LVLS	B-LVLS	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_9_38	0.02	0.32	0.06	0.04	0.04	5.43	0.45	43.38	8.28	13.24	0.13	103.67	32.71	32.57	−0.41%
GL30c	HB	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_10_39	0.039	0.27	0.063	0.036	0.007	5.50	0.44	44.02	8.63	12.77	0.14	115.65	20.46	20.51	0.22%
GL30c	HB	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_11_40	0.02	0.315	0.06	0.036	0.008	5.43	0.46	44.43	8.62	13.61	0.13	110.96	29.83	29.34	−1.65%
GL30c	OSB	S-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_12_41	0.033	0.315	0.063	0.036	0.012	5.85	0.40	39.03	7.53	12.44	0.18	112.13	18.09	18.12	0.14%
GL30c	OSB	B-LVLQ	Type2	None	4.5	8	0.8	45	Arms	0_13_42	0.02	0.315	0.06	0.036	0.018	5.46	0.45	43.50	8.40	13.36	0.14	105.00	27.01	27.28	1.00%
GL30c	GL28c	S-LVLQ	Type2	1	4.5	8	1.2	45	Arms	0_0_43	0.033	0.36	0.033	0.036	0.036	5.65	0.58	40.56	7.78	17.43	0.11	84.46	12.73	12.80	0.53%
GL30c	S-LVLS	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_1_44	0.033	0.36	0.033	0.036	0.027	5.40	0.61	44.59	8.64	18.58	0.10	85.05	13.28	13.31	0.23%
GL30c	B-LVLS	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_2_45	0.033	0.36	0.033	0.036	0.04	5.82	0.55	37.62	7.12	16.54	0.13	89.02	16.42	16.45	0.18%
GL30c	GL28c	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_3_46	0.02	0.45	0.02	0.048	0.036	5.54	0.77	42.28	8.09	25.73	0.08	79.48	15.67	15.37	−1.90%
GL30c	S-LVLS	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_4_47	0.02	0.495	0.02	0.036	0.027	5.72	0.74	39.59	7.54	25.46	0.09	79.75	16.44	15.99	−2.73%
GL30c	B-LVLS	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_5_48	0.02	0.45	0.02	0.036	0.04	5.78	0.75	38.10	7.22	24.01	0.09	82.38	20.00	19.27	−3.64%
S-LVLS	S-LVLS	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_6_49	0.033	0.36	0.033	0.027	0.027	5.38	0.63	44.75	8.68	18.58	0.09	85.93	14.41		
B-LVLS	B-LVLS	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_7_50	0.033	0.32	0.039	0.04	0.04	5.40	0.56	43.45	8.24	16.19	0.11	93.85	20.75	20.62	−0.64%
S-LVLS	S-LVLS	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_8_51	0.02	0.4	0.03	0.027	0.033	5.41	0.72	43.65	8.39	21.39	0.08	83.79	19.99		

(*continued on next page*)

Table C5 (*continued*)

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	Wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	CO2	error
B-LVLS	B-LVLS	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_9_52	0.02	0.44	0.02	0.04	0.04	5.62	0.75	39.96	7.47	24.23	0.09	86.34	25.63	24.25	−5.39%
GL30c	HB	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_10_53	0.033	0.36	0.033	0.036	0.007	5.58	0.58	41.70	8.00	17.47	0.11	99.05	17.35	17.38	0.18%
GL30c	HB	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_11_54	0.02	0.45	0.02	0.036	0.007	5.85	0.72	38.02	7.19	24.50	0.10	91.77	21.00	21.01	0.08%
GL30c	OSB	S-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_12_55	0.033	0.36	0.033	0.036	0.012	5.51	0.59	42.66	8.20	17.72	0.11	96.61	14.64	14.65	0.11%
GL30c	OSB	B-LVLQ	Type2	1	4.5	8	0.8	45	Arms	0_13_56	0.02	0.45	0.02	0.036	0.012	5.78	0.73	38.82	7.36	24.80	0.10	89.58	17.61	17.61	0.01%

Table C6
Global ECO2 optimum compared to MISLP for second generation of Eurocode 5 (Transient response).

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	Wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	CO2	error
GL30c	GL28c	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_0_1	0.033	0.36	0.063	0.036	0.036	8.09	0.34	26.52	4.57	12.57	0.44	102.26	16.63	16.70	0.41%
GL30c	S-LVLS	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_1_2	0.033	0.405	0.063	0.036	0.027	8.35	0.32	24.86	4.10	12.58	0.51	105.10	17.54	17.55	0.09%
GL30c	B-LVLS	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_2_3	0.033	0.36	0.063	0.036	0.04	8.31	0.33	24.44	4.07	11.83	0.49	106.82	20.32	20.35	0.15%
GL30c	GL28c	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_3_4	0.02	0.45	0.06	0.036	0.036	8.50	0.32	22.97	3.63	12.14	0.53	102.71	26.05	25.78	−1.03%
GL30c	S-LVLS	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_4_5	0.02	0.45	0.06	0.048	0.027	8.00	0.32	26.00	4.40	13.18	0.46	105.21	27.01	26.78	−0.86%
GL30c	B-LVLS	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_5_6	0.02	0.405	0.06	0.036	0.04	8.09	0.34	24.68	4.17	12.26	0.44	105.13	29.90	29.93	0.07%
S-LVLS	S-LVLS	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_6_7	0.033	0.4	0.063	0.027	0.027	8.24	0.34	25.45	4.28	12.71	0.46	105.66	18.75	18.70	−0.29%
B-LVLS	B-LVLS	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_7_8	0.033	0.36	0.063	0.04	0.04	8.24	0.32	24.55	4.02	11.58	0.49	110.81	25.06	24.90	−0.65%
S-LVLS	S-LVLS	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_8_9	0.02	0.4	0.06	0.027	0.051	8.07	0.35	24.94	4.23	12.30	0.42	105.90	29.19	28.67	−1.75%
B-LVLS	B-LVLS	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_9_10	0.02	0.44	0.06	0.04	0.04	8.62	0.29	21.31	3.20	11.18	0.59	111.83	36.28	34.90	−3.81%
GL30c	HB	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_10_11	0.033	0.36	0.063	0.036	0.007	8.09	0.34	26.69	4.58	12.39	0.44	116.80	21.25	21.28	0.15%
GL30c	HB	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_11_12	0.02	0.405	0.06	0.036	0.007	8.02	0.34	26.08	4.44	12.75	0.43	114.75	30.87	30.89	0.07%
GL30c	OSB	S-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_12_13	0.033	0.36	0.063	0.036	0.018	8.21	0.32	25.42	4.27	11.88	0.48	112.07	18.63	18.55	−0.43%
GL30c	OSB	B-LVLQ	Type1	None	8	NA	0.8	45	Vrms	0_13_14	0.02	0.405	0.06	0.036	0.015	8.04	0.34	25.64	4.34	12.53	0.44	114.58	28.26	28.26	0.00%
GL30c	GL28c	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_0_15	0.033	0.45	0.033	0.036	0.036	8.28	0.43	23.98	3.91	16.26	0.37	89.33	13.31	13.36	0.39%
GL30c	S-LVLS	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_1_16	0.033	0.495	0.033	0.036	0.027	8.47	0.40	22.89	3.61	16.37	0.42	91.82	14.36	14.36	0.05%
GL30c	B-LVLS	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_2_17	0.033	0.45	0.033	0.036	0.04	8.51	0.40	21.97	3.45	15.25	0.42	94.49	17.92	17.93	0.08%
GL30c	GL28c	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_3_18	0.02	0.63	0.02	0.036	0.048	9.72	0.45	16.96	2.12	19.49	0.51	91.70	17.15	16.25	−5.21%
GL30c	S-LVLS	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_4_19	0.02	0.63	0.02	0.036	0.045	9.57	0.45	17.37	2.24	19.82	0.48	91.97	19.33	19.11	−1.17%
GL30c	B-LVLS	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_5_20	0.02	0.63	0.02	0.036	0.04	9.62	0.44	16.82	2.14	19.29	0.50	93.33	23.00	22.67	−1.40%

(*continued on next page*)

Table C6 (*continued*)

Constraints										Manual cost optimum														MISLP	
Material			overlay	Ceiling system	f1		Deflection [mm]		Type	ID	Dimensions					Constaint level						Objective		[0.5, 1.5]	
edge	fldJst	flg			min	max	w_{1kN} [mm]	Wfin [mm]			topFlg_tck	cvtyHgt	btmFlg_tck	edgJst_w	fldJst_w	f1	w	wFin	Ra	Rv	HC	cost	CO2	CO2	error
S-LVLS	S-LVLS	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_6_21	0.039	0.4	0.039	0.027	0.045	8.02	0.41	24.76	4.18	14.29	0.36	97.77	17.58	17.14	−2.46%
B-LVLS	B-LVLS	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_7_22	0.033	0.44	0.033	0.04	0.04	8.27	0.40	23.00	3.64	15.20	0.39	98.45	23.55	23.33	−0.92%
S-LVLS	S-LVLS	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_8_23	0.03	0.4	0.03	0.027	0.075	8.11	0.45	23.20	3.82	16.13	0.33	99.71	25.44	28.67	12.72%
B-LVLS	B-LVLS	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_9_24	0.02	0.48	0.03	0.04	0.04	8.11	0.47	23.53	3.78	18.15	0.32	95.42	29.48	29.24	−0.83%
GL30c	HB	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_10_25	0.033	0.45	0.033	0.036	0.007	8.16	0.43	24.84	4.11	16.31	0.36	103.77	18.92	18.93	0.09%
GL30c	HB	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_11_26	0.02	0.495	0.03	0.036	0.007	8.26	0.47	23.86	3.85	19.00	0.33	100.94	24.45	24.46	0.03%
GL30c	OSB	S-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_12_27	0.033	0.45	0.033	0.036	0.012	8.03	0.43	25.52	4.30	16.58	0.34	101.64	15.53	15.53	0.01%
GL30c	OSB	B-LVLQ	Type1	1	8	NA	0.8	45	Vrms	0_13_28	0.02	0.495	0.03	0.036	0.012	8.13	0.48	24.49	4.03	19.30	0.32	98.16	20.72	20.71	−0.03%
GL30c	GL28c	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_0_29	0.033	0.495	0.063	0.036	0.036	8.50	0.23	18.21	2.54	8.97	0.72	109.57	17.50	17.55	0.25%
GL30c	S-LVLS	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_1_30	0.033	0.54	0.063	0.036	0.027	8.46	0.22	18.36	2.57	9.35	0.73	111.85	18.62	18.26	−1.90%
GL30c	B-LVLS	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_2_31	0.033	0.45	0.063	0.036	0.04	8.18	0.25	19.31	2.83	8.96	0.62	112.29	21.82	21.83	0.07%
GL30c	GL28c	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_3_32	0.02	0.54	0.06	0.036	0.036	8.20	0.25	19.10	2.76	9.51	0.62	107.58	26.63	26.34	−1.07%
GL30c	S-LVLS	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_4_33	0.02	0.585	0.06	0.036	0.027	8.14	0.24	19.37	2.81	9.91	0.63	112.36	28.28	27.45	−2.94%
GL30c	B-LVLS	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_5_34	0.02	0.495	0.06	0.036	0.04	8.02	0.26	19.61	2.90	9.37	0.57	110.60	31.40	31.41	0.02%
S-LVLS	S-LVLS	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_6_35	0.051	0.4	0.063	0.027	0.051	8.02	0.24	19.90	2.97	8.43	0.60	121.02	22.69	21.82	−3.83%
B-LVLS	B-LVLS	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_7_36	0.033	0.44	0.069	0.04	0.04	8.08	0.22	19.51	2.83	8.30	0.66	119.82	28.23	27.23	−3.54%
S-LVLS	S-LVLS	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_8_37	0.04	0.4	0.06	0.027	0.063	8.10	0.25	18.81	2.71	8.55	0.61	121.98	35.30	37.48	6.16%
B-LVLS	B-LVLS	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_9_38	0.02	0.52	0.06	0.04	0.04	8.32	0.23	18.00	2.47	8.85	0.69	117.26	38.67	38.40	−0.71%
GL30c	HB	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_10_39	0.039	0.45	0.063	0.036	0.007	8.15	0.24	19.78	2.91	8.94	0.64	125.06	23.60	23.60	0.00%
GL30c	HB	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_11_40	0.02	0.54	0.06	0.036	0.007	8.34	0.23	18.52	2.60	9.27	0.68	122.32	33.23	33.23	0.00%
GL30c	OSB	S-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_12_41	0.033	0.495	0.063	0.036	0.012	8.30	0.23	19.08	2.74	9.04	0.68	121.63	19.88	19.87	−0.04%
GL30c	OSB	B-LVLQ	Type2	None	8	NA	0.8	45	Vrms	0_13_42	0.02	0.54	0.06	0.036	0.012	8.19	0.24	19.16	2.75	9.47	0.64	119.47	29.16	29.14	−0.05%
GL30c	GL28c	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_0_43	0.033	0.585	0.033	0.036	0.036	8.55	0.30	17.43	2.36	11.99	0.57	96.65	14.18	13.93	−1.79%
GL30c	S-LVLS	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_1_44	0.033	0.585	0.033	0.036	0.027	8.00	0.32	19.93	2.95	13.17	0.46	99.00	15.55	15.54	−0.06%
GL30c	B-LVLS	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_2_45	0.033	0.54	0.033	0.036	0.04	8.31	0.31	18.08	2.54	11.87	0.51	99.97	19.42	19.42	−0.01%
GL30c	GL28c	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_3_46	0.02	0.63	0.02	0.036	0.048	8.08	0.45	19.42	2.84	17.76	0.33	91.70	17.15	16.54	−3.56%
GL30c	S-LVLS	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_4_47	0.02	0.63	0.02	0.048	0.045	8.00	0.43	19.69	2.88	17.99	0.34	95.11	19.72	18.10	−8.22%
GL30c	B-LVLS	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_5_48	0.02	0.63	0.02	0.036	0.04	8.05	0.44	19.23	2.81	17.57	0.33	93.33	23.00	22.24	−3.30%
S-LVLS	S-LVLS	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_6_49	0.051	0.4	0.069	0.027	0.063	8.02	0.21	18.67	2.67	7.37	0.68	127.88	24.26	23.38	−3.65%
B-LVLS	B-LVLS	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_7_50	0.033	0.52	0.039	0.04	0.04	8.18	0.29	18.43	2.57	10.94	0.54	107.43	26.71	25.66	−3.94%
S-LVLS	S-LVLS	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_8_51	0.05	0.4	0.05	0.027	0.075	8.01	0.24	18.23	2.57	8.23	0.61	125.28	36.09	38.64	7.06%
B-LVLS	B-LVLS	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_9_52	0.02	0.6	0.03	0.04	0.04	8.35	0.33	17.52	2.36	13.57	0.48	103.56	33.06	30.07	−9.05%
GL30c	HB	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_10_53	0.033	0.54	0.039	0.036	0.007	8.08	0.30	19.55	2.84	11.58	0.50	112.52	21.27	21.27	−0.01%
GL30c	HB	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_11_54	0.02	0.63	0.02	0.036	0.009	8.02	0.43	19.85	2.91	18.03	0.34	105.28	26.27	25.19	−4.11%
GL30c	OSB	S-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_12_55	0.033	0.585	0.063	0.036	0.012	9.02	0.19	15.19	1.85	7.83	1.01	126.11	20.77	16.85	−18.89%
GL30c	OSB	B-LVLQ	Type2	1	8	NA	0.8	45	Vrms	0_13_56	0.02	0.63	0.02	0.036	0.022	8.07	0.41	19.19	2.77	17.41	0.36	107.97	21.65	20.70	−4.39%

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