Estimates on degenerate jump-diffusion processes and regularity of the related valuation equation

Abstract   Many risk-neutral pricing problems proposed in the finance literature require to be dealt with by solving the corresponding Partial Integro-Differential Equation. Unfortunately, neither the standard Sobolev spaces theory, or the present literature on viscosity solution theory is able to deal with some problems of interest in finance. A recent result presented by Costantini, Papi and D'Ippoliti accepted for pubblication on Finance and Stochastics [17], shows that, under general conditions on the coefficients of the stochastic integro-differential equation, whenever a Lyapunov-type condition is satisfied, the stochastic process does not reach the boundary of the domain where is defined. Furthermore, in the same work it has been proved that there exists a unique viscosity solution to the pricing problem when we deal with the corresponding pricing problem for European-type derivatives. The viscosity solution theory ensures just the continuity of the solution, when data are continuous, but does not guarantees that such a solution has some additional regularity.

The aim of this work is to improve, for the pure differential case, the results existing in literature dealing with the regularity of both the solutions X of the underlying stochastic differential equations, and the solutions of the corresponding PDE. In particular we will provide some estimates related to dependence with respect to the initial data for the process X. Furthermore, dealing with the pricing problem, we improve our understanding on the assumptions that ensure the viscosity solution to have additional regularity properties beside the mere continuity.

A Lipschitz-type dependence result with respect to initial data, until a stopping time τ, is shown whenever the coefficients are locally Lipschitz continuous, and a Lyapunov-type condition is satisfied. Such a result can be improved if a suitable weight function is put in place.

A standard result in PDE theory ensures that, if the assumptions we assume in our work are satisfied, then in each compact subset where the diffusion matrix is positive defined, there exists a unique classical solution to the localized problem if initial data are continuous (see e.g. [35] or [9]). We make use of such a result in order to prove that this classical solution coincides, in the same subset, with the unique viscosity solution found in [17].

We give an application of such results, applying our evidences to the stochastic volatility model proposed by Ekström and Tysk in [29]. In such a case all the hypotheses we are currently assuming are satisfied, and the expression of the Lyapunov function can be explicitely provided for different final payoff. As a consequence, we are able to get the results of the existence and uniqueness of a classical solution to the pricing problem presented in

[29] in an independent way. Furthermore it is possible to consider weakened assumptions on the final payoff. On the other hand we try to consider a generalization of the model, allowing the process exhibits sudden jumps provided that the jump measure satisfies some suitable properties. In such a case, the expression for the Lyapunov function is provided as well, hence we are able to state that the considered valuation problem admits one and only one viscosity solution.