CHAPTER 7

ABSTRACT

Gravity is the fundamental interaction that rules the Universe at large scales but also defines the space-time microstructure. Its very successful current description, Einstein's theory of General Relativity, brings with it many fundamental theoretical and observational problems at all scales: the problem of quantization of matter fields in presence of Gravity (possibly related to another issue, the cosmological constant problem) and of Gravity itself (Quantum Gravity), the prediction of singularities and event horizons in classical solutions, the black-hole information puzzle, the apparent need to include unknown forms of matter and energy to reconcile predictions with observations (dark matter, dark energy), and so on. All this suggests that General Relativity may not be the final theory of Gravity and that we do not yet understand this interaction correctly. Our goal will be to improve our understanding of Gravity from the Planck scale to cosmological scales through the search for answers to those problems using observational, computational and theoretical methods. This is a very timely project: our technology can now provide the required observational tools, and theoretical research on Gravity is flourishing with new ideas and approaches.

KEYWORDS

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CHAPTER 7

GRAVITY

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1. INTRODUCTION

Gravity is one of the four fundamental interactions of Nature, but its current description (Einstein's General Relativity) is completely different from that of the other three (Quantum Gauge Field Theories). General Relativity provides the space-time framework for the other interactions but we do not yet know how to quantize the latter on curved spaces (the cosmological constant and vacuum energy problems) nor do we know yet how Gravity interacts quantum mechanically with them. On top of this, conventional General Relativity predicts the existence of singularities (mainly within black holes and at the origin and perhaps at the end of the Universe); a prediction that raises many important questions that remain unanswered.

Thus, even though many (but not all) of the predictions of General Relativity have been successfully checked through experiments and observations [Will, 2014], the general consensus is that General Relativity may not be the final description of Gravity. There is a growing body of theoretical and observational evidence showing that we do not yet understand the true nature of Gravity at all scales. It is necessary to keep testing General Relativity and searching for viable alternatives and solutions to the above problems.

This is an exceptional moment in the history of Physics in which our technology can provide us with the tools that we need to make the observations and experiments that will allow us to deepen our knowledge of Gravity and test the current paradigm (General Relativity) and the alternatives. The direct detection of gravitational waves (LIGO, Virgo, Pulsar Timing Array, LISA, Einstein Telescope) and the direct observation of black holes (Event Horizon Telescope) open the door to the study of Gravity in astronomical settings we have never observed before, with very strong gravitational fields or possible cumulative effects beyond Einstein's theory. On the other hand, the advance in quantum control technologies will allow us to test the Equivalence Principle with an unprecedented precision. At intermediate and cosmological scales, the data provided by gravitational wave astronomy will be supplemented by instruments such as the Square Kilometer Array or the Cherenkov Telescope Array and the very large galaxy surveys EUCLID, the Wide Field Infrared Survey Telescope and the Large Synoptic Survey Telescope.

At the same time, there are many new theoretical ideas to study and test, like the surprising connections between Gravity and seemingly unrelated areas of Physics such as thermodynamics, fluid dynamics, Quantum Information theory and Yang-Mills gauge theories.

We will undertake the challenge of understanding Gravity at all scales using all the observational, experimental, numerical and theoretical tools available. In what follows, we concisely review the problems that arise in the study of Gravity at different scales.

The central role played by Gravity in Nature and the broad scope of this enterprise makes it akin to several of the challenges included in Strategic Topic 9. The reader will find many synergies with more than half of them, but we would like to mention specially the challenges 4 "Origin and fate of the Universe", on the theoretical, phenomenological and experimental side, and 8, "New instrumentation and techniques for understanding the Universe, its structure and evolution" on the instrumental side.

1.1. Gravity at the Planck scale

The main problem at this scale is how to combine Gravity and Quantum Mechanics in a consistent way. If Gravity must be quantized, what would the theory of Quantum Gravity be like? A consistent field-theory quantization of General Relativity using the conventional methods has long been known to be at least extremely troublesome and thought to be outright impossible. The quantization of Gravity is a long-standing problem that is fair to call the *Holy Grail of Theoretical Physics*. It has been addressed in many ways. We are going to review below those on which CSIC groups are working. Some of what is known about a hypothetical theory of Quantum Gravity has been learned through the study of quantized fields on classical curved spacetimes and gives rise to many questions and new problems.

On the experimental side, gravitational wave astronomy can test the non-linear and dynamic radiative regime of Gravity, giving us access to the degrees of freedom of the theory, which cannot be studied in any other way. Combining observations in several bands of the spectrum we can also test the very foundations of the current Physics paradigm: Is Lorentz symmetry violated? What is the propagation speed of Gravity? Does the graviton have mass? What is the microscopic structure of spacetime?

Some approaches to a theory of Quantum Gravity

Quantum Field Theory and Gravity A first step towards the reconciliation of Gravity and Quantum Mechanics, simpler than constructing a full-fledged theory of Quantum Gravity, is to study the quantization of fields in a classical curved spacetime [Wald, 1995]. This approach led to the discovery that particle creation occurs in the very early Universe, with very important implications in cosmology, and in the vicinity of black holes (the famous Hawking radiation). The theoretical discovery of Hawking radiation gave rise to the field of black-hole thermodynamics and gave meaning to the Bekenstein–Hawking entropy of black holes, but it also led to the idea of the “evaporation” of black holes with the apparent loss of all its information content (the so-called *black-hole information paradox*) and to the problem of the identification of the statistical (or microscopic) interpretation of the black-hole entropy. The relation between the area of the black hole horizon and its entropy has also inspired the holographic paradigm that we will review below. All these problems and ideas seem to be ultimately related to Quantum Information [Harlow, 2016] with quantum entanglement of states playing the role of the *éminence grise* behind it all.

Another very important problem that arises in this approach is related to the interpretation of the cosmological constant (and dark energy) as the vacuum energy of its matter field content, which, when computed using the Standard Model, gives a number which is off by dozens of orders of magnitude [Weinberg, 1989; Martin, 2012] with respect to its recently measured experimental value, which is very small, but drives the accelerated expansion of the Universe. Either the calculation is affected by some yet unknown mechanism that makes this number small or we do not understand how the vacuum energy couples to Gravity: it is assumed that it obeys the Equivalence Principle but nobody has ever weighted it and checked this assumption (see Section 7.3).

A full-fledged theory of Quantum Gravity is expected to solve all these problems, but, still, important non-perturbative aspects of Quantum Field Theory induced by Gravity remain to be explored and related to current and near-future observations in cosmology and gravitational waves.

Analog Gravity and analog Hawking Radiation. The Hawking radiation of astrophysical mass black holes is too weak to be detectable. However, in 1981 Unruh showed that this prediction of Quantum Field Theory in curved spacetime could be tested in the lab, opening a new field of indirect research on Quantum Gravity.

Steinhauer and his group provided the first experimental evidence of (quantum) analog Hawking radiation from acoustic black holes in Bose–Einstein condensates, toy models for curved spacetime, using a method (density correlator) first proposed by collaboration including CSIC researchers. They could also study the time dependence of the evaporation process. A theoretical understanding of such results requires the study of the backreaction of the Hawking flux on the condensate itself. In the context of Gravity, such an analysis is relevant to address the information loss paradox.

The propagation of surface waves in water provides another interesting toy model. Unruh's and Rousseaux's groups have provided interesting experimental results on stimulated Hawking radiation from white-hole-like flows. The first scattering experiment of surface waves on an analogue black hole flow made by Rousseaux's group opens up the possibility of observing the analogue of the Hawking effect from a black hole horizon in such a system, complementing Steinhauer's experiments on Bose–Einstein condensates.

Modified Gravity and extensions of General Relativity There are several important motivations for searching and studying modifications of General Relativity:

1. If General Relativity cannot be quantized by conventional methods, perhaps a modification of this theory can. It might also happen that the modified Gravity theory solves some of the problems of General Relativity mentioned at the beginning.
2. One should consider the possibility that the anomalous (from the General Relativity point of view) motion of matter in galaxies and of the galaxies in the Universe which are usually attributed to the presence of unknown (*dark*) forms of matter an energy are, instead, a sign that

General Relativity has to be modified. Observe that the dark-matter and dark-energy issues arise at intermediate and cosmological scales.

3. Some modified Gravity theories (especially those with terms of higher order in curvature) could also be regarded as the effective theory of a yet unknown full-fledged theory of Quantum Gravity beyond General Relativity. From a phenomenological perspective it might describe both the phenomena typically associated to the quantum gravitational regime and those proper of astrophysical and cosmological scenarios.
4. A modification of General Relativity (such as *unimodular Gravity*) in which vacuum energy does not couple directly to Gravity may help to solve the cosmological constant problem (page 137).
5. Modifications of Gravity have also been suggested to provide mechanisms for the large energy density fluctuations which are needed to produce primordial black holes (see page 143).

Although General Relativity is manifestly difficult to modify without spoiling its internal consistency and beauty, this is an extremely active field of research. In order to make progress in it, it is essential to determine the number and type of degrees of freedom that describe gravitational interactions within the foreseeable experimentally/observationally accessible regimes. The construction of specific models beyond General Relativity, the analysis of their theoretical consistency, and their confrontation with observations will provide useful information in this direction.

Loop Quantum Gravity If General Relativity cannot be quantized consistently using the conventional methods of perturbative Quantum Field Theory, perhaps other approaches can give a deeper insight. Loop Quantum Gravity is a non-perturbative and background independent program for the quantization of General Relativity based on the Hamiltonian formalism. Its application to cosmological spacetimes has opened a new area of research known as Loop Quantum Cosmology. A relevant part of its foundations have been established by CSIC researchers, who developed the real SU(2) connection formalism used in the formulation of the theory, quantized for the first time a gravitational system with an infinite number of degrees of freedom in the framework of Loop Quantum Cosmology, and introduced the hybrid quantization scheme used to study cosmological perturbations.

Non-local Quantum Gravity Another alternative proposal of Quantum Gravity theory is based on a perturbative field theory of Gravity whose dynamics is governed by weakly non-local operators, kinetic terms with infinitely many derivatives. This promising theory, proposed in the 1980s but developed systematically and rigorously in the 2010s, has non-singular solutions, is unitary and renormalizable and, on a cosmological background, provides a robust justification of successful inflationary models such as Starobinsky's. The phenomenology of non-local Quantum Gravity is under very active study.

Multi-scale Gravity In the great majority of theories of Quantum Gravity, the geometry of spacetime changes with the probed scale. This phenomenon, called dimensional flow, and its observational consequences have been studied both as a universal feature and within an independent proposal, called multi-fractional spacetimes, emphasizing its wide observational impact. From cosmic to particle-physics scales, constraints on the fundamental scales of the geometry have been extracted.

Superstring theory Superstring theory is an internally consistent theory of Quantum Gravity which is not based on standard Quantum Field Theory, where quantum fields get excited and decay by absorption or emission of point-particles. The quanta of Superstring Theory are vibrating one-dimensional objects ("strings"), whose interactions (the joining and splitting of the strings) do not cause the ultraviolet divergences that ruin the quantization of General Relativity (a standard field theory). This single interaction describes in a unified way the four fundamental interactions; all the observed particles being seen as different states of a single object (the string).

Superstring Theory has provided the first microscopic interpretations (including precise calculations) of the black-hole entropy proposed by Bekenstein and Hawking and has also inspired the paradigm of *holography* which can be described as the art of deriving local quantum physics (the framework of both Gravity and Quantum Field Theory) from quantum systems living on boundaries (see page 143). Further- more, its internal consistency has inspired the *swampland program* (see below) on which CSIC researchers are at the forefront of current research.

At low energies, when the small strings can be approximately seen as point-particles, Superstring Theory is effectively described by a modified Gravity theory of the kind described above. In most cases, the leading term is a theory of *supergravity*.

Supergravity Supergravity theories are theories of General Relativity in which Gravity is coupled in a very subtle way to bosonic and fermionic matter fields so that the theory has *supersymmetry*. Supersymmetry is the largest spacetime symmetry consistent with the foundations of Quantum Field Theory (and, therefore, of the Standard Model) and it can incorporate in its structure all fields and symmetries. Therefore, it is not surprising that some of them arise as low-energy effective field theories of Superstring Theories and provide the main tools to study (apart from theories beyond the Standard Model) black holes from the superstring point of view.

Some of these theories are, however, very interesting by themselves. For instance, it is possible that the theory known as $N = 8$ supergravity can be quantized consistently in the conventional fashion. The absence of ultraviolet divergences has been proven to five loops in [Bern et al., 2018].

The swampland program A quantum description of Gravity is also essential to have a proper understanding of fundamental physics because the implications of a theory of Quantum Gravity go well beyond gravitational systems. There is a wide consensus that the features of Quantum Gravity also affect other domains in High Energy Physics, and that this could even lead to predictions to be tested in future experiments. The *swampland program* aims to characterize the constraints that a consistent theory of Quantum Gravity such as Superstring Theory places on Quantum Field Theories like those describing the Standard Model of Particle Physics and Cosmology. "The swampland" is the set of Quantum Field Theories which are perfectly consistent by themselves but inconsistent when seen as effective field theories of the type stemming from a theory of Quantum Gravity such as Superstring Theory. In the last decade, there has been a sustained effort in trying to find the structure of the swampland, mapping its boundaries. During this time, the program has quickly gained command of the fundamental understanding of open questions in particle physics and cosmology, ranging from the hierarchy of fundamental scales in Nature, to the origin and final fate of the universe.

A clear example of development within the swampland program is the so-called *Weak Gravity Conjecture*, which states that Gravity must be the weakest fundamental force in any consistent theory of Quantum Gravity. Albeit motivated by the physics of black holes, the Weak Gravity Conjecture has direct implications for many early-universe cosmological models, which are formulated in terms of effective field theories. The same approach can be applied to enhance our understanding of the properties of the Standard Model of Particle Physics. For instance, detailed constraints on the number and character of low-energy species of neutrinos have been derived in this approach.

Holography Gravitational Holography has been a dominating paradigm in the quest for a theory of Quantum Gravity for more than two decades now. It was inspired by the celebrated Bekenstein–Hawking formula for black holes, Entropy=Area/4, which establishes a deep connection between geometry and information theory. The basic idea of holography is to take this formula literally as a measure of the fundamental degrees of freedom in Quantum Gravity. It follows that they must be associated to boundaries rather than volumes of spacetime and that one should be able to deduce local quantum physics from the boundary description.

The most popular and best studied realization of this idea, called *AdS/CFT*, is a string-theoretical duality between a gravitational theory on negatively curved space- times and a non-gravitational Conformal Field Theory living on the boundary at infinity. AdS/CFT is a well-defined mathematical laboratory for holography. In recent years, important progress has been made by importing ideas from Quantum Information Theory. In broad terms, the holographic emergence of the interior space is realized via specific entanglement patterns in the boundary theory. Concepts such as quantum error correction codes and computational complexity are becoming routine notions for the Gravity theorists working in this realm, but the quantum information community is also sensitive to these results as well.

These fascinating ideas represent main research lines in the top Quantum Gravity groups around the world, but one should not forget that, ultimately, we do not live in a spacetime with negative cosmological constant. The extension of these ideas to the flat-space and De Sitter settings are long-term challenges in the field.

1.2. Gravity at astronomical scales

Black holes are quickly becoming central objects of study in astrophysics and cosmology but, are the objects that we observe those predicted by General Relativity? Do they have the same properties?

This fundamental question may be answered by multi-band gravitational wave astronomy. For instance, the gravitational wave spectrum in the ring down phase of the collision of neutron stars or the coalescence of massive black holes can be compared directly with the predictions of General Relativity. The coalescence of binary systems with intermediate or extreme mass ratios will be detectable by LISA in the low-frequency band and in the Hz band in future detectors (the Einstein Telescope project). The gravitational waves emitted and detected will contain a map of the geometry of supermassive black holes and it will be possible to compare the multipole structure that will be extracted from it with, again, the predictions of General Relativity.

The Event Horizon Telescope will also be able to study General Relativity in supermassive black holes through repeated observations of increasing precision. In the near future we will have better ground-based antennae testing General Relativity with increased precision in a neighborhood of the supermassive black hole at the center of the Milky Way, whose mass and position we have measured very precisely. In the long term, antennae in orbit around the Earth (the Millimetron project, for instance) in combination with the Event Horizon Telescope will give us angular resolutions below the micro-arc-second level.

In summary, all these instruments will allow us to test the current General Relativity paradigm with unprecedented precision and in unexplored situations, both from the phenomenological and fundamental points of view. The construction and study of alternative, consistent theories is an absolute necessity in order to make real progress.

We should recall now the question of the existence and nature of dark matter and dark energy discussed in the previous section as a motivation for modified Gravity theories (page 138). It has also been suggested that the existence of primordial black holes could account for some of the dark matter. In any case, the understanding of dark matter requires the understanding of Gravity at these scales.

Black hole alternatives, theory and phenomenology Ultracompact configurations with no horizons, which the current observations cannot discern from General Relativity's black holes, have been proposed as an interesting alternative to the latter. This kind of proposals stimulate progress in the field since they demand improvements in the observations and tests of General Relativity to distinguish between different candidates. For these alternative models to be taken seriously, the formation and stabilization mechanisms of these ultracompact objects have to be studied, all the while paying special attention to the phenomenological characteristics that could make these models verifiable or falsifiable by near-future observations.

Primordial black holes and gravitational waves The old idea that the dark matter of the Universe could be composed by black holes formed in the early Universe by a non-astrophysical mechanism has regained momentum in the last five years, becoming a hot topic in the context of Gravity and cosmology. There are several interesting issues, directly related to our understanding of Gravity, that arise in this context. The following is an incomplete list:

- Hawking and others proposed in the 1970's that black holes may originate from the collapse of large overdense regions of the Universe such as those caused by density fluctuations in the early Universe above a certain thresh- old. Black holes produced by this mechanism would be called *primordial*. Roughly speaking, the correlation length and the size above the threshold of these fluctuations would determine their mass and abundance, but a detailed understanding of the collapse process and the physical and statistical properties of the ensuing black holes is missing. A proper understanding of the formation of primordial black holes would clarify the relevance of these objects for cosmology and it would undoubtedly augment our understanding of Gravity. This challenge requires different expertise, from theoretical cosmology to numerical simulations of Gravity in the deeply non-linear regime.
- A possible source of the required density fluctuations could be primordial inflation. Although the spectrum of inflationary fluctuations inferred from the Cosmic Microwave Background and Large Scale Structure data has a much lower amplitude than required for abundant primordial black hole formation, these observations only allow us to study a small fraction of the inflation needed to solve the horizon and flatness problems. Therefore, in the period of inflation that we cannot observe by those means, much larger fluctuations may have been produced. Several mechanisms have been proposed for the generation of such large perturbations, some of which amount to modifications of Gravity in the early Universe. Thus, primordial black holes could become a unique probe of the inner workings of Gravity at high energies.
- A signature of the formation of primordial black holes from inflation is the generation of an associated stochastic background of gravitational waves. The detection of such a background with LISA and its proper identification as a relic from the formation of primordial black holes after inflation, on top of proving their existence, would have momentous implications for our knowledge of Gravity and, possibly, of dark matter.
- Black hole evaporation through Hawking radiation (page 137) is negligible for astrophysical-mass black holes, but relevant for those with small masses. Primordial black holes lighter than $\sim 10^{15}$ g would have

mostly evaporated already, but the detection of Hawking radiation from heavier primordial black holes evaporating today would open a new window to study Gravity and (perhaps even the information loss paradox) in a currently inaccessible regime.

- Since 2015, LIGO and Virgo have identified gravitational waves emitted in the merging of several binary black holes. There is a chance that some of those that will be detected will be of primordial origin, with huge implications for our exploration of Gravity. The statistics of such mergers could provide us with yet another window for learning about Gravity in the very early Universe.

Visualizing black holes: the Event Horizon Telescope The Event Horizon Telescope is the virtual Earth-size telescope that captured the first image of the "shadow" of a supermassive black hole. The expansion of the Event Horizon Telescope in the next decade (the so-called *next-generation Event Horizon Telescope*) will be capable of making the first real-time movies of supermassive black holes and their emanating jets. These movies will resolve the complex structure and dynamics at the event horizon, bringing into focus not just the persistent strong-field Gravity features predicted by General Relativity, but also details of active accretion and relativistic jet launching that drive galaxy evolution and may even affect large scale structures in the Universe. The next-generation Event Horizon Telescope will turn the extreme environment of their event horizons into laboratories where astronomers, physicists and mathematicians can actively study the black hole boundary in real-time, and with a sensitivity and angular resolution that will allow them to attack longstanding fundamental questions of how Gravity works from completely new directions.

1.3. Gravity at cosmic scales

Two of the main problems in our understanding of the Universe bear a direct relation with Gravity: the existence of dark matter and dark energy. Could a Modified Gravity theory (see page 138) account for them? Is dark energy just the cosmological constant? Are they the vacuum energy? (See page 138). The very large galaxy surveys and the study of the 21 cm absorption lines will in a near future provide us with information about these questions that can be used to test alternative theories. Another, more recent, problem that might be solved by modifications of General Relativity is that of the tension in the value of the Hubble constant H_0, described below (see page 146).

A big unknown in our current theory of the origin of structure in the Universe is that of the value of the ratio between scalar and tensor modes in the primordial fluctuations that gave rise to those structures. Tensor modes are associated with the presence of a gravitational-wave background and its value could validate or falsify the (typically inflationary) models proposed to explain the origin of the inhomogeneities. The imprints of these waves on the Cosmic Microwave Background are being actively searched for by the experiments QUIJOTE and BICEP2, but they may be directly detected by LISA (in which CSIC participates) if that gravitational wave background is large enough (see below). LISA will also be able to discriminate between General Relativity and other models.

Search and characterization of cosmological backgrounds in LISA Since the official approval of the LISA mission in 2017, a big effort has been put in predicting and characterizing stochastic gravitational wave backgrounds from the primordial universe and check its detectability. In particular, the shape of the energy density spectrum of gravitational wave backgrounds from 1.- cosmic inflation, 2.- the non- disturbing processes of particle production (such as post-inflationary overheating), 3.- possible first-order phase transitions in physics beyond the Standard Model, 4.- the formation and evolution of cosmological defects, and in particular of cosmic strings, 5.- several models stemming from Quantum Gravity theories or scenarios, and 6.-the formation of primordial black holes, is being studied by numerical and analytical techniques, as well as the non-Gaussianity and chirality of these backgrounds.

Beyond Einstein's theory in LISA The production and the propagation of astrophysical and primordial gravitational waves are affected by the underlying theory of Gravity. Through the observation of standard sirens (sources of gravitational waves with an optical counterpart) and their luminosity distance, LISA will allow us to detect these effects in models where the cosmic expansion and the dispersion relations do not agree with the predictions of General Relativity. Many models of dark energy or High Energy Physics are currently under scrutiny, stimulating questions of wider scope such as the types of parametrization of the luminosity distance one can use to interpret data correctly.

The Hubble constant tension and modifications of Gravity The measurements of the Hubble parameter H_0 (describing the expansion of the Universe today), that are inferred from the Cosmic Microwave Background and from the study of supernovae and other observations at "small" redshift are very different (in statistical terms, they are separated by approximately "4.5 sigmas"). This is very uncommon in present-day cosmology and may point to the need of a profound revision of our description of the Universe at large spacetime scales. No satisfactory theoretical explanation has emerged so far, but the possibility exists that the problem might be fixed by an adequate modification of General Relativity at the time of the CMB formation. Although it is perfectly conceivable that so far overlooked systematics in the observations could also resolve the current situation, it is fair to assume that in the next few years there will be a surge of activity around this topic, with potentially revolutionary consequences. Understanding if and how a modification of Gravity could address the issue is an ideal playground for testing Gravity at large scales. This line of research could be nicely woven with observational efforts (LISA and Euclid, to give just two examples).

1.4. The mathematics of Gravity

The search for alternative theories of Gravity has stimulated the exploration of new mathematical tools and theories of increasing complexity. The twistor theory, introduced in order to solve Einstein equations has had a great impact in mathematics relating to fundamental problems in complex algebraic geometry and differential geometry, including the study of moduli spaces of bundles, Yang-Mills instantons, monopoles, hyper-Kähler manifolds, etc. The Superstring/Supergravity theory approach to Gravity has also motivated the study of mirror symmetry, Calabi-Yau manifolds, Langlands duality, and created a new mathematical field, sometimes referred as quantum geometry. Conformal Field Theory holographic theorems, complex systems, fractal geometry constrained Hamiltonian systems, gauge theories, non-commutative algebras and non-commutative geometry, generalized functions and the theory of partial differential equations are just some among the many fields touched upon during our quest.

General Relativity, linked from the very beginning with Riemannian geometry, has also motivated a tremendous body of mathematical work, including the recent fundamental work on Kähler-Einstein manifolds and the relation to stability criteria in algebraic geometry. However, General Relativity itself constitutes a challenge at the numerical level when new computing techniques are required to determine, for instance, the wave form of a gravitational-wave source at the precision achieved by near-future interferometers.

2. IMPACT ON BASIC SCIENCE AND POTENTIAL APPLICATIONS

Out of the four fundamental interactions it can be argued that Gravity is the most fundamental one insofar as it governs space and time, the playground for the rest of Physics. Any improvement in our understanding of gravity and any change of paradigm in its description will have a huge impact not just in Physics (Cosmology, Particle Physics etc.) but in all fundamental disciplines (including Philosophy). As we have explained in the Introduction (page 137), there seems to be a deep connection between Gravity and Quantum Information theory which is pushing research in both fields. The consequences of new discoveries concerning this connection simply cannot be foreseen.

On the other hand, it is clear that the most immediate applications of any improvement in our understanding of Gravity will take place in the study of environments in which extreme gravity plays an important role: the early Universe, active galactic nuclei, black- hole mergers, and so on. However, we have mentioned a number of problems in which Gravity plays or may play a crucial role even in non-extreme conditions: the clarification of the nature of dark matter and dark energy (the cosmological constant and the vacuum energy), the nature of event horizons, the luminosity distance of gravitational waves, etc. The technological impact will probably remain mostly indirect for a long time.[3] However, it has to be taken into account that all the experiments in this field (just as in Particle Physics), spatially LIGO-Virgo and LISA, demand technology (software, hardware, mathematical tools) that, often, does not yet exist and has to be developed *ex professo*, acting as powerful drivers of innovation. These cutting-edge technologies will undoubtedly have many spin-offs in its due time.

3. KEY CHALLENGES

Within the overall challenge of achieving a better understanding of Gravity at all scales there are several intermediate challenges which are keys to reach the main goal. We have formulated them as problems and questions in the introduction: the cosmological constant problem, the dark matter and dark energy problems, the black-hole information paradox, the microscopic interpretation of the black-hole entropy, the real existence of event horizons and/or singularities in nature, the existence of primordial black holes, the Hubble parameter tension, the measurement of the tensor-to-scalar ratio, etc. Far from being mere intermediate steps in our progress to the main goal, many of these challenges are extremely hard questions that have occupied for a long time the scientific community working on Gravity. They are, indeed, our long-term key challenges as well.

Nevertheless, it is convenient to formulate challenges that the research groups working on Gravity will face in shorter terms. Among many, we have chosen the following:

- Develop and put to the test a specific model of black hole alternative based on semi- classical and emergent Gravity ideas, as well as on any of the abovementioned proposals for Quantum Gravity or theories beyond General Relativity. Is semiclassical physics able to produce and stabilize ultracompact objects, or does one need additional contributions? Is the singularity resolved? What wave-form would produce the inspiralling and merger of two such objects, and could it be tested with gravitational-wave interferometers?
- To prove or disprove the viability of an emergent theory of Gravity based on condensed- matter systems. It has been shown that many laboratory systems exhibit curved spacetime properties. Is it possible to design a system in which those spacetimes satisfy the Einstein equations?
- Scrutinize the effects that new gravitational physics beyond Einstein’s theory could have on self-gravitating systems with well understood composition as a way to minimize the uncertainties that affect compact objects.
- Understanding in detail if and how primordial black holes can form from the col- lapse of large overdense regions in the early Universe and if they can do so abundantly enough to account for the dark matter. Developing the tools for calculating precisely their abundance. Determining the electromagnetic and gravitational tests that may reveal their existence unequivocally or rule them out.
- Clarifying whether the current so-called Hubble tension has anything to do with the properties of Gravity at different distance, time or curvature scales. Determining if gravitational wave observations may help to unravel this tension.
- To determine if metastable De Sitter vacua belong to the swampland or not. In other words, to determine if they are compatible with Superstring Theory’s type of Quantum Gravity.

- In the last ten years it has become clear that massive quantum entanglement is key to the emergence of Einstein Gravity in AdS/CFT models. A crucial question is whether other ingredients are needed as well, such as concepts derived from the theory of computational complexity, a notion that so far has remained elusive, despite considerable effort in its elucidation. In other words, is entanglement enough?
- AdS/CFT is the only mathematically well-defined working model, but the real world is definitely not AdS. Rather, at large scale, it is better approximated by a De Sitter spacetime with positive dark energy. The generalization of holography to such cosmological situations is the most important open problem.
- To develop new techniques in Numerical Relativity to access the full non-linear and dynamical regime of Gravity, in General Relativity and in other alternative theories. The goal would be to provide tools to tackle a number of problems: Modeling of sources of gravitational waves in the different bands of the spectrum; dynamics of black holes in different theories and scenarios, applications to physical problems where the AdS/CFT correspondence can play a major role etc.
- To produce the first real-time movies of black holes with an angular resolution capable of discerning any deviation from the Kerr metric near the horizon. In particular, the next-generation Event Horizon Telescope main science objectives for the next decade are aimed to answer some of the most fundamental questions related to Gravity and accretion onto supermassive black holes, namely: Are supermassive black holes described by the Kerr metric? Does General Relativity break down near the event horizon? How do supermassive black holes form and evolve? What is the black hole mass function across cosmic time? Are there horizon-less compact objects? What drives black hole accretion? How do black holes form and power relativistic jets?
- Starting from well-developed formalisms of Quantum Gravity such as String Theory, Supergravity, Loop Quantum Gravity, Non-local Quantum Gravity, Multi-scale Gravity, or any other proposal described above, extract falsifiable predictions for cosmology and for gravitational-wave Physics which can be confronted with observations of ongoing or planned missions that explore the behavior of Gravity at astronomical and cosmic scales.