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Beating no-go theorems by engineering defects in quantum spin models

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E-mail: ujjwal@hri.res.in**Keywords:** disordered quantum systems, quantum dense coding, bell inequality, No-go theorems in quantum mechanics, multiparty communication networks, monogamy in quantum mechanics

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**Abstract**

Diverse no-go theorems exist, ranging from no-cloning to monogamies of quantum correlations and Bell inequality violations, which restrict the processing of information in the quantum world. In a multipartite scenario, monogamy of Bell inequality violation and the exclusion principle of dense coding are such theorems which impede the ability of the system to have quantum advantage between all its parts. In ordered spin systems, the twin restrictions of translation invariance and monogamy of quantum correlations, in general, enforce the bipartite states to be neither Bell inequality violating nor dense codeable. We show that it is possible to conquer these constraints imposed by quantum mechanics in ordered systems by introducing quenched impurities in the system while still retaining translation invariance at the physically relevant level of disorder-averaged observables.

1. Introduction

Quantum mechanics places strict restrictions in the form of ‘no-go theorems’, like no-cloning [1] and monogamy of quantum correlations [2, 3], on information processing tasks (see also [4]). In this paper, we concentrate on two restrictions imposed by the quantum mechanical principles—monogamy of Bell inequality violation [5] and the exclusion principle of classical information transmission [6]. In a multipartite scenario, with a boss and several subordinates, the laws state that if the shared quantum state between the boss and a single subordinate exhibits quantumness, either by violating Bell inequality or by being dense codeable, then the other channels between the boss and the subordinates are prohibited from possessing the same quantum advantage, and hence, enforce limitations upon the quantum information processing tasks possible in that scenario.

It is easy to see, therefore, that the two-qubit states obtained from translationally invariant systems, which include the ground states of one-dimensional translation-invariant quantum spin models (without disorder), neither violate Bell inequality, nor have quantum advantage in dense coding [7]. The two-pronged restriction imposed by monogamy and translation invariance causes *all* two-qubit states of such multipartite systems to be devoid of the quantum advantages. The same arguments are true for an arbitrary isotropic higher-dimensional lattice. We now ask the following question: Is it possible to regain the quantum advantages in these two-qubit states in some physical many-body system, while still retaining the translation invariance of the system, at least at the level of observables under study, i.e. at the level of the amount of Bell-inequality violation and the capacity of dense coding? We answer the question in the affirmative by using quenched disordered spin systems.

Defects, in general, reduce the physical properties like magnetization, conductivity, classical correlation, and quantum correlation of the system [8]. Thereby the system may lose its ability to perform in a better way than its classical counterpart. It has been reported that disorder reduces the fidelity of quantum state transmission as well as of quantum gate implementation [9]. However, thermal fluctuation or impurities in the system may lead to a counterintuitive enhancement of physical properties, known as ‘disorder-induced order’ or ‘order-from-disorder’ [10–12]. We show that defects can give rise to more radical advantages. It may be noted that defects can appear naturally in physical systems and can also be artificially engineered [13]. Interesting phenomena obtained in disordered systems include those in [14].

We consider quenched disordered one-dimensional quantum spin-1/2 systems. We show that even though translation symmetry is present in these systems after quenched averaging, such disordered models can overcome the hurdle of Bell monogamy and the exclusion principle of dense coding. First, we show that in the disordered quantum XY spin glass and in the random field quantum XY model, the quenched averaged quantities for the amount of Bell inequality violation as well as the capacity of dense coding, of the nearest-neighbor zero-temperature state, can attain nonclassical values and thereby overcome the monogamy constraints, despite the fact that the post-quenched averaged quantities are translation-invariant. The analysis is carried out by applying the Jordan-Wigner transformation to the disordered XY models [15–17]. The phenomena observed is potentially generic, in that we have also demonstrated them in quenched disordered quantum Heisenberg spin glasses, which are not analytically tractable, and for which the investigation is performed via the density matrix renormalization group (DMRG) technique [18]. Finally, finite-size scaling analyses are carried out for both the quenched observables in all the models considered.

2. Monogamy of Bell inequality and dense coding capacity

It is known from the celebrated Bell theorem [19] that the violation of the Bell inequality by a two-party state guarantees that the state cannot have a local realist description. Given any two-qubit state, ρ , violation of the Bell-CHSH inequality [20] occurs if and only if [21]

$$M(\rho) > 1, \quad (1)$$

where $M(\rho) = u_1 + u_2$, with u_1 and u_2 being the two largest or the largest and the second-largest eigenvalues of $U = T_\rho^T T_\rho$. Here, $T_\rho^{mn} = \text{Tr}(\sigma_m \otimes \sigma_n \rho)$ are the elements of the corresponding correlation matrix, T_ρ . In the case of multipartite states, *if the quantum state shared by any two subparts of a multipartite system leads to a Bell inequality violation, then it precludes its violation for the states which the subparts share with the other parts of the total system.* This is referred to as monogamy for Bell inequality violation for the multipartite quantum states [5]. We define a quantity $\delta(\rho_{AB}) = \max\{0, M(\rho_{AB}) - 1\}$, which quantifies the amount of Bell inequality violation for the two-qubit states, and investigate its behavior while exploring different physical many-body systems.

On the other hand, the quantum dense coding protocol [22] incorporates a sender-receiver scheme for communicating classical information over a quantum channel. If we consider that our conventional sender, Alice, and receiver, Bob, initially share a state ρ_{AB} , with d_A and d_B being the dimensions of the Hilbert spaces corresponding to Alice's and Bob's parts respectively, then the dense coding capacity turns out to be [23]

$$C(\rho_{AB}) = \log_2 d_A + C^{adv}(\rho_{AB}) \quad (2)$$

bits. The quantity $C^{adv}(\rho_{AB}) = \max\{0, S(\rho_A) - S(\rho_{AB})\}$ is referred to as the 'quantum advantage' of dense coding of the state ρ_{AB} over the classical channel. This is justified by the fact that $\log_2 d_A$ bits of classical information can be transmitted by sending a d_A -dimensional quantum system without using prior shared entanglement. A bipartite quantum state is said to be dense codeable if it has a positive quantum advantage of dense coding. In a multipartite scenario, the 'exclusion principle' for quantum dense coding demands that *if any two subsystems of a multipartite quantum system shares a dense codeable state, then they can't share any such quantum state efficient for dense coding, simultaneously, with other parts of the system* [6].

Let us illustrate the above no-go theorems for a three-party state. When a tripartite state ρ_{ABC} is shared between A, B, and C, the monogamy of Bell inequality violation and the exclusion principle implies that if the reduced state ρ_{AB} violates local realism or has a quantum advantage in dense coding, i.e. if $\delta(\rho_{AB})$ or $C^{adv}(\rho_{AB})$ is positive, then the reduced state at BC will have $\delta(\rho_{BC}) = 0$ or $C^{adv}(\rho_{BC}) = 0$, respectively.

3. The model and the methodology

The Hamiltonian for the one-dimensional disordered quantum XY spin chain in a random transverse field is given by

$$H = \kappa \left[\sum_{i=1}^N \frac{J_i}{4} \left((1 + \gamma) \sigma_i^x \sigma_{i+1}^x + (1 - \gamma) \sigma_i^y \sigma_{i+1}^y \right) + \sum_{i=1}^N \frac{h_i}{2} \sigma_i^z \right], \quad (3)$$

where κJ_i is the coupling strength between the i^{th} and $(i + 1)^{\text{th}}$ site, κh_i represents the field strength at the i^{th} site, and γ is the anisotropy constant. κ is a constant and has the unit of energy, while J_i , h_i , and γ are dimensionless. Here, σ^j , for $j = x, y, z$, correspond to the Pauli spin matrices. For the ordered system, all the J_i and h_i are separately equal, and are denoted by J and h , respectively. Here we have assumed the cyclic boundary condition, so that the $(N + 1)^{\text{th}}$ and the 1^{st} sites are equivalent.

The ordered model is exactly solvable via successive use of the Jordan-Wigner, Fourier, and Bogoliubov transformations [15–17], while the disordered model is not. However, the same procedure can again lead us to the one- and two-site reduced density matrices for the disorder case, which is enough for our study. For completeness, we briefly review the mechanism here. First, we map the Pauli spin operators to the spinless fermions via the Jordan-Wigner transformation, so that equation (3) becomes

$$H = \kappa \left[\sum_{i,j=1}^N c_i^\dagger A_{ij} c_j + \frac{1}{2} \sum_{i,j=1}^N (c_i^\dagger B_{ij} c_{j+1}^\dagger + h.c.) \right], \quad (4)$$

where A and B are symmetric and antisymmetric real $N \times N$ matrices, respectively, and are given by

$$A_{ij} = h_i \delta_{ij} + \frac{J_i}{2} \delta_{i+1,j} + \frac{J_j}{2} \delta_{i,j+1}; \quad B_{ij} = \frac{\gamma}{2} (J_i \delta_{i+1,j} - J_j \delta_{i,j+1}), \quad (5)$$

with $A_{1N} = A_{N1} = J_N$ and $B_{1N} = -B_{N1} = -\frac{\gamma}{2} J_N$ for the cyclic boundary condition. Here, the c_i^\dagger , c_i are spinless fermionic operators obtained via the Jordan-Wigner transformation. Defining Φ_k^T via the eigen-equation

$$(A - B)(A + B)\Phi_k^T = \Lambda_k^2 \Phi_k^T, \quad (6)$$

with eigenvalue Λ_k and obtaining the corresponding Ψ_k from the equation

$$\Psi_k^T = \Lambda_k^{-1} (A + B)\Phi_k^T, \quad (7)$$

we can calculate the correlation matrix G , defined as

$$G_{ij} = -\sum_k \psi_{ki} \phi_{kj} = -(\Psi^T \Phi)_{ij}, \quad (8)$$

where Φ and Ψ are the matrices ϕ_{ki} and ψ_{ki} , with ϕ_{ki} (ψ_{ki}) being the i th element of Φ_k (Ψ_k). Finally, one can show that the magnetizations and two-point correlation functions of the zero-temperature state can be easily obtained from the correlation matrix G . We get $m_i^z = -G_{ii}$ and $m_i^x = m_i^y = 0$. The diagonal correlations are given by

$$T_{i,i+1}^{xx} = G_{i,i+1}; \quad T_{i,i+1}^{yy} = -G_{i+1,i}; \quad T_{i,i+1}^{zz} = G_{i,i} G_{i+1,i+1} - G_{i,i+1} G_{i+1,i}, \quad (9)$$

while all off-diagonal correlations vanish. The one- and the two-site density matrices can now be easily constructed from the one- and two-point correlation functions and consequently, the Bell inequality violation (equation (1)) and the dense coding capacity (equation (2)) can be computed.

4. Quenched averaging

In the present work, for all purposes, the type of disorder that has been used is ‘quenched’. Spin glass states are those which emerge due to the presence of such a type of disorder in the system and the term ‘glass’ comes from the analogy with the chemical glass which is formed by quenching a liquid. The term ‘quenched’ signifies that the time over which the dynamics of the system takes place is much smaller than the time scale over which there is a change in a particular realization of parameters governing the disorder in the system. This leads to the fact that while calculating the quenched averaged value of a physical quantity, we need to perform the averaging of several expectation values of that quantity, each of which is obtained for a fixed configuration over the relevant probability distribution of the configurations of the disorder.

5. Anisotropic XY spin glass

Let us now consider the quantum XY model of N spins interacting via site-dependent nearest-neighbor exchange interactions, J_i , which are identically and independently distributed (i.i.d.) with Gaussian probability distribution, while the field strength, h_i , at each lattice site is kept constant. The corresponding Hamiltonian follows from equation (3) by setting $h_i = h$ for all $i = 1, 2, \dots, N$ and by letting J_i follow the probability distribution

$$P(J_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{J_i - \bar{J}}{\sigma}\right)^2\right], \quad (10)$$

where \bar{J} and σ are, respectively, the mean and the standard deviation of the distribution.

Let ρ_{AB} be a two-party state reduced from the N -party zero-temperature state in the ordered case, which can be obtained analytically via the Jordan-Wigner transformation. Here A and B are disjoint collections of lattice

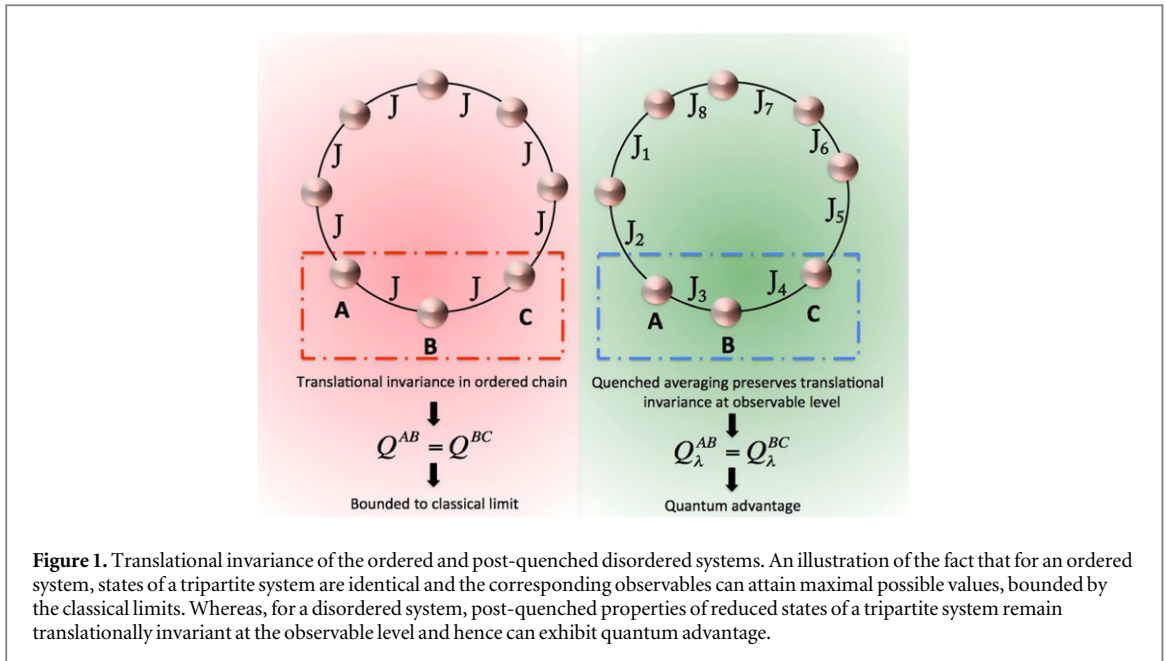


Figure 1. Translational invariance of the ordered and post-quenched disordered systems. An illustration of the fact that for an ordered system, states of a tripartite system are identical and the corresponding observables can attain maximal possible values, bounded by the classical limits. Whereas, for a disordered system, post-quenched properties of reduced states of a tripartite system remain translationally invariant at the observable level and hence can exhibit quantum advantage.

sites of the one-dimensional chain. The translation invariance of the ordered chain (with a periodic boundary condition for finite N) implies that we can always find a collection C of lattices that is disjoint from both A and B such that the reduced states ρ_{AB} and ρ_{BC} of the zero-temperature state are equal (see figure 1). The subtle assumption here is that the chain is sufficiently large, so that C does not overlap with A or B (cf. [7]). In particular if ρ_{AB} is a nearest-neighbor (two-site) density matrix, we only need $N \geq 3$. Applying now the monogamy relations to the state ρ_{ABC} , we obtain the stated results for the ordered case in the translational invariant scenario. While we have considered only the zero-temperature states in one-dimensional systems in this paper, the same arguments hold for any isotropic higher-dimensional lattice, as well as for finite temperature states. Therefore in such systems, we have $\delta = C^{adv} = 0$ for nearest-neighbor spins.

For the spin glass system, the above line of argument cannot be applied. The properties of the system are physically relevant only after quenched averaging has been performed, and post-quenching, these properties are again translationally invariant, just like the ordered case. So $Q_{\lambda}^{AB} = Q_{\lambda}^{BC}$ for any physical property (see figure 1), Q , for the reduced states at AB and BC . Note that throughout this paper, we associate the subscript λ to a quantity if it is quenched averaged. The Q_{λ}^{AB} and Q_{λ}^{BC} , however, do not correspond to a single state of ABC , and so the monogamy argument of the ordered case does not carry over to the disordered ones. We are therefore confronted with the possibility that disordered systems can give rise to situations, which, despite being translationally invariant, will have nearest-neighbor Bell inequality violation and quantum advantage in dense coding. Whether this is actually the case, however, requires explicit investigations.

Figure 2 clearly shows that after quenched averaging, the system violates the Bell-CHSH inequality and has quantum advantage in dense coding for the nearest-neighbor spins, which is a part of any nearest-neighbor three-party state of the N -party state. The violation is in an extreme sense, since both the two-party reduced states of the three-party cluster violate Bell inequality with equal strength. The same is true for dense-codeability.

6. Random field quantum XY model

We now introduce the randomness in the field while keeping the coupling strength uniform. Similar to the case of the XY spin glass, we find quantum advantage in both the quantities for the set of parameters considered here, thereby helping to overcome the restriction put by the monogamy relations.

Figure 3 shows the variation of the quantities δ_{λ} (figure 3(a)) and C_{λ}^{adv} (figure 3(b)) with respect to \bar{h}/J . Similar to the case of the disordered XY spin glass, we find quantum advantage in both the quantities for the set of parameters considered here. Thus, here too, introduction of the quenched disorder in the system helps to overcome the restriction put by the monogamy relations.

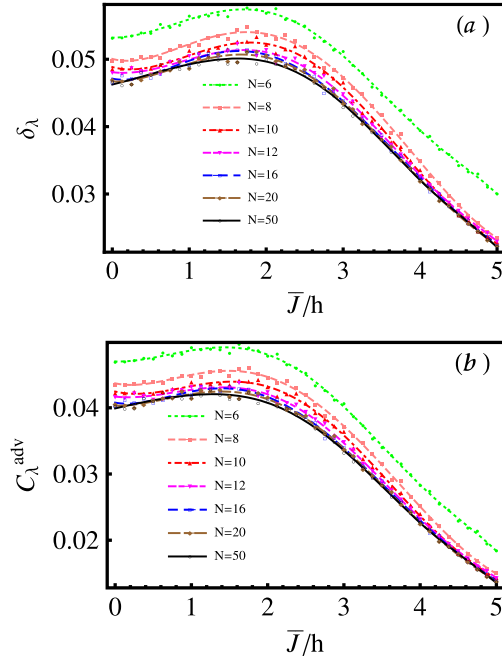


Figure 2. Overcoming monogamy in quantum XY spin glass. (a) Plot of quenched averaged Bell inequality violation (δ_λ) on the vertical axis against \bar{J}/h on the horizontal axis for nearest-neighbor spins of the zero temperature state in the quantum anisotropic XY spin glass for different N . Here we have chosen the uniform field strength $h = 0.4$, the anisotropy constant $\gamma = 0.5$, and the disorder strength $\sigma = 1.0$. (b) This is the same as (a), except that the quenched averaged quantum advantage for dense coding, C_λ^{adv} is plotted on the vertical axis. All quantities are dimensionless, except C_λ^{adv} , which is in bits.

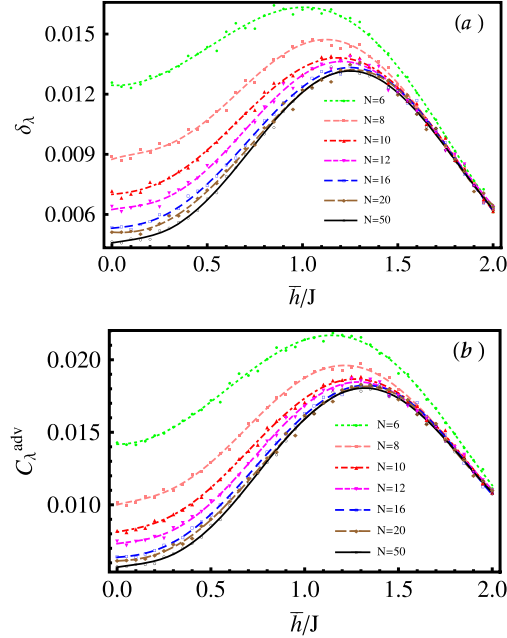
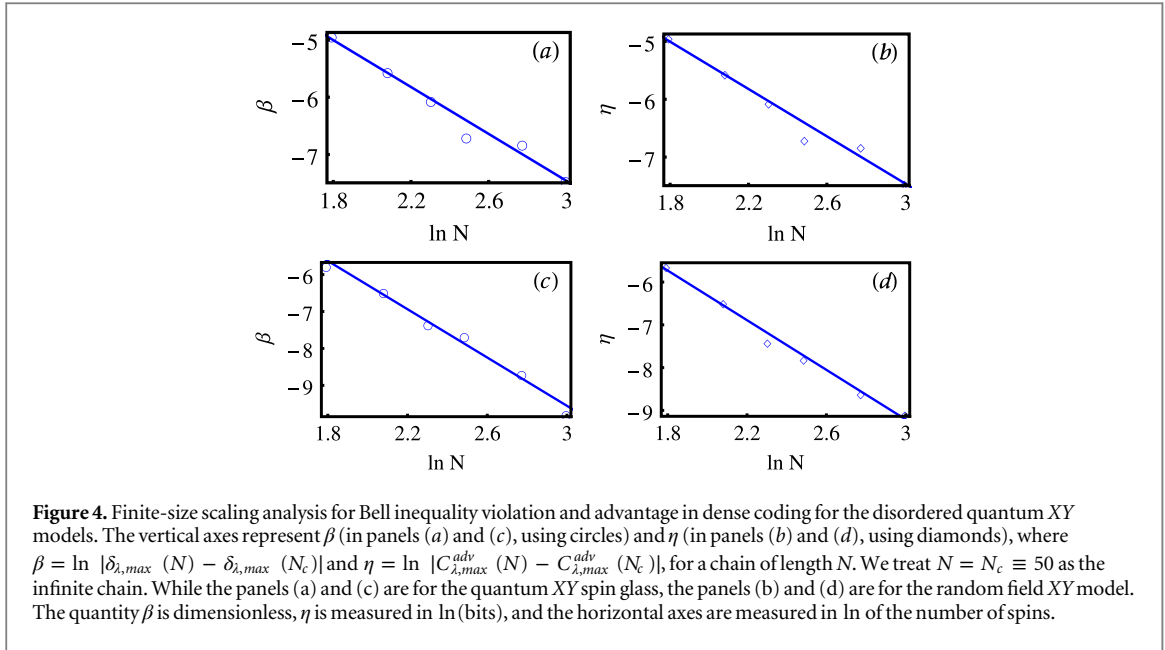


Figure 3. Overcoming monogamy in a random field quantum XY model. The considerations here are exactly the same as in figure 2, except that we are here using the random field quantum XY model with the h_i being i.i.d. Gaussian random variables (mean \bar{h} and unit standard deviation). Also, the horizontal axes represent dimensionless variable \bar{h}/J .

7. Scalings for the XY models

We observe that variation of both the quantities, δ_λ and C_λ^{adv} , for large systems, mimic the pattern obtained for $N = 20$. Hence, the systems with $N > 20$ spins can safely be assumed to serve the purpose of infinite spin chains. We now perform finite-size scaling, where we use the value for $N = 50$ as for the infinite system. We find that



$\delta_{\lambda, \max}$ and $C_{\lambda, \max}^{adv}$ decay as $N^{-2.05}$ and $N^{-2.30}$ for the spin glass while $N^{-3.27}$ and $N^{-2.90}$ for the random filed XY model, respectively. Here the subscript ‘max’ indicates that the scaling is done for the maximum values of both the quantities. The scaling analysis and the overall behavior of the quantities with increasing N clearly indicate that the violation of monogamy and the quantum advantage of classical information transmission will be sustained even in the thermodynamic limit, since for $N > 20$, the overall behavior of the physical quantities do not change with the increase of N , within the accuracy considered.

8. Quantum XYZ spin glass

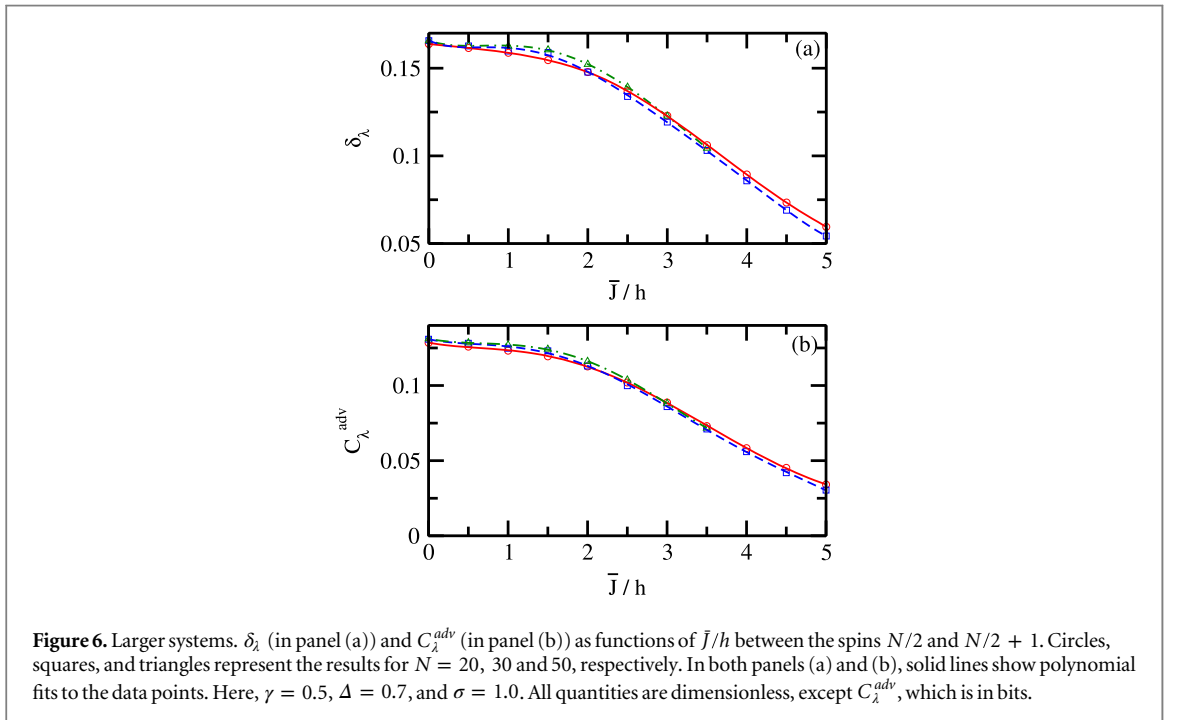
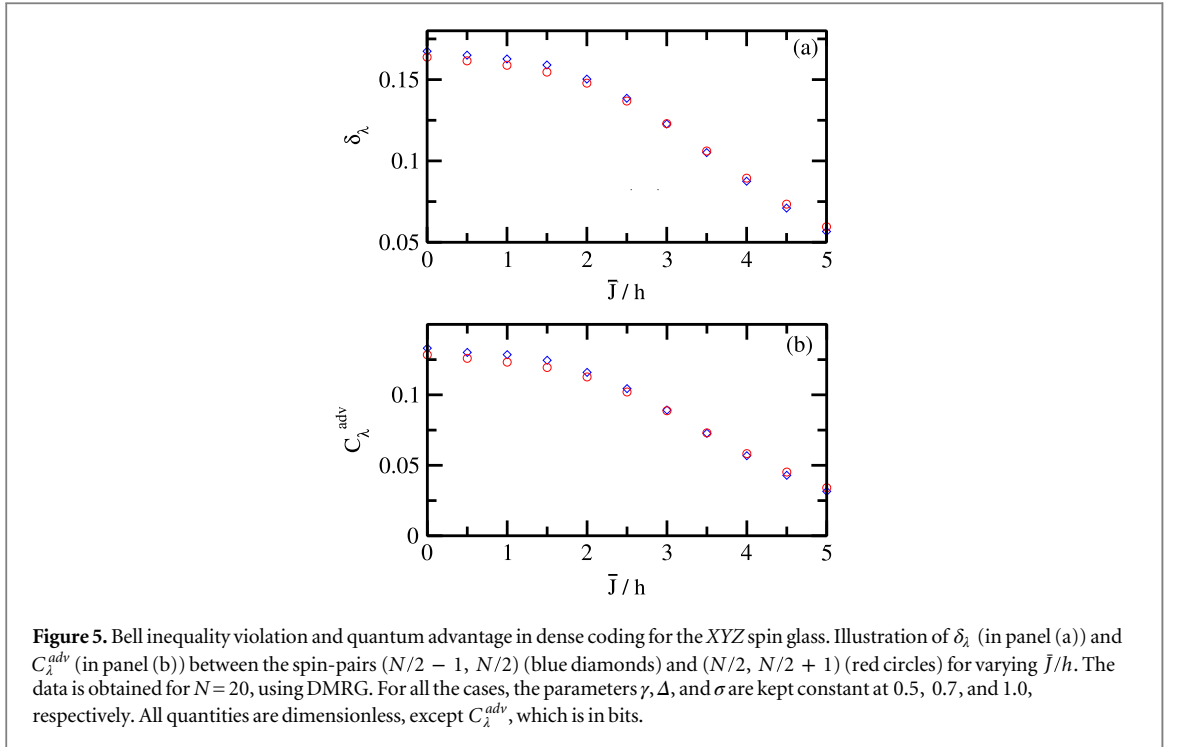
The ordered quantum XY model is exactly solvable. The corresponding quenched disordered systems are also analytically tractable up to a certain extent. To find whether the phenomena considered here are generic, we also consider a non-integrable model, viz. the quenched disordered quantum XYZ spin glass. The one-dimensional quantum XYZ Heisenberg Hamiltonian with random nearest-neighbor couplings is given by

$$H = \kappa \left[\sum_{i=1}^{N-1} \left[J_i \left[(1 + \gamma) \sigma_i^x \sigma_{i+1}^x + (1 - \gamma) \sigma_i^y \sigma_{i+1}^y \right] + \Delta \sigma_i^z \sigma_{i+1}^z \right] + h \sum_i \sigma_i^z \right]. \quad (11)$$

In order to investigate the monogamy relations for Bell inequality violation as well as the exclusion principle for dense coding, the ground state for the system characterized by the Hamiltonian in equation (11) is obtained by the numerical technique called the DMRG method [18].

In the present scenario, first, the infinite-size DMRG method is performed iteratively, where the system size is increased at each iteration by selectively choosing the most relevant basis states important for describing the system while truncating the rest. Afterwards, several finite size DMRG are also carried out on the disordered chain in order to increase the accuracy [18]. The quenched averaged values of the physical quantities are obtained by averaging over 5000–8000 random realizations.

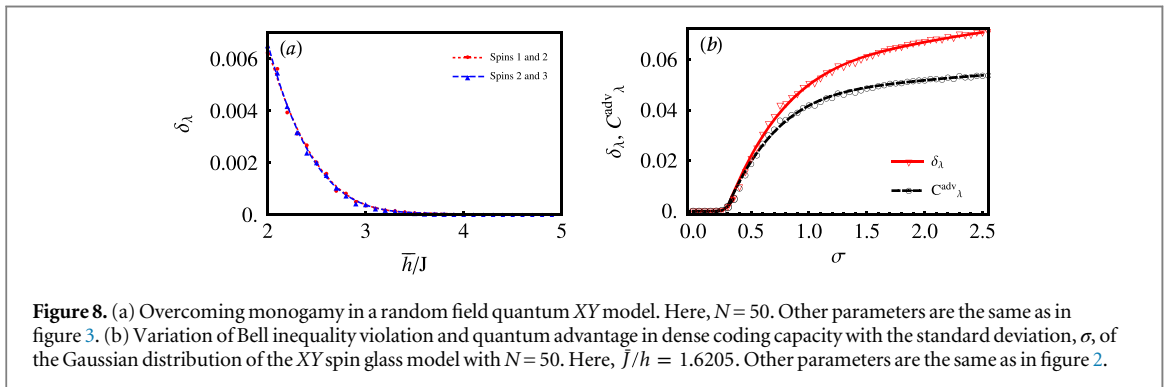
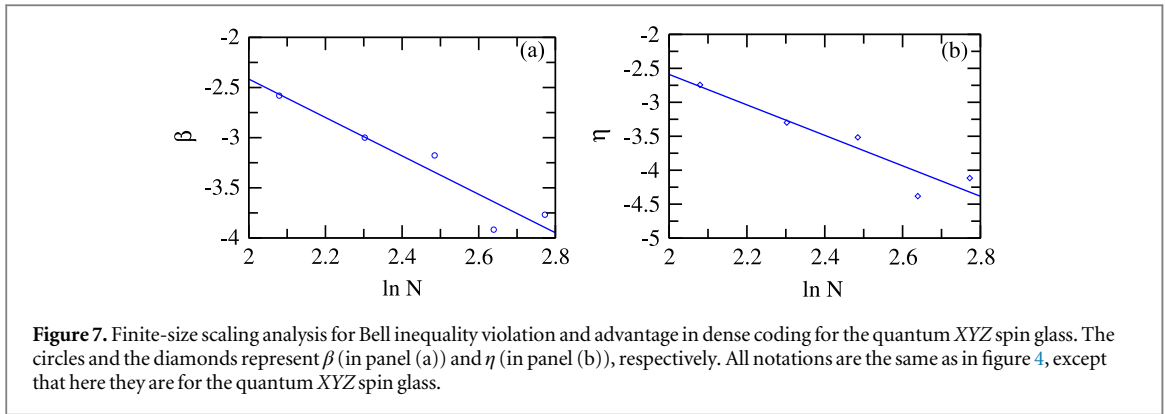
DMRG gives much less accurate results in the case of periodic boundary conditions. However, the advantage of the open boundary condition comes at the expense of the boundary effects. Nevertheless, an adequate description of the Bell monogamy and the exclusion principle for dense coding is possible provided the system size is not too small and the measurement of the observables on either fringe are excluded. In order to forego the boundary effect, we focus on the two adjacent bipartite subsystems at the center, composed of the $(N/2 - 1, N/2)$ and $(N/2, N/2 + 1)$ site pairs (see figure 5) and find that the results for the pairs agree with each other for all \tilde{J} . The consensus of the results demonstrate that the effective environment is essentially similar for both the pairs, ensuring the effective translational symmetry near the centre of the chain of the quenched averaged observables associated with these subsystems—a fact that would naturally be followed in the case of the closed chain. In figure 6, we show the behavior of the quantities δ_{λ} and C_{λ}^{adv} between the spins $N/2$ and $N/2 + 1$ as functions of \tilde{J}/h for $N = 20, 30$, and 50 . We find qualitatively consistent results with the observations previously made for the random XY spin models, except that the post-quenched values are an order



of magnitude higher. Moreover, we observe that quenched Bell inequality violation and advantage in dense coding capacity after quenching increase with the introduction of the zz -interaction, i.e. with the introduction of Δ . We choose $\bar{J} = 0$, where the Bell inequality violation and the dense coding capacity reach their respective maxima (see figure 6), to illustrate the finite-size scalings. We find that $\delta_{\lambda, \text{max}}$ decay as $N^{-1.92}$ and $N^{-2.24}$, respectively, much slower than in the XY disordered models (see figure 7).

9. Discussion

Quenched disordered spin chains are considered for investigating the monogamy of Bell inequality violation and the exclusion principle in dense coding of three-spin nearest-neighbor clusters of large chains in zero-



temperature states. In particular, we focus on the zero-temperature states of the random XY spin models and the random Heisenberg spin glass. Our analysis reveals that although the monogamy of quantum properties and the translational invariance of the Hamiltonian in clean systems force the considered quantum characteristics to attain at most classical values—leading to no-go theorems—the quantum nature can be resurrected by the introduction of quenched disorder in the system. The Hamiltonian itself is not translation invariant in the quenched system but the physically relevant post-quenched observables are so, and it is then possible for the system to overcome the monogamy of Bell inequality violation and quantum advantage for dense coding. The no-go theorems are at the level of observables, and in quenched disordered systems, it is the post-quenched quantities (and not the pre-quenched ones) that are physically meaningful. Finite-size scaling analysis is performed for all the models and for both the quantum characteristics considered, which clearly indicates that the observations sustain even in the thermodynamic limit.

The quenched averages that we consider show that it is possible to tune the system parameters of our physical models so that the quenched averaged quantities significantly violate Bell inequality. This is because we have checked that in all the models considered, the limits (the quenched averages) converge to a nonclassical value as we increase the number of quenched realizations. As is usual in such quenched averaging, we have performed a scaling with respect to the number of quenched realizations. But there can also be a situation where even the quenched averaged quantities do not violate Bell inequality. We consider two such instances here.

The first instance is obtained in figure 8(a), which is an extension of figure 3, showing the portion of the horizontal axis for $2 \leq \bar{h}/J \leq 5$, and for $N = 50$ (for periodic boundary conditions). The two curves correspond to the quenched averaged Bell inequality violations for the spins 1 and 2, and for the spins 2 and 3 of the chain. The figure clearly shows that for $\bar{h}/J \geq 4$ the zero-temperature state does not violate Bell inequality in spite of being quenched averaged. So, it is not possible to guarantee, *a priori*, whether the actual physical models that we consider will violate Bell inequality or not. The second instance is obtained by considering the behavior of the violation of Bell inequality with variation of the strength of randomness in the system. For specificity, we consider the variation of the Bell inequality violation and quantum advantage in dense coding with respect to the change in the standard deviation of the distribution of the disorder in the quantum XY spin glass. The plots are presented in figure 8(b). We find that both the quantities are enhanced with an increase in the standard deviation. Moreover, there is a threshold width (as quantified by the standard deviation) of randomness below which the quenched quantities do not show any quantum advantages. Interestingly, the finite-size scaling exponent is almost constant with the variation of the standard deviation in which there is a nonzero violation of Bell inequality. The same holds for quantum advantage in dense coding. We have observed that the susceptibility in this case reaches a maximum at the same threshold value of standard deviation of the disorder, below which

the disorder-induced leverage is absent for the quenched Bell inequality violation and quantum advantage in dense coding.

There is an ongoing effort in conquering no-go theorems in quantum mechanics either by going beyond the static framework of the quantum formalism [24] or by relaxing quantum dynamical postulates like unitarity [25]. The work presented in this paper shows another path for overcoming the no-go theorems of ordered systems, while still remaining within the quantum realm, by introducing impurities or defects.

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References

- [1] Wootters W K and Zurek W H 1982 *Nature* **299** 802
Dieks D 1982 *Phys. Lett. A* **92** 271
Yuen H P 1986 *Phys. Lett. A* **113** 405
- [2] Bennett C H, Bernstein H J, Popescu S and Schumacher B 1996 *Phys. Rev. A* **53** 2046
Coffman V, Kundu J and Wootters W K 2000 *Phys. Rev. A* **61** 052306
Osborne T and Verstraete F 2006 *Phys. Rev. Lett.* **96** 220503
Adesso G, Serafini A and Illuminati F 2006 *Phys. Rev. A* **73** 032345
Ou Y-C and Fan H 2007 *Phys. Rev. A* **75** 062308
Hiroshima T, Adesso G and Illuminati F 2007 *Phys. Rev. Lett.* **98** 050503
Seevinck M 2007 *Phys. Rev. A* **76** 012106
Lee S and Park J 2009 *Phys. Rev. A* **79** 054309
Kay A, Kaszlikowski D and Ramanathan R 2009 *Phys. Rev. Lett.* **103** 050501
Hayashi M and Chen L 2011 *Phys. Rev. A* **84** 012325 and references therein
- [3] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
- [4] Barnum H, Caves C M, Fuchs C A, Jozsa R and Schumacher B 1996 *Phys. Rev. Lett.* **76** 2818
Pati A K and Braunstein S L 2000 *Nature* **404** 164
Bub J 2001 *Found. Phys.* **31** 735
Piani M, Horodecki P and Horodecki R 2008 *Phys. Rev. Lett.* **100** 090502
- [5] Toner B and Verstraete F 2006 arXiv:[quant-ph/0611001](https://arxiv.org/abs/quant-ph/0611001)
Kurzyński P, Paterek T, Ramanathan R, Laskowski W and Kaszlikowski D 2011 *Phys. Rev. Lett.* **106** 180402
de Oliveira T R, Saguia A and Sarandy M S 2013 *Eur. Phys. Lett.* **100** 60004
- [6] Prabhu R, Pati A K, Sen(De) A and Sen U 2013 *Phys. Rev. A* **87** 052319
- [7] Sun Z-Y, Wu Y-Y, Huang H-L, Chen B-J and Wang B 2013 *Phys. Rev. A* **88** 054101
- [8] Karvonen J T, Taskinen L J and Maasilta I J 2005 *Phys. Rev. B* **72** 012302
Salafranca J and Brey L 2006 *Phys. Rev. B* **73** 214404
Hsu H C, Lee W L, Lin J-Y, Liu H L and Chou F C 2010 *Phys. Rev. B* **81** 212407
Berlijn T, Volja D and Ku W 2011 *Phys. Rev. Lett.* **106** 077005
Jaworowski B, Potasz P and Wojs A 2013 *Superlattices Microstruct.* **64** 44 and references therein
- [9] Bayat A and Karimipour V 2005 *Phys. Rev. A* **71** 042330
Cai J-M, Zhou J-W and Guo G-C 2006 *Phys. Rev. A* **74** 022328
Sen(De) A, Sen U, Ahufinger V, Briegel H J, Sanpera A and Lewenstein M 2006 *Phys. Rev. A* **74** 062309
Burgarth D 2007 *Eur. Phys. J. Special Topics* **151** 147
Burrell C K, Eisert J and Osborne T J 2009 *Phys. Rev. A* **80** 052319
Petrosyan D, Nikolopoulos G M and Lambropoulos P 2010 *Phys. Rev. A* **81** 042307
Yang S, Bayat A and Bose S 2010 *Phys. Rev. A* **82** 022336
Weimer H, Yao N Y, Laumann C R and Lukin M D 2012 *Phys. Rev. Lett.* **108** 100501 and references therein
- [10] Adamska L, Silva Neto M B and Smith C Morais 2007 *Phys. Rev. B* **75** 134507
Niederberger A, Rams M M, Dziarmaga J, Cucchiatti F M, Wehr J and Lewenstein M 2010 *Phys. Rev. A* **82** 013630
Niederberger A, Wehr J, Lewenstein M and Sacha K 2009 *Europhys. Lett.* **86** 26004
Niederberger A, Schulte T, Wehr J, Lewenstein M, Sanchez-Palencia L and Sacha K 2008 *Phys. Rev. Lett.* **100** 030403
Tsomokos D I, Osborne T J and Castelnovo C 2011 *Phys. Rev. B* **83** 075124
Foster M S, Xie H-Y and Chou Y-Z 2014 *Phys. Rev. B* **89** 155140
Martín P V, Bonachela J A and Muñoz M A 2014 *Phys. Rev. E* **89** 012145 and references therein
- [11] López-Sandoval R and Garcia M E 2006 *Phys. Rev. B* **74** 174204
Hide J, Son W and Vedral V 2009 *Phys. Rev. Lett.* **102** 100503
Fujii K and Yamamoto K 2010 *Phys. Rev. A* **82** 042109
Santos L F, Rigolin G and Escobar C O 2004 *Phys. Rev. A* **69** 042304
Mejia-Monasterio C, Benenti G, Carlo G G and Casati G 2005 *Phys. Rev. A* **71** 062324
Lakshminarayanan A and Subrahmanyam V 2005 *Phys. Rev. A* **71** 062334
Karthik J, Sharma A and Lakshminarayanan A 2007 *Phys. Rev. A* **75** 022304
Brown W G, Santos L F, Sterling D J and Viola L 2008 *Phys. Rev. E* **77** 022106
Dukesz F, Zilbergerts M and Santos L F 2009 *New J. Phys.* **11** 043026
Prabhu R, Pradhan S, Sen(De) A and Sen U 2011 *Phys. Rev. A* **84** 042334 and references therein
- [12] Huse D A, Nandkishore R, Oganesyan V, Pal A and Sondhi S L 2013 *Phys. Rev. B* **88** 014206

- [13] Roati G, D'Errico C, Fallani L, Fattori M, Fort C, Zaccanti M, Modugno G, Modugno M and Inguscio M 2008 *Nature* **453** 895
Billy J, Josse V, Zuo Z, Bernard A, Hambrecht B, Lugan P, Clément D, Sanchez-Palencia L, Bouyer P and Aspect A 2008 *Nature* **453** 891
Yu R et al 2012 *Nature* **489** 379
Hv'onen D, Zhao S, Månsson M, Yankova T, Ressouche E, Niedermayer C, Laver M, Gvasaliya S N and Zheludev A 2012 *Phys. Rev. B* **85** 100410(R)
Krinner S, Stadler D, Meineke J, Brantut J-P and Esslinger T 2013 and in preparation (arXiv:1311.5174)
Hazzard K R A et al 2014 *Phys. Rev. Lett.* **113** 195302 and references therein
- [14] Vojta T 2013 *AIP Conf. Proc.* **1550** 188
Vojta T 2006 *J. Phys. A* **39** R143
- [15] Lieb E, Schultz T and Mattis D 1961 *Ann. Phys.* **16** 407
- [16] Barouch E, McCoy B and Dresden M 1970 *Phys. Rev. A* **2** 1075
- [17] Barouch E and McCoy B 1971 *Phys. Rev. A* **3** 786
- [18] White SR 1992 *Phys. Rev. Lett.* **69** 2863
White SR 1993 *Phys. Rev. B* **48** 10345
Schollwock U 2005 *Rev. Mod. Phys.* **77** 259
- [19] Bell J S 1964 *Physics* **1** 195
- [20] Clauser J F, Horne M A, Shimony A and Holt R A 1969 *Phys. Rev. Lett.* **23** 880
- [21] Horodecki R, Horodecki P and Horodecki M 1995 *Phys. Lett. A* **200** 340
- [22] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
- [23] Bose S, Plenio M and Vedral V 2000 *J. Mod. Opt.* **47** 291
Hiroshima T 2001 *J. Phys. A: Math. Gen.* **34** 6907
Bowen G 2001 *Phys. Rev. A* **63** 022302
Horodecki M, Horodecki P, Horodecki R, Leung D and Terhal B 2001 *Quantum Information and Computation* **1** 70
Liu X S, Long G L, Tong D M and Li F 2002 *Phys. Rev. A* **65** 022304
Ziman M and Bužek V 2003 *Phys. Rev. A* **67** 042321
Bruß D, D'Ariano G M, Lewenstein M, Macchiavello C, Sen(De) A and Sen U 2004 *Phys. Rev. Lett.* **93** 210501
Bruß D, Lewenstein M, Sen(De) A, Sen U, D'Ariano G M and Macchiavello C 2006 *Int. J. Quant. Inf* **4** 415
- [24] Barrett J and Pironio S 2005 *Phys. Rev. Lett.* **95** 140401
- [25] Ferrero M, Salgado D and Sánchez-Gómez J L 2004 *Phys. Rev. A* **70** 014101 and references therein