An inventory model of a deteriorating product considering carbon emissions

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An Inventory Model of a Deteriorating Product Considering Carbon Emissions

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Abstract

This paper studies an inventory optimization problem involving perishable products under carbon policy. It considers that the market demand for perishable products is deterministic and it is related to selling price and stock level and that the ordering and storing of perishable products can create carbon emissions. Additionally, preservation technology can change perishable item's deterioration rate is also taken into account. For maximizing the retailer's profit in the finite planning period, inventory models under a carbon tax policy and under a carbon cap-and-trade policy are developed. Firstly properties of the optimal solutions of two inventory models are analysed. Next algorithms are designed to solve the two inventory models and the optimal solutions of the inventory models without carbon constraint, under the two carbon policies are compared. Finally, the theoretical properties of the models are demonstrated by numerical examples, sensitivity analysis of the main model parameters is carried out, and the influence of the parameters on the retailer's inventory policy is discussed.

Keywords: carbon tax policy; carbon cap-and-trade policy; preservation technology; perishable products; inventory model

1. Introduction

Deterioration has been defined as decay or change, by which the products are no longer in their original state (Shah, Soni and Patel 2013). For example, fruits, blood, cereals, alcohol and gasoline are all perishable products. With the development of the global economy, perishable products are playing a more important role in the daily life of people. For example, we consume fruits and vegetables much more than before. Perishable products are liable to deterioration, so retailers selling perishable products can incur higher costs, resulting in lower profit. Therefore retailers need an effective inventory strategy that considering the decay of products so as to obtain their maximum profit.

Many scholars have done a lot of research about inventory replenishment policy for perishable goods. For example, Larissa Janssen, Ali Diabat, Jürgen Sauer, and Frank Herrmann (2018) studied the inventory policy based on time of day (morning, midday, afternoon or evening) with fixed and known lifetime of items, deterministic lead time, fixed given order cycle. To alleviate the deterioration of perishable products, the retailers often use preservation technology (He and Huang 2013). For example, fruit retailers may use temperature control devices to lower the deterioration rate. While this reduces the loss of perishable products, the investment in preservation technology will also result in additional costs. So the application of preservation technology is also an important factor in the model. Furthermore, the best replenishment and preservation technology investment strategies are obtained from inventory models to maximize their profit.

In recent years, with the increase of extreme climate events, global warming has attracted increasing attention. The Intergovernmental Panel on Climate Change claims that global warming may be a result of carbon emissions (IPCC 2007). To shrink the effects of global warming, the Kyoto protocol was signed in 1997, enabling many countries to reach consensus on the issue of reducing greenhouse gas emissions. Therefore, many countries have introduced carbon policies to achieve this goal. One of the most effective carbon reduction policies is the carbon cap-and-trade policy (Qin, Bai and Xia 2015). Firms are given a free carbon emissions quota in a limited period, they can trade these carbon quotas with other organizations in their carbon trade market under this policy(Toptal, Özlü and Konur 2013; He et al. 2015). The European Union Emissions Trading System (EU) is the largest centre to facilitate trade in carbon quotas. The carbon cap-and-trade policy has begun trial operation in seven provinces of China

(Qin, Bai and Xia 2015). The carbon tax policy is another form of carbon regulation, and it imposes a tax on the total emissions (Benjaafar, Li and Daskin 2013; Aviyonah and Uhlmann 2009). The implementation of these carbon policies means that the operating environment of firms has changed significantly. As a result, we need to study the optimal inventory strategy and the optimal preservation technology investment strategy for retailers dealing with the carbon emissions regulations.

This paper is the first paper to study investment in preservation technology and the replenishment strategy of perishable items operating under carbon regulations. The analysis reaches the optimal ordering decision and the optimal preservation technology decision for retailers operating under the carbon tax policy and the carbon cap-and-trade policy. This paper's main contributions are listed as follows. Firstly, the applications of preservation technology and carbon regulations are considered simultaneously in this paper. This aims to fill the gap in the literature for inventory management models for deteriorating products. Next it assumes that the investment of preservation technology is a function of the period of inventory (i.e. the time for which at least one item is present in storage) rather than the entire order cycle. In this way, the model can describe the cost of preservation technology more accurately.

The rest of the article is organized as follows. Section 2 reviews the literature on preservation technology and carbon emissions policy. Section 3 introduces the modelling assumptions and the notation used for the model parameters. Section 4 develops two inventory models: (i) under the carbon tax policy and (ii) under the carbon cap-and-trade policy. Section 5 provides the theoretical results and the proposed solution methods. Section 6 presents some examples to illustrate the inventory models and their optimal solutions. Section 7 carries out sensitivity analysis. Finally, Section 8 presents conclusions and future research.

2. Literature review

The two critical points of this research are the application of preservation technology and the impact of carbon regulations, so the literature review firstly considers the literature on inventory models including preservation technology for deteriorating items and then considers the literature on inventory models with carbon regulations.

2.1. Preservation technology

In recent years, the inventory management of deteriorating items has attracted much interest from researchers and scholars. The research of the inventory strategy for perishable products cannot ignore the loss caused by deterioration. The deteriorating characteristic of perishable products is an essential factor that needs to be considered when studying the optimal inventory strategy of deteriorating items.

Ghare and Schrader (1963) proposed a deteriorating items inventory model with a fixed deteriorating rate. Dipankar, Kumar, and Kumar (2018) considered an inventory models with Weibull distribution deterioration. Chen et al. (2018) studied a deterioration product inventory model which considered a non-decreasing deterioration rate with shortages allowed. Tiwari et al. (2018a, 2018b) considered the inventory model with a constant deterioration rate and demand related to both the inventory level and the product price. Chang, Teng, and Chern (2010) presented the ordering strategies for deteriorating products, assuming the deterioration rate is constant. Shaikh et al. (2019) assumed the demand is a function of inventory level and price; the shortages are partially backordered, thus creating an EOQ model of perishable items. Pando et al. (2018) developed an inventory model for perishable products with market demand related to inventory level. The comparison was made between the model without deterioration, the model minimize total cost and the model with the maximum total profit. The replenishment strategy of perishable goods with stochastic lead time and holding cost has also been studied (Sazvar, Baboli and Akbari Jokar 2013).

However, the studies mentioned above considered the deterioration rates as exogenous variables, which are either constant or change over time; but cannot be changed by the action of decision makers. But as we all know, in practice, preservation technology can change deterioration rate. The application of preservation technology can shrink perishable product's deterioration rate, thus reducing the cost due to deterioration, while increasing the cost of preservation technology. So it is necessary to explore the optimal preservation technology investment to get retailer's maximum profit.

Some scholars have conducted research about the application of preservation technology. Hsu, Wee, and Teng (2010) explored the optimal replenishment strategies when preservation technology can shrink the deterioration rate. The model also allows shortages. Mishra (2013) built a model that considers the application of technology and the salvage values, and provides a graphical analysis to illustrate the optimal solution. Hsieh and Dye (2013) formulated a model for deteriorating products and used a traditional PSO method to solve the problem. Mishra et al. (2017) developed an inventory model that considered the investment in preservation technology. It proved the profit was a concave function of the investment in preservation technology. Dye, Yang, and Lev (2016) established a deteriorating product model with controllable deterioration rate. The pricing and preservation technology investment strategy of the system was studied. Liu, Zhang, and Tang (2015) developed a perishable items model with market demand related to selling price and quality to obtain a preservation technology investment strategy.

Although these scholars taken the investment of preservation technology into account, for some perishable products, it is unreasonable to assume the cost of preservation technology is a fixed cost that is independent of the ordering cycle. For example, if we buy a temperature-controller device or refrigerator, the cost of the capital needs to be replaced equivalently by a time-related cost. Besides, some procedural changes involve each product, such as wrapping them with protective materials or sealing them in airtight containers to reduce deterioration, which will add to the unit cost. Thus, a more reasonable representation of the cost of preservation technology is related to the inventory cycle. Some scholars have studied it in their research.

Dye and Hsieh (2012) formulated a deteriorating item's inventory model with the time-varying deterioration rate. In their models, the investment in preservation technology related to ordering cycle. Lee and Dye (2012) established a perishable products model that permitted shortages with partial backlogging. They assumed the cost of preservation technology was a linear function of ordering cycle. He and Huang (2013) assumed the cost of preservation technology is related to ordering cycle and studied the optimal ordering strategy. Dye (2013) assumed the demand is constant and discussed the optimal ordering strategy and the optimal preservation technology investment when preservation technology can change the deterioration rate of products. While considering the effects of initial reference price, Dye, Yang, and Wu (2017) proposed a deterioration item system with pricing and preservation technology investment decisions.

The research of the above scholars regarded the cost of preservation technology related to the ordering cycle, which is reasonable and relatively close to reality. However, for some deteriorating products, such as meat in a supermarket, it is obviously not reasonable. For these types of deteriorating products, there are two parts to the cost of preservation technology. First, temperature-controllers or refrigerators need to be bought, and the capital cost of equipment can be replaced by time-related costs. Next, the temperature-controller is used during the period of inventory. During the period of shortage, the temperature-controller can be turned off to reduce unnecessary cost. Therefore, the cost of preservation technology for this perishable product should be a function of the period of inventory rather than the entire order cycle. This paper studies this type of deteriorating products, considering the cost of preservation technology dependent on the period of inventory. This cost item is used in the model, and the optimal investment in preservation technology is solved to maximize the total profit. What is more, the above literature also does not take into account the impact of carbon policy. However, carbon policies are increasingly implemented in more countries. Therefore, it is important to study the optimal replenishment and preservation strategy under the carbon policy.

2.2. The carbon policy

There are two main incentive policies to reduce carbon dioxide emissions to slow global warming, carbon cap-and-trade refers to the trading of carbon dioxide emissions as a commodity, and carbon tax is a tax on carbon dioxide emissions. The government often introduces some regulations and policies to guide enterprises to meet the best interests of the whole society on the basis of pursuing their own interests' maximization (Zhong-Zhong Jiang et al. 2019). Hammami, Nouira, and Frein (2015) considered emissions due to transportation, process setup, production, and storage. They established a model that incorporated carbon emissions in a production-inventory model and explored the impact of carbon tax policy and the carbon cap policy on inventory decision-making. Chen, Benjaafar, and Elomri (2013) considered emissions associated with ordering, storage, and production. They developed an EOQ model, solved the optimal inventory strategy under two main carbon policies, and then analysed and compared how inventory strategy was affected by those two policies. Qin, Bai, and Xia (2015) considered carbon emissions of purchasing, delivering, and storage. They considered inventory policies under carbon cap-and-trade policy and carbon tax policy. Chen, Gong, and Wang (2017) considered the emissions of ordering, storage, and production. They studied how a retailer may adjust the optimal ordering policy under the carbon cap-and-trade mechanism, and how the minimum total cost and carbon emissions change. Dye and Yang (2015) assumed carbon emissions come from ordering and holding inventory. They considered sustainability on the background of joint trade credit, where demand relates to the credit period. Hua, Cheng and Wang (2011) focused on the carbon emitted by transportation and storaging. They studied the inventory control decision of products under the carbon tax policy, obtained the optimal ordering quantity. The literature above explores the impact of carbon policy on non-perishable product's inventory decision-making of enterprises. In contrast, our focus is on the impact of carbon policy on inventory management strategies for perishable products.

Tiwari, Daryanto and Wee (2018) considered the carbon emissions generated during the transportation, storage and processing of perishable products, and concluded that the integrated model is more effective both in reducing costs and emissions. Huang, He, and Li (2018) considered emissions caused by production, transportation, and storage. They proposed a Stackelberg model with deterioration in production to study the optimal replenishment and preservation strategies. Hua et al. (2016) assumed that perishable products produce carbon emissions in transportation, storage and deterioration. They established a perishable product's inventory model with freshness-dependent demand. The above literature does not consider the effect of preservation technology on deterioration rate. For perishable products, in practice, preservation technology is often used to lower the deterioration rate so as to improve the total profit. Therefore, it is necessary to study the inventory strategy of perishable goods based on carbon constraints while considering a controllable deterioration rate.

In summary, although so far many scholars have carried out much meaningful research on the inventory problem of perishable products, the following aspects still need to be further explored. First, in the application of preservation technology, the cost function used in past research does not apply to all perishable products. For some perishable products, such as meat, preservation technology mainly involves the application of temperature-control equipment, such as refrigerators. In such cases, it is appropriate to assume that the cost of preservation technology is linear in the period of inventory, but independent of the quantity stored. Past research considering inventory decision-making for perishable products under a carbon constraint policy does not consider the application of preservation technology to control deterioration rate. However, preservation technology is often used in this way and the carbon emissions generated by its application cannot be ignored. This paper develops a model to study inventory strategy for perishable products under carbon emission regulations with investment in preservation technology.

3. Notation and assumptions

3.1. Notation

The notation is listed in table 1.

3.2. Assumptions

（1）The decision maker is a retailer who sells a single deteriorating product which is sourced from a single supplier.

（2） The demand function $D(p, I(t))$ is the function of selling price and instantaneous stock level $I(t)$. The function $D(p, I(t))$ is listed as (Mishra et al. 2017):

$$D(p, I(t)) = \begin{cases} D(p) + \beta I(t), & I(t) > 0 \\ D(p), & I(t) \leq 0 \end{cases}$$

The function $D(p, I(t))$ is given by $D(p) = a - bp$ where a is market demand scale, and b is price sensitivity parameter.

(3)The market demand exists in T.

(4)Shortages are permitted, and they can be completely backordered.

(5)The replenishment process is periodic.

(6)The lead time is 0.

（7） The deterioration rate and preservation technology investment satisfy the function relation $\theta(\xi) = \theta_0 e^{-\delta\xi}$ (Mishra et al. 2017)

4. Mathematical model

This paper studies the optimal ordering strategy and the optimal preservation technology under two type carbon emissions regulations. A single deteriorating product is stocked for sale and shortages are permitted. It is assumed that demand is deterministic and relates to selling price and stock level. The main carbon emissions come from inventory replenishment and inventory holding (Dye and Yang 2015). In the process of replenishment, transportation is the source of the main carbon emissions, which can be expressed in ordering frequency and the ordering quantity. In the process of storage, the main carbon emissions result from the investment in preservation technology, which can be expressed by a function related to stock level.

4.1. The inventory model under the carbon tax policy

The model derives the selling price, ordering frequency and preservation technology investment that maximize the retailer's profit when operating under the carbon tax policy. The model involves two trade-offs. The first one relates to the ordering frequency and order size. As the ordering frequency increases and the order size decreases, the deterioration cost and holding cost decrease, but the fixed ordering cost increases. The second trade-off is between preservation technology cost and deterioration cost. With greater investment in preservation technology, the deterioration cost falls.

The planning horizon is an interval of length T. $I(t)$ represents the inventory level at time t during the interval $[0, T/n]$ where n is the ordering frequency. During $[0, t_1]$, $I(t)$ falls due to both demand and deterioration and it reaches zero at $t = t_1$. Shortages are permitted, and the inventory level continues to fall in the period $[t_1, T/n]$ because demand is fully backordered. From the assumptions mentioned above, the inventory level is described as Fig.1.

The change of inventory level satisfies the equation (1) and equation (2)

$$\frac{dI(t)}{dt} = -\theta(\xi)I(t) - D(p, I(t)), 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -D(p, I(t)), t_1 \leq t \leq T/n \tag{2}$$

From $I(t_1) = 0$, $I(t)$ is given by equation (3);

$$I(t) = \begin{cases} \frac{D(p)}{\theta(\xi)+\beta}\left[e^{[\theta(\xi)+\beta](t_1-t)} - 1\right], & \text{if } 0 \leq t \leq t_1 \\ D(p)(t_1 - t), & \text{if } t_1 \leq t \leq T/n \end{cases} \tag{3}$$

The initial inventory level (S) is expressed as

$$S = I(0) = \frac{D(p)}{\theta(\xi)+\beta}\left[e^{[\theta(\xi)+\beta]t_1} - 1\right] \tag{4}$$

The order quantity per cycle is calculated as

$$Q = S + \int_{t_1}^{T/n} D(p, I(t))\, dt = \frac{D(p)}{\theta(\xi)+\beta}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 + \frac{[\theta(\xi)+\beta]T}{n} - [\theta(\xi)+\beta]t_1\right\} \tag{5}$$

The quantity of deteriorated products between the interval $[0, t_1]$ is computed as

$$P_\tau = S - \int_0^{t_1} D(p, I(t))dt = \frac{\theta(\xi)D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\} \tag{6}$$

The total carbon emissions (TE) in a finite time horizon T is given by equation (7) (Dye and Yang 2015):

$$TE = n\left[A_e + c_1^e Q + h_1^e \int_0^{t_1} I(t)dt\right] = n\left\{A_e + \frac{c_1^e D(p)}{\theta(\xi)+\beta}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 + \frac{[\theta(\xi)+\beta]T}{n} - 1\right\} + \frac{h_1^e D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\}\right\} \tag{7}$$

Then, the components of objective function under the carbon tax policy are computed as follows:

Sales revenue:

$$R = np\int_0^{T/n} D(p, I(t))dt = pTD(p) + \frac{np\beta D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\} \tag{8}$$

Purchase cost:

$$C_1 = ncQ = \frac{ncD(p)}{\theta(\xi)+\beta}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 + \frac{[\theta(\xi)+\beta]T}{n} - [\theta(\xi)+\beta]t_1\right\} \tag{9}$$

Holding cost:

$$C_2 = nh\int_0^{t_1} I(t)dt = \frac{nhD(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\} \tag{10}$$

Shortage cost:

$$C_3 = ns\int_{t_1}^{T/n}[-I(t)]dt = nsD(p)\left[\frac{t_1^2}{2} + \frac{T^2}{2n^2} - \frac{Tt_1}{n}\right] \tag{11}$$

The handling cost of deteriorated products:

$$C_4 = ngP_\tau = \frac{ng\theta(\xi)D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\} \tag{12}$$

Fixed ordering cost:

$$C_5 = nK \tag{13}$$

Preservation technology cost:

$$C_6 = n\xi t_1 \tag{14}$$

The carbon emissions cost:

$$C_7 = \lambda TE = \lambda n\left\{A_e + \frac{c_1^e D(p)}{\theta(\xi)+\beta}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 + \frac{[\theta(\xi)+\beta]T}{n} - 1\right\} + \frac{h_1^e D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\}\right\} \tag{15}$$

Therefore, the total profit of the retailer under the carbon tax policy is calculated by

$$TP_2 = R - C_1 - C_2 - C_3 - C_4 - C_5 - C_6 - C_7$$

$$TP_2(n,\xi,p) = pTD(p) + \frac{np\beta D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\} - nsD(p)\left[\frac{t_1^2}{2} + \frac{T^2}{2n^2} - \frac{t_1 T}{n}\right] - n\xi t_1 - nK - \frac{ncD(p)}{\theta(\xi)+\beta}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 + \frac{[\theta(\xi)+\beta]T}{n} - [\theta(\xi)+\beta]t_1\right\} - \frac{nhD(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\} - \frac{ng\theta(\xi)D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 - 1\right\} - \lambda n\left\{A_e + \frac{c_1^e D(p)}{[\theta(\xi)+\beta]}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 + \frac{[\theta(\xi)+\beta]T}{n} - 1\right\} + \frac{h_1^e D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 - 1\right\}\right\} \tag{16}$$

Let $t_1 = \frac{uT}{n}, 0 < u < 1$, so the profit function under the carbon tax policy is;

$$TP_2(n,\xi,p) = pTD(p) + \frac{np\beta D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - 1 - \frac{[\theta(\xi)+\beta]uT}{n}\right\} - nsD(p)\left[\frac{(uT)^2}{2n^2} + \frac{T^2}{2n^2} - \frac{uT^2}{n^2}\right] - uT\xi - nK - \frac{ncD(p)}{\theta(\xi)+\beta}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - 1 + \frac{[\theta(\xi)+\beta]T}{n} - \frac{[\theta(\xi)+\beta]uT}{n}\right\} - \frac{nhD(p)}{[\theta(\xi)+\beta]^2}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - 1 - \frac{[\theta(\xi)+\beta]uT}{n}\right\} - \frac{ng\theta(\xi)D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - 1 - \frac{[\theta(\xi)+\beta]uT}{n}\right\} - \lambda n\left\{A_e + \frac{c_1^e D(p)}{[\theta(\xi)+\beta]}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} + \frac{[\theta(\xi)+\beta]T}{n} - \frac{[\theta(\xi)+\beta]uT}{n} - 1\right\} + \frac{h_1^e D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - \frac{[\theta(\xi)+\beta]uT}{n} - 1\right\}\right\} \tag{17}$$

Then, this problem is described as follows:

$$\text{Max}\, TP_2(n,\xi,p)$$

$$\text{subject to}$$
$$D(p) > 0$$
$$p > 0$$
$$n \geq 1 \text{ and discrete}$$
$$0 \leq \xi \leq \bar{\xi}$$

To get the results of this problem, we simplify the problem. It is well-known from the Taylor series expansion that, for a small x value, $e^x \approx 1 + x + x^2/2!$. Equation (17) then simplifies as Equation (18).

$$TP_2(n,\xi,p) = pTD(p) + \frac{p\beta D(p)(uT)^2}{2n} - \frac{sD(p)}{2n}[(uT)^2 + T^2 - 2uT^2] - \frac{hD(p)(uT)^2}{2n} - cD(p)\left\{\frac{u^2T^2[\theta(\xi)+\beta]}{2n} + T\right\} - \frac{g\theta(\xi)D(p)(uT)^2}{2n} - \lambda\left\{nA_e + c_1^eD(p)\left\{\frac{u^2T^2[\theta(\xi)+\beta]}{2n} + T\right\} + h_1^eD(p)\frac{u^2T^2}{2n}\right\} - nK - uT\xi \quad (18)$$

Additionally, the retailer's profit function without considering carbon emissions $TP_1(n,\xi,p)$ is obtained from equation (18) when $\lambda = 0$.

$$TP_1(n,\xi,p) = pTD(p) + \frac{p\beta D(p)(uT)^2}{2n} - \frac{sD(p)}{2n}[(uT)^2 + T^2 - 2uT^2] - \frac{hD(p)(uT)^2}{2n} - cD(p)\left\{\frac{u^2T^2[\theta(\xi)+\beta]}{2n} + T\right\} - \frac{g\theta(\xi)D(p)(uT)^2}{2n} - nK - uT\xi \quad (19)$$

4.2. The inventory model with the Carbon cap-and-trade

Firms operating under the carbon cap-and-trade mechanism are given a free emissions quota over a period of time, and they can trade these carbon quotas with other firms in carbon market (Toptal, Özlü and Konur 2013; He et al. 2015).

The carbon emissions cost:

$$C_7' = -c_p\left\{C_c - n\left\{A_e + \frac{c_1^eD(p)}{\theta(\xi)+\beta}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 + \frac{[\theta(\xi)+\beta]T}{n} - 1\right\} + \frac{h_1^eD(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 - 1\right\}\right\}\right\} \quad (20)$$

Therefore, the total profit of the retailer under the carbon cap-and-trade policy is calculated by

$$TP_3(n,\xi,p) = R - C_1 - C_2 - C_3 - C_4 - C_5 - C_6 - C_7'$$

$$TP_3(n,\xi,p) = pTD(p) + \frac{np\beta D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\} - n\xi t_1 - nK - \frac{nhD(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 - [\theta(\xi)+\beta]t_1\right\} - \frac{ng\theta(\xi)D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 - 1\right\} - nsD(p)\left[\frac{t_1^2}{2} + \frac{T^2}{2n^2} - \frac{t_1 T}{n}\right] - \frac{ncD(p)}{\theta(\xi)+\beta}\left\{e^{[\theta(\xi)+\beta]t_1} - 1 + \frac{[\theta(\xi)+\beta]T}{n} - [\theta(\xi)+\beta]t_1\right\} + c_p\left\{C_c - nA_e - \frac{nc_1^e D(p)}{\theta(\xi)+\beta}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 + \frac{[\theta(\xi)+\beta]T}{n} - 1\right\} - \frac{nh_1^e D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{[\theta(\xi)+\beta]t_1} - [\theta(\xi)+\beta]t_1 - 1\right\}\right\} \quad (21)$$

Let $t_1 = \frac{uT}{n}, 0 < u < 1$, so the profit function under the carbon cap-and-trade policy is;

$$TP_3(n,\xi,p) = pTD(p) + \frac{np\beta D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - 1 - \frac{[\theta(\xi)+\beta]uT}{n}\right\} - nsD(p)\left[\frac{(uT)^2}{2n^2} + \frac{T^2}{2n^2} - \frac{uT^2}{n^2}\right] - \xi uT - nK - \frac{ncD(p)}{\theta(\xi)+\beta}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - 1 + \frac{[\theta(\xi)+\beta]T}{n} - \frac{[\theta(\xi)+\beta]uT}{n}\right\} - \frac{nhD(p)}{[\theta(\xi)+\beta]^2}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - 1 - \frac{[\theta(\xi)+\beta]uT}{n}\right\} - \frac{ng\theta(\xi)D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - 1 - \frac{[\theta(\xi)+\beta]uT}{n}\right\} + c_p\left\{C_c - nA_e - \frac{nc_1^e D(p)}{\theta(\xi)+\beta}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - \frac{[\theta(\xi)+\beta]uT}{n} + \frac{[\theta(\xi)+\beta]T}{n} - 1\right\} - \frac{nh_1^e D(p)}{[\theta(\xi)+\beta]^2}\left\{e^{\frac{[\theta(\xi)+\beta]uT}{n}} - \frac{[\theta(\xi)+\beta]uT}{n} - 1\right\}\right\} \quad (22)$$

Then, this problem is calculated as follows:

$$\text{Max}\, TP_3(n,\xi,p)$$
$$\text{subject to}$$
$$D(p) > 0$$
$$p > 0$$
$$n \geq 1 \text{ and discrete}$$
$$0 \leq \xi \leq \bar{\xi}$$

As before, to get the results of this problem under the carbon cap-and-trade policy, we use the Taylor series approximation ($e^x \approx 1 + x + x^2/2!$ for small x values) to simplify the problem. Equation (22) simplifies as

$$TP_3(n,\xi,p) = pTD(p) + \frac{p\beta D(p)(uT)^2}{2n} - \frac{sD(p)}{2n}[(uT)^2 + T^2 - 2uT^2] - \frac{hD(p)(uT)^2}{2n} - \frac{g\theta(\xi)D(p)(uT)^2}{2n} - nK - uT\xi - cD(p)\left\{\frac{u^2T^2[\theta(\xi)+\beta]}{2n} + T\right\} + c_p\left\{C_c - nA_e - c_1^e D(p)\left\{\frac{u^2T^2[\theta(\xi)+\beta]}{2n} + T\right\} - h_1^e D(p)\frac{u^2T^2}{2n}\right\} \quad (23)$$

5. Theoretical results and optimal solution

5.1. Properties and optimal solution of the model under the carbon tax policy

This section gives some theorems and a corollary to explain the properties of objective function. In the end, algorithm 1 is provided to obtain the optimal solution. It should be noted that, in order to simplify the expression, $\Delta_1, \Delta_2, \cdots, \Delta_6$ is used as a simplified notation for the corresponding complex expression, the specific expression is shown in the appendix accordingly.

Theorem 1 If the selling price p and preservation technology investment ξ are fixed: (1) when $\beta < \Delta_1$ then the total profit function $TP_2(n,\xi,p)$ is a concave function of ordering frequency n; (2) when $\beta \geq \Delta_1$ then the total profit function $TP_2(n,\xi,p)$ is a monotonic decreasing function of n.

Proof Refer to 'Appendix 1'.

Theorem 1 suggests that when the stock level has a large enough impact on market demand ($\beta \geq \Delta_1$), the retailer tends to reduce ordering frequency to maintain a relatively high stock level to satisfy more market demand. In this way the retailer can maximize profits.

Theorem 2 For given ordering frequency n and fixed p, the following conclusions are established.

(1) If $\Delta_2(n,p) \leq 0$ then when $\xi^* = 0$, $TP_2(n,\xi,p)$ attains its maximum value.

(2) If $\Delta_3(n,p) \geq 0$ then when $\xi^* = \bar{\xi}$, $TP_2(n,\xi,p)$ attains its maximum value.

(3) If $\Delta_2(n,p) > 0$ and $\Delta_3(n,p) < 0$ then $TP_2(n,\xi,p)$ is a concave function of ξ, and $TP_2(n,\xi,p)$ attains its maximum value at $\xi^* \in (0, \bar{\xi})$ when $\{\partial[TP_2(n,\xi,p)]/\partial\xi\} = 0$

Proof Refer to 'Appendix 2'.

Theorem 2 shows that when the initial deterioration rate is pretty small, or the sensitivity parameter of preservation technology investment to deterioration rate is very low, more preservation technology is not good for total profit. Additionally, if the input of preservation technology is not limited, the retailer might acquire more profit.

Theorem 3 For fixed ordering frequency n and fixed preservation technology ξ, there is a unique p^* that maximizes $TP_2(n, \xi, p)$.

Proof Refer to 'Appendix 3'.

Theorem 4 Given ordering frequency n, there exists a unique (ξ^*, p^*) which maximizes $TP_2(n, \xi, p)$.

Proof The result follows immediately from Theorems 1, 2 and 3.

Integrating the results of theorem 1 to theorem 4, algorithm 1 is developed. It is similar to the algorithm of He and Huang (2013). The optimal solution of the non-linear optimization problem is obtained through the following algorithm 1.

Algorithm 1

Step 1. Initialize $n = 1$.

Step 2. Initialize the iteration times $m = 1$ and set the value of $p^m = p_0$.

Step 3. Compute $\Delta_2(n, p)$, $\Delta_3(n, p)$ and execute one of the three cases.

(1) If $\Delta_2(n, p) \leq 0$, then $\xi_2^m = 0$. Determine p_2^m with Eq. (30).

(2) If $\Delta_3(n, p) \geq 0$, then $\xi_2^m = \bar{\xi}$. Determine p_2^m with Eq. (30).

(3) If $\Delta_2(n, p) > 0$ and $\Delta_3(n, p) < 0$, Compute ξ_2^m with (26) equal to zero. Substitute ξ_2^m into Eq. (30) and compute p_2^m.

Set $p^{m+1} = p_2^m$ and $\xi^m = \xi_2^m$.

Step 4.If $|p^{m+1} - p^m| \leq \tau$ ($\tau = 10^{-4}$,He and Huang 2013), then $(\xi^*, p^*) = (\xi^m, p^{m+1})$ and go to Step 5. Otherwise, set $m = m + 1$ and go to Step 3.

Step 5. Determine $TP_2(n, \xi^*, p^*)$ with Eq. (18) which is the maximum value for a fixed n.

Step 6. Set $n' = n + 1$, iterate Step 2 to 5 and determine $TP_2(n', \xi^*, p^*)$ with Eq. (18). Go to Step 7.

Step 7. If $TP_2(n', \xi^*, p^*) \geq TP_2(n, \xi^*, p^*)$, set $n = n'$. Go to Step 6. Otherwise go to Step 8.

Step 8.Set $(n^*, \xi^*, p^*) = (n, \xi^*, p^*)$ and $TP_2(n, \xi^*, p^*)$ as the optimal solution.

Step 9.Determine order quantity Q with Eq. (5).

Corollary 1 There exist unique n, ξ, p that maximize the total profit function under the carbon tax policy; retailers operating in an environment without carbon emissions regulations can also get their maximum value at unique n, ξ, p.

Combining theorem 1-theorem 4, corollary 1 is developed. Assume that retailers operating under the carbon tax policy get their maximum value $TP_2(n_2^*, \xi_2^*, p_2^*)$ at $n = n_2^*, \xi = \xi_2^*, p = p_2^*$ and that retailers operating in an environment without carbon emissions regulations attain their maximum value $TP_1(n_1^*, \xi_1^*, p_1^*)$ at $n = n_1^*, \xi = \xi_1^*, p = p_1^*$.

Theorem 5 The maximum profit of retailers operating under the carbon tax policy is less than that of retailers operating without carbon emissions, i.e. $TP_1(n_1^*, \xi_1^*, p_1^*) > TP_2(n_2^*, \xi_2^*, p_2^*)$.

Proof:

Case 1: When $n_1^* = n_2^*$, $\xi_1^* = \xi_2^*$, $p_1^* = p_2^*$, we have $TP_1(n_1^*, \xi_1^*, p_1^*) - TP_2(n_2^*, \xi_2^*, p_2^*) = \lambda\left\{c_1^e D(p_2^*)\left\{\frac{u^2T^2[\theta(\xi_2^*)+\beta]}{2n_2^*} + T\right\} + h_1^e D(p_2^*)\frac{u^2T^2}{2n_2^*} + n_2^* A_e\right\} > 0$. In other words, $TP_1(n_1^*, \xi_1^*, p_1^*) > TP_2(n_2^*, \xi_2^*, p_2^*)$ is established.

Case 2: When $n_1^* \neq n_2^*$, $\xi_1^* \neq \xi_2^*$, $p_1^* \neq p_2^*$, we can deduce $TP_1(n_1^*, \xi_1^*, p_1^*) > TP_1(n_2^*, \xi_2^*, p_2^*)$. Additionally as in case 1, it is easy to show that $TP_1(n_2^*, \xi_2^*, p_2^*) > TP_2(n_2^*, \xi_2^*, p_2^*)$, so $TP_1(n_1^*, \xi_1^*, p_1^*) > TP_2(n_2^*, \xi_2^*, p_2^*)$ holds.

5.2. Properties and optimal solution of the model under the carbon cap-and-trade policy

Theorem 6 If the selling price p and preservation technology investment ξ are fixed: (1) when $\beta < \Delta_4$, the total profit function $TP_3(n, \xi, p)$ is a concave function of ordering frequency n; (2) when $\beta \geq \Delta_4$, the total profit function $TP_3(n, \xi, p)$ is a monotonic decreasing function of n.

Proof Refer to 'Appendix 4'.

Theorem 1 indicates that when the stock level has a large enough impact on market demand ($\beta \geq \Delta_4$), the retailer tends to reduce ordering frequency to maintain a relatively high stock level and satisfy more market demand to maximize its profit.

Theorem 7 For given ordering frequency n and fixed selling price p, the following results hold:

(1) If $\Delta_5(n, p) \leq 0$ then when $\xi^* = 0$, $TP_3(n, \xi, p)$ attains its maximum value.

(2) If $\Delta_6(n, p) \geq 0$ then when $\xi^* = \bar{\xi}$, $TP_3(n, \xi, p)$ attains its maximum value.

(3) If $\Delta_5(n, p) > 0$ and $\Delta_6(n, p) < 0$ then $TP_3(n, \xi, p)$ is a concave function of ξ, and $TP_3(n, \xi, p)$ attains its maximum at $\xi^* \in (0, \bar{\xi})$ when $\{\partial[TP_3(n, \xi, p)]/\partial\xi\} = 0$

Proof Refer to 'Appendix 5'.

Theorem 7 suggests that when the deterioration rate without preservation technology is very small or the effect of preservation technology investment on deterioration rate is relatively low, the input of preservation technology is not good for total profit. Furthermore, if the input of preservation technology is not limited, the retailer might obtain more profit.

Theorem 8 For fixed values of ordering frequency n and fixed preservation technology investment ξ, there is a unique p^* that maximizes the total profit.

Proof Refer to 'Appendix 6'.

Theorem 9 Given ordering frequency n, there exist a unique (ξ^*, p^*) that maximizes $TP_3(n, \xi, p)$.

Proof The result follows immediately from Theorems 6, 7 and 8.

Integrating the results of theorem 6 to theorem 9, the following iterative algorithm is developed. Algorithm 2 is similar to algorithm 1. The optimal solution is obtained through algorithm 2.

Algorithm 2

Step 1. Initialize $n = 1$.

Step 2. Initialize the iteration times $m = 1$ and set the value of $p^m = p_0$

Step 3. Compute $\Delta_5(n,p), \Delta_6(n,p)$, and execute one of the three cases.

(1) If $\Delta_5(n,p) \leq 0$, then $\xi_3^m = 0$. Determine p_3^m with Eq. (34).

(2) If $\Delta_6(n,p) \geq 0$, then $\xi_3^m = \xi$. Determine p_3^m with Eq. (34).

(3) If $\Delta_5(n,p) > 0$ and $\Delta_6(n,p) < 0$, Compute ξ_3^m with (31) equal to zero. Substitute ξ_3^m into Eq. (34) and compute p_3^m.

Set $p^{m+1} = p_3^m$ and $\xi^m = \xi_3^m$.

Step 4. If $|p^{m+1} - p^m| \leq 10^{-4}$ (τ=10^{-4}, He and Huang 2013), then $(\xi^*, p^*) = (\xi^m, p^{m+1})$ and go to Step 5. Otherwise, set $m = m + 1$ and go to Step 3.

Step 5. Determine $TP_3(n, \xi^*, p^*)$ with Eq. (23) which is the maximum value of $TP_3(n, \xi, p)$ for a fixed n.

Step 6. Set $n' = n + 1$, iterate Step 2 to 5 and determine $TP_3(n', \xi^*, p^*)$ with Eq. (23). Go to Step 7.

Step 7. If $TP_3(n', \xi^*, p^*) \geq TP_3(n, \xi^*, p^*)$, set $n = n'$. Go to Step 6. Otherwise go to Step 8.

Step 8. Set $(n', \xi^*, p^*) = (n, \xi^*, p^*)$ and $TP_3(n, \xi^*, p^*)$ as the optimal solution.

Step 9.Determine order quantity Q with Eq. (5).

Corollary 2 There exist unique n, ξ, p which maximize the total profit function for retailers operating under the carbon cap-and-trade policy.

Combining Theorem 6 to Theorem 9, corollary 2 is developed. Assume retailers operating under the carbon cap-and-trade policy get their maximum profit $TP_3(n_3^*, \xi_3^*, p_3^*)$ at $n = n_3^*, \xi = \xi_3^*, p = p_3^*$.

Corollary 3 Under the carbon cap-and-trade policy, the following results relating to the maximum profit of the retailer can be derived from equation (25):

(1) $TP_3(n, \xi, p)$ is an increasing function of the cap C_c.

(2) When $C_c > TE(n_3^*, \xi_3^*, p_3^*)$, the retailer needs to sell $C_c - TE(n_3^*, \xi_3^*, p_3^*)$ units of carbon emissions to obtain its maximum profit value; when $C_c < TE(n_3^*, \xi_3^*, p_3^*)$, the retailer needs to buy $C_c - TE(n_3^*, \xi_3^*, p_3^*)$ units of carbon emissions to obtain its maximum profit value; when $C_c = TE(n_3^*, \xi_3^*, p_3^*)$, the retailer does not need to trade with others in the carbon trading market.

Theorem 10 The maximum profit of retailers operating under the carbon cap-and-trade policy is no less than that of retailers operating without carbon emissions when $C_c \geq h_1^e D(p_1^*) \frac{u^2 T^2}{2n_1^*} + n_1^* A_e + c_1^e D(p_1^*) \left\{ \frac{u^2 T^2 [\theta(\xi_1^*) + \beta]}{2n_1^*} + T \right\}$,i.e. $TP_3(n_3^*, \xi_3^*, p_3^*) \geq TP_1(n_1^*, \xi_1^*, p_1^*)$; and no greater than that of retailers operating without carbon emissions when $C_c \leq c_1^e D(p_3^*) \left\{ \frac{u^2 T^2 [\theta(\xi_3^*) + \beta]}{2n_3^*} + T \right\} + n_3^* A_e + h_1^e D(p_3^*) \frac{u^2 T^2}{2n_3^*}$, i.e. $TP_1(n_1^*, \xi_1^*, p_1^*) \geq TP_3(n_3^*, \xi_3^*, p_3^*)$.

Proof Refer to 'Appendix 7'.

Theorem 10 shows that when the carbon emissions cap is large enough, namely, $C_c \geq h_1^e D(p_1^*) \frac{u^2 T^2}{2n_1^*} + n_1^* A_e + c_1^e D(p_1^*) \left\{ \frac{u^2 T^2 [\theta(\xi_1^*) + \beta]}{2n_1^*} + T \right\}$, the retailer gets more profit under the carbon cap-and-trade policy than without carbon emissions regulations. However, when the carbon emissions cap is relatively small, namely, $C_c \leq n_3^* A_e + c_1^e D(p_3^*) \left\{ \frac{u^2 T^2 [\theta(\xi_3^*) + \beta]}{2n_3^*} + T \right\} + h_1^e D(p_3^*) \frac{u^2 T^2}{2n_3^*}$, retailers operating under the carbon cap-and-trade policy cannot get more profit than retailers operating without carbon emissions regulations. This is intuitive because if the cap is very high, retailers will not find their actions restricted and they can benefit from trading the excess carbon emissions quotas. Therefore, the value of carbon cap determines whether the retailers operating under the carbon cap-and-trade policy can make greater profits than those operating under without carbon emissions regulations. Thus it affects their acceptance of the carbon cap-and-trade policy. This conclusion can be used as a reference for the carbon management agencies to allocate the quotas to the firms.

Corollary 4 The maximum profit of retailers operating under the carbon cap-and-trade policy is no less than that of retailers operating under the carbon tax policy when $C_c \geq h_1^e D(p_1^*) \frac{u^2T^2}{2n_1^*} + n_1^* A_e + c_1^e D(p_1^*) \left\{ \frac{u^2T^2[\theta(\xi_1^*)+\beta]}{2n_1^*} + T \right\}$, i.e. $TP_3(n_3^*, \xi_3^*, p_3^*) \geq TP_2(n_2^*, \xi_2^*, p_2^*)$.

Corollary 4 gives a sufficient condition for retailers to prefer the carbon cap-and-trade policy to the carbon tax policy. It is clear that the value of the carbon cap plays a vital role in the comparison of the profit of two carbon policy. Because under carbon cap-and-trade policy retailers can trade excess quotas to the market, so when the cap is large enough, retailers tend to accept the carbon cap-and-trade policy.

6. Numerical examples

To illustrate the models, six numerical examples are given. The examples are related to the inventory model under two carbon policies. The first example is a basis for comparison with the other examples for the case. The first illustrates the situation when the constraint on preservation technology investment is loose; namely, the optimal solution for ξ exists during the interval $[0, \bar{\xi}]$. The second example assumes the initial deterioration rate is very small. The third example considers a case where the sensitivity parameter of preservation technology to deterioration rate is relatively small.

6.1. The carbon tax policy

Example 1 Consider $T = 60$, $a = 30$, $b = 0.2$, $c = 10$, $g = 5$, $h = 0.02$, $s = 1$, $\beta = 0.01$, $u = 0.5$, $\theta_0 = 0.02$, $\delta = 0.5$, $A_e = 100$, $c_1^e = 4$, $h_1^e = 2$, $\lambda = 0.1$, $K = 100$, $\bar{\xi} = 10$. According to Algorithm 1, Step 1. In the case of $n = 3$; Step 2. Initialize the iteration times $m = 1$ and set the value of $p^1 = 40$; Step 3. Compute $\Delta_2(n, p)$, $\Delta_3(n, p)$. Because $\Delta_2(n, p) = \Delta_2(3, 40) = 1494.6 > 0$, and $\Delta_3(n, p) = \Delta_3(3, 40) = -2.076 < 0$, so compute ξ_2^m with (26) equal to zero and get $\xi_2^m = \xi_2^1 = 5.6594$. Substitute ξ_2^m into Eq. (30) and compute p_2^m, we get $p_2^m = p_2^1 = 81.71$. Set $p^2 = p_2^1 = 81.71$ and $\xi^1 = \xi_2^1 = 5.6594$; Step 4. Because $\left| p^2 - p^1 \right| = 81.71 - 40 > \tau = 10^{-4}$, set $m = m + 1 = 1 + 1 = 2$ and go to Step 3. So again and again, when $m = 3$, we get

$\left|p^4 - p^3\right| = 81.7235 - 81.7235 \ < \tau = 10^{-4}$, then $(\xi^*, p^*) = (\xi^3, p^4) = (4.7055, 81.7235)$ and go to Step 5; Step 5. Determine $TP_2(n, \xi^*, p^*)$ with Eq. (18), $TP_2(n, \xi^*, p^*) = 56,867$ is the maximum value for a fixed $n = 3$.

Continue the steps 6 to 9 of algorithm 1, For different n values, the optimal solution is easily obtained. As Table 2 shows, in this situation TP_2 is a concave function of n. Notice that when n = 6, TP_2 achieves its maximum value. It is TP_2 = \$57105. In this numerical example, the upper boundary of preservation technology is not active. Figure 2 shows how the total profit TP_2 varies with ξ and p when n = 6. It shows that TP_2 is a concave function of ξ and p.

The calculation process of other examples is similar to example 1 and will not be repeated.

Example 2 Consider $T = 60$, $a = 30$, $b = 0.2$, $c = \$10$, $g = \$5$, $h = 0.02$, $s = 1$, $\beta = 0.01$, $u = 0.5$, $\theta_0 = 0.0001$, $\delta = 0.5$, $K = 100$, $n = 6$. We assume now that θ_0 is very small, namely $\theta_0 = 0.0001$. From algorithm 1, it is easy to show that $\Delta_2(n, p) < 0$, so the optimal preservation technology investment is $\xi^* = 0$. This indicates that when the initial deterioration rate is very small, the input of preservation technology is not good for the total profit.

Example 3 Let $T = 60$, $a = 30$, $b = 0.2$, $c = \$10$, $g = 5$, $h = 0.02$, $s = 1$, $\beta = 0.01$, $u = 0.5$, $\theta_0 = 0.02$, $\delta = 0.005$, $K = 100$, $n = 6$. From algorithm 1, it is easy to obtain $\Delta_2(n, p) < 0$, so the optimal preservation technology investment is $\xi^* = 0$. This shows that when the sensitivity parameter of deterioration rate to investment (δ) is relatively small, it is harmful to invest the preservation technology.

6.2. The carbon cap-and-trade policy

Example 4 The value of the model parameters are as follows: $T = 60$, $a = 30$, $b = 0.2$, $c = 10$, $g = 5$, $h = 0.02$, $s = 1$, $\beta = 0.01$, $u = 0.5$, $\theta_0 = 0.02$, $\delta = 0.5$, $A_e = 100$, $c_1^e = 4$, $h_1^e = 2$, $c_p = 0.2$, $C_c = 5000$, $K = 100$. For some values of n, the optimal solution is obtained with algorithm 2. As table 3 shows, TP_3 is a concave function of ordering frequency n. The optimal solution is reached at $n = 6$. The optimal total profit is $TP_3 = \$57502$. In this numerical example, the optimal value of does not lie on the boundary of ξ.

Example 5 The model parameter values are as follows: $T = 60$, $a = 30$, $b = 0.2$, $c = 10$, $g = 5$, $h = 0.02$, $s = 1$, $\beta = 0.01$, $u = 0.5$, $\theta_0 = 0.0001$, $\delta = 0.5$, $K = 100$,

$n = 6$. If θ_0 is rather small, namely $\theta_0 = 0.0001$. From algorithm 2, it is easy to show that $\Delta_5(n,p) < 0$, so the preservation technology investment is $\xi^* = 0$. This indicates that when the initial deterioration rate is extremely small, the investment in preservation technology is not good for the profit.

Example 6 For this example the model parameters are $T = 60$, $a = 30$, $b = 0.2$, $c = 10$, $g = 5$, $h = 0.02$, $s = 1$, $\beta = 0.01$, $u = 0.5$, $\theta_0 = 0.02$, $\delta = 0.005$, $K = 100$, $n = 6$. Algorithm 2 indicates that $\Delta_5(n,p) < 0$, so the optimal preservation technology investment is $\xi^* = 0$. This indicates that when preservation technology has little effect on reducing deterioration rate, the input of it is not good for total profit.

7. Sensitivity analysis

The optimization of the total profit function presented above regards the system parameters as static values. It is useful for managers to know how robust the optimal policy is to changes in parameter values. For this purpose, sensitivity analysis is conducted by changing one of $a, b, c, g, h,$ s, $\beta, \theta_0, \delta, \lambda, c_p, C_c$ and keeping others parameters fixed.

7.1. The carbon tax policy

From table 4, the following conclusions are easily obtained.

(1) With the increase of a, β and δ, TP_2^* increases monotonically. With the decrease of parameters $b, c, g, h, \theta_0, s, \lambda$, TP_2^* increases monotonically. Further TP_2^* is more sensitive to a and b, but less sensitive to c, and particularly insensitive to parameters $g, h, s, \beta, \theta_0, \delta$ and λ.

(2) With the increase of parameters a and β, TE_2^* increases monotonically. Along with the decrease of b, c, g, h, s, δ and λ, TE_2^* increases monotonically. TE_2^* is independent of θ_0, but TE_2^* is more sensitive to parameters a, b, s and β. TE_2^* is less sensitive to parameters c and λ and very insensitive to parameters $g, h,$ and δ.

Managerial implications

i) With increase in the demand factor a, the ordering quantity per order Q^*, the preservation technology investment ξ^*, the optimal selling price p^*, the total profit TP_2^*, and the total carbon emissions TE_2^* will increase. This indicates that as the market demand increases, to reduce the deterioration loss, retailers should invest more in preservation technology. At the same time, retailers also increase the selling price to obtain more profits. Similarly, because of the increase in market demand, retailers tend to order and sell more deteriorating items to get more profit which in turn causes more carbon emissions.

ii) As the price elasticity b rises, the ordering quantity per order Q^*, the preservation technology investment ξ^*, the optimal selling price p^*, the total profit TP_2^* and the total carbon emissions TE_2^* decrease. This shows that when the customer demand is more sensitive to price, the retailer tends to lower the selling price to avoid sharp decreases in the market demand. Even so, the market demand still shrinks compared to the market demand before. Retailers will then tend to order and sell less deteriorating items. To reduce unnecessary expenditure, the retailers lower the preservation technology investment. Meantime the total profit decreases because of the reduction in market demand and the lower selling price. As a result, the carbon emissions are also reduced.

iii) The preservation technology investment cost increases with respect to c, g and θ_0. This suggests that when the buying cost, the deteriorating cost and the initial deterioration rate are large, the retailer tends to invest more in preservation technology. The deterioration cost increases with β. This tells us that when the stock level has a more significant effect on market demand, the retailer will put more preservation technology investment on the deal with the operation of deteriorating products.

iv) The ordering frequency n^* increases with increases in s and λ. n^* decreases with an increase in β. This indicates that when the shortage cost per unit is high, the retailers will increase the ordering frequency to avoid excessive shortage costs. When the stock level has enough effect on the market demand, retailers tend to reduce the ordering frequency and increase the ordering quantity per order to maintain a relatively high stock level to stimulate the market demand. When the carbon tax price is high, retailers increase the ordering frequency and reduce the ordering quantity per order to reduce the total carbon emissions to avoid excessive carbon emissions costs.

7.2. The carbon cap-and-trade policy

For this case a sensitivity analysis is performed using Example 4.

From the results of table 5, the following conclusions are easily obtained.

i) When parameters a, β, δ and C_c increase, the total profit TP_3^* increases. With decreasing values of parameters b, c, g, h, θ_0, s and c_p , the total profit TP_3^* also increases. The total profit TP_3^* is more sensitive to a and b; less sensitive to c; and very insensitive to parameters $g, h, s, \beta, \theta_0, \delta, C_c$ and c_p.

ii) Following the increase of parameters a and β, the total carbon emissions TE_3^* increases. However, when the parameters b, c, g, h, δ, s and c_p decrease, the total carbon emissions TE_3^* increases. Particularly the total carbon emissions TE_3^* changes nothing when the parameter θ_0、 C_c increases or decreases. The total carbon emissions TE_3^* is more sensitive to a, b and s ; less sensitive to c, β and c_{p}; and rather insensitive to g, h and δ.

Managerial implications

i) When the demand factor a increases, the ordering quantity per order Q^*, the preservation technology investment ξ^*, the optimal selling price p^*, the total profit TP_3^*, and the total carbon emissions TE_3^* increase. This shows that when the market demand increases, to reduce loss from deterioration, the retailer will invest more in preservation technology. Meanwhile, retailers increase the price to get more profit. Equally, to achieve more significant profit, they tends to order and sell more deteriorating products. As a result, there are more carbon emissions.

ii) With the increase of price elasticity parameter b, the preservation technology ξ^*, the optimal selling price p^*,the total profit TP_3^*, and the total carbon emissions TE_3^* decrease. This indicates that when the customer demand is more sensitive to selling price, retailer tends to lower the selling price to avoid a sharp reduction in demand. Even so, compared with the initial demand, the market demand shrinks. Consequently, the retailer orders and sells fewer deteriorating products and, to reduce unnecessary expenditure, the retailer reduces the investment in preservation technology. Due the reduction in market demand and selling price, the total profit tends to decrease. The total carbon emissions are also smaller than before.

iii) The investment in preservation technology increases with the parameters c, g and θ_0. This suggests that when the buying cost, the processing cost of deteriorated products, and the initial deterioration rate are higher, the retailer will invest more in preservation technology. The cost of preservation technology increases with an increase in β. This shows that when the demand is more sensitive to stock level, retailer tends to put more funds in preservation technology.

iv) The ordering frequency n^* increases with increases in s and c_p, but n^* decreases with increases in β. This indicates that when the shortage cost per unit is high, the retailers will increase the ordering frequency to avoid excessive shortage costs. When the stock level has enough effect on the market demand, the retailers tend to reduce the ordering frequency and increase the ordering quantity per order to maintain a relatively high stock level to stimulate the market demand. When the carbon tax price under the carbon cap-and-trade policy is high, the retailers increase the ordering frequency and decrease the ordering quantity per order to avoid excessive carbon emissions costs.

8. Conclusions

This paper establishes two inventory models that operate under two carbon policies. The models can simultaneously determine the optimal frequency, the selling price, and the preservation technology investment. The first model helps retailers operating under the carbon tax policy to obtain maximum profit. The second model solves the inventory problem for retailers operating under the carbon cap-and-trade policy. From the theorems given in this paper, it is easy to establish the existence and uniqueness of the optimal solution for each inventory model. Additionally, algorithms are developed to solve the optimal solution. Numerical examples are used to illustrate the inventory models and the solution algorithms. Finally, sensitivity analysis is performed and the results provide important managerial insights.

The advantages of this article include: the applications of preservation technology and carbon regulations are considered simultaneously firstly in this paper, which fills the gap in the literature for inventory management models for deteriorating products. Secondly the investment of preservation technology is a function of the period of inventory rather than the entire order cycle. In this way, the model can describe the cost of preservation technology more accurately.

This paper also has some limitations, this future research can further be extended by considering stochastic demand. Additionally, the time value of money can also be included in the research by considering trade credit. Another extension to the research is possible by assuming the lead time is not zero.

References

Aviyonah, R. S., and D. M. Uhlmann. 2009. "Combating Global Climate Change: Why a Carbon Tax is a Better Response to Global Warming Than Cap and Trade." *Ssrn Electronic Journal* 28(3):1-50. doi:10.2139/ssrn.1109167.

Benjaafar, S., Li, Y., and M. Daskin. 2013. "Carbon Footprint and the Management of Supply Chains: Insights From Simple Models." *IEEE Transactions on Automation Science and Engineering* 10(1): 99-116. doi: 10.1109/tase.2012. 2203304.

Chang, C. T., J. T. Teng, and M. S. Chern. 2010. "Optimal Manufacturer's Replenishment Policies for Deteriorating Items in a Supply Chain with Up-Stream and Down-Stream Trade Credits." *International Journal of Production Economics* 127 (1): 197-202. doi:10.1016/j.ijpe.2010.05.014.

Chen, X., W. Gong., and F. Wang. 2017. "Managing Carbon Footprints under the Trade Credit." *Sustainability* 9(7): 1235. doi:10.3390/su9071235.

Chen, L., X. Chen, F. K. Matthew, and G. Li. 2018. "Optimal Pricing and Replenishment Policy for Deteriorating Inventory under Stock-Level-Dependent, Time-Varying and Price-Dependent Demand." *Computers and Industrial Engineering* 135: 1294-1299. doi:10.1016/j.cie.2018.06.005.

Chen, X., S. Benjaafar, and A. Elomri. 2013. "The Carbon-Constrained EOQ." *Operations Research Letters* 41 (2): 172–179. doi:10.1016/j.orl.2012.12.003.

Dipankar, C., J. D. Kumar, and R. T. Kumar. 2018. "Two-warehouse Partial Backlogging Inventory Model with Ramp Type Demand Rate, Three-Parameter Weibull Distribution Deterioration under Inflation and Permissible Delay in Payments." *Computers and Industrial Engineering* 123: 157–179. doi:10.1016/ j.cie.2018.06. 022.

Dye, C. Y., and T. P. Hsieh. 2012. "An Optimal Replenishment Policy for Deteriorating Items with Effective Investment in Preservation Technology." *European Journal of Operational Research* 218 (1): 106-112. doi:10.1016/j.ejor.2011.10.016.

Dye, C. Y. 2013. "The Effect of Preservation Technology Investment on a Non-Instantaneous Deteriorating Inventory Model." *Omega* 41 (5): 872-880. doi:10.1016/j.omega.2012.11.002.

Dye, C. Y., and C. T. Yang. 2015. "Sustainable Trade Credit and Replenishment Decisions with Credit-Linked Demand under Carbon Emission Constraints."

European Journal of Operational Research 244 (1): 187-200. doi:10.1016/j.ejor.2015.01.026.

Dye, C. Y., C. T. Yang, and B. Lev. 2016. "Optimal Dynamic Pricing and Preservation Technology Investment for Deteriorating Products with Reference Price Effects." *Omega* 62: 52-67. doi:10.1016/j.omega.2015.08.009.

Dye, C. Y., C. T. Yang, and C. C. Wu. 2017. "Joint Dynamic Pricing and Preservation Technology Investment for an Integrated Supply Chain with Reference Price Effects." *Journal of the Operational Research Society* 69 (6): 811-824. doi:10.1057/s41274-017-0247-y.

Feng, L. 2018. "Dynamic Pricing, Quality Investment, and Replenishment Model for Perishable Items." *International Transactions in Operational Research* 26(4): 1558–1575. doi:10.1111/itor.12505.

Hammami, R., I. Nouira, and Y. Frein. 2015. "Carbon Emissions in a Multi-Echelon Production-Inventory Model with Lead Time Constraints." *International Journal of Production Economics* 164: 292-307. doi:10.1016/j.ijpe.2014.12.017.

He, Y., and H. Huang. 2013. "Optimizing Inventory and Pricing Policy for Seasonal Deteriorating Products with Preservation Technology Investment." *Journal of Industrial Engineering*, 2013, 1-7. doi: 10.1155/2013/793568.

He, P., W. Zhang, X. Xu, and Y. Bian. 2015. "Production Lot-sizing and Carbon Emissions under Cap-and-trade and Carbon Tax Regulations." *Journal of Cleaner Production* 103: 241-248. doi:10.1016/j.jclepro.2014.08.102.

Hsieh, T. P., and C. Y. Dye. 2013. "A Production Inventory Model Incorporating the Effect of Preservation Technology Investment when Demand is Fluctuating with Time." *Journal of Computational and Applied Mathematics* 239 (1): 25-36. doi:10.1016/j.cam.2012.09.016.

Hsu, P. H., H. M. Wee, and H. M. Teng. 2010. "Preservation Technology Investment for Deteriorating Inventory." *International Journal of Production Economics* 124 (2): 388-394. doi:10.1016/j.ijpe.2009.11.034 .

Hua, G., T. C. E. Cheng, and S. Wang, 2011. "Managing Carbon Footprints in Inventory Management." *International Journal of Production Economics* 132 (2): 178-185. doi:10.1016/j.ijpe.2011.03.024.

Hua, G. W., T. C. E. Cheng, Y. Zhang, and J. L. Zhang. 2016. "Carbon-Constrained Perishable Inventory Management with Freshness-Dependent Demand."

International Journal of Simulation Modelling 15 (3): 542–552. doi:10.2507/ ijsimm15(3)co12.

Huang, H., Y. He, and D. Li. 2018. "Pricing and Inventory Decisions in the Food Supply Chain with Production Disruption and Controllable Deterioration." *Journal of Cleaner Production* 180: 280-296. doi:10.1016/j. jclepro.2018.01.152.

IPCC. Climate Change 2007: The Physical Science Basis, 2007. Available online: http://www.slvwd.com/agendas/Full/2007/06-07-07/Item%2010b.pdf (accessed on 4 December 2015).

Qin, J., Bai. X., and Xia. L. 2015. "Sustainable Trade Credit and Replenishment Policies under the Cap-and-trade and Carbon Tax Regulations." *Sustainability* 7 (12): 16340-16361. doi:10.3390/su71215818.

Larissa Janssen, Ali Diabat, Jürgen Sauer, andFrankHerrmann.2018. "A stochastic micro-periodic age-basedinventory replenishment policy for perishablegoods." *Transportation Research Part E: Logistics and Transportation Review* 118: 445–465. doi.org/10.1016/j.tre.2018.08.009.

Lee, Y. P., and C. Y. Dye 2012. "An Inventory Model For Deteriorating Items under Stock-Dependent Demand and Controllable Deterioration Rate." *Computers & Industrial Engineering* 63 (2): 474-482. doi:10.1016/j.cie.2012.04.006.

Liu, G., J. Zhang, and W. Tang. 2015. "Joint Dynamic Pricing and Investment Strategy for Perishable Foods with Price-Quality Dependent Demand." *Annals of Operations Research* 226 (1): 397-416. doi:10.1007/s10479-014-1671-x.

Mishra, V. K. 2013. "Deteriorating Inventory Model using Preservation Technology with Salvage Value and Shortages." *Advances in Production Engineering and Management* 8 (3): 185–192. doi:10.14743/apem2013.3.166.

Mishra, U., L. E. Cárdenas-Barrón, S. Tiwari, A. A. Shaikh, and G. Treviño-Garza. 2017. "An Inventory Model under Price and Stock Dependent Demand for Controllable Deterioration Rate with Shortages and Preservation Technology Investment." *Annals of Operations Research* 254 (1-2): 165–190. doi:10.1007/s10479-017-2419-1.

Pando, V., L. A. San-Jos, J. Garca-Laguna, and J. Sicilia. 2018. "Optimal lot-size Policy for Deteriorating Items with Stock-Dependent Demand Considering Profit Maximization." *Computers and Industrial Engineering* 117: 81-93. doi:10.1016/ j.cie.2018.01.008.

Sazvar, Z., A. Baboli, and M. R. Akbari Jokar. 2013. "A Replenishment Policy for Perishable Products with Non-Linear Holding Cost under Stochastic Supply Lead Time." *The International Journal of Advanced Manufacturing Technology* 64 (5-8): 1087-1098. doi:10.1007/s00170-012-4042-2.

Schrader, S. F. 1963. "A Model for Exponentially Decaying Inventory." *The Journal of Industrial Engineering* 5(14): 238–243. doi: https://lirias.kuleuven.be/handle/123456789/360242.

Shah, N. H., H. N. Soni, and K. A. Patel. 2013. "Optimizing Inventory and Marketing Policy for Non-Instantaneous Deteriorating Item with Generalized Type Deterioration and Holding Cost Rates." *Omega* 41(2): 421-430. doi:10.1016/j.omega.2012.03.002.

Shaikh, A. A., M. A.-A. Khan, G. C. Panda, and I. Konstantaras. 2019. "Price Discount Facility in an EOQ Model for Deteriorating Items with Stock-Dependent Demand and Partial Backlogging." *International Transactions in Operational Research* 26 (4): 1365–1395. doi:10.1111/itor.12632.

Tiwari, S., L. Eduardo Cárdenas-Barrón, M. Goh, and A. A. Shaikh. 2018a. "Joint Pricing and Inventory Model for Deteriorating Items with Expiration Dates and Partial Backlogging under Two-level Partial Trade Credits in Supply Chain." *International Journal of Production Economics* 200:16-36. doi:10.1016/j.ijpe.2018.03.006.

Tiwari, S., Y. Daryanto. and H. M. Wee. 2018b. "Sustainable Inventory Management with Deteriorating and Imperfect Quality Items Considering Carbon Emission." *Journal of Cleaner Production* 192: 281–292. doi:10.1016/j.jclepro.2018.04.261.

Toptal, A., H. Özlü, and D. Konur. 2013. "Joint Decisions on Inventory Replenishment and Emission Reduction Investment under Different Emission Regulations." *International Journal of Production Research* 52(1): 243–269. doi:10.1080/00207543.2013.836615.

Zhong-Zhong Jiang, Na He, Lei Xiao, and Ying Sheng. 2019. "Government subsidy provision in biomass energy supply chains." *Enterprise Information Systems* 13(10): 1367–1391. doi:10.1080/17517575.2019.1658807.

Appendix

Appendix 1: Proof of Theorem 1

The first partial derivative of $TP_2(n,\xi,p)$ with respect to n is given by equation (24), the second partial derivative is given by equation (25).

Let $G(n)=\frac{\partial TP_2(n,\xi,p)}{\partial n}$

$$\frac{\partial TP_2(n,\xi,p)}{\partial n}=-\frac{p\beta D(p)(uT)^2}{2n^2}+\frac{cD(p)(uT)^2[\theta(\xi)+\beta]}{2n^2}+\frac{hD(p)(uT)^2}{2n^2}+\frac{sD(p)[uT-T]^2}{2n^2}-K+\frac{gD(p)\theta(\xi)(uT)^2}{2n^2}-[A_e-\frac{c_1^eD(p)(uT)^2[\theta(\xi)+\beta]}{2n^2}-\frac{h_1^eD(p)(uT)^2}{2n^2}]\lambda \quad (24)$$

$$\frac{\partial^2 TP_2(n,\xi,p)}{\partial n^2}=\frac{p\beta D(p)(uT)^2}{n^3}-\frac{cD(p)(uT)^2[\theta(\xi)+\beta]}{n^3}-\frac{hD(p)(uT)^2}{n^3}-\frac{sD(p)[uT-T]^2}{n^3}-\frac{gD(p)\theta(\xi)(uT)^2}{n^3}-[\frac{c_1^eD(p)(uT)^2[\theta(\xi)+\beta]}{n^3}+\frac{h_1^eD(p)(uT)^2}{n^3}]\lambda \quad (25)$$

Let $\frac{\partial^2 TP_2(n,\xi,p)}{\partial n^2}<0$, after simplification, it can be shown that

$$\beta<\frac{c\theta(\xi)+h+s\left(1-\frac{1}{u}\right)^2+g\theta(\xi)+[c_1^e\theta(\xi)+h_1^e]\lambda}{p-c-c_1^e\lambda}=\Delta_1(n,p)$$

In this case, TP_2 is a concave function of ordering frequency n.

Let $\frac{\partial^2 TP_2(n,\xi,p)}{\partial n^2}\geq 0$, after simplification, it can be shown that

$$\beta\geq\frac{c\theta(\xi)+h+s\left(1-\frac{1}{u}\right)^2+g\theta(\xi)+[c_1^e\theta(\xi)+h_1^e]\lambda}{p-c-c_1^e\lambda}=\Delta_1(n,p)$$

Then $G(n)$ is a monotonically increasing function with respect to n.

$$\lim_{n\to\infty} G(n)=-K-A_e\,\lambda<0$$

So the function TP_2 is monotonically decreasing with respect to ordering frequency n.

Because n is an integer, so to get the optimal solution, it is only necessary to find the integer optimal solution.

Appendix 2: Proof of Theorem 2

The first partial derivative of $TP_2(n,\xi,p)$ with respect to ξ is given by Equation (26); the second partial derivative is shown in Equation (27).

Let $F(\xi) = \frac{\partial TP_2(n,\xi,p)}{\partial \xi}$

$$\theta(\xi) = \theta_0 e^{-\xi\delta}$$

$$\theta'(\xi) = -\delta\theta_0 e^{-\xi\delta} < 0$$

$$\theta''(\xi) = \delta^2\theta_0 e^{-\xi\delta} > 0$$

$$\frac{\partial TP_2(n,\xi,p)}{\partial \xi} = -(c + g + c_1^e\lambda)\frac{D(p)(uT)^2\theta'(\xi)}{2n} - uT \quad (26)$$

$$\frac{\partial^2 TP_2(n,\xi,p)}{\partial \xi^2} = -(c + g + c_1^e\lambda)\frac{D(p)(uT)^2\theta''(\xi)}{2n} < 0 \quad (27)$$

So the function $F(\xi)$ is monotonically decreasing with respect to ξ.

$$F(0) = \frac{\delta\theta_0 D(p)(uT)^2}{2n}(c + g + c_1^e\lambda) - uT = \Delta_2(n,p)$$

$$F(\bar{\xi}) = \frac{\delta\theta_0 e^{-\bar{\xi}\delta} D(p)(uT)^2}{2n}(c + g + c_1^e\lambda) - uT = \Delta_3(n,p)$$

When $\Delta_2(n,p) \leq 0$, then $F(\xi)$ is always less or equal to zero during the interval $[0, \bar{\xi}]$, In this case, $TP_2(n,\xi,p)$ is monotonically decreasing with respect to ξ and the optimal preservation technology investment $\xi_2^* = 0$.

When $\Delta_3(n,p) \geq 0$, $F(\xi)$ is always greater or equal to zero during the interval $[0, \bar{\xi}]$, In this case, $TP_2(n,\xi,p)$ is monotonically increasing with respect to ξ and the optimal preservation technology investment $\xi_2^* = \bar{\xi}$.

When $\Delta_2(n,p) > 0$ and $\Delta_3(n,p) < 0$, $TP_2(n,\xi,p)$ increases at first and then decreases. There exists $\xi_2 \in (0,\bar{\xi})$ satisfying $F(\xi_2) = 0$. The optimal preservation technology investment is $\xi_2^* = \xi_2$.

Appendix 3: Proof of Theorem 3

The first derivative of $TP_2(n,\xi,p)$ concerning the selling price p is given by Equation (28), and the second derivative is shown in Equation (29).

$$\frac{\partial TP_2(n,\xi,p)}{\partial p} = (a - 2bp)T + \frac{(a-2bp)(uT)^2\beta}{2n} + cb\left\{\frac{(uT)^2[\theta(\xi)+\beta]}{2n} + T\right\} + \frac{hb(uT)^2}{2n} + \frac{bs(uT-T)^2}{2n} + \frac{gb\theta(\xi)(uT)^2}{2n} + \left\{c_1^e b\left\{\frac{(uT)^2[\theta(\xi)+\beta]}{2n} + T\right\} + \frac{h_1^e b(uT)^2}{2n}\right\}\lambda \quad (28)$$

$$\frac{\partial^2 TP_2(n,\xi,p)}{\partial p^2} = -2bT - \frac{\beta b(uT)^2}{n} < 0 \quad (29)$$

Because the second derivative of $TP_2(n,\xi,p)$ is less than zero, TP_2 is a concave function concerning p. In other words, there exists a unique p_2^* that maximizes TP_2 for fixed n and fixed ξ.

Let $\frac{\partial TP_2(n,\xi,p)}{\partial p} = 0$; after simplification, it can be shown that

$$p_2^* = \left\{aT + \frac{a\beta(uT)^2}{2n} + bc\left\{\frac{(uT)^2[\theta(\xi)+\beta]}{2n} + T\right\} + \frac{hb(uT)^2}{2n} + \frac{bs(uT-T)^2}{2n} + \frac{gb\theta(\xi)(uT)^2}{2n} + \left\{c_1^e b\left\{\frac{(uT)^2[\theta(\xi)+\beta]}{2n} + T\right\} + \frac{h_1^e b(uT)^2}{2n}\right\}\lambda\right\} / [2bT + \frac{b\beta(uT)^2}{n}] \quad (30)$$

p_2^* is the price that maximizes TP_2 for fixed n and fixed ξ.

This ends the proof of Theorem 3.

Appendix 4: Proof of Theorem 6

The first derivative of $TP_3(n,\xi,p)$ concerning selling price n is given by Equation (31), and the second derivative is shown in Equation (32).

Let $G'(n) = \frac{\partial TP_3(n,\xi,p)}{\partial n}$

$$\frac{\partial TP_3(n,\xi,p)}{\partial n} = -\frac{p\beta D(p)(uT)^2}{2n^2} + \frac{cD(p)(uT)^2[\theta(\xi)+\beta]}{2n^2} + \frac{hD(p)(uT)^2}{2n^2} + \frac{sD(p)[uT-T]^2}{2n^2} - K + \frac{gD(p)\theta(\xi)(uT)^2}{2n^2} + [-A_e + \frac{c_1^e D(p)(uT)^2[\theta(\xi)+\beta]}{2n^2} + \frac{h_1^e D(p)(uT)^2}{2n^2}]c_p \quad (31)$$

$$\frac{\partial^2 TP_3(n,\xi,p)}{\partial n^2} = \frac{p\beta D(p)(uT)^2}{n^3} + [-\frac{cD(p)(uT)^2[\theta(\xi)+\beta]}{n^3} - \frac{hD(p)(uT)^2}{n^3} - \frac{sD(p)[uT-T]^2}{n^3} - \frac{gD(p)\theta(\xi)(uT)^2}{n^3} - [\frac{c_1^e D(p)(uT)^2[\theta(\xi)+\beta]}{n^3} + \frac{h_1^e D(p)(uT)^2}{n^3}] \quad (32)$$

Let $\frac{\partial^2 TP_3(n,\xi,p)}{\partial n^2} < 0$, after simplification, it can be shown that

$$\beta < \frac{c\theta(\xi) + h + s\left(1 - \frac{1}{u}\right)^2 + g\theta(\xi) + [c_1^e\theta(\xi) + h_1^e]c_p}{p - c - c_1^e c_p} = \Delta_4(n,p)$$

In this situation, TP_3 is a concave function concerning ordering frequency n.

When $\beta < \frac{c\theta(\xi)+h+s\left(1-\frac{1}{u}\right)^2+g\theta(\xi)+[c_1^e\theta(\xi)+h_1^e]c_p}{p-c-c_1^e c_p} = \Delta_4$, or in other words $\frac{\partial^2 TP_3(n,\xi,p)}{\partial n^2} \geq 0$, it can be inferred that $G(n)$ is monotonically increasing.

$$\lim_{n\to\infty} G'(n) = -K - A_e\, c_p < 0$$

So TP_3 is a monotonically decreasing function concerning ordering frequency n.

Because n is an integer, so in order to get the optimal solution, it is only necessary to achieve the integer optimal solution.

This ends the proof of Theorem 6.

Appendix 5: Proof of Theorem 7

The first derivative of $TP_3(n,\xi,p)$ with respect to ξ is given by Equation (33); and the second derivative is shown by Equation (34).

Let $F(\xi) = \frac{\partial TP_3(n,\xi,p)}{\partial \xi}$

$$\theta(\xi) = \theta_0 e^{-\xi\delta}$$

$$\theta'(\xi) = -\delta\theta_0 e^{-\xi\delta} < 0$$

$$\theta''(\xi) = \delta^2\theta_0 e^{-\xi\delta} > 0$$

$$\frac{\partial TP_3(n,\xi,p)}{\partial \xi} = -(c + g + c_1^e c_p)\frac{D(p)(uT)^2\theta'(\xi)}{2n} - uT \tag{33}$$

$$\frac{\partial^2 TP_3(n,\xi,p)}{\partial \xi^2} = -(c + g + c_1^e c_p)\frac{D(p)(uT)^2\theta''(\xi)}{2n} < 0 \tag{34}$$

So $F(\xi)$ is a monotonically decreasing function of ξ.

$$F(0) = \frac{\delta\theta_0 D(p)(uT)^2}{2n}\left(c + g + c_1^e c_p\right) - uT = \Delta_5(n,p)$$

$$F(\bar{\xi}) = \frac{\delta\theta_0 e^{-\bar{\xi}\delta} D(p)(uT)^2}{2n}\left(c + g + c_1^e c_p\right) - uT = \Delta_6(n,p)$$

When $\Delta_5(n,p) \leq 0$, then $F(\xi)$ is never greater than zero in interval $[0, \bar{\xi}]$. Hence, the total profit $TP_3(n,\xi,p)$ is a monotonically decreasing function concerning ξ. In this situation, the optimal preservation technology investment is $\xi^* = 0$.

When $\Delta_6(n,p) \geq 0$, $F(\xi)$ is never less than zero on the interval $[0, \bar{\xi}]$, and so total profit $TP_3(n,\xi,p)$ is a monotonically increasing function concerning ξ. In this situation, the optimal preservation technology is $\xi^* = \bar{\xi}$.

When $\Delta_5(n,p) > 0$ and $\Delta_6(n,p) < 0$, the total profit $TP_3(n,\xi,p)$ increases at first and then decreases. There exists $\xi_3 \in (0, \bar{\xi})$ satisfying the equation $F(\xi_3) = 0$. In this case, the optimal preservation technology cost is $\xi^* = \xi_3$。

Appendix 6: Proof of Theorem 8

The first derivative of the total profit $TP_3(n,\xi,p)$ with respect to p is given by Equation (35), and the second derivative is shown by Equation (36):

$$\frac{\partial TP_3(n,\xi,p)}{\partial p} = (a-2bp)T + \frac{\beta(a-2bp)(uT)^2}{2n} + cb\left\{\frac{(uT)^2[\theta(\xi)+\beta]}{2n} + T\right\} + \frac{hb(uT)^2}{2n} + \frac{bs(uT-T)^2}{2n} + \frac{gb\theta(\xi)(uT)^2}{2n} + \left\{c_1^e b\left\{\frac{(uT)^2[\theta(\xi)+\beta]}{2n} + T\right\} + \frac{h_1^e b(uT)^2}{2n}\right\} c_p \quad (35)$$

$$\frac{\partial^2 TP_3(n,\xi,p)}{\partial p^2} = -2bT - \frac{b\beta(uT)^2}{n} < 0 \quad (36)$$

Owing to the second derivatives of $TP_3(n,\xi,p)$ being less than zero, TP_3 is a concave function concerning p. That is to say; there exists a unique p_3^* that maximizes TP_3 for fixed n and fixed ξ.

Let $\frac{\partial TP(n,\xi,p)}{\partial p} = 0$, it can be shown that

$$p_3^* = \left\{aT + \frac{a\beta(uT)^2}{2n} + bc\left\{\frac{(uT)^2[\theta(\xi)+\beta]}{2n} + T\right\} + \frac{hb(uT)^2}{2n} + \frac{bs(uT-T)^2}{2n} + \frac{gb\theta(\xi)(uT)^2}{2n} + \left\{c_1^e b\left\{\frac{(uT)^2[\theta(\xi)+\beta]}{2n} + T\right\} + \frac{h_1^e b(uT)^2}{2n}\right\} c_p\right\} / [2bT + \frac{b\beta(uT)^2}{n}] \quad (37)$$

p_3^* is the price that maximizes TP_3 calculated by Equation (23) for fixed n and fixed ξ.

This ends the proof of Theorem 8.

Appendix 7: Proof of Theorem 10

Prove：

Case 1: When $n_1^* = n_3^*, \xi_1^* = \xi_3^*, p_1^* = p_3^*$, $TP_3(n_3^*,\xi_3^*,p_3^*) - TP_1(n_1^*,\xi_1^*,p_1^*) = \{C_c - \{h_1^e D(p_1^*)\frac{u^2T^2}{2n_1^*} + n_1^* A_e + c_1^e D(p_1^*)\{\frac{u^2T^2[\theta(\xi_1^*)+\beta]}{2n_1^*} + T\}\}\}c_p$. It follows that if $C_c \geq \left\{c_1^e D(p_1^*)\left\{\frac{u^2T^2[\theta(\xi_1^*)+\beta]}{2n_1^*} + T\right\} + \frac{h_1^e D(p_1^*)(uT)^2}{2n_1^*} + n_1^* A_e\right\}$, then $TP_3(n_3^*,\xi_3^*,p_3^*) \geq TP_1(n_1^*,\xi_1^*,p_1^*)$; otherwise $TP_1(n_1^*,\xi_1^*,p_1^*) \geq TP_3(n_3^*,\xi_3^*,p_3^*)$.

Case 2：When $n_1^* \neq n_3^*, \xi_1^* \neq \xi_3^*, p_1^* \neq p_3^*$ and $C_c \geq \left\{c_1^e D(p_1^*)\left\{\frac{u^2T^2[\theta(\xi_1^*)+\beta]}{2n_1^*} + T\right\} + n_1^* A_e + \frac{h_1^e D(p_1^*)(uT)^2}{2n_1^*}\right\}$, it is easy to show from the optimal solution property of total profit $TP_3(n,\xi,p)$ with respect to n,ξ,p that $TP_3(n_3^*,\xi_3^*,p_3^*) \geq TP_3(n_1^*,\xi_1^*,p_1^*)$. From case 1, in this situation $TP_3(n_1^*, \xi_1^*, p_1^*) - TP_1(n_1^*, \xi_1^*, p_1^*) = c_p \left\{C_c - \left\{c_1^e D(p_1^*)\left\{\frac{u^2T^2[\theta(\xi_1^*)+\beta]}{2n_1^*} + T\right\} + \frac{h_1^e D(p_1^*)(uT)^2}{2n_1^*} + n_1^* A_e\right\}\right\}$, and so $TP_3(n_1^*, \xi_1^*, p_1^*) \geq TP_1(n_1^*, \xi_1^*, p_1^*)$. Combining these two inequalities, $TP_3(n_3^*, \xi_3^*, p_3^*) \geq TP_1(n_1^*, \xi_1^*, p_1^*)$. On the other hand, when $C_c \leq \left\{c_1^e D(p_3^*)\left\{\frac{u^2T^2[\theta(\xi_3^*)+\beta]}{2n_3^*} + T\right\} + n_3^* A_e + \frac{h_1^e D(p_3^*)u^2T^2}{2n_3^*}\right\}$, it is easy to show from the optimal property of total profit $TP_1(n, \xi, p)$ with respect to n, ξ, p that $P_1(n_1^*, \xi_1^*, p_1^*) \geq TP_1(n_3^*, \xi_3^*, p_3^*)$. Similar to case 1, in this situation $TP_3(n_3^*, \xi_3^*, p_3^*) - TP_1(n_3^*, \xi_3^*, p_3^*) = \mathrm{c}_p \left\{C_c - \left\{c_1^e D(p_3^*)\left\{\frac{u^2T^2[\theta(\xi_3^*)+\beta]}{2n_3^*} + T\right\} + \frac{h_1^e D(p_3^*)u^2T^2}{2n_3^*} + n_3^* A_e\right\}\right\}$, and so $TP_1(n_3^*, \xi_3^*, p_3^*) \geq TP_3(n_3^*, \xi_3^*, p_3^*)$. Hence, $TP_1(n_1^*, \xi_1^*, p_1^*) \geq TP_3(n_3^*, \xi_3^*, p_3^*)$.

This concludes the proof of Theorem 10.

Tables

Table 1 List of parameters and their meaning

Decision Variables	Meaning of the variables
n	Ordering frequency
ξ	The cost of preservation technology investment per unit time
	(0 ≤ ξ ≤ ξ)
p	The selling price of products
Dependent Variables	
$I(t)$	Inventory level at time t
Q	Ordering quantity per order
t_1	The moment when the inventory level is drops to zero
P_τ	The quantity of products that deteriorate during the interval $[0,t_1]$
$\theta(\xi)$	Deterioration rate with preservation technology investment
$D(p,I(t))$	The integrated market demand
$D(p)$	The market demand
System Parameters	
T	Inventory cycle length
θ_0	Initial deterioration rate
δ	Sensitivity of preservation technology investment to deterioration rate
β	Sensitivity of stock to consumption ($0<\beta<1$)
a	Demand scale parameter
b	Sensitivity of demand to price
	Note: δ,β,a,b are estimated according to econometrics theory
Cost Parameters	
c	Unit buying cost
g	Per unit processing cost of the deteriorated product
h	Per unit inventory holding cost per unit time
s	Per unit shortage cost
K	Fixed cost per order

Carbon Emission Parameters	
A_e	Fixed carbon emissions of per order
c_1^e	Per unit carbon emission of orders
h_1^e	Per unit carbon emission for inventory per unit time
λ	Carbon tax under the Carbon tax policy(dollar/per unit carbon emission)
c_p	Carbon price under the Carbon cap-and-trade policy(dollar/per unit carbon emission)
C_c	Carbon cap under the Carbon cap-and-trade policy

Table 2 The optimal solution under the carbon tax policy

n	p^*	ξ^*	Q^*	TP^*	TE^*
3	81.7235	4.7055	281.5646	56,867	7,771.4
4	81.3584	4.1409	210.9148	57,036	6,861.0
5	81.1373	3.7010	168.6162	57,097	6,349.6
6	80.9889	3.3406	140.4523	57,105	6,039.9
7	80.8825	3.0354	120.3514	57,080	5,846.1
8	80.8024	2.7707	105.2843	57,035	5,725.2
9	80.7399	2.5369	93.5704	56,977	5,653.0

Table 3 The optimal solution under the carbon cap-and-trade policy

n	p^*	ξ^*	Q^*	TP^*	TE^*
3	82.1677	4.7437	279.7065	57,093	7,722.5
4	81.7427	4.1809	209.7136	57,352	6,824.5
5	81.4852	3.7421	167.7475	57,464	6,319.8
6	81.3125	3.3826	139.7796	57,502	6,014.0
7	81.1886	3.0779	119.8063	57,497	5,823.0
8	81.0954	2.8135	104.8279	57,464	5,704.0
9	81.0227	2.5800	93.1788	57,413	5,633.2

Table 4 Sensitivity analysis for Example 1

Parameters	%changes	ξ^*	n^*	Q^*	p^*	TP_2^*	TE_2^*	% Change TP_2^*	% Change TE_2^*
a	−50	2.1253	5	77.1407	43.6801	11,334	3,169	−80.15	−47.53
a	−25	2.7060	6	102.4502	62.2522	29,936	4,565.1	−47.58	−24.42
a	+25	4.1828	5	214.3224	99.8796	92,834	7,938.8	+62.57	+31.44
a	+50	5.0129	4	325.6142	118.8458	137,170	10,383	+140.21	+71.91
b	−50	6.9599	1	917.9244	159.4889	127,030	16,387	+122.45	+171.31
b	−25	4.1866	4	215.7771	106.3576	79,834	7,010.3	+39.80	+16.07
b	+25	3.2967	6	137.4179	65.9897	43,545	5,922.1	−23.75	−1.95
b	+50	2.9484	7	115.2506	55.8840	34,549	5,627.6	−39.50	−6.83
c	−50	2.6267	6	145.8460	78.4876	61,397	6,244	+7.52	+3.38
c	−25	3.0223	6	143.1172	79.7383	59,230	6,141.2	+3.72	+1.68
c	+25	3.6049	6	137.8243	82.2395	55,018	5,939.4	−3.65	−1.66
c	+50	3.8291	6	135.2197	83.4902	52,971	5,839.4	−7.24	−3.32
g	−50	2.9863	6	140.5843	80.9889	57,115	6,042.9	+0.02	+0.05
g	−25	3.1713	6	140.5124	80.9889	57,110	6,041.2	+0.01	+0.02
g	+25	3.4967	6	140.4012	80.9889	57,100	6,038.7	−0.01	−0.02
g	+50	3.6415	6	140.3574	80.9889	57,096	6,037.7	−0.02	−0.04
h	−50	3.3408	6	140.4648	80.9827	57,115	6,040.3	+0.02	+0.007
h	−25	3.3407	6	140.4032	80.9858	57,110	6,040.1	+0.01	+0.003
h	+25	3.3405	6	140.4460	80.9920	57,099	6,039.6	−0.01	−0.005
h	+50	3.3404	6	140.4398	80.9950	57,094	6,039.4	−0.02	−0.008
s	−50	6.8726	1	878.9117	82.7462	58,031	15,692	+1.62	+159.81
s	−25	4.1476	4	211.6185	81.1283	57,423	6,882.6	+0.56	+13.95
s	+25	3.0316	7	120.1218	81.0150	56,858	5,836.2	−0.43	−3.37
s	+50	2.7640	8	104.9325	81.0347	56,647	5,708.7	−0.80	−5.48
β	−50	3.0353	7	119.6884	80.8861	56,767	5,828.0	−0.59	−3.51
β	−25	3.0354	7	120.0188	80.8842	56,923	5,837.1	−0.32	−3.36
β	+25	3.7011	5	169.2813	81.1338	57,317	6,362.3	+0.37	+5.34
β	+50	4.7061	3	285.4011	81.7051	57,598	7,814.4	+0.86	+29.38

Table 4 (continued)

Parameters	%changes	ξ^*	n^*	Q^*	p^*	TP_2^*	TE_2^*	% Change TP_2^*	% Change TE_2^*
θ_0	−50	1.9543	6	140.4524	80.9888	57,146	6,039.9	+0.07	0
θ_0	−25	2.7653	6	140.4523	80.9889	57,122	6,039.9	+0.03	0
θ_0	+25	3.7869	6	140.3969	80.9898	57,091	6,039.9	−0.03	0
θ_0	+50	4.1515	6	140.4524	80.9888	57,080	6,039.9	−0.04	0
δ	−50	3.9066	6	141.0641	81.0247	57,028	6,052.6	−0.14	+0.21
δ	−25	3.6866	6	140.6552	81.0008	57,074	6,044.1	−0.05	+0.07
δ	+25	3.0297	6	140.3310	80.9817	57,126	6,037.3	+0.04	−0.04
δ	+50	2.7680	6	140.2503	80.9770	57,142	6,035.6	+0.07	−0.07
λ	−50	3.6799	5	169.0509	80.9633	57,415	6,364.6	+0.54	+5.38
λ	−25	3.6905	5	168.8336	81.0503	57,256	6,357.1	+0.26	+5.25
λ	+25	3.3512	6	140.2842	81.0697	56,954	6,033.4	−0.26	−0.11
λ	+50	3.3617	6	140.0609	81.1507	56,803	6,026.9	−0.53	−0.22

Table 5 Sensitivity analysis for Example 4

Parameters	%changes	ξ^*	n^*	Q^*	p^*	TP_2^*	TE_2^*	% Change TP_2^*	% Change TE_2^*
a	−50	2.1541	5	76.2700	44.0288	12,019	3,139.1	−79.10	−47.80
a	−25	2.7444	6	101.7771	62.5760	30,480	4,539.2	−46.99	−24.52
a	+25	3.8654	6	177.7710	100.0548	93,071	7,488.4	+61.86	+24.52
a	+50	4.2541	6	215.7572	118.7998	137,190	8,962.5	+138.58	+49.03
b	−50	6.9984	1	911.9674	160.3868	126,390	16,282	+119.80	+170.73
b	−25	3.7889	5	171.6977	106.4844	80,173	6,457.2	+39.43	+7.37
b	+25	3.0323	7	117.1235	66.1894	43,968	5,708	−23.54	−5.09
b	+50	2.7229	8	100.2092	56.0971	34,978	5,487.6	−39.17	−8.75
c	−50	2.6931	6	145.1530	78.8112	61,773	6,217.7	+7.43	+3.39
c	−25	3.0741	6	142.4373	80.0619	59,618	6,115.2	+3.68	+1.68
c	+25	3.6396	6	137.1561	82.5632	55,426	5,913.7	−3.61	−1.67
c	+50	3.8583	6	134.5544	83.8138	53,389	5,813.8	−7.15	−3.33
g	−50	3.0381	6	139.9043	81.3125	57,512	6,016.9	+0.02	+0.05
g	−25	3.2176	6	139.8366	81.3125	57,507	6,015.3	+0.01	+0.02
g	+25	3.5348	6	139.7311	81.3125	57,497	6,012.9	−0.01	−0.02
g	+50	3.6763	6	139.6893	81.3125	57,493	6,012.0	−0.02	−0.03
h	−50	3.3828	6	139.7922	81.3063	57,512	6,014.5	+0.02	+0.008
h	−25	3.3826	6	139.7860	81.3094	57,507	6,014.3	+0.01	+0.005
h	+25	3.3824	6	139.7734	81.3156	57,497	6,013.8	−0.01	−0.003
h	+50	3.3824	6	139.7672	81.3186	57,492	6,013.5	−0.02	−0.008
s	−50	4.1943	4	211.1213	81.2823	58,122	6,867.8	+1.08	+14.20
s	−25	3.7475	5	168.1979	81.3004	57,773	6,335.4	+0.47	+5.34
s	+25	3.0739	7	119.5765	81.3212	57,276	5,813.1	−0.39	−3.34
s	+50	2.8067	8	104.4761	81.3277	57,077	5,687.5	−0.74	−5.43
β	−50	2.8133	8	104.3231	81.0987	57,191	5,688.2	−0.54	−5.42
β	−25	3.0777	7	119.4746	81.1907	57,341	5,814.0	−0.28	−3.33
β	+25	3.3826	6	140.2367	81.3097	57,684	6,024.6	+0.32	+0.18
β	+50	3.7423	5	169.0801	81.4772	57,900	6,345.1	+0.69	+5.51

Table 5 (continued)

Parameters	%changes	ξ^*	n^*	Q^*	p^*	TP_2^*	TE_2^*	% Change TP_2^*	% Change TE_2^*
θ_0	−50	1.9962	6	139.7797	81.3125	57,543	6,014	+0.07	0
θ_0	−25	2.8072	6	139.7796	81.3125	57,519	6,014	+0.03	0
θ_0	+25	3.8288	6	139.7797	81.3125	57,489	6,014	−0.02	0
θ_0	+50	4.1935	6	139.7796	81.3125	57,478	6,014	−0.04	0
δ	−50	3.3810	7	120.3125	81.2246	57,427	5,835.5	−0.13	−2.97
δ	−25	3.7425	6	139.9766	81.3245	57,471	6,018.1	−0.05	+0.07
δ	+25	3.0632	6	139.6618	81.3053	57,523	6,011.6	+0.04	−0.04
δ	+50	2.7959	6	139.5835	81.3005	57,540	6,009.9	+0.07	−0.07
C_c	−50	3.3826	6	139.7796	81.3125	57,002	6,014	−0.87	0
C_c	−25	3.3826	6	139.7796	81.3125	57,252	6,014	−0.44	0
C_c	+25	3.3826	6	139.7796	81.3125	57,752	6,014	0.43	0
C_c	+50	3.3826	6	139.7796	81.3125	58,002	6,014	0.87	0
c_p	−50	3.3406	6	140.4523	80.9889	57,605	6,039.9	0.18	0.43
c_p	−25	3.3617	6	140.1159	81.1507	57,553	6,026.9	0.09	0.21
c_p	+25	3.0985	7	119.5339	81.3417	57,456	5,811.4	−0.08	−3.37
c_p	+50	3.1189	7	119.2618	81.4947	57,415	5,799.9	−0.15	−3.56

Figures

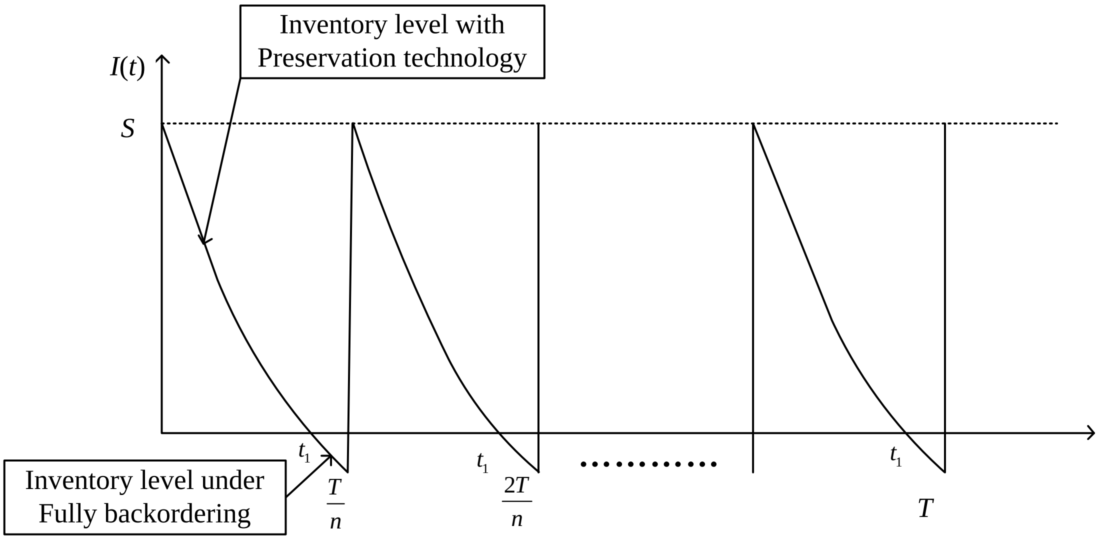

Fig.1 Graphical representation of the inventory system with complete backordering

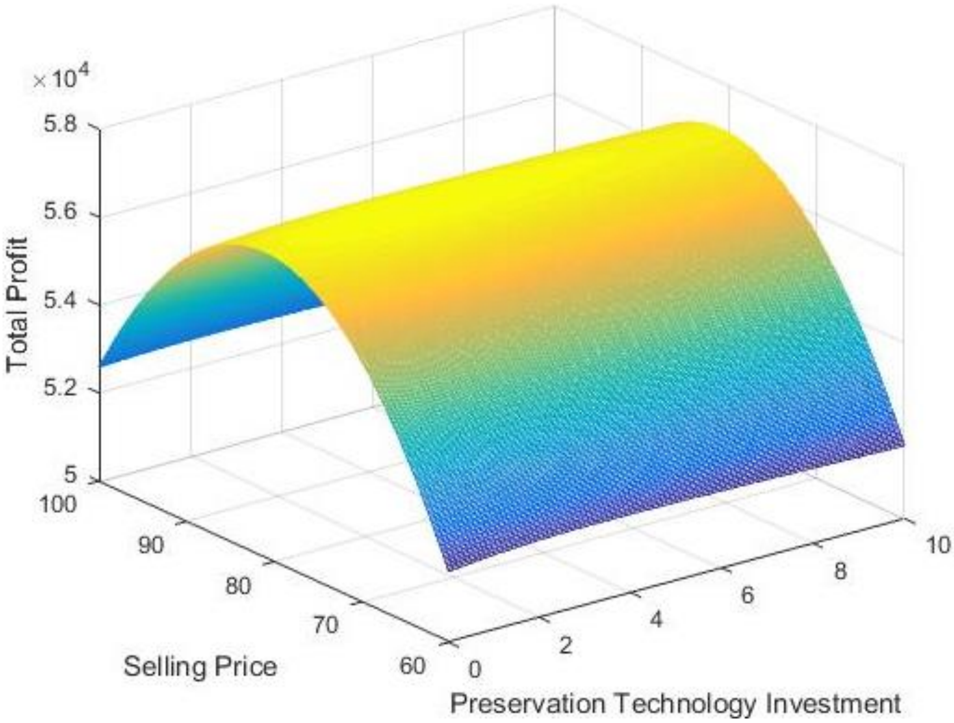

Fig.2 The total profit as a function of preservation technology investment and selling price when the ordering frequency $n = 6$ for example1