Numerical optimisation of mechanical ring reinforcement for bulk high-temperature superconductors

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Abstract. The finite element method has been used extensively in recent years to solve various problems related to applied superconductivity and provides a useful tool for analysing and predicting experimental results. Based on a recently-developed modelling framework, implemented in the finite element software package *COMSOL Multiphysics*, investigations on the minimum ring reinforcement required to prevent mechanical failure in bulk high-temperature superconducting magnets have been carried out. Assuming homogeneous $J_c(B,T)$ across the bulk sample irrespective of its dimensions, the maximum magnetic stresses experienced, and the minimum ring thickness required to prevent the hoop and radial stresses from exceeding the tensile strength of the bulk superconductor have been determined for varying values of the Young's modulus, radius, height and temperature of a representative single-grain Ag-containing Gd-Ba-Cu-O bulk sample. This comprehensive analysis details the influence each of these key parameters has on the magnetic stress and hence their impact on the necessary ring thickness to prevent mechanical failure in any given system, i.e., for any combination of material properties and sample dimensions.

1. Introduction

(RE)-Ba-Cu-O (where RE = rare earth or yttrium; hereafter, (RE)BCO) bulk superconducting magnets are under active consideration for practical applications such as synchronous electrical machines, magnetically targeted drug delivery and magnetic separation, since they are capable of trapping magnetic fields significantly greater than that attainable by the best conventional permanent magnets [1]. This is demonstrated by the current trapped field record of 17.6 T at 26 K [2], which has been achieved by using a stack of two 24 mm diameter Ag-containing Gd-Ba-Cu-O bulks mechanically reinforced by shrink-fit stainless steel rings. The ability to trap fields is dependent on the critical current density, $J_c(B,T)$, and numerical simulations have shown that trapped fields with a magnitude higher than 20 T should be achievable with the current state-of-the-art material properties [3]. However, magnetic stresses that develop as a result of the Lorentz force, $\boldsymbol{F_L} = \boldsymbol{J_c} \times \boldsymbol{B}$, places severe constraints on the potential of (RE)BCO bulk superconductors, as unreinforced YBCO bulks have been shown to fail at around 7 - 9 T due to their brittle, ceramic nature [4]. In this context, developing an appropriate ring, optimised reinforcement system, which could compensate the stresses that develop in the

superconductor, is key to achieve higher trapped fields in bulk superconducting magnets and to exploit their full trapped field capability.

Experimental, investigations can be time and resource intensive. Numerical approaches come in handy by providing a useful, cost-effective tool for analysing experimental results, but their predictive capability can indicate possible directions for further investigations. Hence, the interest in numerical modelling regarding mechanical stresses in bulk superconductors has grown tremendously, which is therefore the main purpose of this paper. The numerical models used in this study are based on the H-formulation, implemented in the commercial finite element software package *COMSOL Multiphysics* [5] and should convey a comprehensive overview of the required reinforcement system required for (RE)BCO bulk superconducting magnets to not exceed the tensile strength, which could cause cracks and by that a significant decrease of the trapped field. The analysis of this work is done by using an Ag-containing Gd-Ba-Cu-O bulk as an example but can be applied to any material in the (RE)BCO family of materials.

This work focusses on investigating the different system parameters like the bulk and the ring reinforcement geometry (radius and thickness sizes), as well as their mechanical properties (Young's-modulus and Poisson's-ratio) and operating conditions, like the temperature and the associated applied magnetic field since lower temperatures allow higher magnetic fields to be trapped in the magnet due to higher critical current densities. The mechanical stresses, the trapped field and the Lorentz forces occurring in the very centre of the bulk determined by the calculations will be used to conclude the minimum reinforcement requirements.

2. Modelling Framework

Based on the modelling framework presented in detail in [4], a 2D axisymmetric model of a single-grain Ag-containing Gd-Ba-Cu-O bulk superconductor was set up to study how the stress evolution during magnetisation is affected by the material properties and sample dimensions. In summary, the 'Solid Mechanics', 'Heat Transfer in Solids' and 'Magnetic Field Formulation' interfaces have been coupled together to determine 1) the thermal stresses from cooling and 2) electromagnetic stresses from the Lorentz force. For more detailed explanations of the electromagnetic H-Formulation, the differential equations of each module, the exact numerical implementation and setup, as well as the values of the material properties the reader is highly encouraged to refer to the paper [4], which this study is extending.

The numerical model is composed of a 2D axisymmetric Ag-containing Gd-Ba-Cu-O bulk with radius R, enclosed by a ring reinforcement system of height D (the same height as the bulk) and thickness
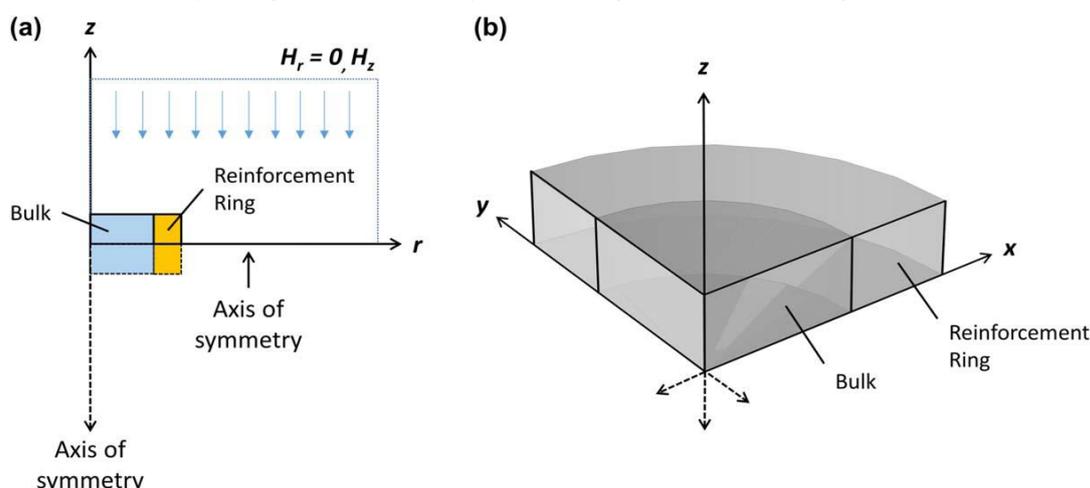

Figure 1. (a) Setup of a 2D axisymmetric numerical model of a (RE)BCO bulk with a reinforcement ring and an applied magnetic field in the z-direction in the r-z-φ cylindrical coordinate system, as evaluated in [4]. (b) The equivalent 3D model [4]. © IOP Publishing. Reproduced by permission of IOP Publishing from [4]. All rights reserved.

r, which is magnetised by the field-cooled magnetisation (FCM) process by an external magnetic flux density B_z applied in the z-direction as shown in Figure 1 [4]. In this case, the model can take advantage of the 2D axis-symmetry to only calculate the upper half of the whole bulk. This is quite similar to reality since experiments are usually carried out by stacking two samples above each other and then measuring the mechanical and electromagnetic data in the gap in between [2] [11]. Hence, the data of the numerical results are collected in the very centre of the bulk (r = 0 m; z = 0 m).

The critical current density, $J_c(B,T)$, which strongly influences the field trapping capability, is included by using experimental data [6], as well as extrapolating the data for higher magnetic fields with the following equation (1), presented in [7],

$$J_c(B) = J_{c1} \exp\left(\frac{-B}{B_L}\right) + J_{c2}\frac{B}{B_{max}} \exp\left[\frac{1}{y}\left(1 - \left(\frac{B}{B_{max}}\right)^y\right)\right] \tag{1}$$

and interpolating the data between the different temperature measurements as described in [8]. The $J_c(B,T)$ for $T > T_c$ is set to $J_c = 10^6$ A/m^2, so the current density is also defined for the normal state, which is non-zero, but significantly lower than the possible superconducting current density. For simplicity, the $J_c(B,T)$ characterisations are assumed to be homogenous in all directions (r,z and φ) of the bulk, but one could implement a position-dependent J_c, such as the φ-dependency investigated in 3D model in [4].

Simulating the FCM process requires having various time-dependent steps during the modelling to change the applied magnetic field density, as well as the temperature, representing the cooling process [4]:

1) The applied field is ramped up linearly until it reaches the required maximum field after 50 seconds at room temperature (300 K).
2) The setup is cooled down at a rate of -1 K/s to the required operating temperature while the applied field stays constant.
3) When the temperature has stabilized after 600 seconds, the applied magnetic field is ramped down to zero at a rate of 2 mT/s.

The Lorentz forces developed due to the electrical current and the magnetic flux density in the bulk lead to stresses and strains in the radial and hoop directions. There is a compressive stress applied by the contraction of the ring during the cooling process that should compensate for the electromagnetic stresses such that they do not exceed the tensile strength of the bulk (for Ag-containing Gd-Ba-Cu-O: $\sigma_{tensile} \approx 34$ MPa [12]). The main material properties used in the numerical model are summarised in Table 1.
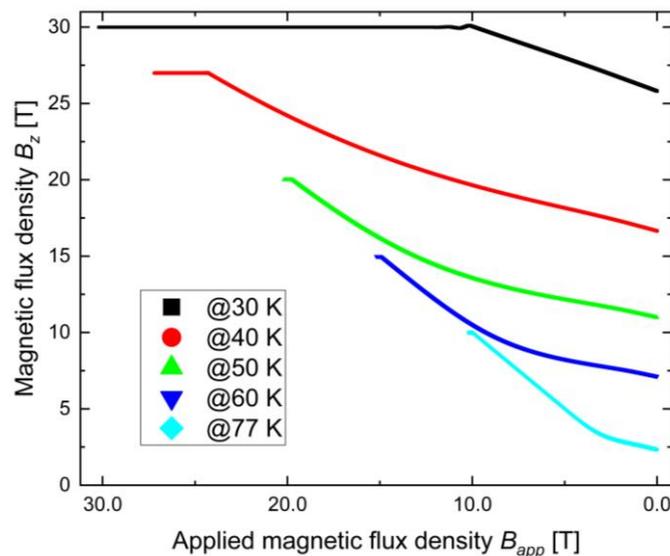
Table 1. Material properties of the Ag-containing Gd-Ba-Cu-O bulk and the stainless steel ring as proposed in [4].

Material	Young's modulus E (GPa)	Poisson's ratio v	Density ρ (kg/m^3)
Gd-Ba-Cu-O (with Ag)	100	0.33	5900
Stainless steel	193	0.22	8000

To extend the study done in [4] and optimise the required ring geometry for different bulk geometries, operating temperatures and applied magnetic fields, five different sweeps, as shown in Table 2, have been carried out. Due to lower temperatures enabling higher critical currents in the bulk and therefore higher trapped fields, the maximum magnitude of the applied field has to be higher for lower temperatures and is limited at 30 T, as shown in Figure 2. Figure 2 shows the magnetic field in the centre of the bulk ($R = 12.5$ mm; $D = 25$ mm) during its magnetisation for different temperatures and reinforced by a stainless steel ring with thickness $r = 3$ mm.

Table 2. List of different sweeps and the associated parameters of the numerical model of the mechanical stresses and magnetic fields in an Ag-containing Gd-Ba-Cu-O bulk during its magnetisation.

Sweep name	Temp-erature T [K]	Radius of the bulk R [mm]	Height of the entire system D [mm]	Thickness of the ring r [mm]	Young's modulus of the ring E [GPa]	Poisson's ratio v
Height - temperature	[30; 40; 50; 60; 77]	[12.5; 17.5]	[15; 20; 25; 30; 35]	[3]	[100]	[0.33]
Radius - temperature	[30; 40; 50; 60; 77]	[10; 12.5; 15; 17.5; 20]	[25; 35]	[3]	[100]	[0.33]
Radius - thickness	[40; 77]	[10; 12.5; 15; 17.5; 20]	[25]	[0; 1; 2; 3; 4; 5]	[100]	[0.33]
Height - thickness	[40; 77]	[12.5]	[15; 20; 25; 30; 35]	[0; 1; 2; 3; 4; 5]	[100]	[0.33]
Material properties	[60]	[12.5]	[25]	[1; 2; 3; 4; 5]	[100, 150, 200, 250, 300]	[0.2; 0.3; 0.4]

Figure 2. Magnetic flux density during the field-cooled magnetization (FCM) of an Ag-containing Gd-Ba-Cu-O bulk of radius $R = 12.5$ mm and height $D = 25$ mm at the centre reinforced by a ring of thickness $r = 3$ mm for different temperatures.

3. Results

In this section, the results of the different sweeps of the numerical model are shown to provide a comprehensive overview of the required ring reinforcement setup for various temperatures, their related applied magnetic flux densities and bulk geometries during the FCM process. Extrapolation or connection of the resulting data has been carried out by applying following equation to the corresponding points:

$$y = y_0 + A * exp(B_0 * x) \tag{2}$$

3.1. Height-temperature-sweep

The first model was carried out using the setup and parameters of the "Height-temperature-sweep" shown in Table 2. The main procedure is to calculate the hoop stress, the Lorentz force and the magnetic flux density in the bulk during the ramp down of the applied field, as shown in Figure 3 and Figure 4. The centre of the bulk is the location at which the hoop stress is expected to be at its highest value in an unreinforced bulk [4]. This stress will increase during the magnetisation due to inwards pointing Lorentz forces in the entire bulk till the outer parts are emerging a negative Lorentz force since the magnetic flux density is lowering there its value first [4]. Hence, the forces at the outer parts are pointing outwards, lowering the stress everywhere in the bulk and the maximum occurs. This event happens as the trend of the hoop stress peaks in Figure 3 display at higher applied fields for lower temperatures because the outer parts have a higher influence due to their greater circumference and, in those cases, higher negative magnetic flux density.

Figure 3 also shows the trend that lower temperatures correlate with higher stress peaks due to higher critical currents and higher magnetic fields in the bulk, leading to higher Lorentz forces, as displayed in Figure 4. Additionally, the higher the peak, the higher the applied field, at which it appears, except for the peak with $R = 17.5$ mm at 30 K since, there, the Lorentz force as indicated in Figure 4 has not reached the centre yet due to its increased radius. Therefore, the top orange peak is not the highest possible stress for this setup since, in this case, the bulk is not fully magnetized, but if so, the peak would occur at a higher applied magnetic flux density and match with the dashed line. Those sharp transitions of the Lorentz forces are occurring, when the magnetic field is changing in the centre as shown in Figure 2 and are causing the kinks of the stress in Figure 3. If the kinks are appearing in later stages of the magnetisation as they do at 30 K, the stress has not reached its potential maximum since the Lorentz force has not influenced the whole bulk for the entire process. This leads to the idea, that the magnetisation of a bulk with a maximum applied field close to the expected trapped field at the end results in a lower stress maximum.
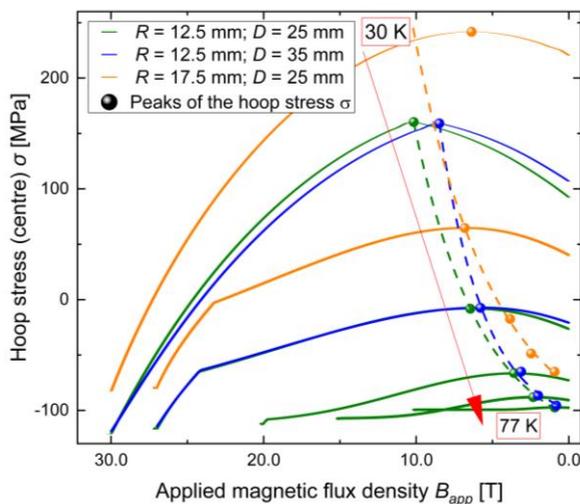
Figure 3. Hoop stress at the centre of an Ag-containing Gd-Ba-Cu-O bulk reinforced by a ring of thickness $r = 3$ mm during the FCM process of the model with its maxima and their trends indicated with dashed lines for different setups:
1) $R = 12.5$ mm; $D = 25$ mm for T = [30 K, 40 K, 50 K, 60 K, 77 K];
2) $R = 12.5$ mm; $D = 35$ mm for T = [30 K, 40 K];
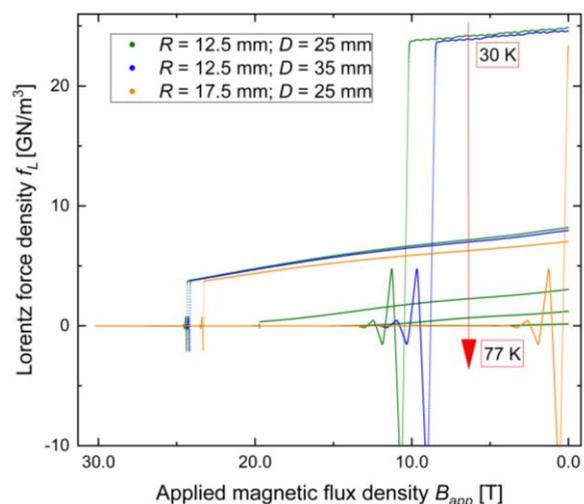3) $R = 17.5$ mm; $D = 25$ mm for T = [30 K, 40 K].

Figure 4. Lorentz force at the centre of an Ag-containing Gd-Ba-Cu-O bulk reinforced by a ring of thickness $r = 3$ mm during the FCM process of the model for different setups:
1) $R = 12.5$ mm; $D = 25$ mm for T = [30 K, 40 K, 50 K, 60 K, 77 K];
2) $R = 12.5$ mm; $D = 35$ mm for T = [30 K, 40 K];
3) $R = 17.5$ mm; $D = 25$ mm for T = [30 K, 40 K].

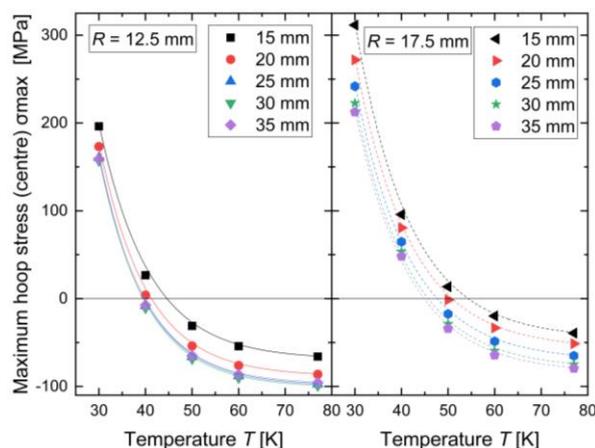**Figure 5.** Maximum hoop stress in the centre of an Ag-containing Gd-Ba-Cu-O bulk of radius R = [12.5 mm; 17.5 mm] reinforced by a ring of thickness r = 3 mm for several values of height D (15 mm – 35 mm) as well as different temperatures (30 K – 77 K).
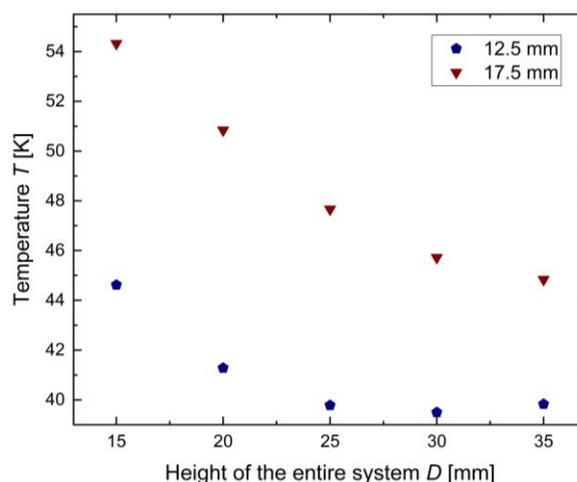
Figure 6. Minimum operating temperature, such that the centre of the Ag-containing Gd-Ba-Cu-O bulk of radius R = [12.5 mm; 17.5 mm] reinforced by a ring of thickness r = 3 mm remains in a compressive state during the entire magnetization process for several values of height D (15 mm – 35 mm).

Furthermore, increasing the radius leads to an increase in the maximum hoop stress. Although the Lorentz force will develop at the centre at lower applied fields and has a similar maximum as the other setups, the increased radius results in forces over a greater region with a greater magnetic flux density at the centre. The changes of the height though have only a small impact on the stress and Lorentz force development. This numerical model includes reinforcing the bulk with a stainless steel ring of thickness r = 3 mm, which directly influences the stress at the start of the magnetisation process. Increasing the thickness of the ring would lower the stress due to a higher compression by thermal contraction and offsets the whole stress profile in a negative direction, which is the main purpose of the ring reinforcement.

Figure 5 shows the steep exponential increase of the maximum hoop stress for lower temperatures and the corresponding magnetic flux densities in the bulk when using a stainless steel ring of thickness r = 3 mm for reinforcement. The comparison of the two diagrams indicates the influence of the enlargement of the radius, especially for lower temperatures: the maximum hoop stress increases from around σ = 200 MPa to σ = 315 MPa. As shown in Figure 5 and Figure 6, the enlargement of the height of the entire system impacts the allowable minimum temperature by providing additional compression and lowering the maximum stress for the same temperature. This can decrease the potential temperature to ensure the bulk still remains in a compressive state ($\sigma \leq 0$ MPa) from around T = 44.5 K to T = 39.5 K for R = 12.5 mm and from T = 54.5 K to T = 45 K for R = 17.5 mm as shown in Figure 6. However, Figure 6 shows, that larger bulk heights, such as D = [25 mm, 30 mm, 35 mm] for R = 12.5 mm, are not increasing their impact much compared to the smaller ones. For R = 12.5 mm, the minimum temperature will even increase with the height D = 35 mm. Hence, having a height:diameter ratio close to one is an optimal value for a bulk to remain in a compressive state and a larger height would not decrease the minimum temperature much even more.
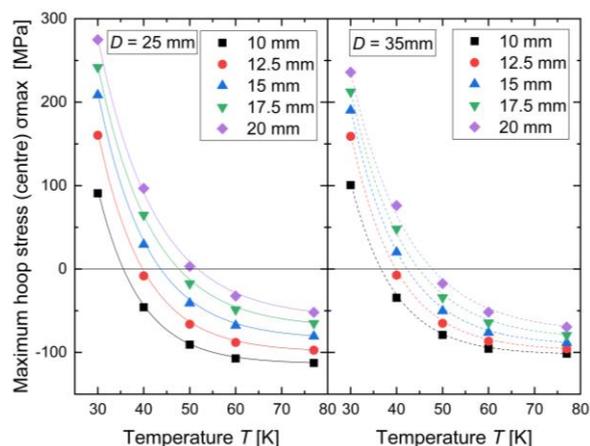
Figure 7. Maximum hoop stress at the centre of Ag-containing Gd-Ba-Cu-O bulk of heights D = [25 mm; 35 mm] reinforced by a ring of thickness r = 3 mm for radii R (10 mm – 20 mm), as well as different temperatures (30 K - 77 K).
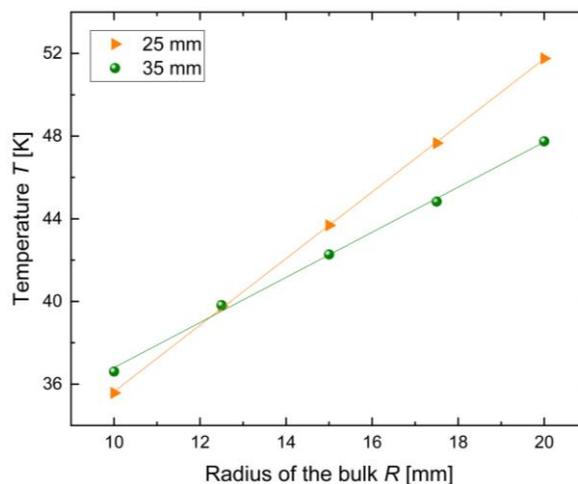

Figure 8. Minimum possible temperature, such that the centre of the Ag-containing Gd-Ba-Cu-O bulk of a radii R (10 mm – 20 mm) reinforced by a ring of thickness r = 3 mm remains in a compressive state during the entire magnetization process for two heights D = [25 mm; 35 mm].

3.2. Radius-temperature-sweep

The second model was carried out using the setup and parameters of the "Radius-temperature-sweep" shown in Table 2. In this case, the maximum hoop stress in Figure 7 is calculated for different values of radii R of the bulk reinforced by a stainless steel ring of radius r = 3 mm and two heights D = [25 mm; 35 mm] for several operating temperatures. Again, the maximum hoop stress increases exponentially for lower temperatures due to a higher J_c and magnetic flux density. But in this case, the radius impacts the stress significantly, since it enables a greater magnetic flux density, as well as a larger region over which the forces act and works against the ring compression. Hence, a bigger radius leads to a higher hoop stress as, shown in Figure 7. As mentioned above, the height of the bulk does not have a huge influence on the hoop stress and has less influence (see Figure 7). For higher radii, the minimum operating temperature increases linearly from T = 37 K to T = 48 K for D = 25 mm and from T = 35 K to T = 52 K and hence decreases the trapped magnetic field if using the same setup with the same ring thickness still providing adequate thermal compression to the bulk to remain in a compressive state (see Figure 8).

3.3. Radius-thickness-sweep

The third model was carried out using the setup and parameters of the "Radius-thickness-sweep" shown in Table 2. The necessary thickness of the reinforcement ring has been determined by calculating the intersections of the maximum hoop stress with the zero-line of the stress in Figure 9 for various bulk radii at 40 K and 77 K and a system height D = 25 mm. At those points, the centre of the bulk remains in a compressive state and is sufficiently reinforced by the ring. Of course, the hoop stresses increase by enlarging the radius R and decreases with the higher compression from the enlarged ring. The required ring thicknesses are shown in Figure 10 and show a tremendous exponential increase in this requirement if the radius R increases linearly. Even though the bulk has a very small tensile stress at 77 K (shown by the very low required ring thickness between r = 40 μm and r = 90 μm), the trend of an exponential increase for bigger radius values is still noticeable in Figure 10. For bulks of radii R = 17.5 mm and above, the required ring thickness for the same setup would be enormous due the high fields in the bulk at this temperature as also shown in Figure 8.
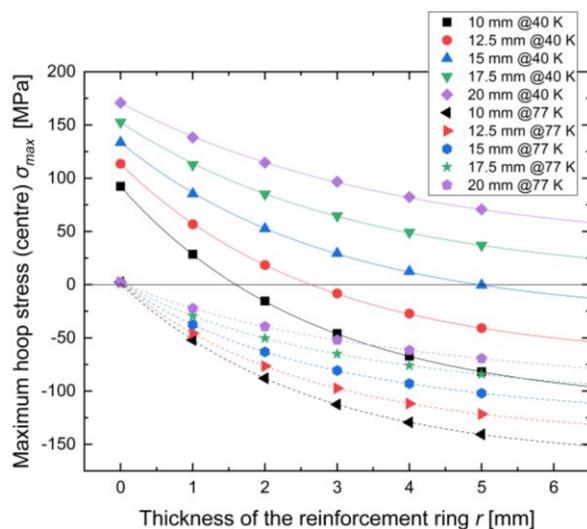
Figure 9. Maximum hoop stress at the centre of an Ag-containing Gd-Ba-Cu-O bulk with of height $D = 25$ mm reinforced by a ring of thicknesses r (0 mm – 5 mm) for several radii R (10 mm – 20 mm), as well as different temperatures $T = [40 \text{ K}; 77 \text{ K}]$.
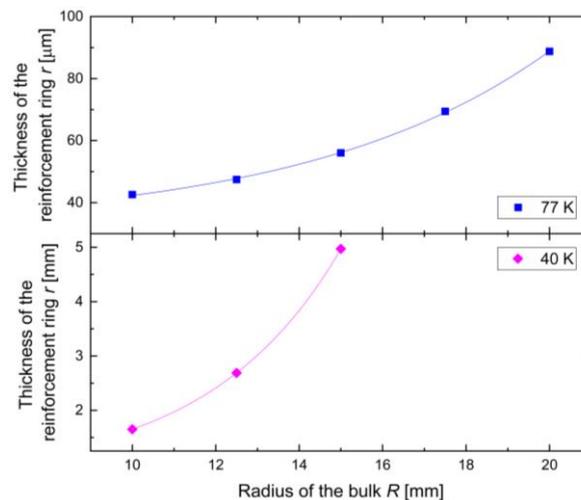

Figure 10. Minimum thickness of the ring reinforcement r required to provide compression at the centre of an Ag-containing Gd-Ba-Cu-O bulk of height $D = 25$ mm for the entire magnetisation process for several radii R (10 mm – 20 mm), as well as different temperatures $T = [40 \text{ K}; 77 \text{ K}]$.

3.4. Height-thickness-sweep

The fourth model was carried out using the setup and parameters of the "Height-thickness-sweep" shown in Table 2. The necessary thickness of the ring has been calculated in the same way as in section 3.3. Figure 11 shows the progress of the maximum hoop stress for different ring thicknesses and several heights (15 mm – 35 mm) at 40 K and 77 K. As mentioned in section 3.1., if the height is at least 25
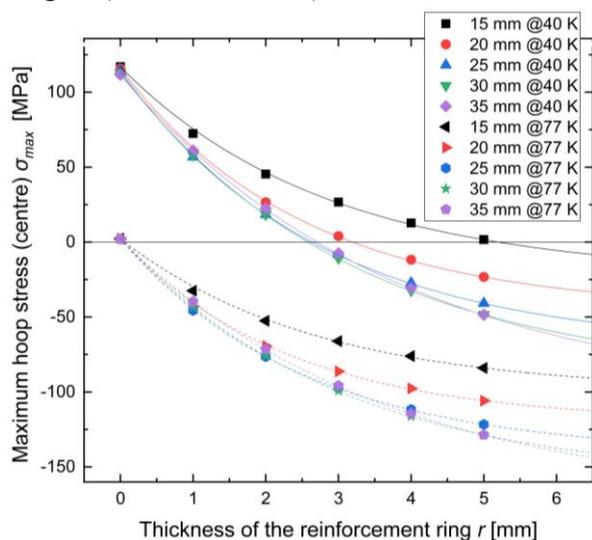

Figure 11. Maximum hoop stress at the centre of an Ag-containing Gd-Ba-Cu-O bulk of radius $R = 12.5$ mm reinforced by a ring of thicknesses r (0 mm – 5 mm) for several bulk heights D (15 mm – 35 mm), as well as different temperatures $T = [40 \text{ K}; 77 \text{ K}]$.
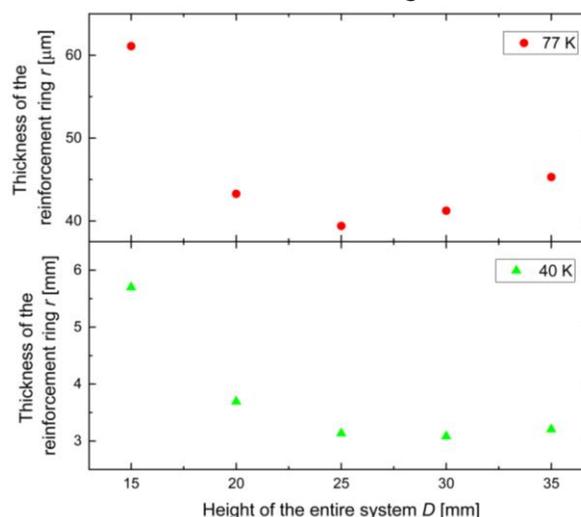

Figure 12. Minimum thickness of the ring reinforcement r required to provide compression at the centre of an Ag-containing Gd-Ba-Cu-O bulk of radius $R = 12.5$ mm for the entire magnetisation process for several heights D (15 mm – 35 mm), as well as different temperatures $T = [40 \text{ K}; 77 \text{ K}]$.

mm, any further increase in height would not impact this trend much. Figure 11 shows this, as well as the trends for D = [25 mm; 30 mm; 35 mm] are not much different, but, of course, the larger ring thicknesses will result in lower maximum stress values. Figure 12 underlines this, showing, that a height
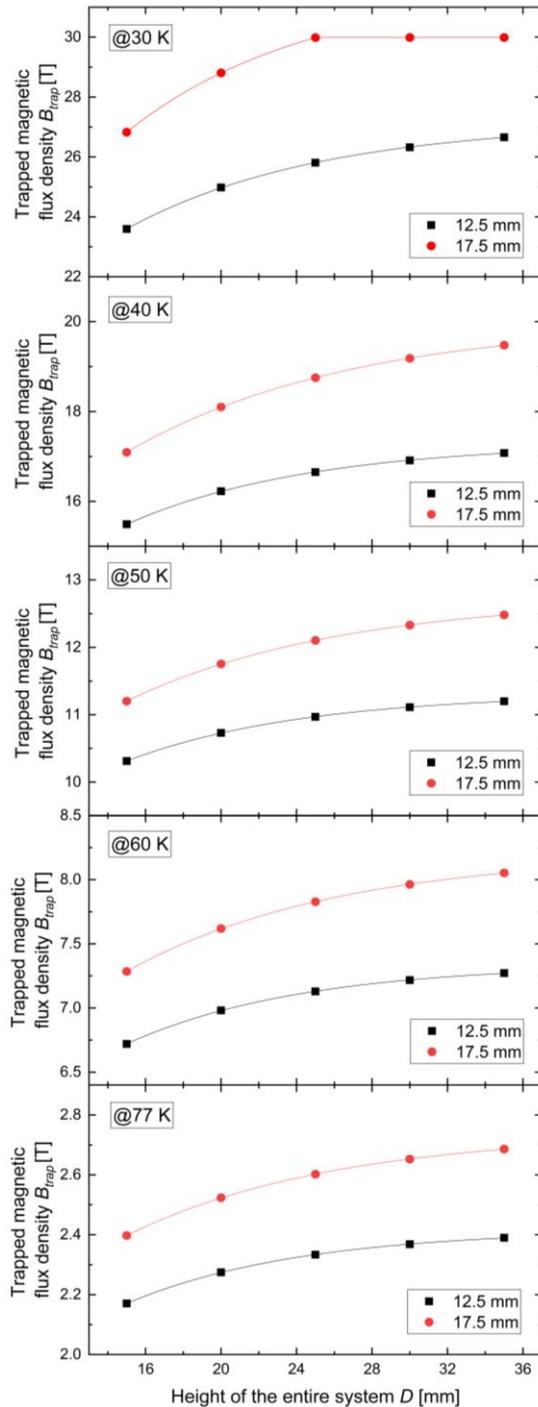

Figure 13: Trapped field at the end of the FCM process of an Ag-containing Gd-Ba-Cu-O bulk of radii R = [12.5 mm; 17.5 mm] reinforced by a ring of thickness r = 3 mm for several heights D (15 mm − 35 mm), as well as different temperatures (30 K − 77 K).
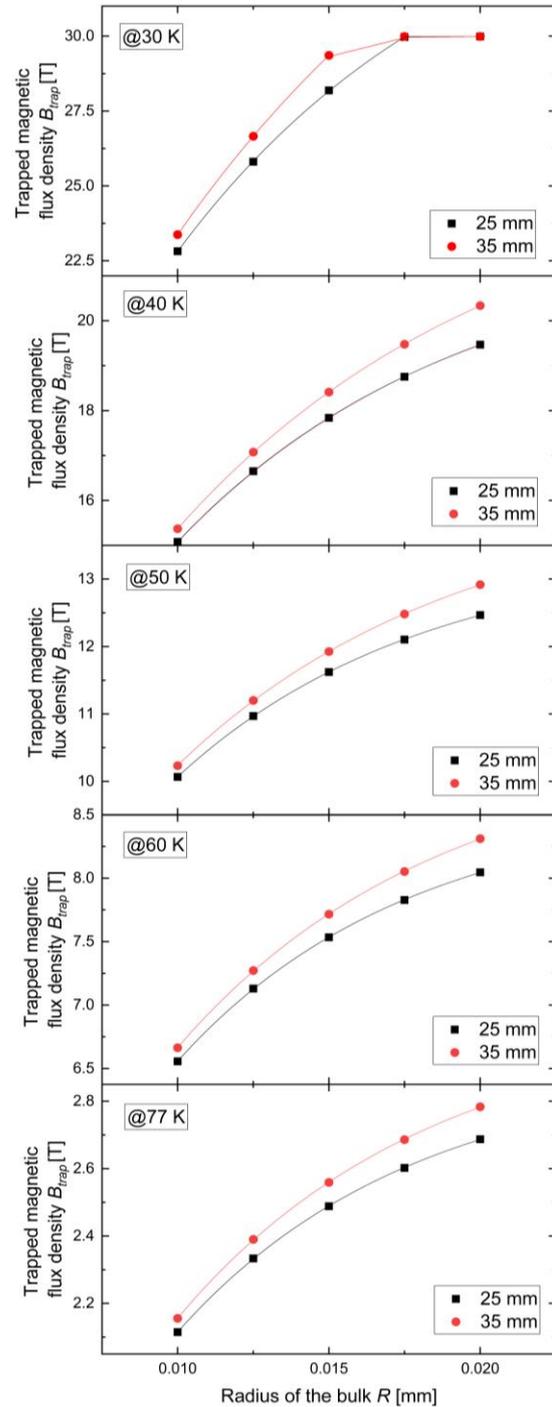
Figure 14: Trapped field at the end of the FCM process of an Ag-containing Gd-Ba-Cu-O bulk of heights D = [25 mm; 35 mm] reinforced by a ring of thickness r = 3 mm for several radii R (15 mm − 35 mm), as well as different temperatures (30 K − 77 K).

greater than 25 mm at 40 K would not decrease the required ring thickness further; even though the bulk has a very small tensile stress at 77 K (shown by the very low required ring thickness between $r = 39$ µm and $r = 62$ µm), the trend is still the same. Hence, a height of $D = 25$ mm minimizes the required ring thickness and is therefore the optimal setup for a bulk with a radius $R = 12.5$ mm. This leads to the idea, presented already in section 3.1., that a height:diameter ratio close to one is the optimal geometry for a (RE)BCO bulk to minimize its stress in the centre and therefore is able to trap higher magnetic fields.

3.5. Trapped field at different temperatures

Figure 13 and Figure 14 show the trapped fields at the centre of the bulk at the end of the FCM process. Since the reinforcement ring has nearly no impact on the electromagnetic aspects of the bulk, these results only show a comparison of the trapped fields for different geometries for various operating temperatures. Obviously, the lower temperatures result in higher trapped fields, as already alluded in Figure 2. The potential of the enlargement of the radius is shown in Figure 14, where the trapped field can increase in magnitude by around 40% by changing from a radius $R = 10$ mm to $R = 20$ mm at 40 K. This will of course have an impact on the hoop stress. The effect of the enlargement of height is not as significant as seen in Figure 13, but still noticeable as it can increase the magnitude of the trapped field at 30 K from around 33.5 T to 36.5 T using a bulk radius $R = 12.5$ mm.
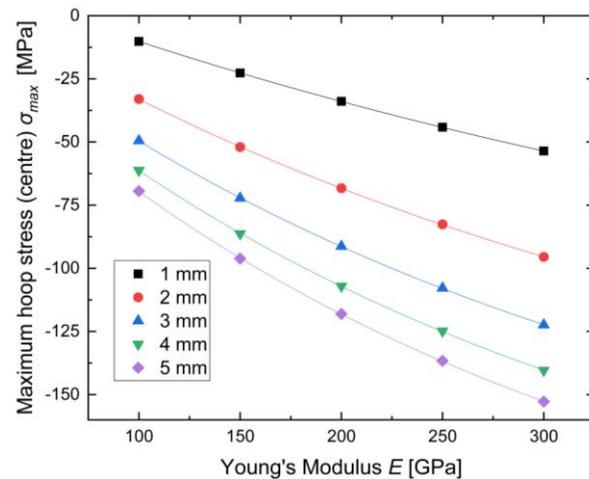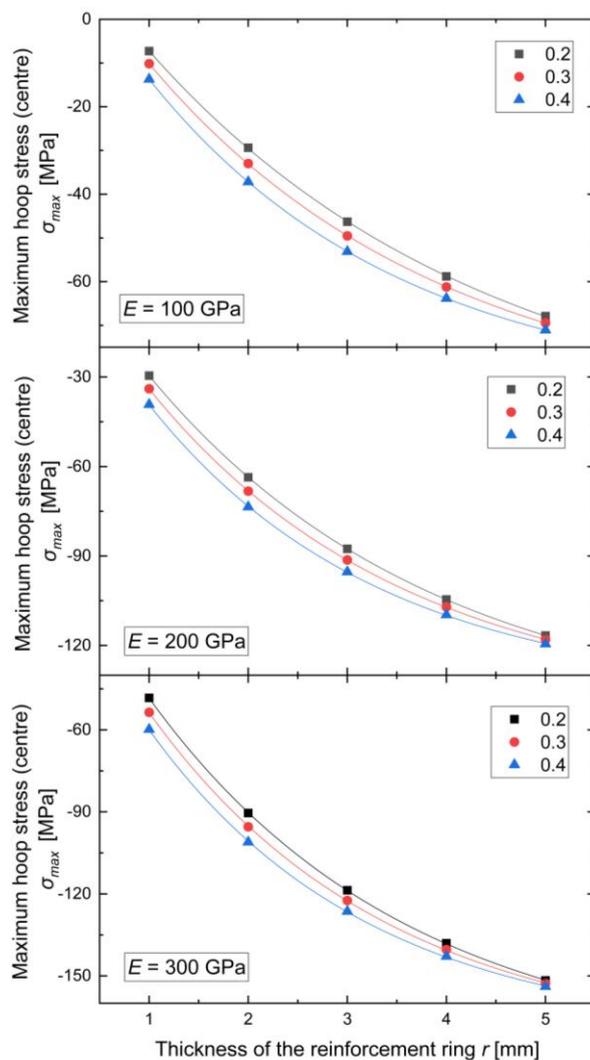

Figure 16: Maximum hoop stress at the centre of an Ag-containing Gd-Ba-Cu-O bulk of radius $R = 12.5$ mm and height $D = 25$ mm reinforced by a ring of thickness r (0 mm – 5 mm) and Poisson's ratio $v = 0.3$ for several values of Young's modulus E (100 GPa – 300 GPa) of the ring material at 60 K.

Figure 15: Maximum hoop stress at the centre of an Ag-containing Gd-Ba-Cu-O bulk of radius $R = 12.5$ mm and height $D = 25$ mm reinforced by a ring of thickness r (0 mm – 5 mm) for several values of Young's modulus $E = [100\ \text{GPa}, 200\ \text{GPa}, 300\ \text{GPa}]$ of the ring material, as well as different values for the Poisson's ratio $v = [0.2, 0.3, 0.4]$ at 60 K.

3.6. Material properties sweep

The last model carried out investigates on the influence of the material properties of the reinforcement ring if it is not made of stainless steel. The Young's modulus and the Poisson's ratio is varied to demonstrate other possible cryogenic materials with different material properties, like Nickel Steel (Fe-2.25 Ni) with a Young's modulus $E = 218$ GPa [10], and the maximum hoop stresses are determined for a bulk with $R = 12.5$ mm and $D = 25$ mm at 60 K. Since the bulk has a low tensile stress at 60K, the additional ring reinforcement leads in all cases to negative maximum stresses, but the trends are still noteworthy. Obviously, the enlargement of the ring thickness decreases the developed stress, as shown in Figure 15. There is also a small decrease in the stress when increasing the Poisson's ratio of the ring. However, an increase in the Young's modulus impacts the stress more, since it relates directly to the stiffness of a material and therefore resists more against the stresses caused by the Lorentz force using the same ring geometry and same thermal expansion, as shown in Figure 15 and Figure 16.

4. Conclusion

Since (RE)BCO bulk superconductors are ceramics and have a low tensile strength around 34 MPa [12], sufficient mechanical reinforcement is necessary to successfully design a superconducting magnet with fields greater than 20 T that will not fail mechanically [9]. In this paper, investigations on the required reinforcement for a representative, Ag-containing Gd-Ba-Cu-O bulk superconducting magnet have been carried out using the finite element software package *COMSOL Multiphysics* for various values of the operating temperature, bulk radius, system height, as well as some material properties of the reinforcement ring. One could also use the presented data to extrapolate the results and get information for lower temperatures, larger thicknesses of the ring or higher Young's Modulus.

(1) The numerical analysis showed that there are many possible bulk geometries and temperature combinations which could achieve a trapped field over 20 T, but the appearing magnetic stresses would cause mechanical failure in it, which are not compensable by the ring reinforcements presented in this paper. Using a stainless steel ring of thickness $r = 5$ mm for reinforcement, this study predicts a maximum trapped field close to 18 T at 40 K with an Ag-containing Gd-Ba-Cu-O of radius $R = 15$mm and thickness $D = 30$ mm. But it might be possible to achieve higher trapped fields with a similar setup, but different parameters, like one model predicted to achieve 28 T with a radius $R = 15$ mm and height $D = 25$ mm at 30 K provided it would be sufficiently reinforced.

(2) Increasing the radius of the bulk can increase the trapped magnetic field by up to 40% at 40 K and even more at lower temperatures. This increases the mechanical stress that develops exponentially due to the increasing Lorentz force. To compensate for those enormous stresses, the necessary reinforcement ring thickness will increase exponentially as well. To achieve $B_{trap} > 20$ T in such a magnet at 40 K would require a bulk of radius $R = 20$ mm, but there is no practicable thickness of the ring with this setup at this temperature.

(3) The numerical results showed that increasing the height of the entire system will not have a great impact on the trapped field at the end (~1 T). Additionally, a height:diameter ratio close to one is the optimal to provide the maximum possible compression and a further enlargement of height does not contribute more to the compression.

(4) Increasing the Poisson's ratio for the reinforcement ring material has very little influence on the mechanical stress, but the Young's modulus of the ring material is a significant factor for the entire mechanical system. A cryogenic material used to reinforce a bulk superconducting magnet should be constant regarding its material parameters for low temperatures and have a high Poisson's ratio and Young's modulus.

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