Submitted exclusively to the London Mathematical Society
doi:10.1112/0000/000000

OBITUARY

Richard Kenneth Guy, 1916-2020

1. *Overview*

Richard Guy was born in Nuneaton on 30 September 1916 to Augusta Adeline Tanner and William Alexander Charles Guy, both school teachers, his father having survived the disastrous Gallipoli campaign. He was a border at Warwick School, where he was put in classes well above his age group. Richard's mathematical talent was apparent from an early age and he was encouraged by his teacher Cyril Caton to study beyond the school syllabus. When 17 he purchased, and was fascinated by, Dickson's *History of the Theory of Numbers* ⟨**2**⟩, and he was left in no doubt that mathematics would be central to his life. Obtaining three scholarships, he went up to Gonville and Caius, Cambridge in 1935 to read Mathematics, and found that he was already familiar with all the first-year material. He recalls that later courses by Albert Ingham and Charles Burkill on number theory and analysis were particularly inspiring. However, he spent a considerable time playing chess and bridge and composing chess problems, with the consequence that he graduated in 1938 with only a second class degree, but also perhaps having fueled his lifetime interest in game theory.

Michael Thirian, a couple of years below Richard at Warwick School, also won a scholarship to Gonville and Caius, and through him Richard met his sister, Louise Thirian. Louise and Michael were close siblings, for example, in their teens they went on a three-week hiking trip in the Swiss Alps, covering up to 25 miles a day, and living very frugally. Louise took a teaching diploma at a Domestic Science College in Leicester; whilst there she invited Richard to a college dance, and this was soon followed by a trip together to the Lake District, where Richard had enjoyed walking holidays with his family. Richard and Louise became engaged in November 1939, and they married in December 1940.

Meanwhile, against the advice of his parents Richard decided to become a teacher. He gained a teaching diploma at the University of Birmingham, and in 1939 took up a mathematics post at Stockport Grammar School. He was not called up until 1941 when he was commissioned as a flight lieutenant in the meteorological branch of the RAF, and was posted first in Scotland, then Iceland, and finally Bermuda. Whilst in Iceland he took up snow and ice climbing, and skiing, which became lifelong pastimes.

Richard managed to get home to Louise from time to time, and their three children were born during the war: Elizabeth Anne, and Michael, who both went on to study mathematics at Cambridge, and Peter, who read Statistics at University College London. Altogether, Richard created quite a dynasty of mathematicians and related specialists, with seven of his eight children and grandchildren pursuing careers in mathematics or computing, see ⟨**11**⟩. Indeed, Richard co-authored papers with son Michael Guy [**107**], and with grandson Andrew Guy [**91**]. Granddaughter Katharine Scott obtained her mathematics degree at the University of Warwick. Emily Booth, one of Richard's two great-granddaughters, said at the age of eight that she wanted to become a pure mathematician 'like Grandpa Richard', and at the time of writing is studying mathematics at the University of York.

2020 *Mathematics Subject Classification* 00000.

After the war Richard resumed teaching at Stockport Grammar School, but the prospect of a lifetime of middle-school teaching did not appeal, and he left in 1947 to take up a post teaching mathematics at Goldsmiths College in London. Goldsmiths was then a teacher training college but most of the teaching was at degree level which suited Richard. He was also able to take a graduate course at Birkbeck College with the thought of perhaps eventually getting a PhD, and he attended seminars at University College London organised by Harold Davenport, where he met several leading number theorists.

However, Richard and Louise decided that they wanted to see more of the world, and Richard obtained a post at University of Malaya in Singapore in 1951. The mathematics department, then led by number theorist Alexander Oppenheim, was small but stimulating with regular visitors from across the world. But after a change in head of department another move became desirable, and in 1962 he went on to help start up the new Indian Institute of Technology in New Delhi where he mainly taught engineers. Whilst in India, Richard and Louise enjoyed mountaineering in the foothills of the Himalayas but the Institute had its frustrations, and they found the New Delhi climate oppressive. In 1965 Richard moved to the Calgary branch of the University of Alberta, founded in 1908, which became the autonomous University of Calgary in 1966, and they remained in Calgary for the rest of their lives.

Calgary was a wonderful centre for mathematics, not least with the presence of Peter Lancaster and Eric Milner, former colleagues at the University of Malaya, whom Richard had been instrumental in attracting to Calgary. But Richard and Louise were also able to enjoy mountaineering to the full, and together they climbed many peaks in the Rockies. They became well-known members of the Alpine Club of Canada who raised $500,000 and coordinated volunteers to build a hut near Mont des Poilus on the Wapta Icefield named the Louise and Richard Guy Hut, which opened in 2016, Richard's 100th year. The short book *Young at Heart – The Inspirational Lives of Richard and Louise Guy* by Chic Scott ⟨**11**⟩ provides a lovely account of their lives, and in particular their mountaineering adventures together.

Louise was supportive of all Richard's activities, and welcomed the many mathematicians who came to stay. As a 90th birthday present for Richard she set up the Richard and Louise Guy popular lecture series at the University of Calgary. Richard was deeply affected by Louise's death in 2010 but vowed to continue with his mathematics and outdoor activities. He raised considerable sums for charity in her memory, and each year until he was 99 he climbed the 802 steps of the Calgary Tower, with a picture of Louise around his neck, to raise money for the Alberta Wilderness Association. In 2014, he donated $100,000 to the Alpine Club of Canada for the training of amateur mountain leaders. Richard celebrated his 100th birthday at Mount Assiniboine Lodge with a hike to Wonder Pass, and the next day he ascended the 7,800ft high Niblet.

Although Richard nominally retired in 1982, he was at his desk in the Calgary mathematics department almost daily until shortly before his death. 'I didn't retire' he commented, 'they just stopped paying me. I was quite happy to retire, I knew that I could go on doing what I wanted to do.' He never took a PhD but in 1991 Calgary awarded him an Honorary Doctor of Laws, 'out of embarrassment', Richard claimed.

Richard's last book *The Unity of Combinatorics* [**130**], co-authored with Ezra Brown, was published by the Mathematical Association of America two months after he died on 9 March 2020, aged 103.

Although Richard was the oldest member of the LMS, he did not join until 1960 so was not the longest-standing member – that distinction goes to Freeman Dyson, a member for 77 years who died nine days before Richard.

2. *Mathematicial style*

Until the 1960s Richard's work was almost entirely devoted to teaching with chess and other games a recreational interest. But in 1960, whilst in Singapore, he met Paul Erdős, the Hungarian mathematician noted for solving, and challenging others with, difficult mathematical problems, in particular in number theory and graph theory. Erdős shared several problems with Richard who later remarked 'I made some progress in each of them. This gave me encouragement, and I began to think of myself as possibly being something of a research mathematician, which I hadn't done before.'

Richard soon published a paper giving a lower bound for the coarseness of the complete graph on n vertices, disproving a conjecture of Erdős [**4**], see Section 3.2. This was followed by a joint paper with Erdős [**22**] obtaining bounds on the number of lattice points that can be chosen from a square lattice with all mutual distances distinct. Aged nearly 50, Richard's research career was just beginning, and this paper bore several hallmarks of his future work: it was jointly authored, as were the vast majority of his research papers, it concerned a problem that could be explained to almost anyone, and it included a number of unsolved problems.

Richard loved meeting other mathematicians and became a frequent and familiar figure at conferences, wearing an old jacket, always with a badge 'Peace is a disarming concept' on his lapel. His conference interactions led to many collaborations, his 50+ collaborators including John Conway, Paul Erdős, Martin Gardner, and Donald Knuth. Many of Richard's conference talks presented unsolved problems which were often published in the proceedings [**21**, **24**, **72**, **75**, **81**, **95**, **100**, **113**].

Many mathematicians enjoyed Richard and Louise's generous hospitality, staying at their Calgary home, sometimes for quite lengthy periods, to work with Richard, with Louise providing support in the way of meals or perhaps entertaining other family members. There were 'days-off' when Richard and Louise would take their visitors to the Rockies setting a brisk pace on hikes and climbs. Andrew Bremner recalls from a visit in 1996: 'An accessible peak in May is Yamnuska, which we had climbed. While we were eating lunch at the summit, a twenty-something couple arrived to share our ridge perch, the girl curiously eyeing our group. "I hope I'm not being rude", she said to us, "but just how old are you?" Richard proffered the reply: "Well, our three ages total 200 years ... ." After a scrunch of brow, "Wow," she said, "you must all be at least 60, that's amazing!" (For the record, I was 44). "And how old are you?" said Richard. "Well we add up to 45." "Aha," said Richard, "you must both be at least 10!".'

Working with Richard could be intensive; if engrossed in a problem he would continue to be focussed through the night. Richard was staying with me in 1987 when my wife went into labour. When I came home from the hospital at 5 am he was still up, and I remarked that Isobel had just given birth to a son. 'Yes,' he replied, 'I have calculated the volume of the extremal polyhedron that we were discussing yesterday.'

Perhaps taking after Erdős, Richard thrived on unsolved problems, particularly those of an 'intuitive' nature, that is problems which can be stated at a level requiring little technical mathematical knowledge, even though their solution may be difficult. Richard collected and promulgated many such problems, particularly in number theory, graph theory, and geometry. For many years he was Unsolved Problems Editor of the *American Mathematical Monthly*, which each month included an account of an 'easily stated unsolved problem dealing with notions ordinarily encountered in undergraduate mathematics'. In particular, every two years he would write an article presenting progress on problems posed in the column [**43**, **93**]. Richard's problem collection became so extensive that he brought them together in books: *Unsolved Problems in Number Theory* [**53**], and, with Kenneth Falconer and Hallard Croft, *Unsolved Problems in Geometry* [**77**], books which have undoubtedly inspired and stimulated many budding mathematicians. But 'unsolved' problems often become 'solved', and before long the number theory volume required a second, and then a third, revised and expanded edition, and the geometry book had an updated reprint. In an introduction Richard writes 'To pose good unsolved problems is a difficult art. The balance between triviality and hopeless unsolvability is delicate. There are many simply stated problems which experts tell us are unlikely to be solved in the next generation. On the other hand, "unsolved" problems may not be unsolved at all, or may be much more tractable than was at first thought.'

Richard was a master of exposition. His style and clarity of writing held the attention of readers, whether in articles for school students, undergraduates, or researchers. He loved words and wordplay, and would polish off the *Globe and Mail* cryptic crossword after dinner each evening. When introducing a new notion he would ensure that the terminology was spot on, for example 'evil' and 'odious' numbers for integers with an even or odd number of 1s, respectively, in their binary expansion. His playful sense of humour runs through his writing, even in titles such as 'The nesting and roosting habits of the laddered parenthesis', 'Catwalks, sandsteps and Pascal pyramids' or 'Don't try to solve these problems!'. He believed that finding the right notation was key to presenting mathematics clearly. Visulisation was important to Richard in both his approach to research and in exposition; he loved illustrating his work with diagrams, at first carefully hand-drawn, then he taught himself to use computer drawing packages including LaTeX graphics packages. The superb illustrations in *Winning Ways* [**54**] are a testament to his skills. Wide dissemination of mathematics was important to Richard; he did not care particularly about publishing in top research journals, but would rather explain his many clever ideas in expository journals such as the *American Mathematical Monthly* or *Mathematics Magazine* where they would be more widely enjoyed.

Richard was a gifted teacher and lecturer, and he retained his clear and entertaining style until the end of his life, for example his talk 'A triangle has eight vertices' at the age of 98 was recorded for posterity ⟨**4**⟩. He set high standards for himself, and encouraged the same in others. In an amusing but thought-provoking article in the *Mathematical Intelligencer* entitled 'How good a mathematician are you?' [**63**] he writes: 'When you read the title did you do a rapid self-assessment of your recent publications ...? But I meant the question in the sense 'How complete a mathematical life do you lead?' He refines this into a long list of questions covering aspects of teaching and service to which a well-rounded mathematician should aspire.

Equally at home with world-leading researchers and keen youngsters, Richard would talk about mathematics with anyone who was interested, and his door was always open. Many were inspired by Richard's encouragement and enthusiasm. As Richard Nowakowski writes, 'As an undergraduate and graduate student at Calgary, Richard generated a sense of inclusion in his students, and a sense of awe and wonder at his breadth of knowledge and his ability and willingness to work.' He coached talented school students, and prepared undergraduates for the Putnam competition - key to his approach was that 'Math is Fun'. The book *The Inquisitive Problem Solver* [**110**], authored with Paul Vaderlind and Loren Larson, contains 256 not-too-difficult problems with hints and solutions designed to challenge and instruct budding mathematicians in the art of problem solving. Richard's many works on 'intuitive' mathematical problems and recreational mathematics, together with his approachable writing style, undoubtedly attracted many students into studying mathematics with many continuing to mathematical careers.

Richard had four official PhD students: Roger Eggleton who went on to become professor at the University of Newcastle, Australia, Richard Nowakowski who became professor at Dalhousie University in Canada, as well as Dan Calistrate, and Jia Shen. But he undoubtedly influenced many others, not least with his annual talk 'How not to be a graduate student'.

The reader may enjoy a YouTube video of a wonderfully eloquent talk at the *Gathering for Gardner* meeting in 2014, marking the centenary of Martin Gardner's birth, in which Richard recalls his life and career ⟨**5**⟩.

3. *Mathematical work*

Richard wrote some 10 books and probably over 300 papers and articles, some in mainstream journals and some in magazines, mathematical or otherwise. The following survey attempts to cover Richard's main research work, but there is certainly much more.

3.1. *Combinatorial game theory, games and recreations*

Combinatorial game theory concerns sequential games where all players (usually two players) have perfect information; well-known examples include noughts and crosses, checkers or draughts, chess, and Go. Moves in such games may be represented by a game tree.

In the 1930s, Roland Sprague and Patrick Grundy independently showed that impartial games, where the same moves are available for each player, are equivalent to a game of Nim, the game where, starting with several piles of objects, the players take turns to remove objects (at least one) all from the same pile, the aim of the game being to take the last object, or perhaps avoid taking it. In particular, there is a natural way to 'add' such games.

From his school days Richard was a serious chess player, and was Endings Editor of the *British Chess Magazine* from 1947-51. He composed over 200 chess puzzles, requiring ingenious logical solutions; indeed in 1966 John Roycroft ⟨**10**⟩ compiled and published a collection of these. In 1947 Thomas Dawson showed Richard a game played with just pawns on a $3 \times n$ board, now known as Dawson's chess, where the last player to move loses. Richard analysed a version of this game, and developed the theory more generally. He made contact with Cedric Smith, who was familiar with the Sprague-Grundy work, and it became clear that Richard had independently rediscovered and generalised this theory. Together they wrote Richard's first paper [**1**] which studies the Sprague-Grundy values $G(P)$, which for a single game is the least number of moves required to finish a game from position P but for a disjunctive sum of games G is given by the nim-sum of the individual games. Richard made a particular study of octal games, introduced in this paper, which are generalisations of Nim where for a turn a player may split a pile into two.

It was John Conway that had the remarkable insight that the constructions of the ordinals, reals, and Donald Knuth's 'surreal numbers' could be unified in a way that provided a natural framework for combinatorial games. Positions of a game can be represented as $\{a, b, \ldots | c, d, \ldots\}$ with the letters on either side representing the options for the two players. Games can be constructed 'inductively' from the game $0 = \{\ |\ \}$ where neither player has any valid moves. Games include 'numbers', the positive integers being defined inductively from 0 as $n + 1 = \{n|\ \}$, with further definitions for the rationals and reals. Games can be added in a manner that represents the two games being played simultaneously with each player choosing which game to play on each turn. This formalism can be used to analyse the structure and outcome of games, for example if a game is positive then the 'left' player has a winning strategy. (See ⟨**7**⟩ for a more detailed summary.)

In 1967, Elwyn Berlekamp suggested to Richard that they write a book about combinatorial games, and Richard suggested that Conway should also be an author. Initially the book was planed to cover impartial games, but as the theory developed it was extended to partisan games where players can move in different ways. Whilst Richard, in characteristically modest manner, would suggest that most of the ideas in the book came from Berlekamp and Conway, and that he merely did the writing 'from Conway's dictation', there is no doubt that Richard contributed significant mathematical input, not least the parts on octal and related games. Certainly, Richard's characteristic wit and clarity of exposition are readily apparent throughout, as is his artistic talent and imagination in the diagrams, often clever cartoons, on almost every page. The two volume, 850 page, book *Winning Ways for your Mathematical Plays* [**54**] finally appeared in 1982 after 15 years' work. The first volume covers the general theory of partisan games, using the formalism indicated above, and is illustrated throughout by Hackenbush positions displayed boldly with red, blue and green edges. (Hackenbush is a game invented by Conway that uses edge-coloured graphs that are equivalent to certain games and numbers.) Volume Two presents a cornucopia of combinatorial games and puzzles such as Dots and Boxes, Turning Turtles, Hare and Hounds, and many others. These are analysed using the theory from Volume 1 alongside many other ideas and techniques.

The book is totally unique in content and style, and appeals as much to enthusiastic amateurs as to research mathematicians. It has been described as a masterpiece by many reviewers, and after 40 years it remains the definitive work on combinatorial games. It was republished in four volumes in 2001-04 with errors corrected, extended 'Extras' sections at the end of each chapter, new references added, and major revision to the chapter on the game Fox and Geese 'combining the theory with innovative computing algorithms'. Sadly, the three authors all died within a year and two days of each other in 2019-20. Richard recalls how he became interested in combinatorial games leading to *Winning Ways* in a video ⟨**6**⟩.

To make some of this material more accessible to the amateur player, Richard published a simpler guide to two-person games with the book *Fair Game: How to Play Impartial Combinatorial Games* [**69**] in 1989 which suggests strategies for playing a range of games, with exercises and solutions to develop the reader's skills.

Richard wrote many papers and articles about puzzles and games, here we mention a couple written with Alex Fink, then a student, which claimed a record of 70 for the greatest age difference of joint authors of a paper. The *Number-Pad game* is played on a 3×3 key-pad of numbers from 1 to 9 as on a calculator or phone [**116**]. Two players take turns to press keys which must be different from, but in the same row or column as, the previous player; the first player whose keys sum to more than a given integer t losing. This game, and some variants, is investigated for various t by setting up a 'tallest tolerable tower'. They also investigated *Rick's tricky Six Puzzle* involving the permutations that can be obtained by sliding 6 objects around the vertices of a graph formed by a hexagon with a pair of opposite vertices joined by two edges with a 7th vertex in the middle [**118**][**130**, Chapter 17]. This rather specific example nevertheless reveals connections with a rich variety of concepts: Steiner systems, the projective plane of order 4, card-shuffling, and binary codes.

The (one-player) game of *Life*, introduced by John Conway in 1970, became very popular amongst recreational players and programmers in the '70s. The game, which is a cellular automaton, is played on an infinite square board. Some initial configuration of squares or 'cells' are deemed to be alive, usually represented by counters on the squares. The configuration mutates in discrete time, a cell being live at time $t + 1$ if and only if it has exactly 2 or 3 live neighbours at time t. This delicately balanced rule leads to many fascinating life cycles, with stable as well as periodic configurations. Richard studied the game extensively, and in particular came up with a 5 cell configuration, known as the 'glider' which replicates itself with period 4 but displaced one square diagonally. Subsequently Bill Gosper invented the 'glider gun' which fires off a glider every 30 steps, giving rise to an unbounded number of live cells. Gliders are an important configuration in the theory of *Life*, and are key in showing that a universal Turing machine can be built out of the game. *Life* is described in detail in 'Winning Ways' [**54**, Chapter 25].

The popularity of *Life* was to a large extent due to Martin Gardner. Gardner (1914-2010) was a legendary American popular mathematics and science writer. He had no formal mathematical training but was enormously influential through his monthly 'Mathematical Games' column in *Scientific American*, which continued for 25 years, and his many books on mathematical recreations and games. Richard, together with John Conway, were welcomed by Gardner at his home in Hastings-on-Hudson, New York, to discuss their ideas on *Life* which Gardner wrote up for the October 1970 *Scientific American*, and later in his books. Gardner was an assiduous correspondent, and Richard valued greatly their exchange of ideas and suggestions on recreational mathematics that lasted for many years. Indeed, Gardner was the hub of a network

'Martin Gardner's Mathematical Grapevine' of leading names interested in games, including M. C. Escher, Solomon Golomb, Donald Knuth, and Roger Penrose, and he facilitated many collaborations across the network.

Richard was a regular participant, and indeed an organiser, for a series of conferences *Games of no Chance* held in MSRI Berkeley, and then at the BIRS Banff. He contributed regularly to the conference proceedings in the *Mathematical Sciences Research Institute* series ⟨**3**⟩.

A particular achievement was bringing the Eugène Strens *Recreational Mathematics Collection* of books and games to the University of Calgary. Eugène Strens (1899-1980) was a Dutch book collector who amassed a considerable library, including some 2,000 volumes on recreational mathematics and its history. Richard became aware of the collection through Martin Gardner, and, after Strens' death, Richard visited the Strens children who agreed to donate a substantial part of the collection to Calgary, on the understanding that it would be kept together, and Richard, with his characteristic generosity purchased the remainder. There have been many further significant donations to the collection, making it the most complete library of recreational mathematics in existence. He organised a conference in 1986 to celebrate the foundation of the collection ⟨**12**⟩.

3.2. *Graph theory and combinatorics*

Graph theory and combinatorics were amongst Richard's favourite areas, but up to the 1960s combinatorics did not have a very high reputation. As Richard points out ⟨**5**⟩ there was the view that 'with combinatorics we are just dealing with counting things and so is obviously trivial and most respectable mathematicians thought that ... combinatorics was not a reputable subject.' But in the late 1960s and early 1970s there were a considerable number of combinatorics conferences, many attended by Richard, and there was a realisation that researchers from a range of areas such as coding theory, experimental design, latin squares, combinatorial game theory, or enumerative combinatorics were all doing similar mathematics, but 'with a completely different vocabulary'.

Throughout his career Richard endeavoured to unify this diversity. A lecture he gave in Tehran in 1994, written up in [**94**], uses examples to show that apparently disparate topics across the area are in fact intimately and elegantly related. This was the genesis of the book *The Unity of Combinatorics* [**130**], written jointly with Ezra Brown, which brings together many of Richard's lifetime interests, including combinatorial games, number theory, projective geometry, and codes. Eight of the chapters are expanded versions of one or other of the authors' papers with three chapters papers from other authors, whilst the other eight chapters are Brown-Guy collaborations. The book was published in 2020 two months after Richard's death.

Many of the specific questions Richard investigated involved the 'thickness' of graphs quantified in some way. Several papers concern the *crossing number* of graphs, that is the least number of edge crossings in any plane drawing of the graph; see [**25**, **29**, **34**] for Richard's updating surveys. Möbius ladders (n-gons with each vertex joined the opposite vertex if n is even, or the opposite two vertices if n is odd) he showed have crossing number 1, and in [**6**] he gave bounds for the crossing number of the complete graph K_n on n vertices, obtaining exact values for $n \leq 10$ in [**30**]. Zarankiewicz had claimed a formula for the crossing number of the bipartite graph $K_{(m,n)}$, but the proof turned out to be flawed, and Richard verified the formula in some special cases [**14**]. With Hill, he obtained bounds for the crossing number of the graph obtained by deleting a Hamiltonian circuit from K_n, getting exact numbers for $n \leq 10$ [**30**]. Bounds for the *toroidal crossing number* (where the graph is drawn on a torus) of complete graphs are obtained in [**7**], and of bipartite graphs in [**13**].

Richard also wrote extensively on the *coarseness* of graphs, that is the maximum number of edge-disjoint nonplanar subgraphs whose union is the graph. By Kuratowski's theorem these nonplanar subgraphs can be taken to be the complete graph K_5 or the bipartite graph $K_{(3,3)}$.

In one of his earliest papers [**4**] he showed that the coarseness of K_n is at least $\binom{r}{2} + \lfloor r/5 \rfloor$ if $n = 3r > 30$ rather than Erdős's conjectured value of $\binom{r}{2}$. Then with Beineke [**9**] he found the coarseness of K_n for all n except when $n = 9r + 7$ or $n \in \{13, 18, 21, 24, 27\}$ where two values remain possible in each case, and they also found the coarseness of $K_{(m,n)}$ for many pairs (m, n) [**17**]. A related concept is the *outercoarseness* of a graph which is the maximum number of parts in an edge partition with each part non-outerplanar, where an outerplanar graph is one that may be drawn in the plane with all vertices on the boundary of a single face. With Nowakowski, Richard found bounds for the outercoarseness of the n-dimensional hypercube [**74**, **76**], and 17 years later, with Fink [**127**], he obtained exact values for all but a small number of values of n .

A nice result with Bollobás [**59**] shows that a tree with n vertices and maximum degree Δ can be *equitably 3-coloured* if $n \geq 3\Delta - 8$ or $n = 3\Delta - 10$, that is, the vertices can be coloured with 3 colours with adjacent vertices of different colours and the number of vertices of each colour differing at most by 1.

On the combinatorial side, Richard, with Austin [**50**], obtained simple recurrence formulae for the number of sequences of 0s and 1s of length n such that every digit 1 occurs in blocks of at least two 1s, and more generally at least k 1s.

3.3. *Number theory*

Richard was fascinated by 'intuitive' problems in number theory, such as the many described in *Unsolved Problems in Number Theory* [**53**]. He was not one for the technical side of algebraic or analytic number theory, except for using their results as tools for solving problems. As well as finding explicit or implicit formulae for solutions, he believed that numbers were needed to get the feel for results, and his papers generally include tables of values, some of which are the result of considerable computation, either by hand or by machine.

He enjoyed investigating Diophantine problems. For example, with Bremner and Nowakowski he classified the integers that are representable as the product of the sum of three integers with the sum of their reciprocals [**87**] and with Bremner he examined other representation problems of this nature [**101**]; these papers include substantial tables of numerical solutions. Further Diophantine problems stemmed from geometrical questions, see Section 3.4.

In his 1891 memoir, Lucas introduced *divisibility sequences* to develop tests for primality of certain numbers of the form $Ar^n \pm 1$ where $r = 2, 3$ or 5, $\gamma = \pm 1$ and r does not divide A. Richard's papers with Roettger and Williams [**122**, **125**] correct some errors in the formulation of these tests, but also show how some of Lucas' ideas can be extended to numbers of this form not covered by Lucas' work. Several other papers with Williams [**119**, **120**, **122**, **124**] also develop the theory of divisibility sequences. An earlier paper with John Selfridge [**26**] proposed procedures based on the converse of Fermat's little theorem for testing primality on small computers.

An *aliquot sequence* is the sequence of iterates $\{s^k(n)\}_{k=1}^{\infty}$, given an initial n, of the function $s(j) = \sigma(j) - j$ where $\sigma(j)$ is the sum of the divisors of j. Specific examples include perfect numbers and amicable numbers, where the sequence is constant or has period 2, respectively. Richard's fascination for these sequences led to several papers in the 1970s, with Selfridge and others [**27**, **37**, **40**, **41**, **91**]. Catalan in 1888, with a modification by Dickson in 1913, had conjectured that all such sequences terminate, that is reach a prime p, so that $s(p) = 1$, or become periodic. From a combination of theory and numerical evidence Guy and Selfridge made a counter-conjecture, that $\{s^k(n)\}_{k=1}^{\infty}$ diverges for many, if not all, even n. Richard's last paper [**129**] co-authored with some computer scientists, provided much stronger numerical evidence of this, including investigating 8,000 sequences with randomly selected initial numbers until they terminated or exceeded 2^{888}.

An elegant paper with Erdős and Selfridge [**56**] considers factorisations of $n! = a_1a_2\cdots a_k$ into integers with $n < a_1 < a_2 < \cdots < a_k \leq 2n$. Erdős had previously shown that such factorisations were only possible for finitely many n, but here all solutions are found; in particular there are none if $n > 239$. A range of further such results are presented, for example, if the inequalities are relaxed to $n < a_1 \leq a_2 \leq \cdots \leq a_k \leq 2n$ then there are solutions for all $n > 13$.

Fermat's little theorem tells us that if q is a prime then $b^q \equiv b \pmod q$ for all b. Variants on this are discussed at length in Richard's book [**53**]. In particular, a *prime pretender to base b* is a composite number q for which this identity holds, and the least such $q(b)$ corresponding to a given b is called the *primary pretender to base b*. The paper with Conway, Schneeberger, and Sloane, [**102**] obtains the remarkable result that there are exactly 132 distinct primary pretenders, with $q(b)$ periodic in b with period given by the 122-digit number $p!_{59}p!_9$ where $p!_k$ is the product of the first k primes.

Many of Richard's contributions originated from observing patterns in sequences of numbers, even though this could often be misleading. He formulated the *Strong Law of Small Numbers* [**68**] as 'You can't tell by looking' or 'There are not enough small numbers to meet the demands placed on them', for which he was awarded the Mathematical Association of America Lester R. Ford award for outstanding exposition. He followed this up with the *Second Strong Law of Small Numbers* [**73**], 'When two numbers look equal it ain't necessarily so!'. These laws are illustrated with 80 clever 'facts' drawn from across mathematics, for example, for the sequence $x_0 = 1,\ x_{n+1} = (1 + x_0^2 + x_1^2 + \cdots + x_n^2)/(n + 1)$, x_n is always an integer – or is it? It is an integer at least for n up to 42. Examples focussed on graph theory are given in [**84**].

For further reflections on Richard's work on number theory, see the account by Michael Jacobson, R. Scheidler and Hugh Williams in $\langle$**1**$\rangle$.

Widespread communication of mathematics was always important to Richard, and his lovely book *The Book of Numbers* [**108**] co-authored with John Conway is aimed at the keen amateur or budding student. Written in Richard's inimitable style, it opens with a chapter on the romance of numbers, and covers the development and generalisations of numbers, gems from number theory and its applications, families of numbers such as primes and Catalan numbers, and special numbers such as π and e.

3.4. *Geometry*

Euclidean geometry, particularly of objects in the plane or in three-dimensional space, provided another plentiful source of intuitive problems and problems with neat solutions; many of these were presented in the book with Croft and Falconer *Unsolved Problems in Geometry* [**77**].

A *unistable* or *monostatic polyhedron* is a solid convex polyhedron which when made of a uniform material always rolls onto the same face. A lovely problem was to show that unistable polyhedra exist, ultimately with as few faces as possible, and Richard designed, and actually constructed, one with 19 faces and 34 vertices [**3**, **19**]. Since then improvements have been made, and a unistable polyhedron with just 14 faces was designed by Reshetov $\langle$**9**$\rangle$ with the aid of a computer.

Richard wrote 'The combination of geometry and number theory is close to my heart.' [**115**]. Indeed, problems in geometry which involve integer sidelengths or areas of triangles or other figures can generally be formulated in number-theoretic terms, perhaps as Diophantine equations. In papers with Andrew Bremner, he investigated tilings of the unit square by triangles with rational sidelengths [**85**], finding points on adjacent sides of a unit square such that the three sides of the triangle formed are rational [**71**], and finding integer right-angle triangle-rectangle pairs with the same area and perimeter (such as with sidelengths (5,5,6) and (2,6,2,6)) [**114**]. Here the geometric questions are converted into problems which can be studied using elliptic curves.

Richard was fascinated by triangles and their various associated circles. The well-known theorem of Morley states that the angle trisectors of a triangle T meet at the vertices of an equilateral triangle. By allowing external trisectors as well there are 18 Morley triangles, all equilateral. A four-author paper [**107**] considers a triangle with rational sidelengths and asks when the Morley triangles have rational sidelengths. Using direct calculation it turns out that if T is equilateral then 6 Morley triangles have rational sides, there is a 1-parameter family of Pythagorean triangles where 2 have rational sides, and a 2-parameter family where all 18 have rational sides. Otherwise none of the Morley triangles have rational sidelengths. The 15 cases of T with integer sidelengths at most 30,000 are listed.

In an extraordinary 45 page paper *The lighthouse theorem, Morley & Malfatti – a budget of paradoxes* [**115**], the Lighthouse Theorem states 'Two sets of n lines at equal angular distances, one set through each of the points B, C, intersect in n^2 points that are the vertices of n regular n-gons. The circumcircles of the n-gons each pass through B and C.' One interpretation is that the point of intersection of the beams from lighthouses located at B and C rotating at the same constant rate traces out a circle. This paper is a fine example of Richard's expository skills, with delightfully crafted wording enhanced by his sense of humour, and many intricate figures drawn by Richard using a computer package.

Putting together these and other ideas that had been brewing in his mind, and adding in considerable further research, computations, and many more detailed diagrams, Richard embarked on a book called simply *The Triangle*. By October 2019 he realised that he would never finish the book, and he uploaded the 240 page manuscript on the *Mathematics arXiv* [**128**] just as it stood, including many interspersed comments and self-reminders; indeed Section 5.5 is simply headed 'So little done, so much to do'. But it gives an insight into the working methods of a remarkable expositor seeking to get every intricate detail just right.

Andrew Bremner writes 'Our last mathematical communications were in October of 2019, shortly before a planned visit to an MSRI workshop commemorating his colleague and co-author Elwyn Berlekamp. He had unearthed a rough draft of something we had talked about many years ago, involving Diophantine properties of the geometric configuration of the Morley triangles of a given triangle and a related Apollonian packing. This brought together his lifelong interest in classical Euclidean geometry, and his love of elementary number theory. Sadly, a final manuscript was never produced. His parting email finished: "Try the degenerate specimen, in which the point $(0, -60/13)$ on the circumcircle of the (right-angled) triangle $A(0, -60/13), B(-25/13, 0), C(144/13, 0), \ldots$" and rounded off with "I can't do arithmetic any more!". Then "See you ere long. 'air long? 'air long gone. R." '

Acknowledgements. I am most grateful to Anne Scott, Richard Guy's daughter, for her comments on this account, and to Andrew Bremner and Richard Nowakowski for their very helpful input.

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Kenneth J. Falconer
School of Mathematics and Statistics,
University of St Andrews,
St Andrews,
Fife, KY16 9SS,
Scotland.

kjf@st-andrews.ac.uk