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Essays in International Finance in Small, Very Open Economies

Shane Rommel Lowe

Submitted in fulfillment of the requirements of the Degree of Doctor of Philosophy in Economics

Adam Smith Business School, College of Social Sciences

University of Glasgow

February 2022

Abstract

The three essays which make up this thesis seek to evaluate how accounting for the unique structure of the world's smallest and most open economies affects standard macroeconomic relationships and models in international finance. These countries' vulnerabilities to external shocks, limited production capacity and their dependence on imports of goods and services for consumption and investment leave them exposed to excessive consumption volatility. Moreover, their choice of exchange rate regime may further complicate how they choose to manage and respond to external shocks.

Consumption theory advocates that consumers will seek to smooth consumption over their lifetime, while international risk sharing implies a positive and perfect correlation between relative consumption and the real exchange rate. However, substantial research has identified numerous examples of low or negative correlations between consumption and the real exchange rate, a phenomenon now known as the Backus-Smith puzzle. While several authors have sought to reconcile what has become an empirical regularity with theory, most perform these tests primarily for members of the Organisation for Economic Co-operation and Development or more developed economies. The first essay (Chapter 2) sought to empirically evaluate whether the degree of home bias for given values of the elasticity of substitution between domestic and foreign tradables helps to explain the Backus-Smith puzzle (as suggested in Corsetti et al. (2008)) for a diverse group of 150 countries. The results suggest that the Backus-Smith puzzle disappears for a group of more open economies. This is particularly evident for countries whose average imports of goods and services relative to nominal gross domestic product exceed 44%. These results highlight the importance of accounting for nonlinearity and country heterogeneity when testing economic theories of small open economies.

Since the 1990s, emerging markets and developing economies have accumulated substantial stocks of foreign exchange reserves to act as buffers against external shocks. However, increasingly, several authors have emphasized a role for foreign exchange reserves in reducing the probability of a sudden stop to capital inflows and the resulting roll-over risk of upcoming debt maturities. Yet, while some research has studied the role of foreign exchange reserves in reducing the marginal cost of borrowing (see Levy Yeyati (2008) and Bianchi et al. (2018) for example) and advocate some role for countries to borrow to 'top

up' foreign exchange reserves for precautionary purposes and reduce default risk and sovereign risk premiums, few identify whether this relationship varies with existing external debt or foreign exchange reserves levels or whether it varies across different types of countries or economic structures. For example, stylized facts presented in the second essay (Chapter 3) suggest that countries with more stable exchange rate regimes appear to exhibit a greater relationship between foreign exchange reserves and sovereign bond spreads than countries with more flexible exchange rate regimes. Thus, Chapter 3 seeks to ascertain the role of foreign exchange reserves in reducing the spreads on external sovereign bonds and to determine whether that effect varies as external debt levels rise and across exchange rate regimes. Leveraging data for 28 emerging markets and developing economies, Chapter 3 finds evidence that a larger stock of foreign exchange reserves reduces sovereign bond spreads, particularly in markets with more stable exchange rates. However, this relationship becomes statistically insignificant once external government debt levels exceed 33% of nominal gross domestic product (GDP). Further, while Chapter 3 presents evidence that countries can borrow to accumulate foreign exchange reserves and reduce bond spreads, countries with less flexible exchange rates stand to benefit more from this than their counterparts with more flexible exchange rate regimes.

The determinants of currency and debt crises are well researched, and several studies have yielded consistent results outlining the major predictors of these events. However, while some authors have sought to distinguish between the most important predictors depending on exchange rate regime, few, if any, seem to have investigated how these relationships vary by economic structure. In fact, the third essay (Chapter 4) hypothesizes that, due to differences in the degree of trade openness or country size, policymakers across economies may face tradeoffs when choosing between nominal exchange rate devaluation and default on external or foreign currency debt when deciding how best to correct for large macroeconomic imbalances. Chapter 4 contributes to the literature in two ways. First, it seeks to determine whether the predictability of real exchange rate overvaluation, a common measure of external imbalances and a key predictor of currency crises, varies, not only by the exchange rate regime, but also by the degree of trade openness and population size. Secondly, it assesses whether a country's choice to default on foreign currency obligations rather than to devalue its nominal exchange rate considering macroeconomic imbalances varies by the country's size or the degree of trade openness. The findings in this essay provide some evidence that for given levels of real exchange rate overvaluation, smaller, more open economies with fixed or managed exchange rates are less susceptible to currency crashes than larger, less open markets. This result is especially evident when the degree of real exchange rate overvaluation exceeds 24% but becomes statistically insignificant for small, open economies when real misalignment exceeds 35%. Further, when faced with real exchange rate overvaluation or other macroeconomic imbalances, the governments of smaller, more open economies are more likely to choose to default on their foreign currency debt rather than to devalue their nominal exchange rates, compared to their larger, less open counterparts.

Finally, the insights gleaned from this thesis suggest that policymakers should account for an economy's unique structure and inherent vulnerabilities when designing economic policies. Moreover, they justify the need for academics to incorporate additional country heterogeneities into their models of the small, very open economy.

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Acknowledgements

First, I would like to thank God for providing me with the opportunity to pursue this PhD with the University of Glasgow and with the ability to successfully complete this thesis, after five and a half years. From gaining acceptance into the programme, being offered a scholarship, to submitting the final thesis, He has proven that everything works in His timing and to His glory.

I wish also to thank the University for the opportunity to pursue this course of study. Not only did the University provide funding via a College of Social Sciences scholarship, but they granted my request to undertake this journey from thousands of miles away. The faith that was placed in me to complete this journey has spurred me on from Day 1, and I continue to sing the University's praises to anyone who will listen.

Of course, I am extremely grateful to my supervisors, Professor John Tsoukalas and Professor Ronald MacDonald, who agreed to supervise this project, recognizing the challenges which distance learning presented. Their guidance, support, and encouragement, whether remotely or on my periodic visits to Glasgow, have helped to push my research along, particularly when I met roadblocks along the way. Moreover, throughout the process, they challenged me to ask many pertinent questions about my research in every effort to produce the best work possible.

I also owe much gratitude to my parents, family, and friends, both in Glasgow and at home, who have all offered their heartfelt and sincerest support and encouragement throughout my journey. Special thanks go to Simon Naitram (my fellow Barbadian), Nabulambo Kilembe, and Johanna Tiedemann for constantly checking on my progress and for always making me feel welcome whenever I visited Glasgow.

Finally, to my wife Terri-Anne, I say a special thank you for putting up with my occasional rants and the many emotions that have accompanied this journey over the past five and a half years. She was always there to support me, especially in my lowest moments, and cheered me on until I reached the finish line. For that, I will be eternally grateful.

Dedication

This thesis is dedicated to my wife Terri-Anne.

Author's Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

Name: Shane Rommel Lowe

Signature:

Chapter 1: Introduction

Research in international finance is replete with a host of puzzles, established relationships and empirical regularities which may not always hold for economies of all structures and exchange rate regimes. Further, many models which have become standard frameworks with which to model the small, open economy are built on assumptions which may not be readily generalised across the very heterogeneous group of economies worldwide. For example, many of the standard real business cycle models of the small, open economy (see for example, Galí and Monacelli (2005)) assume that the elasticity of substitution between domestically produced goods and foreign goods is equal to 1, allowing for an analytical solution to the small, open economy model. This result, according to Obstfeld (2014), is highly controversial, and estimates have suggested that it may be closer to 2 (Feenstra et al., 2014) or less than 1 (Hooper et al., 2000; Bussière et al., 2013). Similarly, for most economies, nominal exchange rate devaluation is an oft-recommended course of action for many policymakers, in response to excessive external imbalances or near-depleted external buffers. However, many small, open economies often lack the production capacities necessary to substitute imported goods and services and take advantage of ‘cheaper’ domestic production. Consequently, nominal (or at times real) exchange rate devaluation is sometimes much less effective for this group of countries, and instead, they usually resort to alternative tools with which to restore macroeconomic stability. In fact, a country’s choice of exchange rate regime may depend on several factors which depend heavily on country-specific characteristics, including the extent of trade openness between two countries and the degree of mobility of both labour and capital (MacDonald, 2007).

The world’s smallest economies are perhaps among those whose unique structures may justify some deviation away from the standard assumptions, calibrations and relationships which underpin many models. These economies are typically more susceptible to the effects of global shocks and often experience larger declines in output because of this. For example, small, open economies, particularly those with highly concentrated export sectors (many of whom are largely dependent on tourism) were among those who suffered from the largest declines in real gross domestic product in 2020, triggered primarily by the onset of the ongoing COVID-19 pandemic. Similarly, of the 25 countries who suffered the steepest declines in real gross domestic product in the two years immediately after the global financial crisis (2009-10), 19 had populations of less than 5 million people.[1] More specifically, 11 of those 25 countries (44%) had populations below 1 million people, even though only about one-fifth of the world's countries have populations of that size.

Many small emerging markets and developing economies are also small island developing states (SIDS). These countries are especially exposed to the effects of natural disasters, such as hurricanes and earthquakes, which impose substantial damages to infrastructure and crops and impair these countries' abilities to earn foreign exchange from the export of goods and services. Acevedo (2016) estimates that between 1950 and 2016, the most destructive tropical cyclones imposed damages of 81.7% of gross domestic product (GDP) on average for twenty Caribbean islands, even in cases where these storms did not make landfall. Consequently, much of the volatility in economic activity witnessed in some of these economies comes from recuring episodes of destruction and rebuilding which exacerbate the effects of other external shocks emanating from the global economy.

Even with more volatile output, emerging markets and developing economies often suffer from excessive volatility of per capita consumption relative to per capita output. This is although greater trade and financial integration should (in theory) facilitate greater risk sharing among consumers across emerging markets and developing economies in response to shocks to domestic income. Small economies and those most dependent on imported goods and services for consumption and investment are probably most prone to this vulnerability. For example, Moore and Walkes (2010) provide evidence that the concentrated economic structures that often characterize small economies expose economies to more volatile consumption growth, and this can be further exacerbated by the extent of trade openness.

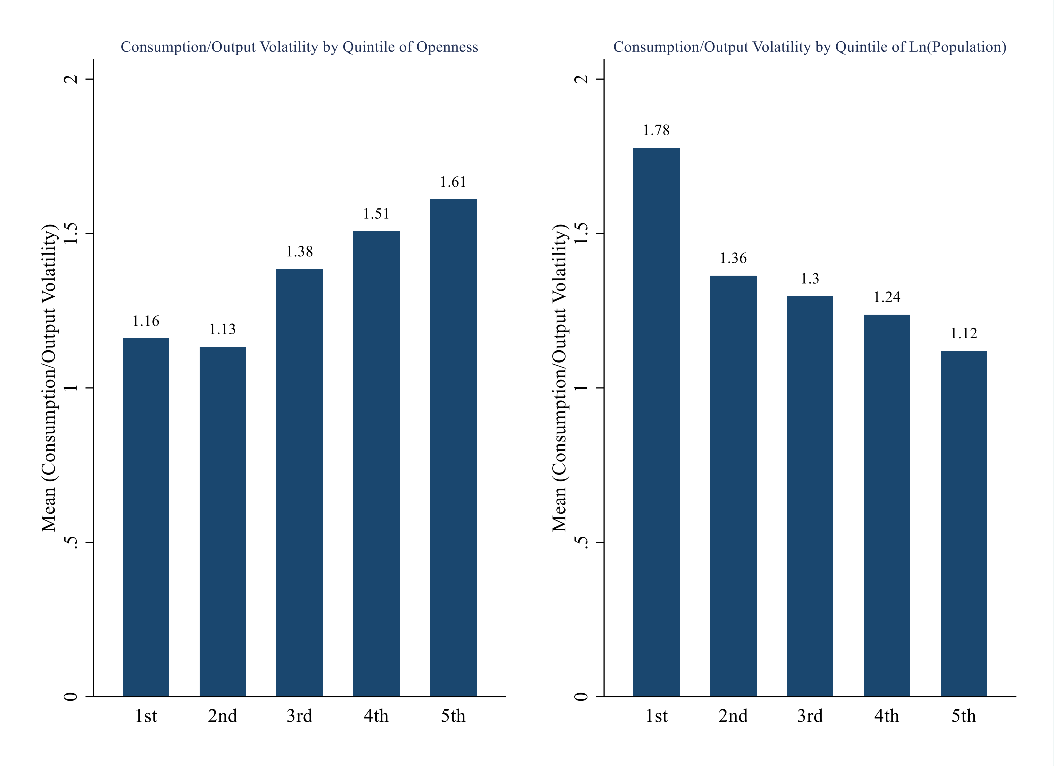
Figure 1.1 illustrates that for a sample of 150 countries (over the period 1980 to 2018), the ratio of consumption volatility to income volatility is increasing with the degree of trade openness (measured as the average ratio of imports of goods and services to nominal gross domestic product) and declining in the size of the population.[2] Those economies in the 4^{th} and 5^{th} quintiles of trade openness exhibit average per capita consumption volatility that is at least one and a half times that of per capita output. Moreover, the smallest 20% of

[1] This assessment was derived from data using the International Monetary Fund's World Economic Outlook. The total number of countries considered was 193, of which the median and mean population sizes were 7 million people and 35 million people, respectively.

[2] Volatility is measured by taking the standard deviations of the deviations of logged real household consumption per capita and logged real gross domestic product per capita away from their respective logged Hodrick-Prescott-filtered trends.

countries exhibits significantly more volatile consumption than output than their larger counterparts. In contrast, volatility in per capita consumption is much closer to that of per capita output for the largest and least open group of countries.

Figure 1.1: Ratios of Per Capita Consumption Volatility to Output Volatility by Quintiles of Openness (left panel) and Quintiles of ln(Population) (right panel) Across 150 Countries (1980 – 2018)



Source(s): World Bank's World Development Indicators, United Nations National Accounts Statistics, author's calculations.

Accounting for the unique economic characteristics of small, open, developing economies goes some way to explaining statistical phenomena such as these. Generally, small, very open economies face binding foreign exchange constraints and typically import most of their consumption needs; even domestic production and investment depend heavily on imported intermediate goods (Worrell et al., 2013). Import-dependent economies may face sharp reductions in consumption (perhaps even greater than the contraction in income), if they face unexpected constraints in their ability to finance imports (Kodama, 2013).

One of the recurring puzzles that has been a feature of international macroeconomics over the last three decades is the Backus-Smith puzzle. Backus and Smith (1993) illustrate that one of the features of international risk sharing is a perfect, positive co-movement between relative, cross country consumption and the real exchange rate. Essentially, as a consumer's basket of goods and services gets cheaper (depreciates) relative to the rest of the world, it is expected to consume more goods and services relative to everyone else. However, several authors (including Backus and Smith (1993)) find that this fails to hold in practice. In fact, most find that these correlations are often low or even negative. Although a host of reasons has been posited for why this may be the case (see for example, Selaive and Tuesta (2003), Chari et al. (2002), Corsetti et al. (2008), Akkoyun et al. (2017)), most of the empirical literature that has attempted to reconcile theory with reality has focused almost exclusively on understanding this issue for larger, more developed economies. Consequently, the assumptions that have underpinned these approaches (for example, a high degree of home bias) often reflect those that are most appropriate for these economies. Smaller, more open economies have received little attention, and the assumptions that are used in these studies usually reflect that. Because of that, the results of the research that has been conducted to date should not be extrapolated to draw general conclusions about smaller, more open economies.

Chapter 2 of this thesis expands the investigation into the Backus-Smith puzzle to capture a much more diverse and heterogeneous group of countries. The analysis leverages the work of Corsetti et al. (2008) who illustrate that the nature of the relationship between relative per capita consumption and the real exchange rate may depend on the elasticity of substitution between domestically produced goods and those produced abroad, and on the degree of home bias in the consumer's consumption basket. As highlighted earlier, the calibrated parameters that have been used in models of the small open economy before have often failed to capture the realities of the smallest countries in the world. The results of this chapter highlight that, by accounting for these economies' high degrees of trade openness (and low degrees of home bias) and their relatively inelastic response to changes in relative import prices, the Backus-Smith puzzle becomes less prevalent (and virtually disappears) for smaller, more import-dependent economies. Specifically, these effects are most stark for economies for whom imports of goods and services account for over 44% of nominal gross domestic product and whose population sizes are no more than 5.4 million people.

Even if emerging markets and developing economies opt to share risks, many still opt to protect themselves from severe shocks which may disrupt economic activity. Several authors (see for example, Moore and Glean (2016), Prabheesh (2013)) have posited and explained a role for the accumulation of foreign exchange reserves as buffers to balance of payments shocks, including those emanating from the terms of trade and sudden stops to capital inflows. Additionally, central banks in emerging markets also worry about excessive exchange rate volatility and its cost to consumption and general economic activity. Quite possibly, the role of foreign exchange reserves as an appropriate policy tool may take on greater relevance for small, open economies, given these countries' dependence on imports for consumption and the naturally heavy weighting that foreign goods and prices have in the overall household consumption basket of goods and services. Further, using foreign exchange reserves as buffers to limit exchange rate volatility and output losses may in turn achieve the targets of price stability and full employment in a way very much like what the traditional Taylor rule intends. The International Monetary Fund also recognizes the need to account for the unique characteristics of small, open economies when assessing the adequacy of a country's foreign exchange reserves. In its 2016 guidance note on reserve adequacy, it specifically refers to the use of a special metric for small islands with access to international financial markets which accounts for the vulnerabilities which are unique to these countries (International Monetary Fund, 2016; Mwase, 2012). The International Monetary Fund (2015) also recognizes that emerging markets and developing economies may hold foreign exchange reserves for different reasons or in response to different shocks, depending on the exchange rate regime that the country chooses to employ.

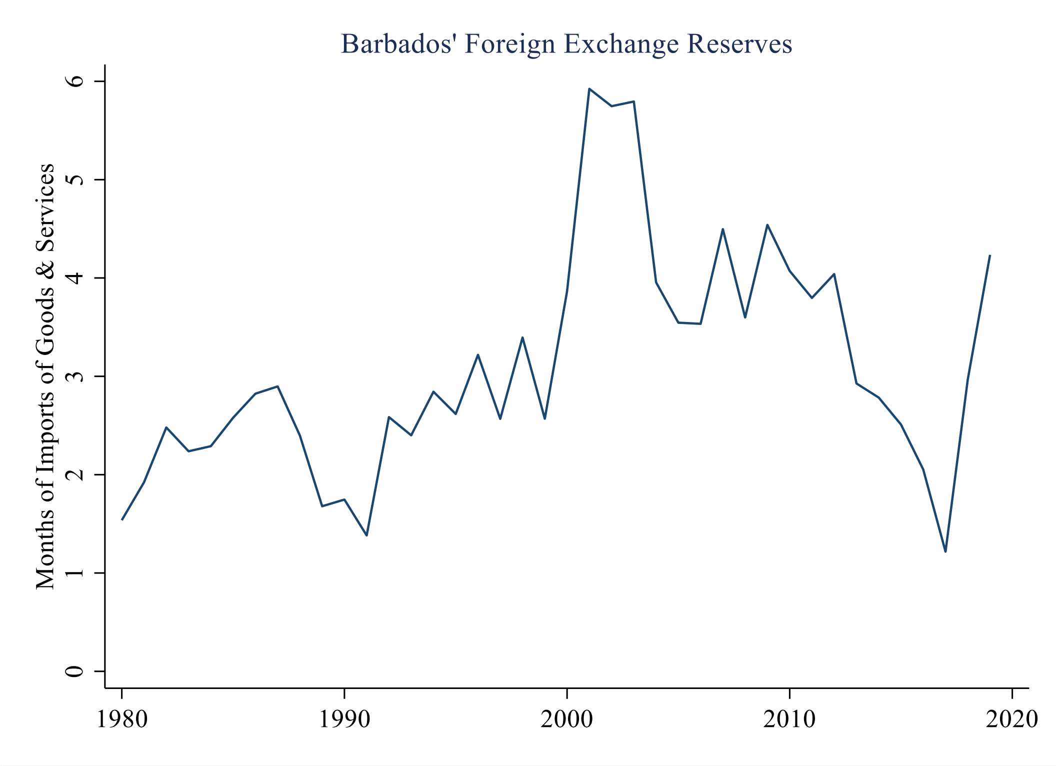
The way in which the accumulation of foreign exchange reserves is financed, and the risks associated with doing so are probably not homogenous across country groups, however. Holding foreign exchange reserves often comes at an opportunity cost because the return that central banks earn on the risk-free assets in which they tend to hold foreign exchange reserves is typically very low, compared to the coupons paid on the debt that is sometimes used to accumulate them. Initially, it may seem practical to suggest that governments should use any excess foreign exchange reserves to pay down as much external debt as possible. However, in practice, most countries hold both foreign exchange reserves and external debt, with some holding more of the former than the latter.

Chapter 3 attempts to determine to what extent issuing additional debt to accumulate more foreign exchange reserves improves or worsens a country's susceptibility to a sudden stop in capital inflows. Governments and central banks need to strike a delicate balance between the additional default risk that greater debt brings and the additional security that a larger stock of foreign exchange reserves provides. Understanding how the accumulation of both affects the spreads that governments pay on their external debt can give us a clue about how investors perceive that the net effect of these changes affects their assessments of a government's probability of default. Leveraging data for 28 emerging markets and developing economies, the analysis finds that accumulating foreign exchange reserves via greater debt issuance may not increase bond spreads as long as external government debt is no more than 33% of nominal gross domestic product. Beyond that threshold, foreign investors become increasingly concerned and demand a greater spread on the government's debt. However, economies with less flexible exchange rates appear to benefit more from the accumulation of foreign exchange reserves than those with more flexible exchange rate regimes.

Still, even governments with access to financial markets and large external buffers face large external shocks or macroeconomic imbalances which require sharp policy adjustments. For example, large imbalances can sometimes evolve into an overvalued real exchange rate or an unsustainably large and negative net foreign asset position after years of persistently wide current account deficits. To correct these imbalances, governments may opt either to, among other things, default on their stock of external debt or devalue the nominal exchange rate, especially in cases where the exchange rate is typically fixed or managed.

The choice between devaluation and default, and the implications which ensue, may differ depending on (again, among other things) the nature of the economy and the exchange rate regime. For example, exchange rate devaluation may appear less favorable for economies with limited production capacities and those whose dependence on imports may trigger a sharp passthrough to inflation. Over the past three decades, Barbados, a small, import-dependent island economy with a long-standing exchange rate peg, has faced at least two instances where exchange rate devaluation seemed almost inevitable – once in 1991/1992 and again in 2017/2018. On both occasions, large imbalances reduced the country's stock of foreign exchange reserves which reached precariously low levels (Figure 1.2). On neither occasion did the government adjust its exchange rate peg with the United States dollar, however. Instead, the authorities opted for a sharp fiscal adjustment (1992) and a default on the government's debt (both domestic and external) combined with a fiscal adjustment (2018) (both times accompanied by a financing programme with the International Monetary Fund) to restore external balance.

Figure 1.2: Barbados Foreign Exchange Reserves Measured in Months of Imports of Goods and Services (1980 – 2019)



Source(s): International Monetary Fund's International Financial Statistics, author's calculations

Chapter 4 models the choice between a sharp exchange rate devaluation and a default on foreign currency debt and tries to determine to what extent this decision varies by a country's size or degree of openness. As a preview, the results suggest that smaller, more open economies with fixed or managed exchange rates are less likely to devalue their currencies (and experience a currency crash) than their larger, less open peers. Moreover, when faced with the option to devalue or default, they are more inclined to choose the latter.

Ultimately, this thesis contributes to the literature on small, very open economies and attempts to highlight why the economic issues facing these markets should be viewed differently to those facing their larger, less open peers. Specifically, the analysis in each chapter highlights the heterogeneities and nonlinearities which are present in many economic relationships, and is intended to help academics, investors and policymakers better understand the peculiarities that these markets face. Finally, the research in this thesis addresses issues that are relevant to today's environment. The large economic shocks that have emanated from the ongoing pandemic and the policy responses in small, open economies worldwide will serve as a further test of whether the hypotheses posited within and the conclusions and policy implications inferred in Chapter 5 hold true.

Chapter 2: Revisiting the Backus-Smith Puzzle – Is there a Role for Country Heterogeneity?

2.1 Introduction

Established consumption theory suggests that consumers seek to smooth their consumption over their lifetimes, even in the face of temporary shocks to or lifetime variations in income (see Friedman (1957), Modigliani and Brumberg (1954) and Modigliani and Ando (1957)). Further, in an open economy context with complete markets, countries may choose to save or borrow on international markets and share risks to smooth consumption in the face of external shocks to income, leading to almost perfectly positive consumption correlation across countries (Backus et al., 1992). Empirically, however, these correlations are weak, and Backus and Smith (1993) illustrate that introducing non-traded goods to an otherwise single, traded good model goes a long way to explaining why these international correlations are weak.

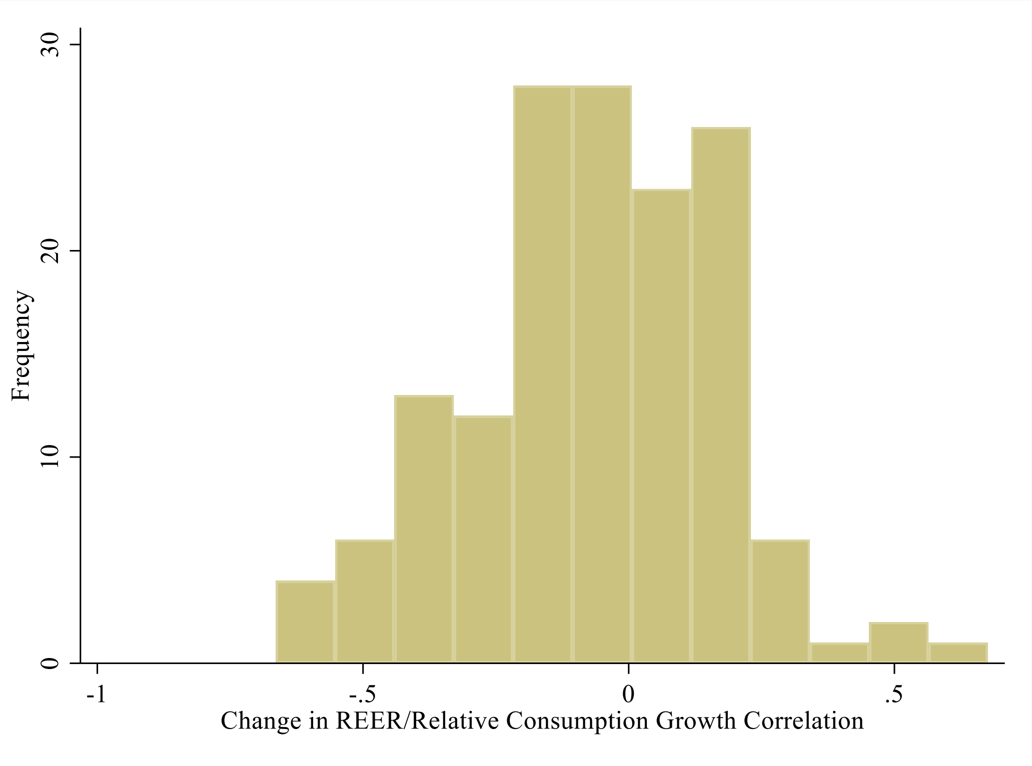
Backus and Smith's (1993) theoretical model also implies that real exchange rates and cross-country consumption should move together in the same direction. As real exchange rates depreciate (increase), the country which benefits from a cheaper consumption basket should take advantage of this and consume more relative to their trading partners. Similarly, appreciation (decrease) of the real exchange rate should prompt a reduction in consumption relative to foreign consumption as the domestically consumed basket of goods and services becomes more expensive. Ultimately, relative consumption and the real exchange rate should also exhibit similar moments. Backus and Smith's (1993) empirical findings across eight Organisation for Economic Co-operation and Development countries suggest that this does not hold in practice, however, providing some evidence against international risk sharing. In fact, they find quite the opposite: real exchange rates are generally more volatile than relative consumption, while correlations between relative consumption and real exchange rates are low or negative. Others have since documented similar results for primarily large or advanced economies (see for example Chari et al. (2002) and Corsetti et al. (2008)), and this puzzle has subsequently been labeled the consumption-real exchange rate anomaly or the Backus-Smith puzzle.

Several authors have proposed various solutions (with differing degrees of success) to reconcile the negative correlation between relative consumption and real exchange rates with international risk sharing. These include the presence of non-traded sectors (Selaive and Tuesta, 2003) and incomplete asset markets (Chari et al., 2002), the introduction of distribution costs (Corsetti et al., 2008), and cointegrated productivity shocks (Akkoyun et al., 2017). Further, Hess and Shin (2010) test the standard risk sharing equation and find that, while less than half of consumers in their sample shares risks, accounting for the volatility of the nominal exchange rate goes a long way to explaining the Backus-Smith puzzle for the remaining consumers as relative bilateral inflation rates move as expected with relative per capita consumption growth.

However, these studies have focused almost exclusively on a relatively homogenous group of more developed economies, with little or no emphasis on smaller, less homogenous, developing economies. The assumptions made and parameter estimates or calibrations used are therefore less representative of a wider group of countries, and so, any conclusions from their empirical findings cannot necessarily be readily extrapolated to smaller, less developed economies. For example, Hevia (2014) documents material differences in empirical regularities between advanced economies and emerging markets. Compared to advanced economies, emerging markets exhibit higher consumption volatility relative to output, more volatile real gross domestic product and trade balances, and a larger negative correlation between the trade balance and gross domestic product. Still, in his sample of 13 emerging markets, all but one are currently classified as high or upper-middle income countries.[3] Below, Figure 2.1 plots the distribution of correlations between growth in real effective exchange rates and per capita consumption growth relative to its top 20 trading partners for 150 countries using annual data spanning 1981 to 2018. On average, these correlations (hereafter referred to as the Backus-Smith or BS correlation) are negative, with mean and median correlations of -0.067 and -0.053, respectively. However, in some cases, correlations are positive, and materially so – the largest correlation across all countries is 0.676. Therefore, these relationships cannot and should not be generalised across countries.

[3] The 13 emerging markets were Argentina, Brazil, Chile, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Slovakia, South Africa, Thailand, and Turkey.

Figure 2.1: Distribution of Backus-Smith Correlation Across 150 Countries (1981 – 2018)



Source(s): World Bank's World Development Indicators, United Nations National Accounts Statistics, author's calculations.

Corsetti et al. (2008) offer a potential explanation for this heterogeneity. They suggest that their ability to reproduce the Backus-Smith correlation depends on the persistence of endowment shocks, the assumed value of the trade elasticity parameter, and the degree of home bias in consumption. These parameters (particularly the latter two) determine whether the substitution or income effects dominate consumers' response to productivity or supply shocks and influence the nature of the correlation between the real exchange rate and relative consumption. Therefore, for economies with low trade elasticities and high degrees of home bias, or high trade elasticities and low degrees of home bias, their theoretical model implies that the Backus-Smith correlation should be negative. In all other extreme cases, the correlation could be positive. This occurs because, in the first scenario, a positive domestic supply shock reduces the price of the domestically produced good and in turn depreciates (increases) the terms of trade. The value of domestic output declines, generating a negative income effect on domestic demand. Even though the consumer may desire slightly more of the comparatively cheaper domestic good, the substitution effect is unable to offset the effects of weaker income effects, due to a low elasticity of substitution. Because of home bias in consumption, the terms of trade and real exchange rates move in the same direction (depreciate) but relative consumption falls. Alternatively, with a low elasticity of substitution between tradable goods produced at home and abroad and a low degree of home bias in the domestic economy, the positive supply shock also reduces the prices of domestically produced goods and depreciates the terms of trade. Again, the negative income effects more than offset the positive substitution effect from lower domestic prices and reduces domestic consumption. However, because the degree of home bias is low, and consequently, domestic consumers include a larger share of foreign goods in their consumption baskets, the terms of trade and real exchange rate move in opposite directions, in response to domestic supply shocks. In this scenario, relative consumption and the real exchange rate (which appreciates) move in the same direction.

This chapter leverages Corsetti et al.'s (2008) work and seeks to empirically evaluate the effects of home bias (or inversely trade openness) on the relationship between relative consumption and real exchange rates. More generally, it puts some of the hypotheses derived from their work to the test and takes their model to the data. Unlike other research in this area, this chapter broadens the analysis to a panel of 150 countries of varying population sizes, degrees of openness and per capita income levels, to draw more general conclusions about the nature of the Backus-Smith relationship and the extent to which it varies across different country groups. Econometric results suggest that trade elasticities in this sample of countries are low (less than one in absolute value), in line with other estimates using macroeconomic data. Moreover, while the Backus-Smith correlation is still low or negative for most countries, the relationship becomes positive and statistically significant for more open economies (that is, those with less home bias). Finally, the results of a panel threshold regression suggest that the sign and statistical significance on the real exchange rate switches from negative and statistically insignificant to positive and statistically significant when the ratio of imports to nominal gross domestic product (the proxy for openness) exceeds 44%.

The rest of the chapter is structured as follows. Section 2 reviews relevant theoretical and empirical literature on the Backus-Smith puzzle and risk sharing, while sections 3 and 4 provide descriptions of the theoretical and empirical frameworks, respectively upon which the study is based. Section 5 presents and discusses the empirical results and section 6 offers conclusions and recommendations.

2.2 Literature Review

2.2.1 The Backus-Smith Puzzle and Evidence of International Risk Sharing

The Backus-Smith Condition and the Consumption-Real Exchange Rate Anomaly

A common theoretical feature of global risk sharing is the one-to-one relationship between relative consumption and the real exchange rate (see for example Backus and Smith (1993), Kollmann (1995), Chari et al. (2002)). This relationship stems from the efficient allocation of international consumption which satisfies the condition that the marginal utility of an extra unit of consumption is equal to its marginal cost. In other words,

$\frac{P_t^*}{P_t} U_{c,t} = U^*_{c^*t}$ or

$$\frac{P_t^*}{P_t} = \frac{U^*_{c^*t}}{U_{c,t}} \qquad (2.1)$$

where P_t^* and P_t are the foreign and domestic consumer price indices at time t expressed in the units of the home country by using the nominal exchange rate. Thus, $\frac{P_t^*}{P_t}$ gives the real exchange rate. Similarly, $U^*_{c^*t}$ and $U_{c,t}$ are the marginal utilities of domestic (C_t) and foreign (C_t^*) consumption. For this relationship to hold, any increase (depreciation) in the real exchange rate would theoretically lead to an increase in domestic consumption and an associated reduction in its marginal utility. Denoting $\frac{P_t^*}{P_t}$ (expressed in units of the home country) as the real exchange rate (RER_t) and taking logs of both the real exchange rate and relative, per capita consumption, equation 2.1 can be expressed as the standard risk sharing condition:

$$c_t - c_t^* = \frac{1}{\sigma} rer_t \qquad (2.2)$$

where lower case letters denote those that have been logged and σ is the risk aversion parameter from the constant relative risk aversion (CRRA) utility function.[4]

[4] See Appendix 2.A.1 for a fuller derivation of this relationship leveraging Galí and Monacelli's (2005) small, open economy model.

However, Backus and Smith (1993) discovered that empirical data for the Organisation for Economic Co-operation and Development appeared at odds with the predictions of standard, international business cycle models to which non-traded goods were introduced to reconcile the puzzle of low cross country consumption correlations. Instead of the positive and strong correlations expected between relative consumption and real exchange rates, Backus and Smith (1993) found quite the opposite. The empirical correlations between consumption and the real exchange rate were low or negative, and this result is robust in the case where only consumption of non-durables and services is measured. This anomaly became known as the "Backus-Smith puzzle".

Kollmann (1995) finds comparable results to Backus and Smith (1993) and further illustrates that consumption and real exchange rates do not exhibit a long run, cointegrating relationship as suggested by theory. However, they do suggest that the assumption of complete asset markets may be unnecessarily strong, and some deviation from this may yield more positive results.

Early Attempts to Reconcile the Data with Theory

Chari et al. (2002) take Kollmann's (1995) advice on board and allow for deviations away from the law of one price and a form of incomplete asset markets where they restrict the set of globally traded assets in an attempt to explain this consumption-real exchange rate anomaly. However, in contrast with Backus and Smith (1993), they chose not to include a specific role for non-traded goods as they deem it unnecessary to explain fluctuations in the real exchange rate. While they illustrate that a household risk aversion parameter of 5 is sufficient to reproduce the level of US/Europe real exchange rate volatility evidenced between 1973 and 2000, their two-country model is unable to match the empirical persistence of real exchange rates and the negative correlation between the exchange rate and consumption. They suggest that "future research should focus on incorporating richer forms of asset market frictions" (Chari et al., 2002) to better explain this puzzle.

Despite what Chari et al. (2002) suggest, most research since theirs has included a role for the relative prices of traded and non-traded goods to help explain the Backus-Smith puzzle. Most also build on their assumption of incomplete asset markets, but some research has shown that this may not be necessary to replicate the aforementioned empirical anomalies. Selaive and Tuesta (2003) take Chari et al.'s (2002) suggestion of richer forms of asset market frictions on board and assume that the cost of holding global bonds, an accumulation of net foreign assets (NFA) and the assumption that uncovered interest rate parity does not hold characterise imperfect asset and international financial markets. Additionally, to permit deviations away from the law of one price, they assume only partial pass-through of exchange rate shocks to domestic prices and the presence of distribution costs which produces a wedge between producer and consumer prices and the elasticities of demand in the home and foreign countries. While the assumption of partial exchange rate pass through appears not to assist in addressing the Backus-Smith puzzle, the other assumptions do. The size of the net foreign asset position determines the country's cost of holding bonds and the interest rate differential (risk premium) with other countries – greater net foreign asset holdings suggest a lower risk premium and smaller cost of holding bonds. Further, in response to positive productivity shocks, the real exchange rate and net foreign assets are negatively correlated, as holding more foreign assets leads to an appreciation of the real exchange rate and leads to a violation of the risk sharing condition. The authors describe their mechanism as such: a positive productivity shock to the tradable sector reduces the prices of domestically produced tradables relative to imported goods (the terms of trade) and grows total output faster than consumption through greater exports. This generation of foreign exchange leads to greater foreign asset accumulation, additional domestic wealth, and a reduction in labour to the nontraded sector (the Balassa-Samuelson effect). The subsequent rise in the relative price of nontraded goods appreciates the real exchange rate and induces the negative correlation between relative consumption and the exchange rate.

Corsetti et al. (2008) also build on the assumptions of incomplete markets and the violation of the law of one price due to the presence of distribution costs and find that international business cycle models can produce results consistent with the "…evidence on the lack of risk sharing…" (Corsetti et al., 2008). Like Selaive and Tuesta (2003), they suggest that large wealth/income effects (for both domestic and foreign consumers) and reduced supply of labour stemming from positive productivity shocks can induce the empirical results characteristic of the Backus-Smith puzzle, but this depends on the value of the price elasticity of tradable goods. With a high degree of home bias, a model with low trade elasticity and incomplete asset markets produces a negative correlation between the real exchange rate and consumption of -0.24 for the United States of America and its Organisation for Economic Co-operation and Development trading partners compared to -0.71 in the data. Further, the model is generally able to match other key empirical facts as it produces real exchange rate volatility 3.0 times that of real gross domestic product (compared to 3.9 in the data), generates significant terms of trade volatility relative to gross domestic product (2.4 relative to 1.7 in the data) and replicates the negative correlation between real net exports and gross domestic product (-0.4 compared to -0.5 for the United States of America). However, significantly increasing the value of the trade elasticity parameter without any additional adjustments fails to reproduce any of these facts and leads to high, positive consumption/real exchange rate correlation, comparatively low real exchange rate and terms of trade volatilities and a positive correlation between net exports and gross domestic product. Corsetti et al. (2008) then suggest that significantly increasing the persistence of (transitory) productivity shocks to the tradable sectors goes some way to correcting these discrepancies, but still leaves us with comparatively stable real exchange rates and an even greater positive correlation between net exports and output. Benigno and Thoenissen (2008) also suggest that the inclusion of non-traded goods and imperfect financial markets produces an appreciation of the real exchange rate and a negative correlation with consumption, but their assumption of fully flexible prices results in a level of real exchange rate volatility which is significantly below that observed in the data.

More Recent Attempts to Reconcile the Data with Theory and Alternative Explanations of the Puzzle

Akkoyun et al. (2017) and Lambrias (2020) also reproduce the negative Backus-Smith correlation in their respective versions of small, open economy models. Both studies investigate the roles of productivity shocks to achieving empirical regularities. Akkoyun et al. (2017) build on Corsetti et al.'s (2008) framework, but model shocks to total factor productivity as "…cointegrated processes instead of transitory processes…" (Akkoyun et al., 2017). Specifically, cointegrated productivity shocks produce the magnitude of income effects necessary to generate the negative relationship between relative consumption and the real exchange rate for the United States of America and a weighted average of 15 European Union countries for both low and high values of the trade elasticity parameter. In contrast, Lambrias' (2020) international business cycle model with complete markets assumes imperfect substitutability of capital and total factor productivity news shocks. He illustrates that the Backus-Smith puzzle "…is not necessarily connected to market inefficiency and limited risk sharing, rather it can arise as the efficient allocation in an economy where wealth effects on labour supply are low, the price of non-tradables plays an important role for real exchange rate determination and innovations to technology are anticipated..." (Lambrias, 2020).

Though the literature reviewed thus far appears to confirm and reconcile Backus and Smith's (1993) finding of limited risk sharing, most studies to date have focused primarily on the more developed economies of the Organisation for Economic Co-operation and Development who tend to have quite flexible nominal exchange rates and exhibit consumption volatility on par with output volatility. However, Hess and Shin (2010) suggest that the Backus-Smith puzzle exists primarily for countries with volatile nominal exchange rates as "…bilateral inflation differentials are negatively correlated with bilateral consumption movements…" (Hess and Shin, 2010) as predicted by theory. Thus, some degree of global risk sharing is indeed possible, and the puzzle is expected to disappear for countries with fixed nominal exchange rates, a hypothesis supported using data for all 50 US states. Hadzi-Vaskov (2008) confirmed Hess and Shin's (2010) findings but used data instead for the 12 original members of the Eurozone. In contrast, Petrović (2016) produced evidence that the Backus-Smith puzzle exists for the 27 member countries of the European Union, but unlike Hadzi-Vaskov (2008), suggested that changes in the nominal exchange rate are not solely responsible for this. Devereux et al. (2012) also test the standard Backus-Smith relationship for a range of 28 upper-middle income, high income, and advanced economies, but use projections from professional forecasters rather than realized observations. Not only do they find no positive relationship between forecasts of consumption and real exchange rates, but they also suggest that this outcome is robust across countries with "…floating nominal exchange rate regimes, fixed exchange rates, or common currencies…" (Devereux et al., 2012). Finally, Pavlidis et al. (2015) evaluate the causal relationship between real exchange rates and consumption for 14 Organisation for Economic Co-operation and Development countries and find that non-linear Granger causality tests find more cases of causality between the two variables than standard linear tests like those mostly used in the empirical literature. The latter finding implies that accounting for non-linearities in the Backus-Smith relationship may shed additional light on the wide dispersion in the correlations illustrated in Figure 2.1.

Further, while consensus seems to be building on the roles of non-traded goods and imperfect capital markets in explaining part of the Backus-Smith puzzle and the limited evidence of global risk sharing, a number of recent studies have delved even deeper into the topic by investigating more granular characteristics of consumption bundles. Engel and Wang (2011) decompose the basket of consumption goods into durables and non-durables, with the former being traded internationally along with a stock of capital goods. While they can replicate the negative correlation between the real exchange rate and consumption after a positive shock to the durable (tradable) sector, their model is unable to maintain the positive correlation between the real exchange rate and the terms of trade evident in the data. However, they provide some insight into how the distinction of durable and non-durable goods may influence the way consumption is measured and tests of the Backus-Smith puzzle are conducted. The authors noted that unlike with non-durable goods, the utility derived from the purchase of durable consumption goods is experienced even beyond the initial purchase. This creates a discrepancy between the measurement of consumption in national accounts databases and the utility maximised by households in standard utility functions (Engel and Wang, 2011). In addition, further relaxing the assumptions and permitting global trade of non-durables produces comparable results to their baseline model but reduces the volatility of both imports and exports. Further permitting greater volatility of tradable non-durables relative to non-traded non-durables produces a level of trade volatility more akin to data for the United States of America.

Burnside and Graveline (2012) and Hamano (2013) then suggest that the Backus-Smith puzzle may purely be an empirical one and need not be associated with no or limited international risk sharing. Burnside and Graveline (2012) discuss specific conditions under which fluctuations in the real exchange rate can be related to the degree of international risk sharing. They suggest that, even under incomplete asset markets, "…to explain how exchange rates are determined, it is necessary to make specific assumptions about preferences, goods market frictions, the assets agents can trade, and the nature of endowments or production…" (Burnside and Graveline, 2012). Meanwhile, Hamano (2013) makes a distinction between an empirical real exchange rate and a welfare-based exchange rate and reconciles the Backus-Smith puzzle with evidence of international risk sharing. Essentially, the study posits that consumers love variety and derive additional wealth from having a higher number of product varieties from which to choose. A positive productivity shock induces more firms (each offering a different variety of the consumption goods) to enter the domestic market and this increased variety increases consumer wealth and depreciates the welfare-based real exchange rate as home bias for domestically produced goods makes domestic consumption more attractive. At the same time, the wealth effect generates an appreciation of the empirical real exchange rate in line with the mechanisms discussed earlier. The result is a strong, positive correlation between the welfare-based real exchange rate and consumption, and a much smaller correlation between the empirical real exchange rate and consumption as found in the data.

Caporale et al. (2015) illustrate that the Backus-Smith puzzle becomes even more severe when one considers an advanced economy and an emerging market (China), even in the context of complete asset markets and even after stock market liberalisation in the latter. In fact, what seem to be more important are the presence of recursive preferences in consumption, a bias toward home consumption and a strong correlation among long-run shocks. These produce levels of real exchange rate volatility, correlations between real exchange rates and consumption, and cross-country consumption correlations all in line with the empirical data.

2.2.2 Imperfect Risk Sharing

Notwithstanding mixed results regarding the general existence of international risk sharing, numerous authors have highlighted that, within countries, many consumers may opt to share risks while others do not. Further, the extent to which this occurs may vary across countries. In addition to finding an empirical solution to the Backus-Smith puzzle, Hess and Shin (2010) estimate the extent to which consumers in their sample of Organisation for Economic Co-operation and Development countries self-insure against financial shocks versus share risks globally. They find that between 20% and 50% of consumers could be classified as risk sharers, with the others opting not to. Crucini (1999) also estimates the extent to which consumers in the United States of America and Canada share risks globally and find that, while most consumers share risks across regions within their country of residence, only about half shares risks with their G7 counterparts. Hevia and Servén (2018) and Fuleky et al. (2017) also find evidence of imperfect risk sharing and go further to investigate the extent to which this varies across countries. Both studies find that more developed economies tend to engage in risk sharing more than their developing peers. Hevia and Servén (2018) also find that greater risk sharing is positively associated with a country's global financial integration and degree of exchange rate flexibility and negatively related with a country's size and its degree of trade openness. Further, Fuleky et al. (2017) posit that cross-country consumption may move out of sync if consumption parameters, including the degree of risk aversion, differ across countries.

2.2.3 Summary of Review

This summary of the literature on the Backus-Smith puzzle and risk sharing suggests that the assumptions made in these models and the nature and correlation of shocks play very important roles in explaining apparent puzzles in international trade and finance data. In fact, accounting for the specific structural characteristics evident in different economies appears paramount to solving these anomalies as Corsetti et al. (2008), Hess and Shin (2010), Caporale et al. (2015) and studies on heterogeneity in risk sharing illustrate. However, except for Devereux et al. (2012) and Caporale et al. (2015), most research to date has focused on the nature of the Backus-Smith puzzle and other international trade and finance puzzles in more developed economies, with little focus on developing economies. For example, several papers considered assume that consumers have a bias toward and primarily consume goods produced at home (see for example Corsetti et al. (2008), Engel and Wang (2011), Hamano (2013) and Caporale et al. (2015)). This assumption aligns with the realities of large, more developed economies such as the United States of America where imports account for a minor portion of overall consumption (see for example Caporale et al. (2015)), but this will likely not be the case in smaller, open economies with very limited production capacities that depend heavily on imports for overall consumption and investment. Testing the Backus-Smith relationship across economies with alternative structural characteristics may therefore yield results that differ slightly from those highlighted in the empirical literature.

2.3 Theoretical Framework

The theoretical framework which underpins the analysis in this chapter leverages work from Corsetti et al. (2008). Appendix 2.A.2 offers a detailed, but yet simple explanation of their original simplified two country, two good, endowment economy model that illustrates the potentially heterogenous relationship between the real exchange rate and relative consumption. Ultimately, they show that the relationship between relative per capita consumption and the real exchange rate can be expressed as:

$$\left(\hat{C} - \widehat{C^*}\right) = \frac{2\alpha_H\omega - 1}{2\alpha_H - 1}\widehat{RER} \qquad (2.3)$$

where, for ease of reference, C and C^* are domestic and foreign consumption, respectively, while RER is the real exchange rate. α_H captures the home consumer's degree of home bias, ω (where $\omega > 0$) captures the trade elasticity parameter which is a function of the elasticity of substitution between domestically produced and foreign produced tradable goods, and variables with $\wedge$ are measured in deviations away from their equilibrium values. Equation 2.3 above is also the same as $\left(\hat{C} - \widehat{C^*}\right) = \frac{1}{\sigma}\widehat{RER}$ or equation 2.2, if $\omega = \frac{2\alpha_H + \sigma - 1}{2\alpha_H\sigma}$ (see also appendix 2.A.2)

Taking the first partial derivative of equation 2.3 with respect to $\widehat{RER}$ illustrates that the Backus-Smith relationship depends both on the degree of home bias of the domestic consumer and the elasticity of substitution between goods produced at home and those produced abroad. In the extreme cases where both α_H and ω are very high or very low, $\frac{2\alpha_H\omega - 1}{2\alpha_H - 1} > 0$ and the Backus-Smith correlation conforms to theory. However, a scenario may exist where a low degree of home bias (that is, $\alpha_H < 0.5$, such that $2\alpha_H - 1 < 0$) combines with a very high value of ω (such that $2\alpha_H\omega - 1 > 0$) to generate a negative relationship between relative consumption and the real exchange rate. For example, in cases where α_H = 0.45 and ω = 1.5, $\frac{2\alpha_H\omega-1}{2\alpha_H-1}$ becomes negative (-3.5). Similarly, a very low trade elasticity parameter could counter a high degree of home bias (α_H > 0.5) to force $2\alpha_H\omega - 1 < 0$ and $2\alpha_H - 1 > 0$ and generate the Backus-Smith puzzle evident in most empirical data. Table 2.1 below summarizes these extreme outcomes under four potential scenarios.

Table 2.1: Matrix Illustrating the Direction of Theoretical Correlations Between Relative Consumption and the Real Exchange Rate

	High degree of home bias *High* α_H	Low degree of home bias *Low* α_H
High trade elasticity *High* ω	Scenario A +	Scenario B −
Low trade elasticity *Low* ω	Scenario C −	Scenario D +

Source(s): Corsetti et al. (2008), author's calculations

The assumptions which underpin scenarios A and C are in line with the two scenarios that Corsetti et al. (2008) analysed in their paper. Specifically, the authors assume that the domestic consumer has a bias for goods produced at home, and they analyse the cases where the trade elasticity parameter is low (in line with estimates produced from macroeconomic data) or high (in line with estimates produced from microeconomic data). Scenarios B and D, in contrast, depart from the assumption of a high degree of home bias.

Under scenario A, the consumer in the home country has a high degree of home bias and the elasticity of substitution between tradable goods produced at home and abroad is high. In this scenario, a positive supply shock reduces the prices of domestically produced goods. In turn, this depreciates the terms of trade, but also leads to greater consumption of domestically produced goods because the substitution effect from the high trade elasticity more than offsets the negative income effects from lower domestic prices. The real exchange rate also depreciates, because of higher home bias in consumption relative to the foreign consumer. Therefore, both the real exchange rate and consumption move together.

Under scenario B, the elasticity of substitution between tradable goods produced at home and abroad remains high but the consumer's degree of home bias is low relative to that in the foreign country. Again, the positive supply shock reduces the prices of domestically produced goods and depreciates the terms of trade. As in scenario A, this also leads to greater consumption of domestically produced goods because of higher substitution effects compared to the negative income effects. However, in this scenario, the real exchange rate appreciates, because of lower home bias in consumption relative to the foreign consumer. Therefore, the real exchange rate and consumption move in opposite directions, reminiscent of the Backus-Smith puzzle.

Under scenario C, the consumer depends heavily on domestic consumption relative to the foreign consumer, but the trade elasticity between tradable goods is low. In this case, a depreciated terms of trade leads to lower consumption of domestically produced goods because the substitution effects from lower prices fail to offset the negative income effects. The real exchange rate also depreciates because of higher home bias in the consumption basket relative to the foreign consumer. Again, the real exchange rate and consumption move in opposite directions, consistent with the negative correlation associated with the Backus-Smith puzzle.

Finally, under scenario D, both consumer home bias and the trade elasticity between tradable goods are low. Assume that the positive endowment shock leads to lower prices of domestically produced goods. This depreciates the terms of trade but leads to lower consumption of domestically produced goods because the substitution effects from lower prices fail to offset the negative income effects, due to a low trade elasticity parameter. However, in this case, the real exchange rate appreciates, because of the low degree of home bias relative to the foreign consumer. Consequently, both the real exchange rate and consumption move together in the same direction.

The empirical analysis below will seek to determine (a) which one of these scenarios best describes the Backus-Smith relationship across countries and (b) whether the theoretical outcomes predicted by each scenario hold in practice.

2.4 Empirical Methodology and Data

2.4.1 Empirical Methodology

Equations 2.2 and 2.3 lend themselves to standard statistical tests by running regressions like those evaluated in Hess and Shin (2010). Recall equation 2.3: $\left(\hat{C} - \widehat{C^*}\right) = \frac{2\alpha_H\omega-1}{2\alpha_H-1}\widehat{RER}$. Replacing deviations away from equilibrium with changes in logged values and setting $\frac{2\alpha_H\omega-1}{2\alpha_H-1} = \frac{2(1-\alpha_F)\omega-1}{2(1-\alpha_F)-1} = \delta$ yields:

$$\Delta c_{it} - \Delta c_{it}^* = \delta \Delta rer_{it} + v_{it} \tag{2.4}$$

where v_{it} is the standard error term which captures shocks to the consumer's preferences and measurement errors in country i. Moreover, again, variables denoted in lower case denote logged variables. For given values of the trade elasticity parameter ω, the sign and statistical significance of δ will depend on the degree of home bias or alternatively the reliance on imports of goods and services in domestic consumption.

Hess and Shin (2010) posit that the relatively low correlation of consumption across countries may suggest that some consumers do not share risks and consume solely out of their incomes. Assuming a certain percentage of consumers θ completely shares risks across countries and the other percentage $1-\theta$ does not, the authors then augment equation 2.4 with ($\Delta y_{it} - \Delta y_{it}^{*}$), the differential between the per capita income growth rates of the home and foreign countries to give:

$$\Delta c_{it} - \Delta c_{it}^{*} = \theta\delta\Delta rer_{it} + (1-\theta)(\Delta y_{it} - \Delta y_{it}^{*}) + v_{it} \quad (2.5)$$

Equations 2.4 and 2.5 can be expressed as testable equations to determine the nature of the Backus-Smith relationship. These become:

$$\Delta c_{it} - \Delta c_{it}^{*} = \beta_{i0} + \beta_1\Delta rer_{it} + v_{it} \quad (2.6)$$

$$\Delta c_{it} - \Delta c_{it}^{*} = \beta_{i0} + \beta_2\Delta rer_{it} + \beta_3(\Delta y_{it} - \Delta y_{it}^{*}) + v_{it} \quad (2.7)$$

where β_1, β_2 and β_3 are equivalent to δ, $\theta\delta$ and $(1-\theta)$, respectively and β_{i0} captures a possible country specific effect. Positive and statistically significant values of β_1 and β_2 are consistent with a scenario where the degree of home bias and trade elasticity are either both high or low. Alternatively, scenarios where both coefficients are negative and statistically significant represent cases where the degree of home bias is high and trade elasticity is low or the degree of home bias is low and trade elasticity is high. Cases where either coefficient is not statistically different from zero may represent other intermediate cases. For example, the degree of home bias may be close or equal to 0.5 and ω could be close or equal to 1. Finally, β_3 = 0 suggests that θ = 1 and so all consumers completely share risks across countries. However, β_3 > 0 implies θ < 1 and suggests that 1 – θ percent of consumers consumes only out of their incomes.

Determining which scenarios β_1 and β_2 correspond to first requires an assessment of whether the trade elasticity parameter is high or low. To see this, the chapter leverages Cubeddu et al. (2019) and estimates the elasticity of imports with respect to changes in relative import prices (as in Bussière et al., 2013). The change in import volumes is modelled as a function of lagged changes in import volumes, changes in the logged ratio of import to domestic prices, and growth in domestic production as a proxy for domestic demand. Specifically,

$$\Delta \ln\left(C_{F_{it}}\right) = \beta_{i0} + \beta_4 \Delta \ln\left(C_{F_{it-1}}\right) + \beta_5 \Delta \ln\left(P_{F_{it}}/P_{H_{it}}\right) + \beta_6 \Delta \ln\left(Y_{H_{it}}\right) + v_{it} \qquad (2.8)$$

where β_5 captures the trade elasticity parameter. If $0 > \beta_5 > -1$, then imports are relatively inelastic to changes in relative import prices and the trade elasticity parameter is low. As Corsetti et al. (2008) highlight, studies using macroeconomic data tend to find relatively low estimates for the price elasticity of tradables. Alternatively, if $\beta_5 < -1$, then the elasticity of tradables is high.

2.4.2 Data

The data for this chapter span 150 economies over the period 1981 to 2018. Throughout the chapter, except otherwise specified, the foreign country's aggregates are assumed to be trade-weighted sums of each country's top 20 trading partners, where constant trade weights are calculated as that trading partner's share of the home country's total exports and imports of merchandise goods during 2008-2012. Hence, real exchange rates are measured in real effective terms.

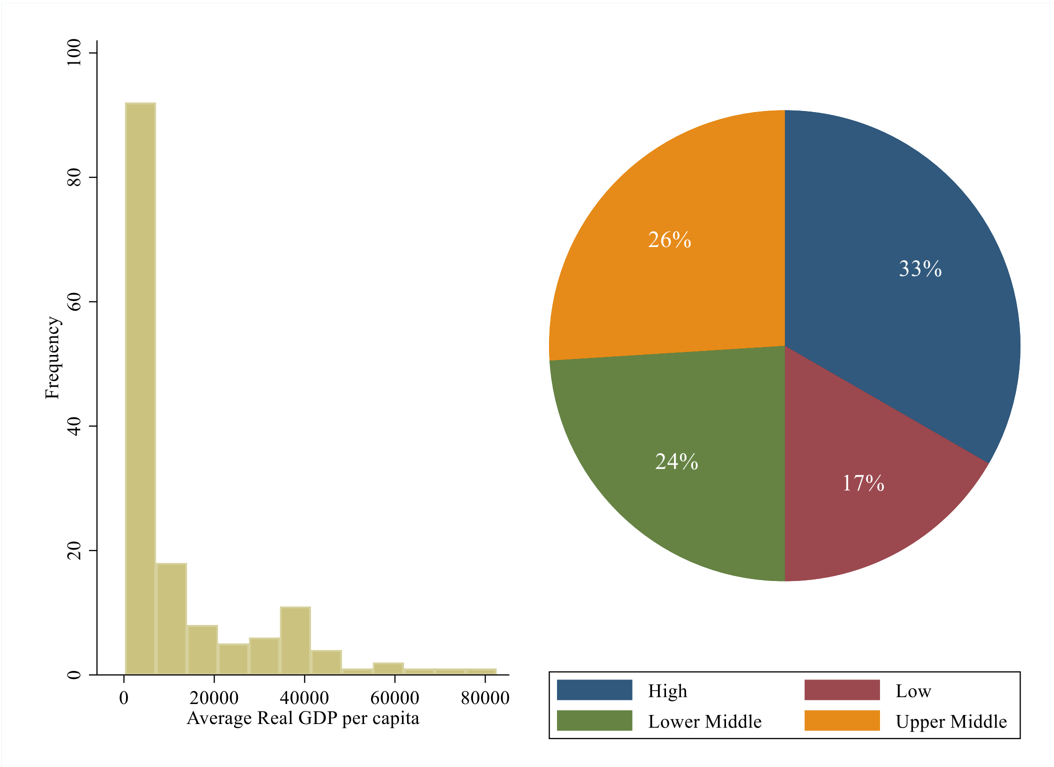
For the sake of brevity, Table 2.11 in Appendix 2.A.3 describes, in detail, the measurement and calculation of each variable and identifies its source. In summary though, data on gross domestic product, consumption and other expenditure components of real output are all sourced from the World Bank's World Development Indicators (WDI) and the United Nations (UN) National Accounts Main Aggregates Database, while exchange rate and price data are sourced from the World Development Indicators and the International Monetary Fund's World Economic Outlook (WEO). Data on import unit prices and volumes are also sourced from the World Development Indicators, while data on bilateral trade are sourced from the International Monetary Fund's Direction of Trade Statistics database.

2.5 Results

2.5.1 Preliminary Data Analysis

The countries in the sample cover a multitude of nations with various levels of economic development, population size and reliance on imports, proxied here by the ratio of total imports of goods and services to nominal gross domestic product. Based on the World Bank's income classification criteria, 41% of the countries in this sample were low or lower-middle income countries as at the end of 2018, while another 26% were upper-middle income countries (see Figure 2.2). Moreover, of the 50 high income countries included in the sample, 9 were still eligible to source World Bank financing in 2018. The other 41 include traditional advanced economies and other wealthy countries. Overall, this represents a departure from most studies which have tested the Backus-Smith puzzle to date, which have primarily focused on wealthier economies. While this provides an opportunity to draw conclusions on the nature of this puzzle for primarily developing nations, the sample includes a sufficiently diverse group of countries to permit us to draw more general conclusions than those inferred from the existing literature.

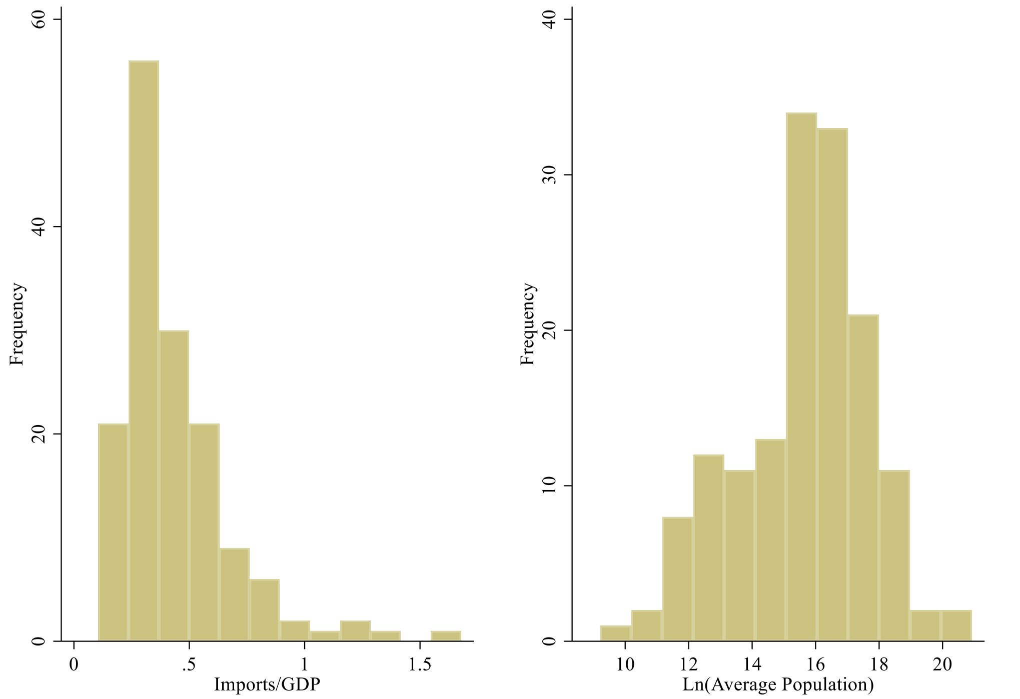
Figure 2.2: Distribution of Average Real Gross Domestic Product per Capita (left panel) and 2018 Income Classification Across 150 Countries (right panel)



Source(s): World Bank's World Development Indicators, United Nations National Accounts Statistics, author's calculations.

Similarly, the populations and import dependence of countries included in the sample are very widely dispersed (see Figure 2.3). In 2018, half of the countries had average population sizes less than 10.5 million – the smallest country's population (Nauru) was just 12,704 people, while India's and China's were approximately 1.4 billion each. By extension, over the sample period considered, the mean and median import to nominal gross domestic product ratios were 43% and 36%, respectively, but ranged from just 11% on average in Brazil and Argentina to as high as 168% in Singapore.

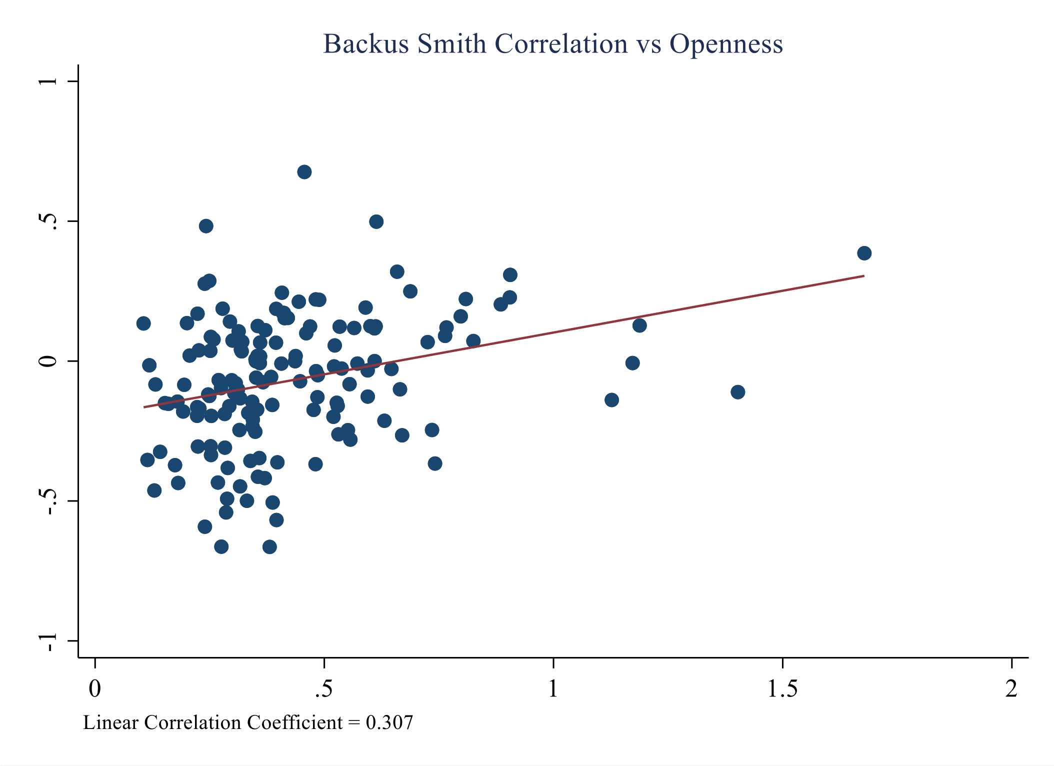
Figure 2.3: Distribution of Average Imports/Nominal Gross Domestic Product (left panel) and Average ln(Population) (right panel)



Source(s): World Bank's World Development Indicators, United Nations National Accounts Statistics, author's calculations.

The matrix in Table 2.1 suggests that countries with lower degrees of home bias or higher degrees of trade openness could exhibit higher or lower Backus-Smith correlations, depending on the size of the trade elasticity parameter. Figure 2.4 suggests that there is some positive, linear but imperfect relationship between the Backus-Smith correlation and the degree of trade openness (proxied by average imports/nominal gross domestic product) for the 150 countries in the sample. This suggests that, quite possibly, trade elasticities for most countries in the sample may be low, in line with estimates using macroeconomic data (see for example Hooper et al. (2000) and Bussière et al. (2013)).

Figure 2.4: Relationship Between the Backus-Smith Correlation and Average Imports/Nominal Gross Domestic Product



Source(s): World Bank's World Development Indicators, United Nations National Accounts Statistics, author's calculations.

2.5.2 Main Regression Results

A panel covering 150 economies over a period spanning from 1981 to 2018 was used to estimate equations 2.6, 2.7 and 2.8. Equation 2.6 and equation 2.7 were estimated using balanced panels of data, with 5,700 observations included for the full sample. In contrast, equation 2.8 was estimated using an unbalanced panel, with the number of years included in each cross section varying from as low as 15 to a maximum of 38. Each model was estimated using a fixed effects (FE) and a random effects (RE) or Generalised Least Squares estimator and the models compared using the Hausman Chi-Squared test. In instances where the Chi-squared statistic failed to reject the null hypothesis at the 10% level of significance or less, the random effects estimates were chosen as they were deemed consistent and more efficient than those produced using fixed effects. Otherwise, in cases where the null hypothesis was rejected at the 10% level of significance, the fixed effects estimates were chosen.

To estimate equations 2.6 through 2.8 however, the error term from each model must display stationary properties. That is, each must have a mean of zero and a constant variance. This is achieved either if each variable in each equation is stationary, or whether the linear combination of non-stationary variables produces a stationary error term and exhibits a cointegrating relationship.

Table 2.2 below provides the results of unit root tests for each variable included in the series of regressions using the Im-Pesaran-Shin test (Im et al., 2003). The test's null hypothesis is that all panels contain unit roots, while the alternative is that, among the series, some panels are stationary. P-values of 0.000 in each case suggest rejection of the null hypothesis in each test at the 1%, 5% and 10% levels of significance. Therefore, because each variable is stationary, estimation of each equation proceeds as is.

Table 2.2: Results of Im-Pesaran-Shin Unit Root Tests

Variable	**W-Stat**	**P-value**
$\Delta c_{it} - \Delta c_{it}^{*}$	-50.8868	0.000
Δrer_{it}	-50.3509	0.000
$\Delta y_{it} - \Delta y_{it}^{*}$	-41.7013	0.000
$\Delta \ln({C_F}_{it})$	-48.7138	0.000
$\Delta \ln({P_F}_{it}/{P_H}_{it})$	-45.0138	0.000
$\Delta \ln({Y_H}_{it})$	-43.6581	0.000

Notes: the optimal lag length for each unit root test equation was chosen based on the Bayesian Information Criterion.

The output in Table 2.3 below appears to confirm prior assumptions about the nature of the trade elasticity parameter across the countries in this chapter's sample. Columns 2 and 3 provide estimates using the fixed effects and random effects estimators, respectively and both suggest that the elasticity of import volumes with respect to changes in relative import prices is approximately 0.4, below 1 and statistically different from zero. Moreover, the income elasticity of demand in each case is approximately 1. Given that the Hausman test rejects the null hypothesis that estimates from the random effects specification are consistent and more efficient, equation 2.8 is re-estimated for two subsamples split by the median imports to nominal gross domestic product ratio, using the fixed effects estimator. Again, the results confirm an elasticity of substitution of approximately 0.4, for both less open (imports of goods and services/nominal gross domestic product less than the median) and more open (imports of goods and services/nominal gross domestic product more than the median) economies. These results suggest that, for the latter group of countries, the Backus-Smith correlation should be positive, or at least higher than the former group.

Table 2.3: Baseline Fixed and Random Effects Linear Regression Estimates of Equation 2.8

Variable	**Full Sample – Fixed Effects**	**Full Sample – Random Effects**	**Less Open – Fixed Effects**	**More Open – Fixed Effects**
$\Delta \ln\left(C_{F\,it-1}\right)$	-0.103***	-0.092***	-0.094***	-0.112***
	(0.022)	(0.022)	(0.030)	(0.033)
$\Delta \ln\left(P_{F\,it}/P_{H\,it}\right)$	-0.437***	-0.448***	-0.456***	-0.401***
	(0.044)	(0.043)	(0.047)	(0.083)
$\Delta \ln\left(Y_{H\,it}\right)$	1.284***	1.248***	1.358***	1.183***
	(0.127)	(0.112)	(0.189)	(0.163)
Constant	0.001	0.002	0.002	0.001
	(0.005)	(0.004)	(0.006)	(0.006)
Observations	4,108	4,108	2,214	1,894
R-squared	0.244	0.244	0.298	0.184
Number of countries	150	150	75	75
Hausman Test Statistic	8.03**			

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Similarly, the results in Table 2.4 provide some evidence that this result generally holds across economies of all 4 major income groups. Trade elasticities are all low, and less than 1 and statistically different from zero for high income, upper-middle income, lower-middle income and low income economies. The magnitude of these coefficients varies across groups, however. Changes in imports are generally less sensitive to changes in relative import prices in high and upper-middle income economies than in those classified as lower-middle income or low income. Still, the results suggest that the assumption that trade elasticities are generally low is robust across different types of economies.

Table 2.4: Fixed Effects Linear Regression Estimates of Equation 2.8 – Split by Income Group

Variable	High Income	Upper-Middle Income	Lower-Middle Income	Low Income
$\Delta\ln(C_{F\,it-1})$	-0.087	-0.020	-0.119***	-0.176***
	(0.076)	(0.037)	(0.031)	(0.038)
$\Delta\ln(P_{F\,it}/P_{H\,it})$	-0.259***	-0.366***	-0.576***	-0.420***
	(0.067)	(0.117)	(0.069)	(0.056)
$\Delta\ln(Y_{H\,it})$	1.998***	1.164***	1.104***	1.224***
	(0.324)	(0.184)	(0.201)	(0.171)
Constant	-0.021**	0.003	0.007	0.011*
	(0.009)	(0.008)	(0.009)	(0.006)
Observations	1,032	1,106	1,131	839
R-squared	0.344	0.290	0.215	0.227
Number of countries	50	39	36	25

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Having established the size of the trade elasticity parameter, the results that follow sought to establish the nature of the Backus-Smith relationship and determine to what extent it varies by the degree of home bias or trade openness.

First, I estimate equation 2.6, where the Hausman test suggests that the random effects estimates are consistent and more efficient than the specification with fixed effects. Still, both fixed effects and random effects estimates imply a positive relationship between relative consumption growth and changes in the real (effective) exchange rate, but only in the latter is the result statistically significant (Table 2.5). However, even in the latter case, the P-value associated with that coefficient is only marginally less than 10% (0.088). While these results differ from the negative relationship traditionally found in the literature, they suggest, still, that the correlation between relative consumption growth and the real exchange rate is weak.

Table 2.5: Baseline Fixed and Random Effects Linear Regression Estimates of Equation 2.6

Variable	**Fixed Effects**	**Random Effects**
Δrer_{it}	0.012	0.013*
	(0.009)	(0.008)
Constant	0.001***	0.001
	(0.000)	(0.001)
Observations	5,700	5,700
R-squared	0.002	0.002
Number of countries	150	150
Hausman Test Statistic	0.37	

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

The baseline results from estimates of Equation 2.7 also tell a similar story but fail to find any statistical significance in the relationship between relative per capita consumption and the real exchange rate for the full sample of countries (Table 2.6). What it does imply, however, is that, as Hess and Shin (2010) find, only a portion of consumers shares risks internationally. In this case, columns 2 and 3 imply that just under 40% of consumers share risks, comfortably within the range of 20% to 50% estimated by Hess and Shin (2010). Moreover, splitting the sample between those countries who are eligible to borrow from the World Bank and those who are not (typically advanced economies and high income economies with large per capita incomes) yields results consistent with other authors who have focused their studies on Organisation for Economic Co-operation and Development countries. For those 41 economies not eligible to borrow from the World Bank, the Backus-Smith relationship is negative and statistically different from zero. For the other 109 countries in the sample, however, the relationship is practically nonexistent. Where the results do differ however, is in the estimate of consumers who share risks. In this sample, a higher proportion of consumers in more developed economies that are not eligible for financing from the World Bank shares risks (about two-thirds) versus about 30% in less developed economies.

Table 2.6: Baseline Fixed and Random Effects Linear Regression Estimates of Equation 2.7

Variable	**Full Sample – Fixed Effects**	**Full Sample – Random Effects**	**World Bank Ineligible – Random Effects**	**World Bank Eligible – Random Effects**
Δrer_{it}	0.008	0.009	-0.078***	0.009
	(0.008)	(0.007)	(0.029)	(0.007)
$\Delta y_{it} - \Delta y_{it}^{*}$	0.614***	0.624***	0.324***	0.679***
	(0.069)	(0.064)	(0.076)	(0.068)
Constant	-0.000	-0.000	-0.001	-0.000
	(0.000)	(0.001)	(0.001)	(0.001)
Observations	5,700	5,700	1,558	4,142
R-squared	0.151	0.151	0.088	0.164
Number of countries	150	150	41	109
Hausman Test Statistic	1.53			

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Having established that the low and sometimes negative correlations associated with the Backus-Smith puzzle do still exist, the next step is to determine to what extent this relationship varies by countries' degrees of home bias or openness. Table 2.7 reproduces the more efficient and consistent random effects estimates for the full sample from Table 2.6 and then splits the sample evenly between those economies with imports/nominal gross domestic product ratios above the median and those below. The results appear to confirm this study's hypothesis. Specifically, the Backus-Smith relationship is positive and statistically significant for more open economies but is negative and statistically insignificant for those with a low dependence on imports or a high degree of home bias. Moreover, as expected with these results, a larger proportion of consumers shares risks in more open economies than in those with higher degrees of home bias.

Table 2.7: Random Effects Linear Regression Estimates of Equation 2.7 – Split by Openness

Variable	Full Sample	Less Open	More Open
Δrer_{it}	0.009	-0.006	0.020***
	(0.007)	(0.007)	(0.007)
$\Delta y_{it} - \Delta y_{it}^{*}$	0.624***	0.672***	0.576***
	(0.064)	(0.104)	(0.075)
Constant	-0.000	-0.000	0.001
	(0.001)	(0.001)	(0.001)
Observations	5,700	2,850	2,850
R-squared	0.151	0.213	0.114
Number of countries	150	75	75

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

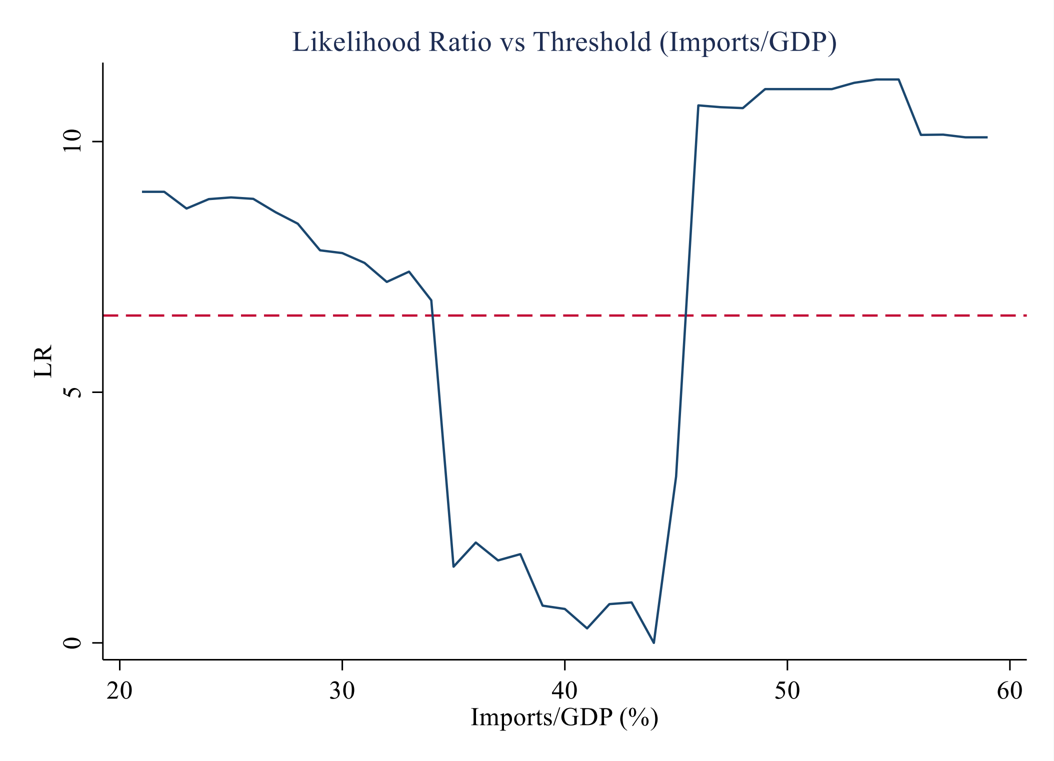
One question that naturally arises from the analysis above is, at what degree of openness are the respective Backus-Smith correlations most different? Or, in other words, for this sample of countries, what threshold clearly defines those countries where the Backus-Smith relationship changes from low or negative to positive and statistically significant? Pavlidis et al. (2015)'s evidence that accounting for nonlinearities in tests of the Backus-Smith relationship yields more positive results lends support for such an approach. To determine this, this section leverages a variant of Hansen's (1999) panel threshold model for non-dynamic panels. Thus, equation 2.7 is augmented to yield:

$$\Delta c_{it} - \Delta c_{it}^{*} = \beta_{i0} + \beta_{2a}\Delta rer_{it} I(x_i \leq \emptyset) + \beta_{2b}\Delta rer_{it} I(x_i > \emptyset) + \beta_3(\Delta y_{it} - \Delta y_{it}^{*}) + v_{it} \quad (2.9)$$

where x_i captures the threshold variable (in this case, the ratio of imports to nominal gross domestic product) and $\emptyset$ represents the value of that threshold. Beyond $\emptyset$, the relationship between changes in relative per capita consumption and the real exchange rate changes. The optimal value of $\emptyset$ is that which minimizes the residual sum of squares (RSS) from a series of regressions that test a range of $\emptyset$ between the 10th and 90th percentiles of the average imports/nominal gross domestic product ratio. Unlike Hansen (1999) however, each specification of equation 2.9 is estimated using the more efficient random effects estimator (for this sample of countries at least), rather than the fixed effects estimator.

Each regression's likelihood ratio (the normalized residual sum of squares) is plotted versus its respective $\emptyset$ in Figure 2.5 below. Values of $\emptyset$ for which the likelihood ratio falls below the dotted red line are statistically significant thresholds with a 90% degree of confidence. [5] In this case, the optimal threshold is at 44% of imports of goods and services to nominal gross domestic product, the point at which the likelihood ratio reaches its lowest point on the graph.

Figure 2.5: Likelihood Ratio (LR) Plot of Equation 2.9 with Imports/Nominal Gross Domestic Product (%) as the Threshold Variable



Source(s):Author's calculations.

Table 2.8 presents the results of the threshold regression with an optimal threshold of 44%. At this point, the difference between the coefficients for degrees of openness above and below the threshold is at its greatest. In summary, for countries whose average imports of goods and services to nominal gross domestic product are at most 44%, the relationship between relative per capita consumption growth and changes in the real exchange rate is small, negative, and statistically insignificant from zero. Above this point, however, the Backus-Smith correlation becomes positive and statistically significant at all conventional confidence levels.

[5] Hansen (1999) defines the likelihood ratio for each threshold as $\left[RSS(\emptyset) - RSS(\widehat{\emptyset})\right] / \hat{\sigma}^2$ where $\widehat{\emptyset}$ and $\hat{\sigma}^2$ are the estimated values of the optimal threshold and residual variance, respectively. Hansen (1999) also shows that the critical value for this test is calculated as: $c(\alpha) = -2\log\left(1 - \sqrt{1-\alpha}\right)$.

Table 2.8: Non-dynamic, Single Threshold Panel Estimates of Equation 2.9

Variable	No Threshold	Single Threshold
Δrer_{it}	0.009	
	(0.007)	
$\Delta rer_{it}\ I(\alpha_{F_i} \leq 0.44)$		-0.007
		(0.007)
$\Delta rer_{it}\ I(\alpha_{F_i} > 0.44)$		0.022***
		(0.007)
$\Delta y_{it} - \Delta y_{it}^{*}$	0.624***	0.623***
	(0.064)	(0.064)
Constant	-0.000	-0.000
	(0.001)	(0.001)
Observations	5,700	5,700
R-squared	0.151	0.153
Number of countries	150	150

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

2.5.3 Robustness Checks

In this subsection, I re-estimated equations 2.6, 2.7 and 2.9 to test the robustness of the results presented thus far. Two alternative approaches were taken. First, the definition of the real exchange rate is replaced with the bilateral real exchange rate relative to the United States of America versus the multilateral real effective exchange rate used up until now in the analysis. Commensurately, the United States of America becomes the foreign country in the analysis, rather than a trade-weighted aggregate of each country's top 20 trading partners. Secondly, I revert to the real effective exchange rate and trade-weighted per capita consumption and real gross domestic product but instead estimate each equation using instrumental variables to account for potential endogeneity between the dependent variable and regressors. As in Hess and Shin (2010), the regressors are instrumented with their lagged values and the lagged value of the dependent variable. The results for both approaches are presented in Table 2.9 and Table 2.10, respectively.

Replacing the real effective exchange rate and trade-weighted per capita consumption and real gross domestic product with the bilateral real exchange rate and US per capita consumption and real gross domestic product yields qualitatively similar results to those presented thus far. Across the entire sample, less than 40% of all consumers share risk, with a higher percentage in more open economies. Also, the Backus-Smith relationship is generally positive for the entire sample, but in this case, is also statistically insignificant in the specification excluding growth in relative real gross domestic product. As before, however, the Backus-Smith relationship becomes statistically significant (in this case at the 5% level) for more open economies but is negative and statistically insignificant for all others. Finally, this specification also finds an optimal threshold of imports to nominal gross domestic product of 44%.

Table 2.9: Random Effects Regression Estimates of Equations 2.6, 2.7 and 2.9 – Using the United States of America as the Foreign Country

Variable	**No Threshold; no $\Delta y_{it} - \Delta y^*_{it}$**	**No Threshold**	**No Threshold – Less Open**	**No Threshold – More Open**	**Single Threshold**
Δrer_{it}	0.009	0.006	-0.008	0.018**	
	(0.009)	(0.007)	(0.008)	(0.008)	
$\Delta rer_{it}\ I(\alpha_{F_i} \leq 0.44)$					-0.010
					(0.008)
$\Delta rer_{it}\ I(\alpha_{F_i} > 0.44)$					0.020**
					(0.008)
$\Delta y_{it} - \Delta y^*_{it}$		0.669***	0.709***	0.627***	0.667***
		(0.064)	(0.105)	(0.076)	(0.064)
Constant	-0.003**	-0.002***	-0.002**	-0.002	-0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	5,700	5,700	2,850	2,850	5,700
R-squared	0.001	0.173	0.239	0.132	0.174
Number of countries	150	150	75	75	150

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

The results from the instrumental variables regressions are starker. The Chi-squared statistic from the first stage generalised two stage least squared regression is statistically significant at all conventional levels, suggesting that the instruments are very strong. Like in Hess and Shin (2010), however, these estimates are more positive (or in their case, less negative) than in the baseline regressions. In fact, they suggest no evidence against the existence of the Backus Smith puzzle across all 150 countries. As before, the relationship differs between economies with imports to nominal gross domestic product ratios above and below the median, but the proportions of consumers that share risks is now higher in less open economies than in their more open counterparts. However, the coefficient on $\Delta y_{it} - \Delta y^*_{it}$ is not statistically significant in the sample with less open markets. Finally, however, the threshold remains the same as in other specifications.

Table 2.10: Random Effects Regression Estimates of Equations 2.6, 2.7 and 2.9 – Using Instrumental Variables

Variable	**No Threshold; no $\Delta y_{it} - \Delta y^*_{it}$**	**No Threshold**	**No Threshold – Less Open**	**No Threshold – More Open**	**Single Threshold**
Δrer_{it}	0.022**	0.018***	-0.022	0.023***	
	(0.008)	(0.004)	(0.061)	(0.006)	
$\Delta rer_{it}\ I(\alpha_{F_i} \leq 0.44)$					-0.017
					(0.040)
$\Delta rer_{it}\ I(\alpha_{F_i} > 0.44)$					0.022***
					(0.006)
$\Delta y_{it} - \Delta y^*_{it}$		0.712***	0.463	0.712***	0.701***
		(0.223)	(0.753)	(0.188)	(0.224)
Constant	0.001	-0.001	-0.001	0.000	-0.000
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	5,550	5,550	2,775	2,775	5,550
R-squared	0.001	0.158	0.228	0.115	0.160
Number of countries	150	150	75	75	150

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

2.5.4 Summary of Results

Overall, the results presented above suggest that the empirical regularities consistent with the Backus-Smith puzzle – low or negative correlations between relative consumption and the real exchange rate – still exist, at least for a subset of countries. The puzzle is more acute for more advanced economies, like those that have been the subject of most of the literature on this topic to date but is less so for less developed markets. Moreover, these results confirm Corsetti et al.'s (2008) proposition and findings that the nature of the Backus-Smith relationship depends on a country's degree of home bias and its trade elasticity parameter. Across the sample of 150 countries analysed in this chapter, economies at different levels of development and openness exhibit low trade elasticities, a result consistent with other studies using macroeconomic data. Thus, those countries that depend heavily on locally produced goods and services for domestic consumption, proxied by a low ratio of imports of goods and services to nominal gross domestic product, exhibit the negative (but statistically insignificant) consumption-real exchange rate correlation that theory suggests is inconsistent with international risk sharing. However, the relationship turns positive (and statistically significant) for a group of countries with comparatively low degrees of home bias.[6] This is

[6] These results are robust to using the size of the country's population as an alternative measure of a country's dependence on foreign goods for domestic consumption. See Appendix 2.A.4 for a review of

most evident for countries whose average imports as a share of nominal gross domestic product exceed 44%. However, not all consumers across these 150 countries share risks. In general, less than 40% of consumers shares risks, well within the range of consumers estimated by Hess and Shin (2010). Nonetheless, most regressions suggest that the percentage of risk-sharing consumers is slightly higher in more open economies than less open countries, a result that is at odds with Hevia and Servén's (2018) when they control for the degree of financial integration in the global economy.

Finally, the results presented above are generally robust to the inclusion of alternative measures of the real exchange rate[7] and to the use of instrumental variables to control for potential endogeneity. One caveat worth noting however, is that splitting the sample and estimating equations 2.6 and 2.7 on each subsample reduces the degrees of freedom available and increases the standard errors of coefficient estimates in regressions in which this is done. In fact, while this permits us to determine the relationships between relative consumption and the real exchange rate in each subsample, it doesn't allow us to make a statement about how this relationship varies along a continuum of trade openness. Introducing an additional variable to the equation, formed by interacting the real exchange rate with the ratio of imports of goods and services to gross domestic product, may allow us to do this.

2.6 Conclusion

Established consumption theory suggests that consumers with access to finance may choose to borrow on international markets and share risks to smooth consumption. However, Corsetti et al. (2008) illustrate that, even without access to international financial markets, consumers can still insure consumption against exogenous shocks. International risk sharing (in theory at least) leads to almost perfectly positive consumption correlation across countries (Backus et al., 1992) and to positive correlation between relative cross-country consumption growth and changes in the real exchange rate as consumers respond positively to reductions in the relative price of their baskets of goods and services. However, most literature covering more developed economies to date suggests that relative consumption growth moves counter to movements in the real exchange rate, suggesting limited to no risk sharing and the presence of what has become known as the Backus-Smith puzzle. Moreover, while several studies have attempted to reconcile this empirical regularity with the theory,

additional results that use population size instead of the ratio of imports of goods and services to nominal gross domestic product.

[7] Also see Appendix 2.A.5 for a review of additional results that use the terms of trade rather than the real exchange rate.

most have focused on a relatively homogenous group of more developed economies, with little or no emphasis on a more diverse group of developing economies. Thus, the results derived from their analyses may not necessarily be generalised across a larger group of countries.

This chapter leveraged Corsetti et al.'s (2008) analytical work to determine whether differences in the degrees of home bias or dependence on imports of goods and services for given values of the trade elasticity parameter could explain the Backus-Smith puzzle. Importantly, the analysis covers a diverse group of 150 countries (mostly emerging markets and developing economies), which permits more general conclusions to be drawn than those inferred from previous studies. To the best of the author's knowledge, this is among the first studies to test Corsetti et al.'s (2008) hypotheses for such a large and diverse group of economies.

The results imply that the price elasticity of substitution between locally produced and foreign produced goods is low, in line with many macroeconomic estimates. Thus, in line with Corsetti et al.'s (2008) propositions, economies who depend comparatively less on imported goods and services exhibit low or negative correlations between relative per capita consumption growth and changes in the real exchange rate. However, for the group of countries that depends more on imported goods and services for their consumption, their correlations are positive and statistically significant. This is particularly evident for countries whose average imports of goods and services relative to nominal gross domestic product exceed 44%. These results are generally robust to whether the foreign country is measured as just the United States of America or a trade-weighted group of countries but are even more stark for regressions estimated with instrumental variables. Overall, however, in most cases, the results suggest that fewer than 40% of consumers across the group of countries share risks internationally.

These results highlight the importance of accounting for country heterogeneity in the analysis of small open economies. Moreover, they support Pavlidis et al.'s (2015) findings that well-established theoretical relationships may exhibit some nonlinearity in practice. In this chapter, accounting for one type of nonlinearity yielded more positive results than a simple linear approach across all 150 countries. Moreover, separate results from regressions run on a sample of more developed economies that are not eligible for development financing from the World Bank produce results that are materially different from less developed economies and more consistent with previous research in the literature. Therefore, future research on small, open economies, and in international trade and finance in general should aim to account for these heterogeneities and nonlinearities, when seeking to draw general conclusions and offer policy recommendations for a wider group of countries. Specifically, future extensions of this work may incorporate the assumptions of limited home bias and other relevant features of small, very open economies in a Dynamic Stochastic General Equilibrium model to see whether the results produced in this chapter hold in that setting.

2.A Appendix

2.A.1 Alternative Derivation of the Risk-Sharing Condition

This appendix illustrates the risk-sharing condition as developed in Galí and Monacelli (2005). They present what has become an often cited framework and foundation for analysing monetary policy in the small open economy. They begin by assuming several of the standard assumptions used in the literature up to that point: the law of one price holds across markets for individual goods and firms set prices in a staggered setting based on the Calvo (1983) framework. Moreover, Galí and Monacelli (2005) model the global economy as:

> *"...a continuum of small open economies represented by the unit interval. Since each economy is of measure zero, its domestic policy decisions do not have any impact on the rest of the world. While different economies are subject to imperfectly correlated productivity shocks, we assume that they share identical preferences, technology, and market structure..." (Galí and Monacelli, 2005)*

Explicitly, households derive and maximise utility from leisure and consumption (equation 2.10) where consumption comprises both domestically produced and imported goods (equation 2.11). Households' total consumption and savings are constrained by wages from labour, nominal earnings from shares and other securities, and a lump-sum transfer from the government (equation 2.12).

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{2.10}$$

$$C_t = \left[(1-\alpha_F)^{\frac{1}{\omega}}\left(C_{H,t}\right)^{\frac{\omega-1}{\omega}} + \alpha_F^{\frac{1}{\omega}}\left(C_{F,t}\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{2.11}$$

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1\int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_tN_t + T_t \tag{2.12}$$

$$P_t = \left[(1-\alpha_F)P_{H,t}{}^{1-\omega} + \alpha_F P_{F,t}{}^{1-\omega}\right]^{\frac{1}{1-\omega}} \tag{2.13}$$

where E_t represents the expectations operator at time *t*, $U(C_t, N_t)$ is the constant relative risk aversion utility function with total consumption C_t and N_t labour as the two inputs and in this model take the form $\frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$, while β^t represents the discount factor. $C_{H,t}$ and $C_{F,t}$ are home consumption of domestically and foreign produced goods, respectively while α_F represents the proportion of consumption allocated to foreign produced goods and can be considered a measure of trade openness. Meanwhile, $P_{H,t}(j)$ and $P_{i,t}(j)$ are the local currency prices of good *j* produced in the home (*H*) and individual foreign (*i*) countries, respectively. W_t and T_t are nominal wages and lump-sum government transfers and D_{t+1} "…is the nominal pay-off in period *t+1* of the portfolio held at the end of period *t* (which includes shares in firms)…" (Galí and Monacelli, 2005). $Q_{t,t+1}$ "…is the stochastic discount factor for one-period ahead nominal pay-offs relevant to the domestic household…" (Galí and Monacelli, 2005). ω > 0 represents the elasticity of substitution between domestically produced and foreign produced goods and, unlike in Corsetti et al. (2008), Galí and Monacelli (2005) assume this equal to unity. Money does not enter the model, but households have frictionless and free access to international securities markets, which permits them to share risks. Further, equation 2.13 illustrates that the consumer price index (CPI) is a weighted average of domestically produced and aggregated foreign produced (*F*) goods.

As mentioned, equation 2.10 can be expressed as equation 2.14, and Galí and Monacelli (2005) derive the optimality conditions and tradeoffs between today's real wages and consumption/labour (equation 2.15) and consumption today and in the future (equation 2.16). The equivalent intertemporal consumption decision expressed in equation 2.16 can also be derived for consumers abroad in country *i* (equation 2.17) where ε^i represents the nominal, bilateral exchange rate with country *i*, expressed in domestic country prices. Finally, the real exchange rate (equation 2.18, RER^i) between the domestic economy and country *i* is expressed as the bilateral nominal exchange rate adjusted by the ratio of prices across countries. In these derivations, the risk aversion parameter (σ) is assumed to be the same across markets as outlined in equations 2.16 and 2.17.

$$\frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \tag{2.14}$$

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{2.15}$$

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1} \tag{2.16}$$

$$\beta \left(\frac{{C_{t+1}}^i}{{C_t}^i}\right)^{-\sigma} \left(\frac{{P_t}^i}{{P_{t+1}}^i}\right) \left(\frac{\varepsilon_t^i}{\varepsilon_{t+1}^i}\right) = Q_{t,t+1} \tag{2.17}$$

$${RER_t}^i = \left(\frac{\varepsilon_t^i {P_t}^i}{P_t}\right) \tag{2.18}$$

Galí and Monacelli (2005) show that, combined and taking logs, equations 2.16 through 2.18 can simplify to the standard risk sharing condition. Still assuming that σ is the same across countries and constant over time, we have:

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = \beta\left(\frac{{C_{t+1}}^i}{{C_t}^i}\right)^{-\sigma}\left(\frac{{P_t}^i}{{P_{t+1}}^i}\right)\left(\frac{\varepsilon_t^i}{\varepsilon_{t+1}^i}\right)$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = \left(\frac{{C_{t+1}}^i}{{C_t}^i}\right)^{-\sigma}\left(\frac{{P_t}^i}{{P_{t+1}}^i}\right)\left(\frac{\varepsilon_t^i}{\varepsilon_{t+1}^i}\right)$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{{P_{t+1}}^i\varepsilon_{t+1}^i}{P_{t+1}}\right) = \left(\frac{{C_{t+1}}^i}{{C_t}^i}\right)^{-\sigma}\left(\frac{{P_t}^i\varepsilon_t^i}{P_t}\right)$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} {RER_{t+1}}^i = \left(\frac{{C_{t+1}}^i}{{C_t}^i}\right)^{-\sigma} {RER_t}^i$$

$$\left(\frac{C_{t+1}}{{C_{t+1}}^i}\right)^{-\sigma} {RER_{t+1}}^i = \left(\frac{C_t}{{C_t}^i}\right)^{-\sigma} {RER_t}^i$$

$$\left(\frac{{C_{t+1}}^i}{C_{t+1}}\right)^{\sigma} {RER_{t+1}}^i = \left(\frac{{C_t}^i}{C_t}\right)^{\sigma} {RER_t}^i$$

$$\left(\frac{{C_{t+1}}^i}{C_{t+1}}\right) {RER_{t+1}}^{i\frac{1}{\sigma}} = \left(\frac{{C_t}^i}{C_t}\right) {RER_t}^{i\frac{1}{\sigma}}$$

$$C_t\left(\frac{{C_{t+1}}^i}{C_{t+1}}\right) {RER_{t+1}}^{i\frac{1}{\sigma}} = {C_t}^i {RER_t}^{i\frac{1}{\sigma}}$$

$$C_t = {C_t}^i {RER_t}^{i\frac{1}{\sigma}}\left[\left(\frac{{C_{t+1}}^i}{C_{t+1}}\right) {RER_{t+1}}^{i\frac{1}{\sigma}}\right]^{-1}$$

Galí and Monacelli assume $\left[\left(\frac{{C_{t+1}}^i}{C_{t+1}}\right) {RER_{t+1}}^{i\frac{1}{\sigma}}\right]^{-1} = \vartheta_i$, implied from "…symmetric initial conditions (that is, zero net foreign asset holdings and an ex ante identical environment), in which case we have $\vartheta_i = \vartheta = 1$ for all i." Galí and Monacelli (2005). Galí and Monacelli (2005) also suggest that the real exchange rate and the terms of trade are both equal to 1 under a symmetric perfect foresight steady state. Thus,

$$C_t = \vartheta_i {C_t}^i {RER_t}^{i\frac{1}{\sigma}} \qquad (2.19)$$

Taking logs, assuming $\vartheta_i = 1$ and aggregating across foreign countries, this simplifies to the standard risk sharing condition:

$$c_t - c_t^* = \frac{1}{\sigma} rer_t \qquad (2.20)$$

where lower case letters indicate that variables have been logged, and * denotes the aggregation of the rest of the world. The expression confirms the theoretical relationship highlighted in the literature, such that, positive changes in relative consumption should perfectly coincide with positive changes in the real effective exchange rate (rer_t).

The analysis above assumes that risk aversion does not vary across countries or consumers. In fact, there is an inconclusive debate in the literature surrounding whether risk aversion does indeed vary across countries and/or consumers. While Gandelman and Hernández-Murillo (2014) find some variation in their estimates of relative risk aversion for seventy-five countries (estimates generally between 0 and 3), they conclude that most countries' risk aversion coefficients are statistically indifferent from 1 and support the assumption of log utility in estimating consumption functions. In contrast however, Liu et al. (2016) and Schneider et al. (2017) both find evidence of variations in risk aversion for their respective samples of consumers. Both studies find that relative risk aversion does vary with consumers' wealth, while the latter confirms that older persons and females are likely to be more risk averse than their younger, male counterparts.

Therefore, for comparison, let us now assume that the relative risk aversion parameters are constant across time but not across consumers in different countries, as Liu et al. (2016) and Schneider et al. (2017) have suggested. Recall equation 2.16 and equation 2.18, but this time, allow the previous risk aversion parameter from equation 2.17 to now vary by country. This produces equation 2.21.

$$\beta\left(\frac{C_{t+1}{}^{i}}{C_t{}^{i}}\right)^{-\sigma_i}\left(\frac{P_t{}^{i}}{P_{t+1}{}^{i}}\right)\left(\frac{\varepsilon_t^i}{\varepsilon_{t+1}^i}\right)=Q_{t,t+1} \qquad (2.21)$$

Again, I simplify:

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)=\beta\left(\frac{C_{t+1}{}^{i}}{C_t{}^{i}}\right)^{-\sigma_i}\left(\frac{P_t{}^{i}}{P_{t+1}{}^{i}}\right)\left(\frac{\varepsilon_t^i}{\varepsilon_{t+1}^i}\right)$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)=\left(\frac{C_{t+1}{}^{i}}{C_t{}^{i}}\right)^{-\sigma_i}\left(\frac{P_t{}^{i}}{P_{t+1}{}^{i}}\right)\left(\frac{\varepsilon_t^i}{\varepsilon_{t+1}^i}\right)$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_{t+1}{}^{i}\varepsilon_{t+1}^i}{P_{t+1}}\right)=\left(\frac{C_{t+1}{}^{i}}{C_t{}^{i}}\right)^{-\sigma_i}\left(\frac{P_t{}^{i}\varepsilon_t^i}{P_t}\right)$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}RER_{t+1}{}^{i}=\left(\frac{C_{t+1}{}^{i}}{C_t{}^{i}}\right)^{-\sigma_i}RER_t{}^{i}$$

$$\frac{C_{t+1}{}^{-\sigma}}{C_t{}^{-\sigma}}RER_{t+1}{}^{i}=\frac{C_{t+1}{}^{i^{-\sigma_i}}}{C_t{}^{i^{-\sigma_i}}}RER_t{}^{i}$$

$$\frac{C_{t+1}{}^{-\sigma}}{C_{t+1}{}^{i^{-\sigma_i}}} RER_{t+1}{}^{i} = \frac{C_t{}^{-\sigma}}{C_t{}^{i^{-\sigma_i}}} RER_t{}^{i}$$

$$\frac{C_{t+1}{}^{i^{\sigma_i}}}{C_{t+1}{}^{\sigma}} RER_{t+1}{}^{i} = \frac{C_t{}^{i^{\sigma_i}}}{C_t{}^{\sigma}} RER_t{}^{i}$$

$$C_t{}^{\sigma} \frac{C_{t+1}{}^{i^{\sigma_i}}}{C_{t+1}{}^{\sigma}} RER_{t+1}{}^{i} = C_t{}^{i^{\sigma_i}} RER_t{}^{i}$$

$$C_t{}^{\sigma} = C_t{}^{i^{\sigma_i}} RER_t{}^{i} \left[\frac{C_{t+1}{}^{i^{\sigma_i}}}{C_{t+1}{}^{\sigma}} RER_{t+1}{}^{i}\right]^{-1}$$

Let us now assume that $\left[\frac{C_{t+1}{}^{i^{\sigma_i}}}{C_{t+1}{}^{\sigma}} RER_{t+1}{}^{i}\right]^{-1} = \mathrm{X}_i$. Then:

$$C_t{}^{\sigma} = \mathrm{X}_i \, C_t{}^{i^{\sigma_i}} RER_t{}^{i} \qquad (2.22)$$

Given that the relative risk aversion parameter now varies across countries, I can no longer assume that $\mathrm{X}_i = \mathrm{X} = 1$ as it varies with the values of the risk aversion parameters where a higher σ or lower σ_i increases X_i. Therefore, taking logs of equation 2.22 yields an alternative risk sharing condition (equation 2.23):

$$\sigma \ln C_t = \ln \mathrm{X}_i + \sigma_i \ln C_t{}^{i} + \ln RER_t{}^{i}$$

$$\ln C_t = \frac{1}{\sigma} \ln \mathrm{X}_i + \frac{\sigma_i}{\sigma} \ln C_t{}^{i} + \frac{1}{\sigma} \ln RER_t{}^{i}$$

$$c_t = \frac{1}{\sigma} \ln \mathrm{X}_i + \frac{\sigma_i}{\sigma} c_t^i + \frac{1}{\sigma} rer_t^i \qquad (2.23)$$

We can assume that $\frac{1}{\sigma} \ln \mathrm{X}_i$ represents a time-invariant term (maybe a fixed effects or random effects term in a panel framework) in a testable regression where X_i also depends on the two relative risk aversion parameters. Equation 2.23 thus becomes a more generalized version of the standard risk sharing condition expressed in equation 2.20 and collapses to that equation if I assume that $\sigma_i = \sigma$ since $\frac{\sigma_i}{\sigma} = \mathrm{X}_i = 1$ under that specific case.

Let us explore the implications of this new relationship. If risk aversion parameters vary across countries, then changes in foreign consumption need not coincide with fluctuations in domestic consumption of the same magnitude. A large value for σ relative to σ_i suggests that c_t would rise by very little in response to a rise in foreign consumption. In contrast, a small value for σ relative to σ_i would imply that consumption at home would respond much more aggressively to a change in foreign consumption.

Equation 2.23 lends itself to time series or heterogeneous panel estimation to uncover estimates for the parameters σ and σ_i once the null hypothesis of $\sigma = \sigma_i$ does not hold. If the null hypothesis does hold, that implies that the relative risk aversion parameters are similar (or statistically the same) across countries. If not, then this justifies the assumption that countries have different risk aversion parameters. However, the presence of different coefficients of risk aversion does not appear (theoretically at least) to affect the correlation coefficient between relative consumption and the real exchange rate.

2.A.2 Explanation of Corsetti et al.'s (2008) Small Open Economy Model

Corsetti et al. (2008) present a framework of a simplified two country, two good, endowment economy that illustrates the potentially heterogenous relationship between the real exchange rate and relative consumption. In their specification, all goods are tradable across borders, in this simplified framework. Therefore, domestic consumption (C) for the representative consumer in the home economy, which is also equal to their total domestic tradable consumption, is a weighted average of consumer goods produced at home (C_H) and abroad (C_F) and is given by:[8]

$$C = C_T = \left[\alpha_H^{1-\rho} C_H^{\rho} + \alpha_F^{1-\rho} C_F^{\rho}\right]^{\frac{1}{\rho}} \tag{2.24}$$

where α_H and α_F measure the shares of goods produced at home and abroad, respectively that are in the consumer's consumption basket, and $\alpha_H + \alpha_F = 1$. α_H represents the consumer's degree of home bias and, conversely, α_F can be considered as a measure of trade openness or reliance on imports for domestic consumption. Finally, $\rho < 1$ is a function of the constant elasticity of substitution between domestically produced and foreign produced tradable goods.

Similarly, the consumer price index (P) is a weighted average of the prices of domestically produced goods (P_H) and foreign produced goods (P_F).

$$P = P_T = \left[\alpha_H P_H^{\frac{\rho}{\rho-1}} + (1-\alpha_H) P_F^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \tag{2.25}$$

[8] This version of the simplified open economy model abstracts from non-traded goods, and so total consumption (C) is equal to consumption of traded goods (C_T).

Again, for simplification, all goods are tradable in this simplified framework, so the consumer's price index is equal to the price index for tradable goods.

By extension, the terms of trade (τ) is equal to the ratio of the prices of goods produced abroad and those produced at home, $\frac{P_F}{P_H}$. Again, the real exchange rate (*RER*) is also defined as the ratio of the weighted average consumer price index for the foreign consumer (P^*), where "*" represents foreign variables, and the weighted average consumer price index for the consumer at home (P). In other words, $RER = \frac{P_t^*}{P_t}$.

The optimal allocation of expenditure on domestically produced goods by the home consumer can be expressed as

$$C_H = \alpha_H \left(\frac{P_H}{P}\right)^{-\omega} C, \quad (2.26)$$

where the trade elasticity (or elasticity of substitution) parameter $\omega = (1-\rho)^{-1}$ and by extension, $\omega > 0$. Therefore, local consumption of domestically produced goods (as a share of total consumption) depends positively on the consumer's degree of home bias but depends negatively on the price of goods produced at home relative to the total consumer price index.

Moreover, total consumption of domestically produced goods, both at home and abroad should equate to total domestic, tradable production $Y_H = C_H + C_H^*$, also referred to as the market clearing condition for domestically produced tradable goods. The same resource constraint (and market clearing condition) holds for consumption and production of foreign, tradable goods ($Y_F = C_F + C_F^*$).

Assuming financial autarky, the consumer's total expenditure is constrained by her income in each period given her inability to save or borrow in financial markets. Thus $\frac{PC}{P_H} = Y_H$ where Y_H captures total domestic output or real gross domestic product.

Leveraging equation 2.26 and $\frac{PC}{P_H} = Y_H$, the domestic demand for consumer goods produced at home can therefore be expressed as:

$$C_H = \alpha_H \left(\frac{P_H}{P}\right)^{-\omega} \frac{P_H Y_H}{P}$$

$$C_H = \alpha_H \frac{{P_H}^{-\omega} P_H}{P^{-\omega} P} Y_H$$

$$C_H = \alpha_H \left(\frac{P_H}{P}\right)^{1-\omega} Y_H$$

$$C_H = \alpha_H \left(\frac{P_H}{[\alpha_H P_H^{1-\omega} + (1-\alpha_H) P_F^{1-\omega}]^{\frac{1}{1-\omega}}} \right)^{1-\omega} Y_H$$

$$C_H = \alpha_H \frac{{P_H}^{1-\omega}}{\alpha_H P_H^{1-\omega} + (1-\alpha_H) P_F^{1-\omega}} Y_H$$

$$C_H \frac{[\alpha_H P_H^{1-\omega} + (1-\alpha_H) P_F^{1-\omega}]}{{P_H}^{1-\omega}} = \alpha_H Y_H$$

$$C_H \left[\alpha_H \frac{P_H^{1-\omega}}{{P_H}^{1-\omega}} + (1-\alpha_H) \frac{P_F^{1-\omega}}{{P_H}^{1-\omega}} \right] = \alpha_H Y_H$$

$$C_H [\alpha_H + (1-\alpha_H)\tau^{1-\omega}] = \alpha_H Y_H$$

$$C_H = \frac{\alpha_H}{\alpha_H + (1-\alpha_H)\tau^{1-\omega}} Y_H \text{ or } C_H = \alpha_H Y_H [\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^{-1} \quad (2.27)$$

Equation 2.27 can be used to determine the response of domestic consumption of home goods (C_H) to a change in the terms of trade (τ). Achieving this requires taking the first, partial derivative of equation 2.27 relative to τ. That yields:

$$\frac{\partial C_H}{\partial \tau} = -\alpha_H Y_H [\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^{-2} (1-\omega)(1-\alpha_H)\tau^{-\omega}$$

$$\frac{\partial C_H}{\partial \tau} = \frac{-\alpha_H (1-\omega)(1-\alpha_H)\tau^{-\omega}}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2} Y_H$$

$$\frac{\partial C_H}{\partial \tau} = \omega \frac{\alpha_H (1-\alpha_H)\tau^{-\omega}}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2} Y_H - \frac{\alpha_H (1-\alpha_H)\tau^{-\omega}}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2} Y_H$$

Whether this expression is more than or less than zero depends on the value of the trade elasticity parameter ω. Where $\omega > 1$, domestic consumption of the home good responds positively to the depreciation (increase) in the terms of trade. Alternatively, if $\omega < 1$, then domestic consumption of the home good reacts negatively to the same. Ultimately, this determines whether the positive substitution effect of lower home prices, $\omega \frac{\alpha_H (1-\alpha_H)\tau^{-\omega}}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2} Y_H$, exceeds the negative income effects, $-\frac{\alpha_H (1-\alpha_H)\tau^{-\omega}}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2} Y_H$, (in absolute value) due to a reduction in home prices. Assume that $\omega < 1$, as is consistent with most estimates using macroeconomic data, and that a positive productivity shock pushes the price of domestically produced goods and services lower and depreciates the terms of trade. Lower prices for domestic goods reduce domestic income and this effect overshadows the substitution effects from lower home prices, because low trade elasticity prohibits a sufficiently strong response to cheaper goods. On the other hand, a much higher trade elasticity parameter, specifically where $\omega > 1$, would drive consumers to buy more of the domestically produced good, despite lower overall income.

Alternatively, Corsetti et al. (2008) show that the same shock will increase consumption of the domestically produced good in the foreign country. Like in equation 2.26 above, the optimal allocation of expenditure on domestically produced goods by the foreign consumer can be expressed as $C_H^* = \alpha_H^* \left(\frac{P_H}{P^*}\right)^{-\omega} C^*$. Combining this with condition $\frac{P^*C^*}{P_H} = \frac{P_F}{P_H} Y_F^*$ or $C^* = \frac{P_H}{P^*} \tau Y_F^*$, which holds in financial autarky, yields:

$$C_H^* = \alpha_H^* \left(\frac{P_H}{P^*}\right)^{-\omega} \frac{P_H}{P^*} \tau Y_F^*$$

$$C_H^* = \alpha_H^* \frac{{P_H}^{-\omega} P_H}{{P^*}^{-\omega} P^*} \tau Y_F^*$$

$$C_H^* = \alpha_H^* \left(\frac{P_H}{P^*}\right)^{1-\omega} \tau Y_F^*$$

$$C_H^* = \alpha_H^* \left(\frac{P_H}{\left[\alpha_H^* P_H^{1-\omega} + (1-\alpha_H^*) P_F^{1-\omega}\right]^{\frac{1}{1-\omega}}}\right)^{1-\omega} \tau Y_F^*$$

$$C_H^* = \alpha_H^* \frac{{P_H}^{1-\omega}}{\alpha_H^* P_H^{1-\omega} + (1-\alpha_H^*) P_F^{1-\omega}} \tau Y_F^*$$

$$C_H^* \frac{\left[\alpha_H^* P_H^{1-\omega} + (1-\alpha_H^*) P_F^{1-\omega}\right]}{{P_H}^{1-\omega}} = \alpha_H^* \tau Y_F^*$$

$$C_H^* \left[\alpha_H^* \frac{P_H^{1-\omega}}{{P_H}^{1-\omega}} + (1-\alpha_H^*) \frac{P_F^{1-\omega}}{{P_H}^{1-\omega}}\right] = \alpha_H^* \tau Y_F^*$$

$$C_H^* [\alpha_H^* + (1-\alpha_H^*) \tau^{1-\omega}] = \alpha_H^* \tau Y_F^*$$

$$C_H^* = \frac{\alpha_H^*}{\alpha_H^* + (1-\alpha_H^*) \tau^{1-\omega}} \tau Y_F^*$$

Partially differentiating foreign consumption of the domestically produced good with respect to the terms of trade yields:

$$\frac{\partial C_H^*}{\partial \tau} = \frac{[\alpha_H^* + (1-\alpha_H^*)\tau^{1-\omega}]\alpha_H^* Y_F^* - \alpha_H^* \tau Y_F^* [(1-\omega)(1-\alpha_H^*)\tau^{-\omega}]}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2}$$

$$\frac{\partial C_H^*}{\partial \tau} = \frac{\alpha_H^* Y_F^* [\alpha_H^* + (1-\alpha_H^*)\tau^{1-\omega} - \tau(1-\alpha_H^*)\tau^{-\omega} + \tau\omega(1-\alpha_H^*)\tau^{-\omega}]}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2}$$

$$\frac{\partial C_H^*}{\partial \tau} = \frac{\alpha_H^* Y_F^* [\alpha_H^* + (1-\alpha_H^*)\tau^{1-\omega} + \tau^{1-\omega}(1-\alpha_H^*)(\omega-1)]}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2}$$

$$\frac{\partial C_H^*}{\partial \tau} = \frac{\alpha_H^* Y_F^* [\alpha_H^* + (1-\alpha_H^*)\tau^{1-\omega}\omega]}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2}$$

$$\frac{\partial C_H^*}{\partial \tau} = (1-\alpha_H^*)\tau^{1-\omega}\omega \frac{\alpha_H^*}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2} Y_F^* + \alpha_H^* \frac{\alpha_H^*}{[\alpha_H + (1-\alpha_H)\tau^{1-\omega}]^2} Y_F^*$$

The positive productivity shock in the home country improves the terms of trade for the foreign consumer and creates a positive income effect which permits greater consumption of the domestically produced good abroad. Lower prices for that good also exacerbate that effect.

The relationships between consumption of the foreign produced good and the home terms of trade, whether by consumers at home or abroad, also depend on the relative income and substitution effects. A depreciation in the terms of trade, driven by an expansion in goods produced at home unambiguously reduces the home consumption of foreign produced goods. This is because the fall in domestic prices automatically generates a negative income effect which reduces the home consumer's capacity to purchase goods and services, whether from at home or abroad. At the same time, the lower price of domestically produced goods relative to foreign goods prompts the domestic consumer to substitute some of her foreign produced goods for the cheaper home good in her consumption basket. Algebraically, this can be expressed as below. Take the optimal allocation of expenditure on foreign produced goods by the domestic consumer, $C_F = \alpha_F \left(\frac{P_F}{P}\right)^{-\omega} C$, and the condition for financial autarky for the domestic consumer, $C = \frac{P_H Y_H}{P}$. Combining the two yields:

$$C_F = \alpha_F \left(\frac{P_F}{P}\right)^{-\omega} \frac{P_H Y_H}{P}$$

$$C_F = \alpha_F \frac{{P_F}^{-\omega} P_H}{P^{-\omega} P} Y_H$$

$$C_F = \alpha_F \frac{{P_F}^{-\omega} P_H}{P^{1-\omega}} Y_H$$

$$C_F = \alpha_F \frac{{P_F}^{-\omega} P_H}{\alpha_H P_H^{1-\omega} + (1-\alpha_H) P_F^{1-\omega}} Y_H$$

$$C_F \frac{[\alpha_H P_H^{1-\omega} + (1-\alpha_H) P_F^{1-\omega}]}{{P_F}^{-\omega} P_H} = \alpha_F Y_H$$

$$C_F \left[\alpha_H \frac{P_H^{1-\omega}}{{P_F}^{-\omega} P_H} + (1-\alpha_H) \frac{P_F^{1-\omega}}{{P_F}^{-\omega} P_H}\right] = \alpha_F Y_H$$

$$C_F \left[\alpha_H \frac{P_H^{1-\omega} P_H^{-1}}{{P_F}^{-\omega}} + (1-\alpha_H) \frac{P_F^{1-\omega} {P_F}^{\omega}}{P_H}\right] = \alpha_F Y_H$$

$$C_F \left[\alpha_H \frac{P_H^{-\omega}}{{P_F}^{-\omega}} + (1-\alpha_H) \frac{P_F}{P_H}\right] = \alpha_F Y_H$$

$$C_F[\alpha_H\tau^{\omega} + (1-\alpha_H)\tau] = \alpha_F Y_H$$

$$C_F = \frac{\alpha_F}{\alpha_H\tau^{\omega}+(1-\alpha_H)\tau} Y_H$$

$$C_F = \alpha_F Y_H[\alpha_H\tau^{\omega} + (1-\alpha_H)\tau]^{-1}$$

Partially differentiating domestic consumption of the foreign produced good with respect to the terms of trade yields:

$$\frac{\partial C_F}{\partial \tau} = -\alpha_F Y_H[\alpha_H\tau^{\omega} + (1-\alpha_H)\tau]^{-2}[\omega\alpha_H\tau^{\omega-1} + (1-\alpha_H)]$$

$$\frac{\partial C_F}{\partial \tau} = -\alpha_F \frac{\omega\alpha_H\tau^{\omega-1}+(1-\alpha_H)}{[\alpha_H\tau^{\omega}+(1-\alpha_H)\tau]^2} Y_H$$

$$\frac{\partial C_F}{\partial \tau} = -(1-\alpha_H)\frac{\omega\alpha_H\tau^{\omega-1}}{[\alpha_H\tau^{\omega}+(1-\alpha_H)\tau]^2} Y_H - (1-\alpha_H)\frac{(1-\alpha_H)}{[\alpha_H\tau^{\omega}+(1-\alpha_H)\tau]^2} Y_H$$

This partial derivative is unambiguously negative for permissible values of α_H and ω.

The effects of a terms of trade depreciation on foreign consumption of the foreign produced good are more complicated. Again, the cheaper domestically produced good prompts the foreign consumer to switch some of their consumption from the foreign produced good to the one produced at home. However, the positive change in the terms of trade increases their capacity to consume both goods. Again, whether they choose to increase or decrease their consumption of the foreign produced good depends on whether the elasticity of substitution (ω) is high (that is, greater than one) or low (that is, less than one). In the extreme case where it is one, consumption does not change. Again, take $\frac{P^*C^*}{P_H} = \frac{P_F}{P_H} Y_F^*$ and combine it with the optimal allocation of expenditure on foreign produced goods by the foreign consumer, $C_F^* = \alpha_F^* \left(\frac{P_F}{P^*}\right)^{-\omega} C^*$. This leads to:

$$C_F^* = \alpha_F^* \left(\frac{P_F}{P^*}\right)^{-\omega} \frac{P_F}{P^*} Y_F^*$$

$$C_F^* = \alpha_F^* \frac{P_F^{-\omega} P_F}{P^{*-\omega} P^*} Y_F^*$$

$$C_F^* = \alpha_F^* \frac{P_F^{1-\omega}}{P^{*1-\omega}} Y_F^*$$

$$C_F^* = \alpha_F^* \frac{P_F^{1-\omega}}{\alpha_H^* P_H^{1-\omega} + (1-\alpha_H^*) P_F^{1-\omega}} Y_F^*$$

$$C_F^* \frac{[\alpha_H^* P_H^{1-\omega} + (1-\alpha_H^*) P_F^{1-\omega}]}{P_F^{1-\omega}} = \alpha_F^* Y_F^*$$

$$C_F^*\left[\alpha_H^*\frac{P_H^{1-\omega}}{P_F^{1-\omega}}+(1-\alpha_H^*)\frac{P_F^{1-\omega}}{P_F^{1-\omega}}\right]=\alpha_F^*Y_F^*$$

$$C_F^*\left[\alpha_H^*\frac{P_H^{1-\omega}}{P_F^{1-\omega}}+(1-\alpha_H^*)\right]=\alpha_F^*Y_F^*$$

$$C_F^*[\alpha_H^*\tau^{\omega-1}+(1-\alpha_H^*)]=\alpha_F^*Y_F^*$$

$$C_F^*[\alpha_H^*\tau^{\omega-1}+(1-\alpha_H^*)]=\alpha_F^*Y_F^*$$

$$C_F^*=\frac{\alpha_F^*}{\alpha_H^*\tau^{\omega-1}+(1-\alpha_H^*)}Y_F^*$$

$$C_F^*=\frac{\alpha_F^*}{\alpha_H^*\tau^{\omega-1}+(1-\alpha_H^*)}Y_F^*$$

$$C_F^*=\alpha_F^*Y_F^*[\alpha_H^*\tau^{\omega-1}+(1-\alpha_H^*)]^{-1}$$

Partially differentiating foreign consumption of the foreign produced good with respect to the home terms of trade yields:

$$\frac{\partial C_F^*}{\partial\tau}=-\alpha_F^*Y_F^*[\alpha_H^*\tau^{\omega-1}+(1-\alpha_H^*)]^{-2}(\omega-1)\alpha_H^*\tau^{\omega-2}$$

$$\frac{\partial C_F^*}{\partial\tau}=\frac{-\alpha_F^*Y_F^*(\omega-1)\alpha_H^*\tau^{\omega-2}}{[\alpha_H^*\tau^{\omega-1}+(1-\alpha_H^*)]^2}$$

$$\frac{\partial C_F^*}{\partial\tau}=-\omega\frac{(1-\alpha_H^*)\alpha_H^*\tau^{\omega-2}}{[\alpha_H^*\tau^{\omega-1}+(1-\alpha_H^*)]^2}Y_F^*+\frac{(1-\alpha_H^*)\alpha_H^*\tau^{\omega-2}}{[\alpha_H^*\tau^{\omega-1}+(1-\alpha_H^*)]^2}Y_F^*$$

Consequently, a depreciation in the terms of trade will reduce foreign consumption of the foreign produced good if the elasticity of substitution between goods produced at home and abroad exceeds 1. Alternatively, if the elasticity parameter is less than 1, positive income effects will trigger greater consumption of the foreign good by foreign consumers.

Corsetti et al. (2008) illustrate how these relationships (primarily the former two) then permit the analysis of the relationship between the global demand for domestic production of consumer goods ($Y_H = C_H + C_H^*$) and positive endowment shocks, depending on the nature of the trade elasticity parameter and the degree of consumer bias for domestically produced goods, both at home and abroad (α_H and α_H^*). A large trade elasticity parameter (where ω is sufficiently greater than 1) and the corresponding demand for cheaper domestically produced goods will more than offset the effects of lower home prices on income and should generate a greater demand for domestic production. In this case, Y_H rises in line with τ. Alternatively, where ω is sufficiently lower than 1 and consumers in both countries are comparatively less responsive to cheaper global prices, Y_H and τ will probably move in opposite directions.

However, this is most likely to occur when the degree of home bias in the home country is high and materially greater than the dependence on domestically produced goods abroad (that is, $\alpha_H > \alpha_H^*$).

The analysis below formalizes this relationship but also illustrates that the home terms of trade and the real exchange rate may not move together. Log-linearizing the expression for the demand for domestically produced goods ($Y_H = C_H + C_H^*$) around a symmetric equilibrium (that is, $Y_H = Y_F^*$ and $\alpha_H = 1 - \alpha_H^*$) yields the following (Corsetti et al., 2008):

$$\hat{\tau} = \frac{1}{1-2\alpha_H(1-\omega)}\left(\widehat{Y_H} - \widehat{Y_F^*}\right) \tag{2.28}$$

$$\widehat{RER} = \frac{2\alpha_H - 1}{1-2\alpha_H(1-\omega)}\left(\widehat{Y_H} - \widehat{Y_F^*}\right) \tag{2.29}$$

where ∧ represents deviations away from equilibrium.

Taking the first partial derivatives of equation 2.28 and equation 2.29, respectively yield:

$$\frac{\partial\hat{\tau}}{\partial\widehat{Y_H}} = \frac{1}{1-2\alpha_H(1-\omega)} = \frac{1}{1-2\alpha_H+2\alpha_H\omega}$$

$$\frac{\partial\widehat{RER}}{\partial\widehat{Y_H}} = \frac{2\alpha_H-1}{1-2\alpha_H(1-\omega)} = \frac{2\alpha_H-1}{1-2\alpha_H+2\alpha_H\omega}$$

In other words, positive supply shocks to domestic endowment which push domestic output higher will positively co-move with both the terms of trade and the real exchange rates, depending on the sizes of both α_H and ω. Corsetti et al. (2008) illustrate that, under the assumption of home bias in consumption ($\alpha_H > 0.5$), the point at which both relationships switch from positive to negative is when $\omega < \frac{2\alpha_H-1}{2\alpha_H}$. When the elasticity of substitution is high and ω exceeds $\frac{2\alpha_H-1}{2\alpha_H}$, the relationship is positive, and positive endowment shocks lead to depreciations in both the home terms of trade and the real exchange rate. However, when ω is less than $\frac{2\alpha_H-1}{2\alpha_H}$, that is, the trade elasticity parameter is low, positive endowment shocks correspond with appreciations in the terms of trade and the real exchange rate.

The terms of trade and real exchange rate need not move together in response to positive supply shocks, however. That depends on the degree of home bias or, conversely, the degree of dependence on foreign goods for domestic consumption. When the degree of home bias is high and $\alpha_H > 0.5$, both the terms of trade and real exchange rate will move in the same direction. However, when $\alpha_H < 0.5$, that is, the consumer relies more on foreign goods than on domestically produced goods for consumption, the two may move in opposite directions. This occurs because of the composition of the consumer price index.

Recall the definition of the real exchange rate and the consumer price index:

$$RER = \frac{P_t^*}{P_t}$$

$$P = \left[\alpha_H P_H^{\frac{\rho}{\rho-1}} + (1-\alpha_H) P_F^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}$$

The corresponding consumer price index for the consumer in the foreign country can be expressed similarly as:

$$P^* = \left[\alpha_H^* P_H^{\frac{\rho}{\rho-1}} + (1-\alpha_H^*) P_F^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}$$

Leveraging the relationship between ρ and ω, $\omega = (1-\rho)^{-1}$, the price indices can be rewritten as:

$$P = [\alpha_H P_H^{1-\omega} + (1-\alpha_H) P_F^{1-\omega}]^{\frac{1}{1-\omega}}$$

$$P^* = [\alpha_H^* P_H^{1-\omega} + (1-\alpha_H^*) P_F^{1-\omega}]^{\frac{1}{1-\omega}}$$

Now, take first partial derivatives of each equation with respect to the price of domestically produced goods. This yields the expressions below:

$$\frac{\partial P}{\partial P_H} = \frac{(1-\omega)\alpha_H P_H^{-\omega}}{1-\omega} [\alpha_H P_H^{1-\omega} + (1-\alpha_H) P_F^{1-\omega}]^{\frac{1}{1-\omega}}$$

$$\frac{\partial P}{\partial P_H} = \alpha_H P_H^{-\omega} [\alpha_H P_H^{1-\omega} + (1-\alpha_H) P_F^{1-\omega}]^{\frac{1}{1-\omega}} \qquad (2.30)$$

$$\frac{\partial P^*}{\partial P_H} = \frac{(1-\omega)\alpha_H^* P_H^{-\omega}}{1-\omega} [\alpha_H^* P_H^{1-\omega} + (1-\alpha_H^*) P_F^{1-\omega}]^{\frac{1}{1-\omega}}$$

$$\frac{\partial P^*}{\partial P_H} = \alpha_H^* P_H^{-\omega} [\alpha_H^* P_H^{1-\omega} + (1-\alpha_H^*) P_F^{1-\omega}]^{\frac{1}{1-\omega}} \qquad (2.31)$$

From equation 2.30 and equation 2.31, $\frac{\partial P}{\partial P_H} > \frac{\partial P^*}{\partial P_H}$ if $\alpha_H > \alpha_H^*$. In other words, if the reliance on goods produced in the home country by the domestic consumer is greater than the reliance on those goods by the foreign consumer, then the consumer price index in the home country increases by more than the consumer price index in the foreign country, in response to higher prices of the domestically produced good. In that case, an appreciation (depreciation) in the terms of trade driven by higher (lower) prices of the domestic good will also appreciate (depreciate) the home country's real exchange rate. In contrast, if the domestic consumer depends more on imported consumer goods than the foreign consumer, that is $\alpha_H < \alpha_H^*$ or $\alpha_F > \alpha_F^*$, then $\frac{\partial P}{\partial P_H} < \frac{\partial P^*}{\partial P_H}$. Therefore, if the degree of home bias is low, or the dependence on foreign produced goods is high at home compared to the dependence on foreign produced goods in the foreign country, the consumer price index in the home country will rise at a slower pace than the consumer price index in the foreign country, in response to higher prices of the domestically produced good. Consequently, an appreciation in the terms of trade driven by higher prices of the domestic good will actually depreciate the home country's real exchange rate.

Clearly then, the nature of this relationship depends heavily on the assumptions around the home consumer's degree of home bias. When home bias is high (for example, where $\alpha_H >$ 0.5), the threshold for the trade elasticity parameter, below which the relationship between the terms of trade or real exchange rate and domestic output turns negative, rises. Therefore, for any given value of the trade elasticity parameter, a higher value for α_H, particularly above 0.5, increases the probability that positive endowment shocks will depreciate the terms of trade and real exchange rate. Conversely, in cases where home bias is low (especially if α_H < 0.5) or the degree of trade openness or reliance on foreign goods is high, the threshold for the trade elasticity parameter below which the relationship between the real exchange rate and domestic output turns negative falls. In other words, lower α_H or higher α_F is more likely to lead to a negative relationship between output shocks and the real exchange rate. However, as outlined above, the real exchange rate may also move inversely with the terms of trade at very low levels of home bias, in response to these shocks.

The next step in this analysis requires understanding how this all relates to the theoretical correlation between relative consumption and the real exchange rate as defined in the Backus-Smith relationship. With financial autarky and no trade in bonds, Corsetti et al. (2008) show that the balanced-trade condition can be used to derive:

$$\tau C_F = C_H^* \text{ or } \frac{C}{C^*} = \frac{\alpha_H^*}{1-\alpha_H}\tau^{\omega-1}\left[\frac{\alpha_H^*+(1-\alpha_H^*)\tau^{1-\omega}}{\alpha_H+(1-\alpha_H)\tau^{1-\omega}}\right]^{\omega/(1-\omega)} \quad (2.32)$$

In other words, relative consumption becomes a function of the terms of trade, the degrees of bias for domestically produced goods, both at home and abroad, and the trade elasticity parameter.

Log-linearizing yields:

$\widehat{RER} = \frac{2\alpha_H - 1}{2\alpha_H \omega - 1}\left(\hat{C} - \widehat{C^*}\right)$ or, expressed differently,

$$\left(\hat{C} - \widehat{C^*}\right) = \frac{2\alpha_H \omega - 1}{2\alpha_H - 1}\widehat{RER} \quad (2.33)$$

Equation 2.33 above is the same as the traditional efficient risk sharing condition $\left(\hat{C} - \widehat{C^*}\right) = \frac{1}{\sigma}\widehat{RER}$ if $\omega = \frac{2\alpha_H + \sigma - 1}{2\alpha_H \sigma}$ (Corsetti et al., 2008), where again σ is the risk aversion parameter from the constant relative risk aversion (CRRA) utility function.

2.A.3 Summary of Variables Used in Chapter 2

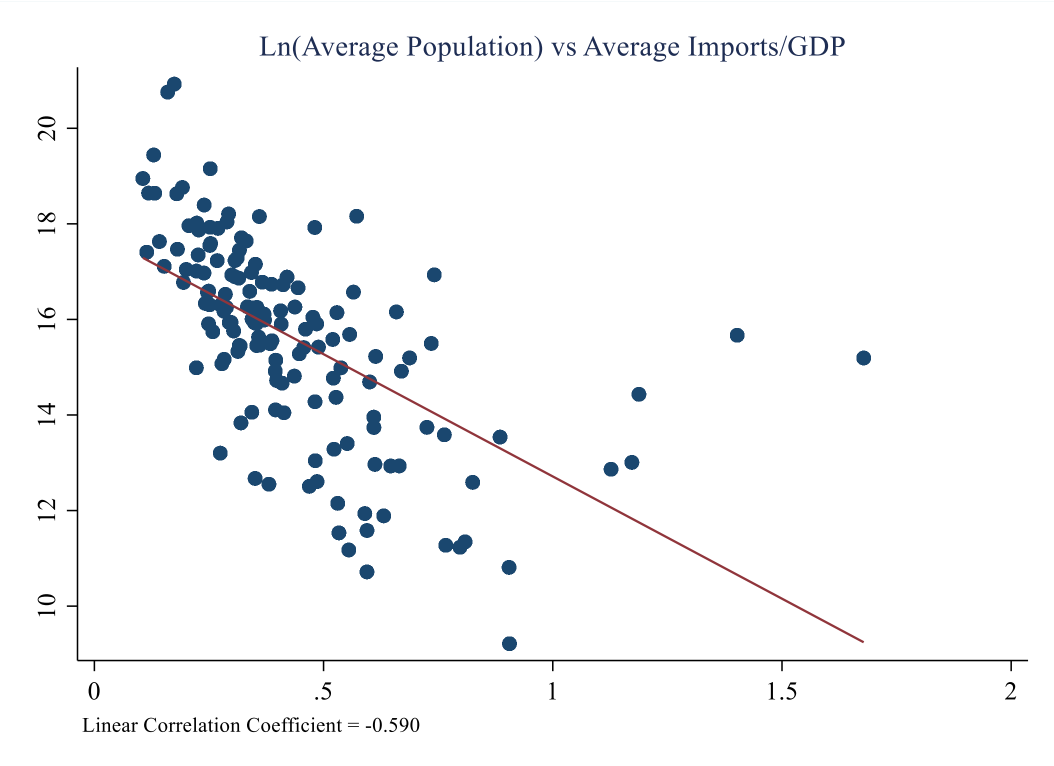
Table 2.11: Definitions and Sources of Variables Used in the Analysis

Variable	Data Definition	Source(s)
Δc_{it}, Δc_{it}^*	First difference of the natural log of real household consumption divided by population for each country; c_{it}^* is a trade-weighted average of real per capita household consumption of each country's top 20 trading partners. Constant trade weights are calculated as that trading partner's share of the home country's total exports and imports of merchandise goods during 2008-2012	World Bank's World Development Indicators, United Nations, International Monetary Fund Direction of Trade Statistics
$\Delta(rer_t)$	First difference of the natural log of the real effective exchange rate (REER) between the home country and its top 20 trading partners. Real effective exchange rates are calculated as the ratio of trade-weighted consumer price indices to the home country's consumer price index, all converted into the same currency units using the trade-weighted nominal exchange rate. Constant trade weights are calculated as that trading partner's share of the home country's total exports and imports of merchandise goods during 2008-2012	World Bank's World Development Indicators, International Monetary Fund Direction of Trade Statistics and World Economic Outlook
Δy_{it}, Δy_{it}^*	First difference of the natural log of real Gross Domestic Product divided by population for each country; y_{it}^* is a trade-weighted average of real per capita Gross Domestic Product of each country's top 20 trading partners. Constant trade weights are calculated as that trading partner's share of the home country's total exports and imports of merchandise goods during 2008-2012	World Bank's World Development Indicators, United Nations, International Monetary Fund Direction of Trade Statistics
${C_F}_{it}$	Import volume index	World Bank's World Development Indicators
${P_F}_{it}/{P_H}_{it}$	Ratio of import unit value index (measured in United States dollars) as a proxy for foreign prices to the GDP deflator (also measured in United States Dollars) as a proxy for the prices of domestic production	World Bank's World Development Indicators, United Nations
${Y_H}_{it}$	Real Gross Domestic Product, sourced directly from the World Bank and United Nations	World Bank's World Development Indicators, United Nations
α_F	Total Imports of Goods and Services all as a ratio of Nominal Gross Domestic Product	World Bank's World Development Indicators, United Nations
Real GDP per Capita	Real Gross Domestic Product divided by Population for each country	World Bank's World Development Indicators, United Nations
Population	Mid-year estimates of the total number of residents, regardless of legal status or citizenship, sourced directly from the World Bank	World Bank's World Development Indicators

2.A.4 Regression Results Using Population Size as a Measure of Dependence on Foreign Goods and Services

The results in this appendix expand on the results in the main body of the chapter, using an alternative measure of openness or dependence on foreign goods and services for consumption. In this case, the country's size may act as a proxy for its ability to produce a range of goods and services domestically, and hence, rely less on imported products. Figure 2.6 illustrates the relationship between the natural log of average population and the average imports of goods and services to nominal gross domestic product ratio for the 150 countries in the sample. The two clearly exhibit a negative relationship, with smaller economies being more open, and larger economies relying less on imports of goods and services as inputs to domestic economic activity.

Figure 2.6: Relationship Between the ln(Average Population) (y-axis) and Average Imports of Goods and Services/Nominal Gross Domestic Product (x-axis)



Source(s): World Bank's World Development Indicators, United Nations National Accounts Statistics, author's calculations

Splitting the sample by the median population size yields very similar results to those in Table 2.7, when the sample was split by the median imports of goods and services to nominal gross domestic product ratio (Table 2.12). Controlling for relative per capita output growth, changes in the real effective exchange rate lead to positive and statistically significant changes in relative per capita consumption growth in the smaller group of countries. In other words, those economies that are expected to be more dependent on imported goods and services for domestic consumption display some evidence of international risk sharing. In contrast, larger economies, those that probably have greater domestic production capacity, exhibit more evidence of the Backus-Smith puzzle – the coefficient on the real effective exchange rate is practically zero for that group of countries. Finally, in both cases, the proportion of consumers that shares risks in each group is under 40%.

Table 2.12: Random Effects Linear Regression Estimates of Equation 2.7 – Split by Average Population Size

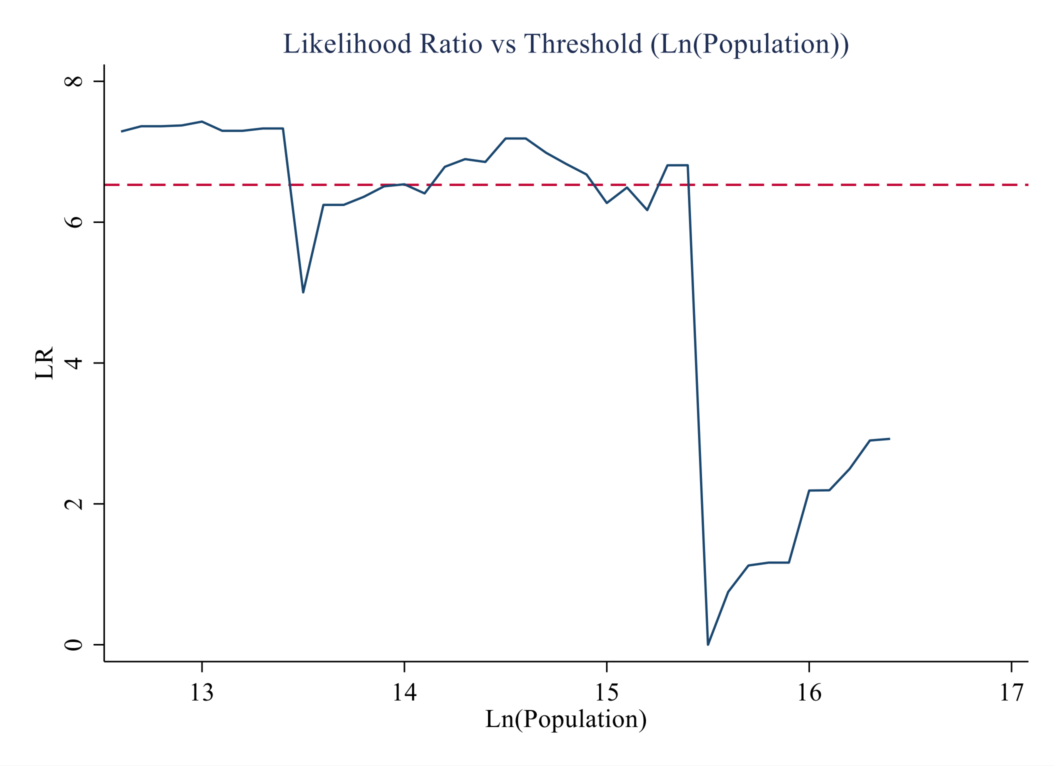
Variable	**Full Sample**	**Smaller**	**Larger**
Δrer_{it}	0.009	0.024**	0.002
	(0.007)	(0.010)	(0.006)
$\Delta y_{it} - \Delta y_{it}^*$	0.624***	0.621***	0.627***
	(0.064)	(0.087)	(0.094)
Constant	-0.000	-0.001	0.000
	(0.001)	(0.001)	(0.001)
Observations	5,700	2,850	2,850
R-squared	0.151	0.144	0.166
Number of countries	150	75	75

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Again, equation 2.9 is re-estimated using the natural log of population as the chosen threshold variable versus the imports to nominal gross domestic product ratio. The expectation is that, below an optimal population size, the real effective exchange rate has a positive and statistically significant effect on cross country consumption growth. Above that size, the effect is expected to illustrate the typical relationship popular with the Backus-Smith puzzle – a negative or low correlation between the real effective exchange rate and relative consumption growth. Figure 2.7 below suggests that the optimal threshold should be at a population size of 5.4 million people (or ln(population) = 15.5). This is the point at which the likelihood ratio of a series of regressions is at its lowest.[9]

[9] Like in the main body of this chapter, the series of possible threshold values excludes the top 10% and bottom 10% of the distribution of the natural log of average population.

Figure 2.7: Likelihood Ratio (LR) Plot of Equation 2.9 with ln(Population) as the Threshold Variable



Source(s):Author's calculations.

Similar to Table 2.8 in the chapter's main body, Table 2.13 illustrates the econometric results of the threshold regression with an optimal population threshold of 5.4 million people (or ln(population) = 15.5). Again, the results suggest that the consumption-real exchange rate anomaly is still prevalent for the larger group of economies. Above the estimated threshold, the relationship is only marginally positive but is still very statistically insignificant. Below this threshold however, the coefficient increases substantially and is statistically insignificant from zero with at least 99% confidence.

Table 2.13: Non-dynamic, Single Threshold Panel Estimates of Equation 2.9 – Using Average Population as the Threshold Variable

Variable	No Threshold	Single Threshold
Δrer_{it}	0.009	
	(0.007)	
$\Delta rer_{it}\ I(Ln(pop) \leq 15.5)$		0.026***
		(0.009)
$\Delta rer_{it}\ I(Ln(pop) > 15.5)$		0.001
		(0.006)
$\Delta y_{it} - \Delta y_{it}^{*}$	0.624***	0.624***
	(0.064)	(0.064)
Constant	-0.000	-0.000
	(0.001)	(0.001)
Observations	5,700	5,700
R-squared	0.151	0.152
Number of countries	150	150

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

2.A.5 Analysis of the Relationship Between the Terms of Trade and Relative Per Capita Consumption

The analysis and results in this appendix offer an alternative view of the Backus-Smith relationship, leveraging the relationship between the real exchange rate and the terms of trade. Unlike in Corsetti et al. (2008) where the authors assume that the consumer's preference for goods produced at home is always relatively high, the analysis below emphasizes that the correlation between the terms of trade and the real exchange rate can switch signs if the degree of home bias at home is too low compared to in the foreign country. The analysis leverages the same 150 countries included in the original analysis.

Theoretical Relationships

First recall equation 2.28 and equation 2.29 which express both the terms of trade and the real exchange rate as deviations away from equilibrium as functions of relative output at home and abroad, both as deviations away from their respective equilibriums.

$$\hat{\tau} = \frac{1}{1-2\alpha_H(1-\omega)}\left(\widehat{Y_H} - \widehat{Y_F^*}\right)$$

$$\widehat{RER} = \frac{2\alpha_H - 1}{1-2\alpha_H(1-\omega)}\left(\widehat{Y_H} - \widehat{Y_F^*}\right)$$

Rearranging each equation to make $(\widehat{Y_H} - \widehat{Y_F^*})$ the subject yields:

$(\widehat{Y_H} - \widehat{Y_F^*}) = [1 - 2\alpha_H(1-\omega)]\ \hat{\tau}$

$(\widehat{Y_H} - \widehat{Y_F^*}) = \frac{1-2\alpha_H(1-\omega)}{2\alpha_H-1}\widehat{RER}$

Setting each equation equal to the other and simplifying further also yields:

$[1 - 2\alpha_H(1-\omega)]\ \hat{\tau} = \frac{1-2\alpha_H(1-\omega)}{2\alpha_H-1}\widehat{RER}$

$\hat{\tau} = \frac{1}{2\alpha_H-1}\widehat{RER}$

$$\widehat{RER} = (2\alpha_H - 1)\hat{\tau} \quad (2.34)$$

Equation 2.34 suggests that, as explained before, deviations of the real exchange rate and the terms of trade away from their respective equilibriums are expected to move together, except when the degree of home bias in the home country is sufficiently low, that is, in this case, where $\alpha_H < 0.5$.

However, this relationship can also help to explain how relative consumption and the terms of trade may move together. Again, recall equation 2.3:

$\left(\hat{C} - \widehat{C^*}\right) = \frac{2\alpha_H\omega-1}{2\alpha_H-1}\widehat{RER}$

Replacing $\widehat{RER}$ with $(2\alpha_H - 1)\hat{\tau}$ from equation 2.34 then yields:

$\left(\hat{C} - \widehat{C^*}\right) = \frac{2\alpha_H\omega-1}{2\alpha_H-1}(2\alpha_H - 1)\hat{\tau}$

Finally, simplifying this equation gives the relationship between changes in relative consumption from their equilibriums and deviations of the terms of trade from its equilibrium:

$$\left(\hat{C} - \widehat{C^*}\right) = (2\alpha_H\omega - 1)\hat{\tau} \quad (2.35)$$

or

$\left(\hat{C} - \widehat{C^*}\right) = [2(1-\alpha_F)\omega - 1]\hat{\tau}$

Taking the first derivative of equation 2.35 with respect to $\hat{\tau}$ implies that, like the Backus-Smith relationship, the relationship between relative consumption and the terms of trade depends both on the degree of home bias of the domestic consumer and the elasticity of substitution between goods produced at home and those produced abroad. In the case where both α_H and ω are very high, $(2\alpha_H\omega - 1) > 0$ and the correlation is positive. At the other extreme however, where both α_H and ω are very low, then $(2\alpha_H\omega - 1) < 0$ and the correlation becomes negative. In the intermediate cases, the correlation can turn positive or negative, depending on the relative sizes of both α_H and ω. Of course, because α_H cannot exceed 1, $(2\alpha_H\omega - 1)$ will be negative if ω, the trade elasticity parameter, is less than 0.5. For example, even if α_H = 0.95 and ω = 0.45, $(2\alpha_H\omega - 1)$ becomes negative (-0.145). Table 2.14 below summarizes the theoretical outcomes under the four potential, extreme scenarios outlined earlier in the chapter.

Table 2.14: Matrix Illustrating the Direction of Theoretical Correlations Between Relative Consumption and the Terms of Trade

	High degree of home bias *High* α_H	Low degree of home bias *Low* α_H
High trade elasticity *High* ω	Scenario A +	Scenario B +
Low trade elasticity *Low* ω	Scenario C –	Scenario D –

Source(s): Corsetti et al. (2008), author's calculations

Under scenario A and scenario C, the directional relationships between relative consumption, and the real exchange rate and terms of trade, respectively are essentially the same as before. In both cases, the consumer in the home country has a high degree of home bias, but the elasticity of substitution between tradable goods produced at home and abroad varies from high to low. In each scenario, a reduction in the prices of domestically produced goods depreciates the terms of trade and the real exchange rate, the latter because of higher home bias in consumption relative to the foreign consumer.

Under scenario A, a high trade elasticity of substitution generates greater consumption of domestically produced goods which more than offsets the negative income effects from a lower value of domestic output. Therefore, the real exchange rate, the terms of trade and consumption move together. In contrast, under scenario C, a low trade elasticity of substitution is unable to offset the negative income effects from a lower value of domestic output. Consumption of goods and services produced at home falls. Consequently, the term of trade (and the real exchange rate) and consumption move in opposite directions.

With scenario B and scenario D however, the relationship between the terms of trade and the real exchange rate turns negative. In each case, because of a low degree of home bias relative to that in the foreign country, lower domestic prices reduce the foreign consumer's price index by more than the domestic consumer's price index. As a result, the real exchange rate actually appreciates, although the home terms of trade depreciates. Therefore, the correlation between relative consumption and the home terms of trade takes the opposite sign to the correlation between relative consumption and the real exchange rate in both scenarios.

Empirical Specifications

The empirical analysis that follows will attempt to determine whether any of these theoretical outcomes holds in practice. Fortunately, equation 2.35 lends itself to be expressed more explicitly as a testable equation using panel data regression analysis. Again, replacing deviations away from equilibrium with changes in logged values and setting $(2\alpha_H\omega - 1)$ = $2(1-\alpha_F)\omega - 1 = \gamma$ yields the familiar expression:

$$\Delta c_{it} - \Delta c_{it}^{*} = \gamma \Delta \ln(\tau_{it}) + v_{it} \tag{2.36}$$

where $\ln(\tau_{it})$ replaces rer_{it}. All other variables and parameters are defined in the same way as earlier in the chapter. The sign and statistical significance of γ will depend on the value of the trade elasticity parameter ω and the degree of home bias α_H or alternatively the reliance on imports of goods and services α_F in domestic consumption.

Moreover, in line with Hess and Shin's (2010) finding that some consumers opt to share risks across borders and others do not, equation 2.36 is augmented with $(\Delta y_{it} - \Delta y_{it}^{*})$, the difference in the per capita income growth rates of the home and foreign countries. As before, a certain percentage of consumers θ completely shares risks across countries and the other percentage $1-\theta$ does not and instead consumes only from their income. This gives equation 2.37:

$$\Delta c_{it} - \Delta c_{it}^{*} = \theta\gamma\Delta\ln(\tau_{it}) + (1-\theta)(\Delta y_{it} - \Delta y_{it}^{*}) + v_{it} \tag{2.37}$$

And, expressing both equation 2.36 and equation 2.37 as testable equations under a panel data framework yields:

$$\Delta c_{it} - \Delta c_{it}^{*} = \beta_{i0} + \beta_1 \Delta \ln\left({P_F}_{it}/{P_H}_{it}\right) + v_{it} \tag{2.38}$$

$$\Delta c_{it} - \Delta c_{it}^{*} = \beta_{i0} + \beta_2 \Delta \ln\left({P_F}_{it}/{P_H}_{it}\right) + \beta_3(\Delta y_{it} - \Delta y_{it}^{*}) + v_{it} \tag{2.39}$$

where β_1, β_2 and β_3 are now equal to γ, $\theta\gamma$ and $(1-\theta)$, respectively, and of course $\ln\left({P_F}_{it}/{P_H}_{it}\right) = \ln(\tau_{it})$. As before, β_{i0} captures the possible country specific effect.

If β_1 and β_2 are both positive and statistically significant, this is consistent with scenario A and scenario B, where the trade elasticity parameter ω is high, no matter the value of the degree of home bias. Conversely, in the cases where both β_1 and β_2 are negative and statistically significant, these are consistent with scenario C and scenario D, where the trade elasticity parameter ω is low, regardless of the value of the degree of home bias. If the degree of home bias is close to or equal to 0.5 and the trade elasticity parameter ω is close to or equal to 1, then neither coefficient will probably be statistically significant from zero. Finally, as was the case earlier, if β_3 = 0, this suggests that θ = 1 and so all consumers completely share risks across countries. However, in the case where β_3 > 0, this implies that θ < 1, and indicates that 1 – θ percent of consumers consumes only out of their incomes.

Regression Results

What follows are the results from estimating equation 2.38 and equation 2.39 to determine whether the key parameters of interest, $\beta_1$and β_2, vary, based on the degree of home bias and the trade elasticity parameter. The analysis leverages data for 150 countries, using an unbalanced panel of data covering the period 1981 to 2018. The minimum number of years included in each cross section was 15 and the maximum number was 38.

Earlier analysis suggested that the trade elasticity parameter for the group of 150 countries included in the sample is low, and this is consistent across different subgroups of countries. Consequently, this narrows the focus of the analysis to scenario C and scenario D. In each case therefore, the relationship between relative consumption and the home terms of trade is expected to be negative, reflecting stronger, negative income effects relative to the substitution effects.

Having already established that each regressor (and the regressand) included in each equation is stationary, the estimations of equation 2.38 and equation 2.39 can proceed as is. Table 2.15 presents both fixed effects and random effects estimates of equation 2.38. Additionally, for comparison purposes only, equation 2.6 from the main body of the chapter is re-estimated using the unbalanced sample used to re-estimate equation 2.38. Moreover, in all cases, the real effective exchange rate is used as the definition of the real exchange rate in the analysis that follows.

The Hausman test statistic confirms that the random effects estimates are consistent and more efficient that those generated using the fixed effects estimator. Still, both the fixed effects and random effects estimates confirm the negative and statistically significant relationship between relative per capita consumption growth and changes in the terms of trade, as predicted by the analysis in Table 2.14. This is consistent with the hypothesis that, regardless of the degree of home bias, the correlation between relative per capita consumption growth and changes in the terms of trade will be negative for low values of the trade elasticity parameter. In contrast, the coefficient on the change in the real effective exchange rate is positive, although statistically insignificant. Again, for low values of the trade elasticity parameter, the coefficient on this variable can take either sign, depending on the degree of home bias.

Table 2.15: Fixed and Random Effects Linear Regression Estimates of Equation 2.38

Variable	**Equation 2.6 – Random Effects**	**Equation 2.38 – Fixed Effects**	**Equation 2.38 – Random Effects**
Δrer_{it}	0.015		
	(0.010)		
$\Delta \ln\left(P_{F\,it}/P_{H\,it}\right)$		-0.046**	-0.048**
		(0.019)	(0.019)
Constant	0.004**	0.003***	0.003**
	(0.002)	(0.000)	(0.002)
Observations	4,271	4,271	4,271
R-squared	0.002	0.004	0.004
Number of countries	150	150	150
Hausman Test Statistic		0.86	

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

The fixed effects and random effects results estimated from equation 2.39 support those produced from equation 2.38 (see Table 2.16). Specifically, the coefficients on $\Delta \ln\left(P_{F\,it}/P_{H\,it}\right)$ are again negative and statistically significant and confirm the negative relationship between relative per capita consumption growth and changes in the terms of trade, given low estimates of the trade elasticity parameter and regardless of the degree of home bias. Once more, for comparison purposes only, equation 2.7 from the main body of the chapter is re-estimated using the unbalanced sample used to re-estimate equation 2.39. The output from that regression confirms that, for the full sample of 150 countries of varying degrees of trade openness or home bias, there is no statistically significant relationship between relative per capita consumption growth and the real effective exchange rate. What is consistent however, whether the real effective exchange rate or the terms of trade is included in the regression, is that the share of consumers who share risks internationally is around 30%. Finally, on this occasion, the Hausman test rejects the null hypothesis that the random effects estimates of equation 2.39 are consistent and more efficient than the estimates produced from the fixed effects specification at the 10% level of statistical significance. Consequently, the analysis that follows uses the fixed effects estimator to produce estimates for equation 2.39.

Table 2.16: Fixed and Random Effects Linear Regression Estimates of Equation 2.39

Variable	**Equation 2.7 – Random Effects**	**Equation 2.39 – Fixed Effects**	**Equation 2.39 – Random Effects**
Δrer_{it}	0.011		
	(0.009)		
$\Delta y_{it} - \Delta y^*_{it}$	0.711***	0.690***	0.710***
	(0.076)	(0.083)	(0.075)
$\Delta \ln({P_F}_{it}/{P_H}_{it})$		-0.036**	-0.038**
		(0.016)	(0.016)
Constant	0.001	0.001**	0.001
	(0.001)	(0.000)	(0.001)
Observations	4,271	4,271	4,271
R-squared	0.177	0.178	0.178
Number of countries	150	150	150
Hausman Test Statistic		5.72*	

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Table 2.17 also suggests that the results outlined in Table 2.16 are largely consistent for countries that are ineligible for World Bank financing and for those that are eligible. The size of the coefficients on the change in the terms of trade is the same across both groups of countries, although a larger standard error (probably due to a smaller sample) renders the coefficient for the former group of countries statistically insignificant from zero. Where the results do differ however, is in the estimates for the share of consumers that share risks internationally. For the countries typically eligible for World Bank financing, the share of consumers that shares risks across borders remains about 30%. However, for all other countries, most of which are advanced economies, almost half of the consumers in these countries share risks internationally.

Table 2.17: Fixed Effects Linear Regression Estimates of Equation 2.39 – Split by World Bank Ineligible and Eligible Countries

Variable	Full Sample	World Bank Ineligible	World Bank Eligible
$\Delta \ln\left({P_F}_{it}/{P_H}_{it}\right)$	-0.036**	-0.036	-0.036**
	(0.016)	(0.029)	(0.017)
$\Delta y_{it} - \Delta y_{it}^{*}$	0.690***	0.535***	0.701***
	(0.083)	(0.120)	(0.087)
Constant	0.001**	0.001***	0.001
	(0.000)	(0.000)	(0.000)
Observations	4,271	816	3,455
R-squared	0.178	0.206	0.178
Number of countries	150	41	109

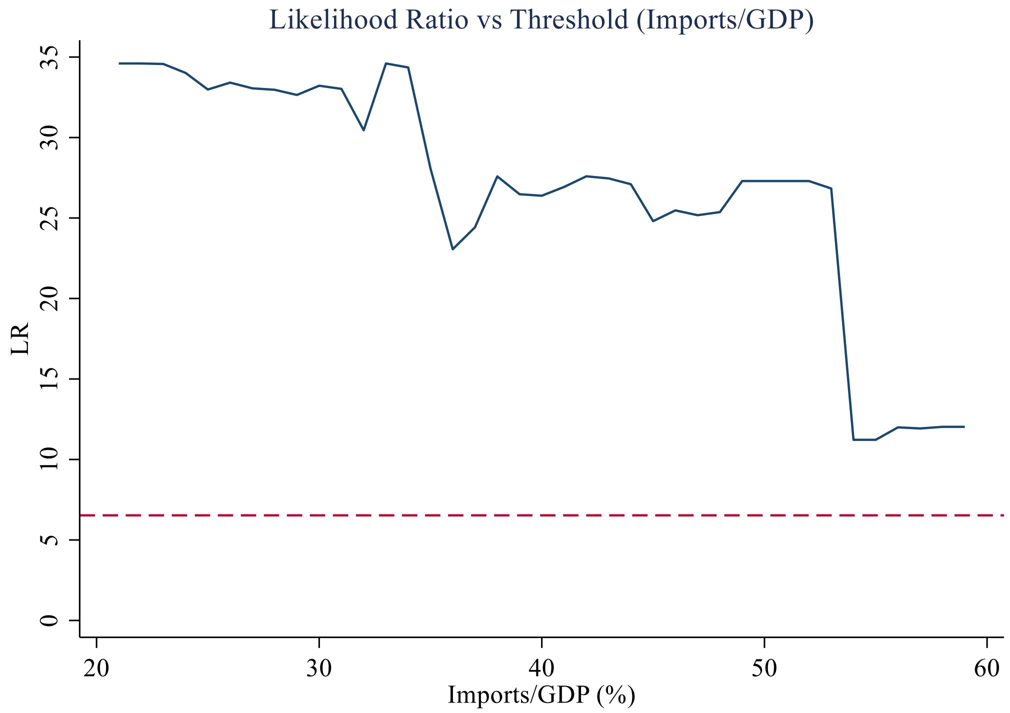
*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

What is perhaps most interesting to determine is whether any threshold exists beyond which the relationship between relative per capita consumption growth and changes in the terms of trade actually changes depending on countries' degrees of home bias or trade openness. To assess that, an alternative specification of equation 2.9, replacing the real effective exchange rate instead with the terms of trade, is estimated for different values of the threshold parameter $\emptyset$ (equation 2.40).

$$\Delta c_{it} - \Delta c_{it}^{*} = \beta_{i0} + \beta_{2\mathrm{a}} \Delta \ln\left({P_F}_{it}/{P_H}_{it}\right) I(x_i \leq \emptyset) + \beta_{2\mathrm{b}} \Delta \ln\left({P_F}_{it}/{P_H}_{it}\right) I(x_i > \emptyset) + \beta_3(\Delta y_{it} - \Delta y_{it}^{*}) + v_{it} \tag{2.40}$$

Again, the optimal threshold of imports of goods and services as a ratio of nominal gross domestic product is the one from the regression with the lowest residual sum of squares, provided that that threshold is found to be statistically significant from zero. In other words, this is the threshold that suggests that estimating equation 2.40 yields statistically more significant results than a regression which estimates equation 2.39. Each estimate of equation 2.40's likelihood ratio (the normalized residual sum of squares) is plotted versus its respective $\emptyset$ in Figure 2.8 below. As in the case where the real effective exchange rate is used as the independent variable, values of $\emptyset$ for which the likelihood ratio falls below the dotted red line are statistically significant thresholds with a 90% degree of confidence. In this case however, the likelihood ratio fails to fall below that line for any value of imports of goods and services as a ratio of nominal gross domestic product. This would suggest that, unlike with the real effective exchange rate, the negative relationship between changes in the terms of trade and relative per capita consumption growth for countries with low elasticities of substitution does not change in a statistically significant way for different degrees of trade openness or home bias.

Figure 2.8: Likelihood Ratio (LR) Plot of Equation 2.40 with Imports/Nominal Gross Domestic Product (%) as the Threshold Variable



Source(s):Author's calculations.

Overall, the results in this appendix provide additional evidence that supports the hypothesis that the relationship between relative per capita consumption growth and changes in the terms of trade or the real effective exchange rate depend on both the elasticity of substitution for tradable goods and the degree of home bias or dependence on imports for domestic consumption.

Chapter 3: Foreign Exchange Reserves, External Debt and Sovereign Bond Spreads

3.1 Introduction

It has been well documented that, since the 1990s, emerging markets and developing economies have accumulated substantial stocks of foreign exchange reserves to act as buffers against external shocks. More generally, foreign exchange reserves have several uses, with the International Monetary Fund (2013) noting that most managers of foreign exchange reserves globally opt to hold foreign exchange reserves for precautionary motives, while several also maintain foreign exchange reserves to manage exchange rate fluctuations in line with monetary policy objectives. In summary, the research reviewed to date has highlighted a critical role for foreign exchange reserves in reducing the cost of financial crises (see literature review for a more comprehensive discussion). Many authors have emphasized the importance of limiting sharp declines in output and subsequently absorption or consumption due to external shocks and support a role for foreign exchange reserves as a tool to minimise consumption volatility within a central bank's or a government's loss function.

However, beyond the benefits of foreign exchange reserves post-external shock or a sudden stop in capital inflows, several authors also note a role for foreign exchange reserves in reducing the probability of a sudden stop in capital inflows and consequently in reducing the chance of a crisis. Jeanne and Rancière (2011) and Kim (2017) are among those who include a role for foreign exchange reserves in restricting the chances of a sudden stop in their models of optimal accumulation of foreign exchange reserves (see the literature review for a more comprehensive list of studies). Moreover, Qian and Steiner (2017) note that greater foreign exchange reserve holdings may reduce a sovereign's cost of borrowing, flatten its yield curve, and encourage greater long-term borrowing. This increases the maturity of external debt, increases available foreign exchange buffers, and consequently reduces roll-over risk. This benefit of holding more foreign exchange reserves implies a role for countries to tap international debt markets to build foreign exchange reserves buffers and protect against both the probability and effects of externally driven shocks to the balance of payments. However, holding more external debt leaves sovereigns susceptible to enhanced default risk and could further increase interest rate spreads and roll-over risk. Thus, debt and foreign exchange reserves managers must strike a delicate balance between building foreign exchange reserve buffers and leaving the country exposed to additional risks.

Yet, while some research has studied the role of foreign exchange reserves in reducing the marginal cost of borrowing (see Levy Yeyati (2008) and Bianchi et al. (2018) for example), few identify whether this relationship varies with debt or foreign exchange reserves levels or whether it varies across different types of countries or economic structures. Levy Yeyati (2008) find that holding greater levels of foreign exchange reserves reduces bond spreads and this effect is even greater for markets with fixed exchange rates. He also illustrates that the cost of borrowing to build foreign exchange reserves is likely overstated given these favourable benefits to spreads. Similarly, leveraging data for Mexico, Bianchi et al. (2018) suggest that borrowing to accumulate foreign exchange reserves may actually increase bond spreads on external debt if the favourable effects of greater foreign exchange reserves on bond spreads do not offset the adverse effects of higher external debt on spreads. However, less focus is placed specifically on understanding how this phenomenon changes depending on an economy's level of external debt. This chapter seeks to provide initial evidence for whether heavily indebted countries benefit more or less from the accumulation of foreign exchange reserves, and whether the relationship varies by exchange rate regime. Specifically, the chapter attempts to shed some light on the following questions:

1. Does issuing external government debt to accumulate foreign exchange reserves reduce average interest rate spreads on external government debt?
2. If yes, above what level of external government debt as a share of nominal gross domestic product does this relationship fall away?
3. Does Levy Yeyati's (2008) finding that economies with less flexible exchange rate arrangements benefit more from the accumulation of foreign exchange reserves still hold once potential non-linear effects of foreign exchange reserves on sovereign bond spreads are accounted for?

The chapter contributes to the literature by building on work by Levy Yeyati (2008) and Bianchi et al. (2018) to test and identify varying non-linear relationships between bond spreads and foreign exchange reserves and external government debt, and to identify specific thresholds above which borrowing to bolster foreign exchange reserve holdings increases sovereign bond spreads and in turn roll-over risk.

The results confirm a common finding in the literature that foreign exchange reserves may reduce the spreads on sovereign debt, while rising external government debt also increases the sovereign risk premium that investors demand on foreign debt. Moreover, the potentially greater effect of foreign exchange reserves (compared to external government debt) on spreads at relatively low levels of debt may reduce (or at least not increase) the risk premium (on a net basis) if the government opts to issue external debt to build foreign exchange buffers. However, as governments accumulate substantially more external debt, the marginal benefit of these foreign exchange reserves to reducing a country's implied probability of default and ultimately sovereign risk premiums falls. For the sample of 28 countries in this study, borrowing to build foreign exchange reserves once external government debt is already above 33% of nominal gross domestic product may increase bond spreads and by extension the risk of rolling over upcoming debt. The results also suggest that economies with less flexible exchange rate arrangements stand to benefit more from the accumulation of foreign exchange reserves via external debt issuances relative to their counterparts with more flexible exchange rates. For the latter set of economies, investors appear more concerned with increases in external debt levels than in the stock of foreign exchange reserves held by the central bank.

The rest of the chapter is structured as follows: section 2 provides an overview of the literature on the role of the accumulation of foreign exchange reserves for precautionary motives, while section 3 highlights some motivating facts on the relationships between sovereign bond spreads on external government debt and country macroeconomic indicators. Section 4 briefly summarizes the theoretical framework mapping reserves, external debt and sovereign bond yields and presents the methodology and data used in the study. Section 5 presents and discusses the results, and section 6 concludes with policy implications and considerations, and future areas of research.

3.2 Literature Review

3.2.1 Motives for Holding Foreign Exchange Reserves

Examples of Holding Foreign Exchange Reserves for Exchange Rate Management

A substantial body of literature has long documented the theoretical and empirical importance of the accumulation of foreign exchange reserves by countries worldwide. The

International Monetary Fund (2015) highlights several uses for foreign exchange reserves, including to support a fixed exchange rate or transfer wealth across generations, among others. Foreign exchange reserves play a vital role in exchange rate management and monetary policy in emerging markets and developing economies. Central banks in emerging markets and developing economies worry about excessive exchange rate volatility and its cost to consumption and general economic activity. Calvo and Reinhart (2002) describe this as the "fear of floating" and suggest that many emerging market economies who have a strong commitment to inflation targeting and claim to have flexible exchange rates use monetary policy tools and the accumulation of foreign exchange reserves to limit the extent of exchange rate volatility (and subsequent pass-through to inflation) to an acceptable level. Ghosh, Ostry and Chamon (2015) suggest that "…emerging market economy central banks, including those with inflation-targeting frameworks, place a premium on exchange rate stability…." The authors advocate that foreign exchange market intervention and changes in interest rates should be used jointly to ensure that both objectives are met (Ghosh, Ostry and Chamon, 2015). Canova (2005) explains that the similarities in the response of output in floating exchange rate economies and non-floating rate economies in Latin America in response to economic shocks emanating from the United States of America may result from central banks in floating exchange rate economies using international reserves to minimise the volatility in their exchange rates. Aizenman and Riera-Crichton (2008) also illustrate that holding large buffers of foreign exchange reserves can reduce the effects of shocks to the terms of trade on exchange rates, while Aizenman et. al (2012) provide evidence that the active management of foreign exchange reserves in response to shocks to the commodity terms of trade in Latin American economies can especially help to support weakening currencies.

Small, open economies also desire exchange rate stability to contain inflation and minimize its impact on consumption and overall welfare. Worrell et al. (2018) emphasize the importance of exchange rate stability to small, open, financially integrated economies whose economies are characterized as having "…(a) high export concentration; (b) a limited range of competitive tradeable production, compared with import needs; and (c) a domestic financial system which is fully integrated into world financial markets..." (Worrell et al., 2018). Further, Worrell (2012) contends that, in a small, very open economy highly dependent on earnings of foreign exchange to facilitate imports of goods and services for most of its domestic consumption, a stable exchange rate supports a low inflation target "…because it does not aggravate the effects of imported inflation…" (Worrell et al., 2018). Active management and accumulation of foreign exchange reserves, and minimising exchange rate volatility may therefore ultimately be essential to reducing the negative effects of currency depreciation and volatility on consumption and overall welfare (Bahmani-Oskooee et al., 2015).

Precautionary versus Mercantilist Motive

Notwithstanding the aforementioned uses of foreign exchange reserves, the primary motives for accumulation have traditionally been segmented into two schools of thought – the mercantilist motive and the precautionary motive. Authors define the mercantilist motive as the byproduct of a policy to promote export competitiveness. Policymakers purchase foreign exchange and accumulate reserves to limit the degree of currency appreciation and encourage export-led economic growth (Aizenman and Lee, 2007). In contrast, the precautionary motive describes a deliberate strategy to build foreign exchange liquidity buffers as self-insurance in anticipation of external shocks (Aizenman and Lee, 2007). Bar-Ilan and Marion (2009) attempt to link the two motives to explain the accumulation of foreign exchange reserves in Asian economies. They believe that it makes little sense to separate the issues of the accumulation of foreign exchange reserves for insurance against shocks and the stabilization of output and/or inflation. They explain that the accumulation of foreign exchange reserves and exchange rate policy are linked, in that the level of foreign exchange reserves affects the level of the exchange rate policymakers choose and the exchange rate in turn influences the level of foreign exchange reserves. Targeting the exchange rate permits the central bank to achieve output and inflation targets via export-led growth, while the subsequent accumulation of foreign exchange reserves reduces the probability of a financial crisis and the associated loss in output. Still, policymakers must balance the need to maintain a weak currency to boost economic growth and the political pressure which naturally arises from a perceived undervalued exchange rate (Bar-Ilan and Marion, 2009).

While there still appears to be some debate in the literature about which motive provides the dominant explanation for the accumulation of foreign exchange reserves since the 1990s, several studies suggest a key role for the precautionary motive in most central banks over the last few decades. Aizenman and Lee (2007) tested the relative importance of each motive for the accumulation of foreign exchange reserves in 53 advanced, emerging and developing economies over the period 1980 to 2000 and found stronger evidence of the precautionary motive than the mercantilist motive in explaining rising foreign exchange reserves over that period. Further, they noted that "…existing patterns of growing trade openness and greater exposure to financial shocks by emerging markets go a long way towards accounting for the observed hoarding of international reserves…" (Aizenman and Lee, 2007). Similarly, the International Monetary Fund (2013) highlights that over 70% of country authorities surveyed identified "precautionary liquidity needs" as the key reason for accumulating foreign exchange reserves, while approximately 40% maintained foreign exchange reserves to manage the exchange rate. Bar-Ilan and Lederman (2007) and Kato et al. (2018) suggest that including foreign exchange reserves as one of the central bank's target variables may permit it to reduce the probabilities that the economy experiences financial and currency crises, respectively. Additionally, Shrestha and Semmler (2014) provide empirical evidence that suggests that, given their foreign exchange constraint and concerns about financial stability, central banks in five eastern and south-eastern Asian economies generally react more strongly to fluctuations in inflation and to foreign exchange reserves than they do to the real effective exchange rate, the foreign interest rate, and the output gap.

Economic Structure and Reserve Adequacy

However, the structure of an economy plays a key role in determining the nature of the shocks each country is susceptible to and the extent to which a specific volume of foreign exchange reserves is deemed adequate to insure against external shocks. For example, Moore and Glean (2016) employed a cost-benefit approach to estimate the appropriate level of foreign exchange reserve holdings for small states vulnerable to natural disasters and other external shocks to reduce output losses associated with a crisis. Considering that holding foreign exchange reserves also comes at an opportunity cost to policymakers, the authors estimated that, depending on the government's fiscal stance, the optimal level of foreign exchange reserve holdings could rise to as high as 25 weeks of imports of goods and services, more than double that of the global rule of thumb of 12 weeks of imports of goods and services. Most notably, the actual level of foreign exchange reserves required depended on the structure and overall policy framework within those economies (Moore and Glean, 2016). Further, Crispolti's (2018) study of small states illustrated that "…the effectiveness of international reserves as a buffer against external shocks depends on the type of shock that is experienced as well as on the structural characteristics of the economy…" (Crispolti, 2018). In fact, he found that small states with fixed exchange rates tended to hold fewer foreign exchange reserve buffers than their floating-rate counterparts (Crispolti, 2018), a result somewhat consistent with Bar-Ilan and Marion (2009) who illustrated that commitment to a fixed exchange rate reduces the level of foreign exchange reserves required to protect against future crises. The International Monetary Fund (2015), in its third of three reports which guide the assessment of reserve adequacy in its member countries, focused on the need to hold precautionary reserves for three types of countries – mature (advanced), countries with global financial market access (typically emerging markets), and countries with limited global financial market access (typically low-income or developing countries). Mature or advanced economies tend to hold foreign exchange reserves to reduce the probability of foreign exchange shortages in the domestic economy. Emerging economies with financial market access worry about mitigating crises emanating from current account or (more particularly) financial account shocks including currency crises and sudden stops to capital inflows. However, low-income countries are concerned with protecting domestic absorption against shocks to the external current account (International Monetary Fund, 2015). While the study emphasized the use of the import coverage ratio as an appropriate method to determine low-income economies' resilience to current account shocks, the International Monetary Fund proposed a revised reserve adequacy metric for emerging markets with financial access. This metric sought to ensure that emerging markets build adequate protection against:

1. Terms of trade shocks which may lead to volatile export revenues,
2. Potential capital flight by residents,
3. Roll-over risk of short-term, external debt, and
4. Other sudden stops or reversals in capital inflows, particularly from previously built-up liabilities.

The International Monetary Fund (2015) weighed each vulnerability to capture its relative importance to emerging markets and determined that roll-over risk and the risk of sudden stops from other financial account liabilities represented the greatest risks to this segment of economies. The recent rise in emerging markets' financial development and openness and their attractiveness as markets for foreign investment (Qian and Steiner, 2017), and the volatility of emerging market interest rates which move counter to the business cycle (Neumeyer and Perri, 2005) appear to validate this determination. Further, Worrell et al. (2018) noted that financial flows, and not trade, dominate the foreign exchange market in small, financially integrated economies in the short run.

3.2.2 Sudden Stops and Foreign Exchange Reserves

The Costs of Sudden Stops and Default

Given the increased dependence of emerging markets on foreign debt funding, a sudden stop to capital inflows or the inability to roll-over upcoming, short-term maturities may increase the probability of sovereign default. This of course comes at a cost. Mendoza and Yue (2012) leveraged the Eaton and Gersovitz (1981) model of default to investigate the effects of sovereign default on external debt on countries' and ultimately firms' access to credit markets and the impact on production. The authors assumed that default increases the cost of firms' access to foreign working capital and forces them to substitute previously imported intermediate inputs for domestic inputs which are imperfect substitutes. The latter's lower productivity leads to output loss for the firm and the country. Further, Na et al. (2018) posit that the decline in output leading up to sovereign defaults may or may not lead to substantial declines in employment, but this depends on the nature of a country's exchange rate regime. Policymakers who are willing and able to adjust their exchange rate to prevent the surge in unemployment which may accompany the fall in output due to downward nominal wage rigidity, may devalue the domestic currency (by at least 35% in their study) to reduce the extent of real exchange rate overvaluation, reduce real wages and keep employment stable (Na et al., 2018). Alternatively, no currency adjustment led to a 20-percentage point rise in unemployment in their model. Ultimately, Mendoza and Yue's (2012) research implies that being more open or relying on external finance to fund imported inputs exacerbates the consequences to sudden stops or loss in capital market access. Thus, open economies without available domestic substitutes are likely to experience greater output loss at the time of default. These countries will default less (at higher debt levels) because they recognize the cost of doing so is much higher than for their less open counterparts (Mendoza and Yue, 2012). Further, Mendoza and Yue (2012) illustrated that exclusion from external markets reduces the capacity for the government to borrow to smooth consumption when output declines compared to borrowing during good times to finance greater consumption. However, throughout the study, the authors assume that the country does not accumulate foreign savings before a default to permit it to draw down on those funds during the period when it is subsequently excluded from global credit markets. This implies that countries cannot build foreign exchange reserves to act as buffers during a future crisis and runs counter to the trends witnessed in emerging markets and developing economies since the 1990s. However, this is likely a function of the Eaton and Gersovitz (1981) assumption that debt matures in one period (see also Bianchi et al. (2018) for the implications of this assumption on required holdings of foreign exchange reserves).

This susceptibility to external shocks and dependence on imports of goods and services create an excessive consumption volatility relative to output volatility (Kodama, 2013). Kodama (2013) illustrates that, notwithstanding access to global financial markets, shocks which disturb developing economies' ability to finance imports of goods and services may directly impact consumption even more so than income. This is because many developing economies depend heavily on imports of goods and services for domestic consumption. Further, the author points out that these economies typically suffer from "…a volatile terms of trade, a volatile borrowing interest rate, the acceptance of aid, and a monocultural economy…" (Kodama, 2013). In the final analysis, Kodama (2013) illustrates that his Dynamic Stochastic General Equilibrium (DSGE) model which appropriately accounts for the characteristics of the small, open, low-income economy can explain 79% of the difference in consumption volatility between Kenya (his sample low-income economy) and Canada (his proxy for a larger, industrialised economy). Again however, like Mendoza and Yue (2012), the authors abstract from the presence of foreign exchange reserves as potential buffers against external shocks.

Foreign Exchange Reserves as Buffers Against Sudden Stops

Several studies have since incorporated a role for foreign exchange reserves in providing a buffer against sudden stops and in reducing the probability of a sudden stop associated with investors choosing not to roll over short-term or maturing debt. Levy Yeyati (2008) emphasize the effects which greater foreign exchange reserves may have on interest rate spreads on existing external debt (see also Tebaldi et al. (2018) who include official reserves in their model of emerging markets' spreads' determinants). He illustrates empirically that, while the accumulation of foreign exchange reserves carries an opportunity cost (since the cost of borrowing or yield from foregone investment usually exceeds the yield from risk-free assets in which foreign exchange reserves are typically invested), greater holdings of foreign exchange reserves actually reduce the spreads on existing debt, especially for fixed exchange rate economies (Levy Yeyati, 2008). In fact, his regression results suggest that the absolute value of the coefficient on foreign exchange reserves may exceed that on sovereign debt, and he notes that the marginal cost of accumulating foreign exchange reserves may be overstated by over 50% if the effects of greater foreign exchange reserves on bond spreads are not accounted for (Levy Yeyati, 2008).

Jeanne and Rancière (2011) develop a small, open economy model where consumers risk losing access to external borrowing markets and may choose to hold foreign exchange reserves relative to short-term debt to insure against losses in consumption arising from a sudden stop to capital inflows. Their model derives an expression for the level of foreign exchange reserves which maximises the consumer's welfare where optimal foreign exchange reserve holdings are positively related to the likelihood, size and output cost of a sudden stop episode and the risk aversion parameter, and negatively related to the cost of accumulating foreign exchange reserves (Jeanne and Rancière, 2011). The authors find that their calibrated model can replicate the average level of foreign exchange reserves relative to nominal gross domestic product for Latin American economies, but they fail to reach the level of foreign exchange reserves accumulated by Asian economies (Jeanne and Rancière, 2011). Solving the latter discrepancy requires an assumption of greater output costs arising from sudden stops and a significant rise in the risk aversion parameter, both of which may arise from the actual experience and lingering fears of East Asian economies coming out of the late-1990s financial crisis (Jeanne and Rancière, 2011). However, it does not explain why China has accumulated the magnitude of foreign exchange reserves it has over the past two decades.

Jeanne and Rancière (2011) also augment their insurance-against-sudden-stops model to allow the probability of a sudden stop to depend negatively on the ratio of foreign exchange reserves to short-term debt to capture the role of foreign exchange reserves in displaying confidence in the economy. While they find that this additional benefit theoretically increases the optimal level of foreign exchange reserves desired by policymakers, Jeanne and Rancière (2011) alternatively find no empirical evidence that foreign exchange reserves reduce the probability of a crisis. Instead, the level of public indebtedness, degree of real exchange rate overvaluation and the degree of financial openness to foreign inflows materially affect a country's probability of experiencing a sudden stop in capital inflows (Jeanne and Rancière, 2011). However, Prabheesh (2013) finds that higher holdings of foreign exchange reserves reduce the probability of sovereign default, which in turn improves the country's credit rating, reduces the cost of borrowing, and maintains access to international capital markets. Using India as an example, Prabheesh (2013) illustrates that the inverse of foreign exchange reserves as a ratio to short-term debt, the size of the government's fiscal deficit, and the "…volatility of foreign institutional investment…" (Prabheesh, 2013) are all significant determinants of that country's sovereign risk premium.

Hur and Kondo (2016) (as do other authors in this strand of literature) leverage the popular Diamond and Dybvig (1983) bank run model to present a framework for determining the optimal level of foreign exchange reserves relative to external debt in the presence of roll-over risk. In this model, investors borrow to finance investment in a technology which yields a certain level of output. However, if production is stalled, the investment may be liquidated, and yields returns less than the uninterrupted value of output. A sudden stop in capital inflows may force pre-mature liquidation and an undesired reduction in output (Hur and Kondo, 2016). Thus, like in Jeanne and Rancière (2011), foreign exchange reserves carry a dual role in their model: they act as a liquidity buffer during a sudden stop but may also reduce investors' probability of not rolling over maturing debt (akin to the effects of deposit insurance on the likelihood of and in the presence of a bank run). Hur and Kondo (2016) endogenize the accumulation of foreign exchange reserves and the occurrence of sudden stops and permit governments to learn of liquidity shocks from each other. They find that global liquidity shocks may generate roll-over risks and produce substantially higher episodes of sudden stops (initially). Policymakers respond by increasing foreign exchange reserves, which reduces the probabilities of crises thereafter (Hur and Kondo, 2016). However, the slower policymakers learn about global liquidity shocks and the increased roll-over risk, the greater the likelihood that policymakers are underinvested in foreign exchange reserves and the greater the chances for sudden stops initially. Countries may in fact learn more slowly if their learning is restricted to the liquidity shocks occurring within their region and not necessarily shocks occurring globally (Hur and Kondo, 2016). Finally, Hur and Kondo (2016) suggest that, since each country's optimal response to higher roll-over risk is to hold more foreign exchange reserves, individual countries tend to hold more foreign exchange reserves than if they opted to pool foreign exchange reserves and share risks. This assumes of course that liquidity shocks to various countries are not perfectly, positively correlated. They note that the International Monetary Fund could potentially act as an option to provide liquidity in times of crisis and reduce the buildup of foreign exchange reserves, but the stigma associated with the International Monetary Fund and their previous programmes may discourage some countries from relying on their assistance and hence promote overinvestment in foreign exchange reserves (Hur and Kondo, 2016).

3.2.3 Joint Determination of External Debt and Foreign Exchange Reserves

While Hur and Kondo (2016) endogenize both foreign exchange reserves and sudden stops, they take the level of external debt as given. In fact, the results presented thus far would seem to imply that, to completely remove roll-over risk, countries should use excess foreign exchange reserves over external debt to pay down outstanding liabilities. Further, the higher-risk nature of emerging markets' debt relative to the safe, liquid assets which comprise many countries' foreign exchange reserve holdings imply an opportunity cost of the accumulation of foreign exchange reserves via external debt. However, empirically, many countries globally choose to hold both external debt and foreign exchange reserves (Kim, 2017), and oftentimes the levels of foreign exchange reserves exceed overall external indebtedness. Kim (2017) also emphasizes that foreign exchange reserves help to reduce both the chances and costs of sudden stops to capital inflows, but their framework also attempts to jointly explain the ratios of foreign exchange reserves and external debt to nominal gross domestic product evidenced in emerging markets. While the model fails to individually explain the levels of foreign exchange reserves and external debt in the sample of developing countries (implying that countries accumulate foreign exchange reserves other than just for precautionary reasons), Kim (2017) points out that the assumption of limited enforcement and the inclusion of default risk are necessary to produce the result of joint holdings of foreign exchange reserves and external debt (Kim, 2017). Holding both external debt and foreign exchange reserves provides the option of default, which would otherwise not be available if governments repaid outstanding external debt with foreign exchange reserves. The latter option is preferred if the cost of a sudden stop is too large, but otherwise, defaulting on external debt during a sudden stop permits the government to "… transfer resources to default states…" (Kim, 2017) and use foreign exchange reserves to smooth consumption (Kim, 2017). However, Kim (2017) does note that while many authors assume an empirically proven role for foreign exchange reserves in reducing the probability and costs associated with a sudden stop, "…their micro-foundation remains to be understood in future studies…" (Kim, 2017).

Most studies to date have also failed to appropriately account for the effects of maturity structure in models of foreign exchange reserve accumulation. Qian and Steiner (2017) investigate the effects of foreign exchange reserves on a country's yield curve's term structure and ultimately the maturity of external debt. While foreign exchange reserves are taken as exogenous, countries' concerns about current and future probabilities of experiencing financial crises prompt governments to hold foreign exchange reserves against these risks. However, foreign exchange reserves are found to both reduce and flatten the government's yield curve, thereby lowering the relative cost of issuing long-term external debt. A government's subsequent bias toward long-term borrowing to build more foreign exchange reserves increases the buffers available during a sudden stop and reduces the ratio of short-term debt relative to long-term debt. The dual effects on numerator and denominator reinforce the effects of foreign exchange reserves on financial stability as measured by the ratio of foreign exchange reserves to short-term debt (Qian and Steiner, 2017). However, quite interestingly, their panel vector-autoregression (VAR) and variance decomposition find virtually no evidence of the share of private and public long-term debt impacting the level of foreign exchange reserves (Qian and Steiner, 2017). Finally, as with Kim (2017), Qian and Steiner (2017) also suggest that countries hold both foreign exchange reserves and external debt pre-default to later permit consumption smoothing when they have defaulted on external debt.

Additionally, Bianchi et al. (2018) seek to determine the optimal level of foreign exchange reserves for emerging markets in the presence of rollover risk from the probability of a sudden stop. Although they do not formally model the government's debt maturity structures in their variant of the Eaton and Gersovitz (1981) model of sovereign default, their results depend on an assumption that the maturity of external debt exceeds one year (Bianchi et al., 2018). Bianchi et al. (2018) highlight the tradeoff that governments face in using external debt to accumulate foreign exchange reserves. In their model, a greater stock of foreign exchange reserves reduces roll-over risk, but (unlike in other studies) incurring more external debt to build foreign exchange reserves actually increases the government's cost of borrowing. They find that the level of external debt and foreign exchange reserves are increasing in the country's level of income but falling with creditors' aversion to risk. During high income periods, governments should incur external debt and build foreign exchange reserves, but once negative income shocks occur, they use foreign exchange reserves to repay external debt (Bianchi et al., 2018). Similarly, an increase in investors' risk aversion increases borrowing costs and reduces the incentive to borrow at higher rates. Governments therefore use foreign exchange reserves to meet maturities of external debt (Bianchi et al., 2018). Hence, borrowing to hold more foreign exchange reserves in good times shifts resources to tough times and reduces future consumption volatility. Finally, Bianchi et al.'s (2018) sensitivity analysis highlights two key results:

1. If overall debt maturity is 1 year or less (in line with the maturity of the risk-free asset for the accumulation of foreign exchange reserves) the required level of foreign exchange reserves falls to almost zero. This implies that governments do not benefit from lower rollover risk as a result of the accumulation of foreign exchange reserves if all external debt matures in the same period as the foreign exchange reserve assets do (Bianchi et al., 2018), and
2. Assuming a greater loss of income from default reduces the required level of foreign exchange reserves (Bianchi et al., 2018) as the government will choose to hold less external debt since the cost of defaulting on that debt is more prohibitive.

Finally, because countries default when foreign exchange reserves are already low and the paper's model does not account for post-default implications, Bianchi et al. (2018) find that an increase in the length of time excluded from financial markets after default does not have a material impact on optimal holdings of foreign exchange reserves. However, Aizenman and Sun (2012) suggest that uncertainty surrounding the duration of a crisis and the chances that the substantial draw down of foreign exchange reserves today increases the probability of future crises (even though it averts a crisis today) jointly create a “fear of losing reserves”. They documented that immediately upon the onset of the global financial crisis, some emerging markets initially allowed their foreign exchange reserves to fall in response to the crisis, primarily to pay short-term external debts. However, as the crisis continued, they slowed the depletion of foreign exchange reserves and instead allowed the exchange rate to adjust (Aizenman and Sun, 2012). Their short model with adjustment costs explains that central banks prefer to smooth the adjustment over multiple periods rather than respond aggressively initially and then lose substantial output or consumption in the latter periods. However, as Na et al. (2018) highlighted, the inability or unwillingness to use devaluation as an adjustment tool during a crisis could substantially increase domestic unemployment.

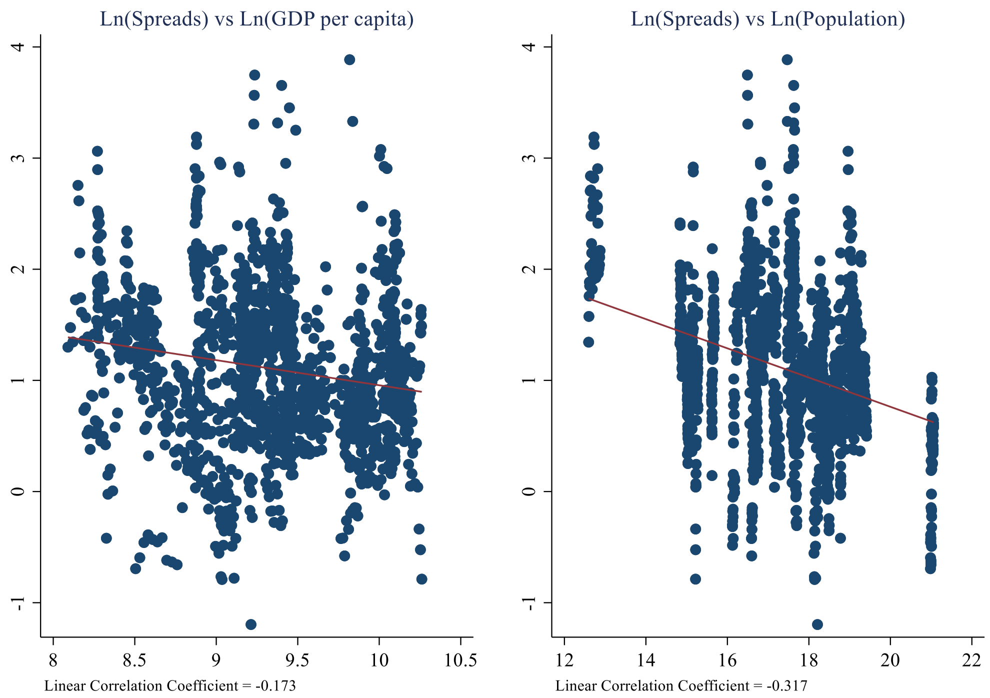
3.2.4 Unresolved Issues

Notwithstanding the substantial literature reviewed to date, several questions and areas for advancement remain. For one, as Bianchi et al. (2018), Qian and Steiner (2017) and others illustrate, paying closer attention to the maturity of external debt can substantially influence the recommended level of foreign exchange reserves that countries may hold against rollover risk and sudden stops. Further, the role of fixed exchange rates (and by extension the degree of exchange rate flexibility) in determining whether economies should hold more or less foreign exchange reserves or can sustain more or less debt than ‘floaters’ remains unclear in the literature. Crispolti (2018) and Bar-Ilan and Marion (2009) imply empirically and theoretically, respectively that emerging markets and developing economies with fixed exchange rates may hold less foreign exchange reserves than emerging markets and developing economies with more flexible exchange rate regimes. However, the International Monetary Fund (2015) implies that lower-income countries without the flexibility of a floating rate to act as a shock absorber should hold more foreign exchange reserves, and they also apply greater weights to fixed exchange rate economies in their reserves adequacy metric for emerging markets with financial market access. Additionally, despite fewer empirical examples of sovereign defaults in fixed exchange rate economies, Na et al. (2018) indicate that these economies have a greater incentive to default and thus pay a higher spread on sovereign debt. This is because default unlocks resources to aid in post-default economic recovery and limits the surge in unemployment associated with external shocks. Jahjah et al. (2013) also illustrate empirically that countries with fixed exchange rates pay higher bond spreads and issue less debt than their floating-rate counterparts. This reduces the level of sovereign debt that these economies hold relative to their floating-rate partners (assuming neither category of economy has access to bailout resources), but also helps to explain why their frequency of default is lower (Na et al., 2018). While this may suggest that fixed exchange rate economies require less foreign exchange reserves against their lower levels of debt, Levy Yeyati's (2008) earlier finding that fixed exchange rate economies benefit more from the impact of foreign exchange reserves on sovereign debt spreads and should therefore hold more foreign exchange reserves than their floating counterparts may run counter to this. Hence, the distinction between fixed and floating exchange rate economies' determinations of the optimal holdings and the accumulation of foreign exchange reserves and external government debt, respectively remains a required area of investigation in this field of research.

3.3 Motivating Facts

The charts and stylized facts in this section illustrate some of the motivating factors for the topic studied in this chapter. Figure 3.1 below illustrates that among the emerging markets and developing countries included in this study, all countries have paid positive spreads on their external government debts over the sample period (2005Q1 – 2019Q4), and smaller, less wealthy countries tend to pay higher spreads than their larger, wealthier peers. Thus, there is a direct cost to the accumulation of external government debt for emerging markets and developing economies, and an opportunity cost if these proceeds are used to invest in lower-yielding foreign exchange reserves.

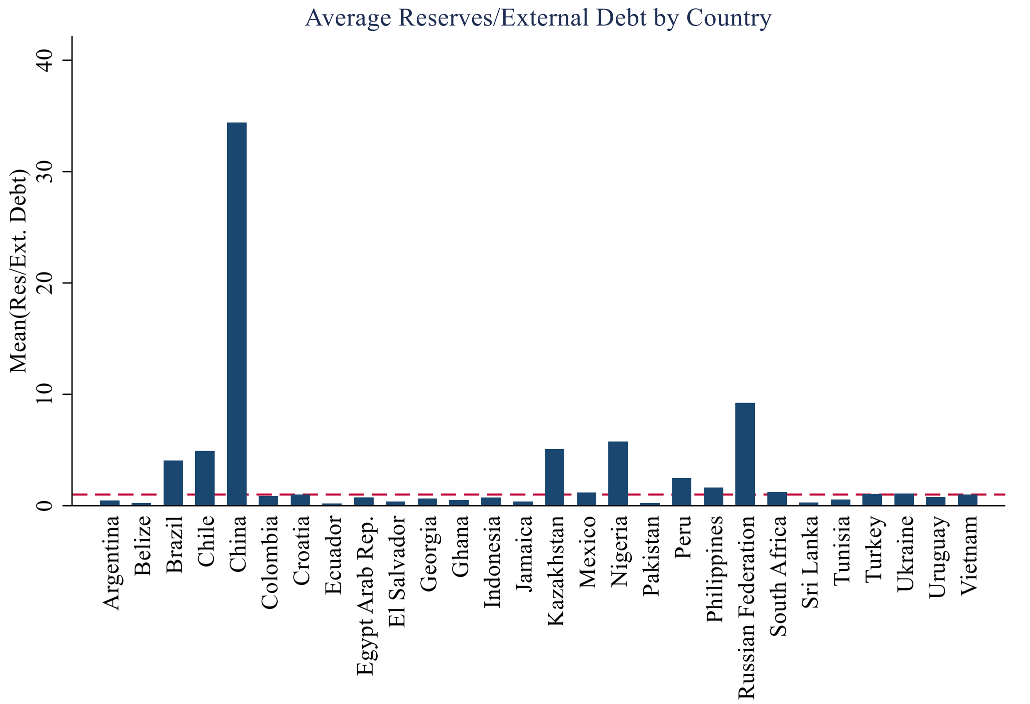
Figure 3.1: Relationships between Natural Log of Emerging Market Bond Index Global (EMBIG) Spreads (y-axis) and ln(Gross Domestic Product per Capita) and ln(Population) (x-axis) – (2005Q1 – 2019Q4)

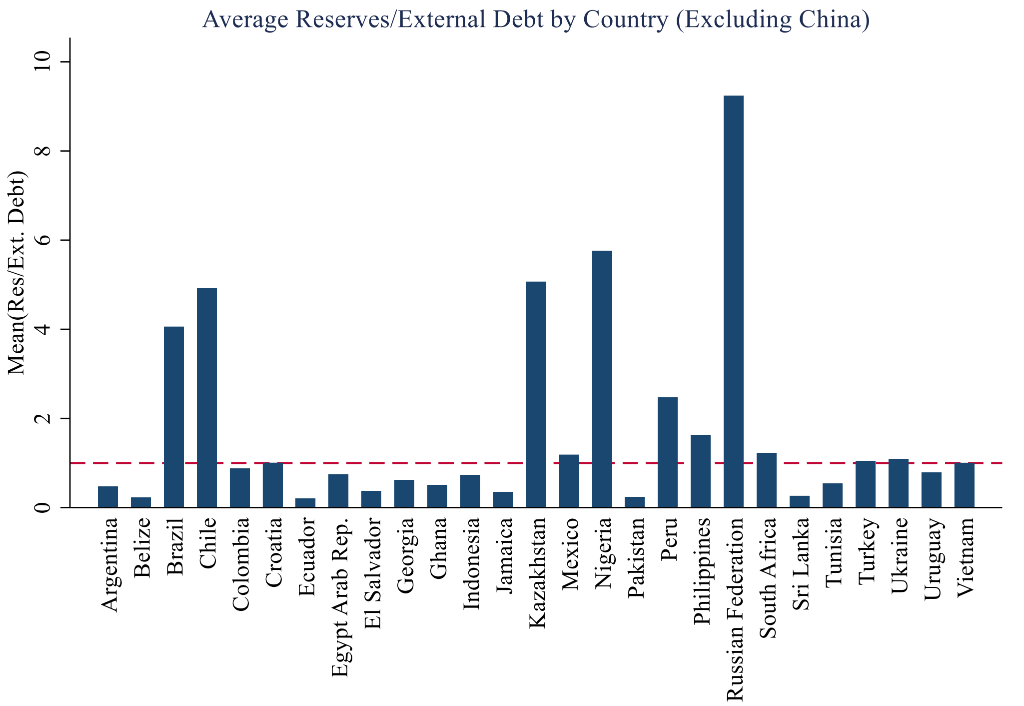


Source(s): JP Morgan, International Monetary Fund, World Bank, author's calculations

However, Figure 3.2 below suggests that, as outlined in the literature, countries still opt to hold both external government debt and foreign exchange reserves, notwithstanding the opportunity cost of doing so. All countries in the sample hold both external government debt and foreign exchange reserves, with 14 of 28 holding at least as much foreign exchange reserves as the stock of external government debt on average (as denoted by the blue bars surpassing the dotted red line at 1). The other countries, however, hold less foreign exchange reserves than external government debt. While this implies some benefit of holding both external government debt and foreign exchange reserves, is there some level of either, above which the positive effects of the accumulation of foreign exchange reserves fade away? Moreover, do these results vary by exchange rate regime?

Figure 3.2: Average Foreign Exchange Reserves/External Government Debt by Country (2005Q1 – 2019Q4)

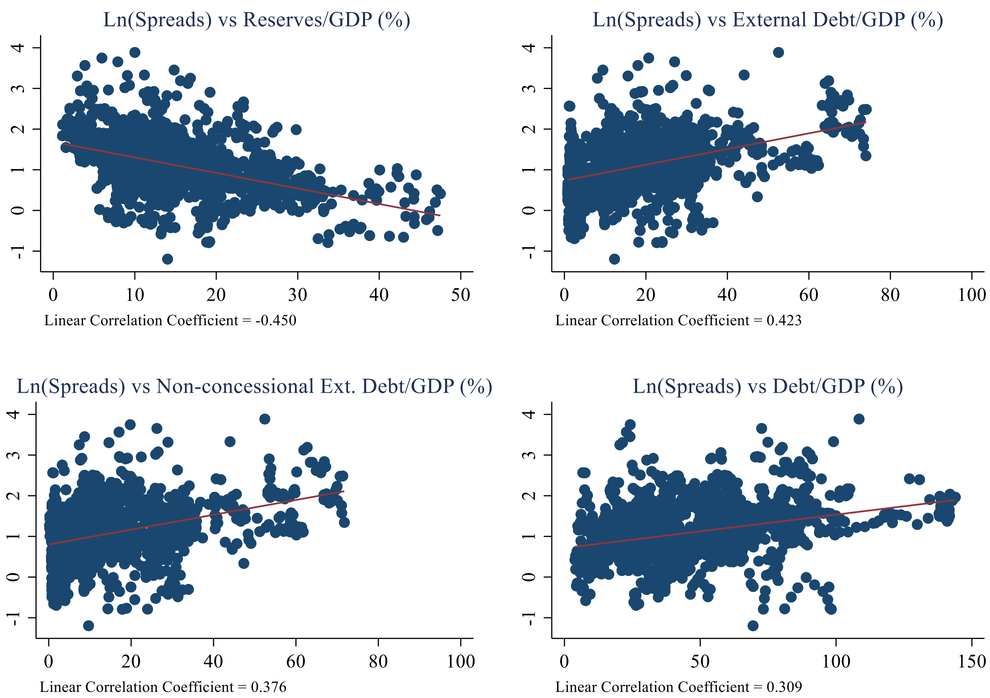




Source(s): International Monetary Fund, World Bank, author's calculations

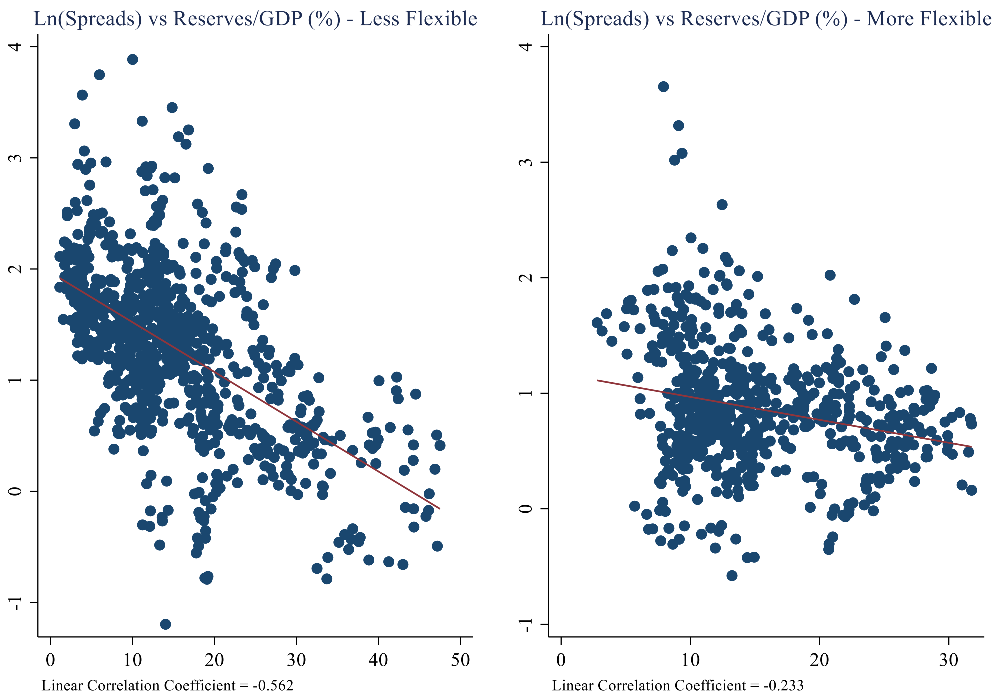
Figures 3.3 and 3.4 attempt to shed some light on the latter hypothesis. The chart in the first quadrant of Figure 3.3 posits a negative, linear relationship between ln(spreads) and foreign exchange reserves as a ratio of nominal gross domestic product (with a correlation coefficient of -0.450) as outlined in much of the literature, while the remaining three charts depict positive relationships between ln(spreads) and the ratios of external government debt, non-concessional external government debt and total gross government debt to nominal gross domestic product, respectively. However, Figure 3.4 suggests that the strength of the relationship between ln(spreads) and foreign exchange reserves as a ratio of nominal gross domestic product may vary across exchange rate regimes. For countries who Ilzetzki et al. (2016) deem to have started the period with an exchange rate regime rated 8 ("de facto crawling band that is narrower than or equal to +/-2%") or below (that is, less flexible exchange rates), bond spreads appear more negatively related to foreign exchange reserves than in the cases where the exchange rate regime is more flexible (rated 9 or higher) (see Appendix 3.A.1 for a description of Ilzetzki et al.'s (2016) exchange rate arrangement classification).

Figure 3.3: Relationships between Emerging Market Bond Index Global Spreads (y-axis), and Debt and Foreign Exchange Reserves as a Ratio of Nominal Gross Domestic Product (x-axis)



Source(s): JP Morgan, International Monetary Fund, World Bank, author's calculations.

Figure 3.4: Relationships between Emerging Market Bond Index Global Spreads (y-axis) and Foreign Exchange Reserves as a Ratio of Nominal Gross Domestic Product (x-axis) by Degree of Exchange Rate Flexibility



Left Panel – less flexible exchange rate regimes; Right Panel – more flexible exchange rate regimes.
Source(s): JP Morgan, International Monetary Fund, Ilzetzki et al. (2016), author's calculations.

The graphical estimates provided by Bianchi et al. (2018) in Figure 9 of their paper and Levy Yeyati's (2008) econometric findings also suggest that non-linearities and heterogeneities are prevalent in the relationship between bond spreads and foreign exchange reserves. The former authors illustrate that, as a country borrows to build its stock of foreign exchange reserves, the (in their case) rise in bond spreads becomes more pronounced with higher levels of external debt and foreign exchange reserves. This suggests that, as both external debt and foreign exchange reserves levels rise, the marginal benefit of higher foreign exchange reserves on bond spreads is unable to keep pace with the negative fallout of higher external debt on bond spreads. Levy Yeyati (2008) on the other hand find a stronger relationship between bond spreads and foreign exchange reserves for economies with pegged exchange rates versus those without. These graphical findings and those in the literature suggest some non-linearity and differentiation in the relationship between foreign exchange reserves and bond spreads and inform the choice of model specification in the empirical section to follow. In this way, the model and estimations in this chapter divert from Levy Yeyati (2008) and Tebaldi et al. (2018).

In summary, while countries find it optimal to hold both external debt and foreign exchange reserves simultaneously, some threshold may exist above which accumulating further foreign exchange reserves with a rising external debt stock yields little or no benefit. Further, competing incentives by national governments and monetary authorities may mean that these relationships may differ depending on a country's preferred exchange rate regime.

3.4 Methodology and Data

3.4.1 Econometric Formulation

The empirical specification employed in this chapter leverages the early work of Feder and Just (1977) and later Edwards (1984, 1986). Appendix 3.A.2 offers a simple explanation of Feder and Just's (1977) original theoretical framework where the authors assume that a bank in the Eurodollar market negotiates the terms of a future loan with a potential borrower. In this framework, the bank (the creditor) assigns some probability of default, *P(X)* to the government (the debtor), where *X* denotes a "…vector of economic indicators related to that probability…" (Feder and Just, 1977).

This framework assumes that borrowing countries acquire debt through bank loans rather than sovereign bonds (which are the subject of this chapter). Edwards (1986) highlights structural differences between the markets for each instrument but finds that some of the major theoretical determinants of sovereign default are key factors which explain interest rate spreads on both developing countries' loans and bonds, even if the empirical sensitivities of these relationships vary by instrument and macroeconomic variable. Moreover, Feder and Just's (1977) model is still useful as a general framework to characterize the relationship between the spreads that countries pay on loans or bonds, the probability of default on either loans or bonds, and the macroeconomic fundamentals that influence these, whatever those fundamentals may be. The derivations from Bianchi et al. (2018) and Levy Yeyati (2008) in appendices 3.A.3 and 3.A.4 complement this work and go further to show how the probability of default on bonds and specific macroeconomic fundamentals are related.

Feder and Just (1977) illustrate that the interest rate spread on loans (r) can be expressed as a function of the elasticity of demand for loans (η), the loss given default (h), the size of the loan (L), and θ, which depends, in part, on the cost of capital (which depends, in part, on the risk-free interest rate):

$$r = \frac{\eta}{\eta-1}\frac{1}{\theta}\frac{P(X)}{1-P(X)}h\frac{U'\{-hL\}}{U'\{rL\theta\}} \qquad (3.1)$$

where $\bar{h} = \int_{\underline{h}}^{1} h\psi(h)dh = E(h)$.

Feder and Just (1977) therefore seek to express the structural equation 3.1 as a testable econometric equation. To do this, they leverage the logistic form to express the probability of a debtor's default, where:

$$P(X) = \{1 + \exp(\beta_0 + \sum_{j=1}^{J} \beta_j x_j)\}^{-1} \exp(\beta_0 + \sum_{j=1}^{J} \beta_j x_j)$$

and j is the number of explanatory economic variables denoted x.[10]

This expression can further simplify to:

$$ln\left(\frac{P}{1-P}\right) = \beta_0 + \sum_{j=1}^{J} \beta_j x_j$$

and thus, equation 3.1 can be rewritten as:

$$ln(r) = \beta_0 + \sum_{j=1}^{J} \beta_j x_j - ln\theta + ln\left(\frac{\eta}{\eta-1}\right) + ln\left[\int_{\underline{h}}^{1} \frac{U'\{-hL\}h\psi(h)dh}{U'\{rL\theta\}}\right] \quad (3.2)$$

Here, $ln\left(\frac{\eta}{\eta-1}\right)$ is unobservable but can probably be captured using country (u_i) and/or time (v_t) specific effects. Finally, $ln\left[\int_{\underline{h}}^{1} \frac{U'\{-hL\}h\psi(h)dh}{U'\{rL\theta\}}\right] \approx ln\bar{h}$ and is immaterial assuming that the loan accounts for just a small percentage of the bank's net worth.

Therefore, the final model may generally be expressed as:

$$lnr_{it} = \beta_0 + \sum_{j=1}^{J} \beta_j x_{ijt} - ln\theta_t + u_i + v_t + w_{it} \quad (3.3)$$

where w_{it} is the standard, white noise error term, i denotes the country or borrower and t denotes the time period. Therefore, excluding unobservable effects, the interest rate spread in this framework is a function of macroeconomic fundamentals and the cost of capital.

3.4.2 Data

Bianchi et al. (2018) articulate the relationships between a country's external debt, its stock of foreign exchange reserves and spreads on sovereign bonds within a framework in which the government chooses optimal levels of external debt and foreign exchange reserves given investors' concerns about default risk (see Appendix 3.A.3 for detailed derivations). The authors' analysis suggests that the net benefits of issuing external debt to accumulate foreign

[10] Edwards (1986) also uses the same standard convention to express the relationship between the probability of default on foreign debt (loans or bonds) and its determinants.

exchange reserves depends on whether the favourable effects of higher foreign exchange reserves on bond spreads outweigh the adverse effects of greater external debt holdings on those bond spreads. Similarly, Levy Yeyati (2008) show theoretically that the marginal cost of issuing external debt to purchase foreign exchange reserves is a function of the current spread on debt, the responsiveness of spreads to greater foreign exchange reserves and external debt, respectively and the ratio of foreign exchange reserves to external debt (see Appendix 3.A.4 for more details). Countries therefore need to consider both the stocks of external debt and foreign exchange reserves when choosing to issue sovereign debt. Thus, there likely comes a point, above which, borrowing to hold foreign exchange reserves provides no additional benefits to the country. However, estimating this effect and threshold depends on the empirical specification of the relationship between bond spreads and the stocks of foreign exchange reserves and external debt.

Specifically, this chapter examines the determinants of sovereign bond spreads for 28 emerging markets and developing economies using an unbalanced quarterly panel over the period 2005Q1 to 2019Q4 (see Appendix 3.A.5 for a complete list of countries and their average corresponding exchange rate arrangement classifications), with the minimum time series for any country being 40 quarters (10 years) in length. This period captures economic and financial developments across numerous markets before, during and since the global financial crisis including the fall and subsequent rise of the global economy and the volatility in global energy prices which has substantially influenced the levels of external government debt and the holdings of foreign exchange reserves in both commodity importers and exporters. The period also excludes the effects of the ongoing global pandemic, which have had significant effects on bond spreads, gross domestic product, external debt, and foreign exchange reserves. Moreover, although the sample captures just 28 emerging markets and developing economies, together, these economies accounted for about three-quarters of emerging markets and developing economies' nominal gross domestic product in 2019.

Sovereign bond spreads RP_{it} for each country are captured by and derived from JP Morgan's Emerging Market Bond Index Global, while the yield on US 10-year Treasury bonds proxies the risk-free interest rate. Further, for the choice of regressors, the chapter leverages some of the regressors captured in Tebaldi et al. (2018) and Prabheesh (2013). International reserves excluding gold as a ratio of nominal gross domestic product (Res_{it}) captures the level of foreign exchange reserve buffers available to the country and is expected to reduce a country's risk profile and its borrowing costs. Meanwhile, external government debt as a ratio of nominal gross domestic product (Ext_{it}) measures the indebtedness of the country's government relative to foreign creditors and a higher level of indebtedness implies a greater probability of default, ceteris paribus. In this case, external debt holdings are proxied by gross external general government debt issued to all creditors. A similar measure (one excluding debt provided by official creditors, typically on concessional terms) is used by Levy Yeyati (2008) in his analysis of bond spreads and changes in this measure may be more closely related to fluctuations in bond spreads than changes in total external debt owed by the government. However, the correlation coefficients calculated in Figure 3.3 suggest that this may not be the case. Growth in real gross domestic product (ΔGDP_{it}) represents changes in economic activity and an increase is expected to reduce bond spreads, while the CBOE Volatility Index (VIX_t) measures the expected volatility of the S&P 500 index and proxies overall global risk aversion. As noted by Bianchi et al. (2018), greater investor risk aversion likely increases sovereign bond spreads for emerging markets. The government's fiscal balance as a percentage of nominal gross domestic product (FB_{it}) and the country's external current account balance as a ratio of nominal gross domestic product (CAB_{it}) are measures of domestic and external imbalances, respectively, and lower values of each are expected to increase bond spreads.

Sovereign bond spreads are all sourced from JP Morgan, international reserves excluding gold (foreign exchange reserves) are sourced from the International Monetary Fund's International Financial Statistics, and gross domestic product at constant prices and gross domestic product at current prices in United States dollars (as a proxy for nominal gross domestic product) are sourced from the World Bank's World Development Indicators (WDI) and the International Monetary Fund's World Economic Outlook (WEO). Gross external government debt (both total and just concessional) is sourced from the World Bank's Quarterly External Debt Statistics (QEDS), in most cases.[11] The fiscal balance as a ratio of nominal gross domestic product and the current account balance as a ratio of nominal gross domestic product are sourced from the International Monetary Fund's World Economic Outlook (WEO), while the CBOE Volatility Index and United States 10-year Treasury yield are captured from the St. Louis Federal Reserve Economic Data (FRED). Most of the classifications of de facto exchange rate arrangements are sourced from Ilzetzki et al. (2016). However, because Ilzetzki et al.'s (2016) classifications end in 2016, 2017 and 2018's de facto classifications are updated, leveraging the International Monetary Fund's Annual

[11] Most data on external government debt were sourced from the World Bank's Quarterly External Debt Statistics (QEDS). However, in a few instances, linear interpolations of annual data sourced from the World Bank's International Debt Statistics and quarterly data sourced from national sources were used to complement the data from the Quarterly External Debt Statistics.

Report on Exchange Arrangements and Exchange Restrictions 2019 (International Monetary Fund, 2020). Except for Pakistan, 2019's classifications are assumed to mirror 2018's.[12] The use of this classification differs from Levy Yeyati's (2008) use of a pegged dummy variable but permits inclusion of countries with exchange rates which may not be pegged but exhibit very little volatility. Further, the use of the within estimator to estimate the fixed effects model in the following section prohibits use of time-invariant dummy variables. Real gross domestic product per capita, based on purchasing power parity (PPP), and population size are both sourced from the World Bank's World Development Indicators. Finally, while quarterly statistics are available for most variables included in the dataset, linear interpolations of annual data (particularly those soured from the International Monetary Fund's World Economic Outlook and the World Bank's World Development Indicators) were used to fill missing gaps.

Equation 3.3 above implies that bond spreads are a non-linear function (and the natural log of emerging market bond spreads are a linear function) of all macroeconomic fundamentals, while Figure 5 of Bianchi et al. (2018) confirms the non-linear relationship between bond spreads and external debt. The analysis suggests that as external government debt levels increase, bond spreads rise, and the slope of this relationship increases at an (apparently) exponential rate as external government debt levels rise.

This chapter's own analysis appears to confirm that view. Figure 3.3 in the previous section suggests a linear (and hence exponential) relationship between the natural log of spreads (hence actual spreads) and a country's ratio of external government debt to nominal gross domestic product. However, the potentially shifting relationship between ln(spreads) and the ratio of foreign exchange reserves to nominal gross domestic product depending on the level of external debt informs the alternative specifications used in the following section.

3.5 Results

Varying panel specifications of equation 3.4 below are estimated to determine the effects of each regressor on sovereign bond spreads.

$$\ln RP_{it} = \beta_{i0} + \beta_1 + \beta_2 Res_{it-1} + \beta_3 Ext_{it-1} + \beta_4 \Delta GDP_{it-1} + \beta_5 FB_{it-1} + \beta_6 CAB_{it-1} + \beta_7 VIX_t + \gamma \ln\left(1 + r_{f_t}\right) + \varepsilon_{it} \quad (3.4)$$

[12] Pakistan's de facto regime was classified as freely falling in 2018, due to a large exchange rate depreciation. The rupee has stabilized since then, however, and so it was reclassified as a de facto crawling peg in 2019.

It is important to note that typically, country specific macroeconomic variables which are likely to be impacted by bond spreads enter each regression with at least one lag as most news on economic indicators are likely to be released and absorbed by investors within a quarter, while the VIX_t and $\left(1 + r_{f_t}\right)$ likely affect the risk premium in the same period with likely no feedback effects. Foreign investors and those who trade sovereign bonds will likely learn of new country specific data with a lag. For example, growth in real gross domestic product for the previous quarter is usually not available until at least a month after the end of the quarter, while news of the levels of foreign exchange reserves also comes with a lag in most countries, albeit usually shorter. In contrast, stock market volatility and yields on global risk-free assets are observable in real time. Thus, except for the latter two variables, and in an effort to reduce any potential feedback effects and effects of endogeneity, all regressors enter the regressions with a quarter's lag.

3.5.1 Preliminary Data Analysis

Before estimating each linear regression, it is normally prudent to test whether each variable entering each regression has stationary properties or not. The presence of stationarity permits me to estimate the regression as specified in equation 3.4, but the presence of non-stationary variables requires one of two treatments: either (1) test for cointegration to determine whether long-run relationships exist between the regressors and a non-stationary regressand or (2) difference each variable until it achieves stationarity. Thus, the panel unit root test developed by Im et al. (2003) is applied to each regressor and regressand used in the estimations to follow. Im, Pesaran and Shin's test presents a number of advantages which are very applicable to this dataset. Firstly, it does not require strongly balanced data, a feature which is not present in this sample. Secondly, the test permits each panel to have its own autoregressive coefficient, and finally, the test statistic is asymptotically normally distributed when the time dimension first tends to infinity and exceeds the number of cross sections (T > N). The test regression also corrects for potential serial correlation by including autoregressive lags, with the lag length chosen by the Bayesian Information Criterion (BIC). Table 3.1 contains these results and suggests that non-concessional external government debt and total government debt (both as percentages of nominal gross domestic product) contain unit roots and are integrated of order 1, as the test fails to reject the null hypothesis that the panels contain unit roots. All other variables are deemed to be stationary. Thus, as a baseline, in this chapter, each variable enters the regression in levels, with external government debt as a ratio of nominal gross domestic product used as the preferred measure of indebtedness.

However, robustness tests later in the study alternatively include non-concessional external government debt as a ratio of nominal gross domestic product as a regressor and compare with the results estimated from the baseline regressions. As a preview, the results are qualitatively and largely quantitatively similar.

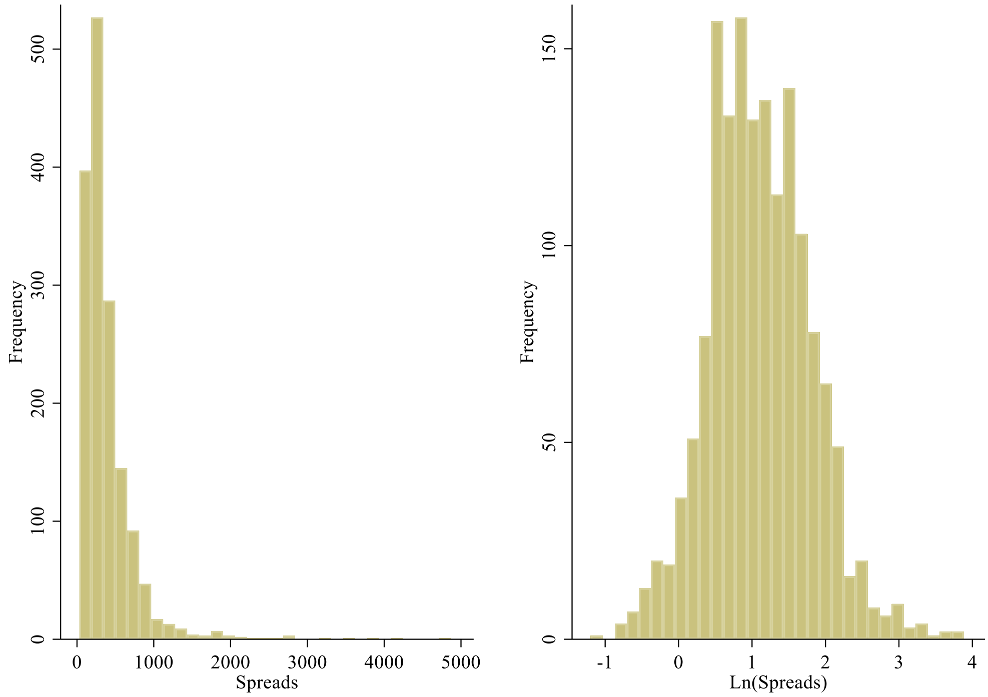
Table 3.1: Results of Im-Pesaran-Shin Unit Root Tests

Variable	W-Stat	P-value	Order of Integration
ln(Spreads)	-5.300	0.000***	I(0)
External Government Debt/GDP	-2.547	0.005***	I(0)
Non-Concessional External Government Debt/GDP	-0.025	0.490	I(1)
Total Government Debt/GDP	1.142	0.873	I(1)
Foreign Exchange Reserves/GDP	-2.919	0.002***	I(0)
VIX	-13.704	0.000***	I(0)
Gross Risk-Free Interest Rate	-1.949	0.026**	I(0)
Real GDP Growth	-8.610	0.000***	I(0)
Current Account Balance/GDP	-6.003	0.000***	I(0)
Fiscal Balance/GDP	-4.841	0.000***	I(0)

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.*

A quick glance at histogram plots of the regressand (see Figure 3.5) and key regressors and indicators (see Figure 3.6) tells a story of its own. The median Emerging Market Bond Index Global spread over the sample period was 291 basis points (bps), but a long right tail pushes the mean spread to 393 bps. Most of the data is clustered around the median, but episodes of market volatility and unrest in some countries pushed the spreads closer to 5,000 bps on a few occasions.

Figure 3.5: Histogram Plots of Emerging Market Bond Index Global Spreads (bps) – left panel and ln(Spreads) (bps/100) – right panel

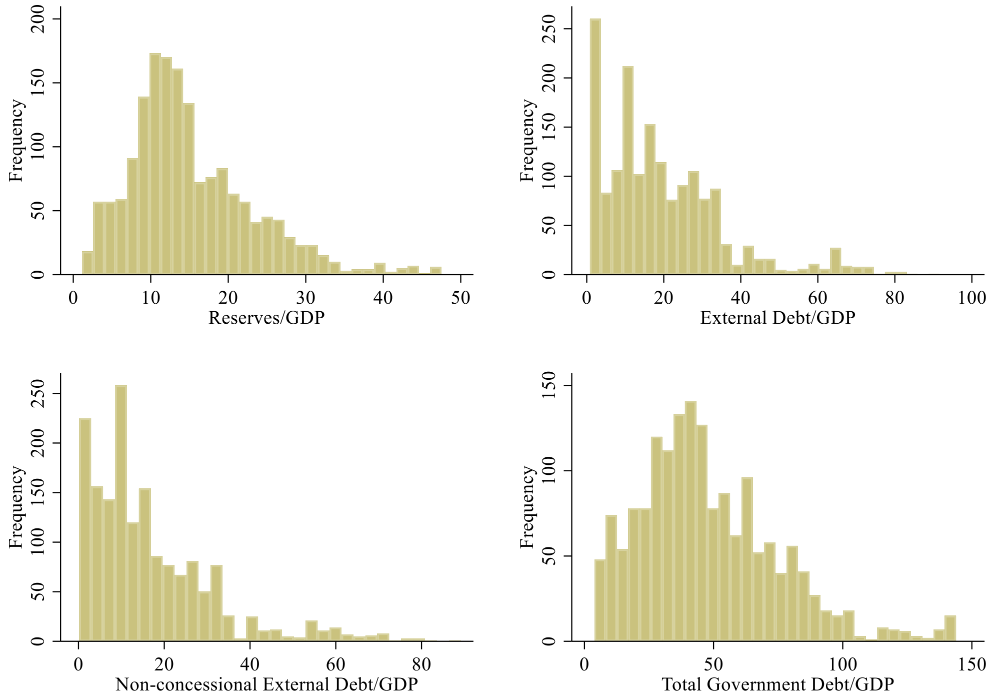


Source(s): JP Morgan, World Bank, author's calculations.

Similarly, foreign exchange reserves as a ratio of nominal gross domestic product and each measure of country indebtedness have similarly-shaped distributions – long right tails. The median ratios of foreign exchange reserves to nominal gross domestic product, external government debt to nominal gross domestic product, non-concessional external government debt to nominal gross domestic product, and total government debt to nominal gross domestic product are 13.4%, 16.3%, 12.3% and 43.1%, respectively, suggesting that countries may hold similar levels of foreign exchange reserves and non-concessional external government debt, but confirming the earlier finding that total external government debt levels often exceed short-term holdings of foreign exchange reserves.

Appendix 3.A.6 includes similar histogram plots, with the sample split between those countries who had less flexible exchange rate arrangements at the start of each period and those with more flexible exchange rate arrangements. In general, while both categories of countries held similar levels of foreign exchange reserves over the sample, on average, countries with less flexible exchange rate arrangements held more debt and paid much higher bond spreads than countries with more flexible exchange rate regimes. The median Emerging Market Bond Index Global spread for economies with more stable exchange rates was 386 bps compared to 227 bps for other markets. Further, for the former set of economies, the median ratios of foreign exchange reserves to nominal gross domestic product, external government debt to nominal gross domestic product, non-concessional external government debt to nominal gross domestic product, and total government debt to nominal gross domestic product are 13.5%, 20.2%, 14.1% and 44.3%, respectively, while for countries with more flexible regimes, the median ratios of foreign exchange reserves to nominal gross domestic product, external government debt to nominal gross domestic product, non-concessional external government debt to nominal gross domestic product, and total government debt to nominal gross domestic product are 13.2%, 11.1%, 10.5% and 41.6%, respectively. Moreover, in this sample, the distributions of external government debt to nominal gross domestic product and non-concessional external government debt to nominal gross domestic product for countries with more stable exchange rate arrangements are more widely spread than those for countries with more flexible regimes.

Figure 3.6: Histogram Plots of Foreign Exchange Reserves/GDP (%), External Government Debt/GDP, Non-concessional External Government Debt/GDP (%) and Total Government Debt/GDP (%)



Source(s): International Monetary Fund, World Bank, author's calculations.

3.5.2 Main Regression Results

Table 3.2 below presents linear, fixed effects (FE) estimates of equation 3.4. The fixed effects specification captures unobserved effects unique to each country and the Hausman test rejects the null hypothesis that the estimates from the FE model and an alternative random effects (RE) specification are similar. Further, robust standard errors correct for potential serial correlation and heteroscedasticity given the length of the dataset. As discussed earlier, previous studies have used a measure of external government debt excluding concessional financing instead of total external government debt as used in this study. However, because the unit root test deemed non-concessional external government debt as a ratio of nominal gross domestic product to be non-stationary, the more general measure of external government debt was preferred. Nonetheless, the results from Table 3.2 below imply that the results using total external government debt as a ratio of nominal gross domestic product, non-concessional external government debt as a ratio of nominal gross domestic product or even total gross government debt as a ratio of nominal gross domestic product are qualitatively and mostly quantitatively similar across debt measures. This result is not surprising given that the linear correlation between external government debt as a ratio of nominal gross domestic product and non-concessional external government debt as a ratio of nominal gross domestic product is 95% while that between external government debt as a ratio of nominal gross domestic product and total government debt as a ratio of nominal gross domestic product is 71%. Thus, the author feels confident in proceeding with total external government debt as a ratio of nominal gross domestic product as an appropriate measure of external indebtedness.

Generally, most of the explanatory variables included in the model appear to carry their expected signs. The estimates generally confirm that greater foreign exchange reserves as a ratio of nominal gross domestic product reduce sovereign risk premiums while higher external government debt levels (also as a share of nominal gross domestic product) increase bond spreads. Growth in real gross domestic product carries the expected negative and statistically significant coefficient, while the volatility in US equity markets – a proxy for global investor risk aversion – has a positive and statistically significant impact on fluctuations in sovereign spreads in all regressions. Further, the risk-free interest rate has a negative and statistically significant impact on sovereign spreads, while a higher current account balance reduces sovereign spreads. However, the coefficient on the fiscal balance is statistically insignificant and close to zero. Of note, the absolute value of the coefficient on foreign exchange reserves as a ratio of nominal gross domestic product appears larger than (or at the very least no smaller than) that on external government debt as a ratio of nominal gross domestic product, potentially suggesting some role for countries to borrow to finance foreign exchange reserves and either reduce or at least maintain bond spreads at the same time.

Table 3.2: Fixed Effects Linear Regression Estimates of Equation 3.4 including Total External Government Debt, Non-concessional External Government Debt and Total Government Debt Alternatively as Measures of External Debt

Variable	**External Government Debt**	**Non-concessional External Government Debt**	**Total Government Debt**
Res_{it-1}	-0.022***	-0.021***	-0.021***
	(0.007)	(0.007)	(0.007)
Ext_{it-1}	0.021***	0.019***	0.009***
	(0.005)	(0.005)	(0.003)
VIX_t	0.026***	0.026***	0.027***
	(0.002)	(0.002)	(0.003)
$ln\left(1 + r_{f_t}\right)$	-0.564***	-0.537***	-0.522***
	(0.122)	(0.124)	(0.132)
ΔGDP_{it-1}	-0.030***	-0.031***	-0.032***
	(0.006)	(0.006)	(0.005)
CAB_{it-1}	-0.032***	-0.033***	-0.026**
	(0.009)	(0.009)	(0.009)
FB_{it-1}	-0.001	-0.001	0.007
	(0.013)	(0.013)	(0.012)
Constant	1.362***	1.385***	1.266***
	(0.253)	(0.245)	(0.289)
Observations	1,525	1,525	1,529
R-squared	0.757	0.754	0.755
Adjusted R-squared	0.752	0.748	0.749
Hausman Test Statistic	43.41***	38.92***	29.21***

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

The outputs in Table 3.3 and Table 3.4 below seek to establish whether these results are robust to both the degree of trade openness (measured as the ratio of imports of goods and services to nominal gross domestic product) and country size (measured as the size of the total population). Equation 3.4 is re-estimated by splitting the sample between countries whose average degrees of trade openness and size fall on either side of their respective medians. In general, except for a few cases where the degree of statistical significance becomes weaker (or disappears) in the smaller subsamples, the results are qualitatively the same. Higher foreign exchange reserves improve bond spreads, while higher external government debt worsens them. Again, in most cases, the magnitude of the former relationship is at least as large as that of the latter. The lone exception may lie with the case of the larger group of economies, but both coefficients are not statistically different from each other. However, the size of the coefficient on external government debt is smaller for smaller, more open economies and becomes statistically insignificant in the case of the latter group. These results conform somewhat with Mendoza and Yue's (2012) results which imply that more open economies suffer from larger output losses after default, and thus have less incentive to default.

Table 3.3: Fixed Effects Linear Regression Estimates of Equation 3.4 – Less Open Economies vs. More Open Economies

Variable	Full Sample	Less Open	More Open
Res_{it-1}	-0.022***	-0.022*	-0.024**
	(0.007)	(0.011)	(0.009)
Ext_{it-1}	0.021***	0.022***	0.017
	(0.005)	(0.006)	(0.010)
VIX_t	0.026***	0.025***	0.026***
	(0.002)	(0.002)	(0.004)
$ln\left(1+r_{f_t}\right)$	-0.564***	-0.614***	-0.646***
	(0.122)	(0.170)	(0.204)
ΔGDP_{it-1}	-0.030***	-0.019*	-0.041***
	(0.006)	(0.010)	(0.009)
CAB_{it-1}	-0.032***	-0.014	-0.041***
	(0.009)	(0.012)	(0.012)
FB_{it-1}	-0.001	-0.010	0.003
	(0.013)	(0.019)	(0.019)
Constant	1.362***	1.478***	1.491***
	(0.253)	(0.339)	(0.490)
Observations	1,525	792	733
R-squared	0.757	0.735	0.780
Adjusted R-squared	0.752	0.728	0.774

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Table 3.4: Fixed Effects Linear Regression Estimates of Equation 3.4 – Smaller vs. Larger Countries

Variable	**Full Sample**	**Smaller**	**Larger**
Res_{it-1}	-0.022***	-0.018*	-0.026**
	(0.007)	(0.008)	(0.010)
Ext_{it-1}	0.021***	0.017**	0.030**
	(0.005)	(0.007)	(0.010)
VIX_t	0.026***	0.027***	0.026***
	(0.002)	(0.003)	(0.003)
$ln\left(1+r_{f_t}\right)$	-0.564***	-0.641***	-0.480***
	(0.122)	(0.178)	(0.149)
ΔGDP_{it-1}	-0.030***	-0.030***	-0.026***
	(0.006)	(0.008)	(0.008)
CAB_{it-1}	-0.032***	-0.045**	-0.023*
	(0.009)	(0.015)	(0.012)
FB_{it-1}	-0.001	0.020	-0.025
	(0.013)	(0.017)	(0.017)
Constant	1.362***	1.423***	1.174***
	(0.253)	(0.268)	(0.340)
Observations	1,525	735	790
R-squared	0.757	0.783	0.699
Adjusted R-squared	0.752	0.777	0.691

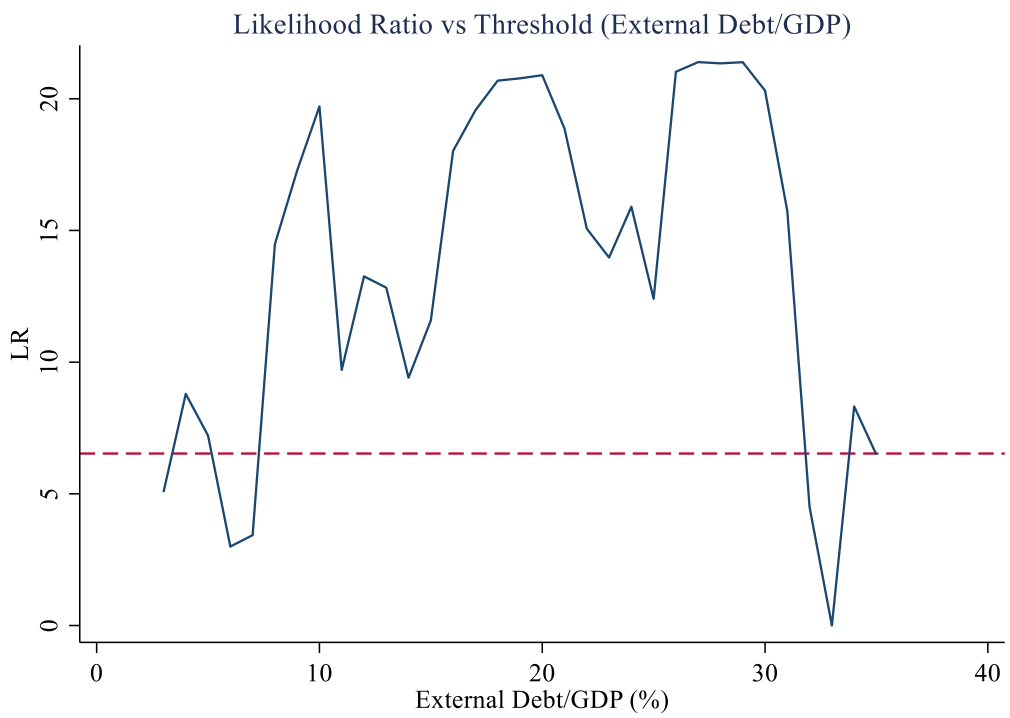
*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

The results produced thus far beg the question of whether Bianchi et al.'s (2018) finding that issuing external debt to hold more foreign exchange reserves increases bond spreads holds, and if so, above what level of external government debt does this relationship commence. To test this, the chapter develops an approach akin to Hansen's (1999) non-dynamic, panel threshold model, estimated with fixed effects to determine whether a threshold exists above which the relationship between foreign exchange reserves and bond spreads changes. Specifically, the model estimates:

$$\ln RP_{it} = \beta_{i0} + \beta_1 + \beta_{2a}Res_{it-1}I(q_{it-1} \leq \delta) + \beta_{2b}Res_{it-1}I(q_{it-1} > \delta) + \beta_3 Ext_{it-1} + \beta_4\Delta GDP_{it-1} + \beta_5 FB_{it-1} + \beta_6 CAB_{it-1} + \beta_7 VIX_t + \gamma\ln\left(1+r_{f_t}\right) + \varepsilon_{it} \quad (3.5)$$

where q_{it} represents the indicator or threshold variable, above whose value δ, the relationship between bond spreads and foreign exchange reserves changes. The model first estimates the optimal value of δ by minimizing the residual sum of squares (RSS) of a series of models estimated across a range of possible δ. In the threshold regressions that follow, the ratio of total external government debt to nominal gross domestic product is used as the threshold variable. However, Appendix 3.A.7 provides alternative estimates which compare the results that follow to those which use non-concessional external government debt as a ratio of nominal gross domestic product as the threshold variable. To ensure that sufficient observations are included in each subsample above and below the thresholds, the sample of external government debt to nominal gross domestic product is trimmed and values in the bottom and top 12.5% of the sample are not considered for use as possible thresholds. Figure 3.7 below plots the likelihood ratio (LR) statistics (a renormalization of the residual sum of squares) for each threshold tested and the area below the dotted red line highlights the 90% confidence interval (based on a 10% level of significance (α)) for the optimal threshold estimate.[13] Interestingly, the likelihood ratio statistic dips below the 10% critical value on two separate occasions: once at external government debt of around 33% of nominal gross domestic product (the optimal, single threshold), but also again at a much smaller external government debt level (6% of nominal gross domestic product). This provides some clue that a second, much lower total external government debt to nominal gross domestic product threshold may also exist. However, first I discuss the results of the single threshold model.

Figure 3.7: Likelihood Ratio (LR) Plot of Equation 3.5 with External Government Debt as a ratio of Nominal Gross Domestic Product (%) as the Threshold Variable



Source(s):Author's calculations.

[13] Hansen (1999) defines the LR statistic for each threshold as $\left[RSS(\delta) - RSS(\hat{\delta})\right]/\hat{\sigma}^2$ where $\hat{\delta}$ and $\hat{\sigma}^2$ are the estimated values of the optimal threshold parameter and residual variance, respectively. Further, Hansen (1999) illustrates that the critical value for this test is calculated as: $c(\alpha) = -2\log\left(1 - \sqrt{1-\alpha}\right)$.

Table 3.5 illustrates the results of the single threshold model. Again, the results are qualitatively similar to the linear versions of the model, except that the statistical significance of foreign exchange reserves as a ratio of nominal gross domestic product now varies on either side of the threshold. Overall, the results imply that, below total external government debt of 33% of nominal gross domestic product, higher foreign exchange reserves reduce bond spreads. However, above that level of external government debt, while the ratio of foreign exchange reserves to nominal gross domestic product maintains its negative sign, it has no material effect on bond spreads, with the coefficient becoming smaller (in absolute value) and statistically insignificant. Of note, the size of the coefficient on the ratio of foreign exchange reserves to nominal gross domestic product below the estimated threshold is statistically larger than that above the threshold, in absolute value.

Table 3.5: Non-dynamic, Panel, Single Threshold Model Estimates of Equation 3.5

Variable	**Single Threshold**
Ext_{it-1}	0.017***
	(0.006)
VIX_t	0.026***
	(0.002)
$ln\left(1 + r_{f_t}\right)$	-0.550***
	(0.115)
ΔGDP_{it-1}	-0.032***
	(0.006)
CAB_{it-1}	-0.032***
	(0.009)
FB_{it-1}	0.000
	(0.012)
$Res_{it-1}\, I(Ext_{it-1} \leq \delta)$	-0.023***
	(0.007)
$Res_{it-1}\, I(Ext_{it-1} > \delta)$	-0.010
	(0.009)
Constant	1.405***
	(0.252)
Observations	1,525
R-squared	0.761
Adjusted R-Squared	0.755
Threshold (δ)	33%

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Given that the likelihood ratio dips below the 10% critical value in two distinct regions in the threshold regression, I now estimate the double threshold regression specified in equation

3.6 to determine at what level of indebtedness a second threshold may exist, keeping the first threshold fixed at 33% of nominal gross domestic product.

$$\ln RP_{it} = \beta_{i0} + \beta_1 + \beta_{2a}Res_{it-1}I(q_{it-1} \leq \delta_1) + \beta_{2b}Res_{it-1}I(\delta_1 < q_{it-1} \leq \delta_2) + \beta_{2c}Res_{it-1}I(q_{it-1} > \delta_2) + \beta_3 Ext_{it-1} + \beta_4 \Delta GDP_{it-1} + \beta_5 FB_{it-1} + \beta_6 CAB_{it-1} + \beta_7 VIX_t + \gamma \ln\left(1 + r_{f_t}\right) + \varepsilon_{it} \quad (3.6)$$

Table 3.6 illustrates these results and compares them with the single threshold version estimated in Table 3.5. The results generally match those from the single threshold model, but now suggest that a second, smaller threshold may exist at 14% of nominal gross domestic product (δ_1), a much lower level of external indebtedness. The coefficients on $Res_{it-1}\, I(Ext_{it-1} \leq \delta_1)$ and $Res_{it-1}\, I(\delta_1 < Ext_{it-1} \leq \delta_2)$ are both negative and statistically different from zero, with $|\beta_{2a}| > |\beta_{2b}|$, while the former is statistically greater than the effect of external government debt on sovereign bond spreads (in absolute value).

Table 3.6: Non-dynamic, Panel Threshold Model Estimates of Equation 3.6 using External Debt as a ratio of Nominal Gross Domestic Product as the Threshold

Variable	**Single Threshold**	**Double Threshold**
Ext_{it-1}	0.017***	0.012**
	(0.006)	(0.006)
VIX_t	0.026***	0.026***
	(0.002)	(0.002)
$ln\left(1+r_{f_t}\right)$	-0.550***	-0.552***
	(0.115)	(0.114)
ΔGDP_{it-1}	-0.032***	-0.031***
	(0.006)	(0.005)
CAB_{it-1}	-0.032***	-0.031***
	(0.009)	(0.009)
FB_{it-1}	0.000	0.001
	(0.012)	(0.012)
$Res_{it-1}\,I(Ext_{it-1} \leq \delta)$	-0.023***	
	(0.007)	
$Res_{it-1}\,I(Ext_{it-1} > \delta)$	-0.010	
	(0.009)	
$Res_{it-1}\,I(Ext_{it-1} \leq \delta_1)$		-0.028***
		(0.008)
$Res_{it-1}\,I(\delta_1 < Ext_{it-1} \leq \delta_2)$		-0.019**
		(0.007)
$Res_{it-1}\,I(Ext_{it-1} > \delta_2)$		-0.004
		(0.009)
Constant	1.405***	1.503***
	(0.252)	(0.261)
Observations	1,525	1,525
R-squared	0.761	0.764
Adjusted R-squared	0.755	0.758
Single Threshold (δ)	33%	
Threshold 1 (δ_1)		14%
Threshold 2 (δ_2)		33%

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Overall, the results seem to provide some evidence that foreign exchange reserves reduce bond spreads, and there may be some role for issuing external debt to accumulate foreign exchange reserves without increasing the marginal cost of borrowing. However, for the single threshold model, above total external government debt of 33% of nominal gross domestic product, the accumulation of foreign exchange reserves has no material effect on bond spreads, and if done via additional external debt issuances by the government, may actually increase the marginal cost of borrowing. The double threshold model provides potentially more interesting insights, however. For very low external government debt levels (below 14% of nominal gross domestic product), building foreign exchange reserves reduces bond spreads but this effect becomes weaker up to an external government debt threshold of 33% of nominal gross domestic product. However, again, beyond this point, the benefit of the accumulation of foreign exchange reserves virtually disappears.

3.5.3 Do Regression Results Vary by Exchange Rate Regime?

Next, the chapter seeks to shed some light on whether the relationships discovered between sovereign bond spreads and foreign exchange reserves hold across exchange rate arrangements as defined by Ilzetzki et al. (2016). Specifically, the analysis splits the sample between periods where countries ended the previous period with an exchange rate regime rated 8 ("de facto crawling band that is narrower than or equal to +/-2%") or below (less flexible exchange rates) and those periods where countries ended the previous period with the exchange rate regime rated at 9 or higher (more flexible exchange rates) (again, see Appendix 3.A.1 for a detailed definition of exchange rate arrangement classifications and the shares of each observation in this sample under each regime).

Table 3.7's results present a stark difference in the linear relationships between the natural log of bond spreads and foreign exchange reserves, and the natural log of bond spreads and external government debt across exchange rate regimes. In economies with less flexible exchange rate regimes, foreign exchange reserves (as a share of nominal gross domestic product) continue to have a statistically significant and negative effect on sovereign bond spreads, with a coefficient that is larger (in absolute value) than for the subsample with more flexible exchange rates and marginally larger (in absolute value) than that on external government debt as a ratio of nominal gross domestic product. The latter is now statistically indifferent from zero. However, while foreign exchange reserves continue to carry their negative coefficient for the more flexible sample, the coefficient is now statistically insignificant and is substantially smaller than in the case of the less flexible sample and relative to the coefficient on external government debt. The coefficient on the ratio of external government debt to nominal gross domestic product is much larger for emerging markets and developing economies with more flexible exchange rate regimes. Most other coefficients generally maintain their signs and/or degrees of statistical significance.

Table 3.7: Fixed Effects Linear Regression Estimates of Equation 3.4 – Less Flexible Exchange Rate Arrangements vs. More Flexible Exchange Rate Arrangements

Variable	**Full Sample**	**Less Flexible**	**More Flexible**
Res_{it-1}	-0.022***	-0.018**	-0.008
	(0.007)	(0.008)	(0.011)
Ext_{it-1}	0.021***	0.014	0.035***
	(0.005)	(0.008)	(0.007)
VIX_t	0.026***	0.028***	0.023***
	(0.002)	(0.003)	(0.003)
$ln\left(1+r_{f_t}\right)$	-0.564***	-0.816***	-0.321***
	(0.122)	(0.176)	(0.102)
ΔGDP_{it-1}	-0.030***	-0.031***	-0.035***
	(0.006)	(0.008)	(0.011)
CAB_{it-1}	-0.032***	-0.037***	-0.024
	(0.009)	(0.011)	(0.014)
FB_{it-1}	-0.001	0.003	0.005
	(0.013)	(0.017)	(0.016)
Constant	1.362***	1.860***	0.594*
	(0.253)	(0.305)	(0.301)
Observations	1,525	868	657
R-squared	0.757	0.752	0.770
Adjusted R-squared	0.752	0.744	0.761

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Similarly, re-estimating equation 3.5 and reproducing the results from Table 3.5 for the established threshold and for two separate samples yields a similar distribution of results (see Table 3.8). External government debt as a ratio of nominal gross domestic product continues to be positive and statistically significant for markets with more flexible exchange rate regimes but is insignificant for other markets. Similarly, just one of the coefficients on foreign exchange reserves as a ratio of nominal gross domestic product (the one below the external government debt threshold of 33% of nominal gross domestic product) is statistically significant in the less flexible subsample. Further, $|\beta_{2a}| > |\beta_{2b}|$ in the regression covering the less flexible subsample as in the full sample. However, in the more flexible subsample, the coefficients on foreign exchange reserves are smaller (in absolute value) than that on external government debt, and neither is statistically different from zero.

Table 3.8: Non-dynamic, Panel, Single Threshold Model Estimates of Equation 3.5 – Less Flexible Exchange Rate Arrangements vs. More Flexible Exchange Rate Arrangements

Variable	**Full Sample**	**Less Flexible**	**More Flexible**
Ext_{it-1}	0.017***	0.007	0.039***
	(0.006)	(0.008)	(0.007)
VIX_t	0.026***	0.029***	0.024***
	(0.002)	(0.003)	(0.003)
$ln\left(1 + r_{f_t}\right)$	-0.550***	-0.741***	-0.304***
	(0.115)	(0.163)	(0.101)
ΔGDP_{it-1}	-0.032***	-0.034***	-0.034**
	(0.006)	(0.008)	(0.012)
CAB_{it-1}	-0.032***	-0.036***	-0.027*
	(0.009)	(0.011)	(0.013)
FB_{it-1}	0.000	0.008	0.007
	(0.012)	(0.016)	(0.017)
$Res_{it-1}\, I(Ext_{it-1} \leq \delta)$	-0.023***	-0.022***	-0.009
	(0.007)	(0.007)	(0.011)
$Res_{it-1}\, I(Ext_{it-1} > \delta)$	-0.010	-0.002	-0.021
	(0.009)	(0.010)	(0.013)
Constant	1.405***	1.914***	0.523
	(0.252)	(0.276)	(0.320)
Observations	1,525	868	657
R-squared	0.761	0.761	0.773
Adjusted R-Squared	0.755	0.753	0.764
Threshold (δ)	33%	33%	33%

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Finally, to summarize, emerging markets and developing economies who have less flexible exchange rate regimes appear to benefit more from holding and increasing foreign exchange reserves than their more flexible counterparts. In contrast, the levels and changes in external government debt appear comparatively less relevant in the former set of economies. The sizes of the coefficients and statistical significance would suggest that economies with less flexible exchange rate regimes are therefore better positioned to reduce their marginal cost of borrowing via a policy of issuing external government debt to purchase and stock up on foreign exchange reserves buffers, particularly at external government debt levels below 33% of nominal gross domestic product. Sovereigns with more flexible exchange rate arrangements on the other hand appear not to benefit as much from the accumulation of foreign exchange reserves and stand to benefit primarily from minimising overall, external indebtedness.

3.5.4 Discussion of Results

Finally, the results presented in this section provide some evidence that while greater accumulation of external government debt generally increases sovereign bond spreads, a larger stock of foreign exchange reserves could aid in reducing the marginal cost of borrowing. In fact, the results imply that borrowing externally to build a country's stock of foreign exchange reserves may not be as costly as some may expect, since the net effect may be a lowering (or at least, no change) in that sovereign's yield curve and a reduction in sovereign roll-over risk, depending on both the level of external government debt and the exchange rate regime in place. These results support Levy Yeyati's (2008) finding that many studies have overestimated the marginal cost of the accumulation of foreign exchange reserves, and the findings give empirical support to the assumption that foreign exchange reserves reduce not only the cost (as found in the literature), but the probability of a sudden stop to capital inflows. The results also support his finding that economies with pegged exchange rates are more likely to benefit from the accumulation of foreign exchange reserves than economies with more flexible exchange rate regimes. These results are not surprising as central banks and monetary authorities in economies with fixed or more stable exchange rates likely hold foreign exchange reserves primarily for a dual purpose. As mentioned earlier, the International Monetary Fund's (2013) survey suggested that precautionary liquidity needs and exchange rate management were the main reasons that reserve managers cited as holding foreign exchange reserves. As the International Monetary Fund (2015) suggests, the latter objective is likely to take greater weight in economies with fixed exchange rates than in countries with more flexible exchange rate regimes. Moreover, as Na et al. (2018) posit, economies with fixed exchange rates have a greater incentive to default on external obligations to release resources during a crisis than economies with floating exchange rates, given the former's commitment to maintaining the exchange rate regime. Thus, investors in sovereign external debt will likely view the adequacy of foreign exchange reserves as more important in those economies where the exchange rate does not act as an automatic stabilizer during adverse economic shocks.

However, the results suggest that, even in economies with more stable exchange rate arrangements, borrowing to purchase foreign exchange reserves becomes less valuable as external government debt levels rise. Therefore, sovereign debt managers should be careful about utilizing this medium to build foreign exchange reserve buffers. At very low levels of external government debt, the accumulation of foreign exchange reserves becomes more valuable. However, above total external government debt levels of 33% of nominal gross domestic product, foreign investors begin to look incrementally less favourably upon governments (particularly those with less flexible exchange rates) who build more foreign exchange reserve buffers (particularly via the accumulation of external debt). Thus, many international investors may begin to charge comparatively higher spreads on future sovereign borrowing.

3.6 Conclusion

The accumulation of foreign exchange reserves has become a common feature of the macroeconomic policy framework of many emerging markets and developing economies worldwide as governments and central banks seek to protect consumption or manage exchange rate fluctuations in the face of external (and sometimes domestic) shocks. However, with emerging markets and developing economies increasing their financial exposure to global capital markets, several authors (see Kim (2017) as an example) now propose a role for the accumulation of foreign exchange reserves in preventing sudden stops in capital inflows, reducing roll-over risks, and curtailing the chance of a full-blown crisis. Further, some research (see for example Levy Yeyati (2008)) has suggested that the absolute value of the effects of foreign exchange reserves on sovereign risk premiums may exceed that of higher external government debt levels on sovereign risk premiums. This implies a role for greater external debt issuance to finance the accumulation of foreign exchange reserves and simultaneously reduce a country's marginal cost of borrowing. However, to date, empirical work has paid little attention to the level of external debt above which borrowing to finance foreign exchange reserves yields little additional benefit, and apart primarily from Levy Yeyati (2008), has placed little emphasis on how these relationships vary by exchange rate regime.

Therefore, this chapter sought to answer the following questions in the context of 28 emerging markets and developing economies:

1. Does issuing external government debt to accumulate foreign exchange reserves reduce average bond spreads on external government debt?
2. If yes, above what level of external government debt as a ratio of nominal gross domestic product does this relationship fall away?
3. Does Levy Yeyati's (2008) finding that economies with less flexible exchange rate arrangements benefit more from the accumulation of foreign exchange reserves still

hold once potential non-linear effects of foreign exchange reserves on bond spreads are accounted for?

The results in this chapter imply that greater foreign exchange reserves do lower spreads on external government debt. This implies a role for borrowing to 'top up' foreign exchange reserves which may lead to a net reduction in the overall risk premium, lower the marginal cost of borrowing and reduce roll-over risk. However, the benefits of this exercise fade and are even reversed as external government debt levels exceed 33% of nominal gross domestic product. Moreover, the empirical analysis also seems to suggest that markets with more stable exchange rate regimes benefit more from the accumulation of foreign exchange reserves than those with more flexible exchange rate regimes, while international investors respond more aggressively to changes in external government debt levels in the latter set of economies than the former. Overall then, emerging markets and developing economies, particularly those with fixed or stable exchange rates, should consider their levels of foreign exchange reserves and external government debt when determining whether they will issue new external debt for precautionary purposes.

Despite supportive results thus far, the analysis does suffer from some limitations and several caveats are worth highlighting. First, the analysis leverages an unbalanced panel of 40 – 60 quarters, mostly post-global financial crisis and may be representative of the specific events which occurred during that time rather than the general relationship among foreign exchange reserves, external government debt and sovereign bond spreads. Secondly, while the data captured 28 emerging markets and developing economies, this sample may not be large enough to draw inferences about emerging markets and developing economies in general, especially the very smallest developing countries, many of whom were not included in the analysis and many who traditionally do not issue sovereign debt on international capital markets. Having said that, these 28 countries accounted for approximately 75% of emerging markets and developing economies' nominal gross domestic product in 2019. Splitting the sample by degrees of openness, population size and exchange rate flexibility also compounds the issue of the small sample size and reduces the precision of parameter estimates across the subsamples. While the parameter estimates generally maintain their statistical significance across subsamples, interacting the external debt and reserves variables with the degrees of openness, population size and exchange rate flexibility could allow us to retain the sample size and preserve the precision of the parameter estimates in the regressions run on the full sample. Also important to mention is that, while most regressors entered the regressions with a quarter's lag in an effort to reduce any potential feedback effects and effects of endogeneity, this may not guarantee that potential feedback effects are completely eliminated.

Additionally, most studies which model bond spreads tend to exclude observations of periods in which a country is in default as including these periods may skew the relationship between bond spreads and the chosen macroeconomic indicators. Of the 28 countries included in the sample, 4 (Argentina, Belize, Ecuador, and Ukraine) have experienced at least one sovereign default on external government debt over the sample period, with each country's spreads briefly rising substantially at some point during the sample period. However, these spikes in bond spreads are short-lived, and removing entire countries from this already small sample or creating gaps in the time series by removing observations would further substantially reduce the dataset available to produce reliable estimates. Nonetheless, the results in Appendix 3.A.8 suggest that the results presented in this chapter are generally robust to controlling for the spikes in bond spreads leading up to or around the quarters surrounding each external default for each of the 4 countries.[14] Finally, sovereign bond spreads may also suffer from common shocks that may affect several economies at once. The inclusion of the CBOE Volatility Index attempts to control for these common shocks, but this may reflect volatility specific to the US equity market rather than to emerging markets' bond markets. More generally, while the regressors included in each regression are among the key determinants of bond spreads, they are not an exhaustive list of possible explanatory variables and so, regression estimates could suffer from some degree of omitted variable bias.

Finally, given the limitations cited above, future work could focus on extending the sample of investigation to an even larger group of countries. Doing such may permit the author to draw conclusions about a wider set of emerging markets and developing economies but may also permit an analysis of whether the thresholds of external government debt as a ratio of nominal gross domestic product vary by exchange rate regime. A future study could also investigate the extent to which reserves and external debt directly affect the frequency of default on external debt rather than just on sovereign bond spreads (which act as a proxy for investors' perceptions of the probability of default), and whether a level of external debt exists, above which the relationship between reserves and the probability of default changes, using this approach. This would allow the study to be extended to a larger sample of

[14] Although not presented in the chapter, the results are also generally robust to the inclusion of the percentage deviation of the real effective exchange rate away from its HP-filtered trend as an additional regressor which has also been used as an explanatory variable in previous studies (see for example, Jahjah et al. (2013)).

countries, particularly those whose bonds do not typically trade internationally and so could not be included in this chapter's sample. Finally, while the author still recognizes that building foreign exchange reserve buffers via export-led growth and persistent current account surpluses is likely a preferred method for the accumulation of foreign exchange reserves for small open economies, future work could also focus on investigating whether other threshold variables exist above which the accumulation of external government debt for foreign exchange reserve buffers becomes less attractive.

3.A Appendix

3.A.1 Exchange Rate Arrangement Classifications

Ilzetzki et al. (2016) provide an assessment of countries' de facto exchange rate arrangements to determine the extent of flexibility in the exchange rate. Their "fine" classification ranges from 1 to 15, with higher values indicating a more flexible exchange rate regime, while their "coarse" classification ranges from 1 to 6 and can be easily mapped to the more granular "fine" classification. As discussed before, because Ilzetzki et al.'s (2016) classifications end in 2016, 2017 and 2018's de facto classifications are updated, leveraging the International Monetary Fund's Annual Report on Exchange Arrangements and Exchange Restrictions 2019 (International Monetary Fund, 2020). 2019's classifications are assumed to mirror 2018's for all countries, except Pakistan.

Table 3.9 below defines each classification and provides a frequency distribution of the number of regimes captured during 2005Q1 – 2019Q4 for the 28 emerging markets and developing economies included in this chapter. For the purpose of this chapter, economies with less flexible exchange rate arrangements are defined as those with a fine score of 8, that is, those countries with a "de facto crawling band that is narrower than or equal to +/-2%", or less. This corresponds to a coarse score of 2 or less and captures countries with crawling pegs and pegged exchange rates. Fine ratings of 9 or more are therefore considered as more flexible. Based on this classification, there were more classifications of less flexible exchange rate arrangements in the sample than more flexible regimes. Managed floats (classified as 12) and de facto crawling pegs (classified 7) were the two most popular exchange rate arrangements.

Table 3.9: Frequencies and Definitions of Exchange Rate Regime Classifications as Defined by Ilzetzki et al. (2016)

Fine Classification	Frequency	%	Cumulative %	Definition	General/Coarse Classification
1	118	7.7	7.7	No separate legal tender or currency union	Pegged (1)
2	51	3.3	11.1	Pre-announced peg or currency board arrangement	Pegged (1)
3	0	0.0	11.1	Pre-announced horizontal band that is narrower than or equal to +/-2%	Pegged (1)
4	84	5.5	16.6	De facto peg	Pegged (1)
5	32	2.1	18.7	Pre-announced crawling peg; de facto moving band narrower than or equal to +/-1%	Crawling Peg (2)
6	0	0.0	18.7	Pre-announced crawling band that is narrower than or equal to +/-2% or de facto horizontal band that is narrower than or equal to +/-2%	Crawling Peg (2)
7	310	20.3	39.0	De facto crawling peg	Crawling Peg (2)
8	273	17.9	56.9	De facto crawling band that is narrower than or equal to +/-2%	Crawling Peg (2)
9	0	0.0	56.9	Pre-announced crawling band that is wider than or equal to +/-2%	Managed Float/ Crawling Band (3)
10	216	14.2	71.1	De facto crawling band that is narrower than or equal to +/-5%	Managed Float/ Crawling Band (3)
11	31	2.0	73.1	Moving band that is narrower than or equal to +/-2% (that is, allows for both appreciation and depreciation over time)	Managed Float/ Crawling Band (3)
12	362	23.7	96.9	De facto moving band +/-5%/ Managed floating	Managed Float/ Crawling Band (3)
13	0	0.0	96.9	Freely floating	Freely floating (4)
14	48	3.2	100.0	Freely falling	Freely falling (5)
15	0	0.0	100.0	Dual market in which parallel market data is missing.	Dual market in which parallel market data is missing (6)
Total	**1,525**	**100.0**			

Source(s): Ilzetzki et al. (2016), International Monetary Fund, author's calculations

3.A.2 Theoretical Framework from Feder and Just (1977)

As highlighted in the main body of this chapter, Feder and Just (1977) assume that a bank in the Eurodollar market negotiates the terms of a future loan with a potential borrower. The bank recognizes that there is a probability that the principal and/or interest on the loan will not be repaid in full. The bank therefore assigns some probability of default, *P(X)* to the borrower, where *X* denotes a "…vector of economic indicators related to that probability…" (Feder and Just, 1977). Similarly, the loss on the loan given default, h, is also uncertain and carries a subjective probability of $\psi(h)$, where h falls within the range [$\underline{h}$, 1], with $\underline{h}$ equal to the minimum expected loss. Therefore,

$$\int_{\underline{h}}^{1} \psi(h)dh = 1 \tag{3.7}$$

Under normal circumstances where the borrower does not default, he or she pays the bank revenue equivalent to $rL(r)$ annually, where r is the interest rate spread or risk premium on the loan and L captures the size of the loan which is a negative function of the spread:

$$L \equiv L(r); \frac{\partial L}{\partial r} \equiv L_r < 0 \tag{3.8}$$

Discounting the annual stream of net revenues based on the bank's cost of capital, r^c yields:

$$\sum_{i=1}^{N} \frac{rL}{(1+r^c)^i} = rL \sum_{i=1}^{N}(1 + r^c)^{-i}$$

$$\sum_{i=1}^{N} \frac{rL}{(1+r^c)^i} = rL \frac{1-(1-r^c)^{-N}}{r^c}$$

$$\sum_{i=1}^{N} \frac{rL}{(1+r^c)^i} = rL\theta \tag{3.9}$$

where $\theta = \theta(r^c, N) = \frac{1-(1-r^c)^{-N}}{r^c}$ and N measures the duration of the loan.

Now assume that the bank possesses a utility function on net discounted revenue (π) where:

$$U = U(\pi), U' > 0, U'' \leq 0. \tag{3.10}$$

Given the uncertainty surrounding repayment, the bank thus maximises its utility by choosing the optimal interest rate spread, *r* from:

$$Max\ \mathrm{U} = [1 - P(X)]U\{rL\theta\} + P(X) \int_{\underline{h}}^{1} U\{-hL\}\psi(h)dh \tag{3.11}$$

Deriving the first-order maximization condition with respect to *r* and rearranging yields:

$$r = \frac{\eta}{\eta - 1} \frac{P}{[1-P]\theta} \int_{\underline{h}}^{1} \frac{U'\{-hL\} h \psi(h) dh}{U'\{rL\theta\}} \tag{3.12}$$

where η is defined as $\left|\frac{rL_r}{L}\right|$, given $L_r < 0$.

η measures the borrower's elasticity of demand for loans, in absolute terms.

Assuming that $\eta > 1$ at the point at which the bank maximises its utility (given that the interest rate spread for risky loans should be more than or equal to zero and $U' > 0$) and assuming that $h = \underline{h}$, this gives:

$$r = \frac{\eta}{\eta - 1} \frac{1}{\theta} \frac{P(X)}{1-P(X)} \underline{h} \frac{U'\{-hL\}}{U'\{rL\theta\}} \tag{3.13}$$

Importantly, the last term $\frac{U'\{-hL\}}{U'\{rL\theta\}}$ represents an additional risk premium that the banker imposes due to his or her risk aversion. In the event of risk neutrality, the term simplifies to 1. Feder and Just (1977) highlight, however, that the additional risk premium is likely not material if the size of the loan is tiny compared to the bank's overall wealth.

Assuming risk neutrality, the expression simplifies to:

$$r = \frac{\eta}{\eta - 1} \frac{\bar{h}}{\theta} \frac{P}{1-P} \tag{3.14}$$

where $\bar{h} = \int_{\underline{h}}^{1} h\psi(h)dh = E(h)$.

Thus, the risk premium in the Eurodollar market can be expressed as a function of the elasticity of demand for loans, the expected loss given default and the probability of default of the debtor country.

3.A.3 Theoretical Framework from Bianchi et al. (2018)

Bianchi et al. (2018) articulate the relationships among a country's government debt, its stock of international reserves and sovereign spreads within a framework in which the government chooses optimal levels of debt and international reserves given investors' concern about default risk. The authors define an economy whose endowment (y_t) is given by the autoregressive process: $log(y_t) = (1 - \rho)\mu + \rho log(y_{t-1}) + \varepsilon_t$, where ρ, the autoregressive coefficient is constrained to be less than 1 in absolute value and μ represents the drift term in a random walk process. The government's preferences over private consumption (c) are denoted as:

$$\mathrm{E}_t \sum_{j=t}^{\infty} \beta^{j-t}\, u(c_j)$$

where β captures the appropriate discount rate and E_t is the expectations operator.

Further, the government's debt (b_t) evolves as follows:

$$b_{t+1} = (1 - \delta)b_t + i_t \qquad (3.15)$$

with $i_t =$ the number of bonds issued in period *t*.

When the government has access to borrowing markets, its budget faces the following constraint:

$$c_t + g + \delta b_t + q_a a_{t+1} = y_t + a_t + i_t q_t \qquad (3.16)$$

with δ = the exogenous rate of decline of bond or coupon payments, q_t = the price of bonds issued at period *t*, q_a = the constant price of international reserve assets and a_t = the quantity of international reserve assets (one year asset) held at the beginning of period *t*. The international reserve asset pays 1 unit of each consumption good, and so $a_t \geq 0$. Therefore, $q_a a_{t+1}$ = the accumulation of international reserves and $i_t q_t$ = new issuances of government debt.

In other words, households' consumption, the government's fixed expenditure (g), coupon payments on debt and the value of the accumulation of international reserves are financed via total income, the starting level of international reserves and new issuances of debt during the period.

On the flip side, when the government loses access to capital markets because of its decision to default on its debt, its budget constraint collapses and simplifies to:

$$c_t + g + q_a a_{t+1} = y_t + a_t$$

$$c_t = y_t + a_t - q_a a_{t+1} - g$$

Thus, consumption is financed solely from the domestic and foreign resources left back after the government makes its rigid and fixed outlays and accumulates additional international reserves.

Investors price cashflows from sovereign bonds using the stochastic discount factor defined as:

$$m_{t,t+1} = e^{-r-\left(\kappa_t \varepsilon_{t+1} + 0.5\kappa_t^2 \sigma_\varepsilon^2\right)} \qquad (3.17)$$

where r is the discount factor for foreign lenders and represents the risk-free rate, while κ_t is the parameter governing the risk premium shock with $\kappa_t \geq 0$. $\kappa_L = 0$ is the value of the shock in good times, while $\kappa_H > 0$ is the value of the shock in bad times. π_{LH} and π_{HL} are the corresponding transition probabilities for the risk premium shock which follows a two-state Markov process. Finally, ε_t captures income shocks where $\varepsilon_t > 0$ is a positive shock to income.

The authors note that the time-varying risk premium produced from the above will "…be endogenous to the gross portfolio positions chosen by the government, which determine default risk…." (Bianchi et al., 2018). In other words, the government's choices of their stocks of debt and international reserves produce a risk premium which varies over time and reflects the country's probability of default.

The government's optimization problem is therefore defined as the maximum value of the payoffs between repaying its debts and defaulting on its debts:

$$V(a, b, s) = max\{V^R(a, b, s), V^D(a, s)\}$$

where $V^R(a, b, s)$ = the value to the government of repaying its debts, $V^D(a, s)$ = the value to the government of defaulting on its debts and s = the current exogenous state of the world where $s = \{y, \kappa\}$.

Bianchi et al. (2018) highlight that "…for any bond price function q, the function V satisfies…" the above equation. Obvious, yet important to note, is that the government cannot borrow during the default period.

The value of repaying the government's debt today is determined by:

$$V^R(a, b, s) = \underbrace{max}_{c\geq 0, a'\geq 0, b'\geq 0} \{u(c) + \beta \mathrm{E}_{s'|s} V^R(a', b', s')\}$$

Subject to:

$$c = y - \delta b + a + q(a', b', s)[b' - (1 - \delta) b] - q_a a' - g$$

Where ($'$) denotes the next period value of that variable.

From equation 3.16 above,

$$c_t + g + \delta b_t + q_a a_{t+1} = y_t + a_t + i_t q_t$$

$$c_t = y_t - \delta b_t + a_t - g - q_a a_{t+1} + i_t q_t$$

Therefore, the value of government bonds issued is equivalent to:

$i_t q_t = q(a', b', s)[\, b' - (1 - \delta)\, b]$

since from equation 3.15,

$i_t = b_{t+1} - (1 - \delta) b_t$

Further,

$q_t = q(a', b', s)$

which is the pricing function for new debt which depends on the level of international reserves, government debt and the state of the world.

Similarly, the value of defaulting on the government's debt today is determined by:

$$V^D(a, s) = \underbrace{max}_{c \geq 0, a' \geq 0} \{ u(c) - U^D(y) + \beta \mathrm{E}_{s'|s} V^D(a', 0, s') \}$$

subject to:

$c = y + a - q_a a' - g$

Bianchi et al. (2018) suggest that the solution to this problem yields the following decision rules:

1. $\hat{d}(a, b, s)$: default
2. $\hat{b}(a, b, s)$: debt
3. $\hat{a}^D\ (a, s)$: international reserves in default
4. $\hat{a}^R(a, b, s)$: international reserves when not in default
5. $\hat{c}^D(a, s)$: consumption in default
6. $\hat{c}^R(a, b, s)$: consumption when not in default

Consistency with lenders' portfolio conditions necessitates that the bond price schedule satisfies the following equation (Bianchi et al., 2018):

$$q(a', b', s) = \mathrm{E}_{s'|s}[m(s', s)[1 - \hat{d}(a', b', s')][\, \delta + (1 - \delta) q(a'', b'', s')]] \qquad (3.18)$$

where:

$b'' = \hat{b}(a', b', s')$

$a'' = \hat{a}^R(a', b', s')$

In the above equation, $1 - \hat{d}(a', b', s') = 0$ if the sovereign has defaulted on its debt, while $1 - \hat{d}(a', b', s') = 1$ if the sovereign has not defaulted.

Therefore, if $\hat{d}(a', b', s') = 1$, then $q(a', b', s) = 0$ and the bond (under this framework) is worth nothing since the debtor has not repaid.

Under no default, the price of the bond is equal to the coupon paid today (δ) and the price of a bond issued in the future, less the payment of that coupon. In other words, the bond's price is the equivalent of the present value of the cash flows associated with the bond, both paid today and paid in the future.

Further, from equation 3.17 above, the price of the risk-free asset, or the international reserves collapses to:

$q_a = e^{-r}$

From $m_{t,t+1} = e^{-r-(\kappa_t \varepsilon_{t+1} + 0.5\kappa_t^2 \sigma_\varepsilon^2)}$, the risk premium shock, κ_t is equal to zero, so $m_{t,t+1} = e^{-r-(0)} = e^{-r}$. Thus, from equation 3.18, if $\delta = 1$ as is the case of a one period, risk-free bond or asset:

$q_a = \mathrm{E}_{s'|s}[e^{-r}[1 - 0][1 + (1 - 1)\, q'_a]]$

$q_a = e^{-r}$

The government's problem is solved using value function iteration and Bianchi et al. (2018) compute the limit of the finite horizon version of the economy.

The consumer's utility function is given by:

$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}$

while the utility loss function is given by:

$U^D(y) = a_0 + a_1 \log(y)$

The spread on sovereign debt is defined as the difference between the yield on the bonds and the risk-free interest rate:

$r_t^s = i_b - r$

and the yield satisfies the following expression which defines the return on the bond, assuming that the bond is held to maturity and that the government does not default:

$q_t = \sum_{j=1}^{\infty} \delta(1-\delta)^{j-1} e^{-j i_b}$

where i_b = the yield on debt if it is held to maturity.

Further, the country's levels of government debt are determined as the present value of future payments discounted at the risk-free interest rate and are given by:

$$\frac{\delta}{1-(1-\delta)e^{-r}} b_t$$

Bianchi et al. (2018) illustrate how borrowing to accumulate international reserves benefits the economy. Assume that the debt and international reserve combination which yields consumption equal to its target $\bar{c}$ satisfies the equation:

$$q_a a' = y - \bar{c} - g - \delta b + a + q(a', b', s)[\, b' - (1-\delta)\, b] \tag{3.19}$$

Equation 3.19 mirrors the government's budget constraint during non-default periods (equation 3.16). Further, define $\tilde{a}(b', x)$ as the amount of international reserves that can be purchased when the government borrows b', for a given x and the level of international reserves (a') consistent with equation 3.19 (Bianchi et al., 2018).

From equation 3.19, replace a' with $\tilde{a}(b', x)$ and apply the implicit function theorem:

$$q_a \tilde{a}(b', x) = y - \bar{c} - g - \delta b + a + q(\tilde{a}(b', x), b', s)[\, b' - (1-\delta)\, b]$$

$$0 = y - \bar{c} - g - \delta b + a + q(\tilde{a}(b', x), b', s)[\, b' - (1-\delta)\, b] - q_a \tilde{a}(b', x)$$

The implicit function theorem suggests that:

$$\frac{\partial \tilde{a}(b',x)}{\partial b'} = -\frac{\frac{\partial F}{\partial b'}}{\frac{\partial F}{\partial \tilde{a}(b',x)}}$$

$$\frac{\partial F}{\partial b'} = \frac{\partial q(\tilde{a}(b',x),b',s)}{\partial b'}[\, b' - (1-\delta)\, b] + q(\tilde{a}(b', x), b', s)$$

$$\frac{\partial F}{\partial \tilde{a}(b',x)} = -q_a + \frac{\partial q(\tilde{a}(b',x),b',s)}{\partial a'}[\, b' - (1-\delta)\, b]$$

Therefore, combining the two yields the following two equations, the latter of which is equation 3.20:

$$\frac{\partial \tilde{a}(b',x)}{\partial b'} = -\frac{\frac{\partial q(\tilde{a}(b',x),b',s)}{\partial b'}[\, b' - (1-\delta)\, b] + q(\tilde{a}(b',x),b',s)}{-q_a + \frac{\partial q(\tilde{a}(b',x),b',s)}{\partial a'}[\, b' - (1-\delta)\, b]}$$

$$\frac{\partial \tilde{a}(b',x)}{\partial b'} = \frac{\frac{\partial q(\tilde{a}(b',x),b',s)}{\partial b'}[\, b' - (1-\delta)\, b] + q(\tilde{a}(b',x),b',s)}{q_a - \frac{\partial q(\tilde{a}(b',x),b',s)}{\partial a'}[\, b' - (1-\delta)\, b]} \tag{3.20}$$

Finally, equation 12 of Bianchi et al. (2018) defines the net benefits from increasing gross positions (both debt and international reserves) as:

$$\frac{d\mathrm{E}_{s'|s}V(\tilde{a},b',s')}{db'} \leq \frac{i}{q_a}\frac{d\tilde{q}}{db'}$$

where, finding the total derivative of the bond price with respect to bond holdings ($\frac{d\tilde{q}}{db'}$) and leveraging equation 3.20 where the value of a bond equals $q(\tilde{a}(b', x), b', s)$ yields:

$$\frac{d\tilde{q}}{db'} = \frac{\partial q(\tilde{a},b',s)}{\partial a'}\frac{\partial \tilde{a}}{\partial b'} + \frac{\partial q(\tilde{a},b',s)}{\partial b'}\frac{\partial b'}{\partial b'}$$

$$\frac{d\tilde{q}}{db'} = \frac{\partial q(\tilde{a},b',s)}{\partial a'}\frac{\frac{\partial q(\tilde{a},b',s)}{\partial b'}i + q(\tilde{a},b',s)}{q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i} + \frac{\partial q(\tilde{a},b',s)}{\partial b'}$$

$$\frac{d\tilde{q}}{db'} = \frac{\left[\frac{\partial q(\tilde{a},b',s)}{\partial b'}i + q(\tilde{a},b',s)\right]\frac{\partial q(\tilde{a},b',s)}{\partial a'} + \frac{\partial q(\tilde{a},b',s)}{\partial b'}[q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i]}{q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i}$$

$$\frac{d\tilde{q}}{db'} = \frac{q(\tilde{a},b',s)\frac{\partial q(\tilde{a},b',s)}{\partial a'} + \frac{\partial q(\tilde{a},b',s)}{\partial b'}q_a}{q_a - \frac{\partial q(\tilde{a},b',s)}{\partial a'}i}$$

Thus, ultimately, Bianchi et al.'s (2018) analysis leads us to the conclusion that the value of $\frac{i}{q_a}\frac{d\tilde{q}}{db'}$ and thus the net benefits of issuing additional government debt to accumulate international reserves depends on whether the positive effects on the value of debt (and inversely, the negative effects on yields and spreads) from increasing international reserves outpaces the negative effects of greater government debt holdings on the value of existing debt. Thus, there likely comes a point, above which, borrowing to hold international reserves provides no additional benefits to the country.

3.A.4 Theoretical Framework from Levy Yeyati (2008)

Levy Yeyati (2008) shed some light on a potential framework for analysing the relationship between bond spreads, sovereign debt, and international reserves. He highlights that the marginal cost of issuing debt depends on the responsiveness of the risk premium to international reserves and debt, respectively, as well as to the respective levels of international reserves and debt. According to Levy Yeyati (2008), the risk neutral investor holds a bond such that:

$$(1 + r_f) = (1 + r_f + \rho)[1 - p(R, D)] + (1 + r_f + \rho)(1 - H) \times p(R, D)$$

where r_f represents the risk-free interest rate, ρ is the risk premium, H is the haircut applied to the debt in percentage terms in default and $p(R, D)$ captures the probability of default which depends on both international reserves (R) and debt (D). Rearranging gives:

$$(1+r_f) = (1+r_f+\rho)[(1-p(R,D)) + (1-H)\times p(R,D)]$$

$$(1+r_f+\rho) = \frac{(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]}$$

$$\rho = \frac{(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]} - (1+r_f)$$

$$\rho = \frac{(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]} - \frac{(1+r_f)\,[(1-p(R,D)) + (1-H)\times p(R,D)]}{[(1-p(R,D)) + (1-H)\times p(R,D)]}$$

$$\rho = \frac{(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]} - \frac{(1+r_f)(1-p(R,D)) + (1+r_f)\times(1-H)\times p(R,D)]}{[(1-p(R,D)) + (1-H)\times p(R,D)]}$$

$$\rho = \frac{(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]} - \frac{1+r_f-p(R,D)-r_f\,p(R,D)+p(R,D)(1+\ r_f-H-r_fH)}{[(1-p(R,D)) + (1-H)\times p(R,D)]}$$

$$\rho = \frac{(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]} -$$
$$\frac{1+r_f-p(R,D)-r_f\,p(R,D)+p(R,D)+r_fp(R,D)-H\times p(R,D)-r_fH\times p(R,D)}{[(1-p(R,D)) + (1-H)\times p(R,D)]}$$

$$\rho = \frac{(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]} - \frac{1+r_f-H\times p(R,D)-r_fH\times p(R,D)}{[(1-p(R,D)) + (1-H)\times p(R,D)]}$$

$$\rho = \frac{(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]} - \frac{(1+r_f)-H\times p(R,D)(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]}$$

$$\rho = \frac{(1+r_f)-(1+r_f)+H\times p(R,D)(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]}$$

$$\rho = \frac{H\times p(R,D)(1+r_f)}{[(1-p(R,D)) + (1-H)\times p(R,D)]}$$

$$\rho = \frac{H\times p(R,D)(1+r_f)}{[1-p(R,D)+p(R,D)-H\times p(R,D)]}$$

$$\rho = \frac{H\times p(R,D)(1+r_f)}{[1-H\times p(R,D)]}$$

$$\rho = \frac{H\times p(R,D)}{[1-H\times p(R,D)]}(1+r_f)$$

Finding the first partial derivatives of the risk premium with respect to international reserves and debt, respectively yields:

$$\frac{\partial\rho}{\partial R} = \rho_R(R,D) = \frac{[1-H\times p(R,D)]\times H\times p_R(R,D) - H\times p(R,D)(-H\times p_R(R,D))}{[1-H\times p(R,D)]^2}(1+r_f)$$

$$\frac{\partial\rho}{\partial R} = \rho_R(R,D) = \frac{H\times p_R(R,D) - H\times p_R(R,D)(H\times p(R,D)) + H\times p_R(R,D)(H\times p(R,D))}{[1-H\times p(R,D)]^2}(1+r_f)$$

$$\frac{\partial\rho}{\partial R} = \rho_R(R,D) = \frac{H\times p_R(R,D)}{[1-H\times p(R,D)]^2}(1+r_f)$$

and,

$$\frac{\partial \rho}{\partial D} = \rho_D(R, D) = \frac{[1 - H \times p(R,D)] \times H \times p_D(R,D) - H \times p(R,D)(-H \times p_D(R,D))}{[1 - H \times p(R,D)]^2}(1 + r_f)$$

$$\frac{\partial \rho}{\partial D} = \rho_D(R, D) = \frac{H \times p_D(R,D) - H \times p_D(R,D)\big(H \times p(R,D)\big) + H \times p_D(R,D)(H \times p(R,D))}{[1 - H \times p(R,D)]^2}(1 + r_f)$$

$$\frac{\partial \rho}{\partial D} = \rho_D(R, D) = \frac{H \times p_D(R,D)}{[1 - H \times p(R,D)]^2}(1 + r_f)$$

Simplifying and setting equal to each other, we have:

$$\rho_R(R, D) \div \frac{H \times p_R(R,D)}{[1 - H \times p(R,D)]^2} = (1 + r_f)$$

$$\rho_D(R, D) \div \frac{H \times p_D(R,D)}{[1 - H \times p(R,D)]^2} = (1 + r_f)$$

$$\rho_R(R, D) \div \frac{H \times p_R(R,D)}{[1 - H \times p(R,D)]^2} = \rho_D(R, D) \div \frac{H \times p_D(R,D)}{[1 - H \times p(R,D)]^2}$$

$$\rho_R(R, D) \times \frac{[1 - H \times p(R,D)]^2}{H \times p_R(R,D)} = \rho_D(R, D) \times \frac{[1 - H \times p(R,D)]^2}{H \times p_D(R,D)}$$

$$\rho_R(R, D) = \rho_D(R, D) \times \frac{[1 - H \times p(R,D)]^2}{H \times p_D(R,D)} \div \frac{[1 - H \times p(R,D)]^2}{H \times p_R(R,D)}$$

$$\rho_R(R, D) = \rho_D(R, D) \times \frac{[1 - H \times p(R,D)]^2}{H \times p_D(R,D)} \times \frac{H \times p_R(R,D)}{[1 - H \times p(R,D)]^2}$$

$$\rho_R(R, D) = \rho_D(R, D) \times \frac{H \times p_R(R,D)}{H \times p_D(R,D)}$$

$$\rho_R(R, D) = \rho_D(R, D) \times \frac{p_R(R,D)}{p_D(R,D)} \leq 0$$

Assuming that international reserves reduce the probability of default, then the above expression will be less than or equal to 0.

Additionally, the last expression above suggests that the marginal change in the risk premium from a unit change in international reserves may equal the absolute value of the marginal change in the risk premium from a unit change in debt if the marginal changes in the probabilities are equal (and the ratio of them is equal to 1). However, if an additional unit of international reserves reduces the probability of default more than a unit of debt increases the probability of default, then the marginal effect of international reserves on interest rate spreads on government debt is greater than the marginal effect of debt on interest rate spreads on government debt.

The government's loss function can be expressed as:

$$L(R, D) = [r_f + \rho(R, D)]D + p(R, D)K - r_f R + k$$

where K captures the expected cost of a crisis and k measures other factors independent of international reserves and government debt stocks.

Assuming each additional unit of international reserves is financed via an additional unit of external government debt then:

$$\frac{\partial D}{\partial R} = 1$$

Therefore, the first partial derivative of the government's loss function of a change in international reserves financed via a change in external debt gives:

$$\frac{\partial L(R,D)}{\partial R} = L_R(R, D) + L_D(R, D)$$

$$L_R(R, D) = \rho_R(R, D)D + p_R(R, D)K - r_f$$

$$L_D(R, D) = r_f + \rho(R, D) + \rho_D(R, D)D + p_D(R, D)K$$

$$\frac{\partial L(R,D)}{\partial R} = L_R(R, D) + L_D(R, D) = \rho_R(R, D)D + p_R(R, D)K - r_f + r_f + \rho(R, D) + \rho_D(R, D)D + p_D(R, D)K$$

$$\frac{\partial L(R,D)}{\partial R} = \rho_R(R, D)D + p_R(R, D)K + \rho(R, D) + \rho_D(R, D)D + p_D(R, D)K$$

$$\frac{\partial L(R,D)}{\partial R} = p_R(R, D)K + p_D(R, D)K + \rho(R, D) + \rho_R(R, D)D + \rho_D(R, D)D$$

$$\frac{\partial L(R,D)}{\partial R} = [p_R(R, D) + p_D(R, D)]K + \rho(R, D) + [\rho_R(R, D) + \rho_D(R, D)]D$$

This last expression gives the net marginal benefit of more international reserves (possibly a net reduction in the probability of a default) plus the marginal cost of borrowing which is the spread on the debt and the marginal change in the spread on existing debt. The net marginal cost of borrowing to buy an additional unit of international reserves is therefore:

$$C'(R, D) = \rho(R, D) + [\rho_R(R, D) + \rho_D(R, D)]D$$

Defining the percentage changes in interest rate spreads on government debt with respect to changes in international reserves and debt as:

$$\frac{\partial \rho_R(R,D)}{\partial \rho(R,D)} \times R = \beta_R$$

$$\frac{\partial \rho_D(R,D)}{\partial \rho(R,D)} \times D = \beta_D$$

then, the sensitivities of bond spreads to international reserves and government debt each depend on the existing levels of international reserves and debt, respectively (Levy Yeyati, 2008). So, if the levels of government debt are already high, the responsiveness of spreads to changes in debt are higher. Similarly, the responsiveness of spreads to international reserves varies with the existing levels of international reserves.

Thus:

$$C'(R, D) = \rho(R, D) + [\frac{\beta_R \times \rho(R,D)}{R} + \frac{\beta_D \times \rho(R,D)}{D}]D$$

$$C'(R, D) = \rho(R, D) + [\frac{\beta_R \times \rho(R,D)}{R}D + \frac{\beta_D \times \rho(R,D)}{D}D]$$

$$C'(R, D) = \rho(R, D) + [\frac{\beta_R \times \rho(R,D)}{R}D + \beta_D \times \rho(R, D)]$$

$$C'(R, D) = \rho(R, D) + \rho(R, D) \times [\frac{\beta_R}{R}D + \beta_D]$$

Let $\omega = \frac{R}{D}$ then:

$$C'(R, D) = \rho(R, D) \times [1 + (\frac{\beta_R}{\omega} + \beta_D)]$$

Hence, the marginal cost of issuing government debt to purchase international reserves is a function of the current spread on external government debt, the responsiveness of spreads to international reserves and external government debt, respectively and the ratio of international reserves to external government debt. It would make sense to issue external government debt until the marginal cost of doing so is equal to the marginal benefit of issuing external government debt to purchase international reserves.

Levy Yeyati's (2008) framework also illustrates that a government can reduce the marginal cost of issuing additional external debt to build additional international reserves, but this depends on the sensitivities of interest rate spreads to international reserves and government debt, respectively, and the relative stocks of international reserves and government debt. In the scenario where $\frac{\beta_R}{\omega} + \beta_D < 0$, the marginal cost of issuing new debt will fall below the existing spreads on the government's debt. At the very extreme, and holding the marginal benefit constant, a country may not want the marginal cost of issuing additional debt to be positive. In that scenario, countries would wish to issue external government debt to purchase international reserves up until the point where the marginal cost is equal to 0. However, from the marginal cost equation above, this requires: $1 + (\frac{\beta_R}{\omega} + \beta_D) = 0$ or $\frac{\beta_R}{\omega} + \beta_D = -1$.

3.A.5 Participating Countries in the Chapter's Analysis

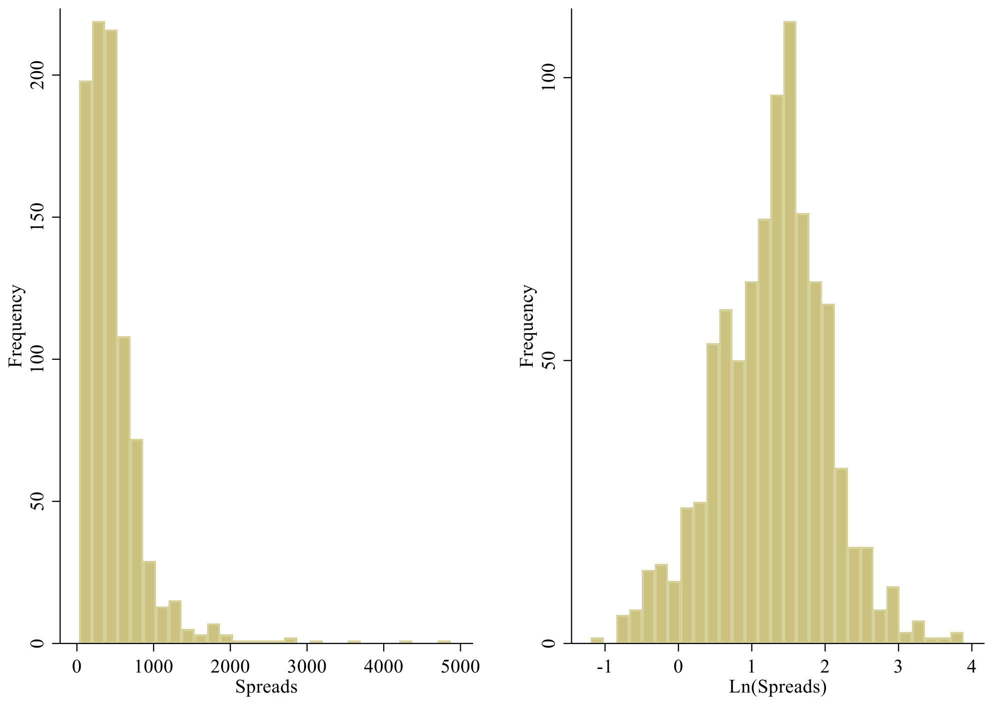
Table 3.10: Sample of Countries and Their Average Exchange Rate Classification

Country Name	**Average Exchange Rate Arrangement Classification**
Argentina	9
Belize	2
Brazil	12
Chile	12
China	6
Colombia	12
Croatia	4
Ecuador	1
Egypt	7
El Salvador	1
Georgia	8
Ghana	9
Indonesia	9
Jamaica	8
Kazakhstan	8
Mexico	12
Nigeria	8
Pakistan	8
Peru	10
Philippines	10
Russia	10
South Africa	12
Sri Lanka	7
Tunisia	8
Turkey	12
Ukraine	7
Uruguay	10
Vietnam	7

Source(s): Ilzetzki et al. (2016), International Monetary Fund, author's calculations

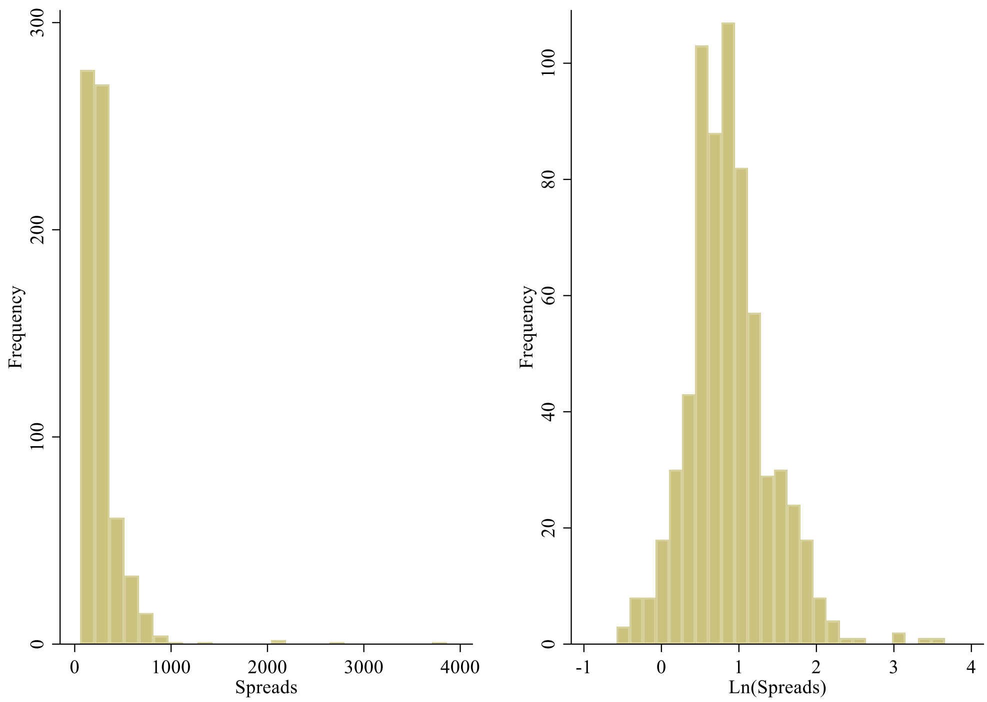
3.A.6 Histogram Plots of Key Financial and Economic Indicators for Economies with Less and More Flexible Exchange Rate Arrangements

Figure 3.8: Histogram Plots of Emerging Market Bond Index Global Spreads (bps) – left panel and ln(Spreads) (bps/100) – right panel (Economies with Less Flexible Exchange Rate Arrangements)



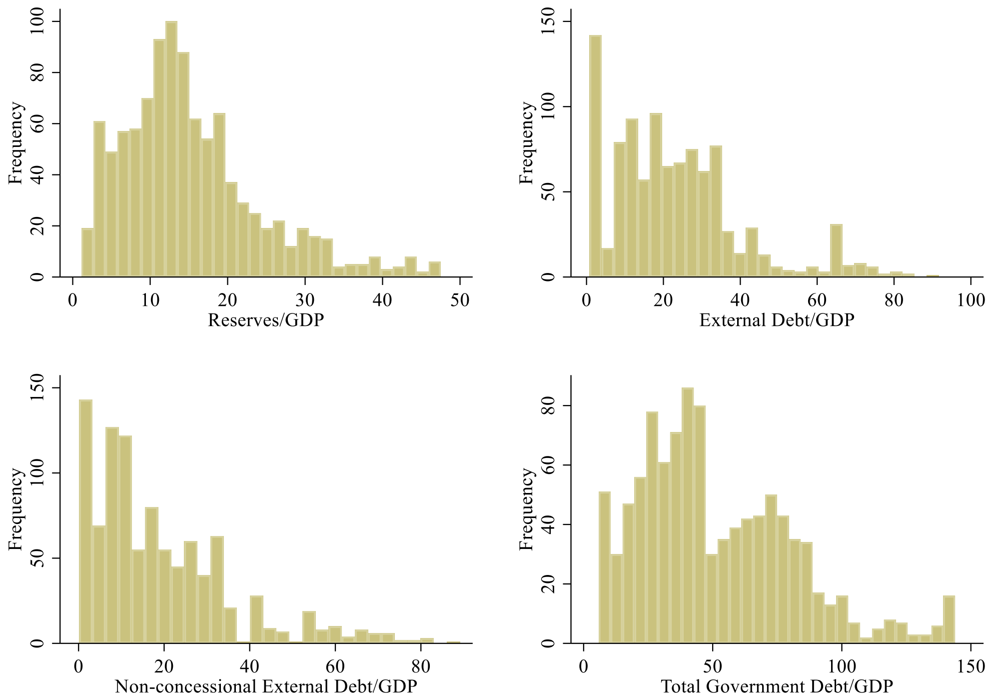
Source(s): JP Morgan, World Bank, author's calculations.

Figure 3.9: Histogram Plots of Emerging Market Bond Index Global Spreads (bps) – left panel and ln(Spreads) (bps/100) – right panel (Economies with More Flexible Exchange Rate Arrangements)



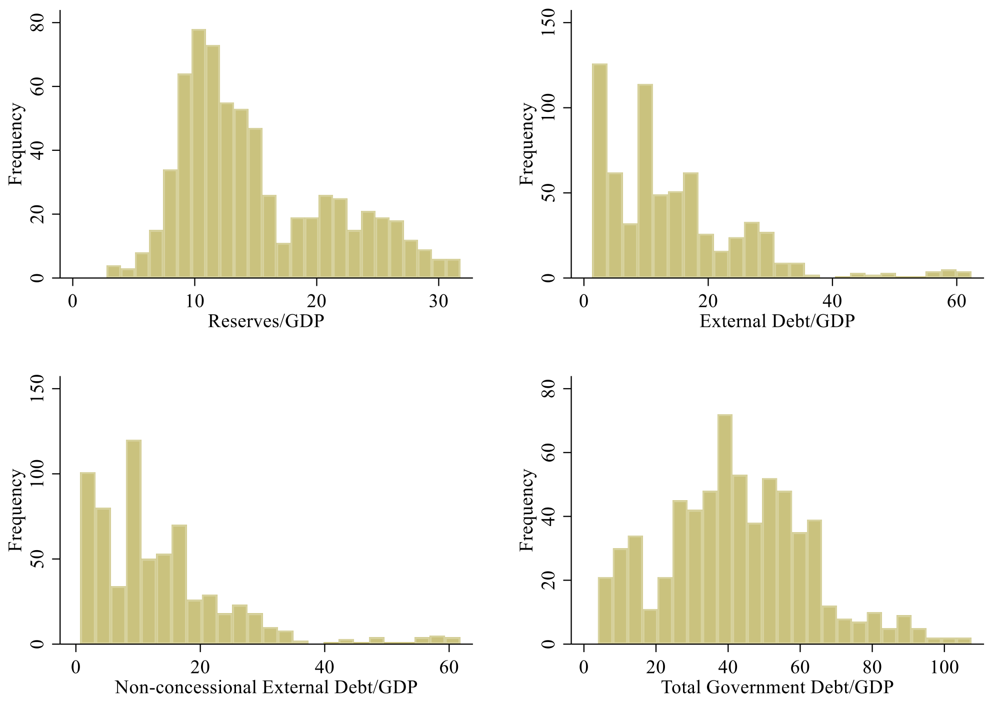
Source(s): JP Morgan, World Bank, author's calculations.

Figure 3.10: Histogram Plots of Foreign Exchange Reserves/GDP (%), External Government Debt/GDP (%), Non-concessional External Government Debt/GDP (%) and Total Government Debt/GDP (%) (Economies with Less Flexible Exchange Rate Arrangements)



Source(s): International Monetary Fund, World Bank, author's calculations.

Figure 3.11: Histogram Plots of Foreign Exchange Reserves/GDP (%), External Government Debt/GDP (%), Non-concessional External Government Debt/GDP (%) and Total Government Debt/GDP (%) (Economies with More Flexible Exchange Rate Arrangements)



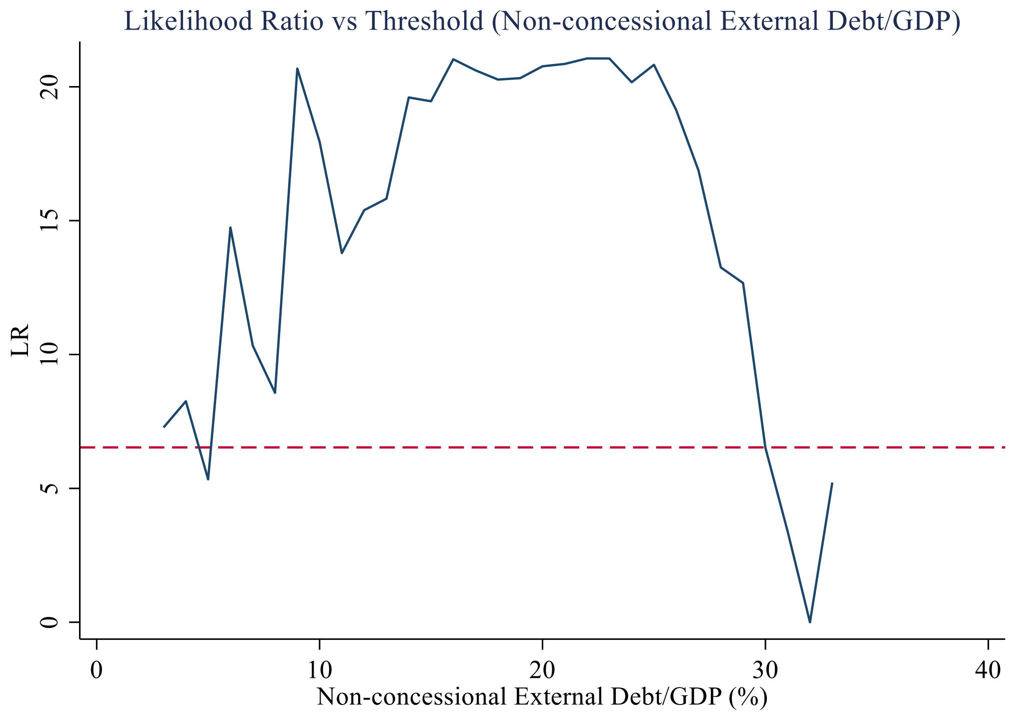
Source(s): International Monetary Fund, World Bank, author's calculations.

3.A.7 Comparative Estimates of Variants of Equation 3.5 Assuming That Non-Concessional External Government Debt/GDP Is the Threshold Variable

In this appendix, the estimates of equation 3.5 presented in the main body of the chapter are reproduced assuming that non-concessional external government debt to nominal gross domestic product is the appropriate threshold variable instead of total external government debt to nominal gross domestic product. Given the similarity of estimates produced using equation 3.4, external government debt as a ratio of nominal gross domestic product is retained as the appropriate, stationary regressor.

As in the main body of the chapter, the observations of non-concessional external government debt as a ratio of nominal gross domestic product are trimmed and the values in the bottom and top 12.5% each of the distribution are not considered for use as possible thresholds. Figure 3.12 below plots the likelihood ratio statistics for each threshold tested and the area below the dotted red line highlights the 90% confidence interval for the optimal threshold estimate. Unlike in the case where external government debt as a ratio of nominal gross domestic product represents the threshold variable, these plots imply a marginally lower threshold of 32% of non-concessional external government debt as a ratio of nominal gross domestic product with a relatively tight confidence interval around that estimate.

Figure 3.12: Likelihood Ratio (LR) Plot of Equation 3.5 with Non-concessional External Government Debt/GDP (%) as the Threshold Variable



Source(s):Author's calculations.

Table 3.11 compares the results of the single threshold models produced using alternative definitions of the threshold variable. Except for the marginally lower threshold, the results using non-concessional external government debt as the threshold variable are qualitatively and quantitatively similar to the specification where the ratio of external government debt to nominal gross domestic product is assumed to be the threshold. Overall, the results imply that, below non-concessional external government debt of 32% of nominal gross domestic product, higher foreign exchange reserves reduce bond spreads. However, above that threshold, foreign exchange reserves have no statistically significant effect on emerging markets bond spreads, with the coefficient becoming small and insignificant. Once more, the size of the coefficient on the ratio of foreign exchange reserves to nominal gross domestic product below the estimated threshold is approximately twice as large as that above the threshold in both specifications.

Table 3.11: Non-dynamic, Panel, Single Threshold Model Estimates of Equation 3.5 Assuming External Government Debt and Non-Concessional External Government Debt are Threshold Variables

Variable	**Total External Government Debt**	**Non-concessional External Government Debt**
Ext_{it-1}	0.017***	0.017***
	(0.006)	(0.006)
VIX_t	0.026***	0.026***
	(0.002)	(0.002)
$ln\left(1+r_{f_t}\right)$	-0.550***	-0.546***
	(0.115)	(0.116)
ΔGDP_{it-1}	-0.032***	-0.032***
	(0.006)	(0.005)
CAB_{it-1}	-0.032***	-0.032***
	(0.009)	(0.009)
FB_{it-1}	0.000	-0.000
	(0.012)	(0.012)
$Res_{it-1}\, I(Ext_{it-1} \leq \delta)$	-0.023***	-0.023***
	(0.007)	(0.007)
$Res_{it-1}\, I(Ext_{it-1} > \delta)$	-0.010	-0.011
	(0.009)	(0.009)
Constant	1.405***	1.393***
	(0.252)	(0.251)
Observations	1,525	1,525
R-squared	0.761	0.761
Adjusted R-Squared	0.755	0.755
Threshold (δ)	33%	32%

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Similarly, re-estimating equation 3.5 and reproducing the results from Table 3.8 for both estimated thresholds and for countries with less flexible versus more flexible exchange rate arrangements yields a similar distribution of results (see Table 3.12). Again, the size and statistical significance of all parameters are very similar across specifications that use external government debt as a ratio of nominal gross domestic product and non-concessional external government debt as a ratio of nominal gross domestic product as the chosen threshold variables. In economies with less flexible exchange rate regimes, $|\beta_{2a}| > |\beta_{2b}|$, suggesting that the benefits of the accumulation of foreign exchange reserves are greater for lower levels of non-concessional external government debt as a ratio of nominal gross domestic product. Overall, then, specifications using either measure of sovereign external indebtedness yield mostly similar results.

Table 3.12: Non-dynamic, Panel, Single Threshold Model Estimates of Equation 3.5 Assuming External Government Debt and Non-Concessional External Government Debt are Threshold Variables – Less Flexible Exchange Rate Arrangements vs. More Flexible Exchange Rate Arrangements

Variable	**Less Flexible (q_{jt-1}= Total External Debt)**	**More Flexible (q_{jt-1}= Total External Debt)**	**Less Flexible (q_{jt-1}= Non-concessional External Debt)**	**More Flexible (q_{jt-1}= Non-concessional External Debt)**
Ext_{it-1}	0.007	0.039***	0.009	0.038***
	(0.008)	(0.007)	(0.008)	(0.008)
VIX_t	0.029***	0.024***	0.029***	0.024***
	(0.003)	(0.003)	(0.003)	(0.003)
$ln\left(1+r_{f_t}\right)$	-0.741***	-0.304***	-0.736***	-0.305***
	(0.163)	(0.101)	(0.167)	(0.102)
ΔGDP_{it-1}	-0.034***	-0.034**	-0.034***	-0.034***
	(0.008)	(0.012)	(0.008)	(0.012)
CAB_{it-1}	-0.036***	-0.027*	-0.036***	-0.027*
	(0.011)	(0.013)	(0.011)	(0.013)
FB_{it-1}	0.008	0.007	0.006	0.006
	(0.016)	(0.017)	(0.016)	(0.017)
$Res_{it-1}\ I(Ext_{it-1} \leq \delta)$	-0.022***	-0.009	-0.022***	-0.008
	(0.007)	(0.011)	(0.007)	(0.011)
$Res_{it-1}\ I(Ext_{it-1} > \delta)$	-0.002	-0.021	-0.005	-0.018
	(0.010)	(0.013)	(0.010)	(0.014)
Constant	1.914***	0.523	1.873***	0.534
	(0.276)	(0.320)	(0.287)	(0.319)
Observations	868	657	868	657
R-squared	0.761	0.773	0.759	0.772
Adjusted R-Squared	0.753	0.764	0.751	0.762
Threshold (δ)	33%	33%	32%	32%

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

3.A.8 Comparative Estimates of Variants of Equations 3.4 and 3.5 Controlling for Periods of External Default

In this appendix, the estimates of equation 3.4 and equation 3.5 presented in the main body of the chapter are reproduced, controlling for the quarters around or leading up to the periods of external defaults on sovereign debt for Argentina, Belize, Ecuador and Ukraine, where bond spreads exceeded 1,000 basis points.[15] To achieve this, a dummy variable ($Default_{it}$) which equals 1 in those periods for those countries (and zero otherwise) is included in each regression. The results presented below in Tables 3.13 to 3.16 are generally robust to those presented in the main body of this chapter. Specifically, augmented versions of equation 3.4 and equation 3.5 are now expressed as equation 3.21 and equation 3.22 below:

$$\ln RP_{it} = \beta_{i0} + \beta_1 + \beta_2 Res_{it-1} + \beta_3 Ext_{it-1} + \beta_4 \Delta GDP_{it-1} + \beta_5 FB_{it-1} + \beta_6 CAB_{it-1} + \beta_7 VIX_t + \gamma \ln\left(1 + r_{f_t}\right) + \beta_8 Default_{it} + \varepsilon_{it} \quad (3.21)$$

$$\ln RP_{it} = \beta_{i0} + \beta_1 + \beta_{2a} Res_{it-1} I(q_{it-1} \leq \delta) + \beta_{2b} Res_{it-1} I(q_{it-1} > \delta) + \beta_3 Ext_{it-1} + \beta_4 \Delta GDP_{it-1} + \beta_5 FB_{it-1} + \beta_6 CAB_{it-1} + \beta_7 VIX_t + \gamma \ln\left(1 + r_{f_t}\right) + \beta_8 Default_{it} + \varepsilon_{it} \quad (3.22)$$

Table 3.13 illustrates results from estimates of equation 3.4 (including the default dummy) for the full sample and for the subsamples split by the median degree of trade openness. Across all three samples, an increase in the ratio of foreign exchange reserves to nominal gross domestic product has a negative and statistically significant effect on the spreads on external government bonds. Higher external government debt as a ratio of nominal gross domestic product increases bond spreads across the full sample and each subsample, but these effects are only statistically different from zero for the full sample and for less open economies. For more open economies, the coefficient is smaller and not statistically different from zero. Most of the other regressors carry the expected signs and degrees of statistical significance. Greater global risk aversion, and lower risk-free interest rates both increase bond spreads and all coefficients are statistically different from zero. Similarly, faster growth in real gross domestic product reduces the spread that emerging markets and developing economies pay on external government debt. Again, these effects are statistically different from zero for the full sample and across subsamples. A larger current account surplus (or smaller deficit) as a ratio of nominal gross domestic product also reduces bond spreads, but the effect is not statistically significant for less open economies. Meanwhile, changes in the fiscal balance as a ratio of gross domestic product has virtually no effect on bond spreads across subsamples. Finally, the dummy variable which captures abnormally high bond spreads is positive, as expected, but is statistically significant only for more open economies.

[15] For Argentina, these are 2015Q1 – 2015Q2 and 2019Q3 – 2019Q4; for Belize, these are 2012Q1 – 2012Q4 and 2016Q1 – 2017Q1; for Ecuador, these are 2008Q4 – 2009Q3; for Ukraine, these are 2014Q3 – 2015Q3.

Table 3.13: Fixed Effects Linear Regression Estimates of Equation 3.4 Controlling for External Defaults – Less Open Economies vs. More Open Economies

Variable	**Full Sample**	**Less Open**	**More Open**
Res_{it-1}	-0.022***	-0.022*	-0.024**
	(0.007)	(0.011)	(0.009)
Ext_{it-1}	0.021***	0.022***	0.017
	(0.005)	(0.006)	(0.010)
VIX_t	0.026***	0.025***	0.026***
	(0.002)	(0.002)	(0.004)
$ln\left(1 + r_{f_t}\right)$	-0.519***	-0.600**	-0.579**
	(0.136)	(0.220)	(0.202)
ΔGDP_{it-1}	-0.029***	-0.019*	-0.039***
	(0.006)	(0.010)	(0.009)
CAB_{it-1}	-0.032***	-0.014	-0.041***
	(0.009)	(0.012)	(0.012)
FB_{it-1}	-0.002	-0.010	0.001
	(0.012)	(0.019)	(0.019)
$Default_{it}$	0.044	0.012	0.063**
	(0.031)	(0.055)	(0.028)
Constant	1.283***	1.453***	1.372**
	(0.273)	(0.419)	(0.483)
Observations	1,525	792	733
R-squared	0.758	0.735	0.781
Adjusted R-squared	0.752	0.728	0.775

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Splitting the sample by population size produces similar results (see Table 3.14), but instead maintains more consistent levels of statistical significance across subsamples. Specifically, foreign exchange reserves and external government debt, both expressed as ratios of nominal gross domestic product, have statistically significant effects on sovereign bond spreads across both subsamples. The results for global risk aversion, risk-free interest rates, growth in real gross domestic product, the current account balance, and the fiscal balance (the latter two each expressed as ratios of nominal gross domestic product) are also mostly consistent in terms of signs and statistical significance across subsamples. Finally, the default dummy carries positive coefficients for the full sample and the subsample with smaller economies but is only statistically significant for smaller economies.

Table 3.14: Fixed Effects Linear Regression Estimates of Equation 3.4 Controlling for External Defaults – Smaller vs. Larger Countries

Variable	Full Sample	Smaller	Larger
Res_{it-1}	-0.022***	-0.018**	-0.026**
	(0.007)	(0.008)	(0.010)
Ext_{it-1}	0.021***	0.017**	0.030**
	(0.005)	(0.007)	(0.010)
VIX_t	0.026***	0.027***	0.026***
	(0.002)	(0.003)	(0.003)
$ln\left(1 + r_{f_t}\right)$	-0.519***	-0.537**	-0.481**
	(0.136)	(0.202)	(0.187)
ΔGDP_{it-1}	-0.029***	-0.027***	-0.026***
	(0.006)	(0.008)	(0.008)
CAB_{it-1}	-0.032***	-0.044**	-0.023*
	(0.009)	(0.015)	(0.012)
FB_{it-1}	-0.002	0.018	-0.025
	(0.012)	(0.017)	(0.017)
$Default_{it}$	0.044	0.096*	-0.001
	(0.031)	(0.047)	(0.044)
Constant	1.283***	1.269***	1.176**
	(0.273)	(0.284)	(0.404)
Observations	1,525	735	790
R-squared	0.758	0.786	0.699
Adjusted R-squared	0.752	0.780	0.691

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

When split by the de facto flexibility of the exchange rate, the results in Table 3.15 and Table 3.16 are almost quantitatively the same as in Table 3.7 and Table 3.8. In general, increases in foreign exchange reserves as a ratio of nominal gross domestic product have no statistically significant impact on bond spreads for economies with more flexible exchange rate regimes, but the effects are larger (in absolute value) and statistically significant for economies with less flexible exchange rate regimes. However, the statistically significant effects for economies with less flexible exchange rate regimes disappears beyond thresholds of external government debt to nominal gross domestic product of 33%. Most other variables carry the same signs and degrees of statistical significance as in Table 3.7 and Table 3.8.

Table 3.15: Fixed Effects Linear Regression Estimates of Equation 3.4 Controlling for External Defaults – Less Flexible Exchange Rate Arrangements vs. More Flexible Exchange Rate Arrangements

Variable	**Full Sample**	**Less Flexible**	**More Flexible**
Res_{it-1}	-0.022***	-0.018**	-0.007
	(0.007)	(0.008)	(0.010)
Ext_{it-1}	0.021***	0.014	0.035***
	(0.005)	(0.008)	(0.007)
VIX_t	0.026***	0.028***	0.023***
	(0.002)	(0.003)	(0.003)
$ln\left(1+r_{f_t}\right)$	-0.519***	-0.778***	-0.267**
	(0.136)	(0.204)	(0.108)
ΔGDP_{it-1}	-0.029***	-0.031***	-0.032***
	(0.006)	(0.007)	(0.011)
CAB_{it-1}	-0.032***	-0.037***	-0.022
	(0.009)	(0.011)	(0.013)
FB_{it-1}	-0.002	0.002	0.003
	(0.012)	(0.017)	(0.016)
$Default_{it}$	0.044	0.035	0.051**
	(0.031)	(0.053)	(0.022)
Constant	1.283***	1.797***	0.489
	(0.273)	(0.329)	(0.287)
Observations	1,525	868	657
R-squared	0.758	0.753	0.771
Adjusted R-squared	0.752	0.745	0.762

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Table 3.16: Non-dynamic, Panel, Single Threshold Model Estimates of Equation 3.5 Controlling for External Defaults – Less Flexible Exchange Rate Arrangements vs. More Flexible Exchange Rate Arrangements

Variable	**Full Sample**	**Less Flexible**	**More Flexible**
Ext_{it-1}	0.017***	0.007	0.039***
	(0.006)	(0.008)	(0.007)
VIX_t	0.026***	0.028***	0.024***
	(0.002)	(0.003)	(0.003)
$ln\left(1+r_{f_t}\right)$	-0.508***	-0.708***	-0.245**
	(0.130)	(0.194)	(0.106)
ΔGDP_{it-1}	-0.031***	-0.033***	-0.031**
	(0.005)	(0.008)	(0.011)
CAB_{it-1}	-0.031***	-0.036***	-0.026*
	(0.009)	(0.011)	(0.013)
FB_{it-1}	-0.001	0.008	0.004
	(0.012)	(0.016)	(0.017)
$Default_{it}$	0.041	0.031	0.054**
	(0.030)	(0.053)	(0.022)
$Res_{it-1}\, I(Ext_{it-1} \leq \delta)$	-0.023***	-0.022***	-0.008
	(0.007)	(0.007)	(0.011)
$Res_{it-1}\, I(Ext_{it-1} > \delta)$	-0.010	-0.002	-0.020
	(0.009)	(0.010)	(0.013)
Constant	1.330***	1.858***	0.409
	(0.270)	(0.300)	(0.301)
Observations	1,525	868	657
R-squared	0.761	0.761	0.774
Adjusted R-Squared	0.755	0.753	0.765
Threshold (δ)	33%	33%	33%

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Chapter 4: Economic Structure, Exchange Rate Misalignment, and Currency and Debt Crises

4.1 Introduction

Several studies have investigated the causes of currency crises over the past four decades (see for example MacDonald (2007) for a summary of some of the literature on currency crisis models) given the large output costs often associated with these episodes. In fact, much empirical work has gone into identifying the determinants which best predict these occurrences. While the literature has been relatively aligned on some of the key variables which explain these events (e.g., real exchange rate overvaluation, insufficient foreign exchange reserves), the likelihood that a country experiences a currency crisis seems to depend, in part, on its exchange rate regime. In fact, some research has suggested that economies with pegged exchange rates appear less likely to experience a currency crisis (Tarashev and Zabai, 2019) but may have a greater incentive to default on external debt (Na et al., 2018). Thus, countries within each exchange rate regime classification may face a tradeoff between choosing to default on its foreign currency or external obligations and choosing to devalue its nominal exchange rate or allow it to depreciate sharply.

Further, while economies with pegged exchange rates tend to be smaller than those with more flexible regimes, many large economies continue to or at some point in the past have chosen to maintain de facto currency pegs. Thus, economies with fixed or relatively stable exchange rates should probably not be treated as a homogenous group of countries and range from small, import-dependent island economies including Barbados, the Bahamas, and the economies of the Eastern Caribbean Currency Union, to larger, commodity exporters including Saudi Arabia and Oman, to the even larger, more advanced economies that make up the Euro Area. These economies are structurally different, and the occurrences and the costs and benefits of exchange rate devaluation and external default probably vary amongst them. For example, smaller, more open economies tend to rely more on imported inputs and final goods and services and will likely suffer from greater inflation and a smaller impetus to growth in real gross domestic product as a result of a large nominal devaluation aimed at correcting large macroeconomic imbalances. Conversely, those economies may benefit more in relative terms from defaulting on external or foreign currency debt by releasing much needed resources during a crisis (Na et al., 2018). Therefore, the levels of concern for common indicators which have typically been used to predict the occurrences of both currency and debt crises may vary by both exchange rate regime and the structure of the economy.

This chapter has two objectives. First, it aims to evaluate whether the ability of the degree of real exchange rate overvaluation to predict currency crises varies by country size and by the degree of trade openness, supporting the hypothesis that smaller, more open economies have less incentive to devalue their nominal exchange rates in the face of external imbalances. Secondly, it seeks to determine whether, when controlling for real exchange rate misalignment and other macroeconomic imbalances, smaller, more open economies are more likely to choose to default on their foreign currency debt obligations rather than to devalue their nominal exchange rates (relative to their larger, less open peers), again testing the hypothesis that economies sometimes face a tradeoff when choosing between sovereign default and nominal exchange rate devaluation. Real exchange rate overvaluation is chosen as the primary, desired measure of macroeconomic imbalances in most cases as it is routinely found to be one of the best predictors of currency and external debt crises in the literature (see for example Reinhart and Rogoff (2009) and Holtemöller and Mallick (2013)) and it is usually considered to be a reliable measure of external imbalances.

The chapter contributes to the existing literature by being among the first (to the best of the author's knowledge) to empirically evaluate the role of a country's size and dependence on imports of goods and services or trade openness on the likelihood of currency crises and the tradeoff that policymakers may make between sovereign default and exchange rate devaluation. The results appear to support these roles and suggest that, at given levels of real exchange rate overvaluation, smaller, more open economies with de facto fixed or managed exchange rate regimes appear less likely to experience sharp nominal exchange rate devaluations than larger, less open economies. This relationship is statistically significant and particularly strong when the level of positive real exchange rate misalignment exceeds 24% but declines after real exchange rate overvaluation of 35%. In fact, for smaller, more open economies, the relationship becomes statistically insignificant at such high levels (above 35% real exchange rate overvaluation). Further, controlling for real exchange rate misalignment and other predictors of currency and debt crises, smaller, more open economies appear more likely to choose to default on their foreign currency debt than to choose to devalue their nominal exchange rate, compared to their larger, less open counterparts.

The rest of the chapter will be structured as follows. Section 2 will review the relevant theoretical and empirical literature on currency crises, exchange rate regimes and real exchange rate overvaluation. Sections 3 and 4 will describe the theoretical framework and previous work on the estimation of real exchange rate misalignment, respectively. Section 5 will discuss the data and methodology used in the study, section 6 will present and discuss the empirical results and section 7 will offer conclusions and areas for future research.

4.2 Literature Review

4.2.1 Generations of Currency Crisis Models

The modern literature on currency crises dates to Krugman's (1979) seminal work on the theory behind balance of payments crises. His model, among the first in a line of papers deemed the first generation of currency crises, explained that a speculative attack on a pegged currency is likely to ensue when economic fundamentals appear out of sync with those necessary to sustain an exchange rate peg. In his framework, a government running a persistently high fiscal deficit that needs to be monetized triggers a decline in international reserves. Notwithstanding efforts to borrow externally or monetize assets to reverse the net capital outflow, unless the fundamental problem of excess money creation to fund the fiscal deficit is corrected, international reserves will continue to fall. Rational investors, understanding that the exchange rate cannot be maintained if these policies continue, launch a speculative attack on the currency before the international reserves have been depleted in the hope that they will profit from an eventual devaluation of the exchange rate. The attack itself extinguishes the remaining stock of international reserves and the country loses the peg.

First-generation models explain some of the balance of payments crises witnessed in Latin America in the 1970s and 1980s very well but fail to explain crises where economic fundamentals appear generally sound. An example of the latter is the case of the near collapse of the European Exchange Rate Mechanism (ERM) in 1992/1993 (MacDonald, 2007). Instead, a second generation of models, pioneered by Obstfeld (1986, 1994), highlights the potential dilemma facing policymakers regarding their willingness to maintain the policies necessary to preserve a peg or exchange rate target. If investors sense that the output costs of maintaining the exchange rate peg (for example, via high interest rates, contractionary fiscal policy) are high enough for policymakers to question their commitment to maintaining the existing regime, a speculative attack could occur. In the case of the Exchange Rate Mechanism cited above, higher interest rates to fight rising inflation in Germany appeared incompatible with weak economic growth in other countries in the rest of the European Monetary System. While tighter monetary policy would appear more consistent with maintaining the exchange rate arrangement, this obviously imposed a cost on economies wishing to fight off recession. Speculative attacks on several currencies ensued and many central banks were forced to devalue their currencies or entirely abandon their pegs. More recently, third- and fourth-generation models have included a role for the balance sheet effects of exchange rate depreciations on firms' and banks' solvency and help to explain the Asian crisis of the late 1990s (MacDonald, 2007). They focus on the effects of currency mismatches on banks' or corporates' balance sheets and their implication for banking crises and/or declines in investment (see for example Krugman (1999), Krugman (2001)).

4.2.2 Empirical Work on the Determinants of Currency Crises

General Determinants of Currency Crises

Given several theoretical approaches to modelling currency crises, numerous authors have attempted to empirically identify the factors which explain or predict historical episodes. Frankel and Rose (1996) famously conducted such an exercise for over 100 emerging markets and developing economies between 1971 and 1992. They define a currency crash (as opposed to a currency crisis) as "…a nominal depreciation of the currency of at least 25% that is also at least a 10% increase in the rate of depreciation." (Frankel and Rose, 1996). They identify and test 4 groups of variables (internal domestic macroeconomic variables, foreign or global variables, variables measuring the composition of the debt stock, and variables capturing international indebtedness and external variables) using an event study approach and a probit model. Additionally, they incorporate and test other variables relevant to the literature on currency crises. In all, they identified 117 currency crashes over the sample period and found that currency crashes are more likely when output is low relative to trend (although the direction of causality was left to be determined), credit growth and foreign interest rates are high, foreign direct investment as a share of debt and foreign exchange reserves are low, and the real exchange rate (RER) is overvalued. However, contrary to expectations, governments' fiscal positions nor the size of the current account deficit did not appear to significantly affect the chance of a currency crash.

Kaminsky et al. (1997) sought to identify early warning indicators and their respective thresholds which may help to signal an impending crisis 24 months in advance. They assess the variables' and their thresholds' predictive abilities by minimizing the ratio of noisy signals (that is, the indicator suggests that a crisis is pending but a crisis does not materialize) to successful signals (that is, the indicator successfully predicts a crisis within 24 months). Overall, they find that several indicators including the deviation of the real exchange rate away from its trend, output growth and changes in broad money as a ratio of international reserves are among the best at signaling upcoming currency crises (Kaminsky et al., 1997; MacDonald, 2007).

Currency Crises and Exchange Rate Regimes

One particularly striking feature of the empirical literature is the varying likelihood and costs of currency crises across countries with different exchange rate regimes. Nakatani (2018) finds that both country-specific risk premium and productivity shocks can prompt currency crises, with the latter particularly important for severe crises. Capital controls can help to mitigate the effects of the latter shocks in the case of economies with exchange rate pegs but raising interest rates may actually prompt a currency crisis given the negative signal sent to investors. Further, countries with floating exchange rates are more prone to shocks which lead to currency crises than countries with exchange rate pegs (Nakatani, 2018). In the latest in a series of papers, Nakatani (2019) concludes that both productivity and risk-premium shocks have material effects on growth in real gross domestic product during both currency and banking crises, but that output costs are greater in economies with floating exchange rates.

Real Exchange Rate Overvaluation and Currency Crises

Real exchange rate overvaluations have long been viewed as a sign of weak competitiveness and external imbalances and Grekou (2015) suggests that, for a sample of 12 CFA (Communauté Financière Africaine) Zone countries, real exchange rate overvaluations hurt growth in real gross domestic product. Perhaps not surprisingly then, among the many indicators identified as key to explaining or predicting currency crises, the extent of real exchange rate overvaluation or positive deviation of the real exchange rate away from its equilibrium stands out as among the more common, statistically significant predictors (see for example Frankel and Rose (1996), Nakatani (2019)). In fact, Reinhart and Rogoff (2009), in an effort to compare housing prices' performance as an indicator of predicting currency and banking crises using the signaling approach leveraged in studies such as Kaminsky et al. (1997), found that housing prices performed no better than their measure of real exchange rate overvaluation in both instances. Further, the latter variable proved to be the most successful among all variables tested in predicting both types of crisis (Reinhart and Rogoff, 2009). Moreover, Isard (2007), in assessing methodologies for estimating equilibrium exchange rates and real exchange rate misalignments, suggests that appropriate choices of estimating equilibrium exchange rates should give greater weight to those approaches that can identify a country's ability or willingness to maintain the currency at existing levels. This thus speaks to the role of real exchange rate overvaluation in predicting or signaling currency crises. Further, Zhao et al. (2014) find that the appropriate predictors of currency crashes vary across exchange rate regimes, with external indicators (especially those from the first-generation models) being especially important for fixed exchange rate regimes relative to intermediate and floating exchange rate regimes. However, variables related to monetary policy and credibility (proxied by inflation and credit growth) are more important in floating regimes. The deviation of the real exchange rate from trend appears important across all regimes.

Finally, Holtemöller and Mallick (2013) confirm the relationship between real exchange rate misalignment and the probability of currency crises but find that this relationship is nonlinear and does vary by exchange rate regime. They first suggest that misalignment of the real effective exchange rate (REER) and the volatility of misalignment are greatest in economies with less flexible exchange rates. Further, the magnitude of real exchange rate overvaluation increases the probability of a currency crisis, but as the degree of misalignment increases, the probability increases at a decreasing rate (Holtemöller and Mallick, 2013). In fact, the peak probability of a currency crisis for their sample is highest for economies with floating exchange rates (at lower levels of real exchange rate overvaluation) but declines below the probability for economies with pegs or intermediate exchange rate regimes at higher levels of real exchange rate misalignment. Further, at higher levels of real exchange rate misalignment, the probability of a currency crisis in economies with intermediate regimes significantly exceeds that in fixed exchange rate economies. This begs the question of whether economies with exchange rate pegs can sustain higher levels of real exchange rate overvaluation than their counterparts with intermediate regimes or floating exchange rates before experiencing currency crises.

4.2.3 Choice of Devaluation or Default

Real Exchange Rate Overvaluation and Debt Crises

Positive real exchange rate misalignments have also been found to explain the risk of sovereign default in emerging markets and developing economies, and as with currency risk, the strength of the relationship varies with the exchange rate regime. Jahjah et al. (2013) find that greater real exchange rate overvaluation increases sovereign bond spreads, but also increases the chances of issuing debt. Greater real exchange rate overvaluation suggests less international competitiveness and a greater chance of sovereign default, but real exchange rate overvaluation may also suggest good (but not sustainable) economic times and encourage excessive borrowing which leads to higher bond spreads in the future. Further, the effects of real exchange rate overvaluation are stronger for economies with less flexible exchange rates as those markets will be less willing to devalue their currencies to achieve a more "competitive" exchange rate (Jahjah et al., 2013).

The Tradeoff Between Devaluation and Default

The final sentence in the previous paragraph and previous research suggesting that economies with pegged exchange rates are less prone to currency crises even at the same level of real exchange rate overvaluation, point to a clear tradeoff which central bankers attempting to maintain an exchange rate peg may make. Tarashev and Zabai (2019) suggest that central banks who are committed to an exchange rate peg, have more aversion to inflation, and benefit from domestic policy credibility, are more likely to keep the peg in the face of pressure on the exchange rate. However, governments committed to maintaining a fixed exchange rate even in the presence of external imbalances have greater incentives to default on their debts versus those with flexible exchange rate regimes due to the unleashing of resources to aid in post-default economic recovery (Na et al., 2018).

Therefore, Bauer et al. (2003) model the conditions under which a government may choose to default on its foreign currency debt, devalue its exchange rate, do both or neither, depending on the economic fundamentals and debt levels existing in the country and on private investors' expectations about the likelihoods of sovereign default and exchange rate devaluation. In the model (analogous to the second generation of currency crisis models), policymakers evaluate the relative costs and benefits of sovereign default versus exchange rate devaluation and determine the optimal choice in order to maximize welfare in the economy. The model produces unique equilibria (clear decision rules on whether to default on debt or devalue the exchange rate, do both or neither) when the fundamentals or debt levels are either clearly good or bad, but multiple equilibria in intermediate cases. Bauer et al. (2007) then test their 2003 hypotheses empirically and seek to determine whether the factors driving pure currency crises, pure debt crises or joint debt and currency crises differ. They find, in fact, that they do, but that in all cases, inflation and real exchange rate overvaluation (with the latter except in the case of a twin crisis) positively increase the chance of one or both crises occurring. Further, modelling pure debt, pure currency and twin debt and currency crises separately yields better predictability of these crises than the traditional approach of just looking at debt and currency crises without considering that they may occur simultaneously (Bauer et al., 2007).

The Relative Costs of Exchange Rate Devaluations and Default by Economic Structure

Given that policymakers may be forced to make a tradeoff between defaulting on their debts and devaluing their exchange rates, understanding the costs associated with these decisions sheds better light on the ease (or lack thereof) with which they are made. For example, countries who default on external debt are often subject to higher interest rate spreads or a default premium (Catão and Mano, 2017) some years after the default has been resolved. Similarly, countries whose investors suffer from larger net present value (NPV) haircuts pay a premium on their external debt post-restructuring and are excluded from borrowing from external markets for a longer period (Cruces and Trebesch, 2013).

The costs of default may also vary by economic structure. Mendoza and Yue (2012) model the cost of default as lost access to working capital to finance imported intermediate inputs and thus suggest that being more open or relying on external finance to fund imported inputs creates greater vulnerability to sudden stops or loss in access to capital markets. Thus, more open countries without available domestic substitutes are likely to experience greater output losses at the time of default. Finally, the nature of an exchange rate regime may also determine the cost of sovereign default and Kuvshinov and Zimmermann (2019) suggest that, because economies with fixed exchange rates often carry large current account deficits prior to sovereign defaults, the sharp external adjustment required post-default (which is not facilitated via a nominal exchange rate devaluation) induces larger costs of default than presumably under flexible exchange rate regimes. This occurs because exports do not rise in the short-run (the authors suggest they fall after default) and most of the adjustment thus occurs via declining domestic demand, and by extension, imports (Kuvshinov and Zimmermann, 2019).

Equally, the costs of nominal exchange rate devaluation may prove prohibitive to some economies, and some peggers more than others. The choice of a currency peg (along with free capital mobility) prohibits economies with fixed exchange rates from pursuing independent monetary policies. However, this could present an advantage for countries opposed to inflation as they may "import the credibility and low inflation environment" (MacDonald, 2007) from the country to whom they have pegged the exchange rate. Unfortunately though, small, open economies have a high pass through of external prices to domestic prices. This reduces their ability to use the nominal exchange rate as a tool to adjust aggregate demand in the short run, in response to some external shock, without inducing large spikes in inflation (Worrell et al., 2018). It does mean however, that the inflexibility of the nominal (and to some extent real) exchange rate in peggers increases the output costs of terms of trade shocks relative to economies with floating exchange rates or those who choose to devalue the exchange rate (Broda, 2004; Schmitt-Grohe´ and Uribe, 2017).

The Relative Effectiveness of Exchange Rate Devaluations

Further, even beyond higher inflation, the effectiveness of nominal exchange rate devaluations in correcting real exchange rate misalignments may vary across countries. Grekou (2019) finds that nominal exchange rate devaluations are more effective at devaluing real effective exchange rates in the medium run when fiscal and monetary policies are not expansive, nominal exchange rate devaluations are large enough, are not accompanied by a switch to a floating exchange rate regime and are preceded by real exchange rate overvaluation in the year prior. Further, he notes that, the socio-political environment does not actually materially affect the effectiveness of nominal exchange rate devaluations. The efficacy of nominal exchange rate devaluations also varies by the size and structure of the economy. For example, while Iossifov and Fei (2019) find that, in the case of Turkey, depreciations of the real effective exchange rate can positively improve trade balances, Prakash and Maiti (2016) find that, because small, island economies (such as their example of Fiji) have little capacity to export and depend heavily on imports of goods and services, nominal exchange rate devaluation proves less effective in improving the trade balance as it appreciates the real exchange rate (via high exchange rate pass through to inflation) after the initial nominal devaluation of the exchange rate and this higher inflation does not permit a bigger boost to aggregate demand. They also find a weak link between services exports and the real exchange rate and suggest that this is indicative of the muted effects of real exchange rate depreciation on tourism to small islands (see also Culiuc (2014)).

4.2.4 Summary of Review

The literature surveyed thus far suggests that the probability of an economy suffering a crisis may depend, not only on the exchange rate regime, but on the structure of that economy. These structural elements, via the respective costs and benefits of sovereign default and nominal exchange rate devaluation, may influence that policymaker's decision to adjust or abandon its peg or default on external or foreign currency debt when faced with external imbalances, including an overvalued real exchange rate. Further, the extent of real exchange rate misalignment which triggers a sovereign default or an exchange rate devaluation likely varies across countries. However, to date, while the empirical literature has made differentiations by exchange rate regime, it has, to date, yet to investigate the roles which country size or trade openness play in determining the probabilities of currency and debt crises. Indeed, a small, service-exporting, import-dependent economy with a fixed exchange rate may respond differently to a real exchange rate misalignment caused by a productivity or terms of trade shock compared to a larger pegger with substantially greater domestic production capacity and lower exchange rate pass through to prices. These issues represent a clear gap in the literature and remain to be researched.

4.3 Theoretical Framework

4.3.1 Bauer et al. (2003)

Bauer et al.'s (2003) model seeks to describe a government's decision-making process when seeking to determine whether to exit a pegged exchange rate regime, default on its foreign currency debt or both. Their model leverages and builds on the escape clause approach to currency crises popularized by Jeanne (2000). In their model, the government issues foreign currency debt with a maturity of one period and maintains a fixed exchange rate. At the beginning of the period *t*, the authorities decide whether to exit the exchange rate regime (with $\lambda = 1$ if the government devalues the nominal exchange rate and $\lambda = 0$ if it keeps the peg) and whether to default on its foreign currency debt due at the end of the period (with $\eta = 1$ if the government defaults on its debt and $\eta = 0$ if it meets its upcoming debt obligations in foreign currency).

Let B_t denote foreign currency debt owed and coming due in period *t* but incurred at the beginning of period *t-1*. The authors assume that if, at the beginning of period *t,* the authorities opt to repay all of their foreign currency debt, it can borrow additional debt of

B_{t+1} during period t, at a price of $q_t = \frac{1}{1+R_t}$ where R_t represents the effective interest rate charged on the debt. Naturally, if the government defaults on its foreign currency obligations, it is unable to access financing internationally.

As with Jeanne (2000), Bauer et al. (2003) presume that, for as long as the exchange rate peg remains, the rate of growth in the money supply ($\widehat{M}_t$, where M_t is the money supply and the hat denotes the growth rate) equals zero and implies a similar growth rate in prices (where P_t denotes the price level) or an inflation rate of $\hat{P}_t = 0$. In contrast, if the authorities devalue the exchange rate, the growth in the money supply will now equal $\vartheta > 0$ and so will the inflation rate. Assuming also that purchasing power parity (PPP) holds, then the change in the nominal exchange rate $\hat{S}_t$ (where S_t is the nominal exchange rate expressed as domestic currency units of one unit of foreign currency) is also equal to the growth in the money supply and the inflation rate. In summary then,

$$M_t = (1 + \lambda\vartheta)\, M_{t-1} \quad (4.1)$$

$$P_t = (1 + \lambda\vartheta)\, P_{t-1} \quad (4.2)$$

$$S_t = (1 + \lambda\vartheta)\, S_{t-1} \quad (4.3)$$

$$\widehat{M}_t = \hat{P}_t = \hat{S}_t = \lambda\vartheta \quad (4.4)$$

Let Y_t, E_t, F_t and cT_t represent real gross domestic product, the real costs of exchange rate devaluation, the real costs of default on foreign currency debt and the total costs (including administration and deadweight loss) of government taxation (where T_t denotes real tax revenue), respectively. In turn, real gross domestic product is deemed to be a function of the natural level of output Y^N and the difference between actual ($\hat{P}_t$) and expected (${\hat{P}^e}_t$) inflation. This version of the Phillips curve is thus written as $Y_t = Y^N + \alpha(\hat{P}_t - {\hat{P}^e}_t)$ and, combining with equation 4.4 gives,

$$Y_t = Y^N + \alpha(\lambda\vartheta - {\hat{P}^e}_t) \quad (4.5)$$

Inflation expectations are assumed to be set and fixed from the end of the previous period, and so, any unexpected rise in prices and money via a nominal devaluation of the exchange rate could boost real gross domestic product above its natural level.

Ultimately, the government's welfare function W_t which it seeks to maximize, takes the form:

$$W_t = Y_t - \lambda E_t - \eta F_t - cT_t \quad (4.6)$$

From equation 4.4, the rates of nominal exchange rate devaluation, inflation and money growth are all proxied by ϑ which is endogenous and depends on the health of economic fundamentals existing in the economy. Worse fundamentals create a greater incentive for exchange rate devaluation (and a larger devaluation at that) to boost real gross domestic product above its natural level. Similarly, the level of government debt helps to inform the government's choice about whether to default on its foreign currency obligations or not. Therefore, economic fundamentals are classified into two groups: the level of government debt and the extent of exchange rate devaluation.

Both the costs of exchange rate devaluation and default on foreign currency government debt comprise fixed and variable components. In the case of the nominal exchange rate devaluation,

where $E_t = A_t + (1 - \eta)\, \Delta\rho B_t$, (4.7)

A_t represents the fixed costs of nominal exchange rate devaluation and Bauer et al. (2003) suggest that this can represent the government's loss of reputation, increased uncertainty and even the costs of higher inflation. Additionally, $\Delta\rho B_t$ represents the variable costs of nominal exchange rate devaluation which matter only if the government does not default on its debt. $\Delta\rho B_t$ captures the rise in the government's interest payments (where $\Delta\rho$ represents the change in the interest rate risk premium ρ) associated with nominal exchange rate devaluation and may represent the effects of a credit rating downgrade or investors' change in their assessment of the economy following the decision to exit the exchange rate peg. Thus, a larger exchange rate devaluation leads to larger increases in the risk premium and interest payable on foreign currency debt if the government chooses to repay its creditors:

$\Delta\rho = f(\vartheta) = \vartheta$, (4.8)

where the changes in the risk premium are a function $f(\vartheta)$ of exchange rate devaluation, ϑ.

Equations 4.7 and 4.8 together therefore give:

$E_t = A_t + (1 - \eta)\, \vartheta B_t$ (4.9)

where again, the variable costs of nominal exchange rate devaluation matter only if the government does not default on its foreign currency debt. If it does, it has no intention or capacity to borrow in future periods and higher costs to borrow do not matter. This assumption about the additional interest rate risk premium associated with a nominal exchange rate devaluation becomes particularly vital to the analysis later.

Similarly, the default on foreign currency debt imposes both fixed and variable costs on the government which equate to:

$$F_t = D_t + (1 - h)\,\rho(h)B_t \qquad (4.10)$$

where D_t represents the fixed costs associated with sovereign default and could be due to an associated fall in output or trade, while the variable costs $(1 - h)\,\rho(h)B_t$ represent the additional risk premium $\rho(h)$ that investors may charge to roll-over the foreign currency debt if the government partially defaults on their obligations. In this case, h (where $0 \leq h \leq 1$) denotes the loss given default and equates to 1 if the government fully defaults and less than 1 if it defaults only partially. The additional risk premium required by investors rises with higher losses given default where $\rho'(h) > 0$ and $\rho(0) = 0$. Bauer et al. (2003) focus their analysis on the case where, if the government does default, it defaults on the entire portion of its debt.

The government's budget constraint stipulates that its fixed expenditures and repayment of foreign currency debt due at the beginning of the period (if the government chooses to repay its obligations) must be financed via tax revenues, new borrowing (if the government has access to financial markets) and seigniorage:

$$\bar{G}_t P_t + (1 - \eta)\, S_t B_t = T_t P_t + (1 - \eta) S_t \frac{1}{1+R_t} B_{t+1} + (M_t - M_{t-1}) \qquad (4.11)$$

where $\bar{G}_t$ represents the government's real, fixed expenditure, $S_t B_t$ is the local currency value of debt previously incurred and now due at the start of period *t*, $T_t P_t$ are the nominal revenues from taxes and $(M_t - M_{t-1})$ is the additional seigniorage earned. If the government chooses not to repay its debt, its fixed expenditures must be financed entirely by seigniorage and taxes. The authors further assume that the government's existing level of foreign currency debt has reached the upper bound $\bar{B}$ up to which it can borrow internationally, and beyond which the financial markets determine the government to be insolvent. Then, merging equations 4.11, 4.2 and 4.3 gives:

$$\bar{G}_t(1 + \lambda\vartheta)P_{t-1} + (1 - \eta)(1 + \lambda\vartheta)S_{t-1}\bar{B} = T_t(1 + \lambda\vartheta)P_{t-1} + (1 - \eta)(1 + \lambda\vartheta)\, S_{t-1}\frac{1}{1+R_t}\bar{B} + \lambda\vartheta M_{t-1} \qquad (4.12)$$

Assuming absolute purchasing power parity implies that the real exchange rate Q_t always equals 1, and so letting $\frac{S_t}{P_t} = \frac{S_{t-1}}{P_{t-1}} = 1$ (assuming that foreign prices = 1) and solving equation 4.12 for T_t yields:

$$\bar{G}_t P_{t-1} + (1-\eta)S_{t-1}\bar{B} = T_t P_{t-1} + (1-\eta)S_{t-1}\frac{1}{1+R_t}\bar{B} + \frac{\lambda\vartheta}{(1+\lambda\vartheta)}M_{t-1}$$

$$T_t P_{t-1} = \bar{G}_t P_{t-1} + (1-\eta)S_{t-1}\bar{B} - (1-\eta)S_{t-1}\frac{1}{1+R_t}\bar{B} - \frac{\lambda\vartheta}{(1+\lambda\vartheta)}M_{t-1}$$

$$T_t = \bar{G}_t + (1-\eta)\frac{S_{t-1}}{P_{t-1}}\bar{B} - (1-\eta)\frac{S_{t-1}}{P_{t-1}}\frac{1}{1+R_t}\bar{B} - \frac{\lambda\vartheta}{(1+\lambda\vartheta)}\frac{M_{t-1}}{P_{t-1}}$$

$$T_t = \bar{G}_t + (1-\eta)\bar{B} - (1-\eta)\frac{1}{1+R_t}\bar{B} - \frac{\lambda\vartheta}{(1+\lambda\vartheta)}\frac{M_{t-1}}{P_{t-1}}$$

$$T_t = \bar{G}_t + \frac{(1-\eta)\bar{B}(1+R_t)-(1-\eta)\bar{B}}{1+R_t} - \frac{\lambda\vartheta}{(1+\lambda\vartheta)}\frac{M_{t-1}}{P_{t-1}}$$

$$T_t = \bar{G}_t + \frac{\bar{B}+\bar{B}R_t-\eta\bar{B}-\eta\bar{B}R_t-\bar{B}+\eta\bar{B}}{1+R_t} - \frac{\lambda\vartheta}{(1+\lambda\vartheta)}\frac{M_{t-1}}{P_{t-1}}$$

$$T_t = \bar{G}_t + (1-\eta)\frac{R_t}{1+R_t}\bar{B} - \frac{\lambda\vartheta}{(1+\lambda\vartheta)}\frac{M_{t-1}}{P_{t-1}} \qquad (4.13)$$

Now, combining equations 4.6, 4.9, 4.10 and 4.13 with equation 4.5 yields:

$$W_t = Y^N + \alpha(\lambda\vartheta - \hat{P}^e{}_t) - \lambda[A_t + (1-\eta)\vartheta B_t] - \eta[D_t + (1-h)\,\rho(h)B_t] - c[\bar{G}_t + (1-\eta)\frac{R_t}{1+R_t}\bar{B} - \frac{\lambda\vartheta}{(1+\lambda\vartheta)}\frac{M_{t-1}}{P_{t-1}}]$$

Recalling as before that $B_t = \bar{B}$ and $h = 1$, then

$$W_t = Y^N + \alpha\lambda\vartheta - \alpha\hat{P}^e{}_t - \lambda[A_t + (1-\eta)\vartheta\bar{B}] - \eta D_t - c\bar{G}_t - c(1-\eta)\frac{R_t}{1+R_t}\bar{B} + c\frac{\lambda\vartheta}{(1+\lambda\vartheta)}\frac{M_{t-1}}{P_{t-1}}$$

$$W_t = \alpha\lambda\vartheta + c\frac{\lambda\vartheta}{(1+\lambda\vartheta)}\frac{M_{t-1}}{P_{t-1}} - \lambda[A_t + (1-\eta)\vartheta\bar{B}] - c(1-\eta)\frac{R_t}{1+R_t}\bar{B} - \eta D_t + Y^N - \alpha\hat{P}^e{}_t - c\bar{G}_t$$

(4.14)

Equation 4.14 above outlines the full welfare benefits and costs of nominal exchange rate devaluation and default on sovereign debt. Bauer et al. (2003) explain each term's contribution to overall welfare. $\alpha\lambda\vartheta$ measures the benefit of unexpected exchange rate devaluation on output via the Phillips curve effect, while $c\frac{\lambda\vartheta}{(1+\lambda\vartheta)}\frac{M_{t-1}}{P_{t-1}}$ represents the increase in seigniorage from greater nominal exchange rate devaluation. From equation 4.9, $\lambda[A_t + (1-\eta)\vartheta\bar{B}]$ captures the total cost of nominal exchange rate devaluation (both fixed and variable) if the authorities opt to devalue the exchange rate. $-c(1-\eta)\frac{R_t}{1+R_t}\bar{B}$ and ηD_t are the benefits and fixed costs of default on sovereign debt, respectively, where $\frac{R_t}{1+R_t}\bar{B}$ captures the interest payments forgone if the government chooses to default on its debt, but that must be paid if it does not. As Bauer et al. (2003) suggest, forgone interest payments reduce the taxes required to be raised and this increases overall welfare. Finally, $Y^N - \alpha \hat{P}^e{}_t - c\bar{G}_t$ represent all of the terms exogenous to the authorities' decisions.

Thus, the government must jointly choose whether to devalue the nominal exchange rate, default on its foreign currency debts, do both or do neither to maximise its welfare function. Table 4.1 below illustrates the net benefits of choosing one of these four options (excluding the exogenous terms).

Table 4.1: Net Benefits of the Government's Decisions to Devalue, Default, Do Both or Do Neither

	No default ($\eta = 0$)	Default ($\eta = 1$)
No devaluation ($\lambda = 0$)	No crisis $W_t = -c\frac{R_t}{1+R_t}\bar{B}$	Debt crisis $W_t = -D_t$
Devaluation ($\lambda = 1$)	Currency crisis $W_t = \alpha\vartheta + c\frac{\vartheta}{(1+\vartheta)}\frac{M_{t-1}}{P_{t-1}} - A_t - \vartheta\bar{B} - c\frac{R_t}{1+R_t}\bar{B}$	Twin crisis $W_t = \alpha\vartheta + c\frac{\vartheta}{(1+\vartheta)}\frac{M_{t-1}}{P_{t-1}} - A_t - D_t$

Source(s): Bauer et al. (2003)

Bauer et al. (2003) go on to illustrate how their results may differ depending on whether investors expect a default on the government's debt or not. They further define the interest rate paid on the government's debt as $R_t = r_t + \rho_t$ where r_t captures the international risk-free rate and ρ_t captures the risk premium paid above and beyond the risk-free rate. If investors do not expect a default on the government's debt, then $\rho_t = 0$ and $R_t = r_t$. However, if investors expect the government to renege on its obligations to repay its debt, then $\rho_t = \infty$ and $R_t = \infty$. Whether investors expect a default on the government's debt or not affects the government's decision to devalue the exchange rate or not by changing the respective costs of those decisions. However, the cost to the government of defaulting on sovereign debt remains unchanged.

For the purpose of the analysis that follows however, assume that investors do not expect a default on the government's debt and so $R_t = r_t$. The government will choose to devalue the nominal exchange rate (and not default on its debt) versus do nothing if the benefits of exchange rate devaluation and the ensuing currency crisis exceed those of no crisis:

$$\alpha\vartheta + c\frac{\vartheta}{(1+\vartheta)}\frac{M_{t-1}}{P_{t-1}} - A_t - \vartheta\bar{B} - c\frac{R_t}{1+R_t}\bar{B} > -c\frac{R_t}{1+R_t}\bar{B}$$

There then probably exists a minimum level of required exchange rate devaluation above which a currency crisis becomes inevitable. Equating the two costs and solving for 0 gives:

$$\alpha\vartheta + c\frac{\vartheta}{(1+\vartheta)}\frac{M_{t-1}}{P_{t-1}} - A_t - \vartheta\bar{B} = 0 \qquad (4.15)$$

which suggests, at this point, that the government is indifferent between nominal exchange rate devaluation and no devaluation at all.

Following on from the literature review, a country's incentive to devalue its nominal exchange rate or exit the fixed exchange rate arrangement may vary across types of economies and may depend on the relative benefits of doing such. Therefore, it may be interesting to understand how the magnitudes of nominal exchange rate devaluation required to trigger a currency crisis vary by economic structure. One way to analyse this is to investigate the implication of varying α in equation 4.15. From equation 4.5, higher values of α suggest that real gross domestic product will increase more in those economies after a rise in the rate of nominal exchange rate devaluation and the unexpected increase in prices relative to expectations. At the same time, in economies where α is much smaller, the benefits to real gross domestic product from nominal exchange rate devaluation are smaller. Implicitly differentiating the rate of nominal devaluation in equation 4.15 with respect to α yields:

$$\frac{d\vartheta}{d\alpha} = -\frac{\vartheta}{\alpha - \bar{B} + c\frac{1}{(1+\vartheta)^2}\frac{M_{t-1}}{P_{t-1}}} \qquad (4.16)$$

Therefore, assuming that $0 \leq \alpha \leq 1$ and $c, \bar{B}, \frac{M_{t-1}}{P_{t-1}} \geq 0$, then for reasonable and small enough values of $\bar{B}$ and large enough values of $\frac{M_{t-1}}{P_{t-1}}$, $\frac{d\vartheta}{d\alpha} < 0$. Moreover, if international credit rating agencies do not downgrade a government's credit ratings and prompt a higher interest rate premium, $\frac{d\vartheta}{d\alpha} < 0$ no matter the debt levels. Thus, countries that benefit more from an increase in real gross domestic product after exchange rate devaluation likely require smaller levels of exchange rate devaluation to choose to exit the peg, while countries who benefit less from a rise in real gross domestic product relative to its potential require larger levels of exchange rate devaluation before they opt to exit the peg. If smaller countries that are more dependent on imports of goods and services are those who benefit less from nominal exchange rate devaluation, their incentive to devalue the exchange rate will be lower (and the minimum threshold above which they will devalue the exchange rate will be higher) than larger, less open countries for any given level of exchange rate devaluation.

Similarly, the government's choice of nominal exchange rate devaluation only over sovereign default only will depend on if:

$$\alpha\vartheta + c\frac{\vartheta}{(1+\vartheta)}\frac{M_{t-1}}{P_{t-1}} - A_t - \vartheta\bar{B} - c\frac{R_t}{1+R_t}\bar{B} > -D_t$$

Therefore, equating the two benefits and implicitly differentiating the rate of nominal exchange rate devaluation with respect to α yields the result in equation 4.16 as well. In other words, depending on the value of the other parameters and variables, the smaller the marginal benefit from a surprise nominal devaluation of the exchange rate, the greater the minimum magnitude of nominal exchange rate devaluation required for an exit of the peg to make sense relative to sovereign default on foreign currency debt.

The implication of this latter result is that, at lower levels of required exchange rate devaluation, a government may choose sovereign default over devaluation of the exchange rate, but as the magnitude of required exchange rate devaluation rises, devaluation may become (relatively) more attractive.

4.3.2 Romer (1993)

An obvious question which arises from the preceding analysis is thus: does α vary by country size and/or a country's degree of trade openness or dependence on imports of goods and services for domestic consumption and production? In fact, both theoretical and empirical evidence provide some support that the marginal benefit of nominal exchange rate devaluation may vary with a country's size and/or its degree of trade openness. In one of the earliest papers to illustrate this, Romer (1993) describes a framework where, because smaller, more open economies benefit less from surprise monetary expansion than larger, less open economies, they have less incentive to do so and as such, inflation in the former set of economies is lower.

Romer (1993) considers a two-country model where each country's citizens consume a variety of differentiated goods, a fraction ω of which is imported. ω can then be considered the economy's degree of trade openness with $0 \leq \omega \leq 1$. p, p^* and e measure the change in the log price of domestically produced goods in domestic currency, the change in the log price of foreign produced goods in foreign currency, and the change in the log nominal exchange rate (measured in local currency units per one foreign currency unit), respectively.[16] Thus, the domestic inflation rate (denoted by x) is given by:

[16] In Romer's (1993) framework, an asterisk (*) denotes a foreign variable.

$$x = \omega(e + p^*) + (1 - \omega)\, p \quad (4.17)$$

The author assumes that a citizen's utility function takes the constant elasticity of substitution (CES) form, with $\theta < 1$ representing the inverse of the elasticity of substitution between two goods. Further, domestically produced and foreign produced goods are imperfect substitutes and greater output leads to a real exchange rate depreciation. Thus,

$$e + p^* - p = \theta(y - y^*) \quad (4.18)$$

where y and y^* capture the changes in log domestic output and changes in log foreign output, respectively. Additionally, some share f of local prices is flexible in the short run, with the remaining $1 - f$ being fixed in the short run. Therefore,

$$p = f\tilde{p} + (1 - f)\, \bar{p} \quad (4.19)$$

where $\tilde{p}$ and $\bar{p}$ measure the changes in the log level of flexible prices and changes in the log level of fixed prices, respectively. Specifically, the log levels of real flexible prices ($\tilde{P} - X$) are a positive function of domestic output (Y), with $\emptyset$ capturing how responsive these prices are to output:

$$\tilde{P} - X = \emptyset Y$$

Taking changes in log levels and assuming prices are initially at equilibrium gives:

$$\tilde{p} - x = \emptyset y \quad (4.20)$$

while the demand for money domestically is expressed as:

$$m - p = y \quad (4.21)$$

with m capturing the change in the log stock of money.

The foreign country or rest of the world is modelled similarly and given as:

$$x^* = \omega p^* + (1 - \omega)(\, p - e) \quad (4.22)$$

$$p^* = f\tilde{p}^* + (1 - f)\, \bar{p}^* \quad (4.23)$$

$$\tilde{p}^* - x^* = \emptyset y^* \quad (4.24)$$

$$m^* - p^* = y^* \quad (4.25)$$

Taking equations 4.17 through 4.25 and finding total derivatives yields equations 4.26 through 4.29 and provides the key theoretical insights from this model (Romer, 1993):

$$\frac{dy}{dm} = \frac{(1-f)[(1-f)+[(1-\omega)\theta+\emptyset]f]}{[(1-f)+\emptyset f][(1-f)+(\emptyset+\theta)f]} \tag{4.26}$$

$$\frac{dp}{dm} = \frac{f[(1-f)\emptyset+f(\emptyset+\theta)\emptyset+(1-f)\omega\theta]}{[(1-f)+\emptyset f][(1-f)+(\emptyset+\theta)f]} \tag{4.27}$$

$$\frac{dx}{dm} = \frac{\emptyset f[(1-f)+f(\emptyset+\theta)]+(1-f)\omega\theta(1+f\emptyset)}{[(1-f)+\emptyset f][(1-f)+(\emptyset+\theta)f]} \tag{4.28}$$

$$\frac{d(e+p^{*}-p)}{dm} = \frac{(1-f)[(1-f)+\emptyset f]\theta}{[(1-f)+\emptyset f][(1-f)+(\emptyset+\theta)f]} \tag{4.29}$$

The central ideas from these derivatives are that a monetary impulse increases output but at a smaller pace in more open economies, while monetary impulses also increase overall consumer price inflation and inflation in domestically produced goods but at greater magnitudes in more open economies. Therefore, assuming that there are costs associated with higher inflation, central banks or monetary authorities must consider how open or dependent on imports of goods and services their respective economies are when determining whether the economy will benefit materially from an expansion in the money supply or a surprise devaluation of the nominal exchange rate. Finally, the degree of trade openness does not have any effect on the relationship between money growth and the real exchange rate.

Romer (1993) also provides empirical evidence which confirms his theoretical findings. He hypothesized that, because output in more open economies benefits less from a monetary impulse, "…policy-makers' incentives to expand are thus lower in more open economies, and equilibrium inflation under discretionary policy is therefore smaller..." (Romer, 1993). Estimating the effects of openness on a cross-section of 114 countries after the end of the Bretton Woods system, he finds a strong, statistically significant, and negative relationship between country openness and average inflation rates. These results hold for a large group of countries (including both with fixed and flexible exchange rates) except for the more developed (non-major, oil-producing) economies.

Lane (1997) also provides both theoretical and empirical support to Romer's (1993) results but also suggests that this relationship holds even when the economy is "…too small to affect international relative prices..." (Lane, 1997). His results are also robust (and in fact even stronger) to the inclusion of a country's size in the empirical regression "…which suggests that openness is not just working through a terms of trade effect…" (Lane, 1997).

Other empirical work also suggests some relationship between inflation and openness, but the statistical significance and direction of this relationship is not always clear cut. For example, Eijffinger and Qian (2016) suggest that the relationship between openness and the response of inflation to changes in the output gap in the Phillips curve may not be homogenous across a sample of countries that are members of the Organisation for Economic Co-operation and Development. Further, the direction in which the sign changes appears not to be uniform. This inconclusiveness of these results appears to support Romer's (1993) finding that the relationship between openness and inflation was statistically insignificant for a sample of developed economies. Nonetheless, they highlight that "…relaxing the parameter homogeneity assumption, we find that trade openness has significantly changed the slope of the Phillips curve in several major industrial countries. In our model with both trade and financial openness, a significant effect of trade openness is found in Canada, France, Italy, Sweden and the United States." (Eijffinger and Qian, 2016).

Therefore, if one interprets the required magnitude of exchange rate devaluation outlined in Bauer et al.'s (2003) framework as the minimum magnitude of nominal exchange rate devaluation necessary to correct a country's estimated degree of real exchange rate overvaluation or misalignment, and if in fact, as some literature suggests, α varies by country size and/or trade openness or the degree of dependence on imports of goods and services, the next obvious questions to ask seem to be:

1. Does the magnitude of real exchange rate overvaluation or misalignment which predicts a currency crisis or crash vary by country size and/or openness or the degree of dependence on imports of goods and services?
2. Empirically, and all else equal, are defaults on foreign currency debt more likely than nominal exchange rate devaluations at given levels of real exchange rate overvaluation or misalignment for smaller economies or those more dependent on imports of goods and services than their larger, less open counterparts?

4.4 Estimation of Real Exchange Rate Misalignment

While pronouncements and estimates of the degree of real exchange rate misalignment are often made and discussed in academia, policy settings and among investors in financial markets, the literature to date has not identified one set way to determine whether an exchange rate is over- or under-valued. Determining the extent of real exchange rate misalignment first requires estimating an equilibrium real exchange rate, away from which the current level of the real exchange rate has strayed. Several approaches to estimating equilibrium exchange rates over various time horizons have been explored, with some appearing more popular than others (see Driver and Westaway (2004) for a comprehensive review of approaches used to estimate equilibrium real exchange rates).

Clark and MacDonald (1998) describe an economy's real exchange rate in terms of its long-run and medium-run (over the business cycle) fundamentals, and its transitory drivers. Specifically,

$$q_t = \beta_1' Z_{1t} + \beta_2' Z_{2t} + \tau' T_t + \varepsilon_t \tag{4.30}$$

where q_t captures the actual real exchange rate, Z_{1t} and Z_{2t} capture the long-run economic fundamentals and medium-run economic fundamentals, respectively, T_t represents short-run or transitory drivers and ε_t is a random error term. β_1', β_2' and τ' capture coefficients of the respective variables and the subscript t represents time. Clark and MacDonald (1998) also define the current equilibrium real exchange rate $q_t{}'$ as the level of the real exchange rate consistent with the current values of the medium-run and long-run economic fundamentals and where the transitory or short-run and random components are zero:

$$q_t{}' = \beta_1' Z_{1t} + \beta_2' Z_{2t} + 0 \tag{4.31}$$

The current misalignment (denoted as cm_t) of the real exchange rate is thus determined as the difference between the actual level of the real exchange rate and its current equilibrium or the short-term deviations away from current fundamentals:

$$cm_t = q_t - q_t{}'$$

$$cm_t = \beta_1' Z_{1t} + \beta_2' Z_{2t} + \tau' T_t + \varepsilon_t - (\beta_1' Z_{1t} + \beta_2' Z_{2t})$$

$$cm_t = \tau' T_t + \varepsilon_t \tag{4.32}$$

However, Clark and MacDonald (1998) also point out that economic fundamentals themselves may deviate from sustainable or acceptable levels. Thus, defining $\bar{Z}_{1t}$ and $\bar{Z}_{2t}$ as the long-run or sustainable values of the long-run and medium-run economic fundamentals, the authors describe a total real exchange rate misalignment (denoted as tm_t) as the deviation of the real exchange rate away from the level implied by the long-run or sustainable values of the country's economic fundamentals:

$$tm_t = q_t - (\beta_1' \bar{Z}_{1t} + \beta_2' \bar{Z}_{2t}) \tag{4.33}$$

Moreover, total real exchange rate misalignment can be further characterized as the combination of the current misalignment of the real exchange rate and the deviation of economic fundamentals away from their sustainable or long-run values:

$$tm_t = (q_t - q_t') + [\beta_1'(Z_{1t} - \bar{Z}_{1t}) + \beta_2'(Z_{2t} - \bar{Z}_{2t})]$$

$$tm_t = \tau' T_t + \varepsilon_t + [\beta_1'(Z_{1t} - \bar{Z}_{1t}) + \beta_2'(Z_{2t} - \bar{Z}_{2t})] \quad (4.34)$$

The measure of real exchange rate misalignment can therefore be defined as either a medium-term or long-run phenomenon, and the choice of model to estimate the equilibrium real exchange rate will depend on the time horizon over which the researcher cares about and their definition of sustainability. Three popular approaches to estimating real exchange rate misalignment in the literature are the internal-external balance approach, the behavioural equilibrium exchange rate (BEER) approach and the atheoretical permanent equilibrium exchange rate (APEER) approach. With the internal-external balance approach, the equilibrium exchange rate is determined as the rate at which the economy is both in internal (economy at full employment and low inflation) and external (the current account balance or savings less investment is equal to a level determined to be sustainable) balance. The fundamental equilibrium exchange rate (FEER) model (a medium-run approach to estimating equilibrium exchange rates) is an example of this, and here the total real exchange rate misalignment is determined as the deviation of the real exchange rate away from the level required for the current account to be equal to a sustainable capital (or financial in more recent editions of the International Monetary Fund's Balance of Payments manual) account. However, this approach requires some judgement of what is deemed to be a sustainable financial account (it is normative by nature) and the estimated "…FEER is likely to be sensitive to the choice of the sustainable capital account…" (MacDonald, 2007).

A more flexible regression approach which has been popularized in the literature (perhaps most prominently by Clark and MacDonald (1998)) is the behavioural equilibrium exchange rate approach. Unlike the internal-external balance approach which is typically based on a model of the current account or savings less investment, the behavioural equilibrium exchange rate is not based on any one model of the real exchange rate (MacDonald, 2007). In its initial form, the behavioural equilibrium exchange rate approach has been used to estimate short-run or medium-run measures of real exchange rate misalignment by considering the current values of economic fundamentals and does not require researchers to make any judgements about the sustainable levels of these fundamentals. Nonetheless, the behavioural equilibrium exchange rate model's flexibility permits researchers to substitute current values of economic fundamentals for their long-run values to derive estimates of total real exchange rate misalignments. This approach, referred to as the permanent equilibrium exchange rate (PEER) approach, can provide additional insight into the drivers of real exchange rate misalignment over time (MacDonald, 2007).

Finally, while the fundamental equilibrium exchange rate and behavioural equilibrium exchange rate approaches require researchers to consider the real exchange rate as a function of macroeconomic variables, the atheoretical permanent equilibrium exchange rate model simply applies statistical techniques to extract the long-run or permanent component of the real exchange rate from the cyclical or more transient components. As a result, unlike the fundamental equilibrium exchange rate approach, this approach requires no judgement about what levels of economic fundamentals are considered sustainable or unsustainable. However, it offers no insight about what factors are driving the level of real exchange rate misalignment and the estimate of the equilibrium real exchange rate is likely sensitive to the statistical technique used (for example, the Hodrick-Prescott (HP) filter, simple linear or quadratic trends, or a long-run average).

Table 4.16 in Appendix 4.A.1 summarises the methods used to estimate equilibrium real exchange rates in some of the research covered in the literature review and other relevant studies. Of these approaches, variants of the behavioural equilibrium exchange rate and atheoretical permanent equilibrium exchange rate appear to be most popular in the currency crisis literature.

4.5 Empirical Methodology and Data

As discussed in the literature review, several authors have investigated the determinants of currency crises, with special emphasis on the effects of real exchange rate misalignments on the probability of these crises occurring. Among the many specifications used, this paper leverages the approach and some of the variables used in Kose et al. (2019). Using a panel logit approach, the authors model the probability of debt, banking, and currency crises as functions of several variables that have been previously identified in the early warning crisis literature. Specifically, a binary crisis indicator Y_{it} (where *i* denotes each cross-sectional unit and *t*, time) is expressed as a function of crisis determinants X_{it-1}, unobserved country heterogeneity μ_i which can be modeled either as random effects or fixed effects, and an error ε_{it} (Kose et al., 2019). β captures the coefficients which measure the common sensitivities of the crisis indicator to its determinants.

$$Y_{it} = \beta' X_{it-1} + \mu_i + \varepsilon_{it} \tag{4.35}$$

Further, as in Kose et al. (2019), the probability of a crisis can be expressed as

$$\Pr(Y_{it} = 1 \mid X_{it-1}, \beta, \mu_i) = \psi(\beta' X_{it-1} + \mu_i) \tag{4.36}$$

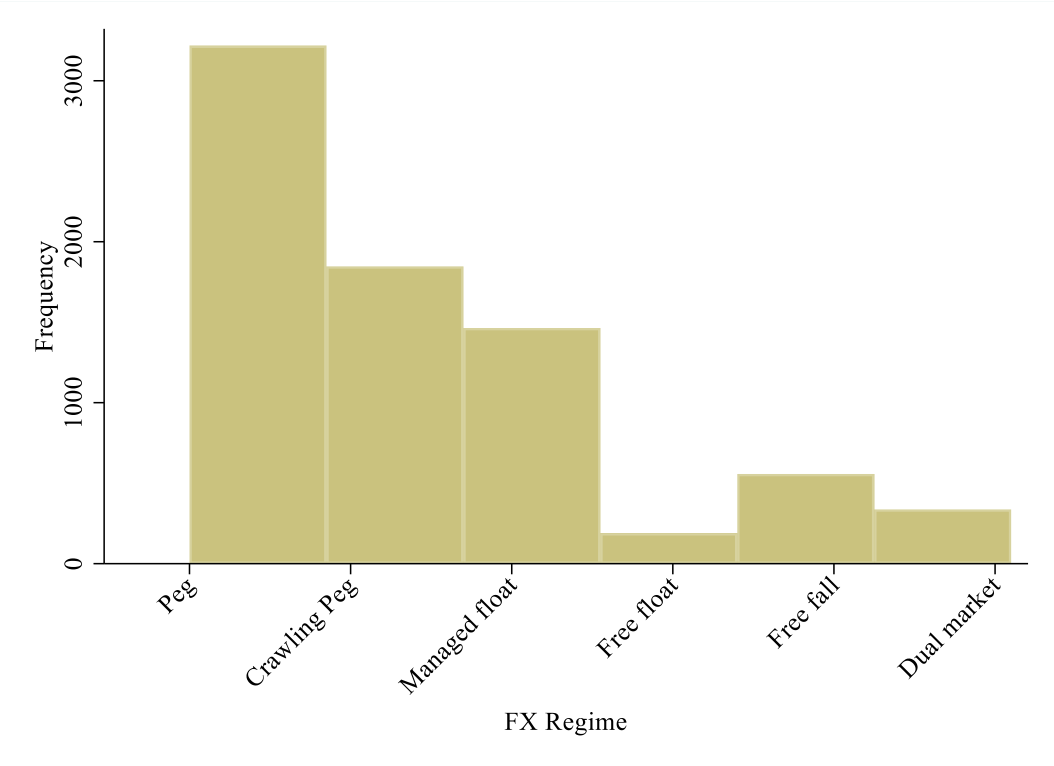
where Y_{it} = 1 if a crisis occurs in that year and Y_{it} = 0 otherwise. Whether the logit (logistic distribution) or probit (normal distribution) model is used to estimate the unknown parameters depends on the assumed distribution of the error term.

Kose et al. (2019) leverage the currency crisis indicator produced by Laeven and Valencia (2018) which identifies 236 currency crashes between 1970 and 2017. Laeven and Valencia (2018) leverage Frankel and Rose's (1996) approach to defining a currency crash and identify a crisis as a period during which the nominal exchange rate depreciates versus the United States dollar by at least 30% year-over-year and this depreciation is at least 10 percentage points greater than the previous year. Additionally, to avoid double counting the same crisis which may extend over several years, the crisis is defined as the first year within a 5-year window to meet the above criteria.

Kose et al.'s (2019) vector of potential currency crisis regressors X_{it-1} includes changes in real US interest rates, growth in real gross domestic product, short-term debt as a share of external debt, the ratio of debt service on external debt to exports, international reserves measured in months of imports of goods and services, changes in government debt and private debt respectively as shares of nominal gross domestic product and their interaction, net foreign direct investment inflows (FDI) as a share of gross national income, the ratio of foreign liabilities to foreign assets and a measure of real exchange rate overvaluation measured as the percentage deviation of the real effective exchange rate away from an HP-filtered trend. Of these, only changes in real US interest rates, international reserves, changes in government and private debt and their interaction, foreign direct investment and the level of real exchange rate overvaluation proved to be statistically significant determinants of currency crises in their analysis. Similarly, Frankel and Rose (1996) identified concessional debt, public sector debt, the ratio of foreign direct investment to debt, international reserves as a ratio of imports of goods and services, exchange rate overvaluation, domestic credit, and foreign interest rates as significant predictors of currency crises, while growth in real gross domestic product was contemporaneously associated with currency crashes.

The sample used in this chapter consists of an unbalanced panel of annual observations for 114 countries spanning 1974 to 2017. Like Kose et al. (2019), Y_{it} is defined as Laeven and Valencia's (2018) binary currency crash indicator. In this chapter, Y_{it} is defined as a currency crash rather than using other currency crisis definitions which focus on exchange market pressure (including pressure on foreign exchange reserves and/or nominal, domestic interest rates) because the objectives in this chapter focus on countries' willingness to maintain a stable exchange rate or limit sharp fluctuations in their managed exchange rates in the face of significant exchange rate misalignment rather than whether their foreign exchange reserves experience sharp, but temporary declines. Thus, the dataset includes only periods in which economies started the year with either a de facto fixed exchange rate regime (hard peg) or a de facto intermediate regime (crawling peg or managed float) as defined by Ilzetzki et al. (2016). Including observations where economies started the period with freely floating or dual exchange rate regimes, or where exchange rates were already in free fall may bias the results since these currencies are, by definition, more likely to experience sharp movements. In any event, periods of freely floating or falling or dual exchange rates account for a small share (less than 15% in total) of the original sample (Figure 4.1).

Figure 4.1: Distribution of Sample Observations by Exchange Rate (FX) Regime



Source(s): Ilzetzki et al. (2016), author's calculations

Because the coverage of the determinants of currency crises highlighted two paragraphs prior varies widely across countries, and in an effort to provide the appropriate balance between including as many relevant regressors as possible and ensuring sufficient data and country coverage, several, but not all, of these regressors are included in the currency crisis regressions in this chapter. Thus, the vector of regressors X_{it-1} for use in the baseline regressions includes:

$$X_{it-1} = \{\overset{+}{\Delta r_t^{US}}, \overset{-}{Res_{it-1}}, \Delta Gov\overset{+}{\ Debt_{it-1}}, \overset{-}{FDI_{it-1}}, \overset{+}{Mis_{it-1}}\} \quad (4.37)$$

where Δr_t^{US}, Res_{it-1}, $\Delta Gov\ Debt_{it-1}$, FDI_{it-1}, Mis_{it-1} represent changes in real US interest rates, foreign exchange reserves measured in months of imports of goods and services, changes in the ratio of government debt to nominal gross domestic product, foreign direct investment as a share of gross national income, and the deviation of the real effective exchange rate away from its equilibrium level, respectively. The superscripts highlight prior expectations about the nature of the relationship between each regressor and the probability of a currency crisis based on Kose et al.'s (2019) and Frankel and Rose's (1996) findings. Greater foreign exchange reserves provide more liquidity with which to defend or support the exchange rate, while foreign direct investment typically represents a more stable form of foreign financing which typically leads to greater domestic capital accumulation and productive capacity (Frankel and Rose, 1996). Sharper increases in real US interest rates may induce foreign investors to divert capital from the domestic economy to assets denominated in United States dollars elsewhere which may put downward pressure on the exchange rate. Finally, the faster accumulation of government debt may signal a large (and potentially unsustainable) fiscal deficit or create concerns about the government's ability to repay debt denominated in foreign currency, potentially leading to speculative attacks on the currency. To control for potential feedback effects and endogeneity, all variables except changes in real US interest rates enter each regression with a period's lag. Table 4.2 below explains how each variable is constructed and its source(s).

Table 4.2: Definition of Variables and Sources

Variable	Definition	Sources
Currency Crisis	Currency Crash	Laeven and Valencia (2018)
Δr_t^{US}	Percentage point change in real lending rate in the United States of America, deflated by the GDP deflator	World Bank's World Development Indicators
Res_{it-1}	Total international reserves excluding gold (foreign exchange reserves) in months of imports of goods and services	World Bank's World Development Indicators
$\Delta Gov\ Debt_{it-1}$	Percentage point change in general government debt to nominal gross domestic product ratio[17]	Global debt database; International Monetary Fund World Economic Outlook
FDI_{it-1}	Net inflows of foreign direct investment as a ratio of gross national income	World Bank's World Development Indicators
Mis_{it-1}	Percentage deviation of the real effective exchange rate from the equilibrium real exchange rate	Couharde et al. (2018)

[17] In cases where general government debt is not available, the change in central government debt as a share of GDP is used.

Finally, instead of the HP-filtered estimates of real exchange rate misalignment used in Kose et al. (2019), this chapter leverages the estimates of real exchange rate misalignments produced by Couharde et al. (2018) using the behavioural equilibrium exchange rate approach. Couharde et al. (2018) calculate real exchange rate misalignments as the percentage deviation of the real effective exchange rate from its equilibrium value. The latter is estimated as the fitted value of the real effective exchange rate from a long-run function of economic fundamentals. Couharde et al. (2018) partially follow Clark and MacDonald (1998) and model the real effective exchange rate as a long-run function of the country's net foreign asset position, its terms of trade, and real gross domestic product per capita relative to its trading partners as a measure of the Balassa-Samuelson effects, using the Pooled Mean Group estimator (Pesaran et al., 1999). Each of these three variables is expected to positively affect the real effective exchange rate in the long run. The net foreign assets represent the accumulation of current account surpluses over time. A negative net position suggests accumulated deficits and a build-up of external liabilities to fund these. To repay creditors requires the future accumulation of current account surpluses which may require a depreciation in the real effective exchange rate. Conversely, a positive net foreign asset position implies previously accumulated surpluses over time which permits real exchange rate appreciation in the long run (Couharde et al., 2018). The Balassa-Samuelson effect describes the process via which greater productivity in the traded versus non-traded sectors of the domestic economy generates higher wages in both sectors and push relative prices in the non-traded sector higher, thereby appreciating the real exchange rate. Similarly, the positive income effects generated from an improving terms of trade are expected to increase the demand for and prices of non-traded goods and appreciate the real exchange rate. The behavioural equilibrium exchange rate's flexibility permits its use across a wide range of countries without requiring the systematic determination of sustainable current account or capital account levels for each economy. Thus, Couharde et al.'s (2018) EQCHANGE database provides one of the most comprehensive, publicly available coverages of real effective exchange rate estimates, and also includes an unbalanced panel of equilibrium real exchange rates and real exchange rate misalignments for 182 countries over the period 1973-2018.

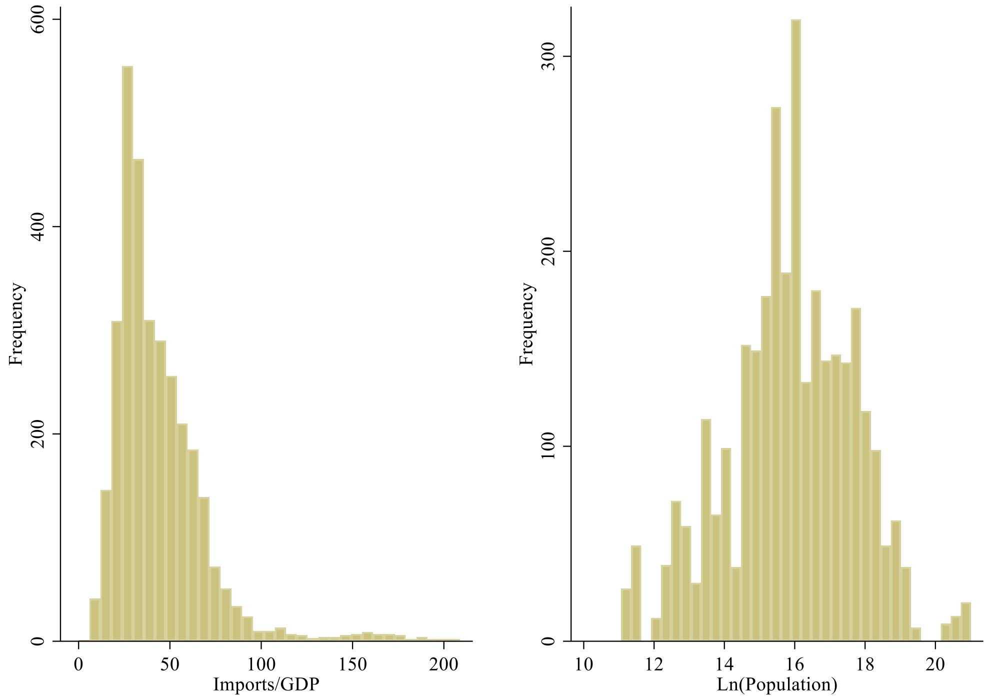
4.6 Results

4.6.1 Preliminary Data Analysis

Before diving into the results from the regression analysis, it is imperative to understand the characteristics of the countries in the sample. Figure 4.2 describes the distribution of observations by the degree of trade openness (measured as imports of goods and services as a ratio of nominal gross domestic product)[18] and the size of the population.[19] On average, imports of goods and services account for about 44% of nominal gross domestic product in the sample, but this metric is widely dispersed, with a standard deviation of 26% and imports of goods and services accounting for over 200% of nominal gross domestic product in the most open economy. Similarly, the average population in the sample is 36.8 million people, but the sample includes countries as small as 64,000 people and as large as 1.3 billion. Thus, the sample captures a wide range of economic structures.

[18] Most data on imports of goods and services as a share of nominal gross domestic product were sourced from the World Bank's World Development Indicators. However, in a few instances where annual observations were missing, estimates from the United Nations National Accounts Statistics were used to complement data from the World Bank's World Development Indicators.
[19] Population data are sourced from the World Bank's World Development Indicators.

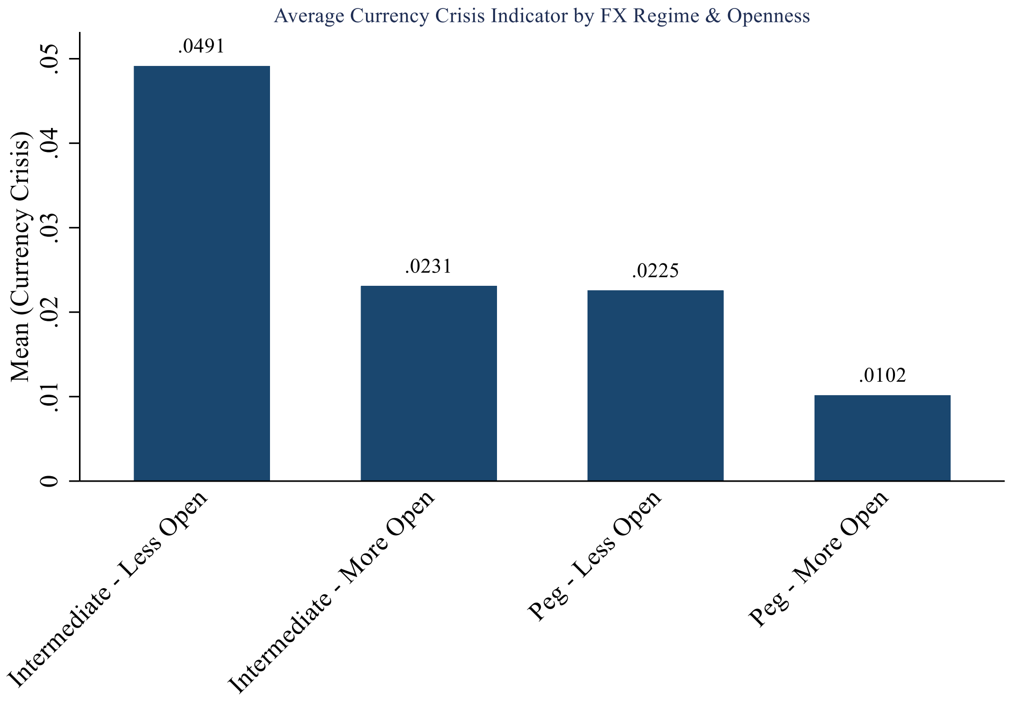
Figure 4.2: Distribution of Imports of Goods and Services as a Ratio of Nominal Gross Domestic Product (%) and ln(Population)



Source(s): World Bank's World Development Indicators, United Nations National Accounts Statistics, author's calculations

Figure 4.3 below calculates mean currency crisis probabilities by exchange rate regime and degrees of trade openness. More open economies are those whose average imports of goods and services as a ratio of nominal gross domestic product exceeds that of the sample median. The crisis probabilities appear to support Tarashev and Zabai's (2019) finding that economies with pegged exchange rates appear less likely to experience currency crises but goes further to suggest that more open economies suffer from fewer currency crises as well. Thus, the occurrence of currency crises appears to be decreasing in both the degree of trade openness and the stability of the de facto exchange rate regime – more open economies with exchange rate pegs have experienced fewer currency crises on average over the past four and a half decades than both less open economies and those with intermediate exchange rate regimes.

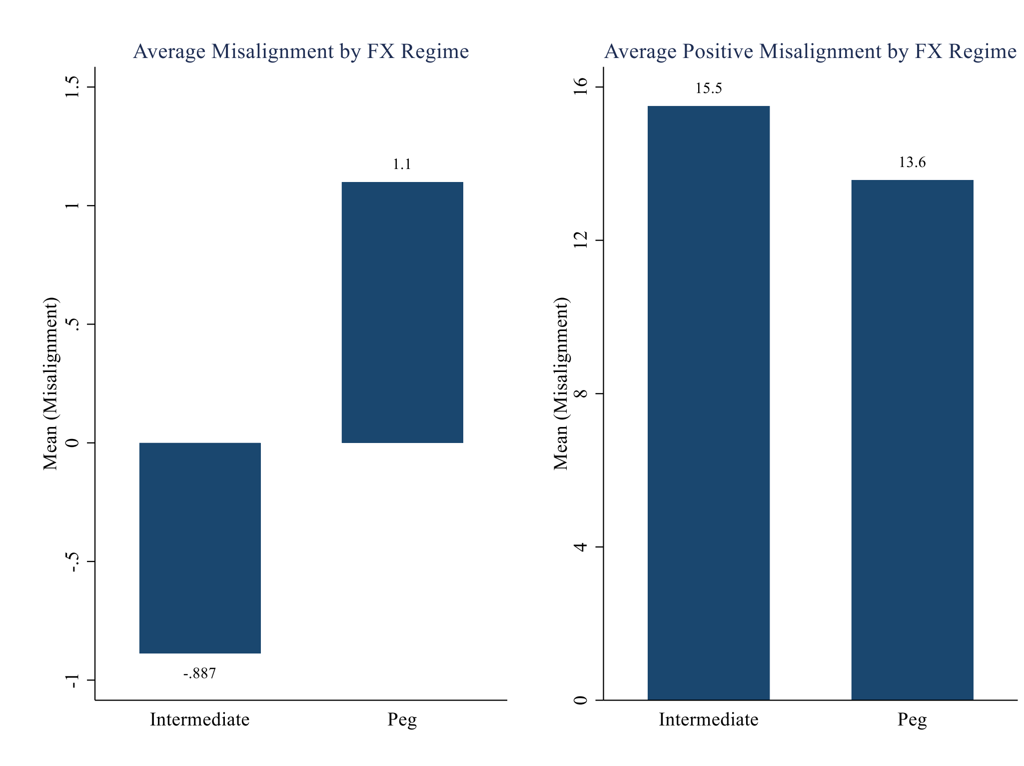
Figure 4.3: Mean Currency Crisis Indicator by Exchange Rate Regime and Degree of Trade Openness



Source(s): Ilzetzki et al. (2016), Laeven and Valencia (2018), World Bank's World Development Indicators, United Nations National Accounts Statistics, author's calculations

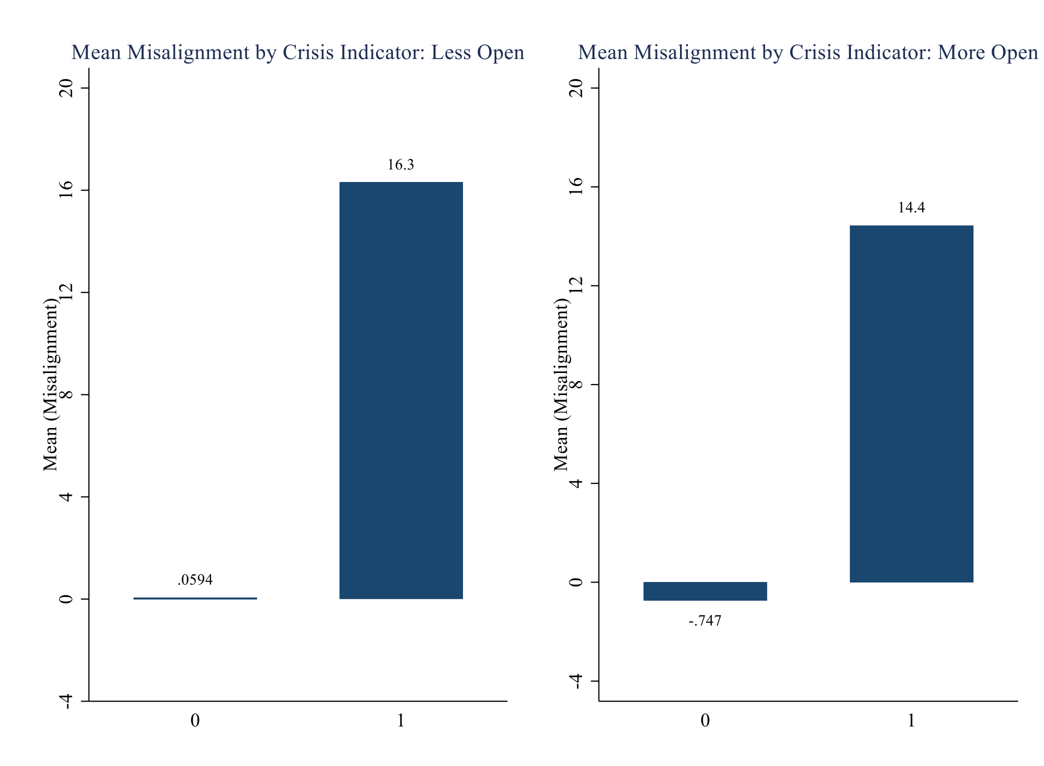
On average, real exchange rates are close to their equilibrium levels across both regimes – periods of real exchange rate undervaluation offset periods of real exchange rate overvaluation (Figure 4.4). However, isolating just positive real exchange rate misalignments, on average, economies with fixed exchange rates have lower levels of real exchange rate misalignment than their counterparts with crawling pegs or managed floats. Further, the extent of average real exchange rate overvaluation immediately prior to a currency crisis is around 15% (though slightly higher in less open economies) (Figure 4.5). Outside of crisis episodes, currencies are also close to their equilibrium levels.

Figure 4.4: Average Real Exchange Rate Misalignment by Exchange Rate Regime (%)



Source(s): Ilzetzki et al. (2016), Couharde et al. (2018), author's calculations

Figure 4.5: Average Exchange Rate Misalignment (One Year Prior) by Crisis Indicator – Sample Segmented by Degree of Trade Openness (%)



Source(s): Laeven and Valencia (2018), Couharde et al. (2018), author's calculations

4.6.2 Main Regression Results

Table 4.3 presents logit and probit estimates of the baseline, currency crisis regression. The signs and statistical significance of the parameter estimates are broadly in line with expectations and across models. Specifically, changes in real US interest rates and government debt to nominal gross domestic product both increase the probabilities of countries experiencing currency crashes of at least 30% one year later. However, only the effects of the former are statistically significant across models. Higher foreign exchange reserves relative to imports of goods and services and greater foreign direct investment relative to gross national income both have negative impacts on the probability of suffering a currency crisis, but only the effects of the former are statistically significant. Finally, as expected, the degree of real exchange rate overvaluation significantly increases the chances that the economy will experience a currency crash within one year.

Table 4.3: Discrete Choice Baseline Logit and Probit Regression Estimates – Dependent Variable: Currency Crash

Variable	**Baseline Logit, RE**	**Baseline Logit, FE**	**Baseline Logit, Pooled**	**Baseline Probit, RE**
Mis_{it-1}	0.028***	0.037***	0.026***	0.012***
	(0.006)	(0.008)	(0.006)	(0.003)
Δr_t^{US}	0.199***	0.204***	0.196**	0.086**
	(0.073)	(0.065)	(0.083)	(0.034)
$\Delta Gov\ Debt_{it-1}$	0.009	0.005	0.010	0.005
	(0.014)	(0.021)	(0.012)	(0.007)
Res_{it-1}	-0.146*	-0.347***	-0.129**	-0.062**
	(0.076)	(0.119)	(0.061)	(0.031)
FDI_{it-1}	-0.029	-0.006	-0.029	-0.012
	(0.018)	(0.077)	(0.021)	(0.009)
Constant	-3.717***		-3.617***	-1.960***
	(0.272)		(0.229)	(0.112)
Total Observations	3,170	1,469	3,170	3,170

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust to potential heteroskedasticity and autocorrelation.*

The results in Table 4.3 above suggest that regression estimates are robust to inclusion of either random, fixed or no (pooled) effects, or the assumption that the error term takes on a logistic or normal distribution. Typically, a Hausman test would determine whether estimates from the fixed effects and random effects specifications are similar, but in this case, the number of observations used under each specification vary substantially. The fixed effects model drops cross sectional units and observations where the dependent variable does not vary over time: that is, for countries in the sample who have never experienced a currency crash and thus exhibit no variation in the binary dependent variable. Thus, in order to keep the sample size as large as possible and understand the characteristics of those countries who have never experienced a currency crash, the random effects specification of the logit model is preferred.

To answer the question of whether the relationship between the degree of real exchange rate misalignment and the probability of currency crises occurring differs by the degree of openness and population size, the baseline random effects, logit model is re-estimated, first by splitting the sample between more open (greater than the median) and less open (lower than the median) economies, and, alternatively, by splitting the sample between larger (populations larger than the median) and smaller (populations smaller than the median) countries (see Table 4.4).

Table 4.4: Discrete Choice Logit Random Effects Estimates by Degree of Openness and Size – Dependent Variable: Currency Crash

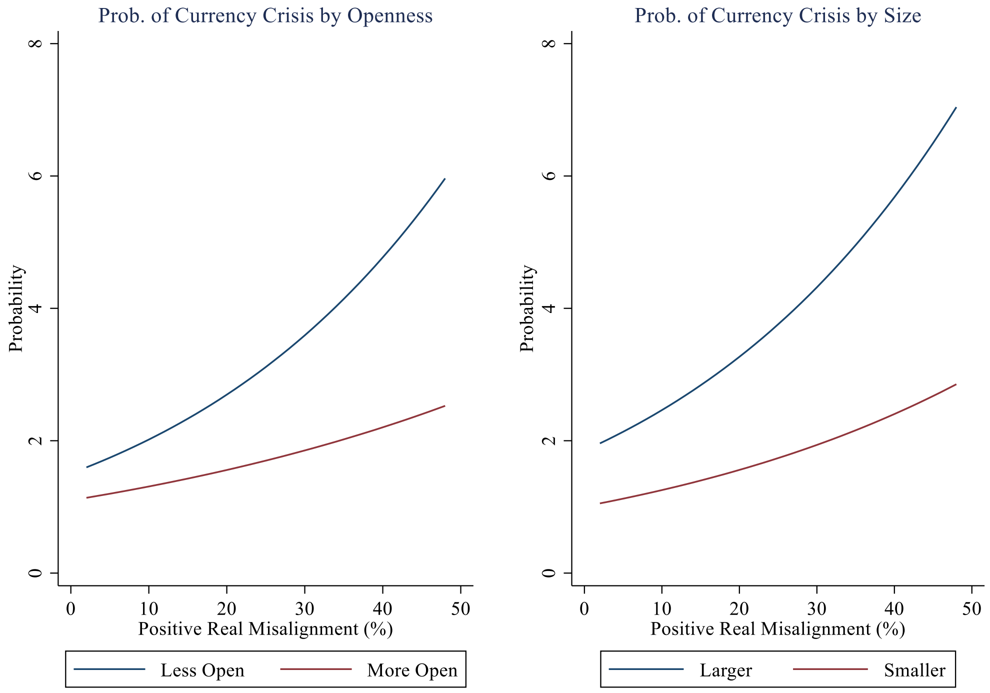
Variable	**Baseline**	**Less Open**	**More Open**	**Larger**	**Smaller**
Mis_{it-1}	0.028***	0.031***	0.018*	0.031***	0.022**
	(0.006)	(0.006)	(0.009)	(0.006)	(0.009)
Δr_t^{US}	0.199***	0.214**	0.182*	0.192**	0.210*
	(0.073)	(0.103)	(0.102)	(0.095)	(0.117)
$\Delta Gov\ Debt_{it-1}$	0.009	0.026	-0.001	0.031	-0.004
	(0.014)	(0.018)	(0.019)	(0.020)	(0.020)
Res_{it-1}	-0.146*	-0.089	-0.359**	-0.124*	-0.257
	(0.076)	(0.076)	(0.168)	(0.074)	(0.185)
FDI_{it-1}	-0.029	-0.084	-0.020	-0.043	-0.020
	(0.018)	(0.105)	(0.015)	(0.033)	(0.020)
Constant	-3.717***	-3.696***	-3.346***	-3.626***	-3.584***
	(0.272)	(0.337)	(0.469)	(0.311)	(0.524)
Total Observations	3,170	1,652	1,518	1,708	1,462

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust to potential heteroskedasticity and autocorrelation.*

In most cases (except for the very small coefficients on $\Delta Gov\ Debt_{it-1}$), the estimated parameters each carry the expected signs, but now, the levels of statistical significance vary relative to the baseline and between groups of countries. In most cases, larger standard errors in the regressions estimated on smaller subsamples mean that the statistical significance of changes in real US interest rates and the level of foreign exchange reserves either vanishes or falls. Quite notably, foreign exchange reserves provide a greater benefit to smaller and more open economies than their larger and less open counterparts, but the effects are no longer statistically significant for smaller economies. Most noteworthy however, is that the size of the coefficient on the measure of real exchange rate overvaluation, while still statistically significant across all regressions, is greater and more statistically significant for larger and less open economies than in smaller, more open markets. In other words, the former groups of economies are more likely to experience or choose sharp devaluations in their currencies in response to growing real exchange rate misalignments.

The results in the previous paragraph provide some evidence that smaller, more open economies are less susceptible to currency crashes, even at the same level of real exchange rate misalignment. These results are confirmed in Figure 4.6 below which plots the probabilities of currency crises occurring for each of the four groups of countries while holding the levels of foreign exchange reserves, foreign direct investment, and changes in government debt and US interest rates constant at their sample averages. For every level of exchange rate misalignment, the probability of a currency crash is greater for less open economies than more open markets, and this gap widens as the level of real exchange rate misalignment grows. This result is robust, and the difference is perhaps even more stark, if we split the sample by the size of the population instead of by trade openness.

Figure 4.6: Estimated Probabilities of Currency Crises by Degree of Real Exchange Rate Overvaluation – Split by Degree of Openness and Population Size[20]



Source: Author's calculations

The literature review suggested that the nature of a country's exchange rate regime may also help to explain cross-country differences in the probability of suffering a currency or debt crisis. Therefore, a reasonable question which may naturally arise from the results presented in this section so far is whether they are robust to controlling for variations in exchange rate regimes. Table 4.5 below illustrates the results if the sample is split further by exchange rate regime. Again, splitting the sample into even smaller subsamples reduces the precision of parameter estimates relative to the baseline specification estimated on the full sample. However, a few interesting results arise. In open economies with fixed exchange rates, changes in real US interest rates have a larger, statistically significant effect than in all other economies, likely reflecting their loss of monetary independence in an environment with perfect capital mobility. If the rise in US rates goes unmatched domestically, foreign portfolio flows may be diverted from the domestic economy and could put pressure on the exchange rate peg. The effects of foreign direct investment are now statistically significant

[20]The illustrative graphs in Figure 4.6 plot the predicted probabilities of a currency crash for different hypothetical values of real exchange rate misalignment by applying the coefficients from the models estimated in Table 4.4 where the sample has been split by the degrees of openness and the size of the population, respectively. In each case, the values for the other regressors from the model (the levels of foreign exchange reserves, foreign direct investment, and changes in government debt and US interest rates) are held constant at their respective sample averages.

for economies with fixed exchange rates, while foreign exchange reserves are most critical in open economies with intermediate exchange rate regimes since central banks in these countries likely need sufficient foreign exchange reserves to actively manage the exchange rate within a pre-announced or pre-determined band. Finally, notwithstanding larger standard errors, the sizes of the coefficients on the estimates of real exchange rate overvaluation are larger for less open economies (regardless of exchange rate regime) than more open economies, suggesting that openness itself matters for the relationship between real exchange rate misalignment and the probability of a currency crisis.

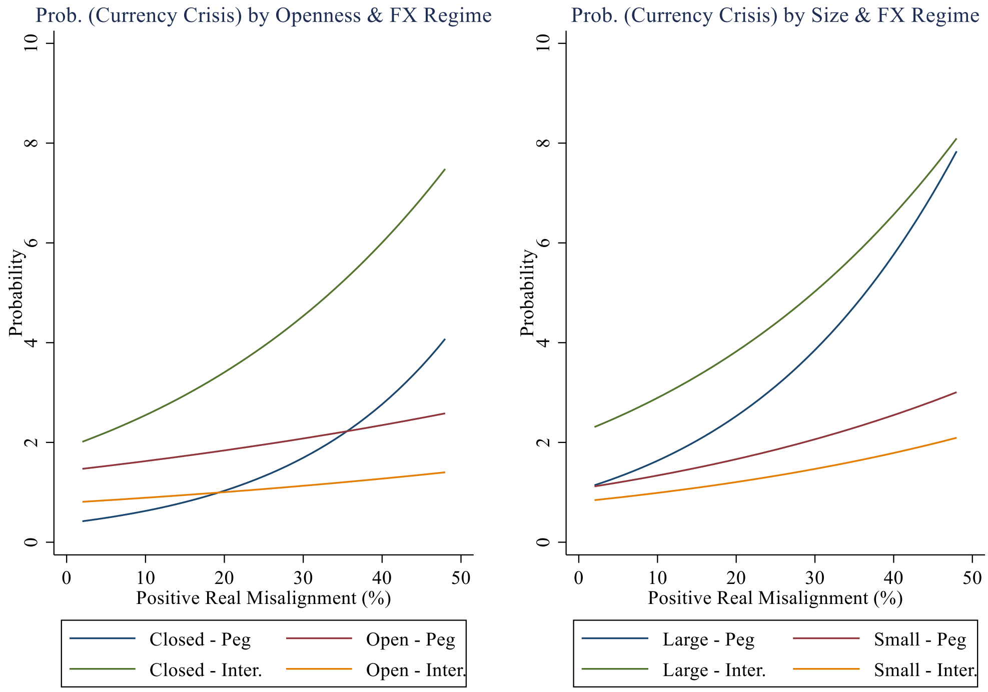
Table 4.5: Discrete Choice Logit Random Effects Estimates by Degree of Openness and Exchange Rate Regime – Dependent Variable: Currency Crash

Variable	**Baseline**	**Less Open & Fixed**	**Less Open & Intermediate**	**More Open & Fixed**	**More Open & Intermediate**
Mis_{it-1}	0.028***	0.050	0.031***	0.014	0.012
	(0.006)	(0.034)	(0.007)	(0.018)	(0.018)
Δr_t^{US}	0.199***	0.182	0.209*	0.584***	0.005
	(0.073)	(0.301)	(0.109)	(0.148)	(0.113)
$\Delta Gov\ Debt_{it-1}$	0.009	0.024	0.026	0.013	-0.003
	(0.014)	(0.049)	(0.019)	(0.037)	(0.011)
Res_{it-1}	-0.146*	0.078	-0.144	-0.239	-0.697***
	(0.076)	(0.117)	(0.093)	(0.195)	(0.259)
FDI_{it-1}	-0.029	-0.346**	-0.045	-0.046**	0.023
	(0.018)	(0.169)	(0.105)	(0.021)	(0.032)
Constant	-3.717***	-4.487***	-3.502***	-4.677***	-2.426***
	(0.272)	(0.608)	(0.396)	(0.727)	(0.647)
Total Observations	3,170	391	1,261	749	769

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust to potential heteroskedasticity and autocorrelation.*

Figure 4.7 also confirms these results. At lower levels of real exchange rate misalignment, the differences between less open (closed) and more open (open) economies are less obvious. However, at higher levels of misalignment, the probabilities of experiencing a currency crisis rise much faster (exponentially) for less open economies and eventually exceed those of more open economies, regardless of exchange rate regime. For more open economies, both with pegs and intermediate exchange rate regimes, the probability of a currency crisis appears relatively insensitive to changes in the degree of real exchange rate misalignment. These results are somewhat similar when the sample is split by the size of the population – the sensitivities of the probability of a currency crisis to changes in real exchange rate misalignment are relatively modest for smaller economies. Moreover, the probabilities are much greater for larger economies than smaller ones (primarily at higher levels of real exchange rate misalignment), irrespective of exchange rate regime.

Figure 4.7: Estimated Probabilities of Currency Crises by Degree of Exchange Rate Overvaluation – Split by FX Regime, Degree of Trade Openness and Population Size[21]



Source: Author's calculations

As a final test, and in order to preserve a sufficiently large sample size, the baseline regression is rerun, but this time including an interaction between real exchange rate misalignment and the degrees of trade openness ($Mis_{it-1} \times Open_{it-1}$) and population respectively ($Mis_{it-1} \times \ln(Pop)_{it-1}$). The sample is then split once, by exchange rate regime. The negative and statistically significant coefficients on the interaction term in Table 4.6 suggest that, across exchange rate regimes, the relationship between real exchange rate misalignment and the probability of a currency crisis is decreasing in the degree of trade openness. Thus, the most open economies, whether with de facto fixed exchange rates, crawling pegs or managed floats, are less likely to experience annual, nominal exchange rate

[21]The illustrative graphs in Figure 4.7 plot the predicted probabilities of a currency crash for different hypothetical values of real exchange rate misalignment. The lines in the first panel are derived by applying the coefficients from the models estimated in Table 4.5 (where the sample has been split by the degrees of openness and exchange rate regime). The lines in the second panel are derived by applying the coefficients from similar models (results not shown, but available) where the sample has been split by the size of the population and exchange rate regime. In each case, the values for the other regressors from the model (the levels of foreign exchange reserves, foreign direct investment, and changes in government debt and US interest rates) are held constant at their respective sample averages.

devaluations of at least 30% than their peers who are less dependent on imports of goods and services. Similarly, in Table 4.7, except for the specification with fixed exchange rate economies only where the coefficient on the interaction term is not statistically different from zero, the coefficients on the interaction term are positive and statistically significant at at least the 10% level. Larger economies, particularly those with intermediate exchange rate regimes, are thus more likely to experience currency crises at higher levels of real exchange rate overvaluation than their smaller counterparts. These results confirm the graphical analysis highlighted in Figure 4.7 above.

Table 4.6: Discrete Choice Logit Random Effects Estimates – Misalignment x Openness – Dependent Variable: Currency Crash

Variable	**Baseline**	**Interaction with Openness**	**Interaction with Openness – Fixed**	**Interaction with Openness – Intermediate**
Mis_{it-1}	0.028***	0.059***	0.078**	0.056***
	(0.006)	(0.008)	(0.034)	(0.009)
Δr_t^{US}	0.199***	0.194***	0.368**	0.144*
	(0.073)	(0.073)	(0.170)	(0.087)
$\Delta Gov\ Debt_{it-1}$	0.009	0.012	0.013	0.014
	(0.014)	(0.014)	(0.034)	(0.011)
Res_{it-1}	-0.146*	-0.179**	-0.116	-0.234**
	(0.076)	(0.073)	(0.145)	(0.101)
FDI_{it-1}	-0.029	-0.029	-0.053**	-0.004
	(0.018)	(0.021)	(0.023)	(0.032)
$Mis_{it-1} \times Open_{it-1}$		-0.001***	-0.001**	-0.001***
		(0.000)	(0.001)	(0.000)
Constant	-3.717***	-3.688***	-4.739***	-3.439***
	(0.272)	(0.251)	(0.495)	(0.333)
Total Observations	3,170	3,170	1,140	2,030

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust to potential heteroskedasticity and autocorrelation.*

Table 4.7: Discrete Choice Logit Random Effects Estimates – Misalignment x Population – Dependent Variable: Currency Crash

Variable	**Baseline**	**Interaction with Population**	**Interaction with Population – Fixed**	**Interaction with Population – Intermediate**
Mis_{it-1}	0.028***	-0.046	0.045	-0.080*
	(0.006)	(0.043)	(0.085)	(0.043)
Δr_t^{US}	0.199***	0.198***	0.417**	0.147*
	(0.073)	(0.072)	(0.178)	(0.085)
$\Delta Gov\ Debt_{it-1}$	0.009	0.011	0.011	0.014
	(0.014)	(0.015)	(0.035)	(0.012)
Res_{it-1}	-0.146*	-0.177**	-0.107	-0.253**
	(0.076)	(0.077)	(0.138)	(0.104)
FDI_{it-1}	-0.029	-0.026	-0.050**	0.001
	(0.018)	(0.019)	(0.019)	(0.032)
$Mis_{it-1} \times \ln(Pop)_{it-1}$		0.005*	-0.001	0.007**
		(0.003)	(0.006)	(0.003)
Constant	-3.717***	-3.643***	-4.689***	-3.339***
	(0.272)	(0.256)	(0.480)	(0.329)
Total Observations	3,170	3,167	1,140	2,027

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust to potential heteroskedasticity and autocorrelation.*

4.6.3 Robustness Checks for Main Regression Results

This subsection evaluates whether the relationships between real exchange rate misalignment, the degree of trade openness, country size and the probability of experiencing a currency crash identified above generally hold when other control variables are considered. It considers whether, when controlling for some of the best predictors of currency crashes highlighted by Reinhart and Rogoff (2009), the size and statistical significance of the coefficient on real exchange rate overvaluation changes.

As discussed earlier, Reinhart and Rogoff (2009) identified real exchange rate overvaluation as one of the best predictors of a currency crash, but also suggested that other macroeconomic and financial indicators do a good job at signaling an impending crisis. Table 4.8 presents alternative results substituting 4 of the 5 additional determinants that they suggest best foretell an upcoming currency crash. Specifically, the list of regressors becomes:

$$X_{it-1} = \{\overset{-}{CAB}_{it-1}, \overset{-}{\Delta Real\ exports}_{it-1}, \overset{+}{M2/Res}_{it-1}, \overset{+}{Bank\ Crisis}_{it}, \overset{+}{Mis}_{it-1}\} \qquad (4.38)$$

where CAB_{it-1}, $\Delta Real\ exports_{it-1}$, $M2/Res_{it-1}$, $Bank\ Crisis_{it}$ represent the current account balance as a share of nominal gross domestic product, the annual percentage change in real exports of goods and services, broad money (M2) over international reserves, and a binary indicator of a banking crisis defined by Laeven and Valencia (2018).[22] $\Delta Real\ exports_{it-1}$and $M2/Res_{it-1}$ are both sourced from the World Bank's World Development Indicators. CAB_{it-1} is also sourced from the World Bank's World Development Indicators and missing observations are supplemented with data from the International Monetary Fund's World Economic Outlook.

Substituting the previous control variables with a new set of regressors reduces the number of observations included in each regression from 3,170 to 2,622. Nonetheless, the regression estimates are consistent with prior expectations and mostly support this chapter's findings thus far. Specifically, faster export growth and a larger current account balance generally reduce the chances that an economy experiences a currency crash, while a larger stock of broad money relative to international reserves places additional pressure on the fixed or managed exchange rate regime. Further, as suggested in much of the literature (see for example Laeven and Valencia (2018)), banking and currency crises often occur simultaneously. Perhaps most importantly, the coefficient on real exchange rate misalignment in the specification without an interaction term is still positive and statistically significant at the 1% level and is similar in size to its value in the baseline regression. Additionally, the coefficient on $Mis_{it-1} \times Open_{it-1}$ is both negative and statistically significant, suggesting that more open economies may be less prone to experiencing currency crashes than less open economies given the same levels of real exchange rate overvaluation. However, the size of this coefficient, while statistically different from zero at the 10% level of significance, is not economically significant. Finally, while the coefficient on $Mis_{it-1} \times \ln(Pop)_{it-1}$ is positive as expected, the coefficients on this variable and Mis_{it-1} in that specification are both statistically insignificant.[23]

[22] Real stock prices, the 5th predictor identified, was excluded due to data limitations.
[23] The coefficient on $Mis_{it-1} \times \ln(Pop)_{it-1}$ is also small and economically insignificant.

Table 4.8: Discrete Choice Logit Random Effects Estimates – Alternative Control Variables – Dependent Variable: Currency Crash

Variable	**Baseline**	**Reinhart and Rogoff Regressors**	**Interaction with Openness**	**Interaction with Population**
Mis_{it-1}	0.028***	0.029***	0.042***	-0.010
	(0.006)	(0.004)	(0.008)	(0.042)
Δr_t^{US}	0.199***			
	(0.073)			
$\Delta Gov\ Debt_{it-1}$	0.009			
	(0.014)			
Res_{it-1}	-0.146*			
	(0.076)			
FDI_{it-1}	-0.029			
	(0.018)			
$M2/Res_{it-1}$		0.019***	0.019***	0.019***
		(0.005)	(0.005)	(0.005)
$\Delta Real\ exports_{it-1}$		-0.019*	-0.019*	-0.020**
		(0.010)	(0.010)	(0.010)
CAB_{it-1}		-0.021**	-0.023**	-0.023**
		(0.011)	(0.010)	(0.011)
$Bank\ Crisis_{it}$		1.425***	1.439***	1.430***
		(0.514)	(0.512)	(0.512)
$Mis_{it-1} \times Open_{it-1}$			-0.000*	
			(0.000)	
$Mis_{it-1} \times \ln(Pop)_{it-1}$				0.002
				(0.002)
Constant	-3.717***	-4.244***	-4.225***	-4.222***
	(0.272)	(0.229)	(0.228)	(0.230)
Total Observations	3,170	2,622	2,622	2,622

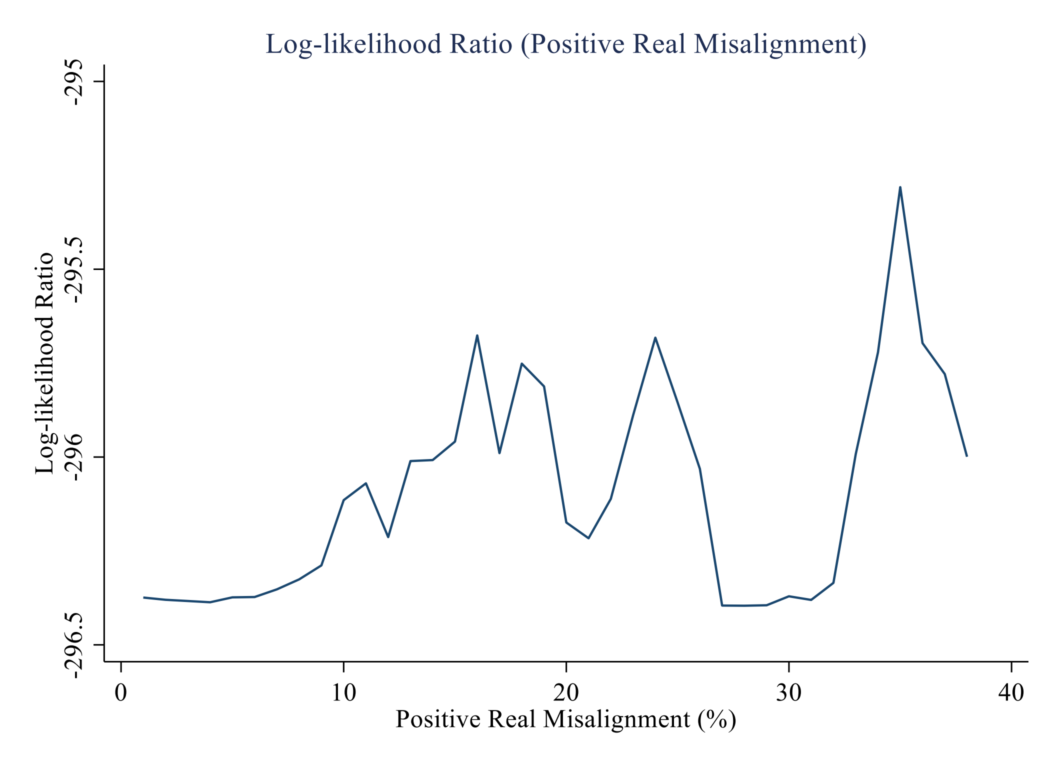
*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust to potential heteroskedasticity and autocorrelation.*

4.6.4 Results of Threshold Models

This subsection investigates how the nature of the relationship between the probability of a currency crash and the magnitude of real exchange rate overvaluation changes above and below estimated thresholds of positive real exchange rate misalignment. Table 4.9 and Table 4.10 present the results of threshold regressions where the sign and size of the coefficient on the extent of real exchange rate misalignment depend on whether the level of real exchange rate overvaluation exceeds or is less than or equal to an estimated threshold (δ). The threshold is estimated by maximizing the log-likelihood ratio from a series of models estimated using a range of possible values of δ, where candidates for δ are restricted to values within the 7.5th and 92.5th percentiles of the distribution of positive real exchange rate misalignments in order to ensure that sufficient data are available to estimate the effects on either side of the threshold.

For various values of the threshold, Figure 4.8 below plots the log-likelihood ratio for each baseline threshold regression estimated using alternative values of the threshold. The graphs suggest that the log-likelihood ratio peaks when the threshold reaches a value of 35% real exchange rate overvaluation, but also has lower spikes at potential thresholds of 16% and 24%.

Figure 4.8: Log-Likelihood Ratio by Real Exchange Rate Misalignment Threshold – Baseline Threshold Currency Crash Regression



Source: Author's calculations

The results from the singe threshold regressions (in Table 4.9 and Table 4.10) mostly confirm those presented thus far, but now suggest that the coefficient on the level of real exchange rate overvaluation gets smaller above a positive real exchange rate misalignment of 35%, both for the full sample, and when the sample is split by degrees of openness and size. However, while the coefficient remains statistically significant above 35% real overvaluation for the full sample and for larger, less open economies, it loses significance at conventional levels for smaller, more open economies. Ultimately though, the results suggest that, while real exchange rate misalignment is still a strong predictor of currency crashes, a sharp nominal exchange rate devaluation actually becomes less concerning (or even less viable) at extremely high levels of real exchange rate overvaluation.

Table 4.9: Discrete Choice Logit Random Effects Estimates with Single Threshold – Split by Degree of Trade Openness – Dependent Variable: Currency Crash

Variable	**Baseline**	**Baseline Threshold**	**Threshold – Less Open**	**Threshold – More Open**
Mis_{it-1}	0.028***			
	(0.006)			
Δr_t^{US}	0.199***	0.199***	0.213**	0.194*
	(0.073)	(0.072)	(0.103)	(0.100)
$\Delta Gov\ Debt_{it-1}$	0.009	0.009	0.026	0.001
	(0.014)	(0.015)	(0.018)	(0.024)
Res_{it-1}	-0.146*	-0.140*	-0.082	-0.390**
	(0.076)	(0.080)	(0.081)	(0.175)
FDI_{it-1}	-0.029	-0.028	-0.086	-0.019
	(0.018)	(0.018)	(0.106)	(0.014)
$Mis_{it-1}\ I(Mis_{it-1} \leq \delta)$		0.040***	0.035***	0.043**
		(0.011)	(0.011)	(0.019)
$Mis_{it-1}\ I(Mis_{it-1} > \delta)$		0.023***	0.029***	-0.003
		(0.008)	(0.008)	(0.019)
Constant	-3.717***	-3.745***	-3.709***	-3.348***
	(0.272)	(0.292)	(0.345)	(0.503)
Threshold (δ)		35%	35%	35%
Total Observations	3,170	3,170	1,652	1,518

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Table 4.10: Discrete Choice Logit Random Effects Estimates with Single Threshold – Split by Population Size – Dependent Variable: Currency Crash

Variable	Baseline	Baseline Threshold	Threshold – Smaller	Threshold – Larger
Mis_{it-1}	0.028***			
	(0.006)			
Δr_t^{US}	0.199***	0.199***	0.228*	0.191**
	(0.073)	(0.072)	(0.119)	(0.096)
$\Delta Gov\ Debt_{it-1}$	0.009	0.009	-0.004	0.031
	(0.014)	(0.015)	(0.026)	(0.020)
Res_{it-1}	-0.146*	-0.140*	-0.296	-0.123
	(0.076)	(0.080)	(0.193)	(0.080)
FDI_{it-1}	-0.029	-0.028	-0.017	-0.043
	(0.018)	(0.018)	(0.017)	(0.033)
$Mis_{it-1}\ I(Mis_{it-1} \leq \delta)$		0.040***	0.052**	0.031***
		(0.011)	(0.021)	(0.011)
$Mis_{it-1}\ I(Mis_{it-1} > \delta)$		0.023***	0.004	0.030***
		(0.008)	(0.018)	(0.008)
Constant	-3.717***	-3.745***	-3.610***	-3.628***
	(0.272)	(0.292)	(0.600)	(0.317)
Threshold (δ)		35%	35%	35%
Total Observations	3,170	3,170	1,462	1,708

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Although Figure 4.8 suggests that a single threshold at 35% maximises the log-likelihood function, the presence of two spikes at 16% and 24% could signify that a second threshold exists. Therefore, Table 4.11 and Table 4.12 present output with two thresholds included in the initial logit regression, rather than just one. The results suggest that, in addition to a threshold at a positive real exchange rate misalignment of 35%, another one may exist at a lower threshold of 24%. However, below 24% real exchange rate misalignment, the probability of a country experiencing a currency crash within the next year is positive, but not statistically significant. In fact, the coefficient falls to almost zero in the case of more open economies. Above 24% and up to 35% however, the size of the coefficient surges (and becomes statistically significant) and is much higher than both coefficients in the specifications with a single threshold. Once again, the size of the coefficient beyond the second threshold (δ greater than 35%) falls, and becomes statistically insignificant for smaller, more open economies. Finally, it is important to note that if the model is estimated with a single threshold at 24% real exchange rate misalignment, the overall sensitivity of a currency crash to an overvalued real exchange rate one year prior is higher for larger, less open economies than smaller, more open ones (see Appendix 4.A.2).

Table 4.11: Discrete Choice Logit Random Effects Estimates with Two Thresholds – Split by Degree of Trade Openness – Dependent Variable: Currency Crash

Variable	**Baseline Threshold**	**Double Threshold**	**Double Threshold – Less Open**	**Double Threshold – More Open**
Δr_t^{US}	0.199***	0.200***	0.211**	0.221*
	(0.072)	(0.073)	(0.101)	(0.115)
$\Delta Gov\ Debt_{it-1}$	0.009	0.009	0.024	0.002
	(0.015)	(0.014)	(0.019)	(0.023)
Res_{it-1}	-0.140*	-0.136*	-0.084	-0.350**
	(0.080)	(0.075)	(0.081)	(0.161)
FDI_{it-1}	-0.028	-0.026	-0.081	-0.014
	(0.018)	(0.019)	(0.107)	(0.015)
$Mis_{it-1}\ I(Mis_{it-1} \leq \delta)$	0.040***			
	(0.011)			
$Mis_{it-1}\ I(Mis_{it-1} > \delta)$	0.023***			
	(0.008)			
$Mis_{it-1}\ I(Mis_{it-1} \leq \delta_1)$		0.017	0.027	0.001
		(0.012)	(0.017)	(0.012)
$Mis_{it-1}\ I(\delta_1 < Mis_{it-1} \leq \delta_2)$		0.066***	0.047***	0.082***
		(0.013)	(0.018)	(0.023)
$Mis_{it-1}\ I(Mis_{it-1} > \delta_2)$		0.025***	0.030***	0.003
		(0.008)	(0.008)	(0.017)
Constant	-3.745***	-3.862***	-3.733***	-3.752***
	(0.292)	(0.280)	(0.343)	(0.551)
Single Threshold (δ)	35%			
Threshold 1 (δ_1)		24%	24%	24%
Threshold 2 (δ_2)		35%	35%	35%
Total Observations	3,170	3,170	1,652	1,518

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Table 4.12: Discrete Choice Logit Random Effects Estimates with Two Thresholds – Split by Population Size – Dependent Variable: Currency Crash

Variable	**Baseline Threshold**	**Double Threshold**	**Double Threshold – Smaller**	**Double Threshold – Larger**
Δr_t^{US}	0.199***	0.200***	0.258**	0.185**
	(0.072)	(0.073)	(0.131)	(0.094)
$\Delta Gov\ Debt_{it-1}$	0.009	0.009	-0.003	0.029
	(0.015)	(0.014)	(0.023)	(0.020)
Res_{it-1}	-0.140*	-0.136*	-0.240	-0.126
	(0.080)	(0.075)	(0.159)	(0.083)
FDI_{it-1}	-0.028	-0.026	-0.013	-0.038
	(0.018)	(0.019)	(0.018)	(0.030)
$Mis_{it-1}\ I(Mis_{it-1} \leq \delta)$	0.040***			
	(0.011)			
$Mis_{it-1}\ I(Mis_{it-1} > \delta)$	0.023***			
	(0.008)			
$Mis_{it-1}\ I(Mis_{it-1} \leq \delta_1)$		0.017	0.016	0.018
		(0.012)	(0.019)	(0.015)
$Mis_{it-1}\ I(\delta_1 < Mis_{it-1} \leq \delta_2)$		0.066***	0.076***	0.053***
		(0.013)	(0.023)	(0.017)
$Mis_{it-1}\ I(Mis_{it-1} > \delta_2)$		0.025***	0.007	0.032***
		(0.008)	(0.018)	(0.009)
Constant	-3.745***	-3.862***	-3.879***	-3.680***
	(0.292)	(0.280)	(0.527)	(0.315)
Single Threshold (δ)	35%			
Threshold 1 (δ_1)		24%	24%	24%
Threshold 2 (δ_2)		35%	35%	35%
Total Observations	3,170	3,170	1,462	1,708

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

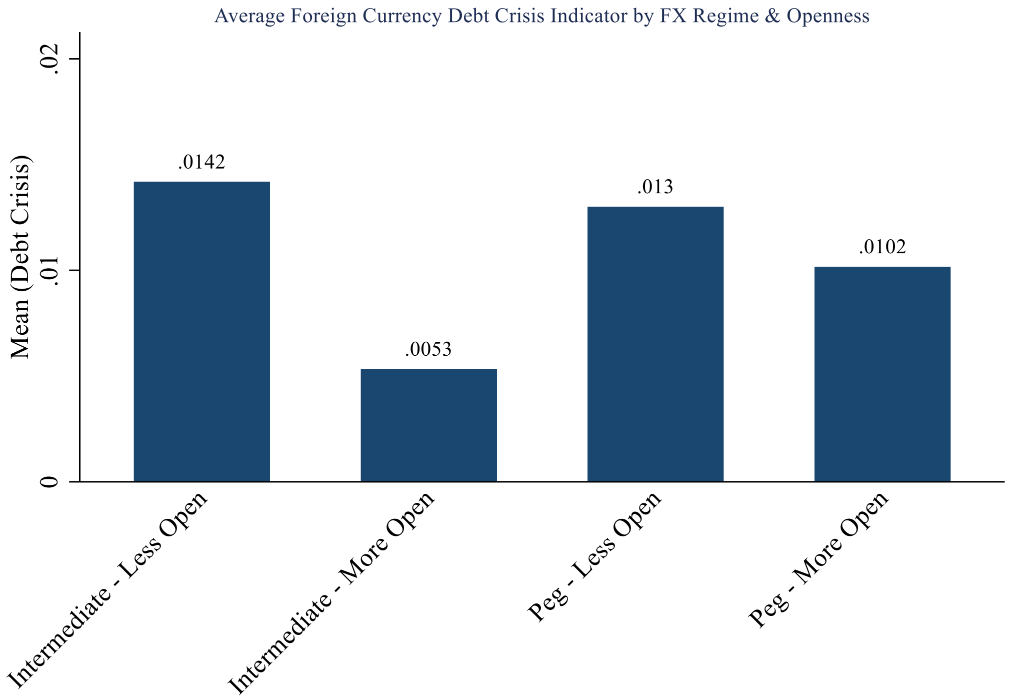
4.6.5 Does the Government's Choice Between Nominal Exchange Rate Devaluation and Sovereign Default Depend on Openness or Size?

So far, the analysis has focused primarily on whether real exchange rate misalignments increase the probabilities that countries experience currency crashes and whether those probabilities vary by country size or the degree of trade openness. This subsection aims to determine, when faced with an overvalued real exchange rate (and other macroeconomic imbalances), whether smaller and/or more open economies with pegged or managed currencies are more or less likely to default on their external obligations (compared to larger, less open economies) rather than substantially deviate from their exchange rate arrangement.

The first task though, is to determine whether greater real exchange rate misalignment significantly increases the probability of default on sovereign debt denominated in foreign currencies. While Laeven and Valencia (2018) also publish a measure of sovereign debt crisis, determined by the "…year of sovereign default to private creditors and/or restructuring..." (Laeven and Valencia, 2018), this series includes defaults on obligations issued in both local and foreign currencies and does not differentiate between defaults on either. At the same time, the Bank of Canada (BoC), in partnership with the Bank of England (BoE), maintains and publishes an annual database of sovereign debt in default. The 2020 edition of this database published the annual US dollar value of stocks of government debt in default over 1960-2019 for 147 countries across a variety of instruments including "…bonds and other marketable securities, bank loans and official loans…." (Beers et al., 2020). While this database does not specify dates of default or restructuring, it does distinguish between stocks of debts in default issued in local and foreign currencies. Thus, in this chapter, dates of sovereign default on foreign currency debt are based on the fulfillment of two criteria: 1. years during which the change in foreign currency debts in default is positive based on the BoC-BoE database and 2. the country experiences a sovereign debt crisis as determined by Laeven and Valencia (2018). These dates are checked against, and in some cases supplemented by, external debt default dates published by Reinhart and Rogoff (2009), and as in the case of the currency crash series, defaults occurring within five years of the initial default are assumed to be part of the initial debt crisis. These criteria ensure that periods of default on domestic currency debt only are excluded. Overall, in the final sample, there are now 30 episodes of foreign currency sovereign defaults or debt crises. As a start, Figure 4.9 illustrates that economies with higher-than-average ratios of imports to gross domestic product have defaulted on their foreign currency debt obligations slightly less frequently than less open economies have. However, the differences between default frequencies between countries of different degrees of openness appear much less stark than the differences in currency crash frequencies between less and more open economies (from Figure 4.3).[24]

[24] Additionally, the data suggest that while smaller economies also suffered from currency crashes less frequently than larger economies, there was little difference in default frequencies between smaller and larger economies.

Figure 4.9: Average Foreign Currency Debt Crisis Indicator by Exchange Rate Regime and Degree of Trade Openness



Source(s): Ilzetzki et al. (2016), Laeven and Valencia (2018), Beers et al. (2020), Reinhart and Rogoff (2009), World Bank's World Development Indicators, United Nations National Accounts Statistics, author's calculations

Having defined the measure of foreign currency debt crisis, Table 4.13 illustrates results from a random effects logit estimation with a binary indicator of a sovereign, foreign currency debt crisis as the dependent variable, both for the full sample as well as subsamples split by population size and the degree of trade openness. In addition to the measure of real exchange rate misalignment, the choice of regressors leverages many of those used in Kose et al.'s (2019) regression analysis of debt crises for which sufficient data are available. These include Δr_t^{US}, Res_{it-1}, $\Delta Gov\ Debt_{it-1}$, which are all included in the currency crash regressions, as well as growth in real gross domestic product ($Growth\ GDP_{it-1}$). The baseline, full sample results suggest that most regressors significantly influence the probabilities of governments defaulting on their obligations denominated in foreign currency. As expected, increases in government debt as a ratio of nominal gross domestic product increase the chances of sovereign default, while higher levels of foreign exchange reserves as a share of imports of goods and services and faster growth in real gross domestic product reduce the chances of default. Further, real interest rate changes in the United States of America have no statistically significant impact on the occurrences of foreign currency debt crises across the sample of 114 countries. Finally, and most importantly, as in the case of currency crashes, real exchange rate overvaluation increases the probability of countries defaulting on their obligations denominated in foreign currency. The effects of the latter are statistically significant for the full sample.

Meanwhile, the results across subsamples suggest that, while the signs all remain the same, most standard errors rise and the sizes and statistical significance of the regressors change across subsamples. Of particular interest, the sizes of the coefficients on real exchange rate misalignment are noticeably greater in larger, less open economies, again suggesting that, like in the case of currency crashes, these economies are likely more sensitive to rising levels of real exchange rate overvaluation.

Table 4.13: Discrete Choice Logit Random Effects Estimates by Degree of Trade Openness and Size – Dependent Variable: Sovereign, Foreign Currency Debt Crisis

Variable	**Baseline**	**Less Open**	**More Open**	**Larger**	**Smaller**
Mis_{it-1}	0.014*	0.021*	0.011	0.020	0.011
	(0.007)	(0.011)	(0.012)	(0.033)	(0.011)
Δr_t^{US}	-0.128	-0.214	-0.044	-0.193	-0.075
	(0.107)	(0.166)	(0.155)	(0.177)	(0.137)
$Growth\ GDP_{it-1}$	-0.064**	-0.062	-0.063	-0.059	-0.060*
	(0.032)	(0.049)	(0.042)	(0.102)	(0.035)
Res_{it-1}	-0.284**	-0.189	-0.420***	-0.473	-0.213
	(0.133)	(0.156)	(0.156)	(0.351)	(0.146)
$\Delta Gov\ Debt_{it-1}$	0.027**	0.052**	0.013	0.017	0.029*
	(0.013)	(0.023)	(0.009)	(0.055)	(0.015)
Constant	-4.006***	-4.277***	-3.869***	-3.936*	-3.982***
	(0.422)	(0.830)	(0.552)	(2.103)	(0.547)
Total Observations	3,254	1,704	1,550	1,752	1,502

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust to potential heteroskedasticity and autocorrelation.*

The next step in the analysis involves modelling the government's implied choices between sovereign default, nominal exchange rate devaluation, and no action. To do this, I leverage a panel, multinomial logit model estimated with random effects. In this case, nominal exchange rate devaluation (currency crash), sovereign default (foreign currency debt crisis), and no action are modelled as 3 distinct alternatives (taking non-ordered values of 1, 2 and 3, respectively), among which policymakers must decide each year. These actions and the utility derived from them depend on the characteristics of each decision and the regressors defined in the baseline regressions used thus far in this analysis. Equations 4.39, 4.40 and 4.41 below define the probabilities that the policy choice (y_{it}) takes values of 1 (devaluation only), 2 (default only) or 3 (no default or devaluation) respectively, where $\beta^{(1)}$, $\beta^{(2)}$, and $\beta^{(3)}$ are coefficient vectors associated with each policy choice or outcome.[25]

$$\Pr(y_{it} = 1) = \frac{e^{X_{it-1}\beta^{(1)}}}{e^{X_{it-1}\beta^{(1)}} + e^{X_{it-1}\beta^{(2)}} + e^{X_{it-1}\beta^{(3)}}} \quad (4.39)$$

$$\Pr(y_{it} = 2) = \frac{e^{X_{it-1}\beta^{(2)}}}{e^{X_{it-1}\beta^{(1)}} + e^{X_{it-1}\beta^{(2)}} + e^{X_{it-1}\beta^{(3)}}} \quad (4.40)$$

$$\Pr(y_{it} = 3) = \frac{e^{X_{it-1}\beta^{(3)}}}{e^{X_{it-1}\beta^{(1)}} + e^{X_{it-1}\beta^{(2)}} + e^{X_{it-1}\beta^{(3)}}} \quad (4.41)$$

However, to identify this model, I must set the coefficients of one of these choices to zero. The coefficients in the other two vectors can then be interpreted relative to the base choice. Assuming that $\beta^{(1)}$, the coefficients corresponding to choosing nominal devaluation only, is set to zero yields:

$$\Pr(y_{it} = 1) = \frac{1}{1 + e^{X_{it-1}\beta^{(2)}} + e^{X_{it-1}\beta^{(3)}}} \quad (4.42)$$

$$\Pr(y_{it} = 2) = \frac{e^{X_{it-1}\beta^{(2)}}}{1 + e^{X_{it-1}\beta^{(2)}} + e^{X_{it-1}\beta^{(3)}}} \quad (4.43)$$

$$\Pr(y_{it} = 3) = \frac{e^{X_{it-1}\beta^{(3)}}}{1 + e^{X_{it-1}\beta^{(2)}} + e^{X_{it-1}\beta^{(3)}}} \quad (4.44)$$

while the probabilities of y_{it} = 2, 3 (default only and no action, respectively) relative to the base outcome (devaluation only) now become:

$$\frac{Pr(y_{it}=2)}{Pr(y_{it}=1)} = e^{X_{it-1}\beta^{(2)}} \quad (4.45)$$

$$\frac{Pr(y_{it}=3)}{Pr(y_{it}=1)} = e^{X_{it-1}\beta^{(3)}} \quad (4.46)$$

Persisting with the assumption that nominal exchange rate devaluation (y_{it} = 1) is the base outcome, Table 4.14 below presents the results of multinomial logit regressions with $Open_{it-1}$ and $\ln(Pop)_{it-1}$ added to the list of regressors. Except for the inclusion of $Growth\,GDP_{it-1}$, $Open_{it-1}$ and $\ln(Pop)_{it-1}$, the results for y_{it} = 3 can almost[26] be interpreted as the reverse of those presented in the second column of Table 4.10, and the results are qualitatively similar. Lower levels of real exchange rate overvaluation, reductions in interest rates in the United States of America, and higher levels of foreign exchange

[25] Twin crises, or episodes of both default and devaluation, were excluded from the estimation. However, such events represented just 5 observations in the entire sample.
[26] Almost because the results in Table 4.10 and before do not account for the presence of twin crises.

reserves relative to imports of goods and services are all associated with lower probabilities of exchange rate devaluation relative to no action at all. Moreover, unlike in Kose et al. (2019), economies experiencing faster rates of economic growth are statistically less likely to devalue their exchange rates than do nothing at all. Finally, compared to their smaller, more open peers, larger, less open economies are more likely to devalue their exchange rates or experience a currency crash than do nothing, when controlling for several macroeconomic and financial factors.

Of particular interest, the results for y_{it} = 2 suggest that, across all countries, higher US interest rates are less likely to lead to default or a foreign currency debt crisis than to lead to exchange rate devaluation (currency crash). However, foreign direct investment as a ratio of gross national income, changes in government debt as a share of nominal gross domestic product, growth in real gross domestic product, foreign exchange reserves as a share of imports of goods and services, and the level of real exchange rate misalignment have no statistically significant effects on the authorities' choices between default on foreign currency debt and exchange rate devaluation. Crucially, once I control for real exchange rate misalignment and other regressors, larger, less open economies are more likely to devalue their nominal exchange rates than smaller, more open markets. Put differently, compared to large, closed economies, small, open economies are more likely to default on foreign currency, sovereign obligations than devalue their exchange rates.

Table 4.14: Multinomial Logit Random Effects Estimates – Includes Openness and Size as Regressors, with y_{it} = 1 as the Base Outcome

Variable	$y_{it} = 2$	$y_{it} = 3$	$y_{it} = 2$	$y_{it} = 3$
Mis_{it-1}	-0.012	-0.028***	-0.015	-0.030***
	(0.011)	(0.005)	(0.011)	(0.005)
Δr_t^{US}	-0.294**	-0.206***	-0.300**	-0.213***
	(0.142)	(0.075)	(0.145)	(0.076)
$Growth\ GDP_{it-1}$	-0.036	0.051*	-0.026	0.060**
	(0.034)	(0.027)	(0.035)	(0.028)
Res_{it-1}	-0.150	0.128*	-0.144	0.133*
	(0.138)	(0.072)	(0.142)	(0.069)
$\Delta Gov\ Debt_{it-1}$	0.021	-0.005	0.019	-0.005
	(0.018)	(0.015)	(0.019)	(0.016)
FDI_{it-1}	0.017	0.007	0.034	0.025
	(0.018)	(0.018)	(0.023)	(0.023)
$Open_{it-1}$	0.023*	0.027**		
	(0.014)	(0.012)		
$\ln(Pop)_{it-1}$			-0.369**	-0.177*
			(0.172)	(0.103)
Constant	-1.233**	2.731***	5.375*	6.526***
	(0.526)	(0.402)	(2.751)	(1.705)
Total Observations	3,162	3,162	3,159	3,159

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust to potential heteroskedasticity and autocorrelation.*

Finally, Table 4.15 reproduces the results for the multinomial regressions but includes interactions between real exchange rate misalignment and openness and size, respectively instead of openness and size on their own. Most control variables (other than, most notably, real exchange rate misalignment) carry the same signs and similar levels of statistical significance for y_{it} = 2 and y_{it} = 3 as in Table 4.14. However, now the degree of real exchange rate misalignment and its interaction terms tell another interesting story. The results in columns 2 and 3 suggest that, not only are more open economies less likely to devalue their nominal exchange rates (relative to the option of doing nothing) than more closed counterparts when faced with higher levels of misalignment, but more open economies are also less likely to devalue than default on foreign currency obligations when faced with the same level of overvaluation. In contrast, the results for Mis_{it-1} and Mis_{it-1} × $\ln(Pop)_{it-1}$ are both statistically insignificant in columns 4 and 5, although the coefficients on the latter carry the expected signs.

Table 4.15: Multinomial Logit Random Effects Estimates – Interacts Openness and Size with Real Exchange Rate Misalignment

Variable	$y_{it} = 2$	$y_{it} = 3$	$y_{it} = 2$	$y_{it} = 3$
Mis_{it-1}	-0.043**	-0.058***	0.042	0.032
	(0.021)	(0.008)	(0.086)	(0.040)
Δr_t^{US}	-0.300**	-0.211***	-0.302**	-0.214***
	(0.144)	(0.076)	(0.143)	(0.076)
$Growth\ GDP_{it-1}$	-0.036	0.052*	-0.033	0.055**
	(0.034)	(0.027)	(0.035)	(0.027)
Res_{it-1}	-0.133	0.144**	-0.139	0.138*
	(0.146)	(0.070)	(0.150)	(0.072)
$\Delta Gov\ Debt_{it-1}$	0.018	-0.008	0.021	-0.005
	(0.019)	(0.014)	(0.019)	(0.015)
FDI_{it-1}	0.040	0.030	0.039*	0.029
	(0.025)	(0.025)	(0.022)	(0.022)
$Mis_{it-1} \times Open_{it-1}$	0.001**	0.001***		
	(0.000)	(0.000)		
$Mis_{it-1} \times \ln(Pop)_{it-1}$			-0.003	-0.004
			(0.005)	(0.002)
Constant	-0.430	3.701***	-0.469	3.660***
	(0.403)	(0.256)	(0.408)	(0.261)
Total Observations	3,162	3,162	3,159	3,159

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust to potential heteroskedasticity and autocorrelation.*

4.6.6 Summary of Results

Finally, the results presented above suggest that both exchange rate regimes and economic structure proxied by either country size or the degree of trade openness influence countries' probabilities of suffering currency crashes for given levels of real exchange rate misalignment. These results, particularly those regarding economic structure, are robust, both to inclusion of alternative control variables, but also to alternative model specifications. Further, while the importance of foreign exchange reserves, changes in foreign interest rates, and of course the level of real exchange rate misalignment implies that the first-generation model of currency crises is still relevant, the importance and differentiated effects of these variables across exchange rate regimes, country size and degrees of openness support the chapter's view that the relationships between economic fundamentals and currency crises is far from homogenous. Overall, these findings support the chapter's hypothesis that smaller economies or those who may be more dependent on imports of goods and services benefit less from nominal exchange rate devaluations and therefore have less incentive to relinquish their pegged or managed exchange rate regimes in order to boost output in the short-term.

These effects are particularly significant at much higher levels of real exchange rate overvaluation. Above 24% but below 35% real exchange rate overvaluation, the probability of a currency crash rises substantially as the degree of real exchange rate misalignment increases. However, this effect becomes smaller beyond real overvaluation of 35%, and becomes statistically insignificant for smaller, more open economies. Therefore, particularly when the extent of real exchange rate overvaluation exceeds 35%, the likelihood that a small, open economy relinquishes its exchange rate regime is typically less than that for larger, more closed economies, all else held equal.

The chapter also confirms that real exchange rate misalignment may be a useful predictor of foreign currency debt crises and supports Jahjah et al.'s (2013) finding that real exchange rate overvaluation increases bond spreads, and by extension, the perceived probability of default. Moreover, smaller, more open economies are (at least relative to larger, less open economies) more likely to default on foreign currency sovereign debt than to devalue their currencies or exit stabilized exchange rate arrangements when faced with increasingly overvalued real exchange rates and controlling for other macroeconomic imbalances. This lends some support to the chapter's hypothesis that they have more incentive to do such given the relative costs of nominal exchange rate devaluation and sovereign default.

4.7 Conclusion

The determinants of currency and debt crises have been well-researched themes over the past four decades. Among measures of external imbalances, one of the most consistent predictors of future (particularly currency) crises has been the degree of real exchange rate overvaluation. While some studies have found that the relationships between currency crises and its determinants vary across exchange rate regimes, none, to the best of the author's knowledge, has studied the roles that country size and/or a country's dependence on imports of goods and services play in this relationship. Further, to date, there is a paucity of literature which has investigated the potential tradeoffs which governments, particularly of small, open economies, may confront when choosing between defaulting on foreign currency debt and devaluing the nominal exchange rate when faced with substantial real exchange rate overvaluation or other macroeconomic imbalances.

This chapter hypothesizes that for given levels of real exchange rate overvaluation, smaller, more open economies who maintain fixed or managed exchange rate regimes are less likely to experience substantial depreciations or devaluations in their exchange rate than their larger, less open counterparts. The former economies often exhibit high pass through from exchange rate depreciations to inflation and benefit less from sharp nominal exchange rate depreciations. Further, while not mutually exclusive, the chapter hypothesizes that, when faced with macroeconomic imbalances or external shocks, particularly an overvalued exchange rate, small, open economies are relatively more likely to choose to default on foreign currency debt obligations than to devalue their nominal exchange rates compared to their larger, less open peers. Therefore, I expect that the smaller the marginal benefit from a devaluation, the greater the level of real exchange rate misalignment or other macroeconomic imbalances required for an exit of the peg or managed exchange rate arrangement to make sense, and the greater the relative benefit of default (all else being equal). This chapter sought to contribute to the literature by illustrating just this.

The results presented in this chapter find some evidence to support these hypotheses. Overall, the probability of suffering a currency crash rises much more slowly in smaller, more open economies in response to increases in the level of positive real exchange rate misalignment. For open economies, this result generally holds across both fixed and intermediate exchange rate regimes, and generally at much larger degrees of real exchange rate overvaluation. It holds more robustly for intermediate exchange rate regimes in the case of population size. Indeed, below or at positive real exchange rate misalignments of 24%, greater real exchange rate misalignments have no statistically significant impacts on the probabilities that economies of either size (large or small) or degrees of trade openness (open or closed) will suffer a currency crash. However, above 24%, this probability increases significantly for economies of all sizes and degrees of openness. Beyond positive real misalignments of 35%, the probability rises significantly only for larger, less open economies. These findings therefore suggest that, for any given level of real exchange rate overvaluation, investors' and policymakers' assessments of a country's willingness and ability to maintain a fixed or managed exchange rate should depend, not only on the de facto exchange rate regime, but also on the size of the economy and/or its reliance on imported goods and services.

Moreover, the chapter's analysis finds some support for Bauer et al.'s (2003) proposition that policymakers may face a tradeoff between choosing nominal exchange rate devaluation and default on the government's foreign currency debt when faced with macroeconomic imbalances. While the results suggest that larger, less open economies are more likely to both devalue and default relative to smaller, more open economies when faced with greater exchange rate misalignment, the latter group of economies appears more likely to choose to default on its debts than to devalue its nominal exchange rate when faced with significant macroeconomic imbalances. Thus, all else (including the degree of real exchange rate misalignment) being equal, while regression estimates and the frequencies of default and devaluation might suggest that higher yields could be justified on both foreign and local currency assets in larger, less open economies than smaller, more open ones, the interest rate spreads or risk premia between these two assets may vary. In smaller, more open economies, international investors may demand much higher yields on sovereign, foreign currency bonds relative to assets denominated in local currency which are likely subject to relatively lower currency risks. Similarly, the spreads between foreign and local currency assets in larger, less open economies may very well be much smaller (or even negative), reflecting their relative tendency to devalue their nominal exchange rate versus default on foreign currency debt. Future research should therefore evaluate whether these differences in interest rate spreads or risk premia exist across assets and country characteristics.

Several caveats are worth highlighting, however. The definition and calculation of a currency crash used in the chapter assumes a depreciation of at least 30% versus the United States dollar, but this measure may capture fewer crises than Frankel and Rose (1996) who use a lower threshold of 25%. Therefore, the results derived in this chapter could be sensitive to the threshold used and the choice of the currency crash indicator. Additionally, throughout the chapter, the analysis does not specifically account for the consecutive nature of currency and debt crises. In some cases, one type of crisis may precipitate another, rather than them being two separate, independent events. Moreover, the analysis in sections 4.6.2 through 4.6.4 does not account for the occurrence of twin crises, although these represent just a small share of the sample. Nonetheless, future versions of this work could model currency and debt crises jointly, including their simultaneous and successive natures. Finally, future work on this topic could alternatively leverage a Dynamic Stochastic General Equilibrium model, rather than the logit regression framework used here, where a policy maker in a very small, open economy chooses the optimal decision between default and devaluation in response to an external shock. This alternative approach would provide additional evidence on whether the results presented in this chapter hold for other approaches used to investigate the same issue.

4.A Appendix

4.A.1 Literature Survey of Approaches Used to Estimate Equilibrium Real Exchange Rates

Table 4.16: Literature Review of Approaches Used to Estimate Equilibrium Exchange Rates or Determinants of REERs

Author(s)	Approach	Determinants	Estimation Method
Couharde et al. (2018)	BEER	Net foreign assets, Terms of trade, Relative productivity	Pooled Mean Group (PMG) Estimation
Clark and MacDonald (1998)	BEER	Net foreign assets, Terms of trade, relative price of traded to non-traded goods, interest rate differential, risk premium (not statistically significant)	Johansen Cointegration
Goldfajn and Valdes (1999)	BEER	Terms of trade, government expenditure, openness, international interest rate	Johansen, Dynamic Ordinary Least Squares (DOLS) – both panel and country by country
	APEER	None	HP-filter
Holtemöller and Mallick (2013)	FEER	World GDP, Domestic GDP, Openness, Terms of trade, Primary current account balance, Output gap (HP-filtered) – latter two not included in the long-run regression since they are stationary	Engle Granger Panel 2-step cointegration approach country by country
Grekou (2015)	BEER/PEER	Productivity, Terms of trade, Government consumption, Openness, Net foreign assets	PMG and Panel cointegration (DOLS)
	APEER	None	HP-filter
Sallenave (2010)	BEER	Productivity, Net foreign assets	Panel DOLS
Caputo (2015)	BEER	Relative productivity between the traded and non-traded sector, Terms of trade, Government spending, Net foreign assets	Panel DOLS
Kaminsky (2006)	APEER	None	Time trend
Nakatani (2017)	APEER	None	5-year moving average of the real effective exchange rate
Zhao et al. (2014)	APEER	None	HP-filter
Kaminsky et al. (1997)	APEER	None	Trend in real bilateral exchange rate
Jahjah et al. (2013)	APEER	None	10-year average of the real effective exchange rate
Frankel and Rose (1996)	PPP	Relative prices	None
Bauer et al. (2007)	APEER	None	Average US Dollar real exchange rate
Vural (2019)	BEER	Real GDP per capita relative to trading partners, openness, central government debt as a percentage of GDP, net foreign assets, oil prices	Johansen Vector Error Correction Model (VECM)

4.A.2 Alternative Threshold Regression Results

The regression results presented in this appendix investigate to what degree the results presented in Tables 4.9 through 4.12 in the main body of the chapter vary when a lower value for the single threshold variable is assumed. The results in Table 4.9 and Table 4.10 suggest that, for a single threshold of 35% real exchange rate misalignment, the degree of real exchange rate overvaluation is statistically significant both above and below the threshold for the full sample. However, the effects are greater below real exchange rate overvaluation of 35%. When the sample is split by population size and the degree of openness, the degree of real exchange rate overvaluation is positive and statistically significant below real exchange rate overvaluation of 35% for all economies but is only statistically significant above the threshold for larger, less open economies. For the double threshold model in Table 4.11 and Table 4.12, below 24% real exchange rate misalignment, the degree of real exchange rate misalignment has a positive but statistically insignificant effect on the probability of experiencing a currency crash one year later. Above that threshold (and below a threshold of 35% real exchange rate misalignment), the size and statistical significance of the effect increases substantially. However, beyond 35%, the size of the coefficient falls again, and even becomes statistically insignificant for those smaller economies that depend more on imports of goods and services.

Table 4.17 and Table 4.18 present the case where the single threshold is a real exchange rate overvaluation of 24%. In this case, real exchange rate misalignments below 24% are not statistically significant predictors of a currency crash one year later. However, above that level of real exchange rate misalignment, the coefficient remains positive and becomes statistically different from zero. This result is consistent across subsamples split by the degree of trade openness and population size. However, the results do suggest that the sensitivities of experiencing a currency crash in response to an increase in real exchange rate overvaluation are greater for larger, less open economies, especially at levels of real exchange rate misalignment above the assumed threshold.

Table 4.17: Discrete Choice Logit Random Effects Estimates with Single Threshold (24%) – Split by Degree of Openness – Dependent Variable: Currency Crash

Variable	**Baseline**	**24% Threshold**	**Threshold – Less Open**	**Threshold – More Open**
Mis_{it-1}	0.028***			
	(0.006)			
Δr_t^{US}	0.199***	0.199***	0.214**	0.182*
	(0.073)	(0.073)	(0.102)	(0.104)
$\Delta Gov\ Debt_{it-1}$	0.009	0.008	0.025	-0.002
	(0.014)	(0.014)	(0.018)	(0.018)
Res_{it-1}	-0.146*	-0.150**	-0.093	-0.340**
	(0.076)	(0.076)	(0.087)	(0.161)
FDI_{it-1}	-0.029	-0.029	-0.083	-0.020
	(0.018)	(0.019)	(0.106)	(0.016)
$Mis_{it-1}\ I(Mis_{it-1} \leq \delta)$		0.016	0.027	0.003
		(0.011)	(0.017)	(0.011)
$Mis_{it-1}\ I(Mis_{it-1} > \delta)$		0.032***	0.032***	0.025**
		(0.007)	(0.009)	(0.012)
Constant	-3.717***	-3.747***	-3.696***	-3.497***
	(0.272)	(0.257)	(0.336)	(0.477)
Threshold (δ)		24%	24%	24%
Total Observations	3,170	3,170	1,652	1,518

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Table 4.18: Discrete Choice Logit Random Effects Estimates with Single Threshold (24%) – Split by Population Size – Dependent Variable: Currency Crash

Variable	**Baseline**	**24% Threshold**	**Threshold – Smaller**	**Threshold – Larger**
Mis_{it-1}	0.028***			
	(0.006)			
Δr_t^{US}	0.199***	0.199***	0.210*	0.193**
	(0.073)	(0.073)	(0.117)	(0.096)
$\Delta Gov\ Debt_{it-1}$	0.009	0.008	-0.004	0.030
	(0.014)	(0.014)	(0.019)	(0.020)
Res_{it-1}	-0.146*	-0.150**	-0.245	-0.140
	(0.076)	(0.076)	(0.178)	(0.087)
FDI_{it-1}	-0.029	-0.029	-0.020	-0.039
	(0.018)	(0.019)	(0.021)	(0.032)
$Mis_{it-1}\ I(Mis_{it-1} \leq \delta)$		0.016	0.015	0.017
		(0.011)	(0.017)	(0.015)
$Mis_{it-1}\ I(Mis_{it-1} > \delta)$		0.032***	0.024**	0.035***
		(0.007)	(0.011)	(0.009)
Constant	-3.717***	-3.747***	-3.628***	-3.632***
	(0.272)	(0.257)	(0.511)	(0.308)
Threshold (δ)		24%	24%	24%
Total Observations	3,170	3,170	1,462	1,708

*Notes: *, **, *** indicate statistical significance at the 10%, 5% and 1% levels respectively; standard errors (se) are robust standard errors to correct for potential heteroskedasticity and autocorrelation.*

Chapter 5: Conclusion

The world's smallest and most open economies are unique. These countries are susceptible to external shocks stemming from various sources (for example, terms of trade, sudden stops in capital inflows, natural disasters) and often suffer from larger contractions in real gross domestic product emanating from these shocks. Moreover, their dependence on imports of goods and services for consumption and investment leave them more vulnerable to shocks which restrict their access to international financing and limit their abilities to import. Consequently, they often exhibit much greater volatility in per capita consumption relative to volatility in per capita income compared to their larger, less open counterparts. Finally, although the range of countries that maintain exchange rate pegs varies by both size and the degree of openness, many small, open economies often opt to maintain fixed or largely stable exchange rates, given their high degrees of openness, limited production capacities and the implications for high exchange rate pass through to inflation.

One would expect that most standard relationships and models in international finance would account for these peculiarities and provide for the heterogeneities present across emerging markets and developing economies. However, many do not, and often make assumptions that are inconsistent with the realities of these economies. Therefore, this thesis sought to illustrate empirically that these heterogeneities and nonlinearities do exist and should justify taking a different approach to modelling these economies.

The analysis in Chapter 2 illustrates that incorporating these heterogeneities and nonlinearities can go some way to explain long-standing puzzles of international macroeconomics. The empirical regularity that has become the Backus-Smith puzzle suggests that relative per capita consumption is either negatively correlated with the real exchange rate, or the two exhibit very little correlation at all. This runs counter to the theoretical predictions which suggest that consumers should take advantage of cheaper baskets of goods and services, thereby producing a strong and positive correlation between relative consumption and the real exchange rate. This would imply a role for international risk sharing. Corsetti et al.'s (2008) work suggests that these correlations may depend on the degree of home bias and the elasticity of substitution between goods produced at home and those produced and imported from abroad. The chapter's analysis leverages data for a diverse group of 150 countries over a period covering 1981 to 2018 to show that the negative correlations associated with the puzzle are consistent with the larger, less import dependent economies that, to date, have primarily been the subjects of this research. However, for smaller, more open economies, the puzzle essentially disappears, especially for countries whose ratio of imports of goods and services to nominal gross domestic product exceeds 44% and countries whose populations do not exceed 5.4 million people.

Although some consumers across small, open emerging markets and developing economies share risks, they still opt to build external buffers with which to shield against or even reduce the likelihood of external shocks. In Chapter 3, the analysis turns to the heterogeneities and nonlinearities which alter the relationships between the accumulation of foreign exchange reserves through debt issuance and the spreads that governments pay on externally issued bonds. Moreover, the analysis questions whether a threshold of external debt exists above which the established negative relationship between foreign exchange reserves and bond spreads disappears. And, it asks whether this result holds across countries with different degrees of exchange rate flexibility. The chapter investigates these issues for 28 emerging markets and developing economies and finds that although larger stocks of external government debt increase bond spreads, this effect is no greater than the negative effect of additional foreign exchange reserves on bond spreads. However, when external government debt exceeds 33% of nominal gross domestic product, the effects of foreign exchange reserves on bond spreads virtually disappear. However, these effects are much stronger for economies with more stable exchange rates than for those with more flexible exchange rate regimes. That is, external bond spreads for economies with less flexible exchange rates benefit more from the accumulation of foreign exchange reserves than in economies whose currencies move more freely.

The results from Chapter 3 suggest that international investors view economies differently, depending both on their exchange rate regime and their underlying economic fundamentals. To investors, foreign exchange reserves appear to matter more for economies with less flexible exchange rates. This is consistent with the many roles that foreign exchange reserves play in maintaining the exchange rate at or close to its pre-determined target and acting as a precautionary buffer in anticipation of shocks to either the current or financial accounts. In contrast, economies with more flexible exchange rates rely less on foreign exchange reserves for exchange rate management. Therefore, the policy advice that may be appropriate to prescribe to and implement in economies with floating exchange rates whose ratios of external government debt to nominal gross domestic product are below 33% will probably differ from those recommended to economies with fixed exchange rates with low levels of external government debt. Countries with fixed exchange rates and low levels of external government debt should be able to borrow internationally to build their stocks of foreign exchange reserves without paying higher rates on any new borrowing.

The analysis in Chapter 4 confirms that international investors should also care about a country's degree of trade openness or its size when attempting to assess both the exchange rate and default risks to their investments. Ultimately, even with the best will in the world, economies of all sizes, degrees of trade openness and exchange rate regimes face external shocks which require an aggressive policy response. The external imbalances which policymakers need to correct sometimes require either (or a combination of) the devaluation of the nominal exchange rate or default on external or foreign currency debt. The policymaker's decision to choose either depends on the relative costs and benefits to the economy. Chapter 4 leverages data for 114 countries over the period 1974 to 2017 to determine whether a country's size or dependence on imports of goods and services influence these decisions, especially when faced with a large, overvalued exchange rate. The results suggest that the degree of real exchange rate overvaluation is a significant predictor of a currency crash (a proxy for a large, nominal exchange rate devaluation) and foreign currency default among economies with fixed or managed exchange rate regimes. Moreover, the relationship between the magnitude of exchange rate overvaluation and the likelihood of experiencing a currency crash is nonlinear – the effects are statistically significant when the degree of overvaluation exceeds 24%, but becomes statistically insignificant for smaller, more open economies when the degree of overvaluation reaches 35%. When faced with a choice between nominal exchange rate devaluation and default on foreign currency debt however, policymakers of smaller, more open economies are typically less likely to choose devaluation than default (relative to either choice, and relative to larger, less open economies), because the net benefits which accrue from the former are probably smaller. The high exchange rate passthrough to inflation from imported goods and services and a limited capacity to produce goods domestically to replace more expensive imports of goods and services reduces the effectiveness of nominal exchange rate devaluation. This helps to explain why many small, open economies (including the example of Barbados in Chapter 1) have opted to maintain their exchange rate pegs for decades, even in the face of recurring balance of payments crises.

International investors may therefore opt to charge a larger risk premium on foreign currency assets than on local currency assets in small, open economies with fixed or managed exchange rates, recognizing that most policymakers' appetite for nominal exchange rate devaluation is limited. Alternatively, the chance of devaluation in much larger economies that depend less on imports of goods and services is probably higher, and so the risk premium demanded on assets denominated in foreign currency (relative to those in local currency) may be smaller or even negative.

Taken together, the results presented in this thesis suggest that policymakers need to design economic policies that cater not only to an economy's macroeconomic fundamentals, but also to its unique structure. For example, the decision of whether or not to devalue the nominal exchange rate and instead opt for other methods of macroeconomic adjustment should consider the legitimate concerns about exchange rate pass through to inflation and limited production capacity. Similarly, a government's decision to default on its foreign currency debt often carries quite a lot of stigma but should be viewed within the context of the lack of viable options which may be at its disposal, given its underlying economic structure and exchange rate regime. Moreover, in light of the vulnerabilities and constraints that small, import dependent economies face (including the range of external shocks to which they are exposed to), a case could be made that these countries could benefit from access to concessional funding which may help to alleviate the foreign exchange constraint that they face. This may be particularly useful in aiding policymakers in these countries to build financial buffers and boost resilience (both to macroeconomic and disaster-related shocks) that may help to reduce the excessive volatility in consumption that plagues many of these economies.

Finally, the findings contained within this thesis offer insights from which academics and those building real business cycle models for small, open economies can incorporate additional heterogeneities into their analysis. The result that the Backus-Smith puzzle disappears for economies with relatively low degrees of home bias should prompt researchers to rethink whether the default assumption of home bias is always an appropriate one. Similarly, many frameworks which model foreign exchange reserves and external debt fail to account for the structure of the economy and the exchange rate regime in place. Perhaps, models which incorporate these features can be among a new wave of frameworks which help us to better understand how small, open economies work.

Bibliography

Acevedo, S. (2016). Gone with the Wind: Estimating Hurricane and Climate Change Costs in the Caribbean. *IMF Working Paper WP/16/199*, 1-40.

Aizenman, J., & Lee, J. (2007). International Reserves: Precautionary Versus Mercantilist Views, Theory and Evidence. *Open Economies Review*, *18*, 191–214.

Aizenman, J., & Riera-Crichton, D. (2008, November). Real Exchange Rate and International Reserves in an Era of Growing Financial and Trade Integration. *The Review of Economics and Statistics*, *90*(4), 812-815.

Aizenman, J., & Sun, Y. (2012). The financial crisis and sizable international reserves depletion: From 'fear of floating' to the 'fear of losing international reserves'? *International Review of Economics and Finance, 24*, 250–269. doi:10.1016/j.iref.2012.03.004

Aizenman, J., Edwards, S., & Riera-Crichton, D. (2012). Adjustment patterns to commodity terms of trade shocks: The role of exchange rate and international reserves policies. *Journal of International Money and Finance, 31*, 1990–2016.

Akkoyun, H. Ç., Arslan, Y., & Kılınç, M. (2017). Risk sharing and real exchange rates: The role of non-tradable sector and trend shocks. *Journal of International Money and Finance*, *73*, 232-248.

Backus, D. K., & Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics , 35*, 297-316.

Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1992). International Real Business Cycles. *Journal of Political Economy*, *100*(4), 745-775.

Bahmani-Oskooee, M., Kutan, A. M., & Xi, D. (2015). Does exchange rate volatility hurt domestic consumption? Evidence from emerging economies. *International Economics, 144*, 53–65.

Bar-Ilan, A., & Lederman, D. (2007). International reserves and monetary policy. *Economics Letters, 97*, 170-178.

Bar-Ilan, A., & Marion, N. P. (2009). A Macroeconomic Perspective on Reserve Accumulation. *Review of International Economics, 17*(4), 802–823.

Bauer, C., Herz, B., & Karb, V. (2003). The other twins: Currency and debt crises. *Review of Economics, 54*(3), 248-267.

Bauer, C., Herz, B., & Karb, V. (2007). Are twin currency and debt crises special? *Journal of Financial Stability, 3*, 59-84.

Beers, D., Jones, E., & Walsh, J. (2020, June). The BoC-BoE Sovereign Default Database: What's New in 2020? *Bank of Canada Staff Analytical Note 2020-13*.

Benigno, G., & Thoenissen, C. (2008). Consumption and real exchange rates with incomplete markets and non-traded goods. *Journal of International Money and Finance, 27*, 926-948.

Bianchi, J., Hatchondo, J. C., & Martinez, L. (2018). International Reserves and Rollover Risk. *The American Economic Review*, 2629-2670.

Broda, C. (2004). Terms of trade and exchange rate regimes in developing countries. *Journal of International Economics, 63*, 31-58.

Burnside, C., & Graveline, J. (2012). Exchange rate determination, risk sharing and the asset market view. *National Bureau of Economic Research, Working Paper 18646*, 1-47.

Bussière, M., Callegari, G., Ghironi, F., Sestieri, G., & Yamano, N. (2013, July). Estimating Trade Elasticities: Demand Composition and the Trade Collapse of 2008-2009. *American Economic Journal: Macroeconomics, 5*(3), 118-151.

Calvo, G. (1983). Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics, 12*(3), 383-398.

Calvo, G. A., & Reinhart, C. M. (2002, May). Fear of Floating. *The Quarterly Journal of Economics, 117*(2), 379-408.

Canova, F. (2005). The Transmission of US Shocks to Latin America. *Journal of Applied Econometrics, 20*, 229–251.

Caporale, G. M., Donadelli, M., & Varani, A. (2015). International capital markets structure, preferences and puzzles: A "US–China World". *International Financial Markets, Institutions and Money, 36*, 85-99.

Caputo, R. (2015). Persistent real misalignments and the role of the exchange rate regime. *Economics Letters, 135*, 112-116.

Catão, L. A., & Mano, R. C. (2017). Default Premium. *Journal of International Economics, 107,* 91-110.

Chari, V. V., Kehoe, P. J., & McGrattan, E. R. (2002, July). Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates? *The Review of Economic Studies*, *69*(3), 533-563.

Clark, P. B., & MacDonald, R. (1998). Exchange Rates and Economic Fundamentals : A Methodological Comparison of BEERs and FEERs. *IMF Working Paper WP/98/67*.

Corsetti, G., Dedola, L., & Leduc, S. (2008). International Risk Sharing and the Transmission of Productivity Shocks. *Review of Economic Studies, 75*, 443-473.

Couharde, C., Delatte, A.-L., Grekou, C., Mignon, V., & Morvillier, F. (2018). EQCHANGE: A world database on actual and equilibrium effective exchange rates. *International Economics, 156*, 206-230.

Crispolti, V. (2018). Do small states need large external buffers? Evidence from an event-study analysis. In L. Briguglio (Ed.), *Handbook of Small States: Economic, Social and Environmental Issues (Europa Perspectives: Emerging Economies).* Taylor and Francis.

Cruces, J. J., & Trebesch, C. (2013). Sovereign Defaults: The Price of Haircuts. *American Economic Journal: Macroeconomics, 5*(3), 85-117.

Crucini, M. J. (1999, February). On International and National Dimensions of Risk Sharing. *The Review of Economics and Statistics, 81*(1), 73-84.

Cubeddu, L., Krogstrup, S., Adler, G., Rabanal, P., Dao, M. C., Hannan, S. A., . . . Wang, Z. (2019, March). The External Balance Assessment Methodology: 2018 Update. *IMF Working Paper WP/19/65*.

Culiuc, A. (2014). Determinants of International Tourism. *IMF Working Paper WP/14/82*.

Devereux, M. B., Smith, G. W., & Yetman, J. (2012). Consumption and real exchange rates in professional forecasts. *Journal of International Economics, 86*, 33-42.

Diamond, D. W., & Dybvig, P. H. (1983, June). Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy, 91*(3), 401-419.

Driver, R. L., & Westaway, P. F. (2004). Concepts of equilibrium exchange rates. *Bank of England Working Papers no.248*.

Eaton, J., & Gersovitz, M. (1981, April). Debt with Potential Repudiation: Theoretical and Empirical Analysis. *The Review of Economic Studies*, *48*(2), 289-309.

Edwards, S. (1984, September). LDC Foreign Borrowing and Default Risk: An Empirical Investigation, 1976-80. *The American Economic Review, 74*(4), 726-734.

Edwards, S. (1986, June). The pricing of bonds and bank loans in international markets: An empirical analysis of developing countries' foreign borrowing. *European Economic Review, 30*(3), 565-589.

Eijffinger, S. C., & Qian, Z. (2016). Trade openness and the Phillips curve: The neglected heterogeneity and robustness of empirical evidence. *International Review of Economics and Finance, 44*, 13-18.

Engel, C., & Wang, J. (2011). International trade in durable goods: Understanding volatility, cyclicality, and elasticities. *Journal of International Economics, 83*, 37-52.

Feder, G., & Just, R. E. (1977). An Analysis of Credit Terms in the Eurodollar Market. *European Economic Review, 9*, 221-243.

Feenstra, R., Luck, P., Obstfeld, M., & Russ, K. (2014). In Search of the Armington Elasticity. *NBER Working Paper No. 20063*.

Frankel, J., & Rose, A. K. (1996). Currency crashes in emerging markets: an empirical treatment. *Journal of International Economics, 41*, 351-366.

Friedman, M. (1957). The Permanent Income Hypothesis. *National Bureau of Economic Research Working Papers*, 20-37.

Fuleky, P., Ventura, L., & Zhao, Q. (2017). Common correlated effects and international risk sharing. *International Finance*, 1-16.

Galí, J., & Monacelli, T. (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies, 72*, 707-734.

Gandelman, N., & Hernández-Murillo, R. (2014, October). Risk Aversion at the Country Level. *Federal Reserve Bank of St. Louis Working Paper Series*, 1-17.

Ghosh, A., Ostry, J., & Chamon, M. (2015). Two targets, two instruments: Monetary and exchange rate policies in emerging market economies. *Journal of International Money and Finance*, 1-25. doi:http://dx.doi.org/10.1016/j.jimonfin.2015.03.005

Goldfajn, I., & Valdes, R. (1999). The Aftermath of Appreciations. *The Quarterly Journal of Economics*, *114*(1), 229-262.

Grekou, C. (2015). Revisiting the nexus between currency misalignments and growth in the CFA Zone. *Economic Modelling, 45*, 142-154.

Grekou, C. (2019). From nominal devaluations to real depreciations. *International Economics, 157*, 68-81.

Hadzi-Vaskov, M. (2008, March). Does the Nominal Exchange Rate Explain the Backus-Smith Puzzle? Evidence from the Eurozone. *Tjalling C. Koopmans Research Institute Discussion Paper Series 07-32*, 1-46.

Hamano, M. (2013). The consumption-real exchange rate anomaly with extensive margins. *Journal of International Money and Finance, 36*, 26-46.

Hansen, B. E. (1999). Threshold effects in non-dynamic panels: Estimation, testing, and inference. *Journal of Econometrics, 93*, 345-368.

Hess, G. D., & Shin, K. (2010). Understanding the Backus–Smith puzzle: It's the (nominal) exchange rate, stupid. *Journal of International Money and Finance, 29*, 169-180.

Hevia, C. (2014). Emerging market fluctuations:What makes the difference? *Journal of International Economics, 94*, 33-49.

Hevia, C., & Servén, L. (2018). Assessing the degree of international consumption risk sharing. *Journal of Development Economics, 134*, 176-190.

Holtemöller, O., & Mallick, S. (2013). Exchange rate regime, real misalignment and currency crises. *Economic Modelling, 34*, 5-14.

Hooper, P., Johnson, K., & Marquez, J. (2000, August). Trade Elasticities for the G-7 Countries. *Princeton Studies in International Economics, 87*.

Hur, S., & Kondo, I. O. (2016). A theory of rollover risk, sudden stops, and foreign reserves. *Journal of International Economics, 103*, 44–63.

Ilzetzki, E., Reinhart, C. M., & Rogoff, K. S. (2016). Exchange Arrangements Entering the 21st Century: Which Anchor Will Hold? *NBER Working Paper No. 23134*, 1-85.

Im, K. S., Pesaran, M., & Shin, Y. (2003). Testing for unit roots in heterogeneous panels. *Journal of Econometrics, 115*(1), 53-74.

International Monetary Fund. (2013). *Assessing Reserve Adequacy - Further Considerations.* Washington, D.C.: International Monetary Fund.

International Monetary Fund. (2015). *Assessing Reserve Adequacy - Specific Proposals.* Washington, D.C.: International Monetary Fund.

International Monetary Fund. (2016). *Guidance Note on the Assessment of Reserve Adequacy and Related Considerations.* Washington, D.C.: International Monetary Fund.

International Monetary Fund. (2020). *Annual Report on Exchange Arrangements and Exchange Restrictions 2019.* Washington, D.C.: International Monetary Fund.

Iossifov, P., & Fei, X. (2019). Real Effective Exchange Rate and Trade Balance Adjustment: The Case of Turkey. *IMF Working Paper WP/19/131*.

Isard, P. (2007). Equilibrium Exchange Rates: Assessment Methodologies;. *IMF Working Paper WP/07/296*.

Jahjah, S., Wei, B., & Yue, V. Z. (2013, October). Exchange Rate Policy and Sovereign Bond Spreads in Developing Countries. *Journal of Money, Credit and Banking, 45*(7), 1275-1300.

Jeanne, O. (2000). Currency crises: A perspective on recent theoretical developments. *Princeton University, International Finance Section, Special Papers in International Economics No. 20*.

Jeanne, O., & Rancière, R. (2011, September). The optimal level of international reserves for emerging market countries: a new formula and some applications. *The Economic Journal, 121*, 905-930.

Kaminsky, G. L. (2006). Currency crises: Are they all the same? *Journal of International Money and Finance, 25*, 503-527.

Kaminsky, G., Lizondo, S., & Reinhart, C. M. (1997). Leading Indicators of Currency Crises. *IMF Working Paper No. WP/97/79.*

Kato, M., Proaño, C. R., & Semmler, W. (2018). Does international-reserves targeting decrease the vulnerability to capital flights? *Research in International Business and Finance, 44*, 64–75.

Kim, Y. J. (2017). Sudden stops, limited enforcement, and optimal reserves. *International Review of Economics and Finance, 51*, 273-282.

Kodama, M. (2013, September). External Shocks and High Volatility in Consumption in Low-Income Countries. *The Developing Economies, 51*(3), 278-302.

Kollmann, R. (1995). Consumption, real exchange rates and the structure of international asset markets. *Journal of lnternational Money and Finance, 14*(2), 191-211.

Kose, M. A., Nagle, P., Ohnsorge, F., & Sugawara, N. (2019). *Global Waves of Debt: Causes and Consequences.* Washington, D.C.: World Bank Group.

Krugman, P. (1979, August). A Model of Balance-of-Payments Crises. *Journal of Money, Credit and Banking, 11*(3), 311-325.

Krugman, P. (1999). Balance Sheets, the Transfer Problem and Financial Crises. In P. Isard, A. Razin, & A. K. Rose (Eds.), *International Finance and Financial Crises, Essays in Honor of Robert P. Flood* (pp. 31-55). Amsterdam: Kluwer.

Krugman, P. (2001). Crises: The Next Generation. *Unpublished Manuscript*.

Kuvshinov, D., & Zimmermann, K. (2019). Sovereigns going bust: Estimating the cost of default. *European Economic Review, 119*, 1-21.

Laeven, L., & Valencia, F. (2018). Systemic Banking Crises Revisited. *IMF Working Paper WP/18/206*.

Lambrias, K. (2020). Real exchange rates and international co-movement: News-shocks and non-tradable goods with complete markets. *Review of Economic Dynamics, 35*, 154-169.

Lane, P. R. (1997). Inflation in Open Economies. *Journal of International Economics, 42*, 327-347.

Levy Yeyati, E. (2008). The cost of reserves. *Economics Letters, 100*, 39–42.

Liu, X., Yang, F., & Cai, Z. (2016). Does relative risk aversion vary with wealth? Evidence from households' portfolio choice data. *Journal of Economic Dynamics & Control, 69*, 229–248.

MacDonald, R. (2007). *Exchange Rate Economics: Theories and Evidence.* London and New York: Routledge.

Mendoza, E. G., & Yue, V. Z. (2012). A General Equilibrium Model of Sovereign Default and Business Cycles. *The Quarterly Journal of Economics, 127*, 889–946. doi:10.1093/qje/qjs009

Modigliani, F., & Ando, A. (1957). Tests of the life cycle hypothesis of saving: comments and suggestions. *Bulletin of the Oxford University Institute of Statistics, 19*, 99-124.

Modigliani, F., & Brumberg, R. (1954). Utility analysis and the consumption function: an interpretation of cross-section data. In K. K. Kurihara (Ed.), *Post Keynesian Economics* (pp. 388-436). New Brunswick: Rutgers University Press.

Moore, W., & Glean, A. (2016). Foreign exchange reserve adequacy and exogenous shocks. *Applied Economics, 48*(6), 490-501. doi:10.1080/00036846.2015.1083085

Moore, W., & Walkes, C. (2010). Does industrial concentration impact on the relationship between policies and volatility? *International Review of Applied Economics, 24*(2), 179-202. doi:10.1080/02692170903424315

Mwase, N. (2012). How much should I hold? Reserve Adequacy in Emerging Markets and Small Islands. *IMF Working Paper WP/12/205*, 1-43.

Na, S., Schmitt-Grohé, S., Uribe, M., & Yue, V. (2018). The Twin Ds: Optimal Default and Devaluation. *The American Economic Review, 108*(7), 1773–1819. doi:https://doi.org/10.1257/aer.20141558

Nakatani, R. (2017). The Effects of Productivity Shocks, Financial Shocks, and Monetary Policy on Exchange Rates: An Application of the Currency Crisis Model and Implications for Emerging Market Crises. *Emerging Markets Finance and Trade, 53*(11), 2545-2561.

Nakatani, R. (2018). Real and financial shocks, exchange rate regimes and the probability of a currency crisis. *Journal of Policy Modeling, 40*, 60-73.

Nakatani, R. (2019). Output Costs of Currency Crisis and Banking Crisis: Shocks, Policies and Cycles. *Comparative Economic Studies, 61*, 83-102.

Neumeyer, P. A., & Perri, F. (2005). Business cycles in emerging economies: the role of interest rates. *Journal of Monetary Economics, 52*, 345–380.

Obstfeld, M. (1986). Rational and Self-fulfilling Balance-of-payment Crises. *American Economic Review, 76*(1), 72-81.

Obstfeld, M. (1994). The Logic of Currency Crises. *NBER Working Paper Series No. 4640*.

Obstfeld, M. (2014). On the use of open economy new Keynesian models to evaluate policy rules. *Journal of Economic Dynamics & Control, 49*, 31-34.

Pavlidis, E. G., Paya, I., & Peel, D. A. (2015). Testing for linear and nonlinear Granger causality in the real exchange rate–consumption relation. *Economics Letters, 132*, 13-17.

Pesaran, M. H., Shin, Y., & Smith, R. P. (1999, June). Pooled Mean Group Estimation of Dynamic Heterogeneous Panels. *Journal of the American Statistical Association, 94*(446), 621-634.

Petrovic, P. (2016). Backus–Smith Puzzle and the European Union: It's Not Just the Nominal Exchange Rate. *Proceedings of Rijeka Faculty of Economics, Journal of Economics and Business, 34*(2), 393-418.

Prabheesh, K. (2013). Optimum international reserves and sovereign risk: Evidence from India. *Journal of Asian Economics, 28*, 76-86.

Prakash, K., & Maiti, D. (2016). Does devaluation improve trade balance in small island economies? The case of Fiji. *Economic Modelling, 55*, 382–393.

Qian, X., & Steiner, A. (2017). International reserves and the maturity of external debt. *Journal of International Money and Finance, 73*, 399–418.

Reinhart, C. M., & Rogoff, K. S. (2009). *This time is different: Eight centuries of financial folly*. Princeton: Princeton University Press.

Romer, D. (1993, November). Openness and Inflation: Theory and Evidence. *The Quarterly Journal of Economics,, 108*(4), 869-903.

Sallenave, A. (2010). Real exchange rate misalignments and economic performance for the G20 countries. *International Economics, 121*, 59-80.

Schmitt-Grohe´, S., & Uribe, M. (2017). *Open Economy Macroeconomics.* Princeton: Princeton University Press.

Schneider, C. R., Fehrenbacher, D. D., & Weber, E. U. (2017). Catch me if I fall: Cross-national differences in willingness to take financial risks as a function of social and state ‘cushioning’. *International Business Review, 26*, 1023–1033.

Selaive, J. D., & Tuesta, V. (2003, May). Net Foreign Assets and Imperfect Pass-through: The Consumption Real Exchange Rate Anomaly. *Board of Governors of the Federal Reserve System International Finance Discussion Papers, 764*, 1-43.

Shrestha, P., & Semmler, W. (2014). Monetary policy and international reserves: empirical evidence from East Asian countries. *International Journal of Finance & Economics, DOI: 10.1002/ijfe.1509.*

Tarashev, N., & Zabai, A. (2019). When pegging is a commitment device: Revisiting conventional wisdom about currency crises. *Journal of International Economics, 118*, 233-247.

Tebaldi, E., Nguyen, H., & Zuluaga, J. (2018). Determinants of emerging markets’ financial health: A panel data study of sovereign bond spreads. *Research in International Business and Finance, 45*, 82-93.

Vural, B. M. (2019). Determinants of Turkish real effective exchange rates. *The Quarterly Review of Economics and Finance, 73*, 151-158.

Worrell, D. (2012). Policies for Stabilization and Growth in Small Very Open Economies. *Group of Thirty, Washington, DC Occasional Paper 85*.

Worrell, D., Moore, W., & Beckles, J. (2018). A new approach to exchange rate management in small open financially integrated economies. In L. Briguglio (Ed.), *Handbook of Small States: Economic, Social and Environmental Issues (Europa Perspectives: Emerging Economies).* Taylor and Francis.

Worrell, D., Naitram, S., & Lowe, S. R. (2013). Growth Forecasts for Foreign Exchange Constrained Economies. *Business, Finance & Economics In Emerging Economies, 8*(1).

Zhao, Y., de Haan, J., Scholtens, B., & Yang, H. (2014). Leading indicators of currency crises: Are they the same in different exchange rate regimes? *Open Economies Review, 25*, 937-957.