Coupling an inviscid IGA – BEM solver with X-Foil's boundary layer model for 2D flows

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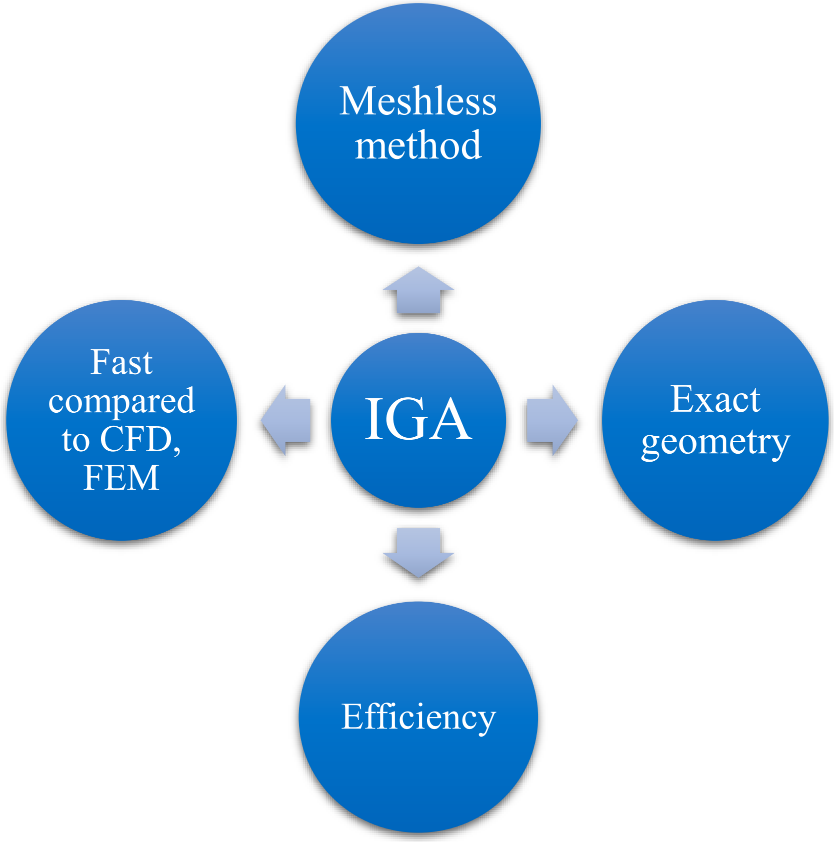
Introduction – Isogeometric analysis



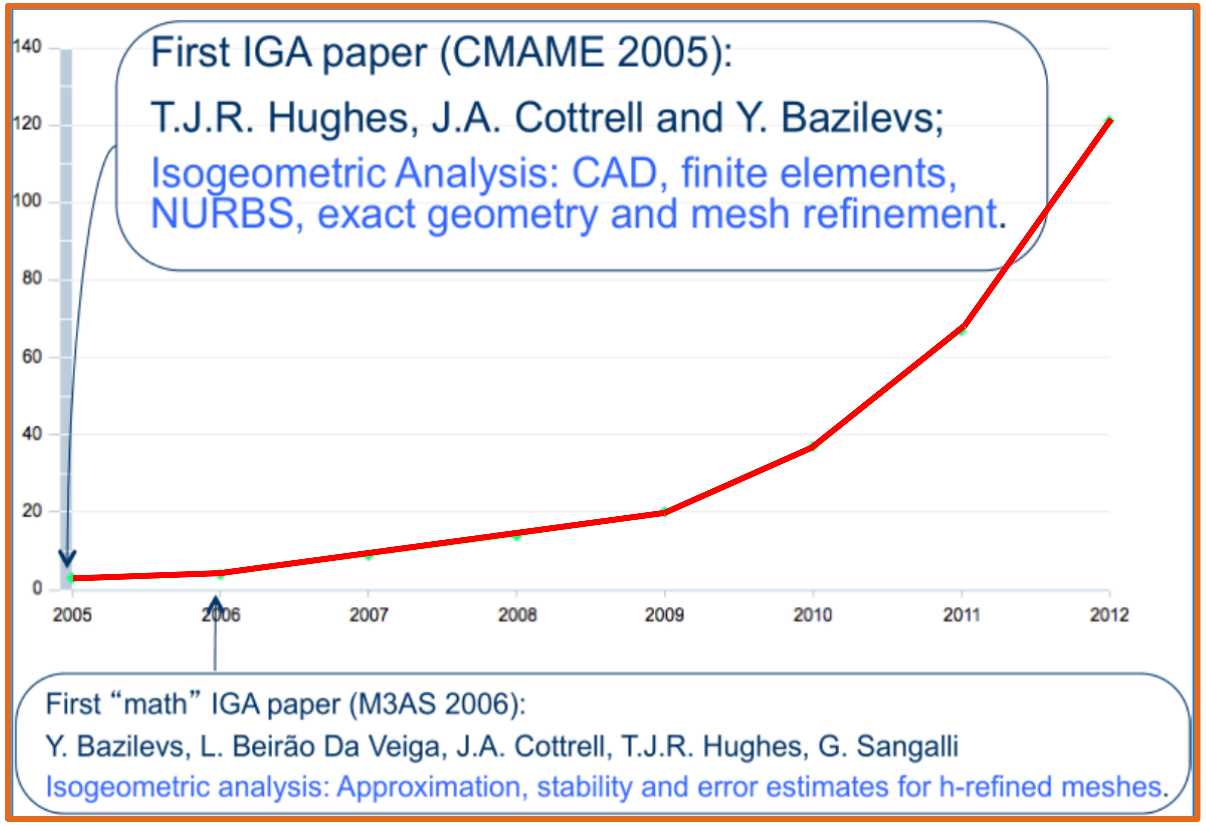
IGA

Meshless method

Exact geometry

Efficiency

Fast compared to CFD, FEM



First IGA paper (CMAME 2005):

T.J.R. Hughes, J.A. Cottrell and Y. Bazilevs;

Isogeometric Analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement.

First "math" IGA paper (M3AS 2006):

Y. Bazilevs, L. Beirão Da Veiga, J.A. Cottrell, T.J.R. Hughes, G. Sangalli

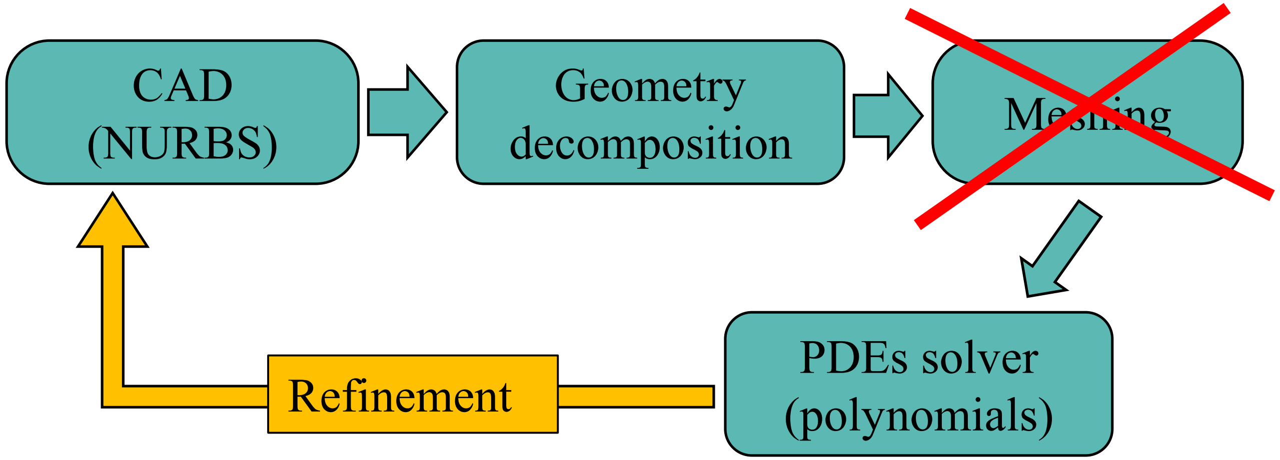
Isogeometric analysis: Approximation, stability and error estimates for h-refined meshes.

Lovadina, C., Reali, A. & Sangalli, G., 2014. What is Isogeometric Analysis?. Milan, Italy: TERRIFIC European Community's Seventh Framework Programme.

Introduction – Isogeometric analysis

Steps in CFD, FEM analysis:

- Model creation (CAD)
- Analysis of the model
- Discretization
- Meshing
- Mesh manipulation
- Solution of the governing PDEs
- Post – Process of the results



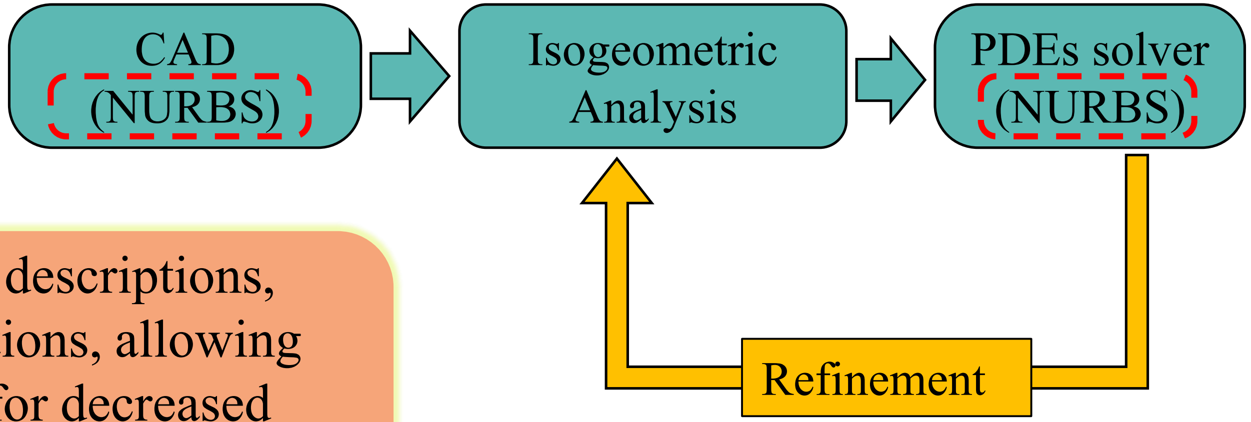
CAD (NURBS)

Geometry decomposition

~~Meshing~~

PDEs solver (polynomials)

Refinement



CAD (NURBS)

Isogeometric Analysis

PDEs solver (NURBS)

Refinement

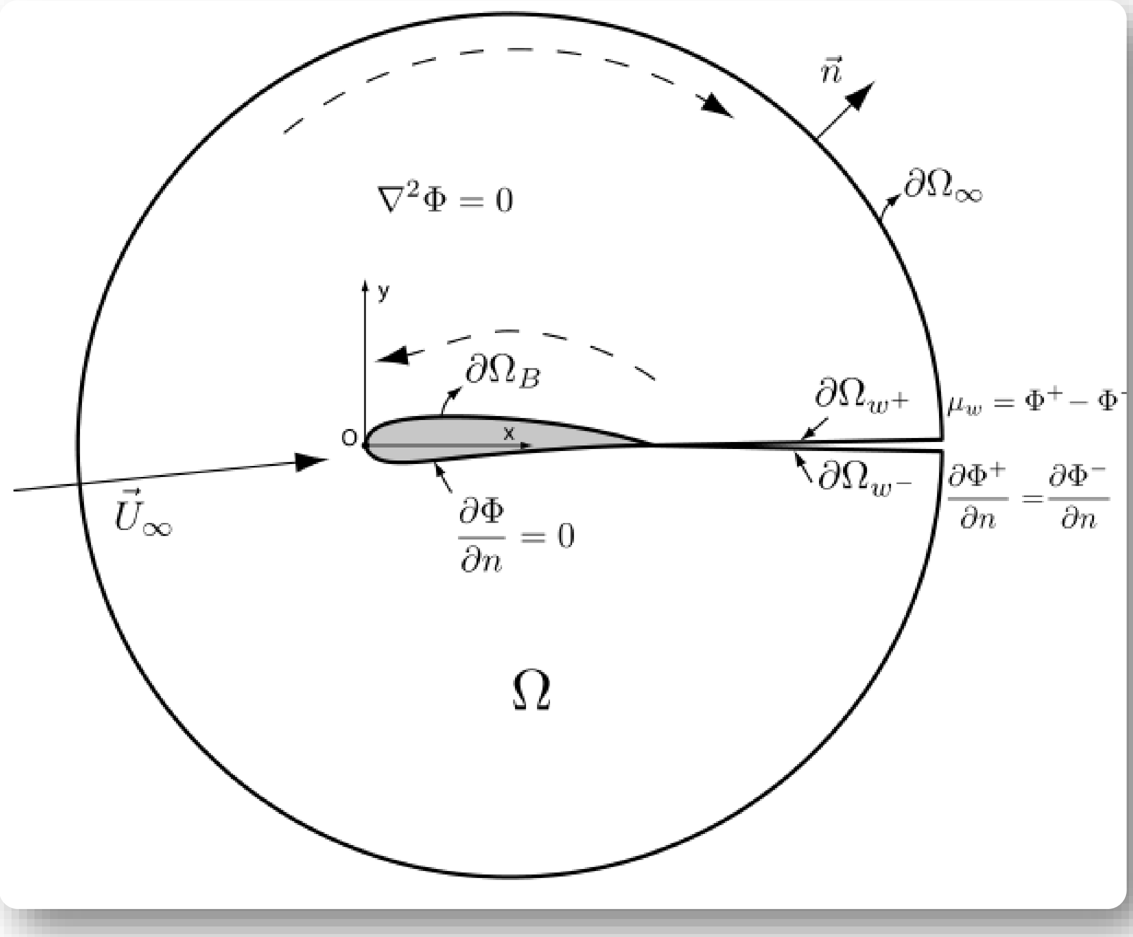
CAD and FEM use different geometry descriptions, while IGA utilizes NURBS representations, allowing for greater accuracy (exact geometry) for decreased computational effort (no meshing).

Introduction – Previous work

- Development of the IGA – BEM inviscid solver and implementation in shape optimization of hydrofoil geometries in 2016 [1].
- One – way coupling with PABLO boundary layer model in 2017 [2].

[1] Kostas, K.V., Ginnis, A.I., Politis, C.G., Kaklis, P.D. (2017) "**Shape-optimization of 2D hydrofoils using an Isogeometric BEM solver**", Computer Aided Design, vol. 82, pp. 79-87.
[2] K.Kostas, A.Ginnis, C.Politis, P.Kaklis (2017) "**Shape – optimization of 2D hydrofoils using one – way coupling of an IGA – BEM solver with a boundary – layer model**", Coupled Problems

Flow formulation



Boundary Value Problem (BVP):

$$\nabla^2\Phi = 0, \qquad \boldsymbol{P} \in \Omega$$

$$\frac{\partial\Phi}{\partial n} = 0, \qquad \boldsymbol{P} \in \partial\Omega_B$$

$$\Phi - \Phi_\infty \to 0, \qquad as\ x^2 + y^2 \to \infty$$

Wake conditions:

kinematic → $\frac{\partial\Phi^+}{\partial n} = \frac{\partial\Phi^-}{\partial n}, \boldsymbol{P} \in \partial\Omega_w$

dynamic → $p^+ = p^-, \boldsymbol{P} \in \partial\Omega_w$

Flow formulation

- Green’s 3rd identity: $\nabla^2\Phi = 0 \implies \Phi(\eta) = \oiint_S G(y,\eta)\frac{\partial\Phi}{\partial \boldsymbol{n}}(y)dS_y - \oiint_S \Phi(y)\frac{\partial G(y,\eta)}{\partial \boldsymbol{n}}dS_y$
- Laplace fundamental solution: $G(P,Q) = \frac{1}{2\pi}ln\|P-Q\|$

Applying Green’s identity between the potential $\Phi(\mathrm{P})$ and the fundamental solution $G(P,Q)$ results in the boundary integral equation (BIE):

$$\frac{\varphi(\mathrm{P})}{2} + \int_{\partial\Omega_B}\varphi(Q)\frac{\partial G(P,Q)}{\partial n_Q}ds_Q - \mu_w\int_{\partial\Omega_w}\frac{\partial G(P,Q)}{\partial n_Q}ds_Q = -\int_{\partial\Omega_B}(\overrightarrow{U_\infty}\cdot\vec{n}(Q))(\partial G(P,Q)ds_Q$$

$$\mu_w = \Phi^+(\boldsymbol{Q}) - \Phi^-(\boldsymbol{Q}), \qquad Q \in \partial\Omega_w$$

The solution of the integral equation results in a known velocity potential **everywhere** in the flow field.

μ_w represents the Kutta condition. By evaluating μ_w, the circulation around the hydrofoil is found, thus the BVP has a unique solution.

IGA implementation

NURBS curve:

$$r(t) = \big(x(t), y(t)\big) = \sum_{i=0}^{n} d_i M_{i,k}(t)$$

IGA requires the representation of the potential in terms of the NURBS basis:

$$\varphi_s(t) = P_s\big(\varphi(\tau)\big) = \sum_{i=0}^{n+l} \varphi_i \, M_{i,k}^{(l)}(t)$$

Now the BIE is transformed into:

$$\frac{1}{2}\sum_{i=0}^{n+l} \varphi_i \, M_{i,k}^{(l)}(t) + \int_I \sum_{i=0}^{n+l} \varphi_i \, M_{i,k}^{(l)}(\tau)\, K(t,\tau) d\tau - \mu_w \frac{1}{2\pi} \arctan\left(\frac{y(t) - y_e}{x(t) - x_e}\right) = g(t)$$

The BIE is solved numerically using a collocation method and eventually the tangential velocity is found using the NURBS basis derivatives as:

$$\boxed{u_t = \overrightarrow{U_\infty} \cdot \vec{t} + \frac{1}{\|\dot{r}(t)\|} \sum_{i=0}^{n+l} \varphi_i \frac{dM_{i,k}^{(l)}(t)}{dt}}$$

X-Foil

- X-Foil^{TM} was developed in MIT by M. Drela (1989) [3].
- Most airfoil software use one – equation methods (such as Thwaites‘) for the representation of the boundary layer region. The disadvantage of this method is the inadequate representation of separated flows.
- For this reason, X-Foil uses a two-equation model [4].

- Integral momentum equation:

$$\boxed{\frac{d\theta}{d\xi} + (2 + H - M_e^2)\frac{\theta}{u_e}\frac{du_e}{d\xi} = \frac{C_f}{2}}$$

  - Momentum thickness:

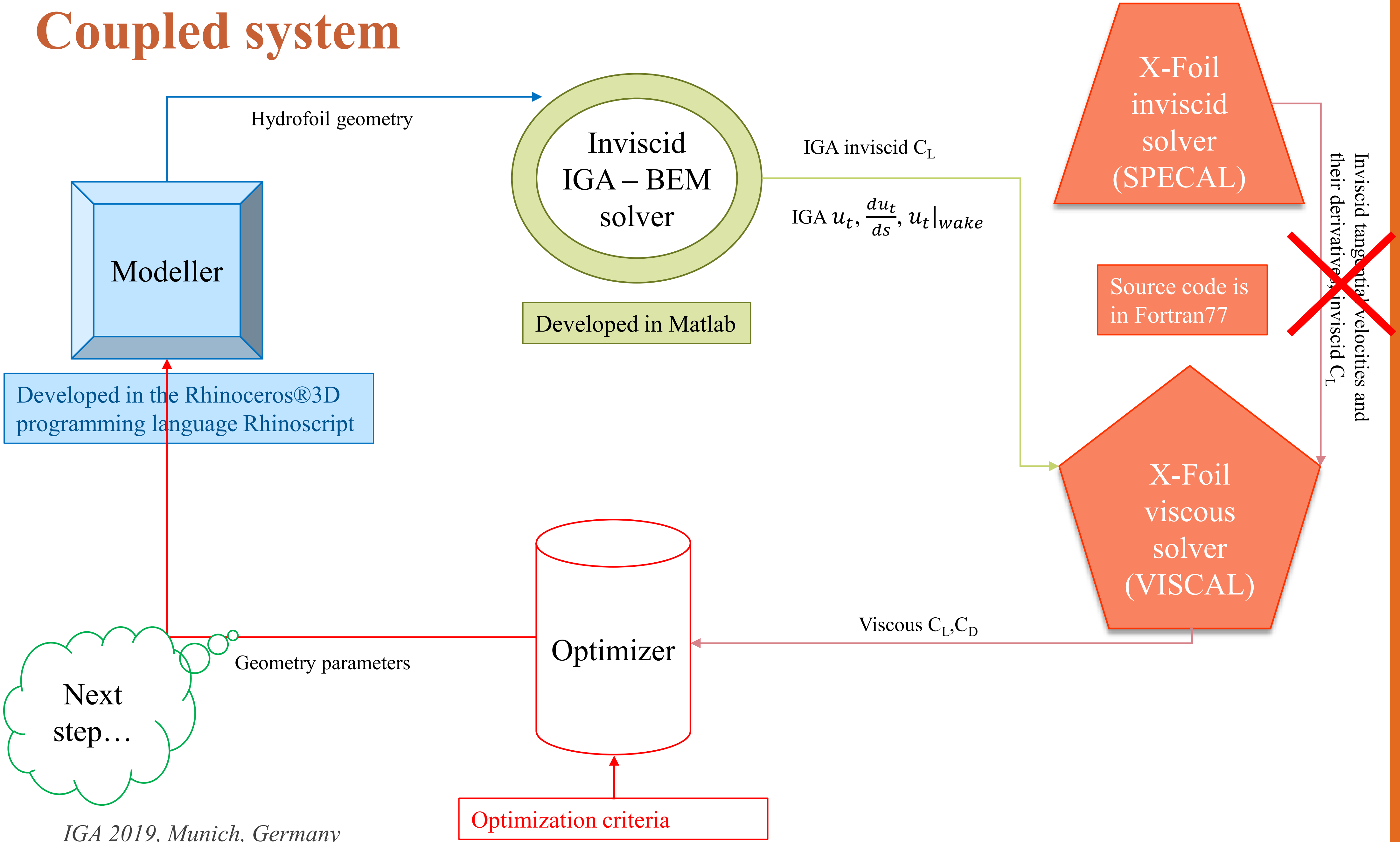
$$\theta = \int \frac{\rho u}{\rho_e u_e}\left(1 - \frac{u}{u_e}\right) d\eta$$

- Kinetic energy shape parameter equation:

$$\boxed{\theta\frac{dH^*}{d\xi} + [2\,H^{**} + H^*(1 - H)]\frac{\theta}{u_e}\frac{du_e}{d\xi} = 2\,C_D - H^*\frac{C_f}{2}}$$

[3] Drela, M. (1989) “**XFOIL: An analysis and design system for low Reynolds number airfoils**”, MIT, Massachusetts, USA.
[4] Drela, M., Giles, M. (1987) “**Viscous – inviscid analysis of transonic and low Reynolds number airfoils**”, *AIAA Journal*, vol. 25(10), pp. 1347 – 1355.

Coupled system



Hydrofoil geometry

Inviscid IGA – BEM solver

Developed in Matlab

Modeller

Developed in the Rhinoceros®3D programming language Rhinoscript

IGA inviscid C_L

IGA $u_t, \frac{du_t}{ds}, u_t|_{wake}$

X-Foil inviscid solver (SPECAL)

Source code is in Fortran77

Inviscid tangential velocities and their derivatives, inviscid C_L

X-Foil viscous solver (VISCAL)

Viscous C_L,C_D

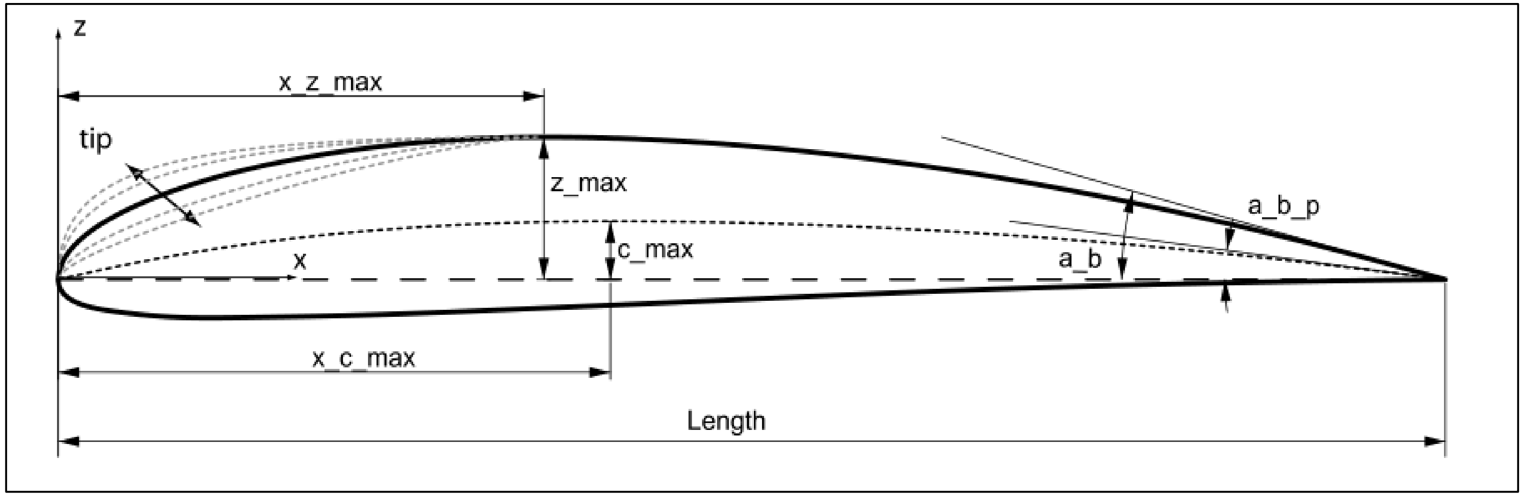
Optimizer

Geometry parameters

Next step…

Optimization criteria

Parametric modeller

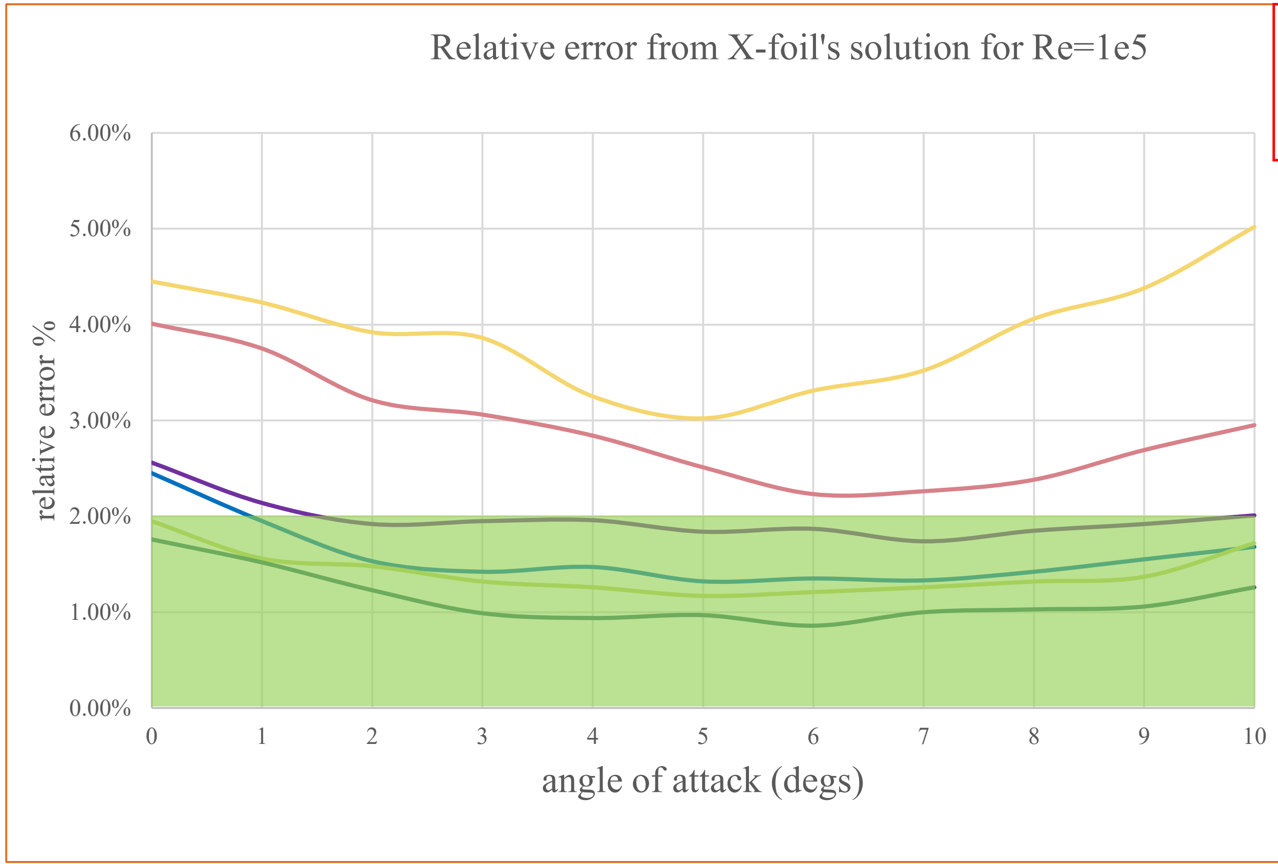


8 geometric parameters:

- Length
- Max width
- Max width position
- Max camber
- Max camber position
- Suction side angle
- Camber angle
- Tip parameter

[1] Kostas, K.V., Ginnis, A.I., Politis, C.G., Kaklis, P.D. (2017) "**Shape-optimization of 2D hydrofoils using an Isogeometric BEM solver**", Computer Aided Design, vol. 82, pp. 79-87.

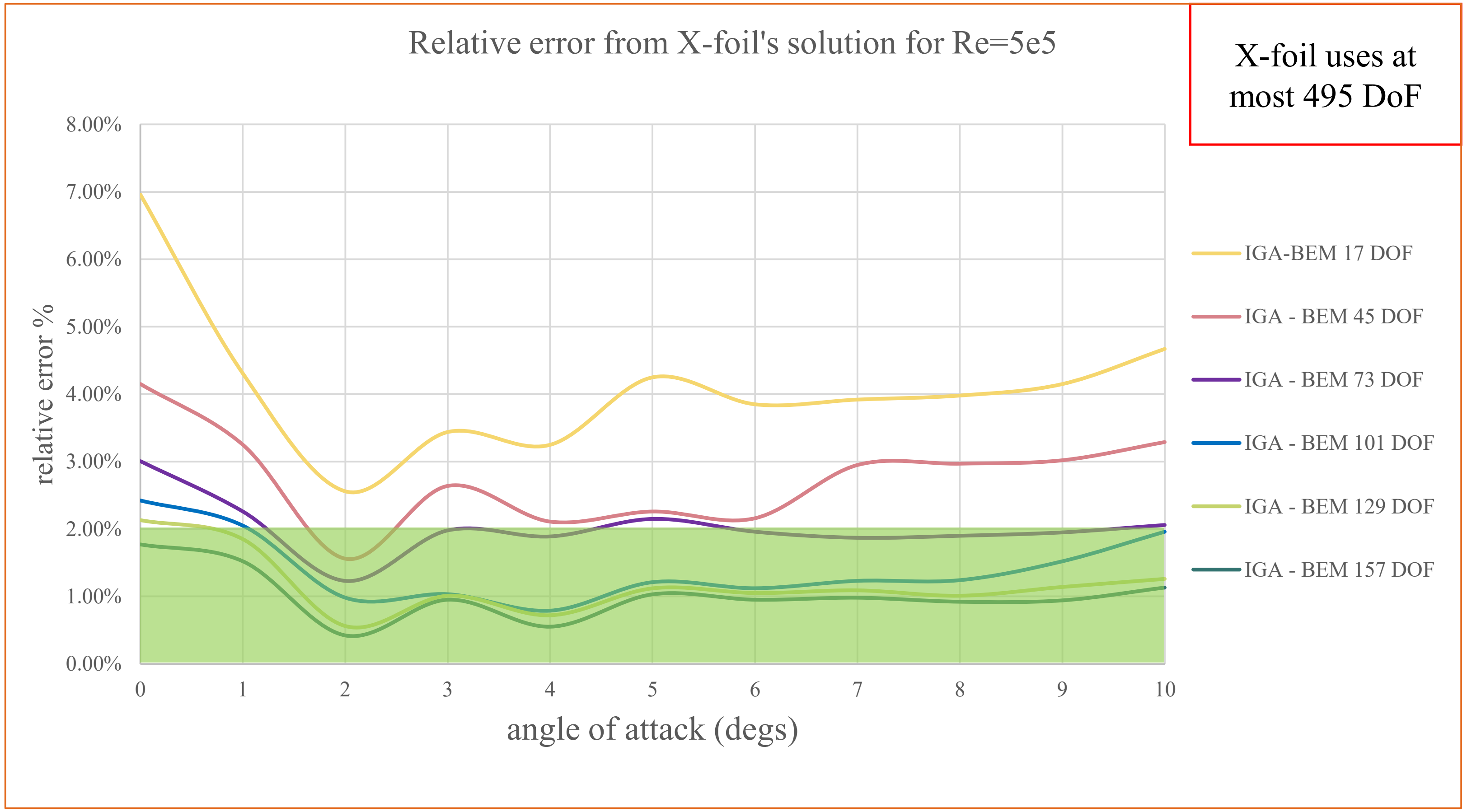
Results



X-foil uses at most 495 DoF

Same accuracy (<2% error) for approximately 1/4 of DoF

Results



X-foil uses at most 495 DoF

Same accuracy (<2% error) for approximately 1/3 of DoF

Method features - conclusions

- X-Foil proved suitable for the coupling process with the IGA – BEM solver → **coupling flexibility.**
- IGA data (lift coefficient, tangential velocity and its derivative) are transferred between the two systems **without loss of accuracy**, since they are evaluated on the exact geometry and not the panels.
- The convergence is acceptable (<2%), with respect to the considerably fewer DoF used in the coupled system → **improved computational efficiency**.

The next step

- Mathematical formulation of $\left.\frac{du_t}{ds}\right|_{wake}$. An extra IGA parameter will be sent in X-Foil’s viscous solver, resulting in stronger IGA coupling.

- Testing for more geometries

- Shape optimization

Thank you!