A significant growth in the volume of high-rise construction gives special relevance and urgency to the problem of safety of such facilities. Scientifically grounded plans of people's evacuation, including all sorts of scenarios for people's evacuation from buildings are developed for such structures. Scenarios include simulations of the motion of human flows along corridors, stairs, using elevators and mobile evacuation vehicles. An unresolved part of the problem is the problem of the rational choice and accommodation of people in stationary and mobile evacuation vehicles.

The MIP model of the rational choice and accommodation of people in mobile vehicles of evacuation from buildings was developed. A particular case of the model – optimization of accommodation of people in the emergency evacuation vehicles according to the sequence of people's arrival from the flow – was explored. The basic features of the model were analyzed: the model of the problem of mixed integer programming with piecewise continuous objective function. The specific features of the model allowed reasonable boiling down the problem to a sequence of sub-problems of accommodation the first objects (people) according to the sequence of their arrival and adapt each of them to the solution employing the multistart method with the application of artificial basis.

A three-component model is considered as objects (of human bodies). The model is subject to restrictions that ensure the conditions for "gluing" the model's components into a single complex object. Continuous rotations of the model components with limitations to the turning angles are allowed.

The proposed models and the solution methods modified in the present research make it possible to find both the configurations of the optimal-local accommodation of complex objects and the spatial shapes of objects

Keywords: mobile evacuation vehicles, accommodation configuration, locally optimal solutions, three-component model of an object, quasi-phi-functions

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DEVELOPMENT OF MODELS FOR THE RATIONAL CHOICE AND ACCOMMODATION OF PEOPLE IN MOBILE TECHNICAL VEHICLES WHEN EVACUATING FROM BUILDINGS

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1. Introduction

A significant increase in the volume of high-rise construction renders special urgency to the problem of safety of such facilities. Because of their specificity, they have a higher degree of potential danger, determined by the following:

– the increased number of floors;

– the presence of a significant number of people and a limited possibility of evacuation and rescue during fires and emergencies;

– complex structures with a large number of engineering communications and the existence of various engineering and technical systems;

– the multifunctionality of high-rise buildings;

– a probability of terrorist acts.

Scientifically grounded plans of people evacuation, including all sorts of scenarios for people evacuation from buildings, are developed for such structures. Scenarios include simulations of the motion of human flows along corridors, stairs, by using elevators and mobile evacuation vehicles during emergencies. If the above means are blocked, people take refuge from hazardous factors of emergencies within specially-protected premises. That is why it is a relevant task to simulate people's motion to allow for the rational choice of paths for evacuation from high-rise buildings, by either mobile or stationary means, provided they are optimally filled.

2. Literature review and problem statement

The applied problem under consideration belongs to the class of problems of geometric design [1], in particular, to Cutting & Packing problems, also called optimal arrangement problems [2, 3]. The problems of optimal arrangement of objects belong to the class of *NP*-complex problems. Heuristic algorithms are typically used to solve problems of this class. That is why there is a need to develop effective algorithms based on the application of the local optimization methods. These methods are based on the analytical description of relations (intersection, touching, non-intersection) between objects, taking into consideration their continuous translations and rotations.

Approach [4] became the first step in the construction of an analytical description of the conditions for mutual non-intersection of objects. The approach is based on the use of R-functions and makes it possible to describe analytically the conditions for the mutual arrangement of geometric objects of a complex shape.

Subsequent research in this direction led to the development of a mathematical apparatus of the dense arrangement function (DAF and its hodograph (HDAF) [5]. Based on the HDAF, a methodology for the sequential-single arrangement in the object arrangement problem was developed. But this methodology allowed finding only approximations to local extremes, typically for objects with piecewise-linear boundaries.

The concept of F-function [6, 7] was consequently introduced, whereby relations among geometric objects are formalized. F-functions for basic objects in a two-dimensional space were plotted [8]. The F-function method allowed describing optimization arrangement problems as the problems of non-linear programming, as well as creating effective methods for finding local extrema for a wider range of geometric objects.

However, for some objects, F-functions have a very complex form, which is why the concept of a quasi-pfi-function [9, 10] was proposed, which simplified the analytical description of relations among objects.

In the considered applied problem, a person is an arrangement object. Papers [11, 12] showed that the most adequate model of the projection of a human body onto a horizontal plane is an ellipse. Article [13] provides a fairly complete overview of the literature on the problems of ellipses arrangement. The problem of optimal arrangement of ellipses allowing continuous rotations was explored in [14]. Pseudo-normalized quasi-phi-functions [9, 10] are used for analytical description of the basic arrangement limitations. It was possible, as reported in paper [11], to simplify the analytical descriptions of relations among ellipses (non-intersection and location at a minimum allowable distance) using the quasi-phi function proposed in the cited paper.

The task of making well-grounded plans of people evacuation from buildings caused the necessity to develop the software packages to model the motion of people's flows. The CITIS software "Flowtech ID" [15], for a simplified analytical and simulation-stochastic model, and "Evatech" [12], for the individual-flow model of the human flow motion, are currently most common. The calculation results obtained from the "Flowtech ID" model give the understated values of the time it takes for a last person to travel along different cross-sections lengthwise the common passage along the evacuation route sections. The time was significantly influenced by the processes of reshaping and spreading of human flows, the description of which is not included in this model.

In the individual-current model "Evatech", the speed of person's motion depends on the flow density, which is computed for each person individually. To do this, an area in the shape of a rectangle, the large side of which is oriented in the direction of the person's motion, is constructed around a person. The area is also shifted in the direction of movement of a person with the coefficient of 0.4, that is, the center of the area is located from the center of a person at the distance equal to the length of the larger side, multiplied by 0.4. The constructed area is split into separate sub-areas (it is impossible to move from one sub-area to another without leaving the area). This approach of motion modeling is explained by the impossibility of analytical description of the process of relocating people, the constituent part of which is the description of conditions of their non-intersection. The values of time it takes for a last person to pass through different cross-sections of evacuation routes, obtained from the "Evatech" program, are near the lower boundary of permissible values when taking into consideration the stochasticity of the evacuation process.

The analysis results [16] show the lack of a model of individual-flow motion of people, adequate to the actual flow of people with limited mobile capabilities of the mixed composition in it in a fairly wide range of public buildings of different classes of functional fire danger.

At a comfortable and patient motion of people, the most adequate model of a person is an ellipse [12]. Paper [11] proposed the model and the method for simulating the motion of people, which are approximated by a set of ellipses, taking into consideration different minimum allowable distances between people and taking into consideration a series of additional technological limitations, among which it is possible to distinguish motion at different speeds, taking into account maneuverability, comfort, transformation processes and spreading of human flows.

When the motion category changes and transfers into the category of an active motion with possible forced actions, flow density increases. There arises a problem of the simulation of the movement of people taking into consideration the natural deformations of a human body, which in the research is accounted for by changing the spatial shape of the object approximating a human body.

In paper [17], it was proposed to consider a complex object represented by the combination of three ellipses as such object. In such a complex object, the basic ellipse allows continuous translations and turnings, and the two auxiliary ones allow continuous rotation within acceptable limits (relative to the turning angle of the basic one) relative to their gluing point.

When simulating the motion of human flows at each discrete moment, there is a configuration of the object arrangement [18], approximating a human body. The problem of arrangement of the considered complex objects can be used to obtain a maximum estimate of the number of people who can be accommodated within both the stationary and mobile means during their evacuation from buildings.

That is why an important and unresolved part of the problem is the development of mathematical models and methods for the rational choice and optimal accommodation of people, represented by a three-component model, within stationary and mobile evacuation vehicles under conditions of an emergency.

3. The aim and objectives of the study

The aim of this study is to develop mathematical models for the rational choice and optimal accommodation of people (complex objects) within mobile evacuation vehicles, which are represented by a set of regions of the predefined size.

To achieve the aim, the following tasks were set:

– to construct an *MIP* model for choosing and optimally accommodating people in mobile vehicles when evacuated from buildings;

– to develop a mathematical model of the rational accommodation of people within a chosen mobile vehicle when evacuees are loaded to them in the predefined order, to analyze its features;

– to construct a geometric model of a human body adequate for evacuation problems, to describe analytically the conditions of non-intersection of objects that are the model of a human;

– to modify the method of searching for a locally optimal solution for the problem of optimization of arrangement of complex three-component objects (people) in a rectangle of the predefined size (an evacuation vehicle) according to the assigned sequence of its filling.

4. *MIP* model of choosing and optimal accommodation of people within mobile vehicles during evacuation from buildings

There is a group of L people l_i, $i=1, 2, ..., n$ with the assigned individual dimensions and set M of mobile TECHNICAL VEHICLES S_j, $j=1, 2,..., m$ with evacuation zones Z_j of cost C_j. It is necessary to ensure evacuation of group L using vehicles from M of minimal cost.

Construct the common "evacuation zone" for the whole set S in the form of $Z=\bigcup_j^m Z_j$, $Z_k \cap Z_j=\varnothing$, $k<j=1,2,..., m$. Then, the *MIP* [19] mathematical model of such a problem can be stated in the form:

$$\sum_{j=1}^{m} b_j C_j \to \min, \tag{1}$$

$$l_i(u_i) \subset Z,\ i=1,\ ...,\ n, \tag{2}$$

$$l_i(u_i) \cap l_s(u_s)=\varnothing,\ i<s=1,2,...,n, \tag{3}$$

$$b_j \in \{0,1\},\ j=1,\ ...,\ m, \tag{4}$$

where $b_j=1$, if component Z_j of zone Z is not empty and is equal to 0, otherwise, u_i are the parameters of accommodation of the i-th object, unambiguously characterizing its position on a plane (the number of such parameters depends on the kind of an object).

Consider a particular case of the presented problem when all the technical vehicles S are the same and evacuees are brought to them in the predefined order.

Under these limitations, the solution to problem (1) to (4) can be reduced to the solution of a sequence of sub-problems that have the following form at the j-th stage.

5. Mathematical model of the rational accommodation of people in the selected mobile vehicles when evacuees are brought to them in the predefined order

Accommodate in zone Z_j a maximum number of people l_i, $i=1,2, ..., n_j$ from ordered group L_j in the assigned sequence.

Here $L_j=n$, at $j=1$,

$$L_j = L \setminus \bigcup_{k=1}^{j-1} L'_k.\ \ n_j = n - \sum_{k=1}^{j-1} n'_k$$

at $j>1$, where L'_k is the group having n'_k, people accommodated in zone Z_k at the k-th step.

Mathematical *MIP* model of the sub-problem can be stated in the following form:

$$\sum_{i=1}^{n_j} b_i \to \max, \tag{5}$$

$$b_i \ge b_{i+1},\ i=1,\ ...,\ n_j-1, \tag{6}$$

$$b_i \Phi^{l_i Z_j^*}(u_i) \ge 0,\ i=1,\ ...,\ n_j, \tag{7}$$

$$b_i b_s \Phi^{l_i l_s}(u_i, u_s) \ge 0,\ i<s=1,2,...,n_j, \tag{8}$$

$$b_i \in \{0,1\},\ i=1,\ ...,\ n_j, \tag{9}$$

where $b_i=1$, object l_i belongs to zone Z_j and is equal to 0, otherwise condition (6) ensures the accommodation of the first objects from ordered set L_j in the assigned sequence, function $\Phi^{l_i Z_j^*}(u_i)$ in (7) is phi-function [7, 8], describing the condition of accommodation of object $l_i(u_i)$ in object Z_j (condition of non-intersection of objects l_i and $Z_j^*=R^2 \setminus \operatorname{int}(Z_j)$, function $\Phi^{l_i l_s}(u_i, u_s)$ in (8) is phi-function, describing the condition of non-intersection of objects $l_i(u_i)$ and $l_s(u_s)$.

Consider the main features of problem (5) to (9):

1. Problem (5) to (9) is the problem of mixed integer programming: variables b_i, $i=1,2, ..., n_j$ are discrete (binary), and variables in vector u_i, $i=1,2, ..., n_j$ are continuous.

2. Objective function (5) is piecewise continuous (equal to the number of objects arranged in the area).

Solution to problem (5) to (9), in its turn, can be reduced to the solution of a sequence of sub-problems, having at the i-th stage the following form: arrange in zone Z_j the first objects $(n_j^*=i)$ form ordered set $\mathbf{L}_j$. If a solution can be found, the attempt is made to solve the next sub-problem of arranging in Z_j the first $(i+1)$ objects, otherwise it is accepted that $n'_j=i-1$ and the process of solving sub-problems is interrupted.

The mathematic model of the sub-problem, presented below, is adapted to the solution by the multistart method with the use of the artificial basis.

$$\lambda \to \max, \quad (10)$$

$$\Phi^{l_i Z_j^*}(u_i,\lambda) \ge 0,\ i=1,\ \ldots,\ n_j^*, \quad (11)$$

$$\Phi^{l_i l_s}(u_i,u_s,\lambda) \ge 0,\ i<s=1,2,\ldots,n_j^*, \quad (12)$$

$$\lambda \in [0,1], \quad (13)$$

where λ is the coefficient of homothety of objects, function $\Phi^{l_i Z_j^*}(u_i,\lambda)$ in (11) is the phi-function, describing the condition of arrangement of object $\lambda l_i(u_i)$ in zone Z_j (condition of non-intersection of objects $\lambda l_i(u_i)$ and $Z_j^* = R^2 \setminus \mathrm{int}(Z_j)$, function $\Phi^{l_i l_s}(u_i,u_s,\lambda)$, in (12) is the phi-function, describing the condition of non-intersection of objects $\lambda l_i(u_i)$ and $\lambda l_s(u_s)$. Optimization begins from artificial admissible point at $\lambda=0$ and random parameters u_i, at which point $\lambda l_i(u_i)$ belongs to zone Z_j. If the global maximum of problem (λ=1) is obtained, the permissible arrangement of objects in the zone is constructed.

Note. For some types of the arranged objects, it is advisable to use quasi-phi-functions [9, 10] instead of phi-functions in expressions (11) and (12). More details on this issue will be explored below.

Thus, to complete the mathematical statement of the problem, it is necessary to specify the type of objects used to model a human body and to plot functions of type (11) and (12) for these objects.

In the future, to simplify the calculation, we will remove index j from model (10)–(13) and will consider the problem of searching for the admissible packing of objects l_i, i=1,2,..., n in zone Z.

6. Construction of a geometric model of a human body that is adequate for evacuation problems

First of all, it should be noted that for the problems of the class in question, it is advisable to use relaxation and move from considering the interactions of three-dimensional objects to the interaction of their projections onto a horizontal plane [12]. At the same time, the representation of the projection of a human body onto a horizontal plane in the form of a rectangle is too rough even for methods, in which the individual transition of human flows in a free mode is simulated. Ellipses are a good model of the projection of a human body onto a horizontal plane [11, 12]. When using ellipses as models of a human body and using convex polyhedrons (rectangles) to model zone Z, the model (10) to (13) takes the form

$$\lambda \to \max, \quad (14)$$

$$\Phi^{E_i Z^*}(u_i,\lambda) \ge 0,\ i=1,\ \ldots,\ n, \quad (15)$$

$$\Phi^{E_i E_s}(u_i,u_s,t_{is},\lambda) \ge 0,\ i<s=1,2,\ \ldots,\ n, \quad (16)$$

$$\lambda \in [0,1], \quad (17)$$

where $\Phi^{E_i Z^*}(u_i,\lambda)$ is the function, describing the conditions of accommodation of object $\lambda E_i(u_i)$ (ellipse) in zone Z,

$$Z^* = R^2 \setminus \mathrm{int}(Z),$$

$$u_i = (x_i, y_i, \theta_i)$$

are the parameters of accommodation of ellipse, E_i, $v_i=(x_i,y_i)$ is the vector of translation, θ_i is the angle of translation of an object;

$\Phi^{E_i E_s}(u_i,u_s,t_{is},\lambda)$ is the quasi-phi-function, describing the condition of non-intersection of ellipses $\lambda E_i(u_i)$ and $\lambda E_s(u_s)$, t_{is} is the auxiliary variable, λ is the coefficient of homothety. Such functions for the construction of the mathematical model of accommodation of ellipses in the polygon zone were proposed in [20].

However, this representation is suitable only for modeling free movements of people and does not make it possible to take into consideration permissible deformations of a human body. This enables making a three-ellipse model of the projection of a human body onto a horizontal surface [17] $l_i = E_{ci} \cup E_{ri} \cup E_{li}$, in which central ellipse E_{ci} models the trunk, and the "right" and "left" ellipses E_{ri} and E_{li}, respectively, the right and left shoulder (Fig. 1).

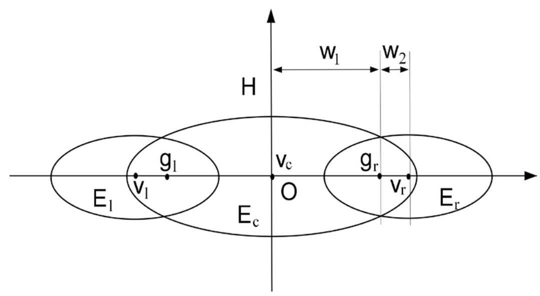


Fig. 1. Three-component model of the projection of a human body onto the horizontal plane

The common "gluing point" g_l is assigned for ellipses E_{ci} and E_{ri}, and "gluing point" g_l is assigned for ellipses E_{ci} and E_{li}. Points g_r and g_l lie on the large axis of ellipse E_{ci} and are located symmetrically in relation to its small axis. The position of points g_r and g_l on the plane is determined exclusively by the parameters of the accommodation of ellipse E_{ci}. The ellipses E_{li} and E_{ri} can only rotate at the angles in the assigned range (in relation to the rotation angle of ellipse E_{ci}) relative to these points (Fig. 2).

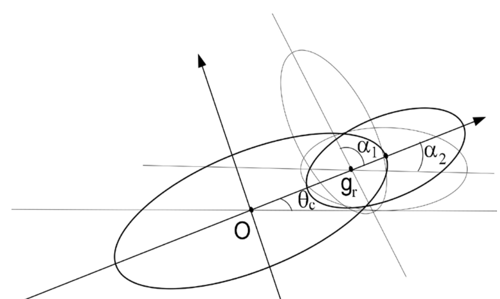


Fig. 2. Illustration of the change of configuration of the composite object

Thus, each of objects l_i on the plane is determined by the vector

$$u_i = (x_{ci}, y_{ci}, \theta_{ci}, x_{li}, y_{li}, \theta_{li}, x_{ri}, y_{ri}, \theta_{ri})$$

at the position of meeting the "gluing" restrictions

$$x_{ci} + w_{i1}\cos\theta_{ci} = x_{ri} - w_{i2}\cos\theta_{ri}, \quad (18)$$

$$y_{ci}+w_{i1}\sin\theta_{ci}=y_{ri}-w_{i2}\sin\theta_{ri}, \quad (19)$$

$$x_{ci}-w_{i1}\cos\theta_{ci}=x_{li}+w_{i2}\cos\theta_{li}, \quad (20)$$

$$y_{ci}-w_{i1}\sin\theta_{ci}=y_{li}+w_{i2}\sin\theta_{li}, \quad (21)$$

and restrictions to the range of turning angles

$$\theta_{ci}-\alpha_{i2}\le\theta_{ri}\le\theta_{ci}+\alpha_{i1}, \quad (22)$$

$$\theta_{ci}-\alpha_{i1}\le\theta_{li}\le\theta_{ci}+\alpha_{i2}. \quad (23)$$

Here w_{i1}, w_{i2}, α_{i1}, α_{i2} are the constants characterizing the physical parameters of a certain person.

With this in mind, model (14) to (17) can be represented in the form

$$\lambda\to\max, \quad (24)$$

$$\Phi^{l_iZ^*}(u_i,\lambda)\ge 0,\ i=1,\ ...,\ n, \quad (25)$$

$$\Phi^{l_il_s}(u_i,u_s,\overline{t}_{is},\lambda)\ge 0,\ i<s=1,2,\ ...,\ n, \quad (26)$$

$$x_{ci}+w_{i1}\cos\theta_{ci}=x_{ri}-w_{i2}\cos\theta_{ri},\ i=1,2,\ ...,\ n, \quad (27)$$

$$y_{ci}+w_{i1}\sin\theta_{ic}=y_{ri}-w_{i2}\sin\theta_{ri},\ i=1,2,\ ...,\ n, \quad (28)$$

$$x_{ci}-w_{i1}\cos\theta_{ci}=x_{li}+w_{i2}\cos\theta_{li},\ i=1,2,\ ...,\ n, \quad (29)$$

$$y_{ci}-w_{i1}\sin\theta_{ci}=y_{li}+w_{i2}\sin\theta_{li},\ i=1,2,\ ...,\ n, \quad (30)$$

$$\theta_{ci}-\alpha_{i2}\le\theta_{ri}\le\theta_{ci}+\alpha_{i1},\ i=1,2,\ ...,\ n, \quad (31)$$

$$\theta_{ci}-\alpha_{i1}\le\theta_{li}\le\theta_{ci}+\alpha_{i2},\ i=1,2,\ ...,\ n, \quad (32)$$

$$0\le\lambda\le 1, \quad (33)$$

where

$$\Phi^{l_iZ^*}(u_i,\lambda)=\min\left\{\Phi^{E_{ci}Z^*}(u_{ci},\lambda),\Phi^{E_{ri}Z^*}(u_{ri},\lambda),\Phi^{E_{li}Z^*}(u_{li},\lambda)\right\}$$

are the phi-function, describing the conditions of arrangement of object $\lambda l_i(u_i)$ in zone Z, $\Phi^{E_{ci}Z^*}(u_{ci},\lambda)$, $\Phi^{E_{ri}Z^*}(u_{ri},\lambda)$, $\Phi^{E_{li}Z^*}(u_{li},\lambda)$ – phi-function, describing the conditions of arrangement of corresponding objects in zone Z.

$$\Phi^{l_il_s}(u_i,u_s,\overline{t}_{is},\lambda)=$$
$$=\min\left\{\begin{array}{l}\Phi^{E_{ci}E_{cs}}(u_{ci},u_{cs},t_{is}^{cc},\lambda),\Phi^{E_{li}E_{cs}}(u_{li},u_{cs},t_{is}^{lc},\lambda),\\ \Phi^{E_{ri}E_{cs}}(u_{ri},u_{cs},t_{is}^{rc},\lambda),\Phi^{E_{ci}E_{ls}}(u_{ci},u_{ls},t_{is}^{cl},\lambda),\\ \Phi^{E_{li}E_{ls}}(u_{li},u_{ls},t_{is}^{ll},\lambda),\Phi^{E_{ri}E_{ls}}(u_{ri},u_{ls},t_{is}^{rl},\lambda),\\ \Phi^{E_{ci}E_{rs}}(u_{ci},u_{rs},t_{is}^{cr},\lambda),\Phi^{E_{li}E_{rs}}(u_{li},u_{rs},t_{is}^{lr},\lambda),\\ \Phi^{E_{ri}E_{rs}}(u_{ri},u_{rs},t_{is}^{rr},\lambda)\end{array}\right\} \quad (34)$$

– quasi-phi-function, describing the condition of non-intersection of objects $\lambda\cdot l_i(u_i)$ and $\lambda\cdot E_s(u_s)$;

$\overline{t}_{is}=(t_{is}^{cc},t_{is}^{cl},t_{is}^{cr},t_{is}^{lc},t_{is}^{ll},t_{is}^{lr},t_{is}^{rc},t_{is}^{rl},t_{is}^{rr})$ is the vector of auxiliary variables of quasi-phi-functions,

$$\Phi^{E_{ci}E_{cs}}(u_{ci},u_{cs},t_{is}^{cc},\lambda),\ \Phi^{E_{li}E_{cs}}(u_{li},u_{cs},t_{is}^{lc},\lambda),$$

$$\Phi^{E_{ri}E_{cs}}(u_{ri},u_{cs},t_{is}^{rc},\lambda),\ \Phi^{E_{ci}E_{ls}}(u_{ci},u_{ls},t_{is}^{cl},\lambda),$$

$$\Phi^{E_{li}E_{ls}}(u_{li},u_{ls},t_{is}^{ll},\lambda),\ \Phi^{E_{ri}E_{ls}}(u_{ri},u_{ls},t_{is}^{rl},\lambda),$$

$$\Phi^{E_{ci}E_{rs}}(u_{ci},u_{rs},t_{is}^{cr},\lambda),\ \Phi^{E_{li}E_{rs}}(u_{li},u_{rs},t_{is}^{lr},\lambda),$$

$$\Phi^{E_{ri}E_{rs}}(u_{ri},u_{rs},t_{is}^{rr},\lambda)$$

are the quasi-phi-functions, describing the conditions of non-intersection of the corresponding pairs of ellipses, equations (26) to (29) assign the conditions of "gluing" the composite object, inequality (31), (32) – the restrictions to the rotation angles of the parts of a composite object.

7. Development of a method to search for a locally-optimal solution to the problem of optimization of people's accommodation in evacuation vehicles

Mathematical model (24) to (33) is the classical problem of non-linear programming, dimensionality of the vector of its variables $u=(\lambda, u_1, u_2,..., u_n, t_{12}, t_{13},..., t_{1n}, t_{23},..., t_{2n},...,t_{n-1,n})$ makes up $27n+\frac{n(n-1)}{2}+1$, assessment of the number of equalities $O(n^2)$.

The problem is solved at two stages:

1) the search for the initial (starting) admissible point;

2) finding the local extremum of the problem.

The method of minimizing for the groups of variables [21] with the use of the homothetic transformations of objects is considered as the method for searching for initial approximation.

Local optimization is performed with the help of the package of the non-linear optimization with the open source code IPOPT [22]. For the number of objects exceeding 5, it is appropriate to apply the decomposition schemes, described in [11], to solve the problem.

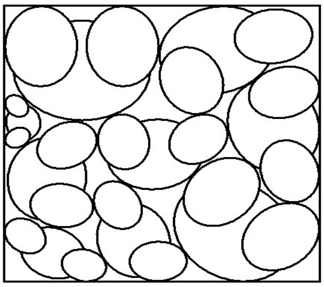
In the case of different mobile technical vehicles, the approach outlined in paper [23, 24], can be used for choosing them.

The algorithmic support for computer simulation of the optimization of objects' accommodation was created. The program was developed in the Visual C 6.0 environment to simulate the process of evacuation of a group of people in the order of assigned sequence with the use of mobile vehicles.

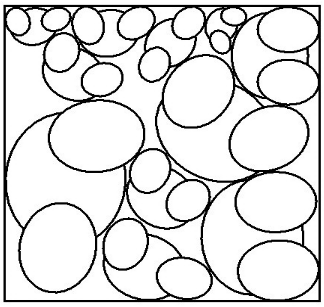
A series of computational experiments were carried out. For the zone having the area of 1 m^2, the problem of determining the maximum number of objects arranged in it was solved by their selection according to the assigned sequence of numbers from the set of 35 objects with the optimization of their arrangement.

Computational experiments were conducted using AMD Athlon 64x2 Dual5200+. The sub-problems of non-linear programming were solved using the IPOPT software, which was developed based on the internal point method and available at an open non-profit resource [25]. The results of computer simulation of the objects' accommodation are shown in Fig. 3, 4.

The time to search for the local extremum for objects represented by a three-component model, with continuous translation of the main ellipses, as well as continuous turnings of both basic and auxiliary ellipses (Fig. 3), is 275.32 s. For the objects presented by a single-component model, with their continuous translation and turnings (Fig. 4), it is 35.53 s.

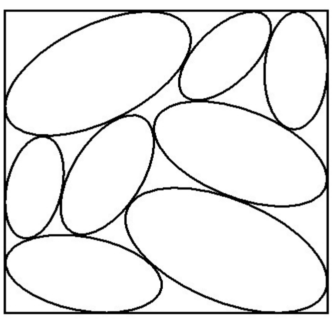


a

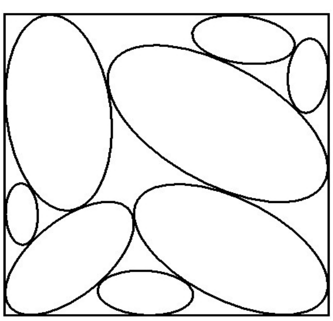


b

Fig. 3. Examples of local extremes [22] of problem (5) to (9) for the objects, represented by a three-component model (18) to (23), Fig. 1, 2: *a* – with continuous translations of the basic ellipses and with continuous rotations of both basic, and auxiliary ellipses and with continuous turnings of both basic and auxiliary ellipses; *b* – with continuous translations and turnings of basic ellipses, continuous turnings of auxiliary ones



a



b

Fig. 4. Examples of two local extremes of problem (5) to (9) for the objects, represented by the one-component model, with their continuous translations and turnings: *a* – ellipses of large and medium dimensions; *b* – ellipses of large, medium, small dimensions, which approximate adults, adolescents and children, respectively

To verify the adequacy of the proposed mathematical models, a computational experiment was conducted.

The process of evacuation of a group of people in the order of the assigned queue was simulated using the same mobile vehicles. Mobile means with the useful area of 2 square meters (1 m by 2 m) having a rectangular shape were considered; the list of admissible forms can be expanded. For each of the zones, the problems with an increase in the number of arranged objects were solved until the problem with an non-admissible system of restrictions (it is not possible to arrange objects in the area) was obtained. The last of the obtained decisions was memorized, which was followed by the transition to filling the next area. Emerging problems of non-linear optimization are solved with the IPOPT software package.

The accommodation of 100 people, approximated by ellipses, with pseudo-randomly generated dimensions of their semi-axes on a first-come, first-served basis was simulated. Ellipses were used as a model of a human body. As a result of the simulation, these ellipses were arranged in seven areas in the number of 16, 14, 16, 17, 16, 13 and 8 objects, respectively. If we do not consider the last area (not fully filled), the accommodation capacity of (100–8)/12/7.666666666666677 people per square meter was achieved. This value corresponds to the experimental estimates for the joint movement of human flows. Thus, it was proved that the model of a human body in the form of ellipses is intended for modeling joint relocations (7–9 people per one square meter) and simulation of dense loading of mobile transport means at non-traumatic deformation of human bodies and does not go beyond its applicability.

The sequence of three-component objects of the form of (18) to (23) with the dimensions (at the zero angle of the "shoulder" turning) coinciding with the dimensions of the ellipses was considered. As a result of the simulation, these objects were arranged in six areas in the quantity of 20, 20, 20, 20, 18, and 2. If we do not consider the last area, the capacity of (100–2)/10=9.8 people per one square meter was achieved. The last estimate corresponds to the experimental estimates (9–10 people per sq. m.) [12], obtained during the study of the process of people's evacuation by mobile vehicles with force influence, implemented by natural deformations of bodies.

It should be noted that compliance with the experimental estimates can be enhanced by selecting parameters that determine the ratio of the dimensions of components of complex objects. However, this does not appear to be significant as the experimental and obtained estimates are fairly well-coordinated.

A three-component model, admitting continuous translating of basic ellipses, as well as continuous turnings of both basic and auxiliary ellipses, is more effective in terms of the considered objective function (5) – the number of arranged objects:

$$\Delta = \frac{b_c - b_o}{b_c} \cdot 100\,\% = \frac{9.8 - 7.7}{9.8} \cdot 100\,\% = 21{,}4\,\%,$$

where b_c is the average number of three-component objects per 1 sq. m; b_o is the average number of arranged ellipses per 1 sq. m.

This result can be used to obtain the maximum estimate of the number of objects, filling the area, for example, when filling the transport vehicles with loads, according to the specified loading sequence. The idea of creating multi-component objects allowing continuous rotations of the components of a model in relation to the basic one can be used, for example, in robotics. A robot consisting of mobile component will be able to enhance its efficiency due to changing its spatial shape during the passage of complex routes arising in the process of emergency development.

Another advantage of this approach is its flexibility. It became possible to obtain quick estimates for the groups of people with non-standard physical parameters (e. g. sumo wrestlers), people with dimensional loads, stretchers, wheelchairs, etc. It is enough just to add to the model the inequalities to simulate non-intersection of new objects with ellipses and among themselves.

6. Discussion of the results of computational experiments obtained by the implementation of the developed mathematical models and their practical use

The *MIP* model of rational selection and accommodation of people in mobile technical vehicles during evacuation from buildings (1) to (4) was constructed. A particular case of the model (5) to (9) – optimization of accommodation of people in both stationary and mobile emergency evacuation vehicles according to the sequence of people's arrival from the moving stream was considered. The following basic features of the model were explored: the problem belongs to the class of problems of mixed integer programming; the objective function is piecewise continuous. The specific features of the model made it possible to reduce reasonably the problem to a sequence of sub-problems of arranging the first objects (people) according to the sequence of their arrival and to adapt each of them to the solution with the use of the multistart method with the application of the artificial basis (10) to (13).

Another very important feature of the model, distinguishing it from the existing ones, is the properties of the explored arrangement objects. An arrangement object is a complex object consisting of three components. The difference from the existing complex objects is that the components of an object can rotate continuously, based on the specified limitations (18) to (23), Fig. 1, 2. To construct a model for such a problem, we obtained an analytical expression in the form of quasi-pfi-function for the conditions of non-intersection of the explored objects (34). This allowed description of the optimization problem of arrangement in the form of non-linear programming problems (24) to (33) and modification of the existing methods for finding local extremes for a wider range of geometric objects. Computer simulation of objects' accommodation presented by a three-component and a single-component models, in Fig. 3, 4, respectively, showed the effectiveness of a multi-component model (an increase in objective function (5) by an average of 20 % is achieved).

In the study, there is a restriction to both the number of the components of an arrangement object and its shape (the shape of an ellipse), based on the terms of the subject area in question. This restriction is not fundamental. Models and methods allow changes in both the number of the model components and their spatial shapes, which will only lead to an increase in labor-intensity of the problem-solving algorithms.

Further research could include the simulation of arrangement of the new types of complex objects with the components allowing turns, as well as modification of the proposed methods in modeling the motion of human flows, robots, obtaining upper estimates of filling the areas with objects.

7. Conclusions

1. The *MIP* model of the rational choice and optimal accommodation of people in mobile technical vehicles during evacuation from buildings was constructed in the research. A sequence of mathematical models, which is a common component, was constructed. Analysis of the features of the proposed mathematical models showed that the solution of the main problem can be represented in the form of two stages. The first stage is the choice of zones of filling (vehicles) on a discrete set, formed by their numbers. The second one is the solution of the problem of optimization of arrangement of complex objects in selected areas of specified dimensions. Thus, based on the features of the mathematical model, a reasonable strategy for solving the main problem was developed.

2. A particular case of the model – optimization of accommodation of people both in stationary and mobile evacuation vehicles according to the sequence of people's arrival from the moving stream, was explored. We considered the following basic specific features of the model: the model of the problem of mixed integer programming, the objective function of which is piecewise continuous. The specific features of the model made it possible to reasonably reduce the problem to the sequence of sub-problems of arranging the first objects (people) according to the sequence of their arrival and to adapt each of them to the solution with the use of the multistart method with the application of the artificial basis.

3. A three-component model is considered as arrangement objects (human body). The model includes limitations: to conditions of sticking the components of the model into a single complex object; to the ratio of angles of rotation of the model components, arising from physical restrictions to the mutual position of body parts. This model allowed simulating the accommodation (relocation) of people taking into consideration their force actions, which occur at their active movement at high arrangement densities. The model can serve in order to obtain the upper estimate of coefficient of area filling. Computer simulation of the objects arrangement, represented by a three-component and a single-component models, showed the effectiveness of a multi-component model (an increase in the objective function on average by 20 % is achieved).

4. The modifications of the methods for obtaining locally-optimal arrangement of complex objects in the area were proposed. The modifications take into consideration both the formation of the sequence of arrangement sub-problems, and the specific features of arrangement objects, for which analytical expressions for the conditions of their non-intersection were obtained. Analytical expressions for non-intersection conditions enable the construction of a mathematical model of arrangement optimization and represent it as a classic problem of non-linear programming, which allowed solving it with the help of the existing IPOPT package of non-linear optimization.

References

1. Stoyan, Yu. G. (1983). Osnovnaya zadacha geometricheskogo proektirovaniya. Kharkiv: In-t problem mashinostroeniya AN USSR, 36.
2. Wäscher, G., Haußner, H., Schumann, H. (2007). An improved typology of cutting and packing problems. European Journal of Operational Research, 183 (3), 1109–1130. doi: https://doi.org/10.1016/j.ejor.2005.12.047
3. Bennell, J. A., Oliveira, J. F. (2008). The geometry of nesting problems: A tutorial. European Journal of Operational Research, 184 (2), 397–415. doi: https://doi.org/10.1016/j.ejor.2006.11.038
4. Rvachev, V. L. (1982). Teoriya R-funktsii i nekotorye ee prilozheniya. Kyiv: Naukova dumka, 552.
5. Yakovlev, S. V., Gil', N. I., Komyak, V. M. et. al..; Rvachev, V. L. (Ed.) (1995). Elementy teorii geometricheskogo proektirovaniya. Kyi: Naukova dumka, 241.
6. Stoyan, Yu. G. (1980). Ob odnom obobshchenii funktsii plotnogo razmeshcheniya. Doklady NAN Ukrainy, 8, 71–74.
7. Stoyan, Yu. G. (2001). Φ-function and its basic properties. Doklady NAN Ukrainy, 8, 112–117.
8. Stoyan, Yu., Gil, N., Romanova, T., Scheithauer, G. (2004). Phi-function for complex 2D object. 40R Quarterly Journal of the Belgian, French and Italian Operations Research Societies, 2 (1), 69–84.
9. Stoyan, Y., Romanova, T., Pankratov, A., Chugay, A. (2015). Optimized Object Packings Using Quasi-Phi-Functions. Springer Optimization and Its Applications, 265–293. doi: https://doi.org/10.1007/978-3-319-18899-7_13
10. Pankratov, A. V., Romanova, T. E., Chugay, A. M. (2015). Optimal packing of convex polytopes using quasi-phi-functions. Engineering problems, 18 (2), 55–64.
11. Komyak, V., Komyak, V., Danilin, A. (2017). A study of ellipse packing in the high-dimensionality problems. Eastern-European Journal of Enterprise Technologies, 1 (4 (85)), 17–23. doi: https://doi.org/10.15587/1729-4061.2017.91902
12. Holshchevnikov, V. V., Samoshin, D. A. (2009). Evakuatsiya i povedenie lyudey na pozharah. Moscow: Akademiya GPS MCHS Rossii, 210.
13. Kallrath, J., Rebennack, S. (2013). Cutting ellipses from area-minimizing rectangles. Journal of Global Optimization, 59 (2-3), 405–437. doi: https://doi.org/10.1007/s10898-013-0125-3
14. Pankratov, A. V., Romanova, T. E., Subbota, I. A. (2014). Optimal'naya upakovka ellipsov s uchetom dopustimyh rasstoyaniy. Journal of Computational & Applied Mathematics, 1, 129–140.
15. Karkin, I. N., Parfenenko, A. P. (2011). Floiwtech VD – computer-simulation method from evacuation calculation. International Scientific and Technical Conference Emergency Evacuation of People from Buildings. Warsaw, 111–118.
16. Kholshchevnikov, V. V., Parfenenko, A. P. (2015). Comparison of different models of the movement of human flows and results of program computer systems. Pozharovzryvobezopasnost', 24 (5), 68–75. doi: https://doi.org/10.18322/pvb.2015.24.5.68-75.
17. Komiak, V. M., Kiazimov, K. T., Danylyn, A. N. (2020). Modeliuvannia aktyvnoho rukhu liudei v pototsi zmishanoho skladu. Materialy Mizhnarodnoi naukovo-prykladnoi konf.: Problemy nadzvychainykh sytuatsiy. Kharkiv: NUTsZU, 97–99.
18. Stoyan, Y. G., Yakovlev, S. V. (2018). Configuration Space of Geometric Objects. Cybernetics and Systems Analysis, 54 (5), 716–726. doi: https://doi.org/10.1007/s10559-018-0073-5
19. Klymenko, V. P., Oksanych, M., Lopushanskyi, A. V. (2018). Data metamodel as a basis for building a unified information environment of a system of situational centers of the Security and Defense Sector. Matematychni mashyny i systemy, 3, 40–47.
20. Kampas, F. J., Castillo, I., Pintér, J. D. (2019). Optimized ellipse packings in regular polygons. Optimization Letters, 13 (7), 1583–1613. doi: https://doi.org/10.1007/s11590-019-01423-y
21. Gill, F., Myurrey, U., Rayt, M. (1985). Prakticheskaya optimizatsiya. Moscow: Mir, 509.
22. Wächter, A., Biegler, L. T. (2005). On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. Mathematical Programming, 106 (1), 25–57. doi: https://doi.org/10.1007/s10107-004-0559-y
23. Komyak, V., Kyazimov, K. (2020). Variantal modeling of evacuation of people from altitude buildings in the event of an emergency situation. Modern Problems of Modeling, 17, 27–35. doi: https://doi.org/10.33842/2313-125x/2019/17/27/35
24. Komyak, V. M., Sobol, A. N., Danilin, A. N., Komyak, V. V., Kyazimov, K. T. (2020). Optimization of Partitioning the Domain into Subdomains According to Given Limitation of Space. Journal of Automation and Information Sciences, 52 (2), 13–26. doi: https://doi.org/10.1615/jautomatinfscien.v52.i2.20
25. Coin-Or. Available at: https://projects.coin-or.org/Ipopt