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DEVELOPMENT OF A GEOMETRIC MODEL OF A NEW METHOD FOR DELIVERING EXTINGUISHING SUBSTANCES TO A DISTANT FIRE ZONE

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A geometric model of a new method of delivering fire-extinguishing substances to a fire zone located at a considerable distance was offered. The idea of delivery is based on the mechanical action of throwing. To this end, a substance (e.g. extinguishing powder) is loaded in a hard shell made as a special container. After delivery by means of a launcher to a fire zone, the container has to release the substance which will promote fire extinguishing.

The known method of remote delivery of extinguishing substances uses a pneumatic gun with a cylindrical container. During delivery, the cylinder must rotate around its axis to ensure flight stability. The cylinder is rotated by a special turbine when passing through the gun barrel. There are difficulties in regulating the distribution of compressed air flows during the turbine operation. In addition, the tightness of the pneumatic part of the gun should be monitored.

The new delivery method uses a container in a form of two spaced loads similar to a sports dumbbell. The dumbbell motion is initiated by simultaneous action of explosion-generated pulses directed at each of its loads in a pre-calculated manner. This results in the rotational motion of the container. To describe the dynamics of the dumbbell motion, a Lagrangian was defined and a system of Lagrange differential equations of the second kind was set up and solved. Examples of modeling trajectories of the centers of masses of the dumbbell loads taking into account air resistance were given.

The proposed method is planned to be a basis of a new fire extinguishing technology. This is evidenced by the new scheme of launching the dumbbell by means of explosion-generated pulses of charges of two pyro cartridges. The obtained results make it possible to estimate magnitudes of explosion-generated pulses necessary for throwing and corresponding distances of the dumbbell delivery

Keywords: geometric modeling, dumbbell-shaped container, Lagrange equation of the second kind, rotational-translational motion of the container

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1. Introduction

Large-scale fires that have occurred in recent years have proved the need to rearm fire departments with qualitatively new equipment. The problem of extinguishing large-scale fires is related to two key points: the development of highly effective extinguishing substances and ways of their delivery to the fire zone. While success in the development of physical and chemical compositions of current extinguishing substances is doubtless, methods of their delivery to the fire zone still need to be improved. In exceptional cases, fire extinguishing substances are delivered by air with the help of planes or helicopters or using heavy equipment such as “fire tanks”. However, such means of delivery of fire-extinguishing substances are not prompt and too expensive.

More often, fire extinguishing means are delivered over long distances by means of throwing devices like an air gun. To do this, the substance (a fire-extinguishing powder) is loaded in a special hard shell serving as a container. It is destroyed when delivered to the fire site and releases the substance thus contributing to fire extinguishing. Gun barrel installations for delivering containers with extinguishing substances over long distances is a well-known example of this method. The container has a cylindrical form. It is delivered to the fire zone with a pneumatic gun as a launcher. During its flight, the container must rotate around its axis for motion stability. A special turbine is used to spin the container when it is propelled through the gun barrel. It must be powerful enough to spin the massive container in a short time.

According to the proposed method, the container has a dumbbell shape, that is, two spaced (for a short distance) loads connected by a rod. For stability during its delivery, the dumbbell must rotate around its center of masses in a vertical plane. The dumbbell motion is initiated due to a directed action of calculated explosion-generated pulses from the combustion of charge of two pyro cartridges. This ensures the rotational motion of the container. It means that the “start” of the dumbbell rotation is easier to realize compared with the spinning of the cylinder by means of an air gun turbine. The explosive is given a shape of a pyro cartridge or a pyro bolt. Such devices are safely used in space technology. Their designs can be taken as prototypes in developing pyrotechnics of the proposed method.

The relevance of the construction of a geometric model of a new method of far delivery of extinguishing substances in dumbbell-shaped shells is proved as follows. The launcher will have a simpler (compared to an air gun) design and will not require efforts and money to maintain it in working order. Inexpensive pyro cartridges (in comparison with the pneumatic system) are used as movers. The container must move within a vertical plane up to the end of the flight while rotating around its center of masses. This will make it possible to use the energy of rotation to destroy the container at the end of the flight by striking against an obstacle with the release of the extinguishant. The considered concept can form a basis of a prospective fire extinguishing technology with a new way of extinguishant delivery on considerable distances.

2. Literature review and problem statement

Large-scale fires include a class of fires characterized by large inflaming areas and significant thermal radiation. These factors forced to take measures to eliminate fires from long distances. In addition to well-known forest fires, large-scale fires occur in warehouses and storage facilities, at enterprises with flammable and explosive substances, airfield facilities, oil and gas production fields, power plants, etc. [1, 2]. Fire aircraft are used in special cases.

Firefighting aircraft are designed to extinguish forest and other large-scale fires with fire extinguishing liquids and firebombs and for spraying reagents on clouds to evoke artificial precipitations over a fire zone.

The study [3] is devoted to theoretical issues of remote fire extinguishing. Fire-fighting missiles [4], explosion-generated shock waves [5, 6], and “fire tanks” [7] are used in remote fire extinguishing.

The listed means of remote delivery of extinguishing substances are used in exceptional cases. Firefighting using air guns is a more common method. Peculiarities of using air guns are considered in [8]. Analysis of the use of extinguishing substances and the methods of their delivery for firefighting are considered in [9]. The issue of preliminary preparation for the use of remote fire extinguishing measures is discussed in [10, 11]. The nature of the hazards that must be taken into account when deploying the firefighting equipment is studied in [12]. Studies [13, 14] report determining the parameters of fire-extinguishing efficiency of ejecting powder mixes from containers in remote fire extinguishing.

The use of an air gun for this purpose [15, 16] can be considered a prototype of the proposed method of a container’s delivery. Studies connected with the design features of a gun barrel installation are listed. Movers that deliver cylindrical containers with extinguishing substances directly into the fire zone are considered. Mechanics of the fire-extinguishing action of powder compositions during their emission due to container destruction under the action of excess pressure created by the products of explosive detonation was quite well studied in [17]. Upon entering the fire zone, the container ruptures under the action of internal forces with a release of inert detonation products and powder composition.

It was shown that according to the basics of ballistics, a cylinder must rotate around its axis for the stability of motion in the process of delivery. In artillery, the spinning of the projectiles of a similar design occurs during their movement along the threaded barrel grooves. This gives an initial axial rotation of the projectile when it enters the external ballistic trajectory. Due to the gyroscopic effect, the projectile motion gets stabilized. In pneumatic guns, the initial pulse of spinning the cylindrical container is ensured by a special turbine [15, 16]. However, there are unresolved issues related to the reliability of the device for the uniform four-directional distribution of air flows in the turbine. Each of the directions blows compressed air to one of the four chambers of the turbine. Compressed air stimulates the translational and rotational motion of the container during its travel down the gun barrel. In addition to the requirements of uniform distribution of compressed air flows, there are requirements for ensuring the tightness of the pneumatic unit of the gun. Failure to comply with these requirements leads to the insufficient spinning of the massive container. Therefore, the cylindrical container may lose its rotation energy in the final flight phase. Gliding flight of the container makes it difficult to destroy it when the fire zone is reached and the release of extinguishant may not occur. Therefore, containers for air guns must be additionally provided with explosives for their destruction. This complicates the design and increases the danger if used. This can be caused by objective difficulties associated with providing the required energy of rotation of the cylindrical container when launching it with a pneumatic gun.

The development of a fundamentally different way of extinguishant delivery in a container may be an option of overcoming the relevant difficulties. This approach was first described in [18] but the method was not substantiated there.

A container shaped like a sports dumbbell is used in the new delivery method. The container must consist of two spaced loads. The dumbbell motion is initiated due to the simultaneous action of directed explosion-generated pulses directed at each of its loads in a pre-calculated manner. As a result, the rotational motion of the container is achieved. To describe the dynamics of the dumbbell motion, a Lagrangian was determined and a system of differential Lagrange equations of the second kind was set up and solved. Examples of modeling trajectories of the center of masses of the dumbbell loads taking into account air resistance were given.

The studies of the revolute translational motion of a dumbbell in a vertical plane can be attributed to the problems of technical mechanics. They are devoted to the calculation of the dynamics of a rigid body with a moving point of the center of mass. Koenig's theorem was used to solve this class of problems [19]. An example of an application of the theorem is given in [20]. The Koenig's theorem enables the expression of the kinetic energy of the system through the kinetic energy of the center of mass. Approximate methods are more convenient for engineering calculations. This paper presents graph-analytical modeling of the dumbbell trajectory taking into account air resistance. Two concepts of the trajectory were used: a calculative one obtained in solving a system of differential Lagrange equations of the second kind and a theoretical obtained by solving a system of differential equations of motion of a point mass when launching at an angle to the horizon. The concept of theoretical trajectory is characterized, inter alia, by the coefficient of air resistance. As a result of the combination of the above concepts, trajectories of motion of a dumbbell and its components were obtained taking into account air resistance. When constructing mathematical models of throwing machines, parameters in this study were set in conditional units.

Examples of modeling trajectories of the center of mass of the dumbbell loads and the center of mass of the whole dumbbell in the process of its rotational motion and images of individual phases of the dumbbell rotation in flight were given. The obtained results make it possible to estimate distances of delivery of the dumbbell-shaped container taking into account the coefficient of air resistance depending on the values of the explosion-generated pulses. All this suggests that it would be appropriate to conduct a study on the development of a new method of distant delivery of extinguishing substances to the fire zone. At the initial stage, it was decided to illustrate such studies on a geometric model.

Thus, as a result of reviewing published data [1–20], issues that have not yet been studied by other authors were identified. This has allowed us to formulate the following study line: suggest a new method of delivering extinguishing substances in a dumbbell-shaped container to a remote area. Moreover, the container must perform rotational and translational motions until the end of the flight within the vertical plane.

3. The aim and objectives of the study

The study objective is to develop a geometric model of a new method of delivering extinguishing substances in a hard dumbbell-shaped shell to remote fire zones.

To achieve this objective, it was necessary to solve the following tasks:

– to substantiate calculations of the methods of throwing loads over long distances on examples of two options of trebuchet mechanisms;

– to simulate the rotational motion of dumbbell-shaped containers in a vertical plane taking into account air resistance: it will make it possible to use the energy of rotation to destroy containers at the end of their flight and release the extinguishing substances;

– to propose a scheme of a launcher for realizing rotational motion of the dumbbell-shaped containers in which explosion-generated pulses of two pyro cartridges aimed at the dumbbell loads should act as movers.

4. Geometric models of throwing units of a trebuchet type as a means of delivering loads over long distances

Before presenting the main material, let us consider two geometric models of throwing units based on the scheme of operation of a mechanism such as a trebuchet [21–23]. Trebuchet throwing machines were used in ancient times as a means of delivering heavy loads over long distances. These were mainly devices for destroying walls of medieval castles. They tried to place them at distances inaccessible to arrows of the castle archers. To ensure the effective dynamics of the mechanism variants, it is necessary to calculate the parameters of its elements. This is done within the framework of Lagrange mechanics which takes into account kinetic and potential energies of the system. As a result of solving the composite Lagrange equation of the second kind, the sought trajectory of the load on a slingshot was obtained which makes it possible to approximate distance of the payload delivery.

4. 1. The geometric model of the trebuchet mechanism in which a vehicle is a counterweight load

A model of a mobile throwing unit of trebuchet type designed for launching (catapulting) of unmanned aerial vehicles is described in [24, 25]. A similar throwing unit can be used to deliver extinguishing substances to a remote fire zone. The mechanism design features the use of a vehicle on which the throwing unit is fixed as a counterweight. This facilitates the mobility of the entire throwing system which dismantles compactly and can be transported by a vehicle. To deploy the installation in a working state, it is necessary to install metal supports and attach to them the trebuchet lever in the form of a "rocker". It is necessary to lift the rear part of the vehicle with the help of an electric winch, that is, create a counterweight at the short end of the mentioned lever (Fig. 1). The scheme of a trebuchet design includes a lever of length l_1+l_2. A lever of length l_3 (denoting a slingshot) and a lever of length l_4 (denoting a lever of fastening the counterweight) are hinged to it. Loads with masses m_1 (car) and m_2 (container with an extinguishant) are attached to the levers in nodal points. The mass m_1 will be several orders of magnitude larger than the mass m_2. For certainty, let the container have a spherical shape. When the vehicle is instantly lowered, the container with the extinguishant will be given an acceleration for throwing [26].

Fig. 1 shows selected generalized coordinates of the trebuchet mechanism: angles $u(t)$, $v(t)$, and $w(t)$. To describe the trebuchet dynamics, expressions for kinetic (T) and potential (P) energies are used [21–23]:

$$T=-m_2 l_3^2\frac{du}{dt}\frac{dw}{dt}-m_2 l_3 l_2\left(\frac{du}{dt}\right)^2\cos\psi+$$
$$+m_1 l_4^2\frac{du}{dt}\frac{dv}{dt}0.5m_1 l_1\left(\frac{du}{dt}\right)^2+0.5m_2 l_2\left(\frac{du}{dt}\right)^2-$$
$$-m_1 l_4 l_1\frac{du}{dt}\frac{dv}{dt}\cos\varphi-m_1 l_4 l_1\left(\frac{du}{dt}\right)^2\cos\varphi+$$
$$+m_2 l_3 l_2\frac{du}{dt}\frac{dw}{dt}\cos\psi+0.5m_2 l_3^2\left(\frac{du}{dt}\right)^2+$$
$$0.5m_2 l_3^2\left(\frac{dw}{dt}\right)^2+0.5m_1 l_4^2\left(\frac{du}{dt}\right)^2+0.5m_1 l_4^2\left(\frac{dv}{dt}\right)^2; \qquad (1)$$

$$P=-m_1 l_1 g\cos u+m_2 l_2 g\cos u-$$
$$m_2 l_3\,g\left(\cos u\cos w+\sin u\sin w\right)+$$
$$m_1 l_4\,g\left(\cos u\cos v-\sin u\sin v\right).$$

where $u(t)$ is the time function of change in the angle of deviation from vertical of the lever of length l_1+l_2; $v(t)$ is the function of change in the angle between the levers of lengths l_4 and l_1+l_2; $w(t)$ is the function of change in the angle between levers of lengths l_3 and l_1+l_2; g=9.81.

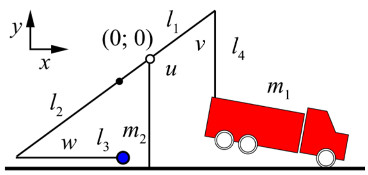


Fig. 1. Schematic view of a throwing unit of a trebuchet type combined with a vehicle

Using the Lagrangian $l=T-P$, a system of differential Lagrange equations of the second kind is obtained in the following form:

$$-m_1 l_1\,g\sin u+m_2 l_2\,g\sin u-m_2 l_3\,g\sin u+$$
$$+m_2 l_3\,g\cos u\sin w+m_1 l_4\,g\sin u\cos v+$$
$$+m_1 l_4\,g\cos u\sin v-m_1 l_1^2\frac{d^2u}{dt^2}+m_1 l_4 l_1\frac{d^2v}{dt^2}\cos v-$$
$$m_1 l_1 l_4\left(\frac{dv}{dt}\right)^2\sin v+2m_1 l_1 l_4\frac{d^2u}{dt^2}\cos v-$$
$$-2m_1 l_1 l_4\frac{du}{dt}\sin v-m_2 l_2^2\frac{d^2u}{dt^2}+2m_2 l_2 l_3\frac{d^2u}{dt^2}\cos w-$$
$$-2m_2 l_2 l_3\frac{du}{dt}\frac{dw}{dt}\sin w-m_2 l_2 l_3\frac{d^2w}{dt^2}\cos w+$$
$$m_2 l_2 l_3\left(\frac{dw}{dt}\right)^2\sin w-m_2 l_3^2\frac{d^2u}{dt^2}+m_2 l_3\frac{d^2w}{dt^2}-$$
$$-m_1 l_4^2\frac{d^2v}{dt^2}-m_1 l_4^2\frac{d^2u}{dt^2}=0;$$

$$m_2 l_2 l_3\left(\frac{du}{dt}\right)^2\sin w-m_2 l_3\,g\cos u\sin w+$$
$$m_2 l_3\,g\sin u\cos w-m_2 l_2 l_3\frac{d^2u}{dt^2}\cos w+$$
$$m_2 l_3^2\frac{d^2u}{dt^2}-m_2 l_3^2\frac{d^2w}{dt^2}=0; \qquad (2)$$

$$m_1 l_1 l_4\left(\frac{du}{dt}\right)^2\sin v+m_1 l_4\,g\cos u\sin v+$$
$$m_1 l_4\,g\sin u\cos v+m_1 l_1 l_4\frac{d^2u}{dt^2}\cos v-$$
$$m_1 l_4^2\frac{d^2u}{dt^2}-m_1 l_4^2\frac{d^2v}{dt^2}=0.$$

The system of equations (2) is solved numerically using the Runge-Kutta method in the Maple environment with the following initial conditions: $u(0)$, $v(0)$, $w(0)$, that is, initial values of the lever deviation; $Du(0)$, $Dv(0)$, $Dw(0)$ are initial rates of change in the deviation angles. Here and further, according to the syntax of the maple language, an expression, e. g. $Dw(0)$ means the value of the derivative function $w(t)$ at time t=0. Using the found approximate solutions for the functions $u(t)$, $v(t)$ and $w(t)$ (denote them, $U(t)$, $V(t)$ and $W(t)$, respectively, the load trajectory must be built in the Cartesian coordinate system xOy according to the formulas:

$$x(t)=-l_2\sin\left(U(t)\right)+l_3\sin\left(U(t)-W(t)\right);$$
$$y(t)=l_2\cos\left(U(t)\right)-l_3\cos\left(U(t)-W(t)\right). \qquad (3)$$

That is, for certain time intervals t, the instantaneous coordinates of the load in a vertical plane can be determined using formulas (3) where there is the system of Cartesian coordinates xOy.

The initial phase of the container trajectory before its delivery to the fire zone is built in a vertical plane with the Cartesian coordinate system xOy using formulas (3).

The following parameter values were selected for the test example: the car weight m_1=4,000; the container weight m_2=50; values of parameters: l_1=0.65; l_2=4.2; l_3=2.5; l_4=1.5; the value of initial conditions $u(0)$=Pi/2–2; $Du(0)$=0; $v(0)$=2; $Dv(0)$=0; $w(0)$=Pi/8; $Dw(0)$=0, g=9.81. Time limits of integration of the system of equations (2) $0<t<1.25$ (all values are given in conditional units).

Fig. 2 shows phase trajectories for functions of generalized coordinates, that is, angles $u(t)$, $v(t)$, and $w(t)$. Fig. 3 shows graphs of rates of change in these angles. Analysis of phase trajectories makes it possible to find out some quantitative estimates of the process of load catapulting. Namely, the maximum value of rapid change in the angle $w(t)$ will be equal to w=80 conditional values which can be considered the container velocity at the time of separation from the slingshot. At the same time, the change in angles $u(t)$ and $v(t)$ will reach the extreme velocity.

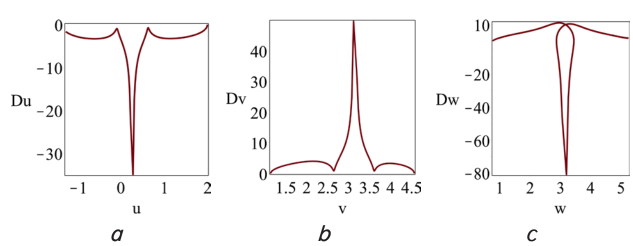


Fig. 2. Phase trajectory for parameters: *a* – $u(t)$; *b* – $v(t)$; *c* – $w(t)$

Let us determine the time moment when the container will reach maximum velocity. For this purpose, it is necessary to construct a graph of dependence of the rate of change in the angle w on time. Fig. 3 shows a corresponding graph where the maximum rate of change in the angle w will occur at t=0.62, which is the recommended moment of the container separation.

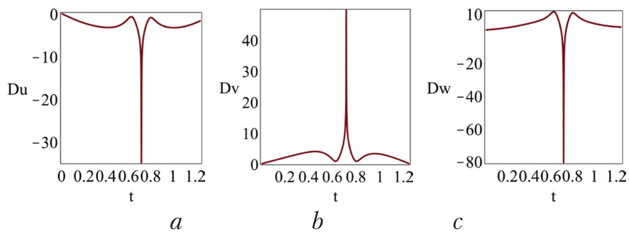


Fig. 3. The graph of change in velocities of angles in time: *a* – *u*(*t*); *b* – *v*(t); *c* – *w*(*t*)

Some qualitative estimates of the process of container catapulting can also be illustrated including determination of the angle of the container departure after its detachment from the slingshot. For this purpose, the program of constructing frames of an animated film of the scheme of trebuchet action was developed. Fig. 4, *a–c* shows separate phases of motion of its elements. Fig. 4 also shows the trajectory of the container center of mass. A computer-animated scheme of container catapulting using a vehicle as a counterweight is given in [40].

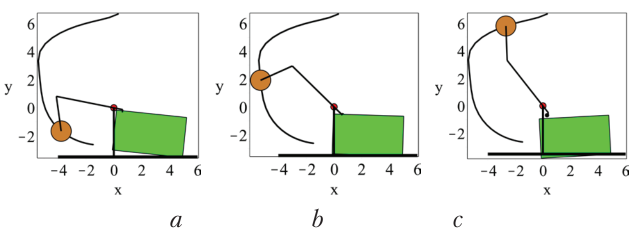


Fig. 4. The obtained images: *a* – the position of elements after start; *b* – the current phase of throwing; *c* – the phase before the moment of load separation

The trebuchet parameters were selected so that the final phase of the trajectory of the center of mass of the container has a rectilinear appearance (Fig. 4). This makes it possible to determine the angle of the container departure at the time of detachment from the slingshot. To do this, coordinates of the endpoints of the trajectory of the container's center of mass must be taken into account.

4. 2. The geometric model of the trebuchet mechanism with a vertical displacement of the counterweight load

Here is a diagram of another throwing unit that differs from the conditional trebuchet design. The main difference is the displacement of the counterweight load vertically downwards during the throwing process (Fig. 5). The considered mechanism is called in literature Floating-Arm Trebuchet [27–30].

The structure consists of the main lever of length l_0=l_1+l_2 to which a lever (or a rope slingshot) of length l_3 is hinged. A counterweight load of mass m_1 is attached to the lever at a node point A and the load of container for throwing having mass m_3 is attached to point D. The mass m_1 is chosen to be several orders of magnitude greater than the mass m_3. The counterweight load with point A must move vertically downwards along the guides under the action of gravity. In this case, the wheel mounted on the lever at point B must roll along the horizontal bar of the structure. Then the payload m_3 will be accelerated causing the effect of throwing after its separation from the slingshot.

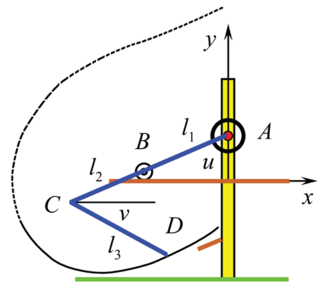


Fig. 5. The scheme of the Floating-Arm Trebuchet throwing mechanism

Fig. 5 shows the Cartesian coordinate system Oxy. Let us choose angles u and v as generalized coordinates. They will determine functions of change in angles over time, namely, changes in the angle $u(t)$ of deviation from the vertical of the lever of length l_1+l_2 as well as changes in the angle $v(t)$ between the horizontal and the lever of length l_1+l_2. The marked points are defined in [27] by coordinates: $A(x_1, y_1)$; $B(x_2, y_2)$; $C(x_3, y_3)$; $D(x_4, y_4)$ where

$$
\begin{aligned}
&x_1 = 0;\quad y_1 = l_1\cos u;\\
&x_4 = -l_1\sin u;\quad y_4 = 0;\\
&x_2 = x_4 - l_2\sin u;\quad y_2 = y_4 - l_2\cos u;\\
&x_3 = x_2 - l_3\cos v;\quad y_3 = y_2 - l_3\sin v;\\
&x_b = x_4 - 0.5(l_2 - l_1)\sin u;\\
&y_b = -0.5(l_2 - l_1)\cos u;\\
&l_0 = l_1 + l_2.
\end{aligned} \tag{4}
$$

Then, to describe the dynamics of the Floating-Arm Trebuchet, it is necessary to use expressions for kinetic (T) and potential (P) energies [27]:

$$
\begin{aligned}
&T = 0.5m_1\dot{y}_1^{\,2} + 0.5m_3\left(\dot{x}_3^{\,2} + \dot{y}_3^{\,2}\right) +\\
&+0.5\left(l_0\left(l_1^2 - l_1 l_2 + l_2^2\right)\dot{u}^2\right);\\
&P = 9.81\left(m_1 y_1 + m_3 y_3 + l_0 y_b\right).
\end{aligned} \tag{5}
$$

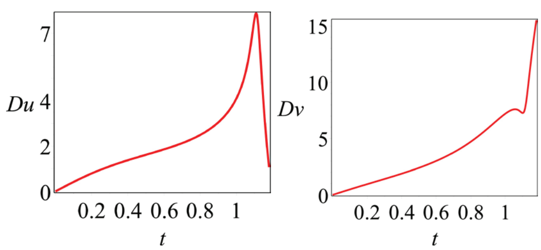
A system of Lagrange equations of the second kind was set up using the Lagrangian L=T–P. It is not given here because of its cumbersomeness. This system was solved numerically using the Runge-Kutta method in an environment of the Maple mathematical processor. Initial conditions $u0$, $v0$ are initial values of the angles of the lever of deviation; $Du(0)$, $Dv(0)$ are initial velocities of change in the angles of deviation. Denote the approximate solutions found for the functions $u(t)$ and $v(t)$ as $U(t)$ and $V(t)$, respectively. These can be expressions in a form of the Maple codes. Then the trajectory of the load displacement in the Cartesian coordinate system xOy must be built according to the formulas:

$$x_4 = -l_1\sin U - l_2\sin U + l_3\cos V;$$

$$y_4 = -l_2 \cos U - l_3 \sin V. \tag{6}$$

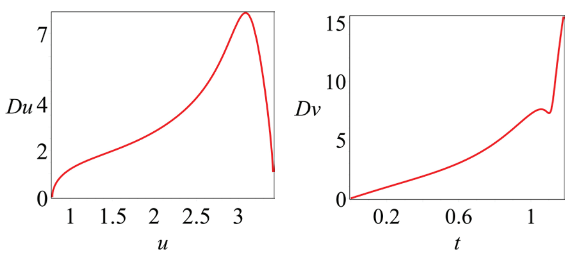
Here is a test calculation of the Floating-Arm Trebuchet model with parameters m_1=2,000; m_2=50; l_1=3; l_2=2.57; l_3=3.6 and initial conditions $u(0)$=Pi/4; $Du(0)$=0; $v(0)$=0; $Dv(0)$=0. All values are in conditional units.

Fig. 6 shows the functional dependences of the rate of change in angles $u(t)$ and $v(t)$ on time. Fig. 7 shows phase trajectories for angles $u(t)$ and $v(t)$. Analysis of the graphs shows that the maximum rate of change in the angle $u(t)$ will be reached at values of t=1.1 and u=3.4. At this point, the rate of change in the angle v will be maximum (about 12.5 conditional units) which will allow the payload to gain maximum velocity at the time of its separation from the slingshot.



a *b*

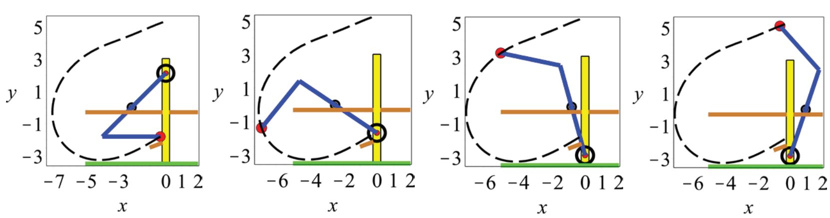
Fig. 6. The graphs of velocity dependence for:
a – $u(t)$; *b* – $v(t)$



a *b*

Fig. 7. Phase trajectory for parameters: *a* – $u(t)$; *b* – $v(t)$

Fig. 8 shows the constructed trajectory of the center of masses of the container. With the help of the developed program, an animated film of Floating-Arm Trebuchet action was created [40]. Some of its frames are shown in Fig. 9.



a *b* *c* *d*

Fig. 8. The obtained images of the throwing phases:
a – initial; *b* – current; *c* – final; *d* – at the time of load separation

It should be noted that the Floating-Arm Trebuchet is easy for transportation as the counterweight load can be fixed on the vertical guideways. A container with water can be used as a counterweight.

Mechanisms of trebuchet type are not used in the practice of real firefighting. But the ideas embedded in their design can be useful and implemented in designing a new throwing mechanism as demonstrated in this section.

5. Modeling of rotary-translational motion of dumbbell-shaped containers in a vertical plane

To ensure the stability of the rotary motion of the dumbbell in a vertical plane, it is necessary to impart rotational motion to it. This can be achieved thanks to two explosion-generated pulses directed at a load of dumbbells in pre-calculated directions.

In the general case, this is the problem of the dynamics of a solid body with a moving center of mass. This class of problems is solved on the basis of Koenig's theorem [19]. An example of such a solution is given in [20]. Some calculations related to the dumbbell motion are given in [33].

Approximate methods are used for engineering calculations. A variant of the graph analytical method is considered in this section.

5. 1. The geometric model of the motion of a dumbbell-shaped object under the condition of initiating its motion analogously to a pendulum

The origin of the dumbbell motion can be explained using the pendulum analogy. To do this, consider a mathematical pendulum of length h with a load A suspended from a stationary load B. Let the load B be separated from the mount at a certain point of the pendulum oscillation. Then the pendulum will lose contact with the fixed point of attachment. Its motion will resemble the motion of a gymnast who performs a somersault from the crossbar. As a result of separation, the newly formed object with two spaced loads will move in the form of a combination of translational and rotational motions. Moreover, the newly formed object will continue to move within the plane of oscillation of the pendulum. At the moment of detachment from the fastening, the object of the study formally ceases to be a pendulum. Therefore, the point of attachment will be called the reference point. Due to the presence of loads attached to the pendulum ends, such a design should be called a dumbbell (or a dumbbell-shaped object) taking the image of a sports dumbbell as a basis.

Let us consider a dumbbell including a weightless rod and two loads of masses m_1 and m_2 at its ends. Distance between the centers of masses of the loads is equal to h. Suppose that the moving reference point has coordinates $x=u(t)$ and $y=v(t)$ in the Cartesian coordinate system Oxy. The center of masses of the first load is superposed with a moving reference point. The rod performs rotational oscillations around the reference point due to the change in the value of the angle $w(t)$.

The problem consists in the development of a geometric model of dumbbell motion within vertical plane Oxy depending on the dumbbell parameters and initial conditions. Note that the conditions contributing to the dumbbell motion can be "explosive" in addition to "pendulum". That is, the motion of the dumbbell can occur due to the explosion-generated pulses directed at the centers of masses of the dumbbell loads. This will be discussed below. It is based on a method of calculation similar to that described in [31, 32].

Let us consider the Cartesian coordinates Oxy of the vertical plane in which the dumbbell should move. Assume the angle $w(t)$ formed by the axis of the rod with the axis Oy as well as coordinates $u(t)$ and $v(t)$ of displacement of the reference point (Fig. 9) be the generalized coordinates of motion.

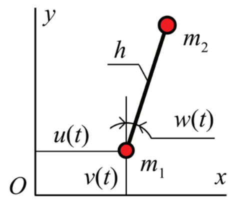


Fig. 9. A rod with a moving reference point

The motion of the rod is started due to the action of the pulse $m_2\,Dw(0)$ set for the angle of deviation $w(t)$. That is, the angle of deviation $w(t)$ is given the initial velocity $Dw(0)$. The vector of direction of setting velocities coincides with the direction of action of the device pulse on the center of masses of the second load directed perpendicular to the axis of the dumbbell. Taking into account the momentum velocity $Dw(0)$ given by pulse, the dumbbell must continue to move mechanically. To start the dumbbell motion simultaneously with actions of the pulse $m_2Dw(0)$, pulses $m_1Du(0)$ and $m_1Dv(0)$ aimed at the reference point with coordinates $u(t)$, $v(t)$ can act simultaneously. That is, the reference point can be given initial velocities $Du(0)$ or $Dv(0)$ in directions of the corresponding coordinates Ox or Oy. Values of all quantities are in conditional units.

To describe the dynamics of motion of a dumbbell with a moving reference point, it is necessary to set up and solve a system of Lagrange equations of the second kind. A Lagrangian for a mathematical pendulum was taken as a basis. It is also necessary to add two generalized coordinates in a form of functions describing the motion of the reference point to the generalized coordinate, that is, angle of the pendulum deviation.

First, using generalized coordinates, calculate "virtual" coordinates of the reference point:

$$x(t)=h\sin\left(w(t)\right)+u(t);$$

$$y(t)=h\cos\left(w(t)\right)-v(t). \qquad (7)$$

In the absence of dissipative forces, the description of the motion of the dumbbell in a vertical plane can be calculated using the Lagrangian $L=T-P$. Expressions for kinetic and potential energies (g=9.81) take the form:

$$T=\frac{m_1}{2}\left(\left(\frac{du}{dt}\right)^2+\left(\frac{dv}{dt}\right)^2\right)+\frac{m_2}{2}\left(\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right); \qquad (8)$$

$$P=-yg\left(2m_1+m_2\right).$$

Using the Lagrangian $L=T-P$, description of the dumbbell motion is obtained in a form of a system of three Lagrange differential equations of the second kind relative to the functions $u(t)$, $v(t)$, $w(t)$.

$$m_2h\cos(w)\frac{d^2u}{dt^2}+m_2h^2\frac{d^2w}{dt^2}+m_2h\sin(w)\frac{d^2v}{dt^2}+$$
$$+gh\sin(w)\left(2m_1+m_2\right)=0; \qquad (9)$$

$$m_2h\sin(w)\left(\frac{dw}{dt}\right)^2+hm_2\cos(w)\frac{d^2w}{dt^2}+\left(m_1+m_2\right)\frac{d^2u}{dt^2}=0;$$

$$hm_2\cos(w)\left(\frac{dw}{dt}\right)^2+hm_2\sin(w)\frac{d^2w}{dt^2}+\left(m_1+m_2\right)\frac{d^2v}{dt^2}+g\left(2m_1+m_2\right)=0.$$

Basic parameters of the dumbbell should be taken into account when solving the system of equations (9), namely, its length h and values of load masses m_1 and m_2 and initial conditions: the values of initial displacements of the reference point and the angle of deviation $u(0)$, $v(0)$, $w(0)$. This requires values of initial velocities given to the point of displacement and the angle of deviation $Du(0)$, $Dv(0)$, $Dw(0)$.

Taking into account corresponding initial conditions, the system of Lagrange equations of the second kind is solved by the Runge-Kutta method in the environment of the Maple mathematical package. The obtained approximate solutions are denoted by symbols $U(t)$, $V(t)$, $W(t)$ which can look like a sequence of the Maple codes. Using approximate solutions, determine coordinates of the centers of masses of the dumbbell loads at time t in the coordinate system Oxy chosen in the plane. To do this, use expressions (7) formally replacing their lowercase letters with uppercase ones: U instead of u, V instead of v, W instead of w.

Taking into account the calculated coordinates of the centers of masses of the dumbbell loads as a function of time, a program of computer animation of the process of dumbbell motion was developed. Options of computer animations can be found on website [40].

The following are examples of the program execution. The dumbbell parameters h=2; m_1=1 and m_2=1 will be the same for them. Recall that according to the syntax of the Maple language, expression, e. g. $Dw(0)$, means the value of the derivative function $w(t)$ at time t=0. The calculation options will differ in initial conditions. It should be noted in advance that it is possible to construct trajectories of the dumbbell loads at arbitrary values of instantaneous velocities $Dw(0)$ and $Du(0)$=10 as initial conditions. For example, Fig. 10 shows the trajectory of motion for values $Dw(0)$=18 and $Du(0)$=13.5.

The disadvantage of a trajectory constructed with arbitrary values of instantaneous velocities consists in its asymmetry relative to half the distance of the dumbbell flight. This will lead to a violation of uniformity of the dumbbell rotation as evidenced by the irregular shape of the phase trajectory for the generalized coordinate $v(t)$ (Fig. 11). Therefore, in the future, the trajectories of the dumbbell motion will be depicted in conjunction with the corresponding phase trajectory.

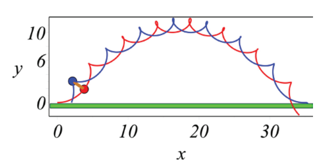


Fig. 10. Trajectories of motion for initial conditions: $w(0)$=Pi/2; $Dw(0)$=18; $u(0)$=0; $Du(0)$=13.5; $v(0)$=0; $Dv(0)$=0; T=2.5

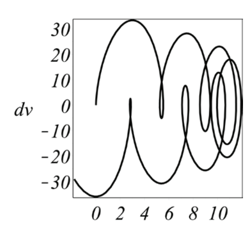


Fig. 11. Phase trajectories for the generalized coordinate v(t) corresponding to the values $Dw(0)$=18 and $Du(0)$=13.5

Example 1. h=2; m_1=1 and m_2=1. Fig. 12–16 show trajectories of the dumbbell loads at different values of initial conditions. In order not to overload the figure, only one instantaneous position of the dumbbell will be conditionally marked.

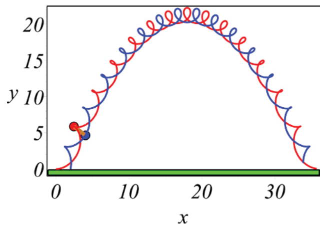


Fig. 12. Plunging trajectories of motion for initial conditions: *w*(0)=Pi/2; *Dw*(0)=25; *u*(0)=0; *Du*(0)=10; *v*(0)=0; *Dv*(0)=0; *T*=3.4

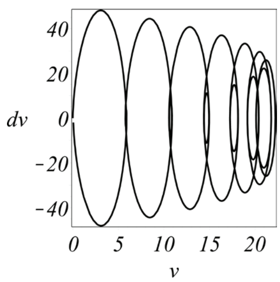


Fig. 13. Phase trajectories for the generalized coordinate *v*(*t*) corresponding to values *Dw*(0)=25 and *Du*(0)=10

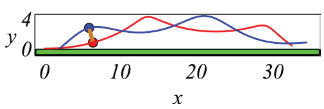


Fig. 14. Flat trajectories of motion for initial conditions: *w*(0)=Pi/2; *Dw*(0)=10; *u*(0)=0; *Du*(0)=25; *v*(0)=0; *Dv*(0)=0; *T*=1.4

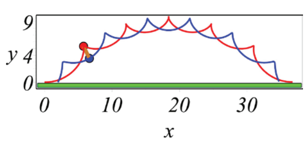


Fig. 15. Trajectories of motion for initial conditions: *w*(0)=Pi/2; *Dw*(0)=16; *u*(0)=0; *Du*(0)=16; *v*(0)=0; *Dv*(0)=0; *T*=2.17

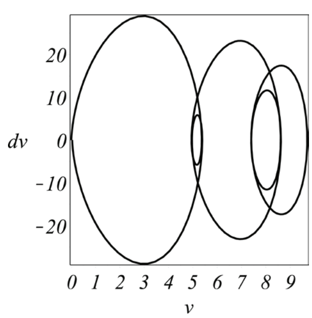


Fig. 16. Phase trajectories for a generalized coordinate *v*(*t*) corresponding to values of *Dw*(0)=16 and *Du*(0)=16

The results of Example 1 make it possible to draw the following conclusions:

1) a change in places of the values of instantaneous velocities $Dw(0)$=25 and $Du(0)$=10 leads to a transition from a plunging to a flat method of dumbbell delivery with different numbers of the dumbbell rotations;

2) deliver a dumbbell to the distance given in Example 1, possibly at the same values of instantaneous velocities $Dw(0)$=16 and $Du(0)$=16;

3) analysis of the phase trajectory for the generalized coordinate $v(t)$ makes it possible to assess the velocities of the dumbbell motion.

Example 2. Fig. 17 shows the trajectory of the dumbbell motion with three identical initial conditions: instantaneous velocities of 9.25 conditional units for parameters h=4; m_1=1 and m_2=1. The phase trajectory of the generalized coordinate $v(t)$ corresponding to this case is shown in Fig. 18.

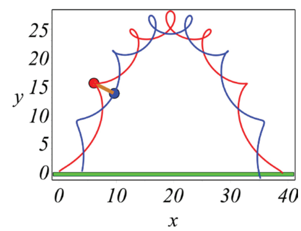


Fig. 17. Trajectories of motion for initial conditions *w*(0)=Pi/2; *Dw*(0)=9.25; *u*(0)=0; *Du*(0)=9.25; *v*(0)=0; *Dv*(0)=9.25

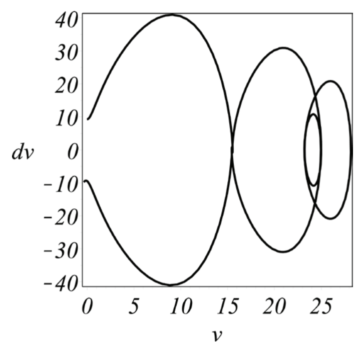


Fig. 18. Phase trajectory of the function *v*(*t*) corresponding to the values of *Dw*(0)=*Du*(0)=*Dv*(0)=9.25

The considered examples contain geometrical modeling of a dumbbell motion as the motion of two separate spaced loads (endpoints of a mathematical pendulum). This does not quite apply to the dumbbell concept as a single whole geometric object. Trajectories of motion of just the load center of mass were determined. The trajectory of the center of mass of the dumbbell as a whole can be determined indirectly using centers of masses of the loads. All this affects the adequacy of geometric modeling of the influence of air resistance on the trajectory of the entire dumbbell. To get rid of this shortcoming, it is necessary to determine the kinetic and potential energy of the dumbbell as a whole. Then, the spaced loads will be combined energetically. This will lead to more adequate computer simulation of the dumbbell motion, and what is most important, to an adequate response to initiation of its motion with the help of explosion-generated pulses of pyro cartridges the action of which is directed at the endpoints of the dumbbell including motion with taking into account air resistance.

5. 2. Development of a geometric model of the rotational-translational motion of the dumbbell in space taking into account air resistance

Here is a calculation variant when the spaced dumbbell loads will be interconnected in the form of a dumbbell as an object when moving. In this case, trajectories of the centers of masses of the loads and the center of mass of the entire dumbbell as a whole will be determined. This will have a positive effect on the adequacy of geometric modeling of the effect of air resistance on the trajectory of movements of the entire dumbbell including different values of masses of the dumbbell loads.

Choose the method of modeling the motion within the vertical plane of the dumbbell described in [34, 35] as a combination of two spaced loads as a basis. Fig. 19 shows a diagram of the initial position of the dumbbell-shaped container in the coordinate system *Oxy*. The container consists of two loads of masses m_1 and m_2 connected by a weightless rod. The center of mass of the first load is located in the origin. The center of mass of the second load is located on the axis *Ox* at a distance *h* from the first load (Fig. 19).

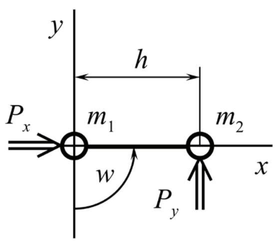


Fig. 19. A diagram of a dumbbell-shaped container

Choose the coordinates $x(t)$ and $y(t)$ of the first load as well as the angle $w(t)$ formed by the dumbbell axis and the negative part of the axis *Oy* as generalized coordinates. Assume that the explosion-generated pulse P_x acts on the mass m_1 and the pulse P_y acts on the mass m_2 simultaneously with the previous one. Using the symbols, initial position of the dumbbell is determined as follows: $x(0)=0$; $Dx(0)=P_x$; $y(0)=0$; $Dy(0)=0$; $w(0)=0$; $Dw(0)=P_y$. Hereinafter, all values are given in conditional units (g=9.81). Air resistance is not taken into account at this stage.

Using the generalized coordinates, calculate "virtual" coordinates of the centers of masses: for the first and second loads as well as the entire dumbbell as a whole:

$$x_1 = x(t);\;\; y_1 = y(t);$$
$$x_2 = x(t) + h\sin w;\;\; y_2 = y(t) - h\cos w; \tag{10}$$
$$x_C = (m_1x_1 + m_2x_2)/(m_1 + m_2);$$
$$y_C = (m_1y_1 + m_2y_2)/(m_1 + m_2).$$

To describe rotational motion of the dumbbell, use the Lagrangian $L=T–P$ where kinetic and potential energies are calculated as follows [34, 35]:

$$T = 0.5m_1\left(\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right) + \\ +0.5m_2\left(\frac{\mathrm{d}x}{\mathrm{d}t} + h\cos(w)\frac{\mathrm{d}w}{\mathrm{d}t}\right)^2 + \\ +0.5m_2\left(\frac{\mathrm{d}y}{\mathrm{d}t} + h\sin(w)\frac{\mathrm{d}w}{\mathrm{d}t}\right)^2; \tag{11}$$

$$P = m_1yg + m_2g\big(y - h\cos(w)\big). \tag{12}$$

The Lagrangian makes it possible to set up a system of Lagrange differential equations of the second kind relative to the generalized coordinates $x(t)$, $y(t)$ and $w(t)$

$$\begin{aligned} &(m_1+m_2)\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - hm_2\sin(w)\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)^2 + \\ &+hm_2\cos(w)\frac{\mathrm{d}^2w}{\mathrm{d}t^2} = 0; \\ &g\sin(w) + \cos(w)\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \\ &+h\frac{\mathrm{d}^2w}{\mathrm{d}t^2} + \sin(w)\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 0; \\ &(m_1+m_2)\left(g + \frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right) + hm_2\cos(w)\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)^2 + \\ &+hm_2\sin(w)\frac{\mathrm{d}^2w}{\mathrm{d}t^2} = 0. \end{aligned} \tag{13}$$

The system of equations (13) is solved by the numerical Runge-Kutta method in the Maple environment. Choose the distance *h* between the centers of masses of the loads, load masses m_1 and m_2 as well as values of initial conditions $x(0)=0$; $y(0)=0$ and $w(0)=\mathrm{Pi}/2$ as the values of parameters. Vary the values of instantaneous velocities $Dx(0)$; $Dy(0)$ and $Dw(0)$.

As a result of solving the system of Lagrange equations of the second kind, approximate expressions for values of the angle $W(t)$ as well as for the coordinates of the first load $X(t)$ and $Y(t)$ are obtained in time *t*. These expressions can be represented as a sequence of the Maple operators in formulas (10) to calculate "virtual" coordinates of the centers of masses.

Here is the sequence of graph-analytical modeling of the trajectory of the dumbbell motion taking into account air resistance. Two notions of the trajectory will be used.

Call motion of the center of masses of the entire dumbbell the calculated trajectory. It is obtained by solving a system of Lagrange differential equations of the second kind based on the kinetic and potential energies of motion.

Call the curve obtained by solving a system of differential equations of motion of a point thrown at an angle to the horizon the theoretical trajectory. It is characterized by the following basic parameters: mass *m* of the point, angle *A* and velocity V_0 of the point departure, and, what is the most important, the coefficient *K* of air resistance.

The relationship of these trajectories is revealed in a sequence of such steps.

Step 1. Under the condition of the Lagrangian definition, construct two trajectories of the centers of masses of the two dumbbell loads after solving the system of the Lagrange equations of the second kind and also construct the calculated trajectory of motion of the center of mass of the dumbbell as a whole. Assume that the air resistance is absent, that is, the coefficient of air resistance R=0.

Step 2. The obtained image of the calculated trajectory of the center of masses of the dumbbell allows one to approximate the angle *A* of the dumbbell departure as well as the range *d* of its flight. The graphic method of study is used here.

Step 3. Using the compiled Maple program, form the theoretical trajectory of the point load and using its analytical description, determine (approximately) the launching velocity V_0 of the dumbbell departure. The analytical method of study is used here.

Step 4. Next, a formal substitution should be made: substitute the theoretical trajectory of the point load for the calculated trajectory of the center of masses of the dumbbell and use its feature to respond to the effect of the coefficient of air resistance.

Step 5. Connect coordinates of the centers of masses of the dumbbell loads with corresponding coordinates of the theoretical trajectory of the point load. Since the theoretical trajectory will react to the change in the coefficient K of air resistance, centers of both dumbbell loads will react to this. As a result, the trajectory of the dumbbell loads is obtained taking into account air resistance.

Let us illustrate this sequence with an example. The values of all parameters were presented in conditional values. The parameters were deliberately exaggerated for the convenience of illustrating the graphics images.

Here is an explanation of the algorithm of “five steps”. Suppose we have a dumbbell of length h=5 with the same masses m_1=25 and m_2=25. It is necessary to estimate the distance to which one can deliver a dumbbell at values of the coefficient of air resistance K=5 or K=10. Initiate the dumbbell motion with the following initial conditions: $x(0)=0$; $Dx(0)=8.23$, $y(0)=0$; $Dy(0)=0$ i $w(0)=\mathrm{Pi}/2$, $Dw(0)=8.23$.

Using the result of solving the system of Lagrange equations of the second kind, construct the calculated trajectory for the value K=0 of the coefficient of air resistance as well as the trajectory of the centers of masses of both loads of the dumbbell (Fig. 20).

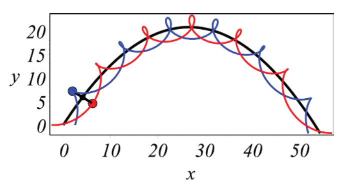


Fig. 20. Trajectories of the centers of masses of the loads in the entire dumbbell (flight time: 4.2 sec)

The phase trajectory of the generalized coordinate $v(t)$ corresponding to this case for the value of K=0 of the coefficient of air resistance is shown in Fig. 21. Its shape indicates symmetry of trajectories of the centers of mass relative to the middle of the distance of the dumbbell delivery.

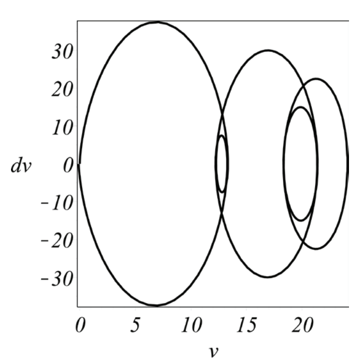


Fig. 21. The phase trajectory of the function $y(t)$ which corresponds to the values of $Dw(0)=Dx(0)=8.23$

The calculated trajectory makes it possible to make an approximate estimate of the dumbbell flight distance (d=52) as well as the angle of its departure (A=1.2 rad). The angle was estimated using a specially compiled program.

To calculate velocity V_0 of the dumbbell, use the well-known [36, 37] description of the theoretical flight trajectory of a physical point with mass M and coefficient K of air resistance

$$X = \frac{V_0 M \cos A\left(-1+\exp\left(-\frac{K\,t}{M}\right)\right)}{K}; \tag{14}$$

$$Y = -M \times \frac{Mg\left(\exp\left(-\frac{K\,t}{M}\right)\right) + V_0 K \sin A\left(\exp\left(-\frac{K\,t}{M}\right)\right) + gK\,t - Mg - V_0 K \sin A}{K^2},$$

where $M=m_1+m_2$ is the mass of the container; A is the throw angle, rad.

Using the compiled program, determine the theoretical flight trajectory of a physical point (Fig. 22). To do this, calculate (approximately) velocity V_0 of the dumbbell departure provided that the calculated trajectory (Fig. 20) and the theoretical trajectory (Fig. 22) do not differ by a percentage of the value of approximation along the axis Oy. In this example, V_0=24 conditional units.

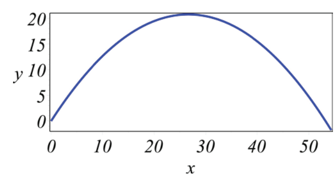


Fig. 22. The theoretical trajectory of flight of a physical point

Next, it is necessary to substitute the theoretical trajectory of the point load for the calculated trajectory of the center of masses of the dumbbell because the shape of the latter is affected by the coefficient of air resistance. To do this, connect the coordinates of the centers of masses of the dumbbell loads with corresponding coordinates of the theoretical trajectory of the point load.

Expressions for calculating coordinates (X_1, Y_1) of the first load depending on coordinates (X_C, Y_C) of the center of masses of the dumbbell take the form

$$X_1 = X_C + \frac{hm_2\, simW}{m_1+m_2};\quad Y_1 = Y_C - \frac{hm_2 \cos W}{m_1+m_2}, \tag{15}$$

where m_1 and m_2 are values of the load masses. Coordinates (X_2, Y_2) of the second load can be calculated by the formulas

$$X_2 = X_1 - h\sin W;\quad Y_2 = Y_1 + h\cos W; \tag{16}$$

As a result of these actions, the image of the trajectories of the dumbbell loads for the values K=10 and K=15 of the coefficients of air resistance are obtained (Fig. 23, 24).

Hence, estimates of dumbbell delivery distances of 52, 42, and 33 conditional values were obtained at values of both explosion-generated pulses of pyro cartridges of 8.23 conditional units and values of the coefficient of air resistance K=0; K=5 or K=10. The angle of departure of the dumbbell was about 45 deg and the departure velocity was 24 conditional values.

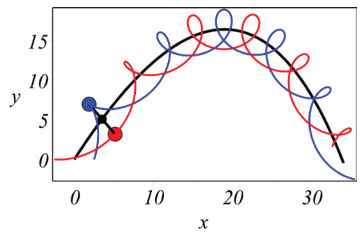


Fig. 23. Trajectories of the dumbbell displacement for the value of K=10 (flight time: 3.7 sec)

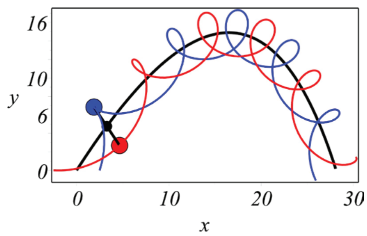


Fig. 24. Trajectories of dumbbell displacement for the value of K=15 (flight time: 3.5 sec)

The example where values of both explosion-generated pulses of pyro cartridges and the load masses are identical was considered above. In the following examples, it is necessary to choose various basic values of parameters and be convinced that changes in their sizes do not lead to contradictions.

Example 1. The values of both explosion-generated pulses of pyro cartridges are the same and masses of loads are different. It is h=5 with masses m_1=25 and m_2=50. The dumbbell motion was initiated with the following initial conditions: $x(0)=0$; $Dx(0)=16$, $y(0)=0$; $Dy(0)=0$ and $w(0)=$ $=$Pi/2, $Dw(0)=16$. The results of geometric modeling are given in Fig. 25–27.

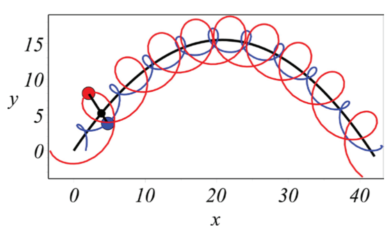


Fig. 25. Trajectories of the dumbbell displacement for the value of K=0.05 (flight time: 3.6 sec)

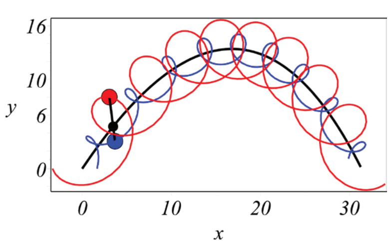


Fig. 26. Trajectories of the dumbbell displacement for the value of K=10 (flight time: 3.3 sec)

Therefore, the following estimates of dumbbell delivery distances were obtained: 42, 32, and 28 conditional values at values of both explosion-generated pulses of pyro cartridges of 16 conditional units and values of the coefficient of air resistance K=0.05; K=5 or K=10. The angle of departure of the dumbbell was about 44.8 deg, and the velocity of departure was 21 conditional units. In the case of geometric modeling, results similar to the previous ones will be obtained at the same values but with masses of m_1=50 and m_2=25.

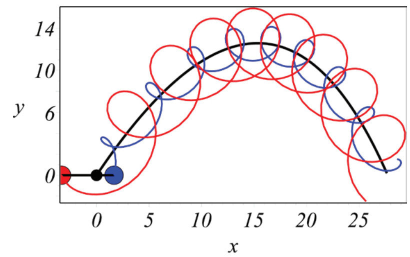


Fig. 27. Trajectories of the dumbbell displacement for the value of K=15 (flight time: 3.2 sec)

Example 2. The values of both explosion-generated pulses of pyro cartridges are different and masses of the loads are the same. There is h=5 at masses m_1=50 and m_2=50. Motion of the dumbbell will be initiated with the following initial conditions: $x(0)=0$; $Dx(0)=16$, $y(0)=0$; $Dy(0)=0$ and $w(0)=$ $=$Pi/2, $Dw(0)=8.24$. The results of geometric modeling are shown in Fig. 28–30.

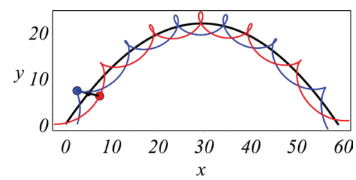


Fig. 28. Trajectories of the dumbbell displacement for the value of K=0.05 (flight time: 4.24 sec)

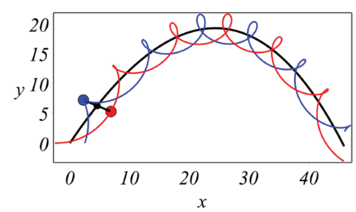


Fig. 29. Trajectories of the dumbbell displacement for the value of K=10 (flight time: 4 sec)

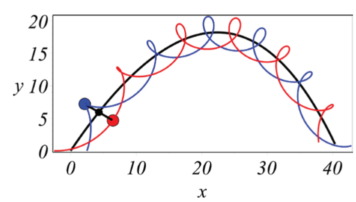


Fig. 30. Trajectories of the dumbbell displacement for the value of K=15 (flight time: 3.8 sec)

Trajectories of the dumbbell for the same values of parameters but with mutually replaced values of explosion-generated pulses of pyro cartridges $Dx(0)=8.24$ and $Dw(0)=16$ are shown in Fig. 31–33. Interestingly, the flight time and delivery distances agree with the previous cases.

Therefore, the following was obtained for masses m_1=50 and m_2=50 at values of explosion-generated pulses $dx(0)=16$ and $dw(0)=8.24$ conditional units. When values of the coefficient of air resistance K=0.05; K=10 or K=15, corresponding estimates of distances of the dumbbell delivery were obtained: 60, 48, and 42 conditional units. The angle of departure of the dumbbell was about 45 deg and the departure velocity was 25 conditional units. In the case of mutual replacement of explosion-generated pulses of pyro cartridges $Dx(0)$=8.24 and $Dw(0)$=16, distances and delivery times similar to the previous ones were obtained at the same values. The difference consists in the more revolutions that the dumbbell will make during the flight. A high frequency of the dumbbell rotation will cause a buzzer-type signal which will contribute to safety.

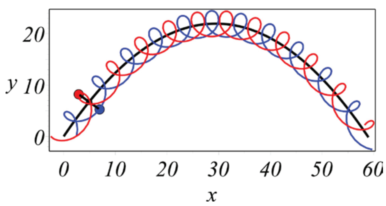


Fig. 31. Trajectories of the dumbbell displacement for the value of K=0.05 (flight time: 4.24 sec)

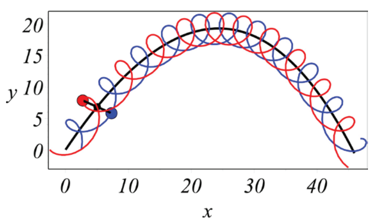


Fig. 32. Trajectories of the dumbbell displacement for the value of K=10 (flight time: 4 sec)

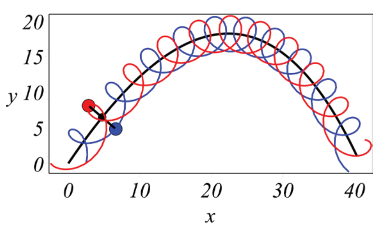


Fig. 33. Trajectories of the dumbbell displacement for the value of K=15 (flight time: 3.8 sec)

Example 3. Investigate the effect of the dumbbell length h on the trajectory of motion taking into account the coefficient of air resistance K=15 at masses m_1=50 and m_2=50. Motion of the dumbbell will be initiated with the following initial conditions: $x(0)$=0; $Dx(0)$=27, $y(0)$=0; $Dy(0)$=0 and $w(0)$=Pi/2, $Dw(0)$=5. The results of geometric modeling are shown in Fig. 34–36.

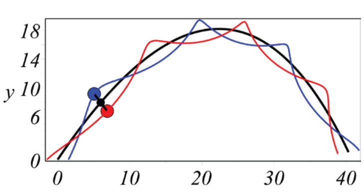


Fig. 34. Trajectories of the dumbbell displacement for the value of h=3 (flight time: 3.8 sec)

Therefore, the following was obtained for masses m_1=50 and m_2=50 at the values of explosion-generated pulses of pyro cartridges $Dx(0)$=27 and $Dw(0)$=5 conditional units. It was shown for the value of the coefficient of air resistance was K=15 conditional units that regardless of the dumbbell length, delivery distance and the flight time will be the same, namely, the distance of 40 conditional values and the flight time of 3.8 sec. The angle of the dumbbell departure was about 45 degrees and the velocity of departure was 25 conditional values.

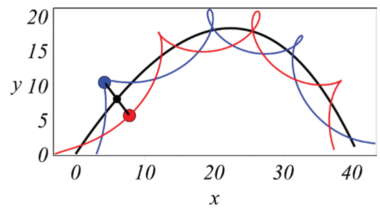


Fig. 35. Trajectories of the dumbbell displacement for the value of h=15 (flight time: 3.8 sec)

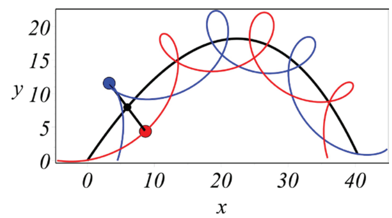


Fig. 36. Trajectories of the dumbbell displacement for the value of h=9 (flight time: 3.8 sec)

5. 3. The scheme of a launcher for the realization of rotational motion of the dumbbell-shaped containers

The launcher is designed to initiate the dumbbell motion in space within the vertical plane. Assume that the dumbbell loads have a spherical form. It is proposed to use explosion-generated pulses of pyrotechnic devices as movers. It is desirable to design the explosive in the form of a pyro cartridge (pyro bolt) capable of creating a directed explosion-generated pulse of a predetermined magnitude [38, 39]. For this purpose, it is necessary to provide fastening of pyro cartridges in the launcher structure and direction of explosion-generated pulses of pyro cartridges to the centers of masses of the dumbbell loads. Fig. 37 presents a schematic view of the launcher [18]. It has a form of a metal angle with two holes shown in the intersection with a vertical plane.

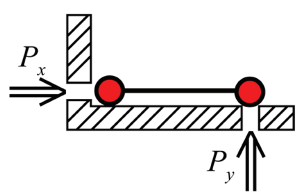


Fig. 37. Schematic view of the launcher in the intersection with a normal plane

Before launching, the dumbbell is installed on the appropriate holes. By means of simultaneous action of pyro cartridges explosion, pulses Px and Py are formed and the dumbbell begins to move in the vertical plane. Its motion will have a rotary-translational form and it will rotate around the center of masses in its flight. The following notation of conditional values of explosion-generated pulses of pyro cartridges with an account of the initial conditions discussed above was taken: Px=$Dx(0)$ and Py=$Dw(0)$. The tightness of the contact of spherical surfaces of the dumbbell loads with the holes is provided by masses of the loads.

Fig. 38 shows stages of launching a dumbbell and its flight in space. Trajectories of motion of the centers of masses of two loads as well as the trajectory of motion of the center of mass of the whole dumbbell are shown. Animation of the start process can be viewed on the website [40].

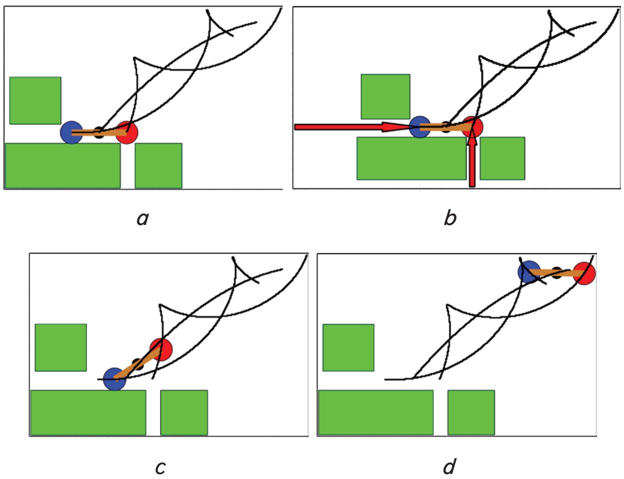


Fig. 38. Schematic view of the stages of dumbbell start: *a* – initial position of the dumbbell; *b* – arrows show directions of the explosion-generated pulses of the pyro cartridges; *c* – the initial phase of the flight; *d* – current phase of the flight

The launcher scheme is modifiable, e. g. by making it for three types of dumbbell sizes. To this end, it is necessary to make three holes in the horizontal part of the scheme corresponding to the lengths of three dumbbell types. It is also necessary to design special locks for pyro cartridges to ensure the safe operation of the launcher.

It should be noted that there is no need to invest in ensuring constant readiness of the launcher to deliver extinguishing substances at long distances. In addition, such a device does not require long-term deployment which is especially important for the prompt actions of firefighters.

6. Discussion of the results of computer simulation of the motion of a dumbbell container taking into account air resistance

The proposed method of remote delivery of extinguishing substances packed in a dumbbell-shaped shell can form a basis for a new firefighting technology. This is evidenced by the new scheme of launching the dumbbell with explosion-generated pulses of charges of two pyro cartridges.

The obtained results can be explained by the possibility of applying the Lagrange variational principle to the calculation of mechanical structures taking into account kinematic connections and the use of energy of a mechanical system. It was possible to simulate the motion of a dumbbell taking into account air resistance based on a combination of results of solving two systems of differential equations, namely, the Lagrange equation of the second kind and equations of motion of a point mass thrown at an angle to the horizon. It was implemented in this study in the form of a five-step algorithm.

To implement this method in actual firefighting practice, the following main issues should be addressed:

1) select the explosive type and develop measures for its safe operation. It is desirable to design the explosive in a form of a pyro cartridge (pyro bolt) capable of creating a directed explosion-generated pulse of a predetermined magnitude;

2) select the material for the manufacture of the dumbbell body. It must be strong enough to resist destruction during start-up. On the other hand, it must be easily destroyed in the fire zone from an impact or under the influence of high temperature;

3) select effective extinguishing substances for delivery to the fire area as fillers for container shells. Such substances may include a fire-extinguishing powder, solid carbon dioxide and organometallic compounds, environmentally friendly freon, or the like;

4) Develop a user-friendly design of the launcher for remote delivery of containers with extinguishing substances. Consider the creation of “batteries” from them, that is, a combination of several launchers;

5) develop hardware implementation of a rangefinder to measure the distance to the fire zone while calculating the required magnitudes of explosion-generated pulses to ensure delivery of the extinguishant to this distance;

6) carry out comprehensive tests of the method of delivering the substance in dumbbell-shaped shells which will provide knowledge of allowable dimensions of the containers and the launcher. This will help correctly move from conditional quantities used in the study calculations to actual physical units.

Difficulties of carrying out studies in this area are mainly related to the use of explosives as a basis for the method of delivery of extinguishing substances to a remote fire zone. A significant disadvantage of the use of explosives consists in an increased danger when using shells with powder charges. Therefore, it was believed that the line of developing means for delivery of extinguishing substances over long distances poses a threat of impact factors during a rupture or destruction of the projectile. However, such threats can be reduced greatly by using pyro cartridges or pyro bolts as explosive devices. Their designs have proven to be reliable and safe in space applications. However, in the future, it is advisable to develop this approach in the direction of other possible movers, e.g. pneumatic or hydraulic movers which would use compressed air or compressed fluid. Solutions in the field of electromagnetic movers also look interesting. This shows the way for further studies.

7. Conclusions

1. The method of calculating the process of throwing goods over long distances was explained by examples of two variants of the trebuchet mechanisms: when the load of the counterweight is a vehicle and when the counterweight moves vertically. To construct the trajectory of the center of mass of the load, it is necessary to determine a Lagrangian and taking it as a basis, set up and solve a system of differential Lagrange equations of the second kind relative to the functions of generalized coordinates. The object trajectory is formed by coordinates of the points of individual solutions of the system of equations.

2. Graphoanalytical modeling of the trajectory of rotational-translational motion of a dumbbell based on a

combination of the result of the solution of two systems of differential equations was offered, namely, the Lagrangian of the second kind and motion of a point mass launched at an angle to the horizon. This has allowed us to simulate the motion of a dumbbell taking into account air resistance. Estimates of the flight range of the dumbbell depending on magnitudes of explosion-generated pulses from the cartridges as a means of initiating its motion were obtained. For example, for masses m_1=50 and m_2=50, the following is obtained at magnitudes of explosion-generated pulses $dx(0)$=16 and $dw(0)$=8.24 conditional units. For values of the coefficient of air resistance K=0.05; K=5 or K=10, corresponding estimates of distances of delivery of a dumbbell were obtained: 60, 52, and 45 conditional units. The angle of departure of the dumbbell was about 45 deg and the velocity of departure was 25 conditional values. Rotational motion of the container will make it possible to use rotational energy to destroy it at the end of the flight and release the extinguishant.

3. A scheme of a launcher for imparting rotational-translational motion to the container of a dumbbell-shaped form was offered. Explosion-generated pulses of two pyro cartridges aimed at the dumbbell loads were used as movers. The generated frames of computer animation of dumbbell motion [40] convince of the adequacy of the geometric model of the method of delivery of a dumbbell-like object.

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