The Formation of Orthogonal Balanced Experiment Designs Based on Special Block Matrix Operations on the Example of the Mathematical Modeling of the Pneumatic Gravity Seed Separator

Vira Malkina, Serhii Kiurchev, Viacheslav Osadchyi and Oksana Strokan

1 Introduction

The experiment is the most important element of scientific research. The qualitative experiment allows obtaining the information about the object of the research. Therefore, the skill full arrangement of the experiment is crucially important. The task of experiment designs is urgent for the mechanism operation process modeling, particularly for the determination of optimal operation parameters.

However, for accurate determination of parameter values based on mathematical equations, it is necessary to obtain the most complete information. For this purpose, the experimental studies with the largest quantity of factor levels (parameters) affecting the separator operation have to be carried out.

In this way, the problem of formation of such design of experimental studies of the separator, which is going to provide the most accurate and complete model of its operation, is of current importance and will allow determining the optimal work parameters.

In the present paper, solved the task of the mathematical model formation of the of a pneumatic gravity separator by means of the organization of the effective experiment conduction scheme with a sufficiently large number of factor levels and simultaneously small number of experiments operation.

V. Malkina (✉) · S. Kiurchev · O. Strokan
Tavria State Agrotechnological University, B. Khmelnitsky Ave.18, Melitopol 72310, Ukraine
e-mail: vira.malkina@tsatu.edu.ua

V. Osadchyi
Bogdan Khmelnitsky Melitopol State Pedagogical University, Getmanska 20, Melitopol 72312, Ukraine

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The formation of the planning matrix, which possesses the specified features that ensure the sufficiently high level of the experimental research efficiency, is the most important stage of the research.

The range of criteria exists that assesses the quality of the experiment design and the criteria of the design optimality. Depending on requirements imposed to the process of the model formation and analysis chosen one or another criterion.

The choice of a criterion depends on the research task. Thus, while studying the influence of separate factors on the object behavior, used the *E*-criterion, while searching the response function optimum chosen the *D*-optimality criterion.

Since the tasks of our research are the effective design formation of experiment conduction with a large number of factor levels and the following determination of the optimal values of the factors based on the resulting model, therefore the most appropriate in this case is the *D*-optimality criterion.

One of the approaches to the formation of *D*-optimal designs is numerical algorithms. Such algorithms are numerical procedures, so-called exchange algorithms. The number of papers [1–3] deals with the design formation algorithms.

In [4], examined the issue of comparison of the experimental design formation algorithms providing the *D*-optimality criteria.

The methodology of the *D*-optimal design formation based on Kronecker operations of special block matrix operations suggested in [5]. In this paper, the algorithm of the orthogonal balanced design formation of the m^n experiment, where m is the number of factor levels, and n is the number of factors is proposed.

In [6, 7], initiated the mathematical model that describes the operation of the pneumatic gravity seed separator. The experiment carried out according to the Box-Bank design with three two-level factors. Based on the experiment, formed the mathematical model, according to which the rational values of the parameters (factors) of the separator operation are determined. The optimal values of the remaining separator parameters are determined experimentally. In this paper, examined the mathematical model of the separator operation formed based on the conduction of experiments with five factors having five levels. For the model, used formation the scheme of orthogonal balanced design formed on special block matrix operations. The optimal values of separator operation parameters found based on the mathematical model formed according to the results of the experiment.

The objective of the article is the algorithm of the formation of orthogonal balanced experiment designs, which are optimal according to the *D*-efficient criterion, the mathematical model development of the pneumatic gravity seed separator, and determination of the optimal separator parameters based on the developed model.

2 Research and Discussion of the Result

It is commonly known that the full factor experiment gives the most complete information about the object of the research with the given number of factors and their levels. In such design, the matrix columns do not correlate; the design is orthogonal and balanced. However, the essential drawback of such design is the necessity of conduction of large number of experiments. Thus, for example, with five factors, each of which varies at five levels, it is necessary to perform $5^5 = 3125$ experiments. This significantly limits the possibilities of application of such designs.

One of the methods of obtaining sufficiently complete information about an object with the reduction of the number of necessary experiments is the fractional design formation. In this case, it is possible to use not the whole matrix of the design of the full factor experiment, but only some of its lines. The matrix of the design, at the same time, formed in such a way that even the part of information sufficiently fully reflects the researched object, in other words, the design must be efficient.

One of the quality indicators of the design completeness is the fact that each factor at each level repeated in the design matrix the same number of times; consequently, the matrix of such design forms an orthogonal array and the design is balanced.

As it is known from [8], the orthogonal design or the orthogonal table $OA_\lambda(t, k, v)$ of size $\lambda v^t \times k$ over the alphabet of v letters is the matrix in which each of the symbol combinations occurs exactly λ times for any set of t columns under the limitation of matrix lines for these t columns.

The design formed based on the orthogonal design is balanced, since it contains the same amount of each level of each factor, each pair of factors occurs the same number of times. If the design is balanced and its matrix is orthogonal, then the criterion of the *D*-efficiency of such design is of the greatest importance.

The *D*-efficient criterion minimizes the determinant of the corresponding normal system of linear equations. According to this criterion, the expected forecast error based on the response function is minimal. The value of the *D*-optimality criterion calculated by the formula:

$$D_{\text{eff}} = \frac{\left|M^{\mathrm{T}} \times M\right|^{\frac{1}{m}}}{n \times m}, \tag{1}$$

where

M the matrix of the experiment;
M^{T} the transposed experiment array;
n the number of factors;
m the number of levels.

Based on the methodology [5], offered the algorithm for the formation of matrix of the orthogonal m^n type design. For the formation of such a matrix, Kronecker operations are used [9].

If A is a matrix of the $m \times n$ size, B is a matrix of the $p \times q$ size, then the Kronecker product is a block matrix of the $mp \times nq$ size

$$A \otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \dots & \dots & \dots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix} \tag{2}$$

Let us examine the Kronecker A matrix product $\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ and $A^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & 2 & 3 \end{bmatrix}$:

$$A \otimes A^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 6 & 9 \end{bmatrix} \tag{3}$$

We perform the "division by remainder" operation with the elements of matrix obtained as the Kronecker product k, that is, the calculation of the remainder from the division by k.

For example, the Kronecker product $A \otimes A^{\mathrm{T}}$ "by remainder 3" is the matrix:

$$D_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

On the analogy of the Kronecker product, the operation of the block matrix summation considered in [5]. The block sum of matrices $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ and $B = \begin{bmatrix} b_{ij} \end{bmatrix}$ is the matrix of the type:

$$A \oplus B = \begin{bmatrix} a_{11} + B & \dots & a_{1n} + B \\ \dots & \dots & \dots \\ a_{m1} + B & \dots & a_{mn} + B \end{bmatrix}$$

$$A \oplus B = \begin{bmatrix} a_{11} + b_{11} & a_{11} + b_{12} & a_{11} + b_{1n} & \dots & a_{1n} + b_{11} & a_{1n} + b_{12} & \dots & a_{1n} + b_{1n} \\ a_{11} + b_{21} & a_{11} + b_{22} & a_{11} + b_{2n} & \dots & a_{1n} + b_{21} & a_{1n} + b_{22} & \dots & a_{1n} + b_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{11} + b_{m1} & a_{11} + b_{m2} & a_{11} + b_{mn} & \dots & a_{1n} + b_{m1} & a_{1n} + b_{m2} & \dots & a_{1n} + b_{mn} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1} + b_{11} & a_{m1} + b_{12} & a_{m1} + b_{1n} & \dots & a_{mn} + b_{11} & a_{mn} + b_{12} & \dots & a_{mn} + b_{1n} \\ a_{m1} + b_{21} & a_{m1} + b_{22} & a_{m1} + b_{2n} & \dots & a_{mn} + b_{q2} & a_{mn} + b_{22} & \dots & a_{mn} + b_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1} + b_{m1} & a_{m1} + b_{m2} & a_{m1} + b_{mn} & \dots & a_{mn} + b_{m1} & a_{mn} + b_{m2} & \dots & a_{mn} + b_{mn} \end{bmatrix} \tag{5}$$

The obtained matrix is the main block for the formation of the orthogonal balanced design scheme for the m^n kind experiment (n is the number of factors, m is the number of levels).

Let us examine the Kronecker product of matrices $L_n = \begin{bmatrix} 0 \\ 1 \\ \dots \\ n \end{bmatrix}$ and $L_n^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & \dots & n \end{bmatrix}$:

$$L_n \oplus L_n^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & 2 & \ldots & n \\ 1 & 2 & 3 & \ldots & n+1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ n & n+1 & n+2 & \ldots & 2n \end{bmatrix} \tag{6}$$

Let us form the D_m matrix as the result of applying the "remainder from division by m" of the $L_n \oplus L_n^{\mathrm{T}}$ matrix elements.

Let us form the F matrix as the result of performing the operation of the block summation of the column-matrix $\begin{bmatrix} 0 \\ 1 \\ \ldots \\ n-1 \end{bmatrix}$ and the D_m matrix:

$$F = \begin{bmatrix} 0 \\ 1 \\ \ldots \\ n-1 \end{bmatrix} \oplus D_m = \begin{bmatrix} 0 + D_m \\ 1 + D_m \\ \ldots \\ n + D_m \end{bmatrix}. \tag{7}$$

Then using operation "remainder of the division by m" for each element of the result matrix.

For example, the matrix of the orthogonal design of the 5^5 type experiment ($m = 5$, which is the number of factor levels, $n = 5$ and is the number of factors), is formed according to the offered scheme and has the type:

$$[l_5, l_5 \oplus D_5] = \begin{bmatrix} l_5,\ 0 + D_5 \\ l_5,\ 1 + D_5 \\ l_5,\ 2 + D_5 \\ l_5,\ 3 + D_5 \\ l_5,\ 4 + D_5 \end{bmatrix}, \tag{8}$$

where l_5—the matrix $\begin{bmatrix} 0 \\ \ldots \\ 4 \end{bmatrix}$.

After the matrix formation row sorting according to the first column, we obtain the experiment design type 5^5 matrix:

$$\begin{matrix} 0 & 0\,0\,0\,0\,0 \\ 0 & 1\,1\,1\,1\,1 \\ 0 & 2\,2\,2\,2\,2 \\ 0 & 3\,3\,3\,3\,3 \\ 0 & 4\,4\,4\,4\,4 \end{matrix}$$

$$
\begin{array}{cc}
1 & 0\,1\,2\,3\,4 \\
1 & 1\,2\,3\,4\,0 \\
1 & 2\,3\,4\,0\,1 \\
1 & 3\,4\,0\,1\,2 \\
1 & 4\,0\,1\,2\,3 \\
2 & 0\,2\,4\,1\,3 \\
2 & 1\,3\,0\,2\,4 \\
2 & 2\,4\,1\,3\,0 \\
2 & 3\,0\,2\,4\,1 \\
2 & 4\,1\,3\,0\,2 \\
3 & 0\,3\,1\,4\,2 \\
3 & 1\,4\,2\,0\,3 \\
3 & 2\,0\,3\,1\,4 \\
3 & 3\,1\,4\,2\,0 \\
3 & 4\,2\,0\,3\,1 \\
4 & 0\,4\,3\,2\,1 \\
4 & 1\,0\,4\,3\,2 \\
4 & 2\,1\,0\,4\,3 \\
4 & 3\,2\,1\,0\,4 \\
4 & 4\,3\,2\,1\,0
\end{array}
\tag{9}
$$

After the formation of the array, the orthogonal coding should be performed in according to [5]. The coding is the process or replacement of our calculated factors by a set of indicators or coded variables. Applied the standardized orthogonal contrast coding:

1 level	1.58	−0.91	−0.65	−0.50
2 level	0	1.83	−0.65	−0.50
3 level	0	0	1.94	−0.50
4 level	0	0	0	2.00
5 level	−1.28	−0.91	−0.65	−0.50

Thus, we obtain the orthogonal balanced design, which has the best D-efficient indicator among designs of the same dimension [5].

So, the following algorithm for the formation of the orthogonal fractional design of the m^n type experiment (m is the number of factor levels, n is the number of factors):

1. To form the matrix—the result of the Kronecker product of two matrices $l_n = \begin{bmatrix} 0 \\ \dots \\ n \end{bmatrix}$ and $l_n^{\mathrm{T}} = \begin{bmatrix} 0 \dots n \end{bmatrix}$, ($n$ is the quantity of the factors).

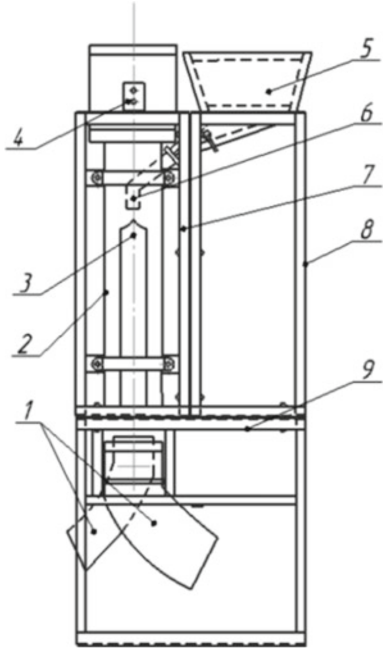

Fig. 1 Pneumatic gravity seed separator: 1—two distributors; 2—the aspirating channel; 3—the divider; 4—the ventilator; 5—the bunker; 6—the supply branch pipe; 7—the left frame part; 8—the upper frame part; 9—the lower frame part

2. To perform the operation "remainder from dividing" by m (m—the number of factor levels) toward the formed matrix and obtain the D_m matrix;
3. Using the block summation operation, to form the matrix $l_m \oplus D_m$;
4. To add the l_n column to the formed matrix and obtain the $[l_n, l_m \oplus D_m]$ matrix;
5. To sort the matrix from step 4 according to the first column;
6. To perform the orthogonal coding operation.

The offered algorithm of the design matrix formation applied while conducting the experimental studies of the optimal parameter values of the pneumatic gravity separator operation [6, 7].

The scheme of the pneumatic gravity seed separator presented in Fig. 1.

Based on the preliminary studies, the degrees of the influence of distinct factors have been determined and established the levels of their variation. The factors that have significant influence on the separation process include:

- the seed drop speed (m/s), v_0;
- the angle of seed input (deg), α;
- the airspeed (m/s), v_v;
- the vertical aspirating channel length (m), l;
- the main aspirating channel diameter (mm), d.

As a response, the size of the average deviation of the seed motion path in the off-loading point (m), y, can be considered.

Table 1 Factor levels

Factors	The real notations	The coded notations	The variability interval	Factor levels				
				5	4	3	2	0
The seed drop speed (m/s)	v_0	x_1	0.2	1	0.8	0.6	0.4	0.2
The angle of seeds input (deg)	α	x_2	20	100	80	60	40	20
The airspeed (m/s)	v_v	x_3	0.5	6	5.5	5	4.5	4
The vertical aspirating channel length (m)	l	x_4	0.25	0.8	0.65	0.5	0.35	0.2
The main aspirating channel diameter (mm)	d	x_5	50	200	175	150	125	100

Each factor modified at five levels. The level values of the factor variation is given in Table 1.

For the determination of rational separator operation parameters, the series of experiments conducted according to the design (9). As the result of the experiment conduction, the following model formed, taking into account only the significant factors:

$$y = -64.829 + 0.067v_0 + 0.002\alpha + 0.92 \times 10^{-5}v_v \\ - 0.042v_0^2 - 9.2 \times 10^{-6}v_v^2 - 2.5 \times 10^{-5}\alpha^2 + 1.115l + 0.32d$$

Based on the formed model, the following optimal values of the separator operation parameters were determined in the range of a variety of factors:

– the seed drop speed $v_0 = 0.8$ M/C;
– the angle of seed input (deg) $\alpha = 40°$;
– the air speed = 5 M/C;
– the vertical aspirating channel length $l = 0.7$ m;
– main aspirating channel diameter $d = 200$ mm.

Achieved the maximum separating capacity of the separator is at the value of the average seed motion path deviation in the off-loading point $y = 0.018$ m.

The results of the full-scale experiments, given in the paper [6, 7], confirm the found theoretical values.

3 Conclusion

The article describes the algorithm of the formation of the orthogonal balanced experiment designs, which are optimal by the *D*-efficient criterion. The algorithm based on the Kronecker matrix product operations the block matrix summations. On the grounds of the offered algorithm, formed and implemented the experiment for the investigation of the optimal parameter values of the pneumatic gravity seed separator. The optimal values of the parameters found from the mathematical model confirmed experimentally.

References

1. Mitchell, T.J.: The algorithm for the construction of D-optimal experimental designs. Technometrics **16**(5), 203–210 (1974)
2. Atwood, C.L.: Sequences converging to D-optimal designs of experiments. Ann. Statist. **1**, 342–352 (1973)
3. Yonchev, H.: New computer procedures for constructing exact D-optimal designs. In: Optimal Design and Analysis of Experiments, pp. 71–80. Holland (1988)
4. Cook, R.D., Nachtsheim, C.J.: A comparison of algorithms for constructing exact D-optimal designs. Technometrics **22**(8), 315–324 (1980)
5. Kuhfeld, W.F.: Experimental Design, Efficiency, Coding, and Choice Designs. SAS Technical Documents TS-694C. SAS Institute Inc., Cary, NC (2010)
6. Kyurchev, S.V., Kolodij, O.S.: The methodology of studying of the parameters of the seeds separator of the offered type. In: Conference 2013. Motorization and Energetic in Agriculture, vol. 15(2), pp. 205–213. Lublin-Rzeszow (2013)
7. Kyurchev, S.V., Kolodij, O.S.: The results of research of rational sizes of the vertical aspirating channel of the seeds separator of agricultural crops. In: Conference 2013. Motorization and Energetic in Agriculture, vol. 15(2), pp. 169–175. Lublin-Rzeszow (2013)
8. Dey, A: Construction of nested orthogonal arrays. Discrete Math. **310**(21), 2831–2834 (2010)
9. Graham, A.: Kronecker Product and Matrix Calculus With Applications. Ellis Horwood Limited, Chichester (1981)