A comprehensive survey on Cultural Algorithms

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Abstract

Cultural Algorithms (CAs) are evolutionary algorithms (EAs) inspired by the conceptual models of the human cultural evolution process. In contrast to the conventional EAs, which work only based on the population space, CAs employ an additional space, called belief space, to collect the information about the behaviour of individuals within the search space. Since the emergence of CAs, they have been extended to solve a wide variety of problems in different branches of science and technology. In this paper, a comprehensive survey on the recent advances in CAs is presented. Literature survey reveals some interesting challenges and future research directions.

Keywords

Cultural algorithms, evolutionary algorithms, belief space, cultural evolution.

Nomenclature

- ABC Artificial Bee Colony
- ACO Ant Colony Optimisation
- ANN Artificial Neural Network
- BA Bat Algorithm
- BB-BC Big Bang-Big Crunch
- BBO Biogeography-Based Optimisation
- BFA Bacterial Foraging Algorithm
- CAs Cultural Algorithms
- CBO Colliding Bodies Optimisation
- COA Coyote Optimisation Algorithm

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- CPSO Cooperative Particle Swarm Optimisation
- CS Cuckoo Search
- CSS Charged System Search
- DE Differential Evolution
- EAs Evolutionary algorithms
- EDP Economic Dispatch Problem
- EHO Elephant herding optimisation
- EP Evolutionary Programming
- FA Firefly Algorithm
- FECO Five-Elements Cycle Optimisation
- GAs Genetic Algorithms
- GP Genetic Programming
- GSA Gravitational Search Algorithm
- GSO Group Search Optimiser
- HICHA Heterogeneous Interactive Cultural Hybrid Algorithm
- HS Harmony Search
- IQPSO Improved Quantum-behaved Particle Swarm Optimisation
- IWO Invasive Weed Optimisation
- JSP Job Shop Problem
- KH Krill Herd
- LCA League Championship Algorithm
- MODE Multi-objective Differential Evolution
- NSGA-II Non-dominating Sorting Genetic Algorithm II
- OIO Optics Inspired Optimisation
- PSO Particle Swarm Optimisation
- QEA Quantum Evolution Algorithm
- rcGA Real-Coded Genetic Algorithm
- SA Simulated Annealing
- SFLA Shuffled Frog Leaping Algorithm
- SI Swarm Intelligence
- SOMA Self-Organising Migrating Algorithm
- TLBO Teaching-Learning-Based Optimisation
- TS Tabu Search

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1 Introduction

With rapid development of science and technology as well as emergence of complex real-world optimisation problems, developing efficient solution-finding approaches has become a research hotspot in the computer science. In most of the practical optimisation problems, due to the presence of highly non-linear objective and constraints functions with mixed types of continuous/discrete variables, finding acceptable solutions is not an easy task. Another source of difficulty stems from the fact that most of the real-world optimisation problems are black-box problems, in which the explicit forms of the objective and constraints functions as well as their derivatives are not available. These have motivated the development of meta-heuristic optimisation techniques to provide optimal/near-optimal solutions for these types of problems in a reasonable time.

Significant progress has been made over the past decades in mimicking the physical, ecological and social phenomena to develop meta-heuristic algorithms as efficient tools of finding acceptable solutions for complex real-world optimisation problems (Del Ser et al., 2019), such as GAs (Mühlenbein, 1997), DE (Storn, Price, 1997), PSO (Eberhart, Kennedy, 1995), HS (Geem et al., 2001), ACO (Dorigo, Di Caro, 1999), (CAs) (Reynolds, 1994), TLBO (Rao et al., 2011), ABC (Karaboga et al., 2014), BBO (Simon, 2008), BB-BC (Erol, Eksin, 2006), LCA (Kashan, 2009; Jalili et al., 2017), CSS (Kaveh, Talatahari, 2010), OIO (Jalili, Husseinzadeh Kashan, 2019), CBO (Kaveh, Mahdavi, 2014), GSO (He et al., 2009), GSA (Rashedi et al., 2009), and KH (Gandomi, Alavi, 2012) algorithm. In contrast to the conventional gradient-based optimisation techniques, the meta-heuristic techniques can be easily implemented to deal with the optimisation problems with different characteristics, such as non-linearity, non-separability, illconditioning, and multi-modality (Ali et al., 2018).

Meta-heuristic techniques can be categorised into different classes based on their

source of inspiration, such as EAs, SI algorithms, and Physics inspired algorithms. EAs are biologically inspired meta-heuristic algorithms based on the Darwinian theory of evolution, in which the survival capabilities of biological species are improved through the natural processes such as selection, reproduction, and survival of the fittest. For example, GAs, DE, and BBO belong to the EAs category. SIs mimic the collective intelligence exhibited by a group or swarm of species in the nature, such as birds, ants, fishes, etc. PSO, ABC algorithm, and ACO are examples of SI methods, which are developed based on the swarming behaviour of a flock of birds and colony of ants. As its name suggests, Physics inspired algorithms simulate the physical processes in the nature through their governing theories and equations to design a search mechanism. For example, CSS simulates the Newtonian laws of classical mechanics and Coulomb law between the charged particles to perform the search process for global optima, OIO method imitates the law of reflection and the optical behaviour of convex and concave mirrors to design a search mechanism for numerical optimisation, GSA performs based on the simulation of the law of gravity and the interactions between the masses, CBO mimics the one dimensional collision process of bodies based on the laws of energy and momentum. The overall framework of the above mentioned meta-heuristic approaches is more or less the same and they usually are referred to as the population-based metaheuristics, in which a population of individual solutions are gradually updated within the search process. Some of the meta-heuristic algorithms, such as SA (Van Laarhoven, Aarts, 1987) and TS (Glover, Laguna, 1998), perform based on the updating of a single solution, which are referred to as single-point or trajectory search methods.

The idea of using multiple sources of knowledge during the search process is emerged with the development of CAs. CAs developed by Reynolds (1994) can be categorised as an EA and is based on the conceptual models of the human cultural evolution process. In contrast to the conventional EAs, which work only based on the population space, CAs employ an additional space, called belief space, to collect the information about the behaviour of individuals in the search space (Reynolds, 2018a,b; Chung, Reynolds, 1996a; Reynolds, Kinnaird-Heether, 2017a; Reynolds, Maletic, 1994; Saleem, Reynolds, 2000; Reynolds et al., 2008c; Reynolds, 1999). CAs model a dual inheritance system observed from the human culture evolution, in which the belief space represents the macro-evolutionary level and the population space performs the micro-evolutionary level. The belief space includes a network of knowledge categories obtained from the evolution process of individuals in the population space, in which each knowledge category indicates a collection of problem-specific knowledge components. The belief and population spaces can interact with each other during the evolution process. Experience exchange between these two spaces is performed based on the communication protocols. Since the emergence of fundamental CAs in 1979 (Reynolds, 1979), they have been successfully extended and widely employed to solve a wide variety of problems in different brunches of science and technology. Hence, a literature review of CAs seems to be crucial to provide a clear picture of their development trends and to highlight challenges and future research directions. The survey reported in this paper covers recent advances in CAs, their general framework, different variants, hybridised versions with other meta-heuristic and search techniques, and their applications in science and engineering.

The general structure of this paper is organised as follows. Section 2 provides the general framework and basic concept of CAs and their prominent variants and formulations. The literature review of CAs in relation to their applications in different

branches of science and engineering is provided in Section 3. The scientific literature with respect to the hybridisation, multi-objective, multi-population, chaotic, fuzzy, and other variants of CAs are reviewed in Section 4. A brief publication analysis is discussed in Section 5. Finally, the concluding remarks and discussion on the future research in the field are presented in Section 6.

2 Cultural Algorithms (CAs)

According to Richerson, Boyd (2005), culture can be defined as a source of information with potential capability of affecting members of a population, which are exchanged between the individuals. This information can be any kind of mental state, such as the beliefs, art, and other things, which can be transmitted to others by means of teaching, imitation, and other forms of social transmission (Richerson, Boyd, 2005). Based on the biocultural evolution theory (Laland, 2008), genes and culture can be viewed as two interacting forms of inheritance. This means that the human behaviour is a product of two different and interacting evolutionary processes, including genetic evolution and cultural evolution. The variations in genes can lead to variations in the culture which can then influence genetic selection, and vice versa *. Reynolds (1979) inspired this concept to model new meta-heuristic optimisation algorithms, called CAs, based on the human cultural evolution process.

Conventional EAs have been indicated to be effective in solving different and complicated optimisation problems. However, the individual solutions exchange the limited amount of knowledge related to the problem between each other and the search process utilised by the standard EAs is unbiased (Engelbrecht, 2007). While the acquired knowledge during the evolution process can be stored and collectively used to perform search more efficiently (Reynolds, 2018a). CAs try to use a set of networks of knowledge sources gained during the evolution process to influence the search process. Fig. 1 shows the general framework and main components of CAs. In contrast to the conventional EAs, which are consisted of a population space, CAs work on two spaces, including population and belief spaces. In CAs, the learning process is performed in both of these spaces simultaneously. The interaction between the population and belief spaces is provided through the communication protocol, i.e., acceptance and influence functions. Like other population-based EAs, the population space of CAs is a genetic component containing individual solutions which are represented on a genotypic and/or phenotypic level. While the belief space represents the cultural information gained during the evolution process.

The general pseudo-code of CAs can be summarised as Algorithm 1. Like other meta-heuristic algorithms, the algorithm starts with a randomly generated population space represented by PS(t). In addition, the general structure of the belief space indicated by BS(t) is initialised by appropriate values. Then, the fitness functions of individuals in PS(t) are evaluated using Obj() function. A given number of individuals with better fitness functions are selected by Accept() function from the PS(t) to update the belief space BS(t) using Update() function. Finally, the function Influence() is used to produce new generation of individuals PS(t + 1). The main components of CAs are explained in the following subsections.

^{*}https://en.wikipedia.org/wiki/Dual_inheritance_theory



Figure 1: Overall framework of CA.

Algorithm 1: Pseudo-code of CAs.				
Set $t = 0$				
Initialise $PS(t)$ and $BS(t)$				
while the stopping criterion is met do				
t = t + 1				
Evaluate the fitness of individuals in $PS(t)$ using $Obj()$				
Accept some elite individuals from the $PS(t)$ using $Accept()$				
Update $B(t)$ using using $Update()$				
Produce new generation $PS(t + 1)$ using $Influence()$				
end				

2.1 Belief Space

During the evolution process, the population space has a set of cultural information which are changing from generation to generation. In order to implement a learning mechanism, the belief space of CAs plays a role of cultural information repository, and stores the knowledge and collective behaviour acquired by the individuals. The primary version of CA was consisted of only one knowledge source (i.e., situational knowledge) (Reynolds, 1979) and it has been complemented by adding four additional knowledge sources, including normative, domain, history, and topographical knowledge sources. These knowledge and cultural information will be used as auxiliary tools to influence the future behaviour of the individuals and move them toward the global optima in the search space. The belief space forms a network of knowledge sources to provide an effective learning capability at different levels of granularity, which are gradually updated. The types of the applied knowledge sources within the belief space depend on the problem type. Literature review reveals that researchers have been used different knowledge sources within the belief space to solve different problems. Table 1

lists the types of knowledge sources implemented by different studies within the belief space. As it is observed from Table 1, the normative and situational knowledge sources are popular knowledge components in literature.

Table 1: The knowledge sources of CA employed by different references in the literature.

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Wu et al. (2010) \checkmark \checkmark Stapley et al. (2019) \checkmark \checkmark	Ali et al. (2016b)	\checkmark	\checkmark		\checkmark	\checkmark
Stapley et al. (2019) \checkmark \checkmark \checkmark	Wu et al. (2010)	\checkmark	\checkmark			
	Stanley et al. (2019)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Awad et al. (2013) \checkmark \checkmark \checkmark \checkmark \checkmark	Awad et al. (2013)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Abdolrazzagh-Nezhad \checkmark \checkmark	Abdolrazzagh-Nezhad	\checkmark	\checkmark			
(2020)	(2020)					
Lin et al. (2009)	Lin et al. (2009)		\checkmark			
Oloruntoba et al. (2019) \checkmark \checkmark	Oloruntoba et al. (2019)	\checkmark	\checkmark			
Sun et al. (2009) \checkmark \checkmark	Sun et al. (2009)	\checkmark	\checkmark			
Mohammadhosseini et al. 🗸 🗸	Mohammadhosseini et al.	\checkmark	\checkmark			
(2019)	(2019)					
Coello Coello, Becerra (2004) \checkmark	Coello Coello, Becerra (2004)	\checkmark	\checkmark			
Chung, Reynolds (1996b) \checkmark \checkmark	Chung, Reynolds (1996b)	\checkmark	\checkmark			
Santos Coelho dos et al. \checkmark \checkmark	Santos Coelho dos et al.	\checkmark	\checkmark			
(2009)	(2009)					
De Freitas et al. (2018) \checkmark \checkmark	De Freitas et al. (2018)	\checkmark	\checkmark			\checkmark
Khan et al. (2014) \checkmark \checkmark	Khan et al. (2014)	\checkmark	\checkmark			
Khan et al. (2017) \checkmark \checkmark	Khan et al. (2017)	\checkmark	\checkmark			
Guo et al. $(2013b)$ \checkmark \checkmark	Guo et al. $(2013b)$	\checkmark	\checkmark			
Guo et al. (2013a) \checkmark	Guo et al. (2013a)	\checkmark	\checkmark			
Gao, Diao (2011)	Gao, Diao (2011)	\checkmark	\checkmark			
Gao, Xu (2013)	Gao, Xu (2013)		\checkmark			
Digalakis, Margaritis (2002) 🗸 🗸	Digalakis, Margaritis (2002)	\checkmark	\checkmark			
Gao et al. (2017a) \checkmark	Gao et al. (2017a)	\checkmark	\checkmark			
Khodabakhshian, Hemmati 🗸 🗸	Khodabakhshian, Hemmati	\checkmark	\checkmark			
(2013)	(2013)					

Reference	Situational	Normative	Domain	History	Topographical
Bhattacharya et al. (2012b)	\checkmark	\checkmark			
Lu et al. (2011)	\checkmark			\checkmark	
Yuan et al. (2009)	\checkmark	\checkmark			
Yuan et al. (2008)	\checkmark	\checkmark			
Cai et al. (2018)	\checkmark	\checkmark			\checkmark
Santos Coelho dos, Mariani		\checkmark			
(2006)					
Xie et al. (2009)	\checkmark	\checkmark			
Ali et al. (2014a)	\checkmark	\checkmark	\checkmark		\checkmark
Zhou et al. (2019)	\checkmark	\checkmark			
Yan et al. (2017b)		\checkmark			
Wang et al. (2017)	\checkmark	\checkmark	\checkmark		
Pan et al. (2010)		\checkmark			
Chung, Reynolds (1998)	\checkmark	\checkmark			
Becerra, Coello (2004)	\checkmark	\checkmark		\checkmark	\checkmark
Pierezan et al. (2019)		\checkmark			
Shah, Kobti (2020)	\checkmark	\checkmark			
Mao et al. (2020)	\checkmark	\checkmark			\checkmark
Goli et al. (2019)	\checkmark	\checkmark			
Mojab et al. (2019)	\checkmark	\checkmark	\checkmark		
Murugadass, Sivakumar	\checkmark	\checkmark			
(2020)					
Xue (2020)	\checkmark	\checkmark			
Muhamediyeva (2020)	\checkmark	\checkmark			
Amalo et al. (2020)		\checkmark			
Selvarajah et al. (2020)		\checkmark			\checkmark
Xue et al. (2011)	\checkmark	\checkmark			
Guo et al. (2011b)		\checkmark			\checkmark
Gao et al. (2006a)		\checkmark			
Gao, Diao (2010)	\checkmark	\checkmark			
Gao et al. (2007)	\checkmark				
Nguyen, Yao (2006)	\checkmark	\checkmark			
Ali et al. (2011a)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Guo et al. (2018)	\checkmark	\checkmark			\checkmark
Kim, Cho (2009)	\checkmark	\checkmark		\checkmark	\checkmark
Guo et al. (2011c)	\checkmark	\checkmark			
Reynolds, Zhu (2001)	\checkmark	\checkmark			
He, Xu (2011)	\checkmark	\checkmark			
Sun et al. (2010)	\checkmark	\checkmark			
Awad et al. (2017)	\checkmark	\checkmark	\checkmark		\checkmark
Ali et al. (2016a)	\checkmark	\checkmark			\checkmark
Ali, Awad (2014)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Ali et al. (2012)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Cortés Rivera et al. (2007)	\checkmark				
Zhang, Zhu (2012)	\checkmark	\checkmark			
Yang, Gu (2014)	\checkmark	\checkmark			\checkmark
Becerra, Coello (2005)	\checkmark				
Alami et al. (2007)		\checkmark			
Lagos et al. (2014)	\checkmark	\checkmark		\checkmark	
Cabrera et al. (2011)	\checkmark			\checkmark	
Srinivasan, Ramakrishnan	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
(2012)					
Crawford et al. (2013)	\checkmark			\checkmark	
Crawford et al. (2007)	\checkmark				
Soza et al. (2011)	\checkmark	\checkmark	\checkmark		
Soza et al. (2007)	\checkmark	\checkmark	\checkmark		
Zadeh, Kobti (2015)	\checkmark	\checkmark			
, , , , , , , , , , , , , , , , , , , ,					

Table 1 – *Continued from previous page*

Table 1 – Continued from previous page

Reference	Situational	Normative	Domain	History	Topographical
Chen, Yang (2015)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Terán et al. (2017)	\checkmark	\checkmark			
Santos Coelho dos, Alotto		\checkmark			
(2009)					
Xue, Guo (2007)	\checkmark	\checkmark		\checkmark	\checkmark
Ali et al. (2014b)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Ali et al. (2011b)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Best et al. (2010)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Reynolds, Ali (2008b)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Reynolds et al. (2008c)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Reynolds, Peng (2005)	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
	Са	ontinued on n	ext page		

The belief space can be mathematically expressed as follows:

$$\boldsymbol{\mathcal{B}}^{t} = \{\boldsymbol{\mathcal{S}}^{t}, \boldsymbol{\mathcal{N}}^{t}, \boldsymbol{\mathcal{C}}^{t}, \boldsymbol{\mathcal{D}}^{t}, \boldsymbol{\mathcal{H}}^{t}\}$$
(1)

where \mathcal{B}^t represents the belief space, \mathcal{S}^t is situational knowledge source, \mathcal{N}^t indicates the normative knowledge source, \mathcal{C}^t is the topographical knowledge source, \mathcal{D}^t is the domain knowledge source, and \mathcal{H}^t represents the history knowledge source. The superscript *t* indicates the iteration number. In the following subsections, each of the knowledge sources are explained in detail.

2.1.1 Situational knowledge

The *situational knowledge* source represented by $S^t = [s_1, s_2, ..., s_n]$ stores the best experience of the individuals in each generation of the population space. Situational knowledge component is updated by Update() function as follows:

$$\boldsymbol{\mathcal{S}}^{t+1} = \begin{cases} \boldsymbol{X}_l^t & \text{if } f(\boldsymbol{X}_l^t) < f(\boldsymbol{\mathcal{S}}^t) \\ \boldsymbol{\mathcal{S}}^t & \text{otherwise} \end{cases}, \quad l = 1, 2, ..., n_{accept} \end{cases}$$
(2)

where \mathbf{X}_{l}^{t} is the *l*th accepted individual at iteration *t* and *n*_{accept} represents the number of accepted elite individuals to update the belief space.

2.1.2 Normative knowledge

The *normative knowledge* source, introduced by Chung, Reynolds (1998), indicates the standards to behaviour of the individuals in each dimension of the problem. This includes a set of intervals representing high quality or socially acceptable individuals, which specify appropriate search ranges in each dimension of the search space. These intervals or norms provide the guidelines for individuals in the population space to follow. Following the cultural norms plays an important role in human social intelligence (Clayton et al., 2000).

In CAs, the normative knowledge stores a set of information for each dimension of the problem, which can be mathematically expressed as follows:

$$\boldsymbol{\mathcal{N}}^{t} = \begin{bmatrix} I_{1}^{t} & I_{2}^{t} & \dots & I_{n}^{t} \\ L_{1}^{t} & L_{2}^{t} & \dots & L_{n}^{t} \\ U_{1}^{t} & U_{2}^{t} & \dots & U_{n}^{t} \end{bmatrix}$$
(3)

where \mathcal{N}^t represents the normative knowledge source at iteration t, $I_j^t = [x_{\min,j}^t, x_{\max,j}^t]$ indicates the belief interval of the *j*th dimension of the problem, $x_{\min,j}^t$ and $x_{\max,j}^t$ represent the lower and upper normative bounds of the *j*th dimension of the problem, respectively, L_j^t and U_j^t are the fitness function values for the lower and upper normative bounds, respectively.

To update the normative knowledge source, CA uses an conservative approach to prevent excessive narrow belief intervals. The components of the normative knowledge source are updated by Update() function in each generation as follows:

$$x_{\min,j}^{t+1} = \begin{cases} x_{l,j}^t & \text{if } x_{l,j}^t \le x_{\min,j}^t & \text{or } f(\mathbf{X}_l^t) < L_j^t \\ x_{\min,j}^t & \text{otherwise} \end{cases}$$
(4)

$$x_{max,j}^{t+1} = \begin{cases} x_{l,j}^t & \text{if } x_{l,j}^t \ge x_{max,j}^t & \text{or } f(\mathbf{X}_l^t) < U_j^t \\ x_{max,j}^t & \text{otherwise} \end{cases}$$
(5)

$$L_j^{t+1} = \begin{cases} f(\mathbf{X}_l^t) & \text{if } x_{l,j}^t \le x_{min,j}^t & \text{or } f(\mathbf{X}_l^t) < L_j^t \\ L_j^t & \text{otherwise} \end{cases}$$
(6)

$$U_j^{t+1} = \begin{cases} f(\mathbf{X}_l^t) & \text{if } x_{l,j}^t \ge x_{max,j}^t & \text{or } f(\mathbf{X}_l^t) < U_j^t \\ U_j^t & \text{otherwise} \end{cases}$$
(7)

In the above equations, $l = 1, 2, ..., n_{accept}$ and \mathbf{X}_l^t represents the *l*th accepted individual at iteration *t* and $x_{l,j}^t$ indicates its *j*th variable.

2.1.3 Topographical knowledge

The *topographical knowledge* was introduced by Jin, Reynolds (1999a,b) as *region-based schemata* within the CAs, which is an multi-dimensional grid illustration of the solution space. The topographical knowledge source assumes the search space as a multi-dimensional grid or array composed of a set of cells in mesh expressed by $C^t = [c_1, c_2, ..., c_k]$, in which each element c_i represents the corresponding records for each cell and k is the number of the belief cells in the topographical knowledge source. Each cell c_i in topographical knowledge stores a set of information, such as information about the feasibility and frequency of cell occupation. For example, Jin, Reynolds (1999a) represented each cell of the topographical knowledge source as follows:

$$\mathbf{c}_i = [Class_i, Cnt1_i, Cnt2_i, W_i, \mathbf{Pos}_i, \mathbf{Csize}_i]$$
(8)

In Equation (8), $Class_i$ represents the information about the feasibility of cell *i*, including feasible, infeasible, semi-feasible or unknown; $Cnt1_i$ and $Cnt2_i$ indicate the number of feasible and infeasible individuals in cell *i*, respectively; W_i is the weight of

cell *i*, which indicates the quality of the cell; **Pos**_{*i*} represents the position of cell in the search space; **Csize**_{*i*} denotes the size of the cell for all dimensions of the problem. It is worth mentioning that the type of the information stored in each cell of the topographical knowledge source depends on type of the optimisation problem. Broadly speaking, this knowledge source can be very useful to extract useful information about the search space and guide the search process. One of the crucial points in using topographical knowledge source during the solution finding process is the computational efficiency. Updating the cells of the topographical knowledge source can be cell sizes are small and the number of cells is huge. To remedy this inefficiency, *k*-*d* ternary tree-based representations were used to describe the cells within the topographical knowledge source (Ali et al., 2016b; Peng, Reynolds, 2004; Ali et al., 2015).

2.1.4 Domain knowledge

The *domain knowledge* source \mathcal{D}^t was first introduced by Reynolds, Saleem (2005) into CAs and is similar to the situational knowledge source. The domain knowledge source stores the positions of the best solutions obtained through different generations and forms an archive of the best individuals since the evolution started (Engelbrecht, 2007).

2.1.5 History knowledge

The *history (or temporal) knowledge* source \mathcal{H}^t was proposed by Saleem to deal with the problems in which the search landscapes change during the evolution. This knowledge source maintains the shifts happened within the search regions during the solution finding process. The history knowledge stores the current best solution (i.e, the best individual before the last shift in the search regions), the directional change in the search region, and the distance change for each dimension of the problem. The data structure for the history knowledge is shown in Fig. 2. In Fig. 2, \mathbf{e}_i represents the best current solution obtained before the last shift in the search regions, the list ($A_{dir,1}, A_{dir,2}, ..., A_{dir,n}$) indicates the directional change in the search region, and $A_{dire,2}, ..., A_{dir,n}$) represents the distance change, and n is the number of variables of the problem.



Figure 2: History knowledge structure (Ali et al., 2016b).

2.2 Communication Protocol

As mentioned earlier, the connection between the population and belief spaces is provided through communication protocols. CAs employ two functions as the communication protocols, including *acceptance*, *update*, and *influence* functions. The acceptance function determines which individuals in the population can affect the belief space, while the effect of the knowledge sources on the population space is determined by the influence function. These two functions are discussed in following subsections.

2.2.1 Acceptance function

In order to update the knowledge sources of the belief space, a set of individuals demonstrating overall behaviour of the population space should be selected. Generally speaking, the acceptance functions used in literature can be classified into static and dynamic categories (Engelbrecht, 2007). In static acceptance functions, the number of accepted individuals is constant during the evolution process. It is very common to select a given percentage of the individuals, ranging from 1% to 100%, to update the belief space depending on the performance of the algorithm in a given application (Chung, Reynolds, 1998). In the dynamic acceptance functions, the number of individuals selected to update the beliefs is not constant and varies from generation to generation. For example, Reynolds, Chung (1997a) proposed a fuzzy acceptance function to select the individuals to shape the beliefs. As another example, Ali et al. (2016b) used an dynamic acceptance function with reset mechanism, in which the number of accepted individuals decreases as generation counter increases.

2.2.2 Influence function

In a real human society, the individuals follow the global beliefs, which are acquired by previous generations and translated to the current generation. In order to model this phenomenon, CAs use the influence functions to affect the population space and create a new generation of individuals. The types of the influence functions depends on the problem type and utilised knowledge sources. When multiple types of knowledge sources are used, the main challenge in designing influence functions is to determine the extent of the impact of every single one of them on the population space. Reynolds, Chung (1997b) proposed four influence functions based on the situational and normative knowledge sources as follows:

- The first influence function uses only the normative knowledge source to determine the step size during the search process:

$$x_{i,j}^{t+1} = x_{i,j}^t + size(I_j^t)N_{i,j}(0,1) \quad j = 1, 2, ..., n$$
(9)

where $x_{i,j}^{t+1}$ indicates the *j*th element of the *i*th individual at iteration t + 1, $size(I_j^t) = x_{max,j}^t - x_{min,j}^t$ represents the size of the normative interval for the *j*th variable, and $N_{i,j}(0,1)$ is the random number generated by normal distribution with mean value of 0 and standard deviation of 1.

- The second influence function determines the search direction based on the situational knowledge source:

$$x_{i,j}^{t+1} = \begin{cases} x_{i,j}^t + |\sigma_{i,j}^t N_{i,j}(0,1)| & \text{if } x_{i,j}^t < s_j^t \\ x_{i,j}^t - |\sigma_{i,j}^t N_{i,j}(0,1)| & \text{if } x_{i,j}^t > s_j^t \\ x_{i,j}^t + \sigma_{i,j}^t N_{i,j}(0,1) & \text{Otherwise} \end{cases}$$
(10)

where s_j^t is the *j*th element of the situational knowledge at iteration *t*, and σ_{ij}^t represents the strategy parameter of the *j*th variable of *i*th individual.

- The third influence function uses the situational knowledge to determine the search direction, while the step size is determined by based on the normative knowl-

edge component. This influence function is similar to Equation 10, which can be stated as follows:

$$x_{i,j}^{t+1} = \begin{cases} x_{i,j}^t + |size(I_j^t)N_{i,j}(0,1)| & \text{if } x_{i,j}^t < s_j^t \\ x_{i,j}^t - |size(I_j^t)N_{i,j}(0,1)| & \text{if } x_{i,j}^t > s_j^t \\ x_{i,j}^t + size(I_j^t)N_{i,j}(0,1) & \text{Otherwise} \end{cases}$$
(11)

- The fourth influence function determines both of the search direction and step size based on the normative knowledge source as follows:

$$x_{i,j}^{t+1} = \begin{cases} x_{i,j}^t + |size(I_j^t)N_{i,j}(0,1)| & \text{if } x_{i,j}^t < x_{min,j}^t \\ x_{i,j}^t - |size(I_j^t)N_{i,j}(0,1)| & \text{if } x_{i,j}^t > x_{max,j}^t \\ x_{i,j}^t + \beta size(I_j^t)N_{i,j}(0,1) & \text{Otherwise} \end{cases}$$
(12)

where $\beta > 0$ is a constant parameter.

2.3 Algorithmic steps of CAs

Now, let us consider a minimisation problem of function $f(\mathbf{X} = [x_1, x_2, ..., x_n])$, in which *n* is the number of variables. The algorithmic steps of CAs for this problem can be summarised as follows:

Step 1: Initialisation of population space

In the first step, the population space PS(t) of CA is initialised within the search space as follows:

$$\mathbf{X}_{i,j}^{0} = \phi(l_j, u_j), \quad i = 1, 2, ..., N, \quad j = 1, 2, ..., n$$
(13)

where $\mathbf{X}_{i,j}^0$ indicates the *j*th element of *i*th individual in the population space, l_j and u_j are the lower and upper bounds for the variable *j*, $\phi(.)$ represents the uniform random function, and *N* is the population size.

Step 2: Initialisation of belief space

In the second step, the iteration number *t* is set to 0 and the initial belief space $\mathcal{B}^0 = \{\mathcal{S}^0, \mathcal{N}^0, \mathcal{C}^0, \mathcal{D}^0, \mathcal{H}^0\}$ is initialised with appropriate values. For example, the normative knowledge source is initialised by assuming $x_{min}^0 = -\infty$, $x_{max}^0 = \infty$, $L_j^0 = \infty$, and $U_j^0 = \infty$ as follows:

$$\boldsymbol{\mathcal{N}}^{0} = \begin{bmatrix} \infty & \infty & \dots & \infty \\ \infty & \infty & \dots & \infty \\ \infty & \infty & \dots & \infty \end{bmatrix}$$
(14)

Step 3: Fitness evaluation

In this step, the fitness functions of the individuals in the population space are evaluated.

Step 4: Update the belief space

In the fourth step, the iteration number is set to t = t+1 and the knowledge sources of the belief space are updated by the information provided by the individuals of population space. For example, Equation (2) is used to update the situational knowledge source and Equations (4-7) are used to update the normative knowledge source.

Step 5: Influence the population space

Then, the information within the knowledge sources of the belief space are used to produce new generation of individuals PS(t + 1). For instance, the new generation of individuals can be determined based on the Equations (9-12).

Step 6: Stop or redo

Repeat steps 3-5 until the stopping criterion is reached and the optimal solution is yielded.

3 Applications of CAs

In recent decades, CAs have been applied to solve a wide variety of problems in science and engineering, such as civil engineering, mechanical engineering, chemical engineering, electrical engineering, and computer science. The numerical results in literature revealed the capabilities of CAs in solving different types of problems. Due to multi-disciplinary nature of the investigated problems in literature, it is quite tricky to categorise them into separate fields. However, this study tries to categorise them into mentioned fields. Table 2 lists the prominent applications of CAs in different areas. In the following subsections, a exhaustive summary of the applications of CAs to each of the mentioned areas will be provided.

Application area	Authors
Civil engineering	
Transportation engineering	Reves et al. (2010), Liu et al. (2012), Arpaia et al. (2007),
1 0 0	Maldonado et al. (2013), Xue (2020)
Structural engineering	Jalili, Hosseinzadeh (2015), Jalili et al. (2019), Jafari et al.
0 0	(2019), Ali et al. (2018)
Environmental engineering	Liu, Lin (2015)
Urban engineering	Jayyousi, Reynolds (2014)
Water engineering	Yan et al. (2017a), Gu, Wu (2010)
Mechanical engineering	
Damage detection	Guedria, Hassine (2018)
Optimum engineering design	Ali, Reynolds (2014), King et al. (2008), Santos Coelho dos,
	Mariani (2006), Reynolds, Ali (2008b), Reynolds, Peng
	(2005)
Optimal vibration control	Wang et al. (2015)
Vehicle safety	Kobti et al. (2006b)
Flight control	Nieng et al. (2011)
Gas turbine operation	Pierezan et al. (2019)
Renewable energy	Amalo et al. (2020)
Chemical engineering	
Process optimisation	Haikal, El-Hosseni (2011)
Ammonia synthesis	Xu et al. (2010), Xu et al. (2012)
J	

Table 2: Applications of CAs and their variants in different fields.

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Application area	2 – Continued from previous page
Molocular crustallography	Tromauno et al. (2000)
Diesel blonding	$C_{20} \text{ of al} (2019)$
Dieser biending	Gao et al. (2019)
Electrical engineering	
Design of high temperature su-	Chen et al. (2013)
perconducting magnets	
Economic dispatch	Goudarzi et al. (2016), Zhang et al. (2013), Bhattacharya
1	et al. (2012a), Santos Coelho dos et al. (2009), De Freitas
	et al. (2018), Ya-Li, Li-ging (2010), Goudarzi et al. (2017)
Network reconfiguration	Haldar Chakraborty (2014) Haldar Chakraborty (2015)
Sensor fault detection and cor-	Khan et al. (2014) Khan et al. (2017)
rection	Khart et al. (2011), Khart et al. (2017)
Wireless sensor networks	Cuo et al. (2013b). Cuo et al. (2013a). Kulkarni. Desai
Whereas sensor networks	(2020) Murugadass Siyakumar (2020)
Signal processing	(2020), with ugadass, Sivakumar (2020) Cap Diag (2010) Cap Diag (2011) Cap Vii (2012)
Maintananaa ashadulina	Gd0, Did0 (2010), Gd0, Did0 (2011), Gd0, Au (2015) Disclabia Managritia (2002)
Maintenance scheduling	Digalakis, Margaritis (2002)
Digital watermarking	Gao et al. (2017a)
Power system stabilizer design	Knodabakhshian, Hemmati (2013), Tavakoli et al. (2014)
Reactive power optimisation	Bhattacharya et al. (2012b)
Optimal operation of cascade	Wang et al. (2009), Xie et al. (2009)
hydropower station	
Optimal design of passive	Guo et al. (2008)
power Filters	
Hydrothermal scheduling	Yuan, Yuan (2006), Lu et al. (2011), Yuan et al. (2009)
Hydro production scheduling	Yuan et al. (2008)
Multi-robot tasks allocation	Yu et al. (2011)
Robot soccer players	Reynolds, Chung (1997c), Ali et al. (2014b)
Sea-surface weak targets detec-	Cai et al. (2018)
tion	
Image matching	Yan et al. (2017b)
Image detection	Wang et al. (2017)
Fault detection	Pan et al. (2010)
PID parameter optimisation	Xiao et al. (2010)
Frequency assignment	Alami El Imrani (2008)
Gesture recognition	Waris, Reynolds (2015)
Computer science	
Function optimisation	Chung, Reynolds (1998), Kobti, others (2014), Kobti, oth-
	ers (2013), Becerra, Coello (2004), Coello, Becerra (2002b),
	Xue et al. (2011), Guo et al. (2011b), Gao et al. (2006a),
	Nguyen, Yao (2008), Gao et al. (2006b), Gao et al. (2007),
	Nguyen, Yao (2006), Li et al. (2010), Ali et al. (2011a),
	Guo et al. (2018), Guo et al. (2011c), Reynolds, Zhu (2001),
	He, Xu (2011), Sun et al. (2010), Sun et al. (2012), Dixit
	et al. (2019), Awad et al. (2017), Ali et al. (2016a), Ali,
	Awad (2014). Ali et al. (2012). Xue, Guo (2007). Coello,
	Becerra (2002a). Ali et al. (2016b). Alami et al. (2007).
	Huang et al. (2008) Wu et al. (2010) Stapley et al. (2009)
	Tang $Ii (2008)$ Awad et al. (2013) Coello Recerra (2003)
	Zhou et al. (2019) Reynolds Kinnaird-Heather (2017a)
	Revnolds Kinnaird-Heether (2017b) Ali et al (2014a)
	Reynolds, Killianu-Heenief (2017), All et al. (2014a), Bormolds, Carrasmah (2012), All $\rightarrow 1$, (2014b), All
	Reynolds, Gawasmen (2012), Ali et al. (2011b), Ali,
	Keynolas (2009), Keynolas, Ali (2008c), Chung, Keynolas
	(1996b), Mao et al. (2020), Murugadass, Sivakumar (2020), Sirah at al. (2018) Muhamadi (2020)
	Singn et al. (2018), Muhamediyeva (2020)
	Continued on next page

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Application area	Authors
Job shop problem	Cortés Rivera et al. (2007), Zhang, Zhu (2012), Kobti, oth-
	ers (2012), Yang, Gu (2014), Becerra, Coello (2005), Niu
	et al. (2011), Ho, Tay (2004)
Facility location	Lagos et al. (2014), Cabrera et al. (2011)
Classification	Srinivasan, Ramakrishnan (2012), Yan et al. (2016), Srini-
	vasan, Ramakrishnan (2013b), Srinivasan, Ramakrishnan
	(2013a), Srinivasan, Muruganandam (2020)
Set covering	Crawford et al. (2013). Crawford et al. (2007)
Expert systems	Sternberg Reynolds (1997)
Semantic network	Rychtyckyj Reynolds (2005) Rychtyckyj Reynolds
Semantic network	(2000) Rychtyckyj, Reynolds (2003), Rychtyckyj, Reynolds
	(2002) Rychtyckyj, Reynolds (2001) Rychtyckyj, Reynolds
	(2002), Kychtyckyj, Reyholds (2001), Kychtyckyj, Reyholds (1002)
Droomen un dersten din e	(1990) Dermolde Grandlik (1905)
Trogram understanding	Reynolds, Sveralik (1995)
limetabling	Soza et al. (2011), Soza et al. (2007)
Mobile agent routing	Jun et al. (2008)
Ancient landscape exploration	Vitale et al. (2011b)
Web search	Zhang (2012)
Social networks	Zadeh, Kobti (2015), Selvarajaha et al. (2019), Selvarajah
	et al. (2020)
Fake news detection	Shah, Kobti (2020)
Intelligent logistics	Ochoa et al. (2010)
Site location decision-making	Reynolds, Nazzal (1997)
Symbolic regression	Zannoni, Reynolds (1997)
Neuro-fuzzy systems	Chen, Yang (2015), Chen et al. (2008)
Multi-aircraft cooperative target	Li et al. (2012)
allocation	
Travelling salesman problem	Unold, Tarnawski (2016), Unold, Tarnawski (2017), Kim,
F	Cho (2009)
Partially observable Markov de-	Prestwich et al. (2008)
cision process	
Support vector regression	Cheng et al. (2009)
Data mining	Abdolrazzagh-Nezhad (2020) Jin Reynolds (2000)
Data mining	Rounolds Al Shahri (1998) Rounolds Al Shahri (1997)
	Deng et al. (2020)
Class descention of	$M_{\text{risk}} = t_{\text{risk}} (2010)$
Cloud computing	Mojad et al. (2019)
rog computing	Hossenhourt et al. (2020)
Neural fuzzy network design	Lin et al. (2009)
reature selection	Oloruntoba et al. (2019)
Dairy product demand	Goli et al. (2019)
Online marketing	Selvarajah et al. (2019)
Quality of service	Sun et al. (2009)
Virtual machine placement	Mohammadhosseini et al. (2019)
Evolutionary programming	Coello Coello, Becerra (2004), Ostrowski, Reynolds (2003),
	Cowan, Reynolds (1999), Chung, Reynolds (1996b)
Games	Vitale et al. (2011a), Fogarty et al. (2011), Reynolds,
	Kinnaird-Heether (2003)
Software testing	Ostrowski, Reynolds (1999)
Image segmentation	Reynolds, Rolnick (1995); Reynolds, Rolnick (1995)
Boolean circuits	Revnolds, Sverdlik (1993)
City security	Ochoa et al. (2012)
<i>.</i> .	
Others	
Officia	
Anthropology	Kobti et al. (2006a)
Anthropology Economic recession	Kobti et al. (2006a) Ostrowski, Reynolds (2005)

Table 2 – Continued from previous page			
Application area	Authors		
Archaeology	Reynolds et al. (2008a)		
Biological science	Alexiou, Vlamos (2012), Judeh et al. (2014), Judeh et al. (2013)		
Industrial engineering Physics	Terán et al. (2017), Rychtyckyj et al. (2003) Santos Coelho dos, Alotto (2009), Pan et al. (2016)		

3.1 Civil engineering

CAs have found some applications in civil engineering areas. The application studies of CAs in civil engineering area can be categorised into five sub-disciplines as follows: i) transportation engineering, ii) structural engineering, iii) environmental engineering, iv) urban engineering, and v) water engineering.

In the transportation area, Reves et al. (2010) employed CA to find the optimal route between two points in the urban public transportation of the Leon city located in Mexico, in which the population has been significantly increased and caused the inhabitants to waste their time on travelling with public transportation. Authors investigated a given part of the city with a set of buses and stations as the case study. Simulation results indicated that CA is able to generate better transportation paths than handbook estimates done by experts. In another study, Liu et al. (2012) applied CA to find shortest path between two points in vehicle routing problem, in which the weather, road quality, traffic, and other uncertain factors were considered. The numerical results obtained from an urban road traffic network showed the capability of CA in finding shortest path. Arpaia et al. (2007) employed CA with DE operators for industrial fault isolation. Experiments were done on industrial problems, such as motor-vehicle fleet remote monitoring and automatic fault isolation of vehicle wear, operating danger, and fraud in a company that transports dangerous goods. Maldonado et al. (2013) developed an android software based on CA and Data Mining to determine the probability of occurring a traffic accident in a selected part of the city. This software can prevent the unnecessary congestion within the cities and reduce the number of traffic accidents in the long term. Xue (2020) employed a CA-based CS algorithm for time-dependent vehicle routing problem with stochastic customers, in which the traffic speed is estimated by an adaptive fractional Kalman filter. Authors investigated a real case study with different business districts in Xiamen, China. Although the literature review revealed the above-mentioned practical applications of CAs in transportation engineering, the performance of them have not yet tested on other types of optimisation problems in traffic engineering, such as signal optimisation (Gao et al., 2017b; Nallaperuma et al., 2020; Liang et al., 2020), battery charging optimisation of electrical vehicles (Karakatič, 2020), urban network design (Gallo et al., 2010), train scheduling (Samà et al., 2015), and transport energy modeling (Ceylan et al., 2008).

In the field of structural engineering, Jalili, Hosseinzadeh (2015) applied CA for optimal design of truss structures under stress and displacement constraints. Authors verified the efficiency of CA through optimising four real-world truss structures. Comparison results between CA and other meta-heuristics, such as BB-BC, PSO, and GA, illustrated that CA is able to provide relatively better structural designs. In another research, Jalili et al. (2019) introduced an improved CA (ICA) for simultaneous size and

shape optimisation of dome-shaped structures under static and frequency constraints. Structural optimisation under frequency constraints is a highly non-linear optimisation problem. Authors investigated the size and layout optimisation of a 276-bar lamella dome structure under stress and displacement constraints and a 708-bar lamella dome structure under frequency constraints. Jafari et al. (2019) developed a hybrid CA and EHO for optimum design of truss structures. Authors applied their algorithm to solve some benchmark structural design problems, including 10-bar truss, 25-bar truss, 72-bar truss, and 120-bar dome structures. The results indicated that the standard CA was able to provide better structural designs than EHO algorithm for all design examples. Ali et al. (2018) presented a balanced fuzzy CA with a modified Levy flight search to solve real-world parameter optimisation. Authors tested the performance of the approach on a set of benchmark functions and a 120-bar dome structure.

In the field of the environmental engineering, Liu, Lin (2015) proposed a CA-based approach to solve the spatial forest resource planning problem, in which the maximising total timber volume harvested over a harvest planning schedule was considered as the objective function. Authors performed a statistical comparison between the CA and SA algorithm by assuming different values for maximum iterations and population sizes. Their statistical comparisons revealed that CA is generally performs better than SA algorithm in terms of efficiency and stability of the results.

CA was also applied to the urban engineering. Jayyousi, Reynolds (2014) applied CA to produce high level structural functional models of cities, in which each individual of CA competes to build up a high level model of the site from a set of building blocks. Their study proved that the application of CA to urban planning can provide new insights into the plans produced by human experts.

In the field of the water engineering, Gu, Wu (2010) used CA to solve multiobjective water resources optimisation problem, in which the correlation of power generation between reservoirs and downstream areas was considered. In another study, Yan et al. (2017a) employed CA to solve the contaminant source identification problem in water distribution networks by assuming different sizes of water supply networks. Authors evaluated the performance of CA based on three water supply networks with 129, 430, and 12,527 nodes. Optimisation results revealed that CA is able to find the approximate pollutant source location with an acceptable level of accuracy. However, it seems that its performance needs to be improved to identify the pollutant sources more accurately in larger scales of water distribution networks.

3.2 Mechanical engineering

CA has also been employed to find solutions for the optimisation problems in mechanical engineering. Guedria, Hassine (2018) applied CA to solve the target optimisation problem of vibration-based damage detection. The performance of CA was investigated on the single and multiple damage detection of a cantilevered beam. Comparisons between CA and other algorithms, such as PSO and DE, showed the superiority of CA in accurately detecting the exact locations and the extents of damages under measurement noise. As the cantilever beam is a relatively simple structure, the performance of CA needs to be investigated on the damage detection of more complicated large-scale structures (Ghiasi et al., 2019; Perera, Ruiz, 2008).

In several studies, CAs have been utilised to solve the mechanical engineering

design problems. Ali, Reynolds (2014) introduced a hybrid CA and TS (CA–TS) algorithm to solve the complex non-linear constrained engineering design optimisation problems. The results reported by authors show that the standard CA performs slightly better than TS. Similarly, King et al. (2008), Reynolds, Ali (2008b), and Reynolds, Peng (2005) applied CA toolkit to solve mechanical engineering design problems.

The vibration control, vehicle safety, and flight control are other interesting applications of CAs in mechanical engineering. Wang et al. (2015) introduced an optimal vibration control strategy for vehicle's active suspension by combining CA and niche algorithm to design Fuzzy-PID controller and optimise control rules. Simulation results revealed that the optimised control rules can improve the ride comfort and handling stability of automobile. However, the convergence curve of CA reported for this problem shows that the CA converged to the solution within 23 generations, which is likely a sign of premature convergence. Kobti et al. (2006b) applied CA to guide driver learning to improve its performance in terms of the child vehicle safety and the injury level. Authors have considered a social networks between the drivers, in which the drivers can exchange their knowledge between each other. Meng et al. (2011) used a CA based on the PSO (CAPSO) to flight control law clearance.

Another interesting application area of meta-heuristics in mechanical engineering is in the performance enhancement of industrial gas turbines (or heavy-duty turbines). Pierezan et al. (2019) employed Cultural COA (CCOA) to improve the performance of heavy-duty gas turbines of a combined cycle power-plant located in Brazil, in which the gas consumption is minimised by taking into account the pollutant emission regulations and the turbine's physical limitations.

3.3 Chemical engineering

CAs have found some interesting applications in chemical engineering area. Haikal, El-Hosseni (2011) developed a modified cultural-based GA (MCBGA) to solve two chemical engineering optimisation problems. Authors solved the optimal control problem of a fed-batch fermentor for Penicillin production, in which maximisation of Penicillin production is considered as the objective function. Haikal, El-Hosseni (2011) also investigated the effectiveness of their algorithm in optimising the performance of the Isothermal continuous stirred tank reactor, which is a common model for a chemical reactor in chemical engineering. The statistical comparisons provided by the authors show that the MCBGA approach performed better than rival methods in terms of best, average, worst, and standard deviation of the results.

Xu et al. (2010) and Xu et al. (2012) proposed a multi-population cultural DE (MCDE) algorithm for optimising the chemical process of Ammonia synthesis. Ammonia has important applications in producing fertilizers in agricultural sector. Authors formulated the Ammonia Synthesis as a optimisation problem, in which the maximising net value of Ammonia is treated as the objective function. Tremayne et al. (2009) applied a hybrid cultural DE (CDE) algorithm to solve the problem of crystal structure determination from powder diffraction data. The main goal of the crystal structure determination is to find the precise spatial arrangements of all of the atoms in a chemical compound in the crystalline state (Massa, 2013). In Ref. (Tremayne et al., 2009), authors performed a kind of sensitivity analysis of internal parameters of the algorithm in order to find the best combination of them. Gao et al. (2019) used a hybrid cultural HS algorithm to optimise diesel blending procedure, in which a series of operations and

units are performed to transform crude oils into various products.

3.4 Electrical engineering

One of the interesting engineering application areas of CAs is electrical engineering. Chen et al. (2013) formulated the design of the high temperature superconducting magnets as the optimisation problem and suggested CA to solve it. In comparison to the conventional design method, the simulation results showed that the approach is able to provide high temperature superconducting magnets with better volume of solenoid coils. According to the results, CA is computationally efficient than the conventional GA for this problem, as it requires less computational time.

EDP is one of the attractive application areas of CA in electrical engineering. The main objective of this problem is minimising total fuel cost of a number of electricity generation facilities to allocate system load demand under transmission and operational constraints. Bhattacharya et al. (2012a) combined CA with EP to solve EDPs. In order to find the feasible solutions, authors used a map of feasible regions of search space to guide the search process. In comparison to PSO, GA, EP, and DE algorithms, the results yielded from a 13-generator system with valve point effects indicated that the CA is able to provide lower optimal fuel costs. Santos Coelho dos et al. (2009) proposed an improved DE approach based on CA and diversity measure to solve EDPs. Authors demonstrated the efficiency of their algorithm on two test systems consisting of 13 and 40 thermal generators. Ya-Li, Li-qing (2010) applied improved cultural PSO algorithm to solve EDP of 3-generators unit with 6 plants under transmission losses and valve-point loading.

In order to consider the released environmental pollutants by the systems of electricity generation through the thermal power plants, researchers have been added the emission effects to the EDP. Goudarzi et al. (2016) suggested four versions of CAs with different influence functions to solve the combined environmental EDP, in which minimising both of the fuel cost and emissions are considered as the objective functions, simultaneously. By using a price penalty factor, authors merged these two objective functions into a single objective function and considered various power system constraints, such as the ramp-rate limit, emission costs, valve-point effect, prohibited operation zone, and the transmission losses. Comparing to other methods in literature, simulation results obtained from three different test case systems demonstrated that CAs are capable to solve combined environmental EDP more efficiently. The convergence diagrams provided by the authors illustrate that two versions of CA are able to converge the optimal solution faster than FA, BBO, ABC, PSO, and GA methods. Similar results were also reported in Ref. (Goudarzi et al., 2017) for this problem. Zhang et al. (2013) developed an enhanced multi-objective CA approach to solve environmental EDP under various constraints, including valve-point effect, prohibited operation zones, and transmission losses. The performance of CA was verified through a sixunit and ten-unit test systems. The comparisons between the non-dominated solutions obtained by the CA, NSGA-II and MODE algorithms indicated that not only CA provides better distributions of Pareto front for environmental EDP, but also requires fewer amount of computational time. De Freitas et al. (2018) suggested combined CA methods with local search techniques for environmental EDPs, in which the fuel cost and gas emission are considered as the objective functions. The performances of the algorithms were investigated by using a real power plant with 10 generators and a system with 13 generating units. The convergence histories illustrated that the standard CA converged to better total cost values than the standard GA.

Reducing the real power loss of the power system networks through network reconfiguration and capacitor installation is another interesting research area in electrical engineering. Haldar, Chakraborty (2014, 2015) introduced a modified CA (MCA) to solve the network reconfiguration and capacitor allocation problems for minimum real power loss. The performance of MCA was investigated on the standard IEEE 16 bus distribution network and a practical transmission network. According to the experiments, MCA was able to provide lower costs for the transmission network and distribution system in comparison to PSO and rcGA methods.

Some applications of CA can be found in the field of the sensor fault detection and correction. Khan et al. (2014) suggested a symmetric sensor failure technique with a hybrid CA and DE (CADE) algorithm for the correction of faulty arrays. In another research, Khan et al. (2017) applied CADE to detect fully and partially defective sensors in a linear array. Authors proposed a symmetrical structure of a linear array to reduce the computational complexity and the partial faults are detected by CADE approach. Numerical results obtained by these studies showed the efficiency of CADE algorithm. Guo et al. (2013b) proposed a quantum-inspired CA method for the coverage optimisation of wireless sensor networks. Simulation results verify the efficiency of the approach in comparison to other algorithms. In another study, Guo et al. (2013a) extended the quantum-inspired CA for the multi-objective energy-efficient coverage optimisation of wireless sensor networks, in which the network coverage rate and the node redundancy rate are considered as the objective functions. The results demonstrated that the approach is able to yield a better pareto-optimal solutions than other multiobjective approaches. Kulkarni, Desai (2020) applied CA for anchor-assisted, rangebased, multi-stage localisation of sensor nodes of wireless sensor networks. The results comparisons between CA, ABC, and PSO revealed the superiority of CA in terms of accuracy and computing time.

CA has been also applied to the signal processing. Gao, Diao (2010) used a differential cultural (DC) algorithm to design finite impulse response and infinite impulse response digital filters. Gao, Diao (2011) suggested a cultural firework (CF) algorithm for digital filters design, in which the firework algorithm and CA are combined. Authors investigated the problem of designing finite impulse response and infinite impulse response digital filters via CF algorithm. Gao, Xu (2013) proposed cultural quantuminspired SFLA for direction finding of non-circular signals. The results showed that the approach can be efficiently used to direction finding of coherent and non-coherent sources of non-circular signals.

Digalakis, Margaritis (2002) presented a parallel co-operating CA (PARCA) method to solve the electrical generator maintenance scheduling problem, in which sum of the overall fuel cost and the overall cost of maintenance is considered as the objective function. By combining CA and IWO algorithm, Gao et al. (2017a) developed a cultural invasive weed algorithm (CIWA) for optimising the robustness and the imperceptibility of the watermarking system, in which the copyright of image data is protected.

Khodabakhshian, Hemmati (2013) applied CAs for tuning parameters of power system stabiliser to improve overall system stability. Simulation results of two typical multi-machine electric power systems containing system parametric uncertainties and various loading conditions verify the efficiency of CAs in improving dynamic stability of the systems. The convergence diagrams and computing times reported by authors for CA and GA methods shows the superiority of CA. Tavakoli et al. (2014) employed CA to design the controllers of the shunt flexible AC transmission system and power system stabiliser coordinately. The numerical results and convergence comparisons between CA, GA, and BFA showed that CA was able to find better solutions within early stages of evolution process. Bhattacharya et al. (2012b) introduced a CA with a single point crossover to solve the reactive power optimisation problem. Wang et al. (2009) applied a chaos cultural PSO (CCPSO) algorithm for the optimal operation of cascade hydropower station. Xie et al. (2009) used a PSO based on CA to solve short-term optimal operation problem of cascade hydropower stations. Authors used the penalty function method to handle the constraints of the problem. Guo et al. (2008) used multiobjective CAs for optimal design of passive power filters by assuming the total harmonics distortion and cost for equipment as the objective functions. The simulation results indicated that the filter designed by CAs has better harmonics suppression effect and lower investment for equipment than the traditional experience-based filter design.

Some researches have been investigated the performance of CA in optimal scheduling of hydrothermal power systems. For example, Yuan, Yuan (2006) utilised CA to solve the daily generation scheduling of hydrothermal power systems. Comparison results between CA, Lagrange, and GA methods verified the fast convergence properties of CA in solving the optimal daily generation scheduling problem of hydrothermal systems. However, the comparison between CA and GA was not supported by a fair statistical analysis. In another work, Lu et al. (2011) proposed a hybrid multi-objective CA for short-term environmental/economic hydrothermal scheduling problem, in which CA and DE are combined. The efficiency of the method was investigated on four coupled hydro plants and three equivalent thermal plants with non-smooth valve-point effects cost function. Yuan et al. (2008) applied CA to solve hydro production scheduling problem in electricity market, in which the profit of the hydro company from selling energy is maximised. In a similar study, Yuan et al. (2009) employed CA to deal with the short-term generation scheduling of hydrothermal systems.

CAs have been applied to solve different problems in the field of the robotics. Yu et al. (2011) employed a heterogeneous interactive cultural hybrid algorithm (HICHA) to solve multi-robot exploration mission planning problem. In their approach, the spatial orthogonal allocation algorithm was used for getting max-min task allocation. Then, the best route was obtained by HICHA. Reynolds, Chung (1997c) applied CA to increase the individual skills and cooperative strategies among a team of autonomous robot soccer players, in which authors assumed a set of constraints on the behaviour of players. Ali et al. (2014b) used CA to evaluate the tactics of robotic soccer team, in which the coordination of groups of agents in complex multi-agent dynamic environments was optimised. Authors evaluated the performance of CA against GA and showed that CA-team performs better than GA-team.

Some other applications for CA in electrical engineering can also be found in literature. Cai et al. (2018) proposed a time-frequency distribution fusion strategy assisted by CA to improve the sea-surface weak targets detection performance of the marine surface surveillance radar systems. Authors reported the advantages of CA method in solving this problem. Yan et al. (2017b) introduced a hybrid isolation niche technology and CA to solve image matching problem. The efficiency of the approach was demonstrated by solving a image matching problem. According to the results reported by the authors, CA provided better match correct rates than those obtained by GA. Wang et al. (2017) applied an adaptive CA with improved quantum-behaved PSO (IQPSO) to detect underwater sonar images. Pan et al. (2010) adapted CA for the size minimisation of binary decision diagrams, in which a binary decision diagram is a directed acyclic graph. Binary decision diagrams are capable of providing canonical representation for Boolean functions and they have important applications in design and verification of digital systems. Authors evaluated the performance of CA against GA on a set of benchmark circuit fault detection problems. Numerical results verified the effectiveness of CA in finding smaller size of binary decision diagram. Xiao et al. (2010) employed a cultural based ant colony algorithm to parameter optimisation of PID controllers, which have important applications in industry process control. Alami, El Imrani (2008) applied a hybrid CA and EP method to solve the fixed spectrum frequency assignment problem, in which the objective is the minimising interference of an assignment plan to efficient use of radio spectrum.

3.5 Computer science

CAs have been attracted much attention to solve a wide range of problems in the field of the computer science, such as function optimisation, Job Shop Problem (JSP), facility location, classification, and set covering. In the following subsections, the references related to these problems will be discussed.

3.5.1 Function optimisation

One of the research areas where the application of CAs is most extensive is the function optimisation. Various researchers have been employed CAs and their variants to solve the unconstrained and constrained function optimisation problems (Chung, Reynolds, 1998; Kobti, others, 2014, 2013; Becerra, Coello, 2004; Coello, Becerra, 2002b; Xue et al., 2011; Guo et al., 2011b; Gao et al., 2006a; Nguyen, Yao, 2008; Gao et al., 2006b, 2007; Nguyen, Yao, 2006; Li et al., 2010; Guo et al., 2018; Ali et al., 2011a; Guo et al., 2011c; Reynolds, Zhu, 2001; He, Xu, 2011; Sun et al., 2010, 2012; Dixit et al., 2019; Awad et al., 2017; Ali et al., 2016a, 2012; Xue, Guo, 2007; Coello, Becerra, 2002a; Ali et al., 2016b; Alami et al., 2007; Huang et al., 2008; Wu et al., 2010; Stanley et al., 2019; Tang, Li, 2008; Awad et al., 2013; Coello, Becerra, 2003; Zhou et al., 2019; Reynolds, Kinnaird-Heether, 2017a,b; Ali et al., 2014a; Reynolds, Gawasmeh, 2012; Ali et al., 2011b; Ali, Reynolds, 2009; Reynolds, Ali, 2008c; Chung, Reynolds, 1996b; Mao et al., 2020; Murugadass, Sivakumar, 2020; Singh et al., 2018; Muhamediyeva, 2020).

3.5.2 JSP

Several researchers applied CAs for solving JSP, in which a set of jobs should be processed through a set of machines. The objective function is the minimisation of the total time required to perform the jobs under some constraints. Cortés Rivera et al. (2007) and Becerra, Coello (2005) employed CA as an alternative optimisation algorithm to solve JSP. Comparison results between CA and rival algorithms, such as GA and its hybridised version with local search, obtained from a set of benchmark JSPs revealed that CA is capable of generating competitive results for this problem. Their results showed that CA is able to provide better results than the standard GA, whereas its performance is comparable to GA with local search. Zhang, Zhu (2012) suggested a cultural PSO algorithm to minimise the maximum completion time in JSP. Kobti, others (2012) applied a multi-population version of CA to solve the JSP and indicated that the convergence speed of CA is remarkable. Yang, Gu (2014) developed a cultural-based genetic tabu algorithm for multi-objective JSP. Niu et al. (2011) employed a CA based on DE (CADE) algorithm to solve the hybrid flow shop scheduling problems with fuzzy processing time. Authors considered the makespan as the objective function, which is modeled as a triangular fuzzy number. The results reported for different instances showed that the standard CA performs statistically better than PSO method. Ho, Tay (2004) applied CA to solve the flexible JSP with re-circulation. Authors indicated the efficiency of CA in comparison to the conventional composite dispatching rule algorithm.

3.5.3 Facility location

The main aim of the facility location problem is to find the optimum placement of facilities of companies to reduce the transportation costs of product distribution between their costumers. Lagos et al. (2014) and Cabrera et al. (2011) applied CAs to solve a biobjective uncapacitated facility location problem, in which the cost minimisation of facility installation and customer allocation as well as maximisation of the customers coverage are considered as the objective functions. Comparing to NSGA-II, the results reported in these references for different benchmarks shows that the CA is capable of generating better solutions with less computational effort.

3.5.4 Classification

Briefly speaking, classification problems deal with the assigning data into one of several known classes. Srinivasan, Ramakrishnan (2012) employed CAs for multi-objective optimisation of classification rules. Yan et al. (2016) proposed a double weighted Naive Bayes and niching CA for multi-lable classification. Srinivasan, Ramakrishnan (2013b,a) proposed extended CA for multi-objective optimisation of classification rules. Their results showed that the extended CA can provide more accurate results than other methods, such as NSGA-II. Recently, Srinivasan, Muruganandam (2020) employed CA to solve multi-objective problem of rule pruning in classification algorithms, in which the unwanted rules are removed to reduce the difficulty of decision making process.

3.5.5 Set covering

The main aim of the set covering problem is to identify the smallest mutually independent subsets of a given set while minimising a cost function defined as the sum of the costs associated to each of the eligible subsets (Chu, Beasley, 1998). Crawford et al. (2007, 2013) applied CA to solve set covering problem. The costs obtained for different benchmarks revealed that CA performs remarkably better than GA and ACO methods.

3.5.6 Others

CAs have also interesting applications in some other types of problems in the field of the computer science. For instance, Reynolds, Sverdlik (1995) applied CA for program understanding, in which CA used to extract functional knowledge from a program object. Sternberg, Reynolds (1997) utilised CA for fraud detection in insurance companies, in which the fraudulent claims received by the insurance companies are identified to reduce the fraud losses. The results showed that CA has an efficient self-adaptive capability in dealing with dynamic optimisation problems. Rychtyckyi, Reynolds (1999) applied CA to solve the problem of semantic network reformulation by analysing the selection of significant attributes for subsumption. Rychtyckyj, Reynolds (1998) suggested a CA-based approach for semantic network reformulation. Rychtyckyj, Reynolds (2000, 2005) suggested CA to improve the performance of the subsumption algorithm in re-engineering large-scale semantic networks. Authors reported that CA is able to provide comparable results to those provided by the manual inspections. It seems that the superiority of CA over manual inspection is more obvious for more complex networks. Rychtyckyj, Reynolds (2002, 2001) applied CA to re-engineer the direct labour management system semantic network at Ford company from a top-down perspective. Authors indicated that CA is able to provide significantly better results comparing to the previously derived results through manual inspections.

Nowadays, the social media has an significant effect on our daily life, in which spreading the fake news generated by anonymous users can make a big challenge for the government and individuals in society. Recently, Shah, Kobti (2020) employed CA to identify the fake news using the text and images in social media. In comparison to the state-of-the-art methods, such as ANN-based methods, the results revealed that CA is able to detect the fake news with higher level of accuracy. Based on CA, Zhang (2012) presented a framework for evolutionary systems to provide an efficient search mechanism for finding useful contents on the web. Zadeh, Kobti (2015) proposed a knowledge-based evolutionary algorithm by using a multi-population CA to solve community detection in social networks. Reynolds, Nazzal (1997) used CAs to discover networks of sites based on the historical database.

Soza et al. (2011, 2007) applied CA to solve timetabling problems. Authors reported that CA performs weaker than SA for timetabling problems. Jun et al. (2008) employed an integrated CA and SA to solve the routing problem of the mobile agent. Vitale et al. (2011b) used CA for ancient landscapes exploration under the lake. Selvarajaha et al. (2019) applied CA to solve the problem of identifying a team of experts in social networks. Numerical results suggested that the efficiencies of CA and GA for this problem are almost same in terms of quality of the results and required computational effort. In a similar study, Selvarajah et al. (2020) used CAs to solve the team formation problems in the industrial organisational settings, in which the objective is forming group of teams with different skills in a social network to accomplish a set of projects. It seems that CA performed better than GA and other algorithms for higher number of projects. Ochoa et al. (2010) developed a software tool to provide an intelligent logistic service for purified water distribution among the costumers, in which the CA was used as optimiser. Authors combined data mining with CA to minimise the delivery costs.

Chen, Yang (2015) designed the neuro-fuzzy system by a DE-based symbiotic CA to solve the nonlinear control problems. Li et al. (2012) employed a hybrid CA and GA, called cultural-GA, to solve the multi-aircraft cooperative target allocation problem, in which the main aim is to select an optimal and reasonable weapon-target allocation plan to achieve the best combat effect.

Unold, Tarnawski (2016, 2017) proposed parallel cultural ACO and multipopulation CA to solve the travelling salesman problem. Kim, Cho (2009) applied a hybrid CA with local search to solve the travelling salesman problem. Ochoa et al. (2012) developed a mobile device based on CA and data mining, which is provided to analyse the risk of being in a part of city. The results showed that the integrated CA and data mining method can perform significantly better than the purely data mining approach.

Prestwich et al. (2008) applied CA to solve partially observable markov decision processes. From the reported diagrams in this reference, it seems that CA can converge to the optimal solutions faster than GA. Abdolrazzagh-Nezhad (2020) applied an enhanced CA to solve the multi-objective attribute reduction problem, which is an prepossessing problem in data mining. Authors provided a fair comparison of CCA against other meta-heuristic algorithms by investigating their performance with different population sizes. The reported resulted showed that CA provides better results than those provided by binary GA, PSO, ACO, ABC, and TS. Jin, Reynolds (2000) used CAs to mine knowledge in large-scale databases, in which the data mining is performed within the belief space and the optimisation is performed by the population space. The approach extracted some new patterns from a large-scale archeological database, which were unknown before. Reynolds, Al-Shehri (1997, 1998) tried to guide the incremental learning decision trees, which have important applications in representation of the results obtained from the data mining.

Oloruntoba et al. (2019) used a clan-based CA for feature selection, which is a process of selecting a subset of relevant features from data sets and has important applications in machine learning. Mojab et al. (2019) proposed an improved CA to solve the general big data workflow scheduling problem, in which the characteristics of cloud computing are optimally used to transfer large amounts of data. Goli et al. (2019) employed CA with multi-layer perceptron neural network to predict the demand for diary products in Iran. Selvarajah et al. (2019) applied CA to analysis the similarities between the users in online marketing. The results showed that CA is able to provide more accurate similarity analysis than GA. Sun et al. (2009) proposed a cultural-SA algorithm to solve the quality of service routing problem. Mohammadhosseini et al. (2019) proposed a virtual machine placement method based on CA to reduce the energy consumption in cloud data centres. Authors reported the efficiency of CA in comparison to a set of stochastic methods in the field of virtual machine placement.

Cowan, Reynolds (1999) applied CA to asses the quality of GP solution programs. Ostrowski, Reynolds (1999) suggested CA to solve the software testing problem, in which the white and black box testing methods were embedded into CA. Reynolds, Rolnick (1995); Reynolds, Rolnick (1995) proposed a image segmentation method based on CAs, in which a set of objects in a digital image are separated from their background. Authors used CAs to support the gradient-based method by finding its parameters.

3.6 Other applications

In the reviewed literature, there are some applications of CA in other research areas. Kobti et al. (2006a) utilised CA to study the emergence of a hub network from two kinship and economic networks, in which CA was used as a solution framework to embed the social intelligence in the system. Reynolds et al. (2008a) employed CA for mining the social fabric of archaic urban centres. Alexiou, Vlamos (2012) suggested CA for the representation of mitochondrial population in mammalian cells. Judeh et al. (2014, 2013) applied a gene set CA to reconstruct networks from unordered gene sets. Terán et al. (2017) utilised CA to optimise the coordination mechanisms in multi-agent

Table 3:	: Hybrid	versions	of CA	۱n ۱	literature.
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Hybrid version	References
CA+GA	Gao et al. (2006a), Gao et al. (2006b), Xue et al. (2011), Haikal,
	El-Hosseni (2011), Li et al. (2012), Tang, Li (2008)
CA+PSO	Lin et al. (2008), Sun et al. (2010), Zhang, Zhu (2012), Sun et al.
	(2012), Guo et al. (2017), Zhang et al. (2013), Lin et al. (2009), Chen
	et al. (2008), Wu et al. (2010), Santos Coelho dos, Mariani (2006),
	Ya-Li, Li-qing (2010), Stanley et al. (2019), Meng et al. (2011), Xie
	et al. (2009), Wang et al. (2017), Yu et al. (2011), Wei, Yan-Ping
	(2012), Guo et al. (2015), Wang et al. (2009), Chen et al. (2013)
CA+DE	Arpaia et al. (2007), Niu et al. (2011), Becerra, Coello (2004), San-
	tos Coelho dos et al. (2009), Tremayne et al. (2009), Dixit et al.
	(2019), Lu et al. (2011), Yuan et al. (2008), Khan et al. (2014), Khan
	et al. (2017), Awad et al. (2017), Ali et al. (2016b)m, Gao, Diao
	(2010), Yuan et al. (2009) , Xu et al. (2012) , Xu et al. (2010) , Chen,
CALL and Course	Yang (2015)
CA+Local Search	All, Reynolds (2014), De Freitas et al. (2018), Nguyen, Yao (2008),
	Nguyen, Yao (2006), Ali, Awad (2014), Ali et al. (2016a), Kim, Cho (2000) Arrad et al. (2012) Sup et al. (2000) Ali et al. (2012) Di
	(2009), Awau et al. (2015), Suit et al. (2009), All et al. (2015), Di-
Otherschames	C_{200} at al. (2014) Cup at al. (2012b) Lafari at al. (2010)
Other schemes	Gao et al. (2014), Guo et al. (2015), Jalair et al. (2019), Gao, Diag (2011) Cao, Yu (2012), Cuo et al. (2013a). Li et al. (2010)
	C_{2011} , Gao, Au (2013), Guo et al. (2013a), El et al. (2010),
	Sao et al. $(2017a)$, Onold, Talilawski (2010) , Gao et al. (2017) , Xiao et al. (2010) Pierezan et al. (2019) Murugadass Siyakumar
	(2020) X11e (2020) Amalo et al. (2020) Hosseinioun et al. (2020)
	Santos Coelho dos Alotto (2009)
	Santos Coeino dos, Alotto (2009)

systems in industrial automation. Pan et al. (2016) used CA with chaotic behaviour for cell allocation in CMOL circuits with the hybrid CMOS/nanodevice structure. Wei, Yan-Ping (2012) applied a CA-based PSO for partner selection of virtual enterprise and minimise time and cost.

4 Variants of CAs

Literature review shows that significant efforts have been done to increase the efficiency of CAs. As results of these efforts, different variants of CAs have been developed, ranging from improved and multi-population versions to hybridised versions and multi-objective versions. In this section, different variants of CAs developed in literature will be reviewed.

4.1 Hybridisation of CAs

Literature survey shows that CAs have been hybridised with other meta-heuristic techniques to increase their efficiency and robustness. Table 3 lists the hybridised versions of CAs with other meta-heuristic techniques, such as GAs, PSO, DE, and local search methods. In the following subsections, different versions of these hybridised schemes of CAs will be reviewed.

4.1.1 Hybrid CA and GA

GA is a basic evolutionary algorithm inspired by the Darwinian theory of evolution. Literature review reveals that researchers have been hybridised CAs with GA to solve various optimisation problems. In most of the cases, they used GA as the evolutionary strategy to improve the fitness of individuals within the population space, while the knowledge components of the belief space of CAs are used as auxiliary tools to guide the operators of GA method.

Gao et al. (2006a) proposed an integrated CA and GA (CGA) algorithm for constrained optimisation. In CGA, an intelligent mutation operator was proposed to guide the search process towards the feasible solutions, in which the topographical knowledge component of belief space consisting of constraint characteristics of different regions of solution space was used as an auxiliary tool. From the reported results, it seems that CGA performed better than the standard GA and real coded GA in terms of solution quality and required computing time. A similar approach was also used by Gao et al. (2006b) to develop a hybrid model of CA and GA for optimisation problems. Xue et al. (2011) suggested a hybrid version of CA and GA, called NSCA, by introducing a novel selection operator in order to increase the population diversity. The approach used GA operators and knowledge components of belief space to generate new individuals. The results obtained for a nonlinear function revealed the superiority of NSCA over the standard CA.

Li et al. (2012) employed a hybrid cultural-GA to solve multi-aircraft cooperative target allocation problem. In cultural GA, the GA operators is used to evolve the population space, while the information of the belief space is employed to influence the selection and mutation operators. The results revealed that the cultural GA performs better than the standard GA. A similar hybrid cultural real-coded GA was developed by Haikal, El-Hosseni (2011) for constrained optimisation problems, in which the information about the higher quality solutions and the ranges of feasible regions are stored in the belief space to generate random offspring solutions around them. Authors compared the effectiveness of their approach against other hybrid algorithms, including gaussian PSO and quantum-behaved PSO. However, the comparisons between their approach and standard CA or GA were not provided. Tang, Li (2008) proposed a triple spaces cultured GA (TSCGA), in which a new space, called anti-culture population, was introduced to prevent premature convergence. In TSCGA, the anti-culture population space tries to generate new individuals outside the current cultural intervals, which can be helpful when the algorithm traps into the local optimum points.

4.1.2 Hybrid CA and PSO

As SI technique, PSO has attracted much attention as a suitable option to develop hybridised versions of CAs. Zhang, Zhu (2012) developed a cultural PSO to solve the JSP. In this algorithm, the adaptive PSO strategy is used to evolve the population space. It seems that authors tried to increase diversity and prevent premature convergence by replacing the global best by some of the poor solutions stored in belief space. Comparative results indicated that the cultural PSO was able to show better performance for JSPs than the standard PSO and its hybridised version. Lin et al. (2008) introduced a hybrid cultural cooperative PSO (CCPSO) for functional-link-based neural fuzzy network model in predictive applications. As illustrated in Fig. 3, CCPSO uses multiple swarms of particles with independent belief spaces, in which the positions of particles

in each swarm are updated based on the knowledge components of the belief space corresponding to the same swarm. Reported convergence diagrams in this reference illustrate that the CCPSO performs remarkably better than the PSO and CPSO algorithms in terms of the convergence speed and solution quality. Similar approaches were also adopted by Lin et al. (2009); Chen et al. (2008). In another study, Sun et al. (2010, 2012) suggested a co-evolutionary cultural based PSO (CECBPSO), in which two sub-populations with different sub-belief spaces perform the optimisation process separately. In CECBPSO, the information collected within the two sub-belief spaces are shared with the global belief space. Then, the global belief space influences both of the population spaces. The general framework of the CECBPSO is given in Fig. 4. From the reported numerical results, it can be seen that the CECBPSO can perform significantly better than the standard PSO and its cooperative versions, such as CPSO.



Figure 3: Framework of the hybrid CCPSO algorithm (Lin et al., 2008).

Guo et al. (2017) proposed cultural PSO algorithm for uncertain multi-objective problems with interval parameters, in which a possibility degree is used to construct a novel dominant comparison relationship. In this approach, the population space of the algorithm is evolved by the PSO algorithm, the flight parameters of PSO are adjusted by considering knowledge components, and the uniformity of non-dominant solutions in objective space is recorded in the topological knowledge component. Zhang et al. (2013) proposed an enhanced multi-objective CA algorithm (EMOCA), which combines PSO and CA framework. In this approach, the population is evolved by PSO, while the situational and history knowledge components are used to improve the convergence



Figure 4: Framework of the hybrid CECBPSO algorithm (Sun et al., 2012, 2010).

speed. From reported Pareto fronts in this reference, it seems that EMOCA can provide more uniform distribution of non-dominate solutions in comparison to NSGAII and multi-objective DE algorithms.

Wu et al. (2010) suggested a hybrid CA and PSO (PSOCCA), in which the population space is evolved based on the modified PSO operators. In PSOCCA, different formulas based on the situational and normative knowledge components of belief space were proposed to update the positions of the particle. Convergence diagrams reported by authors shows that PSOCCA can provide better solutions with lower convergence speed than the standard PSO. This means the applicability of PSOCCA for problems with complex objective functions is limited. Santos Coelho dos, Mariani (2006) proposed a cultural Gaussian PSO (GPSO-CA) algorithm, in which the population space is evolved by the PSO algorithm with Gaussian random numbers. In GPSO-CA, the normative knowledge components of belief space are used to decide whether a variable should be updated.

Stanley et al. (2019) suggested a parallel multi-objective CA and PSO (CAPSO), in which several sub-swarms exchange the cultural information. In CAPSO, each subswarm searches a specific part of search space. Authors also investigated the effect of each knowledge component on the search process. Xie et al. (2009) presented a PSO algorithm based on CA (PSO-CA), in which the population space is evolved by PSO operators considering the normative knowledge component. An adaptive CA with IQPSO (ACA-IQPSO) was developed by Wang et al. (2017), in which IQPSO algorithm and CA are cooperating each other to generate new solutions. Authors proposed a new strategy to update the belief space and redesigned the acceptance and influence functions to enhance the utilisation of cultural information within the population. Yu et al. (2011) developed a HICHA for solving multi-robot exploration mission planning problem, in which the population space is evolved based on the PSO formulation. The general framework of HICHA is very similar to CA, in which the population and evolutionary spaces exchange information through a customer interactive estimation unit.

4.1.3 Hybrid CA and DE

DE developed by Storn, Price (1997) is another population-based meta-heuristic algorithm which uses a set of search operators based on the weighted difference of solution vectors in search space. Arpaia et al. (2007) replaced the influence functions of CA by the mutation operators of DE algorithm to improve the performance of the standard CA. Authors used the normative knowledge components to change the search direction, while the DE operators were used in the algorithm to generate new individuals. The new algorithm was performed better than GA. In another research, Niu et al. (2011) introduced a DE-based CA approach (CADE) to solve the hybrid JSPs. In CADE, the generated new solution vectors by normative-based influence function are randomly updated by DE operator. From the results reported by authors it can be seen that the CADE is statistically more efficient than the standard CA, PSO, and QEA. However, CADE requires slightly more computational effort in comparison to the standard CA. Similar approaches were also proposed by Khan et al. (2017) and Tremayne et al. (2009).

Becerra, Coello (2004) proposed a set of new influence functions for various knowledge components based on the DE strategies. In their approach, each DE-based influence function has a chance to update the position of each individual with a given probability. Similarly, Khan et al. (2014) suggested another hybrid version of CA and DE. By using the situational and normative knowledge components, authors proposed two new influence functions based on DE operators. These new influence functions combine three randomly selected individuals to form new generation of individuals in population space. Santos Coelho dos et al. (2009) presented an improved DE approach based on CA and measure of population's diversity (CDEMD) to solve EDP. In CDEMD, the positions of the individuals are updated using the mutation operators of DE defined based on the normative and situational knowledge components. Comparing to different versions of DE and PSO algorithms, it seems that the performance of CDEMD is remarkable.

Dixit et al. (2019) and Awad et al. (2017) suggested hybrid CA and DE algorithms, in which a participation ratio is introduced for each of CA and DE strategies. In the mentioned studies, the participation factor is updated during the optimisation process. The comparison results showed that the hybrid CA and DE approaches can perform significantly better than the different versions of CA and DE methods. Ali et al. (2016b) developed a hybrid cultural learning theme with a balanced performance for DE frameworks (b-hCA-DE). In b-hCA-DE, the population space is evolved by using the DE operators and the knowledge sources of the belief space, in which the role of each knowledge component is changed over time based on its success in generating high-quality solutions in the previous generations.

Lu et al. (2011) proposed a hybrid multi-objective CA (HMOCA) method, which combines the advantages of CA and DE to solve short-term environmental/economic hydrothermal scheduling problem. In HMOCA, the population space is evolved by the mutation operator of DE algorithm. The results showed that HMOCA is able to provide better distribution of Pareto front solutions than the NSGA-II method. Gao, Diao (2010) developed a differential cultural (DC) algorithm based on the differential-based influence functions. Convergence properties of DC are significantly better than those exhibited by PSO and its variants. Yuan et al. (2008) and Yuan et al. (2009) embedded DE into CA by using the situational and normative knowledge components within the variation operators. Authors used the selection operator of DE to distinguish between

the feasible and infeasible solutions and handle the constraints of the problem. Comparisons showed that the hybrid approach performs better than DE and GA methods in terms of solution quality and required computational effort.

4.1.4 Hybridisation with local search

In some cases, researchers have been hybridised CAs with the local search techniques to improve the exploitation capability. TS and SA algorithms are known examples of such local search techniques. Ali, Reynolds (2014) suggested to use TS algorithm as the local optimiser to search the neighborhood of the optimal solutions obtained by CA in each generation. It seems that the hybrid algorithm performs statistically better than the standard CA and TS. Ali et al. (2013) hybridised the niche CA with TS algorithm. In their hybrid approach, the low-quality solutions are removed before performing local search by TS. A similar approach can be seen in (Ali, Awad, 2014). Sun et al. (2009) developed a cultural-SA (CA-SA) algorithm, in which the step size of the influence function of CA is control by the SA algorithm. Authors also used the temperature parameter of SA to update the situational knowledge of the belief space. The results revealed that CA-SA performs better than GA. However, statistical evidences for effectiveness of CA-SA against GA are not observable in the study. De Freitas et al. (2018) investigated the hybridisation of CA with SA and TS local search techniques for solving EDP. The convergence histories showed that the hybrid CA+TS was yielded significantly better results than CA+SA hybridisation scheme. It can be seen from the convergence histories reported in this reference that the hybridisation scheme of CA+SA has not performed better than the standard CA.

Nguyen, Yao (2006, 2008) hybridised CA with the iterated local search to enhance the efficiency of the standard CA method. Ali et al. (2016a) developed a hybrid CA and modified multiple trajectory search (CA-MMTS) algorithm to improve the exploration and exploitation capabilities of CA. In CA-MMTS, the modified multiple trajectory search plays the role of local search strategy, which uses the knowledge components of belief space to enhance the quality of best solutions obtained by CA. Awad et al. (2013) proposed a hybrid CA and an improved local search (CA-ImLS) approach, in which the improved sub-local search is performed for each knowledge component of belief space. Jun et al. (2008) developed a hybrid CA-SA algorithm, in which the acceptance function works based on the SA formula. The convergence diagrams showed that the hybrid approach can find better solutions with fewer function evaluations than GA ans PSO methods.

4.1.5 Other hybridisation schemes

In recent years, a significant number of new meta-heuristic algorithms have been developed by researchers. Literature survey demonstrates that some studies have investigated the hybridisation schemes of CAs with these new approaches. Gao et al. (2014) introduced a quantum-inspired cultural BFA (QCBFA) by combining the quantum knowledge strategy and BFA, in which a quantum belief space is used to provide a guided cultural behaviour. Guo et al. (2013b) proposed a quantum-inspired CA (QCA) optimisation strategy to properly distribute sensor nodes in wireless sensor networks. In QCA, the population space is evolved by the QEA, and the implicit knowledge extracted from the high-quality solutions stored in belief space is used to update the positions of quantum individuals. Results showed that QCA performs better than both of the standard CA and QEA. A similar study was also performed by Guo et al. (2013a). Following the same idea, Guo et al. (2018) suggested interval multi-objective quantuminspired CA (IMOQCA). In this approach, the population space is evolved by the operators of the QEA, and the knowledge components of the belief space is used to guide the selection and mutation operations of evolutionary individuals. In another study, Gao, Xu (2013) proposed a cultural quantum-inspired SFLA for continuous optimisation. The algorithm uses the quantum knowledge strategy and new quantum leaping equations to enhance the performance of SFLA. As a cultural component, authors constructed a quantum belief space consisting of normative and situational knowledge components to model the cultural behaviour in the leaping movement.

Gao, Diao (2011) suggested a hybrid cultural firework (CF) algorithm based on the combination of firework algorithm and CA to deal with the filter design problem. Authors applied firework algorithm within the population space of CA, and added the cultural knowledge sources to the variation operator of firework evolution. Authors reported the efficiency of CF algorithm against PSO, quantum PSO, and adaptive quantum PSO algorithms. Santos Coelho dos, Alotto (2009) developed a cultural SOMA method for electromagnetic optimisation, in which the search direction in SOMA is obtained by considering the normative knowledge components of belief space. Comparison results showed that the cultural SOMA performs better than the classical SOMA and PSO methods. Jafari et al. (2019) suggested a hybrid EHO and CA (EHOC), in which the belief space of CA is used to improve the efficiency of EHO algorithm. EHO is a meta-heuristic algorithm developed by Wang et al. (2016), which mimics the herding behaviour of elephant groups. In EHOC, the cultural information of high-quality solutions stored in belief space are used to generate new solutions. The convergence diagrams reported in this reference show that the convergence speed of EHOC is faster than the standard CA and EHO methods. Murugadass, Sivakumar (2020) developed a hybrid EHO and CA (HEHO-CA) algorithm for optimal cluster head selection to extend the lifetime in wireless sensor networks. In HEHO-CA, it seems that the worst solutions in the population are replaced by the normative knowledge components in order to provide more accurate exploitation, while the situational knowledge component is used to improve the exploration ability.

Unold, Tarnawski (2016) introduced a paralel cultural ACO (pCACO) to solve the travelling salesman problem, in which CA and ACO-based max-min ant system algorithm are combined. In pCACO, the population space is evolved by ACO algorithm, while the belief space is evolved by a simple GA. In pCACO, the belief space of CA uses the created routes by the ants and tries to find the better ones. Xiao et al. (2010) introduced a cultural based ACO (ACCA), in which the new solutions generated by ACO are re-optimised by CA in each iteration. The results indicated that the ACCA approach has a better performance than GA. However, a fair statistical performance comparison was not carried out by the authors. Gao et al. (2019) developed a hybrid cultural HS algorithm, in which the knowledge components of CA is used to modify the harmonies generated by the HS algorithm. The convergence diagrams reported in this reference suggest that CA can significantly enhance the convergence rate of the HS algorithm. Gao et al. (2017a) suggested a hybrid CA and IWO algorithm (CIWA) to solve the design problem of digital watermarking with wavelet transform. In CIWA, the evolution of the population space is randomly performed by using cultural influence function and IWO operators. In a similar study, Hosseinioun et al. (2020) proposed a hybrid IWO and CA algorithm, in which the population space of CA is evolved by the operators of IWO algorithm.

Pierezan et al. (2019) proposed a cultural COA (CCOA), in which the normative knowledge of CA is used to improve the balance between the exploration and exploitation abilities of COA. Authors used the CA's normative-based influence function (similar to equation(12)) to update the solutions generated by COA. The numerical results obtained for the benchmark functions and gas demand minimisation of turbines reported in (Pierezan et al., 2019) revealed the normative-based influence function can significantly improve the solution qualities generated by the standard COA method. Guo et al. (2010) combined CA with ($\mu + \lambda$) and clonal selection strategies, in which the influence of each selection method is controlled by the implicit knowledge resources. In another study, Mao et al. (2020) hybridised the FECO algorithm and CA for multiobjective numerical optimisation, in which the solutions generated by FECO are updated by randomly using knowledge components of CA. It seems that the hybrid algorithm has different internal parameters, which can affect the optimisation performance. Xue (2020) suggested an adaptive CA-based CS (CACS) algorithm, in which the solutions generated by CS algorithm are updated by CA. Authors reported that the hybrid CACS algorithm can provide better results than CA, CS, and ACO methods. Amalo et al. (2020) introduced a hybrid CA and BA to design a maximum power point tracking for a photo voltaic system, in which the normative knowledge component of CA is used to improve the convergence of BA.

4.2 Communication topology for CAs

One of the important factors that can affect the convergence properties of CAs is the efficiency of the knowledge distribution between the individuals within the population space. A recent survey revealed that the population structures with neighborhood or topological properties can significantly enhance the diversity preservation capacity of the population-based meta-heuristic algorithms (Lynn et al., 2018). Hence, researchers have been tried to control the knowledge dissemination through defining communication topologies within CAs (Ali, 2008; Che et al., 2010; Ali, Reynolds, 2009; King et al., 2008; Reynolds, Ali, 2008b,c,a). Ali, Reynolds (2014) employed simple forms of social network topologies to provide efficient knowledge dissemination and accelerate the convergence speed of CA, including ring, square, and global gBest topologies. Their approach benefits from the TS local search mechanism. The results obtained for function optimisation revealed the effectiveness of square topology. Ali et al. (2011a) proposed a new influence function based on the social fabric metaphor, in which the individuals in the populations are connected to each other by using ring (*lBest*) and global (*gBest*) topologies. Using these connections, individuals are able to share their experience with neighbourhood individuals. Authors reported that the *lBest* topology provides better results than gBest topology. In another study, Ali et al. (2012) proposed a similar approach based on the ring (*lBest*), global (*gBest*), tree, and hybrid tree topologies. The statistical results reported in this reference show that the CA with social fabric influence function performs better than the standard CA and GA.

Reynolds, Gawasmeh (2012) and Ali et al. (2011b) investigated the performance of different homogeneous social networks, such as ring, square and global topologies, to properly spread the useful information among the individuals. From the reported results in these references, it seems that the impacts of communication topologies on CA's performance are more obvious in dealing with complex problems. Ali et al. (2015) developed a tribal sociocultural algorithm with neighbourhood restructuring (T-SCANeR) approach, in which the population is divided into multiple tribes with a given number of individuals for each. For each tribe, a given topology is defined, which shapes its social fabric structure and determines the interactions of individuals. Authors used different topologies to define the connection of individuals within tribes, including the ring, flattened representation of Von Neumann neighbourhood, hybrid-tree, and global topologies. In another study, Reynolds et al. (2015) introduced a sub-cultured topology concept in CAs based on a set of six homogenous topologies, in which the individuals can change their topologies during the evolution process. From the reported graphs in this reference, it can be seen that the sub-cultured concept is computationally more efficient than the homogeneous and heterogeneous approaches for complex problems. It seems that the sub-culture concept explores the search space with small groups of individuals, which is a positive feature from the exploration point of view.

4.3 Improved CA

Review of the literature revealed that several modifications have been made to improve the performance of CA in solving various optimisation problems. In this section, we outline some of the improved versions of CA.

Ali et al. (2014a) enhanced the balance between the exploration and exploitation of CAs through dynamic adjustment of the number of evaluations available for each type of knowledge source. In addition, authors used a local search to improve the solutions provided by the knowledge sources. Jalili et al. (2019) introduced an improved CA (ICA) for structural optimisation by using a new influence function to update the positions of individuals. Authors also used a truncated geometric distribution to simulate the number of dimension changes in each individual. The new influence function uses the normative and situational knowledge components simultaneously. The results showed that the ICA was able to perform significantly better than different versions of CA. Soza et al. (2007) integrated CA with variation operators to improve the exploration ability of the algorithm in solving timetabling problem. The algorithm proposed by the authors uses three cultural variation operators, including interchange, sequencing, and simple mutation operators. Haldar, Chakraborty (2014, 2015) suggested a modified CA (MCA) algorithm, in which the addition, multiplication, replacement and unchanged operators were used as the influence functions. Although the results confirmed that MCA performed better than the standard CA, some abrupt oscillations in convergence diagrams of MCA show that more examinations on the stability of the algorithm are required. Hence, a statistical analysis seems to be necessary to provide a fair comparison between CA and MCA.

Wang et al. (2015) proposed an improved CA for optimal vibration control of a vehicle's active suspension, in which the performance of the standard CA is enhanced by using the niche algorithm. In the improved version of CA, authors used niche algorithm within the acceptance function to accept the changeable number of individuals for updating the belief space. In their approach, the number of accepted solutions is gradually decreased over time. Al-Tirawi, Reynolds (2018) added a common value auction concept to the framework of CA in order to enhance the knowledge distribution between the individuals. The common value auction concept provides a set of parameters for each knowledge component of belief space, which include some infor-

mation about each individual within the social network. Zhou et al. (2019) presented a cultural cognitive evolution optimisation (CCEO) algorithm, in which the cognitive behaviour is added to the CA framework to enhance the exploration and exploitation capabilities. CCEO is a three layer optimisation algorithm consisting of the population, belief, and cognitive spaces. In the cognitive space, the population is evolved by using the Gaussian random walk or Levy flight methods.

Bhattacharya et al. (2012b) suggested to use CA with single-point crossover to enhance the global search capability of CA. In comparison to other methods, the results obtained from a IEEE 14-bus system demonstrated that the approach was able to show comparable performance in terms of real power losses. Authors used a simple normal distribution as a mutation operator to perturb the solutions and increase the diversity. The sudden fluctuation in convergence diagram of the algorithm demands further statistical analysis. Reynolds, Kinnaird-Heether (2013) employed game theoretic mechanisms in order to make the role of the knowledge components more significant, including auction and weighted majority mechanisms. Based on the weighted majority win situation, the knowledge resources are able to predict the future success, while the auction mechanism allows the knowledge source to pro-actively compete for the individual based upon tokens derived from its current performances (Reynolds, Kinnaird-Heether, 2013). Mojab et al. (2019) proposed an improved CA by introducing a comprehensive elite mechanism, in which the information obtained from different knowledge components of the belief space is used for generating new solutions. The results revealed that not only improved CA performed better than the standard version of CA, but also exhibited better performance than other well-known meta-heuristic algorithms, such as GA and PSO. Waris, Reynolds (2018) improved the convergence properties of CA by using a new knowledge distribution mechanism based on the gametheoretic, in which the cooperation and competition between players were simulated.

4.4 Multi-objective CA

Literature review reveals that researchers have been adapted CA to solve multiobjective optimisation problems. Best et al. (2010) extended CA to solve multi-objective problems, in which they have modified different knowledge components of belief space to consider the effects of multiple objective functions. Srinivasan, Ramakrishnan (2012) introduced an extended version of CA for multi objective optimisation of classification rules, in which the individuals are accepted into the belief space based on the Pareto optimisation strategy using the metrics stored in the domain knowledge as vectors. Srinivasan, Ramakrishnan (2013b,a) extended CA to solve the multi-objective optimisation of classification rules. In the extended version of CA, the individuals are categorised into imitator, cautious, and risk taker groups. Then, the new generation of solutions is produced by considering these traits. Cabrera et al. (2011) presented a bio-objective version of CA (BOCA), in which the *S* metric was used as a metric for comparing two non-dominated sets. In BOCA, a given formula is also used to evaluate the fitness function of each individual.

Lagos et al. (2014) proposed a bi-objective CA (BOCA) to solve uncapacitated facility location problem and compared the effect of considering different knowledge components in the belief space. The results reported in this reference shows that the BOCA with history knowledge performs significantly better than the BOCA with normative and situational knowledge components. Guo et al. (2008) employed CA for multi-objective optimisation, in which the distributions of non-dominated individuals in objective space were stored in the implicit knowledge component. Gu, Wu (2010) adapted CA to solve multi-objective problem of water resources optimisation, in which a non-dominate set is added to the general framework of CA. Coello, Becerra (2003) proposed to use CA with Pareto-based selection for multi-objective optimisation problems. Authors reported the competitive performance of CA in comparison to NSGA-II.

4.5 Multi-population CA

Researchers have been tried to develop multi-population versions of CAs. In some cases, the multi-population versions of CAs have been hybridised with other algorithms as well. For example, Gao et al. (2007) developed cultural particle swarm algorithms (CPSAs) by combining CA with PSO algorithm for constrained multi-objective optimisation. In CPSA algorithm, the population space is divided into n+1 subswarms, in which n sub-swarms optimise the n single objective functions separately, and one sub-swarm searches for the solution of multi-objective problem. Then, the knowledge obtained from each sub-swarm are used to update the belief space. Another multi-population version of CA was suggested by Zadeh, Kobti (2015), in which the solutions were divided into several sub-populations with a single belief space. The comparisons results revealed that the approach can provide better results than GA and DE. The general framework of their algorithm is very similar to the hybrid CPSA algorithm developed by Gao et al. (2007).

In another study, Yang, Gu (2014) presented a quadspace cultural genetic tabu algorithm (QSCGTA) to solve the JSP. In QSCGTA, the population space of CA is divided into junior and senior sub-populations with separate belief spaces. In QSCGTA, the junior population space is evolved by GA operators, while the senior population space is formed by TS algorithm. The reported graphs by authors show that the QSCGTA requires less computational time in comparison to NSGA-II and multi-objective version of PSO method. However, QSCGTA seems to be more complicated than the standard CA for implementation. Xu et al. (2012) introduced a multi-population cultural DE (MCDE) algorithm. The MCDE is consisted of two population spaces with separate belief spaces, which are evolving based on the DE operators. In MCDE, these populations exchange the information stored in their belief spaces. A diversity preservation mechanism was also used to alleviate premature convergence. Authors compared the results obtained by the MCDE to those yielded by PSO and GA. However, it seems that further investigations need to be carried out to compare the computational efficiency of MCDE against the standard CA. Unold, Tarnawski (2017) proposed a novel parallel framework for multi-population CA (MPCA), in which each sub-population has a separate belief space. In MPCA, a communication mechanism is provided to exchange the information between the sub-populations. The framework provides the possibility of using any heuristic approach for evolving population and belief spaces. However, the difficulties in implementation cannot be ignored.

Guo et al. (2011b) developed another multi-population version of CA, named multi-population CA adopting knowledge migration (MCAKM), in which a knowledge migration concept is proposed to provide more efficient interaction and cheaper communication between the sub-populations. In MCAKM, the implicit knowledge extracted from the evolution process of each sub-population is shared with other subpopulations based on the knowledge migration. Similar concepts were also adopted by Kobti, others (2012) and Guo et al. (2009) in a multi-population CA (MP-CA). In MP-CA, the sub-populations perform as parallel local CAs and the extracted knowledge by each sub-population is shared with others in the form of structured belief using the knowledge migration. From the results reported in these references, it seems that the MP-CA performs statistically better than the CA with single-population for JSPs. Xu et al. (2010) suggested a multi-population cultural DE (MCDE), in which a knowledge exchange mechanism is defined to improve the evolution process. It seems that authors used a mechanism to prevent cultural fusion and preserve the diversity, in which the main aim is to keep the populations away from each other in the search space.

Digalakis, Margaritis (2002) presented the parallel co-operating CA (PARCA) to solve the maintenance scheduling problem. In PARCA, several sub-populations investigate the different data set, in which the information exchange is performed by the accepted individuals from each sub-population. Authors added different local search mechanisms to PARCA. However, the advantages of PARCA over the standard CA were not investigated. Guo et al. (2011c) developed a multi-population cooperative CA (MCCA) with several sub-population spaces evolving with the competition cooperative GA. Authors verified the efficiency of MCCA against GA and cooperative GA. Kobti, others (2014, 2013) introduced a heterogeneous multi-population CA (HMP-CA) with a set of sub-populations communicated through a shared belief space. In HMP-CA, each sub-population optimises a given variable using DE mutation operator.

Ali, Awad (2014) suggested niche CA (NCA) method based on the parallel niche groups and niche clearing techniques, in which the population space is divided into several parallel sub-populations to perform local search separately. In each iteration, the poor solutions are removed from the population based on the niche-clearing technique. Yan et al. (2016) suggested another niching CA (NCA) with population space divided into several sub-populations. The approach modifies the size of each subpopulation based on the average fitness values. In this approach, the belief space is divided into several parts, in which the knowledge components related to each subpopulation are stored in a given part. In another study, Yan et al. (2017b) introduced an improved CA based on the niche isolation technology, in which each sub-population evolves independently. It seems that the general performance of the approach is significantly better than the standard CA and GA in terms of the solution quality and required computing time. A clan-based CA approach was developed by Oloruntoba et al. (2019) by defining multiple clans (or sub-populations) with independent local belief spaces. Authors defined a migration mechanism to exchange information between the clans. The algorithm has a global belief space to gather the information obtained from the different clans. From the results reported in this reference, it seems that the clan-based CA can provide more accurate results than PSO, ACO, and CS algorithms.

Reynolds, Kinnaird-Heether (2017a) developed a population mechanism as a new knowledge distribution method, which mimics the specialist or professional networks in real human societies. The population mechanism models several evolving networks between the individuals, which lead to development of several subcultures within the system. In another study, Gawasmeh, Reynolds (2014) investigated the impact of the sub-cultures on the performance of CAs, in which they concluded that the presence of sub-cultures can improve the computational efficiency. Singh et al. (2018) proposed a multi-population version of CA, in which the population is divided into several groups evolving separately. Then, some selection operators are used between the groups to generate new solutions. Chen, Yang (2015) developed a DE-based symbiotic CA

(DESCA) to design a fuzzy system. In DESCA, each individual represents a fuzzy system consisting of a set of fuzzy rules. The algorithm divides the population space into several sub-populations, in which the individuals in each sub-population represent a given rule of fuzzy system. The new rules in DESCA are generated by using mutation strategies of DE algorithm and knowledge components of CA. It is observable from the convergence diagrams and results reported in this reference that DESCA performs significantly better than the different versions of DE algorithm.

From the literature review on the multi-population versions of CA, it seems that the solution quality of CA can be enhanced by dividing its population. However, the main question in this area is how these techniques are computationally efficient than the standard CAs? It seems that this question has not been answered in the abovementioned references.

4.6 Chaotic CA

In recent years, the chaotic theory in nonlinear dynamics has attracted much attention from the researchers in the field of the meta-heuristic algorithms. Researchers have been used the chaotic sequences as pseudo-random sequences to improve the performance of meta-heuristic algorithms. There are some studies that the chaotic maps were integrated with CA to achieve better performance. For example, Wang et al. (2009) developed a chaos culture PSO (CCPSO) for function optimisation. In CCPSO, the population space is evolved by the PSO algorithm and a chaotic search based on the logistic chaotic map is performed within the belief space to prevent premature convergence. Pan et al. (2016) designed a CA with chaotic behaviour, in which the individuals are evolving within both of the population and belief spaces. Authors used GA to evolve the population space, while the individuals in the belief space are evolved by chaotic ACO algorithm. It seems that the approach performs computationally cheaper than conventional GA for cell allocation in CMOL circuits with the hybrid CMOS/nanodevice structure. Guo et al. (2011a) proposed a mixed mutation operators based on the Cauchy+chaotic and Cauchy+Gaussian mutation operators within the CA.

Cheng et al. (2009) suggested an adaptive chaotic CA by using a mutation based on the chaotic logistic map within the population space. Although the approach was able to provide accurate results, the effects of applying chaotic map on the performance of CA were not investigated. Chen et al. (2013) presented a cultural evolution algorithm, which uses PSO operators and chaotic maps to evolve the population and belief spaces. He, Xu (2011) developed a chaotic CA (CCA), in which two chaotic-based influence functions using normative and situational knowledge components were proposed. In these chaotic influence functions, chaotic random variables were generated based on the logistic map function.

Although the literature review described in this subsection reveals that researchers have been tried to model the chaotic behaviour in CAs, the questions on the effects of chaotic sequences on the performance of CAs remain unaddressed. Furthermore, all of the above-mentioned references have been used the logistic map function to model chaotic behaviour in CAs. It seems that the suitability of other chaotic maps on the performance of CAs should be investigated as well, such as Logistic, Kent, Bernoulli shift, Sine, and Circle maps.

4.7 Fuzzy CA

Literature review reveals that some researchers combined CA with fuzzy sets theory to speed up its convergence speed. For example, Reynolds, Zhu (2001) developed a fuzzy CA approach with fuzzy acceptance and influence functions, in which the knowledge components of the belief space were represented based on the fuzzy framework. Authors investigated different configurations of fuzzy components on the algorithm's performance. Ali et al. (2018) introduced a balanced fuzzy CA with a modified Levy flight search (b-fCA+mLF) based on the modified fuzzy acceptance and influence functions. Authors designed the fuzzy acceptance function in a way to accept a large number of individuals in initial iterations, while this number is gradually reduced over time. In the influence functions, authors defined a fuzzy parameter to control the search direction. A similar approach was also suggested by Muhamediyeva (2020). Alami et al. (2007) developed a multi-population CA based on the fuzzy clustering, in which the fuzzy clustering is used to divide the population space into a set of sub-populations. Feng, Zhang (2008) proposed a self-adaptive CA based on the fuzzy controller to speed up the convergence of the standard CA. Authors suggested to define the number of accepted individuals as a fuzzy number. Reynolds, Chung (1997a) proposed a new acceptance function using a fuzzy inference engine to select the individuals from the population space. From the results reported in this reference, it can be seen that the CA with fuzzy acceptance function provides significantly better results than those provided by CA with static acceptance function.

4.8 CAs with EP and GP

Some studies have been done on combining CAs with EP and GP approaches. Coello Coello, Becerra (2004) applied CA to improve the performance of EP technique. The study uses the domain knowledge component of belief space to identify the feasible regions of search space and guide the search process. Coello, Becerra (2002b) proposed an approach based on CAs and EP for constrained optimisation. Chung, Reynolds (1996b) used CAs to provide the self-adaptation at the population level for EP systems. Comparisons of the convergence diagrams revealed that CA can significantly accelerate the convergence speed of EP. Huang et al. (2008) developed a CA with multi-layer belief spaces and EP. Ostrowski, Reynolds (2004, 2005) applied a chained CA with GP to evolve strategies for recessionary markets, in which two separate CAs were used for each of the black-box and white-box testings. Reynolds et al. (2008b) simulated the evolution of an ancient urban centre via embedding the GP within CA. Zannoni, Reynolds (1997) proposed a novel CA and GP (CAGP) computational system to solve symbolic regression problem, which is consisted of the population and belief components. In CAGP, the programs and genetic operators follow the constraints provided by the belief space. The convergence diagrams reported by authors illustrate the superiority of CAGP over pure GP approach.

5 Publication analysis

In this section, some information about the publications related to the CAs are provided. This survey paper considers around 245 references related to CAs and their variants in different publications, such as IEEE, Elsevier, Springer, Taylor Francis, and John Wiley. These references are consisting of books, journal and conference papers. Fig. 5 shows the chronological distribution of related publications to the CAs over the recent years. Fig. 5 illustrates that CAs have attracted significant attentions in the last decade. In addition, Fig. 6 illustrates the distribution of CAs related publications in different research areas. From this figure, it can be seen that CAs have been attracted much attention from the researchers in computer science and electrical engineering. Moreover, Fig. 7 presents the distribution of the investigated references in different publishers, including Springer, IEEE, Elsevier, and others. As it can be seen from Fig. 7, most of these references are published by IEEE and Springer.



Figure 5: The number of publications related to CAs in recent years.



Figure 6: Distribution of CA related papers in different fields.



Figure 7: The references of CA in different publications.

6 Conclusions and future research directions

This paper attempted to provide a comprehensive survey of CAs and their applications in science and engineering as well as their development during the recent years. Regarding the applications, CAs have been successfully applied to solve a wide variety of optimisation problems in science and engineering, such as civil engineering, mechanical engineering, chemical engineering, electrical engineering and computer science. Literature review of most relevant references has demonstrated that CAs have been attracted much attention from the researchers in computer science and electrical engineering. The review of this body of literature has revealed that a significant number of variants of CAs have been developed by researchers, which can be categorised into hybridised, improved, multi-objective, multi-population, chaotic, and fuzzy variants. Although the literature review indicates the maturity of CAs as effective optimisation approach, there are significant issues remaining to be addressed in future works, as follows:

- In the last decade, numerous novel meta-heuristic algorithms have been introduced in literature. The review has reveled that most of the hybridised versions of CAs with other meta-heuristic algorithms have been focused on the hybridising CAs with GA, DE, and PSO approaches. Therefore, it may be of interest to investigate hybridisation possibilities of CAs with other recently developed metaheuristic algorithms. The capabilities of the belief space to provide multiple types of knowledge sources can be efficiently combined with other meta-heuristic techniques.
- Literature survey revealed an overall consensus that properly designed influence functions have the potential to use the information of the knowledge sources more efficiently. The key point in designing the influence functions is keeping the diversity of the population space to prevent premature convergence. Therefore, developing efficient influence functions to deal with the different problem environments seems to be vital, and future research should take a few steps forward to this direction.
- Another important component of CAs is the update function, which is used to update the information stored within the knowledge sources of the belief space.

In some cases, the knowledge sources are failed to provide useful information for the population space, which leads to the premature convergence of the algorithm. For example, too narrow normative intervals can easily reduce the convergence speed of the algorithm. Thus, there is a need for future research on improving the update function to keep the cultural information useful during the solutionfinding process.

- Analysis of the literature demonstrated that researchers have been used different types of knowledge sources within the belief space to deal with different problems. However, the current state of research does not provide a clear analysis of the impact of various knowledge sources and their coordination on the performance of CAs. Hence, a rigorous study on the impacts of the knowledge sources in different problem environments seems to be necessary.
- Another important challenge is the adapting CAs to solve multi-objective optimisation problems. Literature survey has also revealed that CAs are put forward to solve multi-objective problems in different areas of science and engineering. However, even if research is successful in applying CAs to solve these types of problems, there will certainly be gaps that require research to address this challenge. Future research will definitely be required to design a unified framework for CAs to deal with these problems.
- Although CAs are capable of dealing with the dynamic optimisation problems, their performance on dynamic problems has been rarely investigated. Hence, the capabilities of CAs in solving the dynamic problems needs to addressed in future researches.

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