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A Subsidy That Is Inversely Related to the Product Price

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Abstract

This study considers a new subsidy design to support the purchase or production of target products. Under the proposed design, subsidy payments are inversely related to product prices. Compared to 'flat' subsidies, this design reduces producers' market power and the subsidy benefits passed on to them, improving the cost-effectiveness of government spending (by up to 50% according to simulations based on an actual subsidy programme). Additionally, this subsidy's cost-effectiveness and incidence can be adjusted flexibly by changing the policy parameters. Finally, the subsidy design can be modified to provide larger payments to higher-quality products, thereby offsetting disincentives for quality improvement.

Keywords: IR subsidy, cost-effectiveness, incidence, imperfect competition, supermodular games

JEL Classification: D43, H21, H22, H23, L13, Q58

1 Introduction

Many subsidy programmes in various countries aim to promote the purchase or production of target goods for such reasons as positive externalities, merit-good characteristics, and distributional concerns. These programmes partially offset the purchase or production costs of these goods through financial incentives offered to consumers or producers, such as grants, rebates, and tax credits or deductions. The target goods of these programmes may be, for example, 'green' technologies (e.g. electric vehicles (EVs) and solar photovoltaic (PV)

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panels), childcare and education (e.g. nurseries), healthcare (e.g. treatment, insurance, and pharmaceuticals), and housing (e.g. purchases and rentals). Typically, the subsidy payment for a unit of a good is either independent of its price (in the case of 'flat' or specific subsidies) or proportional to its price (in the case of *ad valorem* subsidies).[1] Some subsidies use a mixture of the two forms (e.g. an *ad valorem* subsidy with a cap, as in reference pricing for pharmaceuticals). This study proposes a new subsidy form to be used in programmes supporting the purchase or production of target goods. The proposed form includes a mechanism that can reduce producers' incentives to set high prices and thus significantly improve the cost-effectiveness of these programmes. As an illustration, simulations based on an actual EV subsidy in the US indicate that switching from the current specific subsidy to the proposed form would increase the market sales by up to 50%, holding total government spending on the programme constant.

Under imperfect competition, the form of taxation and subsidisation (e.g. a specific or *ad valorem* subsidy) has welfare implications.[2] Many previous studies use theoretical models of imperfect competition in a closed-economy context and analyse the relative efficiency and cost-effectiveness of different policy designs.[3] In the case of taxes, Suites and Musgrave (1953), Delipalla and Keen (1992), Skeath and Trandel (1994), Anderson *et al.* (2001a), and Hamilton (2009), among others, compare specific and *ad valorem* taxes and show that in most settings, an *ad valorem* tax is welfare-superior to a specific tax that raises the same amount of revenue. Myles (1996), Hamilton (1999), and Carbonnier (2014) examine more general tax schemes that include specific and *ad valorem* taxes as special cases. A key determinant of the relative efficiency and cost-effectiveness of different policy designs is how they affect the elasticity of demand faced by producers. In the case of subsidies, Valido *et al.* (2014) and Liang *et al.* (2018) contrast specific and *ad valorem* subsidies. In the policy designs that these studies analyse, the tax or subsidy payment per unit of a product is constant or increasing in the product's price. From an economic policy perspective, the subsidy design that I consider is fundamentally different in that the subsidy payment decreases with the product price.

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[1]In effect, an income tax credit is a specific subsidy that is equivalent to the amount of the credit, and an income tax deduction is an *ad valorem* subsidy at the marginal income tax rate.

[2]In contrast, under perfect competition, in which firms are price takers with no market power, the form of taxation and subsidisation has no effects in equilibrium.

[3]Some studies investigate differences between the specific and *ad valorem* forms of import tariffs or export subsidies in open-economy settings in which domestic and foreign producers are treated differently (e.g. Brander and Spencer, 1984; Collie, 2006). This study focuses on a closed-economy setting in which producers are treated equally regardless of nationality, so export subsidies are outside the scope of this study.

Table 1: *National Subsidy for Residential Solar PV Installation in Japan*

Fiscal year	Rebate (¥/kW)	Condition on pre-rebate price p_{pre} (¥/kW)
2009	0	if $700,000 < p_{\mathrm{pre}}$
	70,000	if $\phantom{700,000 <} p_{\mathrm{pre}} \leq 700,000$
2010	0	if $650,000 < p_{\mathrm{pre}}$
	70,000	if $\phantom{650,000 <} p_{\mathrm{pre}} \leq 650,000$
2011	0	if $600,000 < p_{\mathrm{pre}}$
	48,000	if $\phantom{600,000 <} p_{\mathrm{pre}} \leq 600,000$
2012	0	if $550,000 < p_{\mathrm{pre}}$
	30,000	if $475,000 < p_{\mathrm{pre}} \leq 550,000$
	35,000	if $\phantom{475,000 <} p_{\mathrm{pre}} \leq 475,000$
2013	0	if $500,000 < p_{\mathrm{pre}}$
	15,000	if $410,000 < p_{\mathrm{pre}} \leq 500,000$
	20,000	if $\phantom{410,000 <} p_{\mathrm{pre}} \leq 410,000$

This table shows the rebate rate per kilowatt (kW) of residential solar PV capacity installation. This rebate rate depends on the pre-rebate, per-kW transaction price of the PV system installed by each household.

Source: Japan Photovoltaic Energy Association

This study is motivated by a subsidy programme in Japan that has a distinct feature relative to standard specific or *ad valorem* subsidy schemes. In this national subsidy (rebate) programme for installing residential solar PV systems, the rebate rate per unit of PV capacity decreases with the pre-rebate, unit price of the system, thereby giving sellers and buyers an incentive to trade at lower pre-rebate prices. Specifically, Table 1 shows that as the transaction price of a solar PV system per kilowatt (kW) of capacity (including installation and other related costs) falls, the buyer (i.e. the household) becomes eligible for a greater rebate per kW of capacity. For example, in 2012, a household received no rebate if the pre-rebate, per-kW transaction price of the installed system was above ¥550,000, a rebate of ¥30,000 per kW if this price was between ¥475,001 and ¥550,000, and a rebate of ¥35,000 if this price was equal to or below ¥475,000.[4] The demand for each product discontinuously changes at each price threshold, incentivising sellers to take advantage of this structure. Thus, policymakers expected that this scheme could lower not only consumer prices (i.e. post-rebate out-of-pocket prices paid by households) but also producer prices (i.e. pre-rebate transaction prices received by sellers).

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[4]A UK subsidy scheme for electric and hybrid vehicles adopted a similar scheme in 2016 with a threshold at the vehicle price of £60,000.
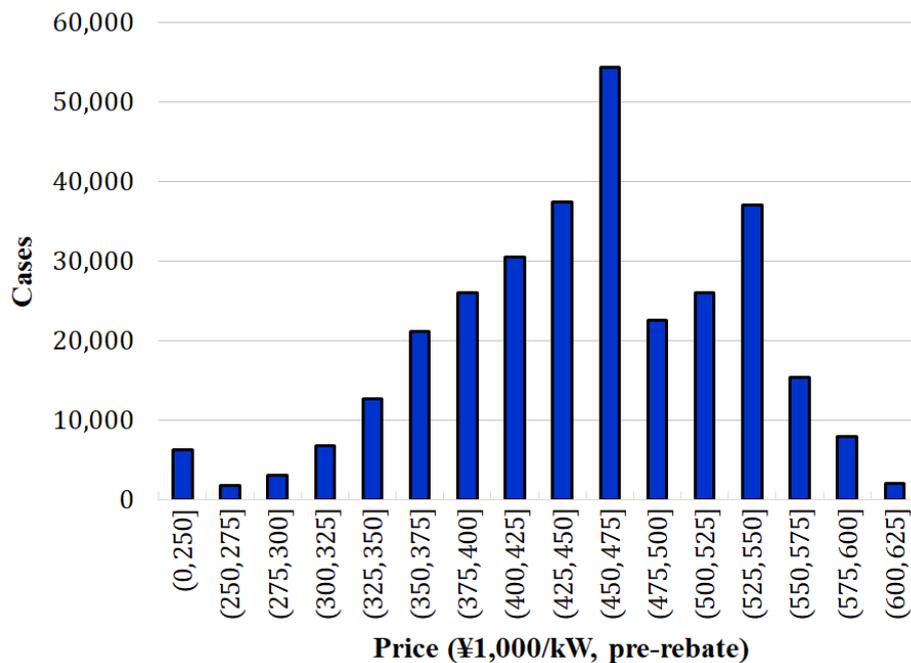
Figure 1: *Distribution of Residential Solar PV System Prices (2012)*
Source: RTS Corporation and the Ministry of Economy, Trade and Industry of Japan

This makes a clear contrast with the case of specific and *ad valorem* subsidies because these standard subsidies typically decrease the consumer prices but *increase* the producer prices. In other words, this unique design was expected to 'overshift' the producer prices, leading to larger reductions in the post-rebate consumer prices and faster diffusion of solar PV systems than in the case of a specific or *ad valorem* subsidy.

Transaction data suggest that this subsidy design indeed worked well in lowering the pre-rebate producer prices in addition to the post-rebate consumer prices. Figure 1 shows the pre-rebate price distribution of installed residential solar PV systems for 2012. The solar PV system prices are bunched in the bins just below the threshold prices (¥475,000 and ¥550,000), indicating that sellers had price-setting power, accounted for the subsidy rule, and traded at lower prices than they would without this subsidy scheme. As the threshold prices were reduced significantly in each year, the subsidy design, along with declining production costs, kept providing downward pressure on the producer prices, further accelerating solar PV diffusion. The subsidy programme was phased out in 2014 following the rapid expansion of residential solar PV capacity.

Despite this thought-provoking observation, to the best of my knowledge, no previous study uses an economics framework to analyse the effects of a subsidy that is inversely related to

a target product's price. This study therefore proposes and evaluates a new subsidy scheme with this property. More specifically, I consider a subsidy that is offered for the purchase or production of a good on the condition that the good's price is below a government-set threshold. Additionally, as the good's price decreases, the per-unit subsidy payment increases in proportion to the difference between the threshold and the price.[5] In other words, the government sets two policy parameters: the price threshold for subsidy eligibility and the rate at which the subsidy payment increases as the price falls (i.e. $\overline{p}_i$ and r in the following sections). Based on a model of imperfect competition (i.e. Bertrand competition with product differentiation) and the theory of supermodular games, I compare this inversely related (IR) subsidy to the benchmark case of no subsidy and to the cases of the widely used specific and *ad valorem* subsidies in terms of various Nash equilibrium (NE) characteristics (e.g. output, price, producer and consumer surplus, and government spending).

From the government's perspective, the proposed design has advantages in cost-effectiveness and flexibility. First, the IR form is more cost-effective than the specific and *ad valorem* forms in the sense that it requires less government spending to induce a target output level through subsidisation. Equivalently, for a given subsidy budget, the IR form can realise more output than the specific and *ad valorem* forms. The IR form is more cost-effective because it makes the demand curve *faced by producers* more elastic. In effect, producers are partially compensated for cutting prices, so a £1 reduction in the consumer price can be achieved with a smaller reduction in the producer price. This means that the IR form increases the elasticity of demand with respect to the producer price. Elastic demand erodes producers' power to maintain high prices, thereby making it easier for policymakers to induce lower prices and more sales.

Second, the use of two policy parameters in the IR form provides an additional advantage of flexibility. In inducing a given output level, the government can also choose the two policy parameters to adjust the share of the subsidy benefit that is passed on to producers (i.e. incidence) and the subsidy budget required to achieve the target (i.e. cost-effectiveness). The government can flexibly make this adjustment in accordance with the policy objectives and market circumstances and within certain limits imposed by producers' rational behaviour. The model in Section 3 shows that, depending on the parameter values, a firm's equilibrium profit under the IR form can be higher or even lower than in the case of no subsidy. That is,

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[5]Under the subsidy schedule in Table 1, the subsidy payment and hence demand are both discontinuous at the threshold prices. For tractability, I consider a subsidy schedule that is continuous in the product price.

in addition to increasing the supply and consumption of subsidised goods, the IR form can be used to financially support producers in an emerging industry (e.g. EV manufacturers) or to lower economic rents due to imperfect competition. Note that the specific and *ad valorem* forms do not offer such flexibility because they each have just one policy parameter.

Counterfactual simulations based on actual market data reveal that the IR scheme has substantial impacts. I construct a hypothetical market using data from the 2017 US EV market, in which buyers were eligible for a specific subsidy of $7,500 from the federal government. I then use this constructed market to simulate the impacts of replacing the original specific subsidy with an IR subsidy so that the market output (50,981 EVs) or total subsidy budget ($382 million) remains constant. The simulations suggest that to induce this output level, the IR form requires up to $4,600 ($5,100) or, equivalently, 61% (68%) less government spending per unit of output than the specific form does, where the two sets of estimates reflect different production cost scenarios. Alternatively, keeping the subsidy budget the same as in the case of the original specific subsidy, the IR form can induce up to 48% (50%) more sales.

Finally, the IR subsidy, like the widely used *ad valorem* tax, may disincentivise product quality improvements. Both schemes make quality improvements more costly because increasing the pre-subsidy (pre-tax) price to reflect the improved quality results in a lower subsidy (higher tax) payment.[6] I show that the IR subsidy can offset this disincentive if the price threshold increases with product quality so that higher-quality products can receive larger subsidy payments. In practice, this result implies that the IR form works better when enough information is available about product characteristics. For example, such products include renewable and energy-efficient technologies (e.g. solar PV systems and EVs) and pharmaceuticals, for which subsidy payments are often quality dependent in existing specific or *ad valorem* subsidy programmes.

The rest of the paper is organised as follows. Section 2 builds a model of Bertrand competition with product differentiation, defines the IR subsidy, and compares the outcomes under various subsidy schemes. Section 3 further analyses the NEs under the IR subsidy, focusing on the roles of the two IR subsidy parameters. Section 4 quantifies the impacts of the IR subsidy through counterfactual simulations based on an actual EV subsidy programme in the US. Section 5 extends the model by incorporating product quality and discusses ways to supplement the IR subsidy to offset disincentives for quality improvement. Section 6 concludes.

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[6]See Keen (1998) for a detailed description of this issue in the case of *ad valorem* taxation.

2 Theoretical Framework

2.1 Basic Setup

Consider a market with n firms (with $n \geq 2$) in which firm i produces a differentiated product with a constant marginal cost c_i (with $c_i > 0$). The demand q_i for firm i's product is given by $q_i = D_i(p_i, \mathbf{p}_{-i})$, where $D_i: [0, p^{\max}] \times [0, p^{\max}]^{n-1} \to \mathbb{R}_+$ is a continuous function, p_i is product i's price, $\mathbf{p}_{-i}$ is a vector of the prices of the other $n - 1$ firms' products, and $p^{\max}$ is large enough to give zero demand for any product regardless of the other products' prices.[7]

For each i, the demand function D_i also satisfies the following properties. It is strictly decreasing in p_i (where $D_i > 0$). It is also increasing in p_j for all $j \neq i$ and is strictly increasing in p_j where $D_i > 0$ and $D_j > 0$, implying that the products are gross substitutes.[8] In addition, I make an assumption that is common in studying games with strategic complementarities such as Bertrand competition with product differentiation (e.g. Milgrom and Roberts, 1990; Vives, 2005; Amir, 2005): if $p_i \geq p_i'$ and $\mathbf{p}_{-i} \geq \mathbf{p}_{-i}'$ (i.e. $p_j \geq p_j'$ for all $j \neq i$), then

$$D_i(p_i, \mathbf{p}_{-i})D_i(p_i', \mathbf{p}_{-i}') \geq D_i(p_i, \mathbf{p}_{-i}')D_i(p_i', \mathbf{p}_{-i}), \tag{1}$$

which means that $\log D_i$ exhibits increasing differences in p_i and $\mathbf{p}_{-i}$ (where $D_i > 0$):

$$\log D_i(p_i, \mathbf{p}_{-i}) - \log D_i(p_i', \mathbf{p}_{-i}) \geq \log D_i(p_i, \mathbf{p}_{-i}') - \log D_i(p_i', \mathbf{p}_{-i}'). \tag{2}$$

If D_i is twice continuously differentiable, then (2) is equivalent to the condition that for each $j \neq i$,

$$\frac{\partial^2 \log D_i(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j} = \frac{1}{D_i(p_i, \mathbf{p}_{-i})^2} \left[D_i(p_i, \mathbf{p}_{-i}) \frac{\partial^2 D_i(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j} - \frac{\partial D_i(p_i, \mathbf{p}_{-i})}{\partial p_i} \frac{\partial D_i(p_i, \mathbf{p}_{-i})}{\partial p_j} \right] \geq 0. \tag{3}$$

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[7]The demand function $q_i = D_i(p_i, \mathbf{p}_{-i})$ can be considered the result of the following optimisation problem of a representative consumer with quasi-linear utility $U(x, q_1, \cdots, q_n) = x + u(q_1, \cdots, q_n)$ (see Vives (1999, Ch. 3) for more details):

$$\max_{x, q_1, \cdots, q_n} x + u(q_1, \cdots, q_n) \text{ s.t. } x + \sum_i p_i q_i \leq I,$$

where x is the numéraire good (i.e. the composite of all other goods besides the n firms' products), and I is income. An interior solution is characterised by $\partial u(q_1, \cdots, q_n) / \partial q_i = p_i \ \forall i$. Thus, the inverse demand function for product i can be expressed as $p_i(q_1, \cdots, q_n) = \partial u(q_1, \cdots, q_n) / \partial q_i \ \forall i$. Inverting the system of inverse demand functions gives the demand function for each product i as $q_i = D_i(p_1, \cdots, p_n)$. Finally, with quasi-linear utility, the representative consumer assumption is not restrictive.

[8]Throughout this paper, a (single-valued) function f is increasing (decreasing) if $x > y$ implies $f(x) \geq (\leq) f(y)$. It is strictly increasing (decreasing) if $x > y$ implies $f(x) > (<) f(y)$.

An intuitive economic implication of (3) is that the own-price elasticity of demand for product i (i.e. $-(\partial D_i / D_i)/(\partial p_i / p_i)$) is decreasing in the price of another product j. That is, the demand for product i becomes less own-price elastic as p_j increases. This condition is satisfied by a large class of demand functions, including linear, logit, CES, and translog demand functions, among others (Milgrom and Roberts, 1990). Note that I make no assumption regarding the concavity or convexity of D_i or $\log D_i$.

I consider Bertrand competition without and with a subsidy. First, in the baseline case of no subsidy, firm i's profit is given by

$$\pi_{iN}(p_i, \mathbf{p}_{-i}) = (p_i - c_i)D_i(p_i, \mathbf{p}_{-i}). \tag{4}$$

Next, suppose that the government offers a subsidy to consumers or producers for buying or selling a unit of a target product. Importantly, this study's results apply regardless of whether consumers or producers are the direct recipients of the subsidy (physical neutrality; see e.g. Weyl and Fabinger, 2013). Throughout this paper, p_i refers to the consumer price, namely, the effective post-subsidy price that a consumer pays out of pocket. Demand depends on the consumer price. The producer price (i.e. the price received by firm i) is denoted by p_i^p and equals the sum of p_i and the subsidy payment. If the government offers a specific subsidy of s_i per unit of good i (where $0 < s_i < c_i$), then firm i's profit is given by

$$\pi_{iS}(p_i, \mathbf{p}_{-i}; s_i) = (p_i + s_i - c_i)D_i(p_i, \mathbf{p}_{-i}). \tag{5}$$

If the government offers an *ad valorem* subsidy of ap_i per unit of good i (where $0 < a$), then firm i's profit is given by

$$\pi_{iA}(p_i, \mathbf{p}_{-i}; a) = [(1 + a)p_i - c_i]D_i(p_i, \mathbf{p}_{-i}). \tag{6}$$

The theory of supermodular games is useful for analysing Bertrand competition with product differentiation (e.g. Milgrom and Roberts, 1990; Vives, 2005; Amir, 2005). It follows from

(1) that $\log \pi_{iN}$, $\log \pi_{iS}$, and $\log \pi_{iA}$ satisfy increasing differences in p_i and $\mathbf{p}_{-i}$,[9] making the corresponding Bertrand competition a log-supermodular game. Thus, in each of these settings, firm i's best response correspondence is increasing in $\mathbf{p}_{-i}$, and at least one (pure-strategy) NE exists.[10] Moreover, if there are multiple NEs in one of these settings, then an NE exists at which every product's price is higher than its price at any other NE in this setting. This coordinate-wise largest NE Pareto dominates other NEs in terms of each firm's profit and thus is the Pareto-best NE for the firms in this setting.

2.2 IR Subsidy

I now define the IR subsidy form proposed in this study. The government conditionally offers a subsidy that is inversely related to the consumer price of the target product. No subsidy is offered if the consumer price is greater than or equal to a certain threshold $\overline{p}_i$ that is set by the government (i.e. no subsidy is available if $p_i \geq \overline{p}_i$). If the price is below $\overline{p}_i$, then the subsidy per unit of the good increases linearly as the price *decreases*. Specifically, if $p_i < \overline{p}_i$, then a subsidy of $r(\overline{p}_i - p_i)$ is provided per unit of the good traded (i.e. the producer price is $p_i^p = p_i + r(\overline{p}_i - p_i)$), where $0 < r < 1$ and $r\overline{p}_i < c_i < \overline{p}_i$.[11] Equivalently, in terms of the producer price p_i^p, the subsidy payment is given by $r^p(\overline{p}_i - p_i^p)$, where $r^p = r/(1-r)$.[12] The threshold $\overline{p}_i$ may vary across i (e.g. Section 5 considers an extension in which $\overline{p}_i$ depends on

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[9]If $p_i \geq p_i' \geq c_i$ and $\mathbf{p}_{-i} \geq \mathbf{p}_{-i}'$, then it follows from (1) that

$$\begin{aligned}
\pi_{iN}(p_i, \mathbf{p}_{-i})\pi_{iN}(p_i', \mathbf{p}_{-i}') &= (p_i - c_i)(p_i' - c_i)D_i(p_i, \mathbf{p}_{-i})D_i(p_i', \mathbf{p}_{-i}') \\
&\geq (p_i - c_i)(p_i' - c_i)D_i(p_i, \mathbf{p}_{-i}')D_i(p_i', \mathbf{p}_{-i}) \\
&= \pi_{iN}(p_i, \mathbf{p}_{-i}')\pi_{iN}(p_i', \mathbf{p}_{-i}).
\end{aligned}$$

Thus, $\log \pi_{iN}$ satisfies increasing differences in p_i and $\mathbf{p}_{-i}$ (where $\pi_{iN} > 0$):

$$\log \pi_{iN}(p_i, \mathbf{p}_{-i}) - \log \pi_{iN}(p_i, \mathbf{p}_{-i}') \geq \log \pi_{iN}(p_i', \mathbf{p}_{-i}) - \log \pi_{iN}(p_i', \mathbf{p}_{-i}').$$

Analogously, $\log \pi_{iS}$ and $\log \pi_{iA}$ satisfy increasing differences in p_i and $\mathbf{p}_{-i}$ (where $\pi_{iS} > 0$ and $\pi_{iA} > 0$, respectively).

[10]Throuout this study, I focus on pure-strategy NEs.

[11]These conditions set the range for the subsidy's generosity. The condition $r < 1$ ensures $dp_i^p/dp_i = 1 - r > 0$, implying that the subsidy is not generous enough that the firm can lower the consumer price without reducing the producer price (i.e. per-unit revenue). The condition $c_i < \overline{p}_i$ means that setting $p_i = c_i$ is profitable, and $r\overline{p}_i < c_i$ means that setting $p_i = 0$ results in a loss.

[12]The subsidy payment $r(\overline{p}_i - p_i)$, which is defined in terms of the consumer price p_i, can alternatively be expressed in terms of the producer price p_i^p as $r^p(\overline{p}_i - p_i^p)$. Here, the parameter r^p differs from r, whereas $\overline{p}_i$ is by construction the same as in the p_i-based definition given above. By rearranging $p_i^p - p_i = r(\overline{p}_i - p_i) = r^p(\overline{p}_i - p_i^p)$, I obtain $r^p = r/(1-r)$. Because the function $g\colon (0,1) \to (0,\infty)$ with $g(r) = r/(1-r)$ is bijective (i.e. one-to-one and onto), it does not matter whether the subsidy is defined in terms of the consumer or producer price.

product i's quality). Let the function π_{iU} be defined by

$$\pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i) = [p_i + r(\overline{p}_i - p_i) - c_i]D_i(p_i, \mathbf{p}_{-i}), \tag{7}$$

which is the profit when the subsidy of $r(\overline{p}_i - p_i)$ (which may be negative) is provided unconditionally (i.e. whether or not p_i is below the threshold $\overline{p}_i$). With the threshold in place, firm i's profit under the IR subsidy, denoted by $\pi_{iI}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$, equals either $\pi_{iN}(p_i, \mathbf{p}_{-i})$ (firm i's profit with no subsidy) or $\pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$:

$$\pi_{iI}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i) = \begin{cases} \pi_{iN}(p_i, \mathbf{p}_{-i}) & \text{if} \quad p_i \geq \overline{p}_i, \\ \pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i) & \text{if} \quad p_i \leq \overline{p}_i. \end{cases} \tag{8}$$

By definition, $\pi_{iN}(p_i, \mathbf{p}_{-i}) = \pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$ if $p_i = \overline{p}_i$, so $\pi_{iI}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$ is continuous.

Because $\pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i) = [(1 - r)p_i + r\overline{p}_i - c_i]D_i(p_i, \mathbf{p}_{-i})$, the IR form can be viewed as a combination of an *ad valorem* tax at a rate of r and a specific subsidy of $r\overline{p}_i$, subject to the non-negativity constraint that the subsidy payment equals $\max\{r(\overline{p}_i - p_i), 0\}$. This constraint ensures that the target product, which is mostly associated with social benefits such as positive externalities, is not taxed even if it is priced highly; it does not make sense for the government to impose a special tax on, for example, solar PV systems when it wants to accelerate their diffusion. In this sense, the IR form is related to the model of Myles (1996) (or the more generalised model of Hamilton (1999)), which analyses such a dual scheme within the context of commodity taxation (i.e. for the case of $r(\overline{p}_i - p_i) < 0$ or $\overline{p}_i < p_i$ without the non-negativity constraint mentioned above).[13]

Leaving aside the non-negativity constraint, from an economic policy perspective, the dual scheme operates fundamentally differently depending on whether it is used to calculate a tax payment, $-r(\overline{p}_i - p_i)$ (which is positive for $p_i > \overline{p}_i$), or a subsidy payment, $r(\overline{p}_i - p_i)$ (which is positive for $p_i < \overline{p}_i$). As a tax scheme, it exhibits the standard property, shared by almost all tax and subsidy schemes, that the tax or subsidy payment is non-decreasing in p_i. As a subsidy scheme, however, it has the unique feature that the subsidy payment is decreasing in p_i, as in the motivating example of the solar PV subsidy described in Section 1.

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[13]With respect to market structure, Myles (1996) and Hamilton (1999) consider homogeneous-product Cournot frameworks with identical firms, whereas I use a differentiated-product Bertrand framework with heterogeneous firms.

As shown in Appendix A.1, $\log \pi_{iU}$ and $\log \pi_{iI}$ satisfy increasing differences in p_i and $\mathbf{p}_{-i}$. Thus, as in the previous settings, the Bertrand game under the IR subsidy has at least one (pure-strategy) NE. In particular, it has a coordinate-wise largest NE, which is the Pareto-best NE from the firms' perspective.

The following proposition gives firm i's maximum profit and best response correspondence under the IR subsidy. The best response correspondences of all firms determine the NE(s) mentioned above. The proposition states that a firm's choice to opt in or out depends simply on the relative sizes of $\max_{p_i} \pi_{iN}(p_i, \mathbf{p}_{-i})$ and $\max_{p_i} \pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$, where maximisation is unconstrained by the subsidy (in)eligibility conditions shown in (8). Thus, the (in)eligibility conditions can be ignored when determining $\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$ because they are implied by the sign of $\max_{p_i} \pi_{iN}(p_i, \mathbf{p}_{-i}) - \max_{p_i} \pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$ (see Lemma 1 in the Appendix).

Proposition 1. *Given r, $\overline{p}_i$, and $\mathbf{p}_{-i}$, suppose that at least one of $\max_{p_i} \pi_{iN}(p_i, \mathbf{p}_{-i})$ and $\max_{p_i} \pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$ is positive, and let $G_i(\mathbf{p}_{-i}; r, \overline{p}_i) = \max_{p_i} \pi_{iN}(p_i, \mathbf{p}_{-i}) - \max_{p_i} \pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$. The maximum profit under the IR subsidy is as follows:*

$$\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i) = \begin{cases} \max_{p_i} \pi_{iN}(p_i, \mathbf{p}_{-i}) & \text{if } G_i(\mathbf{p}_{-i}; r, \overline{p}_i) \geq 0, \\ \max_{p_i} \pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i) & \text{if } G_i(\mathbf{p}_{-i}; r, \overline{p}_i) \leq 0. \end{cases} \tag{9}$$

Let $\psi_{iN}(\mathbf{p}_{-i}) = \arg\max_{p_i} \pi_{iN}(p_i, \mathbf{p}_{-i})$ and $\psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i) = \arg\max_{p_i} \pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i)$. Equation (9) indicates that the best response correspondence ψ_{iI} under the IR subsidy is as follows:

$$\psi_{iI}(\mathbf{p}_{-i}; r, \overline{p}_i) = \arg\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i) = \begin{cases} \psi_{iN}(\mathbf{p}_{-i}) & \text{if } G_i(\mathbf{p}_{-i}; r, \overline{p}_i) > 0, \\ \psi_{iN}(\mathbf{p}_{-i}) \cup \psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i) & \text{if } G_i(\mathbf{p}_{-i}; r, \overline{p}_i) = 0, \\ \psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i) & \text{if } G_i(\mathbf{p}_{-i}; r, \overline{p}_i) < 0. \end{cases} \tag{10}$$

Proof. See the Appendix. ∎

As an illustration, I consider the case of a duopoly with linear demand $D_i(p_i, p_j) = \alpha_i - \beta_i p_i + \gamma_i p_j$, where α_i, β_i, and γ_i are all positive.[14] For this market, Figure 2 shows each

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[14]It is straightforward to show that this demand system satisfies the conditions stated at the beginning of Section 2.1, including the property of increasing differences.
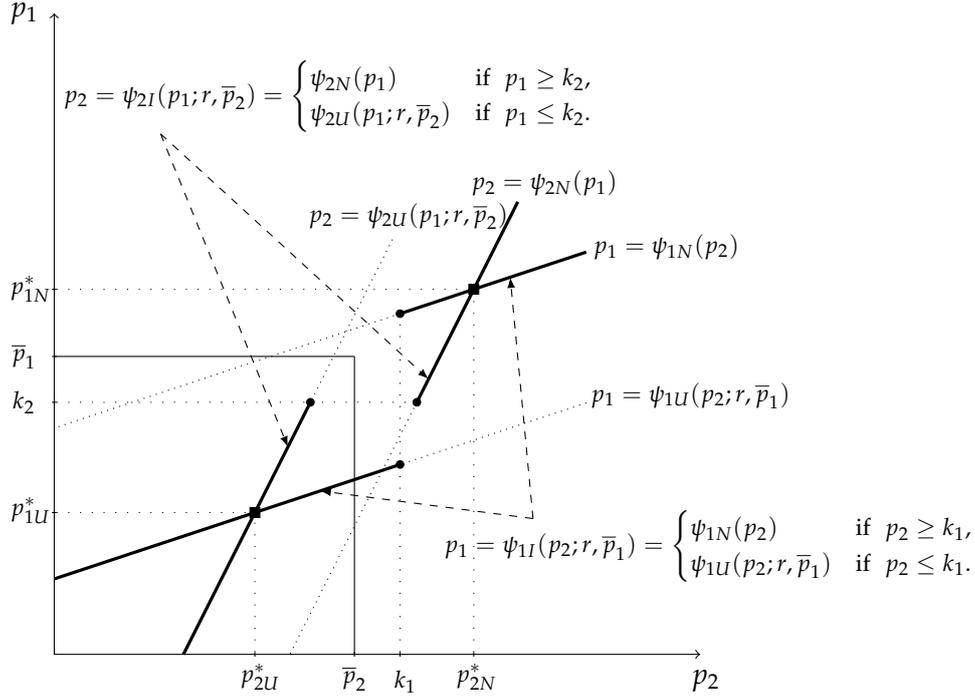
Figure 2: *Duopolists' Best Response Correspondences with an IR Subsidy*

firm's discontinuous best-response correspondence (10) and the resulting NEs. In this setting, $G_i(p_j; r, \overline{p}_i)$ and $p_j - k_i$, where $k_i = \{-\alpha_i + \beta_i[\overline{p}_i + (\overline{p}_i - c_i)(1 - r)^{-0.5}]\}/\gamma_i$, have the same signs. Thus, as shown in Figure 2, firm i's best response $\psi_{iI}(p_j; r, \overline{p}_i)$ switches between $\psi_{iN}(p_j) = \frac{\gamma_i}{2\beta_i} p_j + \frac{\alpha_i}{2\beta_i} + \frac{c_i}{2}$ and $\psi_{iU}(p_j; r, \overline{p}_i) = \frac{\gamma_i}{2\beta_i} p_j + \frac{\alpha_i}{2\beta_i} + \frac{c_i - r\overline{p}_i}{2(1-r)}$ at $p_j = k_i$. Intuitively, given the strategic complementarity of Bertrand competition, if firm j sets a sufficiently high price, firm i should set a price above $\overline{p}_i$ even though it means losing eligibility for the subsidy. A more generous subsidy scheme (i.e. a scheme with a larger r or $\overline{p}_i$) increases k_i, extending the range of p_j in which firm i adopts the scheme.

The NEs of this game are the intersections of $p_1 = \psi_{1I}(p_2; r, \overline{p}_1)$ and $p_2 = \psi_{2I}(p_1; r, \overline{p}_2)$. Figure 2 contains two NEs: (p_{1N}^*, p_{2N}^*), at which both firms opt out, and (p_{1U}^*, p_{2U}^*), at which both firms opt in. The former Pareto dominates the latter. With a different set of $(r, \overline{p}_i)$, ψ_{iU} and k_i would shift, implying that an NE at which both opt in or opt out may not exist. Instead, an NE at which one firm opts in and the other firm opts out may exist (at the intersection of $p_1 = \psi_{1N}(p_2)$ and $p_2 = \psi_{2U}(p_1; r, \overline{p}_2)$ or that of $p_1 = \psi_{1U}(p_2; r, \overline{p}_1)$ and $p_2 = \psi_{2N}(p_1)$). As discussed above, the theory of supermodular games ensures that for a given set of parameters, at least one of these four points is an intersection of $p_1 = \psi_{1I}(p_2; r, \overline{p}_1)$ and $p_2 = \psi_{2I}(p_1; r, \overline{p}_2)$ and thus is an NE.

2.3 Comparative Analysis of Subsidy Forms

This subsection analyses the effects of subsidy policy design on a firm's best response and NEs. For now, I disregard the subsidy eligibility threshold $\overline{p}_i$ and consider $\pi_{iU}(p_i, \mathbf{p}_{-i})$, as defined in (7), rather than $\pi_{iI}(p_i, \mathbf{p}_{-i})$, as defined in (8). That is, the following analysis shows the possible outcomes if the subsidy payment of $r(\overline{p} - p_i)$ (which may be negative) is provided unconditionally without the eligibility threshold. For convenience, this unconditional rule is termed as the UIR form.[15] In Section 3, I focus on $\pi_{iI}(p_i, \mathbf{p}_{-i})$ and the full IR form by accounting for the restriction imposed by the price threshold $\overline{p}_i$.

Given $\mathbf{p}_{-i}$ and the subsidy policy parameters (i.e. $s_i > 0$, $a > 0$, $r \in (0,1)$, and $\overline{p}_i \in (c_i, c_i/r)$), let $\psi_{iN}(\mathbf{p}_{-i})$, $\psi_{iS}(\mathbf{p}_{-i}; s_i)$, $\psi_{iA}(\mathbf{p}_{-i}; a)$, and $\psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i)$ denote the best response correspondences that maximise the respective profits π_{iN}, π_{iS}, π_{iA}, and π_{iU}. In the following discussion, I focus on the non-trivial situations in which the maximum profits are positive.[16] Let $\mathbf{p}_N^*$, $\mathbf{p}_S^*(\mathbf{s})$, $\mathbf{p}_A^*(a)$, and $\mathbf{p}_U^*(r, \overline{\mathbf{p}})$, where $\mathbf{s} = (s_1, \cdots, s_n)$ and $\overline{\mathbf{p}} = (\overline{p}_1, \cdots, \overline{p}_n)$, denote the coordinate-wise largest (and thus Pareto-best) NEs in the respective subsidy settings. The theory of supermodular games ensures the existence of these NEs, as discussed at the end of Section 2.1.

A well-known and intuitive result of applying Topkis's (1978) monotonicity theorem to Bertrand competition with product differentiation is that a reduction in a firm's marginal cost shifts down its best response correspondence and lowers the Pareto-best NE prices of all products. Viewing a subsidy as a reduction in the effective marginal cost, we have the following proposition regarding the impacts of the above subsidies.

Proposition 2. *Providing a subsidy or increasing its generosity shifts down each firm's best response correspondence (points (i) and (ii) below) and lowers the prices of all products at the Pareto-best NE (points (iii) and (iv) below).*

(i) *If $p_{iN} \in \psi_{iN}(\mathbf{p}_{-i})$, $p_{iS} \in \psi_{iS}(\mathbf{p}_{-i}; s_i)$, $p_{iA} \in \psi_{iA}(\mathbf{p}_{-i}; a)$, and $p_{iU} \in \psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i)$, then $p_{iS} \leq p_{iN}$, $p_{iA} \leq p_{iN}$, and $p_{iU} \leq p_{iN}$.*

(ii) (a) *$\psi_{iS}(\mathbf{p}_{-i}; s_i)$, $\psi_{iA}(\mathbf{p}_{-i}; a)$, and $\psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i)$ are strongly decreasing in s_i, a, r, and $\overline{p}_i$,*

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[16]Let $p_i^{\min} = \min\{c_i - s_i, c_i/(1+a), (c_i - r\overline{p}_i)/(1-r)\}$ and $\mathbf{p}_{-i}^{\min}$ be a vector containing $p_j^{\min}$ for all $j \neq i$. If $D_i(c_i, \mathbf{p}_{-i}^{\min}) > 0$ for each i, then firm i's best response to $\mathbf{p}_{-i}$, where $\mathbf{p}_{-i} \geq \mathbf{p}_{-i}^{\min}$, results in a positive profit in each of these subsidy (or no-subsidy) settings.

respectively.[17]

(b) If ψ_{iS}, ψ_{iA}, and ψ_{iU} are (single-valued) functions, then they are decreasing in s_i, a, r, and $\overline{p}_i$,

respectively.

(iii) $\mathbf{p}_S^*(\mathbf{s}) \leq \mathbf{p}_N^*$, $\mathbf{p}_A^*(a) \leq \mathbf{p}_N^*$, *and* $\mathbf{p}_U^*(r,\overline{\mathbf{p}}) \leq \mathbf{p}_N^*$.

(iv) $\mathbf{p}_S^*(\mathbf{s})$, $\mathbf{p}_A^*(a)$, *and* $\mathbf{p}_U^*(r,\overline{\mathbf{p}})$ *are decreasing in* $\mathbf{s}$, a, r, *and* $\overline{\mathbf{p}}$, *respectively.*

Proof. See the Appendix. ∎

Next, I compare the effectiveness of the different subsidy forms in terms of the government spending required to achieve a given target.[18] Given an UIR scheme $(r, \overline{p}_i)$,

$$\pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p}_i) = (1-r)\left[p_i + \frac{r(\overline{p}_i - c_i)}{1-r} - c_i\right] D_i(p_i, \mathbf{p}_{-i}) = (1-r)\pi_{iS}(p_i, \mathbf{p}_{-i}; \frac{r(\overline{p}_i - c_i)}{1-r}),$$
(11)

which implies that $\psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i) = \psi_{iS}(\mathbf{p}_{-i}; r(\overline{p}_i - c_i)/(1-r))$. Thus, under the UIR scheme, the optimising firm sets its product's price as if a specific subsidy of $r(\overline{p}_i - c_i)/(1-r)$ were provided, whereas it actually receives $r(\overline{p}_i - p_{iU})$, where $p_{iU} \in \psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i)$. Because $r(\overline{p}_i - c_i)/(1-r) > r(\overline{p}_i - p_{iU})$,[19] the UIR scheme $(r, \overline{p}_i)$ can induce the same response as the specific subsidy of $s_i = r(\overline{p}_i - c_i)/(1-r)$ can, but it requires a smaller subsidy payment than the specific subsidy scheme does. In this sense, the UIR scheme is more cost-effective than the specific scheme. Because the two schemes result in the same consumer price and thus the same output, the difference between the two schemes in the aggregate subsidy expenditure for product i is, by construction, equal to the difference in firm i's profit. If the condition $s_i = r(\overline{p}_i - c_i)/(1-r)$ holds for all i, then both the specific and UIR schemes lead to the same NEs.

The difference in cost-effectiveness is due to the difference in the price elasticity of demand that the firms face. Firm i faces the demand curve $D_i((p_i^p - r\overline{p}_i)/(1-r), \mathbf{p}_{-i})$ under the UIR scheme (where p_i^p is the producer price), whereas it faces $D_i(p_i^p - s_i, \mathbf{p}_{-i})$ under the specific

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[17]A correspondence f is strongly decreasing (increasing) if $x > x'$ imply $y \leq (\geq) y'$ for any $y \in f(x)$ and $y' \in f(x')$. For example, (ii) (a) means that if $r > r'$, then $p_{iU} \leq p_{iU}'$ for any $p_{iU} \in \psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i)$ and $p_{iU}' \in \psi_{iU}(\mathbf{p}_{-i}; r', \overline{p}_i)$.

[18]Anderson *et al.* (2001a) use a similar approach to compare specific taxation with *ad valorem* taxation.

[19]The assumption $\pi_{iU}(p_{iU}, \mathbf{p}_{-i}; r, \overline{p}_i) > 0$ implies

$$r(\overline{p}_i - c_i)/(1-r) - r(\overline{p}_i - p_{iU}) = [p_{iU} + r(\overline{p}_i - p_{iU}) - c_i]r/(1-r) > 0.$$

scheme. The former demand curve is flatter (on the D_i-p_i^p plane) and more price sensitive than the latter curve. More price-elastic demand induces a lower (producer) price, thereby reducing the subsidy benefit passed on to the firm and the government's cost of increasing the production and sales of the target product through subsidisation.

The following proposition summarises these results and extends the comparison to the *ad valorem* form and the UIR form with different values of r and $\overline{p}_i$.

Proposition 3. *Consider four sets of subsidy policy parameters, (i) $(s_1, \cdots, s_n)$ (specific), (ii) a (ad valorem), (iii) $(r, \overline{p}_1, \cdots, \overline{p}_n)$ (UIR), and (iv) $(r', \overline{p}'_1, \cdots, \overline{p}'_n)$ (UIR'), such that*

$$s_i = \frac{ac_i}{1+a} = \frac{r(\overline{p}_i - c_i)}{1-r} = \frac{r'(\overline{p}'_i - c_i)}{1-r'} \quad \forall i \tag{12}$$

and $r > r'$ (which together imply $\overline{p}_i < \overline{p}'_i \ \forall i$). The four policies result in the same best response correspondence for each firm (i.e. $\psi_{iS}(\mathbf{p}_{-i}; s_i) = \psi_{iA}(\mathbf{p}_{-i}; a) = \psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i) = \psi_{iU}(\mathbf{p}_{-i}; r', \overline{p}'_i))$ and thus the same set of NEs.

At a common NE, denoted by $\hat{\mathbf{p}}$, the four policies are ordered by the government's subsidy expenditure on each product i or, equivalently, by each firm i's profit as UIR < UIR' < specific < ad valorem. More specifically, (13)–(15) below show the differences across the policies in the government's subsidy expenditure on product i (i.e. the leftmost expressions of (13)–(15)) or, equivalently, the differences in firm i's profit (i.e. the second to the left expressions of (13)–(15)):

$$[s_i - r(\overline{p}_i - \hat{p}_i)]D(\hat{p}_i, \hat{\mathbf{p}}_{-i}) = \pi_{iS}(\hat{p}_i, \hat{\mathbf{p}}_{-i}; s_i) - \pi_{iU}(\hat{p}_i, \hat{\mathbf{p}}_{-i}; r, \overline{p}_i) = r\pi_{iS}(\hat{p}_i, \hat{\mathbf{p}}_{-i}; s_i) > 0, \tag{13}$$

$$[r'(\overline{p}'_i - \hat{p}_i) - r(\overline{p}_i - \hat{p}_i)]D(\hat{p}_i, \hat{\mathbf{p}}_{-i}) = \pi_{iU}(\hat{p}_i, \hat{\mathbf{p}}_{-i}; r', \overline{p}'_i) - \pi_{iU}(\hat{p}_i, \hat{\mathbf{p}}_{-i}; r, \overline{p}_i)$$

$$= (r - r')\pi_{iS}(\hat{p}_i, \hat{\mathbf{p}}_{-i}; s_i) > 0, \tag{14}$$

$$(a\hat{p}_i - s_i)D(\hat{p}_i, \hat{\mathbf{p}}_{-i}) = \pi_{iA}(\hat{p}_i, \hat{\mathbf{p}}_{-i}; a) - \pi_{iS}(\hat{p}_i, \hat{\mathbf{p}}_{-i}; s_i) = a\pi_{iS}(\hat{p}_i, \hat{\mathbf{p}}_{-i}; s_i) > 0. \tag{15}$$

Proof. See the Appendix. ∎

Proposition 3 shows that if the firms opt in, the UIR form is more cost-effective than the specific form, and, in turn, the specific form is more cost-effective than the *ad valorem* form,[20]

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[20]Liang *et al.* (2018) show that the specific form is more cost-effective than the *ad valorem* form, using a homogeneous-Cournot model of oligopolistic competition. Unlike the UIR form, the *ad valorem* form reduces the elasticity of demand faced by a firm relative to the specific form, increasing the government's cost of achieving a target consumer price and output through subsidisation.

where cost-effectiveness is defined as achieving a given policy target with lower government spending (or, equivalently, with less of the subsidy benefit being passed on to the firms). The proposition also states that cost-effectiveness increases with r (when r and $\overline{p}_i$ co-move with (12) satisfied).[21]

3 Further Analysis with the Eligibility Condition

Section 2.3 compared the subsidy forms with leaving aside the eligibility condition that the product price must be below $\overline{p}_i$ to receive a subsidy payment. In this section, I take this condition back into consideration to further analyse the properties of the IR subsidy. As discussed in Section 2.2, if the eligibility condition is in place but the IR subsidy scheme is not sufficiently generous, a firm can choose to ignore the scheme and set p_i above the threshold $\overline{p}_i$, thereby subverting the government's goal of increasing the target product's sales. On the other hand, by making the IR scheme less generous to firms, the government can achieve the policy goal more cost-effectively. Then, to what extent can the government improve the IR subsidy's cost-effectiveness by lowering its generosity while keeping firms in the scheme? In this section, I investigate this question by focusing on symmetrically differentiated Bertrand competition.

3.1 Symmetric Games

Consider identical firms that have a common marginal cost c and face a symmetrically differentiated demand system (e.g. Weyl and Fabinger, 2013; Anderson *et al.*, 2001b). These firms also face common subsidy policy parameters (denoted by s, a, r, and $\overline{p}$). The game is therefore symmetric (i.e. unaffected by the permutation of the firms), and the subscript i can be dropped from D_i, π_{iX}, ψ_{iX} (for $X \in \{N, S, A, U, I\}$), and so on. Additionally, I now assume that $\log D_i$ has strictly increasing differences in p_i and $\mathbf{p}_{-i}$, that is, the inequality in (1) or (2) holds strictly if $p_i > p_i'$ and $\mathbf{p}_{-i} > \mathbf{p}_{-i}'$ (i.e. $p_j \geq p_j'$ for all $j \neq i$ and $\mathbf{p}_{-i} \neq \mathbf{p}_{-i}'$).

Under these assumptions, all NEs are symmetric (Vives, 1999, Ch. 2).[22] Thus, to analyse NEs

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[21]Because increasing r (with $\overline{p}_i$ fixed) makes the subsidy more generous, it may seem counterintuitive that increasing r improves the subsidy's cost-effectiveness. However, in Proposition 3, increasing r simultaneously reduces $\overline{p}_i$ to meet (12). Altogether, these changes reduce the subsidy benefits that are passed on to the firms and hence improve the subsidy's cost-effectiveness.

[22]Suppose to the contrary that the best response map $\Psi_X(\mathbf{p}) \equiv \psi_X(\mathbf{p}_{-1}) \times \cdots \times \psi_X(\mathbf{p}_{-n})$ (for $X \in \{N, S, A, U, I\}$) has an asymmetric fixed point. Then, there exist (at least) two firms (denoted by 1 and 2) such that $p_1 \neq p_2$ at the fixed point. Without loss of generality, assume $p_1 < p_2$. Note that $p_1 \in \psi_X(p_2, p_3, \cdots, p_n)$ and $p_2 \in \psi_X(p_1, p_3, \cdots, p_n)$. Because $\log D_i$ has strictly increasing differences in p_i and $\mathbf{p}_{-i}$, the best response correspondence ψ_X is strongly increasing (i.e. every selection of ψ_X is increasing) in each argument (e.g. Vives,

in a subsidy (or no-subsidy) setting, I focus on each firm's best response when all of the other firms set a common price p_0, namely, $\psi_X(p_0, \cdots, p_0)$.[23] An NE is where $p_0 \in \psi_X(p_0, \cdots, p_0)$. For brevity, let $\tilde{D}(p_i, p_0) = D(p_i, p_0, \cdots, p_0)$ and define $\tilde{\pi}_X$, $\tilde{\psi}_X$, and $\tilde{G}$ analogously. I assume that $\tilde{D}$ is twice continuously differentiable and that the profit maximisation problems have interior solutions with positive profits. As discussed in Section 2, for each $X \in \{N, S, A, U, I\}$, the set of NEs, where $p \in \tilde{\psi}_X(p)$, is non-empty and is denoted by E_X (e.g. $E_N = \{p | p \in \tilde{\psi}_N(p)\}$). For clarity, the dependence of E_X on subsidy policy parameters may be shown explicitly (e.g. $E_I(r, \overline{p}) = \{p | p \in \tilde{\psi}_I(p; r, \overline{p})\}$). If E_X contains multiple elements, then the largest element p_X^* in E_X constitutes the (strictly) Pareto-best NE in E_X in terms of π_X (e.g. Milgrom and Roberts, 1990),[24] which is the main focus of this section.

First, the following proposition summarises the results in Section 2.2 about E_I (the set of NEs under the IR subsidy) in the context of the symmetric game.

Proposition 4. *Given an IR subsidy policy $(r, \overline{p})$,*

$$E_I(r, \overline{p}) = \{p | p \in E_N \text{ and } \tilde{G}(p; r, \overline{p}) \geq 0\} \cup \{p | p \in E_U(r, \overline{p}) \text{ and } \tilde{G}(p; r, \overline{p}) \leq 0\}. \quad (16)$$

The firms opt out of the IR subsidy scheme at an NE in $\{p | p \in E_N \text{ and } \tilde{G}(p; r, \overline{p}) \geq 0\}$ and opt in at an NE in $\{p | p \in E_U(r, \overline{p}) \text{ and } \tilde{G}(p; r, \overline{p}) \leq 0\}$.

Moreover, if $E_I(r, \overline{p})$ includes both an opt-out NE (denoted by p_N) and an opt-in NE (denoted by p_U), then $p_U \leq \overline{p} \leq p_N$ (so $p_U < p_N$ unless $p_U = \overline{p} = p_N$), and $\tilde{\pi}_I(p_U, p_U; r, \overline{p}) \leq \tilde{\pi}_I(p_N, p_N; r, \overline{p})$ (with equality if and only if $p_U = \overline{p} = p_N$).

Proof. See the Appendix. ∎

In other words, E_I consists of a subset of E_N for which the price (without the subsidy) is high enough to induce the firms to opt out of the IR subsidy, and a subset of E_U for which the firms opt in to receive the IR subsidy. If an IR subsidy policy $(r, \overline{p})$ can result in an opt-out NE (p_N) and an opt-in NE (p_U), then except in the unlikely case of $p_U = \overline{p} = p_N$, the price and the

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2005, Lemma 1). Therefore, it follows from $p_1 < p_2$, $p_1 \in \psi_X(p_2, p_3, \cdots, p_n)$, and $p_2 \in \psi_X(p_1, p_3, \cdots, p_n)$ that $p_1 \geq p_2$, which is a clear contradiction.

[23]The dependence of ψ_X (for $X \in \{S, A, U, I\}$) on subsidy policy parameters may be suppressed in the following for brevity. The same applies to $\tilde{\psi}_X$, E_X and p_X^* that are defined next.

[24]Given two NEs p and p' in E_X such that $p > p'$, we have $\tilde{\pi}_X(p, p) \geq \tilde{\pi}_X(p', p) > \tilde{\pi}_X(p', p')$, where the last inequality follows because demand is strictly increasing in other products' prices.

profit are always greater at the opt-out NE than at the opt-in NE, and thus the opt-out NE strictly Pareto dominates the opt-in NE.

3.2 Characterising the NEs under the IR Subsidy

Suppose that the government uses a subsidy to induce each firm to lower the consumer price from p_N^* (the Pareto-best NE price with no subsidy policy) to a target price $\hat{p}$. Equivalently, the government aims to raise the output per firm from $\tilde{D}(p_N^*, p_N^*) = D(p_N^*, \cdots, p_N^*)$ to $\tilde{D}(\hat{p}, \hat{p}) = D(\hat{p}, \cdots, \hat{p})$.[25] Note that if no externalities are associated with the consumption or production of the good, inducing marginal cost pricing ($\hat{p} = c$) maximises social surplus (= consumer surplus + producer surplus − government expenditure) by eliminating underproduction due to imperfect competition. If positive externalities, which are often the reason for subsidisation but are not considered explicitly in this study, are present, then the socially optimal $\hat{p}$ is lower than c. The following analysis is *not* about setting $\hat{p}$ optimally, which requires explicitly modelling externalities, but rather holds more generally for a given $\hat{p}$ that is below p_N^*.

Given a specific subsidy of s, the first-order condition (FOC) that is satisfied at the Pareto-best NE $p_S^* \in E_S$ is

$$p_S^* + \frac{\tilde{D}(p_S^*, p_S^*)}{\tilde{D}_1(p_S^*, p_S^*)} = c - s, \tag{17}$$

where $\tilde{D}_1$ is the partial derivative with respect to the first argument of the function $\tilde{D}$.[26,27] The government aims to induce the target $\hat{p}$ with this specific subsidy (i.e. $\hat{p} = p_S^*$). By (17), at this NE $\hat{p} = p_S^* \in E_S$, each firm's profit is $\tilde{\pi}_S(\hat{p}, \hat{p}; s) = -\tilde{D}(\hat{p}, \hat{p})^2 / \tilde{D}_1(\hat{p}, \hat{p})$.

From Proposition 3 and (17), $\hat{p}$ is also the Pareto-best NE in E_U (i.e. $\hat{p} = p_U^*$) if the subsidy parameters r and $\overline{p}$ satisfy

$$\frac{r(\overline{p} - c)}{1 - r} = s = c - \hat{p} - \frac{\tilde{D}(\hat{p}, \hat{p})}{\tilde{D}_1(\hat{p}, \hat{p})}. \tag{18}$$

This condition gives $\overline{p}$ as a function of r conditional on $\hat{p}$, denoted by $\overline{p}(r; \hat{p})$, with $d\overline{p}/dr <$

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[25] I make the standard assumption that the own-price effect on the demand for each product dominates the aggregate cross-price effect on the demand for it. This assumption implies that lowering the consumer price from p_N^* to $\hat{p}$ increases the demand from $\tilde{D}(p_N^*, p_N^*)$ to $\tilde{D}(\hat{p}, \hat{p})$, as is the policymaker's objective.

[26] More precisely, $\tilde{D}_1(p_i, p_0) \equiv \partial \tilde{D}(p_i, p_0)/\partial p_i = \partial D(p_i, p_0, \cdots, p_0)/\partial p_i$.

[27] Additionally, the second-order condition $\tilde{D}(p_S^*, p_S^*)\tilde{D}_{11}(p_S^*, p_S^*) \le 2[\tilde{D}_1(p_S^*, p_S^*)]^2$ needs to hold at this NE. This condition limits the convexity of the demand curve at the optimum and corresponds to the usual condition under Cournot competition that a firm's marginal revenue curve should not be upward sloping at an optimum.

$0.^{28}$ At this NE $\hat{p} = p^*_U \in E_U$, the per-unit subsidy payment is $r(\overline{p} - \hat{p}) = c - \hat{p} - (1 - r)\tilde{D}(\hat{p}, \hat{p})/\tilde{D}_1(\hat{p}, \hat{p})$, and each firm's profit is $\tilde{\pi}_U(\hat{p}, \hat{p}; r, \overline{p}(r; \hat{p})) = -(1 - r)\tilde{D}(\hat{p}, \hat{p})^2/\tilde{D}_1(\hat{p}, \hat{p})$.

I now account for the eligibility threshold and investigate necessary conditions on r and $\overline{p}$ such that an IR subsidy policy $(r, \overline{p}(r; \hat{p}))$ induces $\hat{p}$ as an opt-in NE, particularly the Pareto-best NE, in E_I. First, consider an IR subsidy policy $(r_2, \overline{p}(r_2; \hat{p}))$ under which firm i is indifferent between opting in and out when all of the other firms set $p_j = p^*_N$. That is, firm i is indifferent between $\tilde{\psi}_U(p^*_N; r_2, \overline{p}(r_2; \hat{p}))$ and $p^*_N \in \tilde{\psi}_N(p^*_N)$. Similarly, consider a policy $(r_3, \overline{p}(r_3; \hat{p}))$ under which firm i is indifferent between opting in and out when all of the other firms set $p_j = \hat{p}$. That is, firm i is indifferent between $\hat{p} \in \tilde{\psi}_U(\hat{p}; r_3, \overline{p}(r_3; \hat{p}))$ and $\tilde{\psi}_N(\hat{p})$. In other words, the two policies are endogenously determined through market interactions among the firms and respectively satisfy

$$\tilde{G}(p^*_N; r_2, \overline{p}(r_2; \hat{p})) = \tilde{\pi}_N(p^*_N, p^*_N) - \max_{p_i} \tilde{\pi}_U(p_i, p^*_N; r_2, \overline{p}(r_2; \hat{p})) = 0,$$
$$\tilde{G}(\hat{p}; r_3, \overline{p}(r_3; \hat{p})) = \max_{p_i} \tilde{\pi}_N(p_i, \hat{p}) - \tilde{\pi}_U(\hat{p}, \hat{p}; r_3, \overline{p}(r_3; \hat{p})) = 0. \tag{19}$$

The next proposition shows that r_2 and r_3 are threshold values at which the characteristics of E_I change significantly.

Proposition 5. *Given a government target $\hat{p}$ such that $\hat{p} < p^*_N$, if two policies $(r_2, \overline{p}(r_2; \hat{p}))$ and $(r_3, \overline{p}(r_3; \hat{p}))$ are defined as stated above, then $r_2 \leq r_3$. Moreover, depending on the value of $r \in (0, 1)$, the set of NEs, E_I, under an IR subsidy policy $(r, \overline{p}(r; \hat{p}))$ has the following properties.*

  (i) If $r < r_2$, then $\hat{p} \in E_I$ and $p_N \notin E_I$ for any $p_N \in E_N$ (in particular, $p^*_N \notin E_I$).

   In other words, if $r < r_2$, then the government target $\hat{p}$ is induced as an NE under the IR subsidy policy $(r, \overline{p}(r; \hat{p}))$, but any NE with no subsidy, particularly p^*_N (the Pareto-best NE with no subsidy), is not an NE with this IR subsidy.

  (ii) If $r_2 \leq r \leq r_3$, then $\hat{p} \in E_I$ and $p^*_N \in E_I$.

  (iii) If $r_3 < r$, then $\hat{p} \notin E_I$ and $p^*_N \in E_I$.

The Pareto-best NE in E_I is $\hat{p}$ in case (i) and p^*_N in cases (ii) and (iii).

Proof. See the Appendix. ∎

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^{28}By (18), $\overline{p} = s/r - s + c = [c - \hat{p} - \tilde{D}(\hat{p}, \hat{p})/\tilde{D}_1(\hat{p}, \hat{p})]/r + \hat{p} + \tilde{D}(\hat{p}, \hat{p})/\tilde{D}_1(\hat{p}, \hat{p})$, and $d\overline{p}/dr = -s/r^2 < 0$.

Proposition 5 states that $\hat{p}$ is an opt-in NE under the IR scheme if $r \leq r_3$. In particular, $\hat{p}$ is the Pareto-best NE if $r < r_2$. As Proposition 3 shows, the IR form can achieve the target outcome $\hat{p}$ without passing on as much of the subsidy benefit to each firm as the specific form does:

$$\{r[\overline{p}(r;\hat{p}) - \hat{p}] - s\}\tilde{D}(\hat{p},\hat{p}) = \tilde{\pi}_U(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) - \tilde{\pi}_S(\hat{p},\hat{p};s) = r\frac{\tilde{D}(\hat{p},\hat{p})^2}{\tilde{D}_1(\hat{p},\hat{p})} < 0. \qquad (20)$$

The first expression is the difference in the total government outlay for each product under both subsidy forms, and the second expression is the difference in each firm's profit. Analogously, increasing r (with $\overline{p}$ decreasing to satisfy (18)) has the same effect:

$$\frac{d\{r[\overline{p}(r;\hat{p}) - \hat{p}]\tilde{D}(\hat{p},\hat{p})\}}{dr} = \frac{d\tilde{\pi}_U(\hat{p},\hat{p};r,\overline{p}(r;\hat{p}))}{dr} = \frac{\tilde{D}(\hat{p},\hat{p})^2}{\tilde{D}_1(\hat{p},\hat{p})} < 0.$$

Given $r \leq r_3$, the next proposition compares a firm's profit at this opt-in NE ($p_U^* = \hat{p} \in E_I$) with its profit at the Pareto-best NE with no subsidy ($p_N^* \in E_N$), which is also the Pareto-best, opt-out NE in E_I if $r \geq r_2$ (by Proposition 5). The profit at the latter equilibrium is given by $\tilde{\pi}_N(p_N^*, p_N^*) = -\tilde{D}(p_N^*, p_N^*)^2/\tilde{D}_1(p_N^*, p_N^*)$. Let

$$r_1 = 1 - \frac{\tilde{\pi}_N(p_N^*, p_N^*)}{\tilde{\pi}_S(\hat{p},\hat{p};s)} = 1 - \frac{\tilde{D}(p_N^*, p_N^*)^2/\tilde{D}_1(p_N^*, p_N^*)}{\tilde{D}(\hat{p},\hat{p})^2/\tilde{D}_1(\hat{p},\hat{p})}. \qquad (21)$$

Proposition 6. *If r_1 and r_2 are respectively defined by (21) and (19), then $r_1 < r_2$. Moreover, each firm's profit at an opt-in NE $\hat{p} \in E_I$ can be compared with the firm's profit at $p_N^* \in E_N$ (the Pareto-best NE in the no-subsidy case) as follows:*

$$\tilde{\pi}_U(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) \begin{cases} > \tilde{\pi}_N(p_N^*, p_N^*) & \text{if } 0 < r < r_1, \\ = \tilde{\pi}_N(p_N^*, p_N^*) & \text{if } r = r_1, \\ < \tilde{\pi}_N(p_N^*, p_N^*) & \text{if } r_1 < r \leq r_3. \end{cases} \qquad (22)$$

In particular, if $r_1 < r < r_2$, then each firm's profit is lower at $\hat{p} \in E_I$, which is the Pareto-best NE in E_I by Proposition 5, than at $p_N^* \in E_N$.
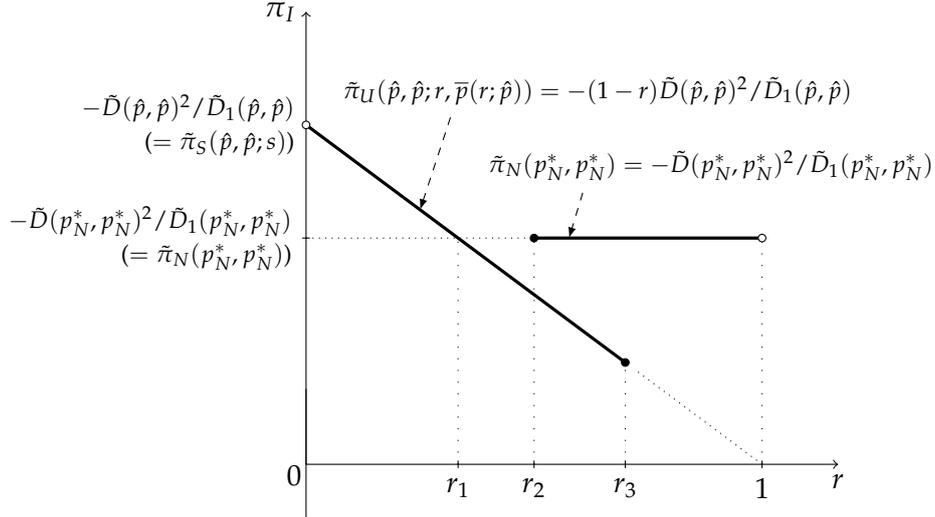
Proof. See the Appendix.

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Figure 3: *NE Profits with Different IR Subsidies (Conditional on $\hat{p}$)*

According to Proposition 6, even an ungenerous IR subsidy policy $(r, \overline{p}(r; \hat{p}))$ with $r \in (r_1, r_2)$ that reduces the equilibrium profit below the no-subsidy level, $\tilde{\pi}_N(p_N^*, p_N^*)$, can still realise the government target ($\hat{p}$ and $\tilde{D}(\hat{p}, \hat{p})$) as the Pareto-best, opt-in NE, thus significantly improving the cost-effectiveness of subsidy spending relative to the specific subsidy. Such a policy substantially increases the price sensitivity of demand that the firms face, thereby eroding their market power. Consequently, the firms reduce the producer price (as well as the consumer price) well below p_N^* ('overshifting') and settle for lower equilibrium profits than in the no-subsidy case despite the increase in sales from $\tilde{D}(p_N^*, p_N^*)$ to $\tilde{D}(\hat{p}, \hat{p})$.

Figure 3 graphically summarises the characteristics of the IR subsidy described in this section. The figure shows a firm's profit at two potential NEs under the IR subsidy (p_N^* and $\hat{p}$) for each value of r (with $\overline{p}$ determined by (18)), conditional on a government target $\hat{p}$. Under the IR scheme, the NE at which each firm i opts out by setting $p_i = p_N^*$ and earns $\tilde{\pi}_N(p_N^*, p_N^*) = -\tilde{D}(p_N^*, p_N^*)^2 / \tilde{D}_1(p_N^*, p_N^*)$ exists if and only if $r \in [r_2, 1)$. For $r \in (0, r_2)$, no opt-out NE exists (i.e. $p_N \notin E_I$ for any $p_N \in E_N$, including p_N^*).

Any pair of IR subsidy parameters $(r, \overline{p})$ with $r \in (0, r_3]$ and $\overline{p}$ determined by (18) can realise $p_U^* = \hat{p}$ as an NE in E_I, at which each firm i opts in by setting $p_i = \hat{p}$. At this NE, each firm earns $\tilde{\pi}_U(\hat{p}, \hat{p}; r, \overline{p}(r; \hat{p})) = -(1 - r)\tilde{D}(\hat{p}, \hat{p})^2 / \tilde{D}_1(\hat{p}, \hat{p})$, and thus its profit decreases as r increases (and $\overline{p}$ decreases according to (18)). Each firm's profit is always lower at $p_U^* = \hat{p} \in E_I$ than at the Pareto-best NE under the specific subsidy ($p_S^* = \hat{p} \in E_S$), which gives each firm $\tilde{\pi}_S(\hat{p}, \hat{p}; \sigma_S(\hat{p})) = -\tilde{D}(\hat{p}, \hat{p})^2 / \tilde{D}_1(\hat{p}, \hat{p})$. Equivalently, $p_U^* = \hat{p} \in E_I$ is realised with less

government spending than $p_S^* = \hat{p} \in E_S$, as seen in (20). Moreover, for $r \in (0, r_1)$ ($r \in (r_1, r_3]$), the opt-in profit (at $\hat{p} \in E_I$) is higher (lower) than the opt-out profit $\tilde{\pi}_N(p_N^*, p_N^*)$.[29] In particular, for $r \in (r_1, r_2)$, the firms are better off if they collude, jointly opt out of the IR subsidy by setting $p_i = p_N^*$ for all i, and earn $\tilde{\pi}_N(p_N^*, p_N^*)$ per firm. However, $p_i = p_N^*$ for all i is not an NE in E_I, so the firms opt in and earn a lower equilibrium profit of $-(1 - r)\tilde{D}(\hat{p}, \hat{p})^2 / \tilde{D}_1(\hat{p}, \hat{p})$ per firm. For $r \in [r_2, r_3]$, both $\hat{p}$ (opt-in) and p_N^* (opt-out) are NEs under the IR scheme, but $p_N^* \in E_I$ gives each firm a higher profit than $\hat{p} \in E_I$ does (i.e. $p_N^* \in E_I$ Pareto dominates $\hat{p} \in E_I$ from the firms' perspective).[30]

Figure 3 shows that the IR subsidy's two policy variables (r and $\overline{p}$) can be utilised to flexibly control its incidence on producers and its cost-effectiveness, unlike the case of the specific and *ad valorem* subsidies. When the government pursues the target outcome ($\hat{p}$ and $\tilde{D}(\hat{p}, \hat{p})$), it can simultaneously adjust the IR subsidy's benefit or burden on producers by changing r in the range of $(0, r_2)$ (and $\overline{p}$ by (18)). In particular, with $r \in (0, r_1)$, the IR scheme increases both consumer surplus and producer surplus at $\hat{p} \in E_I$ relative to the no-subsidy case ($p_N^* \in E_N$). That is, both consumers and producers are subsidised. With $r \in (r_1, r_2)$, consumers receive the same benefits at $\hat{p} \in E_I$ as in the case with $r \in (0, r_1)$. However, with $r \in (r_1, r_2)$, the IR subsidy effectively functions as a tax on producers because their profits at $\hat{p} \in E_I$ are lower than in the no-subsidy case ($p_N^* \in E_N$), and the lost profits are implicitly transferred to the government as a reduction in the government budget required to induce $\hat{p} \in E_I$. In contrast, the specific and *ad valorem* forms offer no such flexibility because they each have only one policy variable (s and a, respectively) and setting the variable simultaneously determines the equilibrium output, profit, and government expenditure.

The IR form's flexibility allows policymakers to adjust the incidence and cost-effectiveness of a subsidy scheme in line with the policy objectives and market situations. For example, if the target products require emerging, innovative technologies (e.g. EVs), producers may incur significant fixed costs (e.g. R&D investment). Under these circumstances, the government

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[29]In the case of a monopoly ($n = 1$), some changes must be made to Figure 3, and r_1 serves as a threshold as follows. The profit functions π_U and π_N are essentially the same as those shown in Figure 3: $\pi_N(p_N^*) = -D(p_N^*)^2 / D_1(p_N^*)$ and $\pi_U(\hat{p}) = -(1 - r)D(\hat{p})^2 / D_1(\hat{p})$. The monopolist sets the price at $\hat{p}$ if $r \in (0, r_1)$, at $\hat{p}$ or p_N^* if $r = r_1$, and at p_N^* if $r \in (r_1, 1)$. This result holds because the firm can opt out of the IR scheme and earn $-D(p_N^*)^2 / D_1(p_N^*)$, and thus the IR scheme cannot make the firm worse off than $\pi_N(p_N^*) = -D(p_N^*)^2 / D_1(p_N^*)$.

[30]Additionally, $(r, \pi_I) = (1, 0)$ in Figure 3 corresponds to the corner (limit) solution that results from the dual tax scheme analysed by Myles (1996), in which the enforceability of the tax eliminates the opt-out option and leads to the Ramsey pricing outcome of transferring all economic rents from the firms to the government through taxation.

may want to support innovative producers by keeping r small (and $\bar{p}$ large) to offer producers significantly higher profits than in the no-subsidy case. Conversely, if the target market is relatively mature prior to government intervention and is served by a small number of firms earning large economic rents owing to imperfect competition, the government can use the IR form to induce more output and simultaneously reduce the oligopolists' profits, thereby substantially improving the cost-effectiveness of the subsidy.

4 Simulating the Impacts of the IR Subsidy

In this section, I illustrate the impact of the IR subsidy in a more empirical setting. I calibrate the symmetrically differentiated Bertrand oligopoly model discussed in the last section, employing actual data from the US EV market, in which buyers are eligible for a specific subsidy.[31] I then use the calibrated model to simulate the results of replacing the specific subsidy with other forms.

4.1 Simulation Model

Following the previous sections, I consider a hypothetical market with a symmetrically differentiated logit demand system:

$$D(p_i, \mathbf{p}_{-i}) = \frac{\alpha \exp(\beta p_i)}{1 + \sum_{j=1}^{n} \alpha \exp(\beta p_j)} M \ \ \forall i,$$

where $\alpha > 0$, $\beta < 0$, M is the market size, and an outside option that can substitute for the n goods is included. This demand system satisfies the conditions given at the beginning of Section 2 and ensures that E_X is a singleton for $X \in \{N, S, A, U\}$ (i.e. p_X^* is a unique and symmetric NE in each of these settings) (e.g. Anderson *et al.*, 1992, Ch. 7). At each of these NEs,

$$q_X^* \equiv \tilde{D}(p_X^*, p_X^*) = \frac{\alpha \exp(\beta p_X^*)}{1 + n\alpha \exp(\beta p_X^*)} M. \tag{23}$$

Thus, with this demand system, an IR subsidy results in at least one and at most two NEs (either p_N^*, p_U^*, or both) depending on the parameter values.

I set the parameters α, β, n, M, and c to reflect an actual market environment, as outlined

below. I calibrate these parameters to US market data for model year 2017 for small or midsize EVs with four to five seats.[32] First, the sales data identify the eight best-selling models in this market, which are produced by eight different manufacturers (BMW, Fiat, Ford, General Motors, Kia, Mercedes, Nissan, and Volkswagen). These models account for almost all of the sales in this category. Their aggregate sales are 50,981, and the (sales-weighted) average price is $34,160 (for vehicles without options, namely, for low-quality and low-cost vehicles) or $38,799 (for vehicles with options, if available, namely, for high-quality and high-cost vehicles). Because EV buyers were eligible for a specific subsidy (more specifically, a federal tax credit) of $s =$$7,500$ in 2017, the (sales-weighted) average consumer price is $26,600 $(= \$34,160 - \$7,500)$ or $31,299 $(= \$38,799 - \$7,500)$.[33] Thus, I set the parameters of the logit model so that the resulting NE reflects these observed values ($p_S^* = \$26,660$ or $31,299 and $\tilde{D}(p_S^*, p_S^*) = 50,981/n$). A reasonable choice for the outside option in the case of these eight EV models is all other hybrid and plug-in EVs, whose total sales are 500,096 (hence $M = 50,981 + 500,096$). On the supply side, based on UBS Evidence Lab's (2017) estimates of the production costs of the Chevrolet Bolt (one of the eight models considered here) with and without options, I set the marginal cost c equal to $27,315 (in the low-quality, low-cost case) or $29,885 (for the high-quality, high-cost case). Lastly, I set n equal to 4.64, which is the equivalent number of firms based on the observed market shares of the eight models.[34] Substituting these values into (17) and (23) determines the demand parameters α and β.

I use two sets of parameter values to check the robustness of the results. One set corresponds to the low-quality, low-cost case ($c = \$27,315$, $\alpha = 1.1688$, and $\beta = -0.1491 \times 10^{-3}$), and the other set corresponds to the high-quality, high-cost case ($c = \$29,885$, $\alpha = 0.7902$, and $\beta = -0.1145 \times 10^{-3}$).

Table 2: *Simulation Results*

	Simulation 1 (sales neutral)		Simulation 2 (budget neutral)	
Quality and cost:	Low	High	Low	High
No Subsidy				
p_N^* (consumer price = producer price; $)	34,071 [*]	38,701 [0]	Same as	
nq_N^* (market sales)	18,004	23,070	Simulation 1	
Specific Subsidy (based on observed data)				
p_S^* (consumer price; $)	26,660 [1]	31,299 [2]		
nq_S^* (market sales)	50,981 [3]	50,981 [3]		
s (subsidy/unit; $)	7,500	7,500		
$p_S^* + s$ (producer price; $)	34,160	38,799	Same as	
ΔConsumer surplus (vs. no subsidy; mil. $)	236 [4]	261 [5]	Simulation 1	
ΔProducer surplus (vs. no subsidy; mil. $)	227	251		
Gov't budget ($s \times nq_S^*$; mil. $)	382 [6]	382 [6]		
ΔSocial surplus (vs. no subsidy; mil. $)	81 [7]	130 [8]		
***Ad Valorem* Subsidy**				
p_A^* (consumer price; $)	26,660 [1]	31,299 [2]	27,665	32,558
nq_A^* (market sales)	50,981 [3]	50,981 [3]	44,462	44,694
a	0.38	0.34	0.31	0.26
ap_A^* (subsidy/unit; $)	10,091	10,487	8,600	8,555
$(1+a)p_A^*$ (producer price; $)	36,751	41,786	36,265	41,113
ΔConsumer surplus (vs. no subsidy; mil. $)	236 [4]	261 [5]	188	201
ΔProducer surplus (vs. no subsidy; mil. $)	359	403	276	298
Gov't budget ($ap_A^* \times nq_A^*$; mil. $)	514	535	382 [6]	382 [6]
ΔSocial surplus (vs. no subsidy; mil. $)	81 [7]	130 [8]	82	117
IR Subsidy with $r = r_0$				
p_U^* (consumer price; $)	26,660 [1]	31,299 [2]	26,616	31,244
nq_U^* (market sales)	50,981 [3]	50,981 [3]	51,287	51,274
r_0	0.013	0.011	0.013	0.011
$\bar{p}_0$ ($)	597,258	703,269	595,346	701,188
$r_0(\bar{p}_0 - p_U^*)$ (subsidy/unit; $)	7,411	7,402	7,455	7,457
$p_U^* + r_0(\bar{p}_0 - p_U^*)$ (producer price; $)	34,071 [*]	38,701 [0]	34,071 [*]	38,701 [0]
ΔConsumer surplus (vs. no subsidy; mil. $)	236 [4]	261 [5]	238	264
ΔProducer surplus (vs. no subsidy; mil. $)	223	246	225	249
Gov't budget ($r_0(\bar{p}_0 - p_U^*) \times nq_U^*$; mil. $)	378	377	382 [6]	382 [6]
ΔSocial surplus (vs. no subsidy; mil. $)	81 [7]	130 [8]	81	131
IR Subsidy with $r = r_1$				
p_U^* (consumer price; $)	26,660 [1]	31,299 [2]	23,808	27,498
nq_U^* (market sales)	50,981 [3]	50,981 [3]	74,345	74,987
r_1	0.65	0.55	0.76	0.70
$\bar{p}_1$ ($)	31,328	35,960	30,547	34,796
$r_1(\bar{p}_1 - p_U^*)$ (subsidy/unit; $)	3,041	2,575	5,143	5,099
$p_U^* + r_1(\bar{p}_1 - p_U^*)$ (producer price; $)	29,701	33,874	28,951	32,597
ΔConsumer surplus (vs. no subsidy; mil. $)	236 [4]	261 [5]	413	498
ΔProducer surplus (vs. no subsidy; mil. $)	0 [9]	0 [9]	0 [9]	0 [9]
Gov't budget ($r_1(\bar{p}_1 - p_U^*) \times nq_U^*$; mil. $)	155	131	382 [6]	382 [6]
ΔSocial surplus (vs. no subsidy; mil. $)	81 [7]	130 [8]	31	116
IR Subsidy with $r = r_2$				
p_U^* (consumer price; $)	26,660 [1]	31,299 [2]	23,714	27,327
nq_U^* (market sales)	50,981 [3]	50,981 [3]	75,251	76,263
r_2	0.67	0.57	0.79	0.73
$\bar{p}_2$ ($)	31,037	35,519	30,180	34,221
$r_2(\bar{p}_2 - p_U^*)$ (subsidy/unit; $)	2,925	2,410	5,081	5,014
$p_U^* + r_2(\bar{p}_2 - p_U^*)$ (producer price; $)	29,585	33,709	28,795	32,341
ΔConsumer surplus (vs. no subsidy; mil. $)	236 [4]	261 [5]	420	511
ΔProducer surplus (vs. no subsidy; mil. $)	-6	-8	-10	-16
Gov't budget ($r_2(\bar{p}_2 - p_U^*) \times nq_U^*$; mil. $)	149	123	382 [6]	382 [6]
ΔSocial surplus (vs. no subsidy; mil. $)	81 [7]	130 [8]	27	113

Entries with a common superscript ([*], [0], [1], ⋯ or [9]) are equal by construction.

4.2   Simulation Results

Using the demand and supply parameters described above, the following simulations derive and compare the NEs in various policy settings (Table 2). Simulation 1, shown in the left column, is sales neutral: the three subsidy schemes (specific, *ad valorem*, and IR) are all designed to induce the equilibrium price and sales targeted by the government (i.e. $\hat{p} = \$26,660$ in the low-quality, low-cost case or $\hat{p} = \$31,299$ in the high-quality, high-cost case, and $\hat{q} \equiv \tilde{D}(\hat{p}, \hat{p}) = 50,981/n$). By construction, the specific subsidy scheme with $s = \$7,500$ achieves the target price and sales. With no subsidy, $p_N^* = \$34,071$ and $nq_N^* = 18,004$ (the market sales) in the low-quality, low-cost case, and $p_N^* = \$38,701$ and $nq_N^* = 23,070$ in the high-quality, high-cost case. Thus, relative to the no-subsidy case, the specific subsidy lowers the consumer price by more than $7,400 and raises the producer price by less than $100, increasing market sales by 183% (121%) (hereafter, the results for the low-quality, low-cost case are shown first, and those for the high-quality, high-cost case are shown next in parentheses). Consumer surplus and producer surplus (i.e. the firms' aggregate profit) increase by similar amounts ($236 million ($261 million) vs. $227 million ($251 million)).[35] As a result, social surplus (= consumer surplus + producer surplus − government expenditure) increases by $81 million ($130 million).[36]

An *ad valorem* subsidy with $a = s/(c-s) = 0.38$ (0.34) induces the same $(\hat{p}, \hat{q})$ (and hence the same consumer and social surplus) as the specific subsidy of $7,500 does. As Proposition 3 indicates, the *ad valorem* subsidy needs an extra subsidy payment of $2,591 ($2,987) per unit compared to the specific subsidy, increasing the producer price by the same amount and producer surplus by $132 million ($152 million). Thus, the (global) incidence of subsidy

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[32]The data set for vehicle sales and prices is constructed using the information available on the US Department of Energy's websites (`www.fueleconomy.gov` and `afdc.energy.gov/data`) and on EV-volumes.com (`www.ev-volumes.com`).

[33]In this calibration, I assume that the tax credit is fully taken up. I also disregard other incentive schemes that may have been available to EV buyers.

[34]That is, a market with 4.64 equal-sized firms gives the same Herfindahl-Hirschman Index as is observed in the data. Simulations with an integer number of firms (i.e. $n = 4$ or $n = 5$) give very similar results (e.g. the difference in the equilibrium consumer price relative to the case with $n = 4.64$ is at most 0.4%). Non-integer values are used for n in calibrations by, for example, Bushnell (2007).

[35]With this logit model, the change in consumer surplus is given by

$$\Delta CS = -M \left[\log(1 + n\alpha \exp(\beta\hat{p})) - \log(1 + n\alpha \exp(\beta p_N^*)) \right]/\beta.$$

[36]As discussed at the beginning of Section 3.2, this increase in social surplus results from correcting underproduction due to imperfect competition. This study does not explicitly model any positive externalities associated with the target product, and accounting for any such externalities would make the increase in social surplus even larger.

spending is more favourable to firms (less favourable to consumers) with the *ad valorem* form than with the specific form.

For the IR subsidy, sales-neutral Simulation 1 shows the outcomes for three thresholds of r that induce $(\hat{p}, \hat{q})$ at the opt-in NE ($p_U^* = \hat{p}$). These thresholds are r_1 and r_2, which are defined in Section 3.2, and r_0, at which the producer prices at the opt-in NE and the NE under no subsidy (p_N^*) are equal. Although Table 2 reports the results for the three representative cases, any $r \in (0, r_2)$ can induce $(\hat{p}, \hat{q})$ as the Pareto-best (and unique in the logit case used here) NE under the IR scheme. A smaller r leads to more profits and government spending in equilibrium (see Figure 3). I examine r_2 rather than r_3 because for $r \in [r_2, r_3]$, the opt-in NE ($p_U^* = \hat{p}$) is Pareto dominated (from the firms' perspective) by the opt-out NE (p_N^*).

When $r = r_0 = 0.013$ (0.011), the IR form results in a unique NE at which the producer price equals that in the no-subsidy baseline (i.e. $\hat{p} + r_0(\bar{p}_0 - \hat{p}) = p_N^*$). For $r > r_0$, the equilibrium producer price is below the no-subsidy equilibrium price (p_N^*), or equivalently, the reduction in the consumer price from the no-subsidy level is greater than the subsidy payment (i.e. $p_N^* - \hat{p} > r(\bar{p} - \hat{p})$), meaning that the producer price overshifts from the no-subsidy baseline. Conversely, the producer price undershifts if $r < r_0$.

The results associated with r_1 can be interpreted as follows. By construction, $r = r_1$ results in the same equilibrium profit as in the no-subsidy case. To induce a unique NE at which each firm opts in and its equilibrium profit is at least as large as that with no subsidy, it must be the case that $r \leq r_1$ and the subsidy payment per vehicle is not less than $r_1(\bar{p}_1 - \hat{p})$, where $\bar{p}_1 = \bar{p}(r_1; \hat{p})$ is defined as in footnote 28. In Simulation 1, r_1 is 0.65 (0.55), and $r_1(\bar{p}_1 - \hat{p})$ is $3,041 ($2,575). In other words, given the constraint that no firm can be worse off than at the no-subsidy NE, switching from the specific form to the IR form can reduce government spending by up to 59% (66%) for the given policy target. Because $r_1 > r_0$, the producer price overshifts: it is lower than p_N^* by $4,370 ($4,827). Equivalently, a subsidy payment of just $3,041 ($2,575) can lower the consumer price by $7,411 ($7,402).

The results associated with r_2 can be interpreted similarly. To induce a unique NE at which each firm opts in (with its equilibrium profit possibly being lower than that with no subsidy), it must be the case that $r < r_2$ and the subsidy payment per vehicle is greater than $r_2(\bar{p}_2 - \hat{p})$. In Simulation 1, r_2 is 0.67 (0.57), and $r_2(\bar{p}_2 - \hat{p})$ is $2,925 ($2,410). This subsidy payment is

slightly lower than the payment with $r = r_1$ because the equilibrium profit can now be lower than at the no-subsidy NE (as is the case for $r \in (r_1, r_2)$). In other words, by switching from the specific form to the IR form so that the firms have no incentive to opt out and the given policy target is met, the government can reduce the subsidy budget by up to 61% (68%). As expected, the producer price overshifts, and producer surplus is somewhat ($6 million ($8 million)) lower than at the no-subsidy NE.

Simulation 2, shown in the right column of Table 2, is budget neutral. As discussed above, Simulation 1 determines the necessary subsidy payment to achieve the target price $\hat{p}$ and sales $n\hat{q} = 50,981$ under each scheme. Budget-neutral Simulation 2 instead computes the sales level that each scheme realises given a fixed subsidy budget of $382 million, which is, by construction, the budget required for the specific subsidy of $s = \$7,500$ to induce $n\hat{q} = 50,981$ ($382 million $= s \times n\hat{q}$). Because the *ad valorem* form is less cost-effective than the specific form, the *ad valorem* form attains 13% (12%) fewer sales with this budget than the specific form.

Conversely, for the given budget, the IR form induces more sales than the specific form. With $r = r_0$, which equalises the equilibrium producer prices between the cases of the IR subsidy and no subsidy, the results are very similar to the corresponding results in Simulation 1.[37] The producer price overshifts (undershifts) if $r > (<) r_0 = 0.013$ (0.011).

According to the simulation with $r = r_1 = 0.76$ (0.70), if the IR scheme with the given budget ($382 million) results in a unique NE at which every firm opts in and earns at least the same equilibrium profit as in the no-subsidy case, then it can induce up to 46% (47%) more sales than the specific scheme (74,345 (74,987) vs. 50,981). Thus, under the IR scheme with the given budget and $r = r_1$, the consumer price ($23,808 ($27,498)) and the subsidy payment per unit ($5,143 ($5,099)) are both lower than under the specific scheme ($26,660 ($31,299) and $7,500, respectively). Consequently, as for the (global) incidence of the subsidy, producer surplus remains the same as in the no-subsidy NE (by construction), meaning that all of the subsidy benefit is passed on to the consumers, whose surplus increases by $177 million ($237 million) relative to the specific subsidy case.

Finally, according to the simulation with $r = r_2 = 0.79$ (0.73), given the weaker constraint that

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[37]The definition of r_0 is slightly different from its definition in Simulation 1 because Simulation 2 is conditional on the subsidy budget, whereas Simulation 1 is conditional on the target consumer price $\hat{p}$. The same comment applies to r_1 and r_2 that are to be discussed next.

the opt-in NE is a unique NE of the game, the IR scheme with the budget of $382 million and with r marginally less than r_2 can further extend the equilibrium sales to be 48% (50%) greater than under the specific subsidy. Thus, the consumer price and the per-unit subsidy payment are even lower, and consumer surplus is even higher, than in the previous case with $r = r_1$. Overshifting is now observed even in terms of producer surplus, as it is $10 million ($16 million) lower than at the no-subsidy NE and the NE with $r = r_1$.

Counterintuitive as it may seem at first, it is not surprising that the equilibrium social surplus in the budget neutral simulation is larger under the specific or *ad valorem* subsidy than under the IR subsidy with $r = r_1$ or $r = r_2$. Social surplus in Table 2 equals (consumer surplus) + (producer surplus) − (government outlay), meaning that externalities are not explicitly considered (as mentioned at the beginning of Section 3.2). Social surplus (without accounting for externalities) is maximised when the consumer price equals the marginal cost $c = \$27,315$ ($29,885), which eliminates underproduction due to imperfect competition. A consumer price below c (as in the cases of $r = r_1$ and $r = r_2$ in Simulation 2) is justified by the positive externalities associated with the sales or consumption of a subsidised good (e.g. reduced pollutant emissions, R&D spillovers and so on in the case of EVs). If these externalities are accounted for, positive externalities above $2,159 ($593) per unit of the good make the IR form with $r = r_1$ preferable to the specific form in terms of social surplus (inclusive of externalities). Note that $2,159 ($593) is much less than the subsidy payment of $5,143 ($5,099) under the IR scheme with $r = r_1$.

Overall, the simulations find that the IR form has substantial impacts, as the theoretical framework in the previous sections suggests. It reduces the government expenditure needed to induce a target sales level ($n\hat{q} = 50,981$) by up to 61% (68%) compared to the specific form. Alternatively, for a given subsidy budget ($382 million), the IR form can induce up to 48% (50%) more sales than the specific form. At the same time, the government can flexibly affect the IR scheme's incidence and cost-effectiveness through its choice of policy variables. When $r \approx 0$, the IR subsidy is (almost) equivalent to the specific subsidy (as shown in Figure 3) and can increase producer surplus by up to $227 million ($251 million) relative to the case of no subsidy. As r increases, producer surplus decreases, cost-effectiveness improves, and a larger share of the subsidy benefit is passed on to consumers. Moreover, as r approaches r_2, the IR scheme starts to function as implicit taxation on firms by reducing producer surplus by up

to \$10 million (\$16 million) relative to the no-subsidy level (according to Simulation 2). The incidence and cost-effectiveness of the subsidy programme can vary flexibly within this range.

5 Extension: Product Quality

The simple IR form discussed thus far may deter product quality improvements by implicitly making them more costly. This is because the per-unit subsidy payment decreases if a firm improves a product's quality and raises its (pre-subsidy) price to reflect the incremental cost. Note that widely used *ad valorem* taxation provides the same disincentive because it levies additional tax payments on better-quality products that are more costly and thus more expensive (before the tax) (Keen, 1998). These policies increase the (producer) price elasticity of demand and induce firms to reduce the product prices, allowing the government to attain a policy target more efficiently or cost-effectively. However, firms are also incentivised to lower product qualities as an easy way to reduce the costs and prices and thus increase (decrease) the subsidy (tax) payments. This issue is shared by the generalised tax forms of Myles (1996), Hamilton (1999), and Carbonnier (2014), which are designed to make the demand faced by a monopolist or oligopolist more elastic, although product quality is outside the scope of these studies. This section considers how the IR subsidy can be adjusted to address the issue of product quality. In short, the disincentive can be curbed by increasing the threshold $\overline{p}_i$ as quality improves.

I introduce product quality into the cost and demand structures of the model in Section 2 with maintaining its basic framework. In this section, I do not assume identical firms and a symmetrically differentiated demand system. The following model considers one-dimensional quality for the sake of simplicity, but similar results can be obtained with multidimensional quality, as discussed in Appendix B. Let the unit production cost of good i depend on its quality $w_i \in \mathbb{R}_+$: $c_i = c_i(w_i)$, with $c_i'(w_i) > 0$ and $c_i''(w_i) > 0$. The demand functions can be derived by adding w_i to the setup in Section 2 (footnote 7). Suppose that one unit of good i with quality w_i increases a representative consumer's utility by $f(w_i)$, where $f'(w_i) > 0$ and $f''(w_i) < 0$. Then, the representative consumer's utility attributed to quality (aggregated over the n products) is $\sum_{i=1}^{n} f(w_i)q_i$. Adding this sum to $U(x, q_1, \cdots, q_n) = x + u(q_1, \cdots, q_n)$ in footnote 7 allows the total utility to be expressed as $U(x, q_1, \cdots, q_n, w_1, \cdots, w_n) = x + u(q_1, \cdots, q_n) + \sum_{i=1}^{n} f(w_i)q_i$.

Given $\{w_i, p_i\}_{i \in \{1, \cdots, n\}}$, the consumer solves the following utility maximisation problem:

$$\max_{x, q_1, \cdots, q_n} x + u(q_1, \cdots, q_n) + \sum_{i=1}^{n} f(w_i) q_i \text{ s.t. } x + \sum_i p_i q_i \leq I.$$

Assuming an interior solution, the FOCs are $\partial u(q_1, \cdots, q_n)/\partial q_i = p_i - f(w_i) \; \forall i$. There-fore, the demand for each good depends on $\{p_i - f(w_i)\}_{i \in \{1, \cdots, n\}}$, that is, $q_i = D_i(p_1 - f(w_1), \cdots, p_n - f(w_n))$ for each i (note that $q_i = D_i(p_1, \cdots, p_n)$ in Section 2). In other words, $p_i = \partial u(q_1, \cdots, q_n)/\partial q_i + f(w_i)$; therefore, $f(w_i)$ is considered the premium attached to product i's quality or, equivalently, the consumer's willingness to pay (WTP) for quality.

[No Subsidy]

First, consider the baseline case of no subsidy. Given the qualities and prices of the other products (denoted by $\mathbf{w}_{-i}$ and $\mathbf{p}_{-i}$), firm i sets w_i and p_i simultaneously to maximise its profit $\pi_{iN}(w_i, p_i, \mathbf{w}_{-i}, \mathbf{p}_{-i}) = [p_i - c_i(w_i)]D_i(h_i, \mathbf{h}_{-i})$, where $h_i = p_i - f(w_i)$ and $\mathbf{h}_{-i}$ is a vector of the h_j's for all $j \neq i$. Assuming an interior solution that gives a positive profit, the following FOCs are satisfied for each i:

$$\frac{\partial \pi_{iN}}{\partial w_i} = -c_i'(w_i)D_i(h_i, \mathbf{h}_{-i}) - [p_i - c_i(w_i)]\frac{\partial D_i(h_i, \mathbf{h}_{-i})}{\partial h_i}f'(w_i) = 0, \tag{24}$$

$$\frac{\partial \pi_{iN}}{\partial p_i} = D_i(h_i, \mathbf{h}_{-i}) + [p_i - c_i(w_i)]\frac{\partial D_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0. \tag{25}$$

Substituting (25) into (24) shows that the optimal quality, denoted by w_{iN}, satisfies

$$f'(w_{iN}) = c_i'(w_{iN}). \tag{26}$$

The optimal quality w_{iN}, which is uniquely determined because $f''(\cdot) - c_i''(\cdot) < 0$, equates the marginal price (= marginal utility) with the marginal cost of quality improvement to maximise the net value of quality $f(\cdot) - c_i(\cdot)$. However, maximising $f(\cdot) - c_i(\cdot)$ is not necessary or sufficient for the social optimum if product quality is associated with positive externalities, which are typically the reason for subsidisation but are not explicitly modelled here. Given $\{w_{iN}, c_i(w_{iN})\}_{i \in \{1, \cdots, n\}}$, the discussion in Section 2 implies that an NE exists, at which (25) is satisfied for each i.

[IR Subsidy]

Suppose that each firm's threshold $\overline{p}_i$ is determined by the government as a function of w_i, that is, $\overline{p}_i = \overline{p}(w_i)$.[38] Assuming that the subsidy scheme is sufficiently generous, I focus on an NE at which all firms opt in and thus $\pi_{iI} = \pi_{iU}$. As above, firm i maximises its profit $\pi_{iU}(w_i, p_i, \mathbf{w}_{-i}, \mathbf{p}_{-i}) = [(1-r)p_i + r\overline{p}(w_i) - c_i(w_i)]D_i(h_i, \mathbf{h}_{-i})$ with respect to w_i and p_i. Assuming an interior solution that provides a positive profit, the following FOCs are satisfied for each i:

$$\frac{\partial \pi_{iU}}{\partial w_i} = [r\overline{p}'(w_i) - c_i'(w_i)]D_i(h_i, \mathbf{h}_{-i}) - [(1-r)p_i + r\overline{p}(w_i) - c_i(w_i)]\frac{\partial D_i(h_i, \mathbf{h}_{-i})}{\partial h_i}f'(w_i) = 0.$$
(27)

$$\frac{\partial \pi_{iU}}{\partial p_i} = (1-r)D_i(h_i, \mathbf{h}_{-i}) + [(1-r)p_i + r\overline{p}(w_i) - c_i(w_i)]\frac{\partial D_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0. \quad (28)$$

Substituting (28) into (27) shows that the optimal quality, denoted by w_{iU}, satisfies

$$(1-r)f'(w_{iU}) = c_i'(w_{iU}) - r\overline{p}'(w_{iU}), \tag{29}$$

where I assume for simplicity that w_{iU} is uniquely determined.[39] Equation (29) shows that the policymaker can affect the realised quality by adjusting the gradient $\overline{p}'(\cdot)$. For example, (26) and (29) imply that if $\overline{p}_i$ is constant (i.e. $\overline{p}'(\cdot) = 0$), as in the previous sections, then the optimal product quality is lower than that in the no-subsidy case ($w_{iU} < w_{iN}$). As another example, increasing $\overline{p}_i$ linearly with w_i (i.e. $\overline{p}'(\cdot) = v > 0$) improves w_{iU} relative to the previous case with a constant $\overline{p}_i$. Furthermore, in principle, setting $\overline{p}'(\cdot) = f'(\cdot)$ makes (26) and (29) equivalent, resulting in the same optimal quality with or without the IR subsidy ($w_{iU} = w_{iN}$).

Given $\{w_{iU}, c_i(w_{iU})\}_{i \in \{1, \cdots, n\}}$, the discussion in Section 2 implies that an NE exists, at which (28) is satisfied for each i. Importantly, the *gradient* $\overline{p}'(\cdot)$ appears in (29), but the *level* $\overline{p}(\cdot)$ does not. Thus, regardless of (29), the policymaker can choose the level of $\overline{p}(\cdot)$ and influence the NE prices through (28). In this sense, the previous results without product quality remain valid when the IR form is supplemented with a quality-dependent eligibility threshold $\overline{p}(\cdot)$ as above.

Following the same steps, it is straightforward to show the effects of the specific and *ad*

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[38] Ito and Sallee (2018) and Barwick *et al.* (2021) provide general discussions on attribute-based policy design.

[39] Assume, for example, that $\overline{p}(\cdot)$ is linear or, more generally, that r and $\overline{p}(\cdot)$ are set so that $(1-r)f(\cdot) - c_i(\cdot) + r\overline{p}(\cdot)$ is strictly concave.

valorem forms on quality. A specific subsidy maintains the same equilibrium quality as in the no-subsidy baseline because it does not distort the effective cost of quality improvement. An *ad valorem* subsidy, by contrast, lowers the effective cost of quality improvement, leading to a better equilibrium quality than in the no-subsidy case.

Based on these results, I consider a few examples in which the IR form can be applied. First, many countries use various subsidy programmes to promote the diffusion of green durable goods, such as renewable-energy technologies, low-emission vehicles, energy-efficient home appliances, and energy-saving building renovations (e.g. the solar PV and EV subsidies discussed in this study). In many cases, information about energy, environmental, and other attributes of these products is available, enabling policymakers to set $\overline{p}_i$ as a function of these attributes (see Appendix B for an extension of this section's model to the case of multidimensional attributes). Often, economic, engineering, or scientific estimates are also available about WTP for or positive externalities of these products' attributes, providing useful benchmarks for linking $\overline{p}_i$ with these attributes. In fact, many specific or *ad valorem* subsidy schemes for these products link subsidy payments to product quality (e.g. US federal tax credits for plug-in hybrid vehicles increase with battery capacity). When the IR form is actually implemented, the low dimensionality of product attributes is helpful for keeping the schedule $\overline{p}(\cdot)$ simple and manageable. In this regard, the IR form can be more suited for solar PV systems than for EVs because solar PV systems have much fewer attributes and their primary function (i.e. solar electricity generation) is the reason for subsidisation.

Second, actual pharmaceutical drug regulations and subsidies are similar to the quality-adjusted IR form discussed in this section. In many countries, the consumer price for a pharmaceutical drug is set below the unregulated level through negotiations between health authorities and pharmaceutical firms. In return, the drug is eligible for a government subsidy. The regulated price and subsidy rate depend on certain factors, such as product quality and characteristics (e.g. clinical effectiveness), production costs, and the prices of similar drugs (Organisation for Economic Cooperation and Development, 2008; Paris and Belloni, 2013). Constructing theoretical models of the negotiation process, Johnston and Zeckhauser (1991) and Wright (2004) suggest that the government can take advantage of its bargaining with pharmaceutical firms and inter-firm strategic interactions to induce lower drug prices (i.e. higher consumer surplus) and flexibly adjust producer surplus at the same time, two features

that the IR form can also achieve.[40]

In addition, many countries adopt reference pricing policies to contain fast-growing public spending on drugs (e.g. Acosta *et al.*, 2014). Under these policies, drugs are clustered based on chemical, pharmacological, or therapeutic equivalence criteria, and a reference price is set for each cluster. The subsidy payment for a drug increases linearly with its price but is capped at the reference price of the corresponding cluster, providing downward pressure on the drug's price when it is above the reference price. This design is similar to the quality-adjusted IR subsidy in that the subsidy rule changes at the threshold price and the threshold depends on product quality. Under reference pricing, however, the subsidy payment is non-decreasing in the product price, unlike under the IR subsidy. Altogether, these observations about government-firm bargaining and reference pricing indicate similarities to the IR subsidy design, suggesting the possibility of applying this design in pharmaceutical drug regulations and subsidies by, for example, letting $\overline{p}_i$ vary across clusters and remain constant within each cluster.

6   Conclusion

This study is motivated by the unique structure of a Japanese subsidy programme in which the rebate payment for buying a target product increases as the product price *decreases*. Interestingly, transaction data suggest that this design helped to lower not only the post-rebate consumer prices but also the pre-rebate producer prices ('overshifting'), thereby boosting sales further. To the best of my knowledge, this type of subsidy design has not been previously studied in the literature. In this study, I provide a theoretical foundation for this policy design by considering a new subsidy form (termed the IR subsidy).

Using a model of imperfect competition (Bertrand competition with product differentiation), I find that the IR form has two unique features relative to the widely used specific and *ad valorem* forms (Sections 2 and 3). First, it is more cost-effective than the other forms in the sense that it can induce a given level of output or sales with less government spending. Equivalently, it can induce more output or sales with a given government budget. Second, in achieving a given output level, policymakers can also flexibly adjust the cost-effectiveness and incidence of

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[40]Wright (2004) points out that, in practice, governments do not exploit the second function well during negotiations, because they often simply benchmark subsidy levels (and thus firm profitability) against foreign markets.

the policy to suit its objectives and the market circumstances. The key mechanism is that the IR form increases the elasticity of demand faced by producers, providing them an incentive to lower producer prices (in addition to consumer prices).

Simulations based on the US EV market demonstrate the substantial magnitude of these advantages (Section 4). For a fixed government budget ($382 million), the IR form can induce up to 48% (50%) more sales than the specific form (the two estimates reflect different scenarios about product quality). Depending on the government's choice of policy parameters, the IR subsidy scheme with this budget can flexibly adjust producer surplus to be between $227 million ($251 million) higher and $10 million ($16 million) lower than in the no-subsidy case.

An issue with the IR form is that it may disincentivise producers from making quality improvements. By extending the theoretical model to include product quality, Section 5 presents a simple way to offset this disincentive: making the price threshold for subsidy eligibility increasing in quality and thus rewarding higher-quality products with larger subsidy payments. This type of quality-adjusted IR subsidy can be applied to, for example, green technologies (e.g. solar PV systems and EVs) and pharmaceutical drugs.

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A Proofs

A.1 $\log \pi_{iI}$ has increasing differences in p_i and $\mathbf{p}_{-i}$ (where $\pi_{iI} > 0$)

Proof. Consider p_i, p_i', $\mathbf{p}_{-i}$, and $\mathbf{p}_{-i}'$ such that $p_i \geq p_i' \geq (c_i - r\overline{p}_i)/(1-r)$ and $\mathbf{p}_{-i} \geq \mathbf{p}_{-i}'$ (i.e. $p_j \geq p_j'$ for all $j \neq i$). Let $\Delta = \pi_{iI}(p_i, \mathbf{p}_{-i})\pi_{iI}(p_i', \mathbf{p}_{-i}') - \pi_{iI}(p_i', \mathbf{p}_{-i})\pi_{iI}(p_i, \mathbf{p}_{-i}')$. As in footnote 9, to prove that $\log \pi_{iI}$ satisfies increasing differences in p_i and $\mathbf{p}_{-i}$ (where $\pi_{iI} > 0$), it suffices to show that $\Delta \geq 0$.

(i) If $p_i' \geq \overline{p}_i$, then $p_i \geq p_i' \geq \overline{p}_i > c_i > (c_i - r\overline{p}_i)/(1-r)$. From (8) and footnote 9,

$$\Delta = \pi_{iN}(p_i, \mathbf{p}_{-i})\pi_{iN}(p_i', \mathbf{p}_{-i}') - \pi_{iN}(p_i', \mathbf{p}_{-i})\pi_{iN}(p_i, \mathbf{p}_{-i}') \geq 0.$$

(ii) If $\overline{p}_i \geq p_i$, then $\overline{p}_i \geq p_i \geq p_i' \geq (c_i - r\overline{p}_i)/(1-r)$. From (8) and (1),

$$
\begin{aligned}
\Delta &= \pi_{iU}(p_i, \mathbf{p}_{-i}; r, \overline{p})\pi_{iU}(p_i', \mathbf{p}_{-i}'; r, \overline{p}) - \pi_{iU}(p_i', \mathbf{p}_{-i}; r, \overline{p})\pi_{iU}(p_i, \mathbf{p}_{-i}'; r, \overline{p}) \\
&= [p_i + r(\overline{p}_i - p_i) - c_i]D_i(p_i, \mathbf{p}_{-i})[p_i' + r(\overline{p}_i - p_i') - c_i]D_i(p_i', \mathbf{p}_{-i}') \\
&\qquad - [p_i' + r(\overline{p}_i - p_i') - c_i]D_i(p_i', \mathbf{p}_{-i})[p_i + r(\overline{p}_i - p_i) - c_i]D_i(p_i, \mathbf{p}_{-i}') \\
&= [p_i + r(\overline{p}_i - p_i) - c_i][p_i' + r(\overline{p}_i - p_i') - c_i][D_i(p_i, \mathbf{p}_{-i})D_i(p_i', \mathbf{p}_{-i}') - D_i(p_i', \mathbf{p}_{-i})D_i(p_i, \mathbf{p}_{-i}')] \\
&\geq 0.^{41}
\end{aligned}
$$

(iii) If $p_i > \overline{p}_i > p_i'$, then it follows from (8), (1), $p_i > \overline{p}_i > c_i$, and $\overline{p}_i > p_i' \geq (c_i - r\overline{p}_i)/(1-r)$ that

$$
\begin{aligned}
\Delta &= \pi_{iN}(p_i, \mathbf{p}_{-i})\pi_{iU}(p_i', \mathbf{p}_{-i}'; r, \overline{p}) - \pi_{iU}(p_i', \mathbf{p}_{-i}; r, \overline{p})\pi_{iN}(p_i, \mathbf{p}_{-i}') \\
&= (p_i - c_i)D_i(p_i, \mathbf{p}_{-i})[p_i' + r(\overline{p}_i - p_i') - c_i]D_i(p_i', \mathbf{p}_{-i}') \\
&\qquad - [p_i' + r(\overline{p}_i - p_i') - c_i]D_i(p_i', \mathbf{p}_{-i})(p_i - c_i)D_i(p_i, \mathbf{p}_{-i}') \\
&= (p_i - c_i)[p_i' + r(\overline{p}_i - p_i') - c_i][D_i(p_i, \mathbf{p}_{-i})D_i(p_i', \mathbf{p}_{-i}') - D_i(p_i', \mathbf{p}_{-i})D_i(p_i, \mathbf{p}_{-i}')] \\
&\geq 0.
\end{aligned}
$$

Cases (i)–(iii) imply that if $p_i \geq p_i' \geq (c_i - r\overline{p}_i)/(1-r)$ and $\mathbf{p}_{-i} \geq \mathbf{p}_{-i}'$, then $\Delta \geq 0$.

■

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[41]This also implies that $\log \pi_{iU}$ satisfies increasing differences in p_i and $\mathbf{p}_{-i}$ (where $\pi_{iU} > 0$).

A.2   Proof of Proposition 1

Let $p_{iN} \in \psi_{iN}(\mathbf{p}_{-i})$ and $p_{iU} \in \psi_{iU}(\mathbf{p}_{-i})$ (I suppress the dependence on r and $\overline{p}_i$ in this proof for brevity of notation). By definition, $G_i(\mathbf{p}_{-i}) = \pi_{iN}(p_{iN}, \mathbf{p}_{-i}) - \pi_{iU}(p_{iU}, \mathbf{p}_{-i})$. First, I state and prove Lemma 1, which is used in the following proofs of Propositions 1 and 4.

Lemma 1. *If $G_i(\mathbf{p}_{-i}) > 0$ or if $G_i(\mathbf{p}_{-i}) = 0$ and $D_i(p_{iN}, \mathbf{p}_{-i}) > 0$, then $p_{iN} \geq \overline{p}_i$. If $G_i(\mathbf{p}_{-i}) < 0$ or if $G_i(\mathbf{p}_{-i}) = 0$ and $D_i(p_{iU}, \mathbf{p}_{-i}) > 0$, then $p_{iU} \leq \overline{p}_i$.*

Proof. If $p_{iN} < \overline{p}_i$, then

$$
\begin{aligned}
\pi_{iU}(p_{iU}, \mathbf{p}_{-i}) &\geq \pi_{iU}(p_{iN}, \mathbf{p}_{-i}) \\
&= [(1-r)p_{iN} + r\overline{p}_i - c_i]D_i(p_{iN}, \mathbf{p}_{-i}) \\
&= (p_{iN} - c_i)D_i(p_{iN}, \mathbf{p}_{-i}) + r(\overline{p}_i - p_{iN})D_i(p_{iN}, \mathbf{p}_{-i}) \\
&= \pi_{iN}(p_{iN}, \mathbf{p}_{-i}) + r(\overline{p}_i - p_{iN})D_i(p_{iN}, \mathbf{p}_{-i}) \\
&\geq \pi_{iN}(p_{iN}, \mathbf{p}_{-i})
\end{aligned}
$$

Thus, $G_i(\mathbf{p}_{-i}) \leq 0$ (with strict inequality if $D_i(p_{iN}, \mathbf{p}_{-i}) > 0$). Therefore, if $G_i(\mathbf{p}_{-i}) > 0$ or if $G_i(\mathbf{p}_{-i}) = 0$ and $D_i(p_{iN}, \mathbf{p}_{-i}) > 0$, then $p_{iN} \geq \overline{p}_i$.

Similarly, if $p_{iU} > \overline{p}_i$, then

$$
\begin{aligned}
\pi_{iN}(p_{iN}, \mathbf{p}_{-i}) &\geq \pi_{iN}(p_{iU}, \mathbf{p}_{-i}) \\
&= (p_{iU} - c_i)D_i(p_{iU}, \mathbf{p}_{-i}) \\
&\geq (p_{iU} - c_i)D_i(p_{iU}, \mathbf{p}_{-i}) + r(\overline{p}_i - p_{iU})D_i(p_{iU}, \mathbf{p}_{-i}) \\
&= [(1-r)p_{iU} + r\overline{p}_i - c_i]D_i(p_{iU}, \mathbf{p}_{-i}) \\
&= \pi_{iU}(p_{iU}, \mathbf{p}_{-i}),
\end{aligned}
$$

Thus, $G_i(\mathbf{p}_{-i}) \geq 0$ (with strict inequality if $D_i(p_{iU}, \mathbf{p}_{-i}) > 0$). Therefore, if $G_i(\mathbf{p}_{-i}) < 0$ or if $G_i(\mathbf{p}_{-i}) = 0$ and $D_i(p_{iU}, \mathbf{p}_{-i}) > 0$, then $p_{iU} \leq \overline{p}_i$.

∎

Next, I give a proof of Proposition 1.

Proof. By the definition of $\pi_{iI}(p_i, \mathbf{p}_{-i})$ in (8),

$$\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}) \leq \max\{\max_{p_i} \pi_{iN}(p_i, \mathbf{p}_{-i}), \max_{p_i} \pi_{iU}(p_i, \mathbf{p}_{-i})\}$$

$$= \max\{\pi_{iN}(p_{iN}, \mathbf{p}_{-i}), \pi_{iU}(p_{iU}, \mathbf{p}_{-i})\}. \tag{A.1}$$

(i) Suppose that $\pi_{iN}(p_{iN}, \mathbf{p}_{-i}) > \pi_{iU}(p_{iU}, \mathbf{p}_{-i})$ or, equivalently, $G_i(\mathbf{p}_{-i}) > 0$. By (A.1),

$$\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}) \leq \pi_{iN}(p_{iN}, \mathbf{p}_{-i}).$$

In addition, because $p_{iN} \geq \overline{p}_i$ by Lemma 1, it follows from (8) that

$$\pi_{iI}(p_{iN}, \mathbf{p}_{-i}) = \pi_{iN}(p_{iN}, \mathbf{p}_{-i}).$$

Thus, $\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}) = \pi_{iI}(p_{iN}, \mathbf{p}_{-i}) = \pi_{iN}(p_{iN}, \mathbf{p}_{-i})$, which is greater than $\pi_{iU}(p_{iU}, \mathbf{p}_{-i})$, and thus $\psi_{iI}(\mathbf{p}_{-i}) = \psi_{iN}(\mathbf{p}_{-i})$.

(ii) Similarly, suppose that $\pi_{iN}(p_{iN}, \mathbf{p}_{-i}) < \pi_{iU}(p_{iU}, \mathbf{p}_{-i})$ or, equivalently, $G_i(\mathbf{p}_{-i}) < 0$. By (A.1), $\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}) \leq \pi_{iU}(p_{iU}, \mathbf{p}_{-i})$. In addition, because $p_{iU} \leq \overline{p}_i$ by Lemma 1, it follows from (8) that $\pi_{iI}(p_{iU}, \mathbf{p}_{-i}) = \pi_{iU}(p_{iU}, \mathbf{p}_{-i})$. Thus, $\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}) = \pi_{iI}(p_{iU}, \mathbf{p}_{-i}) = \pi_{iU}(p_{iU}, \mathbf{p}_{-i})$, which is greater than $\pi_{iN}(p_{iN}, \mathbf{p}_{-i})$, and thus $\psi_{iI}(\mathbf{p}_{-i}) = \psi_{iU}(\mathbf{p}_{-i})$.

(iii) Finally, suppose that $\pi_{iN}(p_{iN}, \mathbf{p}_{-i}) = \pi_{iU}(p_{iU}, \mathbf{p}_{-i}) > 0$. This implies that $G_i(\mathbf{p}_{-i}) = 0$, $D_i(p_{iN}, \mathbf{p}_{-i}) > 0$, and $D_i(p_{iU}, \mathbf{p}_{-i}) > 0$. By (A.1), $\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}) \leq \pi_{iN}(p_{iN}, \mathbf{p}_{-i}) = \pi_{iU}(p_{iU}, \mathbf{p}_{-i})$. In addition, because $p_{iN} \geq \overline{p}_i$ and $p_{iU} \leq \overline{p}_i$ by Lemma 1, it follows from (8) that $\pi_{iI}(p_{iN}, \mathbf{p}_{-i}) = \pi_{iN}(p_{iN}, \mathbf{p}_{-i})$ and $\pi_{iI}(p_{iU}, \mathbf{p}_{-i}) = \pi_{iU}(p_{iU}, \mathbf{p}_{-i})$. As a result, $\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}) = \pi_{iI}(p_{iN}, \mathbf{p}_{-i}) = \pi_{iN}(p_{iN}, \mathbf{p}_{-i})$, and $\max_{p_i} \pi_{iI}(p_i, \mathbf{p}_{-i}) = \pi_{iI}(p_{iU}, \mathbf{p}_{-i}) = \pi_{iU}(p_{iU}, \mathbf{p}_{-i})$. Thus, $\psi_{iI}(\mathbf{p}_{-i}) = \psi_{iN}(\mathbf{p}_{-i}) \cup \psi_{iU}(\mathbf{p}_{-i})$.

∎

A.3 Proof of Proposition 2

Proof. Let the function F_i be defined by $F_i(p_i, \mathbf{p}_{-i}, x_i) = (p_i - x_i)D_i(p_i, \mathbf{p}_{-i})$ and the correspondence ψ_i be given by

$$\psi_i(\mathbf{p}_{-i}, x_i) = \arg\max_{p_i \in [x_i, p^{\max}]} \log F_i(p_i, \mathbf{p}_{-i}, x_i) = \arg\max_{p_i \in [0, p^{\max}]} F_i(p_i, \mathbf{p}_{-i}, x_i). \tag{A.2}$$

Note that $\psi_{iN}(\mathbf{p}_{-i}) = \psi_i(\mathbf{p}_{-i}, c_i)$ by (4), $\psi_{iS}(\mathbf{p}_{-i}; s_i) = \psi_i(\mathbf{p}_{-i}, c_i - s_i)$ by (5), $\psi_{iA}(\mathbf{p}_{-i}; a) = \psi_i(\mathbf{p}_{-i}, c_i/(1+a))$ by (6), and $\psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i) = \psi_i(\mathbf{p}_{-i}, (c_i - r\overline{p}_i)/(1-r))$ by (7).

For each $\mathbf{p}_{-i}$, $\log F_i(p_i, \mathbf{p}_{-i}, x_i)$ has strictly increasing differences in p_i and x_i (where $F_i > 0$) because given $p_i > p_i'$ and $x_i > x_i'$,

$$F_i(p_i, \mathbf{p}_{-i}, x_i)F_i(p_i', \mathbf{p}_{-i}, x_i') = (p_i - x_i)(p_i' - x_i')D_i(p_i, \mathbf{p}_{-i})D_i(p_i', \mathbf{p}_{-i})$$
$$> (p_i - x_i')(p_i' - x_i)D_i(p_i, \mathbf{p}_{-i})D_i(p_i', \mathbf{p}_{-i})$$
$$= F_i(p_i, \mathbf{p}_{-i}, x_i')F_i(p_i', \mathbf{p}_{-i}, x_i),$$

where the inequality holds because

$$(p_i - x_i)(p_i' - x_i') > (p_i - x_i)(p_i' - x_i') + (x_i - x_i')(p_i' - p_i) = (p_i - x_i')(p_i' - x_i).$$

Thus, $\psi_i(\mathbf{p}_{-i}, x_i)$ is strongly increasing in x_i (i.e. if $x_i > x_i'$, then $p_i \geq p_i'$ for any $p_i \in \psi_i(\mathbf{p}_{-i}, x_i)$ and $p_i' \in \psi_i(\mathbf{p}_{-i}, x_i')$) (e.g. Vives, 1999, Ch. 2; Amir, 2005). Hence, (i) holds because $c_i - s_i < c_i$, $c_i/(1+a) < c_i$, and $(c_i - r\overline{p}_i)/(1-r) < c_i$. Likewise, (ii) holds because $c_i - s_i$, $c_i/(1+a)$, and $(c_i - r\overline{p}_i)/(1-r)$ are strictly decreasing in s_i, a, r, and $\overline{p}_i$, respectively.[42]

Following footnote 9, for each x_i, $\log F_i(p_i, \mathbf{p}_{-i}, x_i)$ has increasing differences in p_i and $\mathbf{p}_{-i}$ (where $F_i > 0$). Thus, the game characterised by (A.2) for each i is a log-supermodular game. Additionally, $\log F_i(p_i, \mathbf{p}_{-i}, x_i)$ has (strictly) increasing differences in p_i and x_i, as shown above. Hence, the coordinate-wise largest fixed point of this game conditional on $\mathbf{x} = (x_1, \cdots, x_n)$, denoted by $\mathbf{p}^*(\mathbf{x})$, is increasing in $\mathbf{x}$ (e.g. Milgrom and Roberts, 1990). Therefore, (iii) and (iv) hold because of the properties of $c_i - s_i$, $c_i/(1+a)$, and $(c_i - r\overline{p}_i)/(1-r)$ stated above.

---

[42]Because $r < 1$ and $c_i < \overline{p}_i$,

$$\frac{\partial \frac{c_i - r\overline{p}_i}{1-r}}{\partial r} = \frac{c_i - \overline{p}_i}{(1-r)^2} < 0.$$

■

A.4 Proof of Proposition 3

Proof. If (12) is satisfied, then (11) and an analogous equation for π_{iA} imply that $\psi_{iS}(\mathbf{p}_{-i}; s_i) = \psi_{iA}(\mathbf{p}_{-i}; a) = \psi_{iU}(\mathbf{p}_{-i}; r, \overline{p}_i) = \psi_{iU}(\mathbf{p}_{-i}; r', \overline{p}'_i)$ for all i. Hence, the four policies result in the same set of NEs. Similarly, if (12) satisfied, then (13)–(15) follow from the definitions of π_{iS}, π_{iU}, and π_{iA}.

■

A.5 Proof of Proposition 4

Proof. Equation (16) follows from Proposition 1. Lemma 1 and $p_N \in E_I \cap E_N$ imply that $\overline{p} \leq p_N$, and Lemma 1 and $p_U \in E_I \cap E_U$ imply that $p_U \leq \overline{p}$. Thus, $p_U \leq \overline{p} \leq p_N$. The result for $\tilde{\pi}_I$ holds because the demand for product i and hence firm i's profit are both strictly increasing in the other products' prices (see footnote 24).

■

A.6 Proof of Proposition 5

First, I state and prove Lemma 2, which is used in the following proof of Proposition 5.

Lemma 2. *Given r and $\overline{p}$, if there exists some $\check{p}$ such that $\tilde{G}(\check{p}; r, \overline{p}) = 0$, then*

$$\tilde{G}(p_0; r, \overline{p}) \begin{cases} \leq 0 & \text{for all } p_0 < \check{p}, \\ = 0 & \text{for } p_0 = \check{p}, \\ \geq 0 & \text{for all } p_0 > \check{p}. \end{cases} \tag{A.3}$$

Proof. By the envelope theorem, $d\tilde{\pi}_N(p_N, p_0)/dp_0 = (p_N - c)\tilde{D}_2(p_N, p_0)$, where $p_N \in \tilde{\psi}_N(p_0)$ and $\tilde{D}_2$ represents the partial derivative with respect to the second argument of $\tilde{D}$, and

$d\tilde{\pi}_U(p_U, p_0; r, \overline{p})/dp_0 = [(1-r)p_U + r\overline{p} - c]\tilde{D}_2(p_U, p_0)$. Thus,

$$
\begin{aligned}
\frac{d\tilde{G}(p_0; r, \overline{p})}{dp_0} &= (p_N - c)\tilde{D}_2(p_N, p_0) - [(1-r)p_U + r\overline{p} - c]\tilde{D}_2(p_U, p_0) \\
&= (p_N - c)\tilde{D}(p_N, p_0)\frac{\tilde{D}_2(p_N, p_0)}{\tilde{D}(p_N, p_0)} - [(1-r)p_U + r\overline{p} - c]\tilde{D}(p_U, p_0)\frac{\tilde{D}_2(p_U, p_0)}{\tilde{D}(p_U, p_0)} \\
&\geq \left\{ (p_N - c)\tilde{D}(p_N, p_0) - [(1-r)p_U + r\overline{p} - c]\tilde{D}(p_U, p_0) \right\}\frac{\tilde{D}_2(p_N, p_0)}{\tilde{D}(p_N, p_0)} \\
&= \tilde{G}(p_0; r, \overline{p})\frac{\tilde{D}_2(p_N, p_0)}{\tilde{D}(p_N, p_0)},
\end{aligned}
$$
(A.4)

where the inequality holds because $0 < \tilde{D}_2(p_U, p_0)/\tilde{D}(p_U, p_0) \leq \tilde{D}_2(p_N, p_0)/\tilde{D}(p_N, p_0)$ owing to the property of increasing differences of $\log D$ in p_i and $\mathbf{p}_{-i}$.[43] By (A.4),

$$
\frac{d\tilde{G}(p_0; r, \overline{p})}{dp_0}
\begin{cases}
> 0 & \text{if } \tilde{G}(p_0; r, \overline{p}) > 0, \\
\geq 0 & \text{if } \tilde{G}(p_0; r, \overline{p}) = 0,
\end{cases}
$$

which implies that (A.3) holds.

∎

Next, I give a proof of Proposition 5.

Proof. When all of the other firms set a common price p_0, the best response $p_U \in \tilde{\psi}_U(p_0; r, \overline{p})$ is characterised by the FOC $(1-r)\tilde{D}(p_U, p_0) + [(1-r)p_U + r\overline{p} - c]\tilde{D}_1(p_U, p_0) = 0$. Substituting (18) into the FOC gives

$$
\tilde{D}(p_U, p_0) + \left[p_U - \hat{p} - \frac{\tilde{D}(\hat{p}, \hat{p})}{\tilde{D}_1(\hat{p}, \hat{p})} \right]\tilde{D}_1(p_U, p_0) = 0.
$$
(A.5)

By applying the envelope theorem to $\tilde{\pi}_U(p_U, p_0; r, \overline{p}(r; \hat{p})) = [(1-r)p_U + r\overline{p} - c]\tilde{D}(p_U, p_0)$

---

[43]If $\mathbf{p}_{-ij}$ denotes the vector of the consumer prices of all goods other than i and j,

$$
\begin{aligned}
\frac{\partial \frac{\tilde{D}_2(p_i, p_0)}{\tilde{D}(p_i, p_0)}}{\partial p_i} &= \frac{1}{\tilde{D}(p_i, p_0)^2}\left[\tilde{D}(p_i, p_0)\tilde{D}_{12}(p_i, p_0) - \tilde{D}_1(p_i, p_0)\tilde{D}_2(p_i, p_0) \right] \\
&= \frac{n-1}{D(p_i, \mathbf{p}_{-i})^2}\left[D(p_i, \mathbf{p}_{-i}) \times \frac{\partial^2 D(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j} - \frac{\partial D(p_i, \mathbf{p}_{-i})}{\partial p_i} \times \frac{\partial D(p_i, \mathbf{p}_{-i})}{\partial p_j} \right]\Big|_{\mathbf{p}_{-i}=(p_0, \cdots, p_0)} \\
&\geq 0,
\end{aligned}
$$

where the inequality follows from (3). In addition, $p_U \leq p_N$ by Proposition 2. Thus, $\tilde{D}_2(p_U, p_0)/\tilde{D}(p_U, p_0) \leq \tilde{D}_2(p_N, p_0)/\tilde{D}(p_N, p_0)$.

and noting that $r\overline{p} = c + (r-1)\left[\hat{p} + \tilde{D}(\hat{p}, \hat{p})/\tilde{D}_1(\hat{p}, \hat{p})\right]$ by (18),

$$\frac{d\tilde{\pi}_U(p_U, p_0; r, \overline{p}(r; \hat{p}))}{dr} = \left[-p_U + \hat{p} + \frac{\tilde{D}(\hat{p}, \hat{p})}{\tilde{D}_1(\hat{p}, \hat{p})}\right]\tilde{D}(p_U, p_0)$$

$$= \frac{\tilde{D}(p_U, p_0)^2}{\tilde{D}_1(p_U, p_0)}$$

$$< 0,$$

where the second line follows from (A.5). Thus,

$$\frac{d\tilde{G}(p_0; r, \overline{p}(r; \hat{p}))}{dr} = -\frac{d\tilde{\pi}_U(p_U, p_0; r, \overline{p}(r; \hat{p}))}{dr} > 0. \tag{A.6}$$

Because $\tilde{G}(\hat{p}; r_3, \overline{p}(r_3; \hat{p})) = 0$ and $\hat{p} < p_N^*$, Lemma 2 implies that $\tilde{G}(p_N^*; r_3, \overline{p}(r_3; \hat{p})) \geq 0$. Hence, (A.6), $\tilde{G}(p_N^*; r_2, \overline{p}(r_2; \hat{p})) = 0$, and $\tilde{G}(p_N^*; r_3, \overline{p}(r_3; \hat{p})) \geq 0$ imply that $r_2 \leq r_3$.

Next, (A.6) and $\tilde{G}(p_N^*; r_2, \overline{p}(r_2; \hat{p})) = 0$ imply that $\tilde{G}(p_N^*; r, \overline{p}(r; \hat{p})) < 0$ for all $r < r_2$ and that $G(p_N^*; r, \overline{p}(r; \hat{p})) > 0$ for all $r > r_2$. Thus, Proposition 4 implies that $p_N^* \notin E_I$ for all $r < r_2$ and that $p_N^* \in E_I$ for all $r \geq r_2$. In addition, because $p_N < p_N^*$ for any other $p_N \in E_N$, Lemma 2 and $\tilde{G}(p_N^*; r_2, \overline{p}(r_2; \hat{p})) = 0$ imply that $\tilde{G}(p_N; r_2, \overline{p}(r_2; \hat{p})) \leq 0$. Therefore, $\tilde{G}(p_N; r, \overline{p}(r; \hat{p})) < 0$ for all $r < r_2$ by (A.6), and thus $p_N \notin E_I$ for all $r < r_2$.

Similarly, (A.6) and $\tilde{G}(\hat{p}; r_3, \overline{p}(r_3; \hat{p})) = 0$ imply that $\tilde{G}(\hat{p}; r, \overline{p}(r; \hat{p})) < 0$ for all $r < r_3$ and that $G(\hat{p}; r, \overline{p}(r; \hat{p})) > 0$ for all $r > r_3$. Thus, Proposition 4 implies that $\hat{p} \in E_I$ for all $r \leq r_3$ and that $\hat{p} \notin E_I$ for all $r > r_3$.

Hence, statements (i)-(iii) hold. Given statements (i)-(iii), the statement about the Pareto-best NE in E_I follows from Proposition 4.

∎

A.7 Proof of Proposition 6

Proof. Note that $\tilde{\pi}_U(\hat{p}, \hat{p}; r, \overline{p}(r; \hat{p})) = -(1-r)\tilde{D}(\hat{p}, \hat{p})^2/\tilde{D}_1(\hat{p}, \hat{p})$. By the definition of r_1,

$$\tilde{\pi}_N(p_N^*, p_N^*) = \tilde{\pi}_U(\hat{p}, \hat{p}; r_1, \overline{p}(r_1; \hat{p})) = -(1-r_1)\tilde{D}(\hat{p}, \hat{p})^2/\tilde{D}_1(\hat{p}, \hat{p}). \tag{A.7}$$

When $r = r_2$, both p_N^* and $p_U^* = \hat{p}$ are in E_I (Proposition 5). Because $\hat{p} < p_N^*$, Proposition 4 means that

$$\tilde{\pi}_N(p_N^*, p_N^*) > \tilde{\pi}_U(\hat{p}, \hat{p}; r_2, \overline{p}(r_2; \hat{p})) = -(1 - r_2)\tilde{D}(\hat{p}, \hat{p})^2 / \tilde{D}_1(\hat{p}, \hat{p}). \tag{A.8}$$

By (A.7) and (A.8), $1 - r_2 < 1 - r_1$, so $r_1 < r_2$. Thus, (22) holds because $\tilde{\pi}_U(\hat{p}, \hat{p}; r, \overline{p}(r; \hat{p}))$ decreases with r (given $\hat{p}$).

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B Multidimensional Quality

This appendix extends the model in Section 5 to the case of multidimensional quality. Overall, I can incorporate multiple product attributes in analogous steps and obtain similar results.

The unit production cost of good i depends on its K-dimensional attributes $\mathbf{w}_i \in \mathbb{R}_+^K$ and is expressed as $c_i(\mathbf{w}_i)$. The function $c_i \colon \mathbb{R}_+^K \to \mathbb{R}_+$ is such that $\partial c_i(\mathbf{w}_i)/\partial w_i^k > 0\ \forall \mathbf{w}_i$ for each attribute $k \in \{1, \cdots K\}$ and $\nabla^2 c_i(\mathbf{w}_i)$ is positive definite for all $\mathbf{w}_i$, implying that $c_i(\cdot)$ is strictly convex. For the demand side, one unit of good i with quality $\mathbf{w}_i$ increases a representative consumer's utility by $f(\mathbf{w}_i)$, where the function $f \colon \mathbb{R}_+^K \to \mathbb{R}_+$ is such that $\partial f(\mathbf{w}_i)/\partial w_i^k > 0$ for each k and $\nabla^2 f(\mathbf{w}_i)$ is negative definite for all $\mathbf{w}_i$, implying that $f(\cdot)$ is strictly concave. Following the same steps as in the one-dimensional case, the demand for good i is given by $q_i = D_i(p_1 - f(\mathbf{w}_1), \cdots, p_n - f(\mathbf{w}_n))$, and $f(\mathbf{w}_i)$ is considered the premium attached to product i's quality or, equivalently, the consumer's willingness to pay (WTP) for quality.

[No Subsidy]

In the baseline case of no subsidy, given the qualities and prices of the other products, firm i sets $\mathbf{w}_i$ and p_i simultaneously to maximise its profit $\pi_{iN}(\mathbf{w}_i, p_i, \{\mathbf{w}_j, p_j\}_{j \neq i}) = [p_i - c_i(\mathbf{w}_i)]D_i(h_i, \mathbf{h}_{-i})$, where $h_i = p_i - f(\mathbf{w}_i)$ and $\mathbf{h}_{-i}$ is a vector of the h_j's for all $j \neq i$. Assuming an interior solution that gives a positive profit, the following FOCs are satisfied for each i:

$$\frac{\partial \pi_{iN}}{\partial w_i^k} = -\frac{\partial c_i(\mathbf{w}_i)}{\partial w_i^k}D_i(h_i, \mathbf{h}_{-i}) - [p_i - c_i(\mathbf{w}_i)]\frac{\partial D_i(h_i, \mathbf{h}_{-i})}{\partial h_i}\frac{\partial f(\mathbf{w}_i)}{\partial w_i^k} = 0 \quad \forall k, \tag{B.1}$$

$$\frac{\partial \pi_{iN}}{\partial p_i} = D_i(h_i, \mathbf{h}_{-i}) + [p_i - c_i(\mathbf{w}_i)]\frac{\partial D_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0. \tag{B.2}$$

Substituting (B.2) into (B.1) shows that the optimal quality, denoted by $\mathbf{w}_{iN}$, satisfies

$$\nabla f(\mathbf{w}_{iN}) - \nabla c_i(\mathbf{w}_{iN}) = \mathbf{0}. \tag{B.3}$$

The optimal quality $\mathbf{w}_{iN}$ is uniquely determined because $f(\cdot) - c_i(\cdot)$ is strictly concave. Given $\{\mathbf{w}_{iN}, c_i(\mathbf{w}_{iN})\}_{i \in \{1, \cdots, n\}}$, the discussion in Section 2 implies that an NE exists, at which (B.2) is satisfied for each i.

[IR Subsidy]

Suppose that each firm's threshold $\overline{p}_i$ is determined by the government as a function of $\mathbf{w}_i$, that is, $\overline{p}_i = \overline{p}(\mathbf{w}_i)$. Assuming that the subsidy scheme is sufficiently generous, I focus on an NE at which all firms opt in and thus $\pi_{iI} = \pi_{iU}$. As above, firm i maximises its profit $\pi_{iU}(\mathbf{w}_i, p_i, \{\mathbf{w}_j, p_j\}_{j \neq i}) = [(1-r)p_i + r\overline{p}(\mathbf{w}_i) - c_i(\mathbf{w}_i)]D_i(h_i, \mathbf{h}_{-i})$ with respect to $\mathbf{w}_i$ and p_i. Assuming an interior solution that provides a positive profit, the following FOCs are satisfied for each i:

$$\frac{\partial \pi_{iU}}{\partial w_i^k} = [r\frac{\partial \overline{p}(\mathbf{w}_i)}{\partial w_i^k} - \frac{\partial c_i(\mathbf{w}_i)}{\partial w_i^k}]D_i(h_i, \mathbf{h}_{-i}) - [(1-r)p_i + r\overline{p}(\mathbf{w}_i) - c_i(\mathbf{w}_i)]\frac{\partial D_i(h_i, \mathbf{h}_{-i})}{\partial h_i}\frac{\partial f(\mathbf{w}_i)}{\partial w_i^k} = 0 \quad \forall k, \tag{B.4}$$

$$\frac{\partial \pi_{iU}}{\partial p_i} = (1-r)D_i(h_i, \mathbf{h}_{-i}) + [(1-r)p_i + r\overline{p}(\mathbf{w}_i) - c_i(\mathbf{w}_i)]\frac{\partial D_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0. \tag{B.5}$$

Substituting (B.5) into (B.4) shows that the optimal quality, denoted by $\mathbf{w}_{iU}$, satisfies

$$(1-r)\nabla f(\mathbf{w}_{iU}) - \nabla c_i(\mathbf{w}_{iU}) + r\nabla \overline{p}(\mathbf{w}_{iU}) = \mathbf{0}, \tag{B.6}$$

where I assume for simplicity that $\mathbf{w}_{iU}$ is uniquely determined.[44] Equation (B.6) shows that the policymaker can affect the realised quality by adjusting the gradient $\nabla \overline{p}(\cdot)$. For example, (B.3) and (B.6) imply that if $\overline{p}_i$ is constant (i.e. $\nabla \overline{p}(\cdot) = \mathbf{0}$), then the optimal product quality (denoted by $\tilde{\mathbf{w}}_{iU}$) is lower than that in the no-subsidy case in terms of WTP and the unit production cost: $f(\tilde{\mathbf{w}}_{iU}) < f(\mathbf{w}_{iN})$ and $c_i(\tilde{\mathbf{w}}_{iU}) < c_i(\mathbf{w}_{iN})$.[45] Alternatively, in principle, setting

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[44]Assume, for example, that $\overline{p}(\cdot)$ is linear or, more generally, that r and $\overline{p}(\cdot)$ are set so that $(1-r)f(\cdot) - c_i(\cdot) + r\overline{p}(\cdot)$ is strictly concave.

[45]It follows from (B.3) and (B.6) with $\nabla \overline{p}(\cdot) = \mathbf{0}$ that $\mathbf{w}_{iN} \neq \tilde{\mathbf{w}}_{iU}$. Because $\mathbf{w}_{iN}$ is a unique maximiser of $f(\mathbf{w}_i) - c_i(\mathbf{w}_i)$,

$$f(\mathbf{w}_{iN}) - c_i(\mathbf{w}_{iN}) > f(\tilde{\mathbf{w}}_{iU}) - c_i(\tilde{\mathbf{w}}_{iU}). \tag{B.7}$$

$\nabla \overline{p}(\cdot) = \nabla f(\cdot)$ makes (B.3) and (B.6) equivalent, resulting in the same optimal quality with or without the IR subsidy ($\mathbf{w}_{iU} = \mathbf{w}_{iN}$).

As another illustration, consider $\overline{p}(\cdot)$ that is linear in each argument (and non-constant) (i.e. $\nabla \overline{p}(\cdot) = \mathbf{v} \neq \mathbf{0}$). This linear schedule induces the optimal quality $\mathbf{w}_{iU}$ such that $\overline{p}(\mathbf{w}_{iU}) > \overline{p}(\tilde{\mathbf{w}}_{iU})$, which means that $\mathbf{w}_{iU}$ lies above the hyperplane through $\tilde{\mathbf{w}}_{iU}$ that is perpendicular to $\mathbf{v}$ (i.e. $\mathbf{v} \cdot (\mathbf{w}_{iU} - \tilde{\mathbf{w}}_{iU}) > 0$).[46] In addition, if the policymaker aims to improve a particular attribute k (because of, for example, its positive externalities), increasing v_k (the kth element of $\mathbf{v}$) raises its equilibrium quality (i.e. $\partial w_{iU}^k / \partial v_k > 0$).[47]

Given $\{\mathbf{w}_{iU}, c_i(\mathbf{w}_{iU})\}_{i \in \{1, \cdots, n\}}$, the discussion in Section 2 implies that an NE exists, at which (B.5) is satisfied for each i.

Similarly, because $\tilde{\mathbf{w}}_{iU}$ is a unique maximiser of $(1-r)f(\mathbf{w}_i) - c_i(\mathbf{w}_i)$,

$$(1-r)f(\tilde{\mathbf{w}}_{iU}) - c_i(\tilde{\mathbf{w}}_{iU}) > (1-r)f(\mathbf{w}_{iN}) - c_i(\mathbf{w}_{iN}). \tag{B.8}$$

With (B.7) and (B.8), $(1-r)f(\tilde{\mathbf{w}}_{iU}) - c_i(\tilde{\mathbf{w}}_{iU}) > f(\tilde{\mathbf{w}}_{iU}) - c_i(\tilde{\mathbf{w}}_{iU}) - rf(\mathbf{w}_{iN})$, so $f(\tilde{\mathbf{w}}_{iU}) < f(\mathbf{w}_{iN})$. With (B.8), $c_i(\tilde{\mathbf{w}}_{iU}) - c_i(\mathbf{w}_{iN}) < (1-r)[f(\tilde{\mathbf{w}}_{iU}) - f(\mathbf{w}_{iN})] < 0$, so $c_i(\tilde{\mathbf{w}}_{iU}) < c_i(\mathbf{w}_{iN})$.

[46]More generally, given a (possibly nonlinear) schedule $\overline{p}(\mathbf{w}_i)$ (with $\nabla \overline{p}(\tilde{\mathbf{w}}_{iU}) \neq \mathbf{0}$), the optimal quality $\mathbf{w}_{iU}$ satisfies $\overline{p}(\mathbf{w}_{iU}) > \overline{p}(\tilde{\mathbf{w}}_{iU})$, $\nabla \overline{p}(\tilde{\mathbf{w}}_{iU}) \cdot (\mathbf{w}_{iU} - \tilde{\mathbf{w}}_{iU}) > 0$, and $\nabla \overline{p}(\mathbf{w}_{iU}) \cdot (\mathbf{w}_{iU} - \tilde{\mathbf{w}}_{iU}) > 0$.

[47]Given r, the optimal quality $\mathbf{w}_{iU}$ is determined by the FOC (B.6) with $\nabla \overline{p}(\mathbf{w}_i) = \mathbf{v}$. By the implicit function theorem, $\mathbf{w}_{iU}$ is expressed as a function of $\mathbf{v}$ (let $\mathbf{w}_{iU} = g(\mathbf{v})$), and $\nabla g(\mathbf{v}) = -r[(1-r)\nabla^2 f(\mathbf{w}_{iU}) - \nabla^2 c_i(\mathbf{w}_{iU})]^{-1}$. Because $\nabla^2 f(\mathbf{w}_{iU})$ is negative definite and $\nabla^2 c_i(\mathbf{w}_{iU})$ is positive definite, $\nabla g(\mathbf{v})$ is positive definite. Therefore, the diagonal elements of $\nabla g(\mathbf{v})$ are positive (that is, $\partial w_{iU}^k / \partial v_k > 0$).