Shape-optimization of 2D hydrofoils using one-way coupling of an IGA-BEM solver with a boundary-layer model

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Outline

inviscid flow: continuous formulation

inviscid flow: discrete IGA-BEM formulation

BL (boundary-layer) corrections

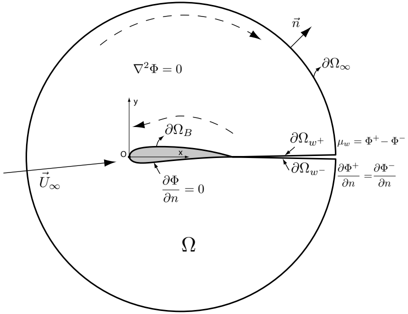
IGA-oriented BL corrections

shape-optimisation environment

shape-optimisation tests

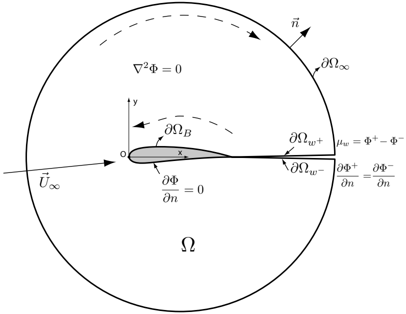
future work

inviscid flow: continuous formulation



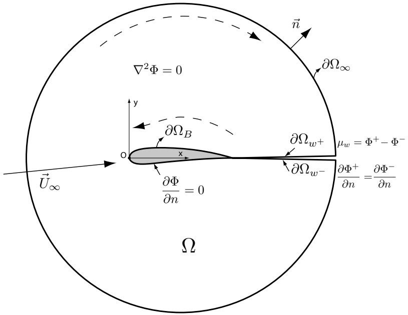
- **"Shape-optimization of 2D hydrofoils using an Isogeometric BEM solver"**, *Computer Aided Design*, **82**, 79-87, (2017)

inviscid flow: continuous formulation



- consider a 2D body whose boundary is $\partial\Omega_B$ moving with constant speed $\vec{U}_B$ in an ideal fluid of infinite extent.

inviscid flow: continuous formulation



- in a body-fixed coordinate system Oxy this problem is equivalent to a uniform stream with velocity $\nabla\Phi_\infty = \vec{U}_\infty = -\vec{U}_B$, where $\Phi_\infty(\mathbf{P}) = u_\infty x + v_\infty y$ is the far-field asymptotic form of the velocity potential $\Phi(\mathbf{P})$ of the resulting flow at point $\mathbf{P}=(x, y)$.

inviscid flow: BVP formulation

boundary-value problem (BVP)

$$\nabla^2 \Phi = 0, \quad \mathbf{P} = (x, y) \in \Omega$$

$$\frac{\partial \Phi}{\partial n} = 0, \quad \mathbf{P} \in \partial \Omega_B$$

$$\Phi - (u_\infty x + v_\infty y) \to 0, \quad \text{as } x^2 + y^2 \to \infty$$

- the above BVP has a unique solution up to an additive constant and, in order to fix a unique solution, we normally consider, for smooth bodies, zero circulation, $\Gamma(C) = \int_C \nabla\Phi \cdot d\mathbf{c} = 0$, over any circuit C surrounding the body

inviscid flow: BVP formulation

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- the difference between potential flows around a smooth body and a hydrofoil is that, in order for the flow around the hydrofoil to have a physical meaning, $\Gamma(C) \neq 0$ and appropriately adjusted until the flow leaves the trailing edge smoothly

inviscid flow: BVP formulation

boundary-value problem (BVP)

$$
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\end{aligned}
$$

- on the basis of Kelvin's theorem, Prandtl concluded that if an airfoil, which started its motion from rest in an ideal fluid, is later found to possess $\Gamma \neq 0$ then the component of the boundary of the fluid which coincided with the airfoil initially, must coincide at a later time with the union of the airfoil surface and a surface, the so-called *wake*, embedded in the fluid which has circulation $-\Gamma$

inviscid flow: BVP formulation

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\Phi - (u_\infty x + v_\infty y) &\to 0, \quad \text{as } x^2 + y^2 \to \infty
\end{aligned}$$

- in contrast to the 3D case the location and shape of the wake in the 2D case can be taken, without loss of generality, to be a straight line emanating from the trailing edge and extending to infinity

inviscid flow: BVP formulation

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\end{aligned}
$$

- this line is a force-free boundary along which the normal fluid velocity and the pressure should exhibit no jump

inviscid flow: BIE formulation

boundary integral equation (BIE)

applying in Ω Green's second identity between the potential $\Phi(\mathbf{P})$, $\mathbf{P}\in\Omega$, and the fundamental solution,

$$G(\mathbf{P},\mathbf{Q}) = (1/2\pi)\ln\|\mathbf{P}-\mathbf{Q}\|$$

of the 2D Laplace equation, we can reformulate the BVP as a 2^{nd}-kind Fredholm integral equation on $\partial\Omega_B$

$$\frac{\Phi(\mathbf{P})}{2} + \int_{\partial\Omega_B} \Phi(\mathbf{Q})\frac{\partial G(\mathbf{P},\mathbf{Q})}{\partial n_Q} ds_Q - \mu_w \int_{\partial\Omega_w} \frac{\partial G(\mathbf{P},\mathbf{Q})}{\partial n_Q} ds_Q = \Phi_\infty(\mathbf{P}),$$

$$\mathbf{P} \in \partial\Omega_B \backslash \mathbf{P}_{TE}$$

$$\mu_w = \Phi^+(\mathbf{P}) - \Phi^-(\mathbf{P}) = constant, \quad \mathbf{P} \in \partial\Omega_w$$

inviscid flow: discrete IGA-BEM formulation

IGA basis

we assume that the body boundary $\partial\Omega_B$ can be (accurately) represented as a closed parametric NURBS curve $\mathbf{r}(t)$, which is regular with the exception of the trailing edge: $\mathbf{r}(0) = \mathbf{r}(1)$, where the derivative vector is not defined

$$\mathbf{r}(t) = (x(t), y(t)) := \sum_{i=0}^{n} \mathbf{d}_i M_{i,k}(t), \quad t \in [0, 1]$$

inviscid flow: IGA-BEM continuous formulation

BIE: an alternative form

$$\frac{\phi(t)}{2}+\int_I \phi(\tau)K(t,\tau)d\tau-\frac{\mu_w}{2\pi}\,\mathrm{arctan}\left(\frac{y(t)-y_e}{x(t)-x_e}\right)=g(t),\ \ t\in(0,1)$$

$\phi(t) := \phi(\mathbf{r}(t))$
$G(t,\tau)) : = G(\mathbf{r}(t),\mathbf{r}(\tau))$
$K(t,\tau) = (\partial G(t,\tau)/\partial n_\tau)\|\dot{\mathbf{r}}(\tau)\|$
$g(t) = -\int_I(\vec{U}_\infty \cdot \vec{n}(\tau))G(t,\tau)\|\dot{\mathbf{r}}(\tau)\|d\tau.$

inviscid flow: IGA-BEM discrete formulation

spline space for the perturbation potential $\phi(t)$

project the perturbation potential $\phi(t)$ on the spline space $\mathcal{S}^k(\mathcal{J}^{(\ell)})$, $\mathcal{S}^k(\mathcal{J}^{(0)}) := \mathcal{S}^k(\mathcal{J})$, expressed in the form:

$$\phi_s(t) := \mathcal{P}_s(\phi(t)) = \sum_{i=0}^{n+\ell} \phi_i M_{i,k}^{(\ell)}(t), \quad t \in I, M_{i,k}^{(0)}(t) := M_{i,k}(t),$$

where $\ell \in \mathbb{N}_0$ denotes the number of knots inserted in I.

inviscid flow: IGA-BEM discrete formulation

BIE: discretisation through collocation

$$\frac{1}{2}\sum_{i=0}^{n+\ell}\phi_i M_{i,k}^{(\ell)}(t_j)+\sum_{i=0}^{n+\ell}\phi_i q_i(t_j)-\frac{(\phi_{n+\ell}-\phi_0)}{2\pi}\arctan\left(\frac{y(t_j)-y_e}{x(t_j)-x_e}\right)$$
$$=g(t_j),\quad j=0,\dots n+\ell$$
$$q_i(t_j)=\int_I M_{i,k}^{(\ell)}(\tau)K(t_j,\tau)d\tau$$

where the collocation points $t=t_j$, are chosen to be the Greville abscissas associated with the knot vector $\mathcal{J}^{(\ell)}$

inviscid flow: IGA-BEM discrete formulation

discretisation through collocation

- since $t \in (0, 1)$, one must consider shifting the values of the first and the last Greville abscissa by a small value $\epsilon > 0$

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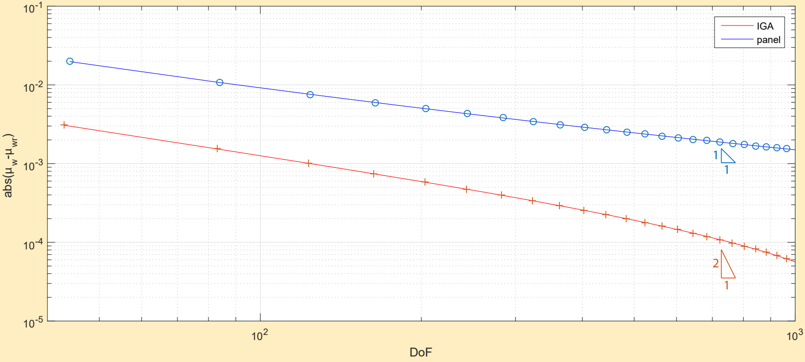
discretisation through collocation

- since $t \in (0, 1)$, one must consider shifting the values of the first and the last Greville abscissa by a small value $\epsilon > 0$
- this shifting process gives rise to the need of proper handling of the numerical evaluation of the integral terms $q_0(1-\epsilon)$ and $q_{n+\ell}(\epsilon)$ appearing in the linear system
- these integrals can be handled by an adaptive numerical integration scheme provided that precautions are taken in order that ϵ does not fall under a threshold value for which the integrals cannot be numerically computed.

inviscid flow: IGA-BEM discrete formulation

error comparison for a NACA-4412 profile



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- for an incompressible turbulent flow past an airfoil viscous effects are important in a small region around the foil
- in this region N-S (Navier-Stokes) equations are approximated by the so-called BL equations
- BL model composes:
  1. a model for the laminar part of the flow
  2. a criterion for the transition point between the laminar and the turbulent flow
  3. a model for the turbulent part of the flow

a one-way coupled computation model)

In this work we adopt a simple one-way coupled computation model consisting of:

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- Thwaites' one equation for laminar flow
- Head's two equations for the turbulent flow
- Michel's criterion for the transition point
- Squire-Young formula for the drag coefficient

Thwaites' one equation

Von-Karman integral momentum equation

$$\frac{d\theta}{dx} + (2 + H)\frac{\theta}{U_e} \cdot \frac{dU_e}{dx} = \frac{1}{2}C_f \tag{1}$$

- x/y are curvilinear coordinates measured tangentially/normal to the airfoil boundary from the stagnation point
- $U_e(x)$ is the free-stream velocity outside the boundary layer

$$\theta = \int_0^\infty \frac{u}{U_e} \cdot (1 - \frac{u}{U_e})dy : \textit{momentum thickness}$$

$$H = \frac{\delta^*}{\theta} : \textit{shape factor},\ \delta^* = \int_0^\infty (1-\frac{u}{U_e})dy : \textit{displacement thickness}$$

$$C_f = \frac{\mu \frac{\partial u}{\partial y}\big|_y = 0}{\frac{1}{2}\rho {U_e}^2} : \textit{skin} - \textit{friction coefficient}$$

laminar model: Thwaites' approach

after some manipulation on (1) we get:

$$\frac{U_e}{\nu} \cdot \frac{d\theta^2}{dx} = 2[(2+H)m + \ell(m)] := L(m)$$

$$m = \left(\frac{\theta^2}{U_e}\right) \frac{\partial^2 u}{\partial y^2}\bigg|_{y=0}, \quad \ell(m) = \frac{\theta}{U_e} \cdot \frac{\partial u}{\partial y}\bigg|_{y=0}$$

Thwaites' argument

there should exist a function relating m and $\ell(m)$ and suggested $L(m) = 0.45 + 6m$, which reduces the Von Karman equation (1) to an ODE for the momentum thickness $\theta(x)$:

laminar model: Thwaites' approach

Thwaites' ODE

$$U_e \frac{d}{dx}\left(\frac{\theta^2}{\nu}\right) = 0.45 - 6\left(\frac{\theta^2}{\nu}\right)\frac{dU_e}{dx}$$

which is integrated to:

$$\theta^2(x) = \left[\frac{U_e(0)}{U_e(x)}\right]^6 \theta^2(0) + \frac{0.45\nu}{U_e^6(x)} \int_0^x U_e^5(x')dx'$$

- laminar separation cannot be predicted

transition point: Michel's criterion

transition should be exptected when

$$Re_\theta > Re_{\theta_max} = 1.174\left(1 + \frac{22.4}{Re_x}\right) {Re_x}^{0.46}$$

where

$$Re_\theta = \frac{U_e \theta}{\nu} \quad Re_x = \frac{U_e x}{\nu}$$

turbulent model: Head's model

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turbulent model: Head's model

- it is a typical integral method, where semi-empitical relations are used to close the system
- starting from Von Karman integral momentum equation and defining a new parameter $H_1 = (\delta - \delta^*)/\theta$, where δ is the thickness of the boundary layer, we get Head's system of equations

turbulent model: Head's model

Head's system of equations:

$$\frac{d\theta}{dx} + (2 + H)\frac{\theta}{U_e} \cdot \frac{dU_e}{dx} = \frac{1}{2}C_f$$

$$\frac{dH_1}{dx} = -H_1\left(\frac{1}{U_e} \cdot \frac{dU_e}{dx} + \frac{1}{\theta} \cdot \frac{d\theta}{dx}\right) + \frac{0.0306}{\theta}(H_1 - 3)^{-0.619}$$

with the closure condition

$$H_1 = \begin{cases} 3.3 + 0.8234(H - 1.1)^{-1.287}, & H \leq 1.6 \\ 3.3 + 1.5501(H - 0.6778)^{-3.064}, & H > 1.6 \end{cases}$$

for the numerical solution a 2nd-order Runge-Kutta is used

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separation criterion

$H_1 = 3.3$

drag coefficient

Squire-Young formula

$$C_d = \left[2\theta_{TE}\left(U_e\right)^{\frac{H_{TE}+5}{2}}\right]_{UP} + \left[2\theta_{TE}\left(U_e\right)^{\frac{H_{TE}+5}{2}}\right]_{LOW}$$

this formula predicts the drag coefficient by relating the momentum defect far downstream to the values of the flow field at the trailing edge (TE)

IGA-oriented BL corrections

PABLO

- stands for *"Potential flow around Airfoils with Boundary Layer coupled One-way"*
- is a subsonic airfoil analysis and design program developed in MATLAB by C. Wauquiez and A. Rizzi

IGA-enhancements of PABLO

- replace PABLO's low-order panel approximations of the free-stream velocity U_e outside the boundary layer
- with its NURBS representation that can be obtained by using the derivatives of the IGA-BEM rational B-spline basis

shape-optimisation environment

components

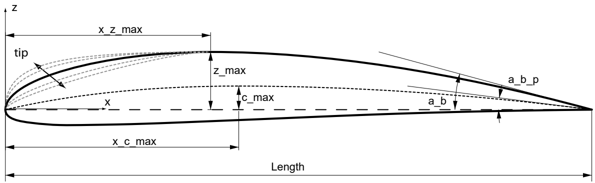
1. the optimization algorithm
2. the IGA-BEM solver
   - inviscid
   - with BL corrections
3. the parametric modeler

parametric modeler

- the parametric model for a general hydrofoil has been materialized within Rhinoceros® 3D modeling software package with the aid of its VBscript-based programming language, Rhinoscript

parametric modeler

- our model generates a closed cubic B-Spline curve that represents a hydrofoil, using a set of 8 parameters



parametric modeler

- all parameters, with the exception of chord's length (L), are defined using appropriate non-dimensional ratios so that their values always lie in $[0, 1]$

parametric modeler

- all parameters, with the exception of chord's length (L), are defined using appropriate non-dimensional ratios so that their values always lie in $[0, 1]$
- this approach eliminates the need of implementing complex interdependent constraints while guaranteeing the robustness of the procedure which is of significant importance in an optimization procedure

parametric modeler

Nr.	Name	description	symbol	actual range
1	Length	Length of hydrofoil's chord	`L`	free
2	Max width	Maximum width of suction side wrt chord	`max_z`	$\left[\frac{\mathrm{L}}{500}, \frac{\mathrm{L}}{5}\right]$
3	Camber width	Camber maximum width wrt chord	`max_c`	$[0, 0.91\mathrm{max\_z}]$
4	Max-width position	Longitudinal position of suction side's max width	`x_z_max`	$\left[\frac{\mathrm{L}}{5}, \frac{7\mathrm{L}}{10}\right]$
5	Max-camber-width position	Longitudinal position of camber's max width	`x_c_max`	$\left[0, \frac{3\mathrm{L}}{10}\right] + \frac{7\mathrm{x\_z\_max}}{10}$
6	Suction-side angle	Suction's side angle at trailing edge wrt chord	`a_b`	$\left[\arctan\left(\frac{\mathrm{z\_max}}{\mathrm{L}-\mathrm{x\_z\_max}}\right), 89\right]$
7	Camber angle	Camber angle at trailing edge wrt chord	`a_b_p`	$[0, \mathrm{a\_b}]$
8	Tip	Leading edge form factor	`tip`	$[0.1, 0.9]$

optimization algorithm

- the selected optimization algorithm belongs to the category of evolutionary ones, as experimentation with gradient and hessian-based algorithms has indicated the existence of multiple local minima that makes their usage problematic
- our optimizer uses the multi-objective optimization method `gamultiobj` which employs a *controlled elitist* genetic algorithm (GA)

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- an elitist GA always favors individuals with better fitness value (rank)
- a controlled elitist GA also favors individuals that can help increase the diversity of the population even if they have a lower fitness value
- it is important to maintain the diversity of population for convergence to an optimal Pareto front

shape-optimization test

optimisation criteria

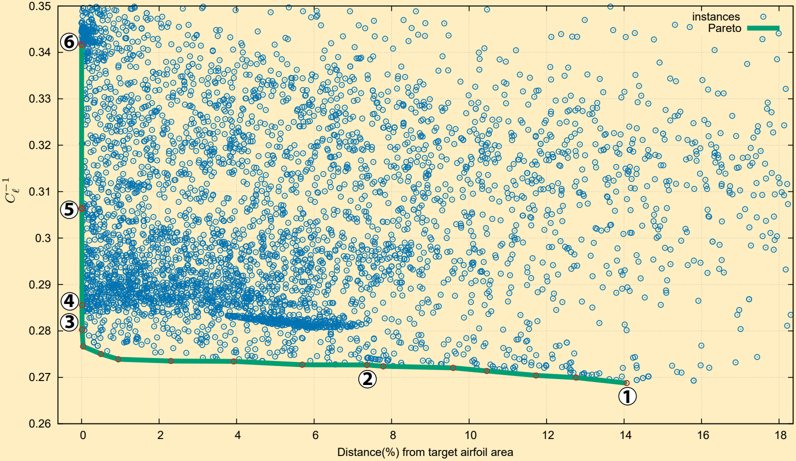
- inviscid model: maximum lift coefficient C_ℓ
- BL model: minimize C_d/C_ℓ
- minimum deviation from a reference area

options made

- the reference area is set to be the one of the NACA-4412 profile ($= 0.0816764523$)
- the IGA-BEM solver produces an average lift coefficient calculated for three angles of attack, namely 1, 3 and 5 degrees
- the parameter Length (`L`) of the hydrofoil parametric model is assumed to be fixed and is regularized to the value of one

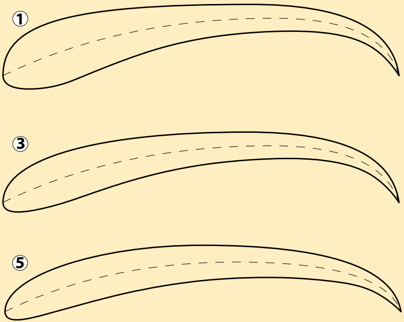
shape-optimisation results

solver: IGA-BEM inviscid



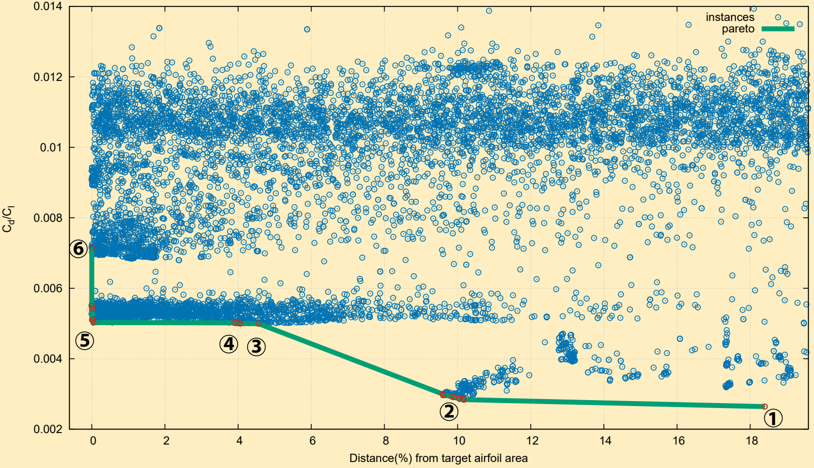
shape-optimisation results

solver: IGA-BEM inviscid: pareto instances



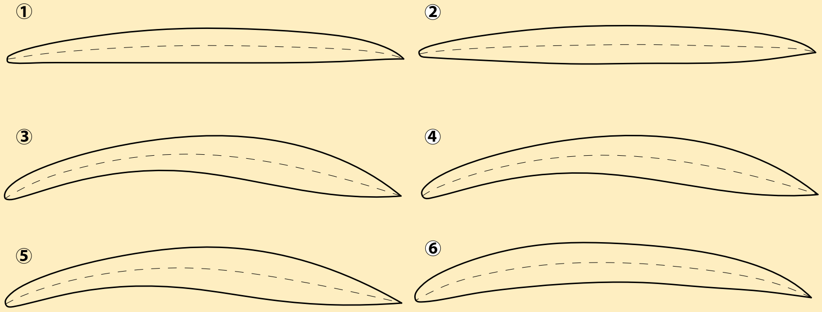
shape-optimisation results

solver: IGA-BEM inviscid with BL corrections



shape-optimisation results

solver: IGA-BEM inviscid with BL corrections: pareto instances



future work

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- BL corrections: IGA-oriented two-way coupling