Fault Detection and Isolation in Controlled Multi-Robot Systems

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Détection et isolation de défauts dans les systèmes multi-robots commandés

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Fault detection and isolation in controlled multi-robot systems

Abstract: Multi-Agent Systems (MASs) have attracted much popularity, since the previous decade due to their potential wide range of applications. Indeed, connected MASs are deployed in order to achieve more complex objectives that could otherwise not be achievable by a single agent. In distributed schemes, agents must share their information with their neighbours, which are then used for common control and fault detection purposes, and thus do not require any central monitoring unit. This translates into the necessity to develop efficient distributed algorithms in terms of robustness and safety. Indeed, the problem of safety in connected cooperative MASs has arisen as a consequence of their complexity and the nature of their operations and wireless communication exchanges, which renders them vulnerable to not only physical faults, but also to cyber-attacks. The main focus of this thesis is the study of distributed fault and attack detection and isolation in connected MASs. First, a distributed methodology for global detection of actuator faults in a class of linear MASs with unknown disturbances is proposed using a cascade of fixed-time Sliding Mode Observers (SMOs), where each agent having access to their state, and neighbouring information exchanges, can give an exact estimate of the state of the overall MAS. An LMI-based approach is then applied to design distributed global robust residual signals at each agent capable of detecting faults anywhere in the network. This is then extended to agents with nonlinear nonholonomic dynamics where a new distributed robust Fault Detection and Isolation (FDI) scheme is proposed using predefined-time stability techniques to derive adequate distributed SMOs. This enables to reconstruct the global system state in a predefined-time and generate proper residual signals. The case of MASs with higher order integrator dynamics, where only the first state variable is measurable and the topology is switching is investigated, where a new approach to identify faults and deception attacks is introduced. The proposed protocol makes an agent act as a central node monitoring the whole system activities in a distributed fashion whereby a bank of distributed predefined-time SMOs for global state estimation are designed, which are then used to generate residual signals capable of identifying cyber-attacks despite the switching topology. The problem of attack and FDI in connected heterogeneous MASs with directed graphs, is then studied. First, the problem of distributed fault detection for a team of heterogeneous MASs with linear dynamics is investigated, where a new output observer scheme is proposed which is effective for both directed and undirected topologies. The main advantage of this approach is that the design, being dependant only on the input-output relations, renders the computational cost, information exchange and scalability very effective compared to other FDI approaches that employ the whole state estimation of the agents and their neighbours as a basis for their design. A more general model is then studied, where actuator, sensor and communication faults/attacks are considered in the robust detection and isolation process for nonlinear heterogeneous MASs with measurement noise, dynamic disturbances and communication parameter uncertainties, where the topology is not required to be undirected. This is done using a distributed finite-frequency mixed $\mathcal{H}_{-}/\mathcal{H}_{\infty}$ nonlinear UIO-based approach. Simulation examples are given for each of the proposed algorithms to show their effectiveness and robustness.

Keywords: Fault Detection and Isolation; Attack Detection; Multi-Agent Systems; Networked Systems; Unknown Input Observers; Linear Matrix Inequalities; Fixed-time Stability; Predefined-time Stability.

Détection et isolation de défauts dans les systèmes multi-agents commandés

Résumé: Les systèmes multi-agents (SMAs) ont beaucoup attiré depuis la décennie précédente en raison de leur large éventail d'applications. En effet, les SMAs connectés sont déployés afin d'atteindre un objectif plus complexe qui pourrait autrement ne pas être réalisable par un seul agent. Dans les approches distribuées, les agents doivent partager leurs informations avec leurs voisins, qui sont ensuite utilisés à des fins de commande et de détection de défaut, et ne nécessitent donc aucune unité de surveillance centrale. Cela se traduit par la nécessité de développer des algorithmes distribués efficaces en termes de robustesse et de sécurité. En effet, le problème de la sécurité dans les SMAs coopérants et connectés est apparu à la suite de leur complexité, de la nature de leurs opérations et de leurs échanges via communication sans fil, ce qui les rend vulnérables non seulement aux défauts physiques, mais également aux cyber-attaques. L'étude de la détection et de l'isolation distribuées de fautes et des attaques dans les SMAs est la principale contribution de cette thèse. Premièrement, une méthodologie distribuée de détection globale des défauts d'actionneur dans une classe de systèmes multi-agents linéaires avec des perturbations inconnues est proposée à l'aide d'une cascade d'observateurs mode glissant à temps fixe, où chaque agent ayant accès à leur état et les échanges d'informations voisins, peuvent donner une estimation exacte de l'état du système global. Une approche basée sur les Inégalités Matricielles Linéaires (IMLs) est ensuite utilisée pour la conception distribuée de résidus robustes au niveau de chaque agent, ces résidus étant capable de détecter des défauts n'importe où dans la flotte. Celle-ci est ensuite étendue aux agents avec une dynamique non linéaire non holonome dans laquelle un nouvel algorithme robuste et distribué de détection et d'isolation de défauts est proposé à l'aide de techniques de stabilité à temps prédéfini afin de construire des observateurs mode glissant distribué. Cela permet de reconstruire l'état du système global dans un temps prédéfini et de générer correctement des signaux résiduels. Le cas des systèmes multi-agents sous forme de chaîne d'intégrateurs, où seule la première variable d'état est mesurable et la topologie est dynamique, est étudié, où une nouvelle approche pour identifier les défauts et les cyber-attaques est introduite. L'algorithme proposé fait en sorte que chaque agent joue le rôle d'une unité de surveillance centrale de l'ensemble des activités du système de manière distribuée, en employant des banques d'observateurs mode glissant basés sur la notion de stabilité prédéfinie, où l'estimation de l'état global et des signaux résiduels duquel ils sont générés se fait avant chaque instant de changement de topologie de communication. Ces résidus sont alors capables de distinguer les cyber-attaques, des fautes physiques malgré la topologie commutée. Le problème de la détection et de l'isolation des attaques et des fautes dans des systèmes hétérogènes connectés avec des topologies dirigées, est ensuite étudié. Premièrement, le problème de la détection distribués des défauts actionneurs pour les systèmes multi-agents linéaires et hétérogènes est traité, où un nouveau système d'observateur de sortie a été proposé pour les topologies dirigées et non dirigées. Le principal avantage de cette approche est que la conception ne dépend que des relations entrées/sorties, ce qui rend le temps de calcul, la quantité d'informations échangées et la flexibilité, très intéressants par rapport à d'autres approches qui utilisent l'estimation de tout l'état des agents et leurs voisins comme base de leur conception. Enfin, un modèle plus général est étudié, où les fautes actionneurs, capteurs, et les défauts/attaques de communication sont considérés dans le processus de détection et d'isolation pour les systèmes multi-agents hétérogènes non linéaires avec bruits de mesures, perturbations dynamiques et incertitudes paramétriques sur la liaison de communication. Pour réaliser cet objectif, une approche de type $\mathcal{H}_{-}/\mathcal{H}_{\infty}$ à fréquence finie est utilisée pour la synthèse des observateurs. Des exemples de simulation sont donnés pour chacun des algorithmes proposés pour montrer leur efficacité et leur robustesse.

Mots-Clés: Détection et Isolation de Défauts; Détection d'attaques; Systèmes Multi-Agents; Systèmes Connectés; Observateurs à Entrées Inconnues; Inégalité Matricielle Linéaire; Stabilité en Temps Fixe; Stabilité en Temps Prédéfini.

Dedicated to

the memory of my father
and to my loving family

Contents

List of Notations

- $\mathbb{C}$ — Set of complex numbers
- $\mathbb{R}$ — Set of real numbers
- $\mathbb{R}^{m\times n}$ — Set of real m by n matrices of entries in $\mathbb{R}$
- $\forall \alpha \in \mathbb{R}^{>0}$, $\forall x \in \mathbb{R}$ — $x \to \lceil x \rfloor^{\alpha}, \quad \lceil x \rfloor^{\alpha} = |x|^{\alpha}\text{sign}(x)$
- For $x = [x_1, x_2, ..., x_N]^T \in \mathbb{R}^N$ — $\lceil x \rfloor^{\alpha} = [\text{sign}(x_1)\lceil x_1 \rfloor^{\alpha}, \text{sign}(x_1)\lceil x_2 \rfloor^{\alpha}, ...,$ $..., \text{sign}(x_N)\lceil x_N \rfloor^{\alpha}]^T$
- $\mathcal{Q}$ — Topology graph
- $\mathcal{A}(\mathcal{Q})$ — Adjacency matrix of $\mathcal{Q}$
- $\mathcal{D}(\mathcal{Q})$ — In-degree matrix of $\mathcal{Q}$
- $\mathcal{L}(\mathcal{Q})$ — Laplacian matrix of $\mathcal{Q}$
- $||(\cdot)||_1$ — Manhattan norm of vector $(\cdot)$
- $||(\cdot)||_2$ — Euclidean norm of vector $(\cdot)$
- $|(\cdot)|$ — Cardinality of the set $(\cdot)$
- $0_{m\times n}$ — Matrix of dimensions $m \times n$ with all 0 entries
- $\mathbf{I}_{m\times n}$ — Identity matrix of dimensions $m \times n$
- $\mathbf{I}$ — Identity matrix of appropriate dimensions
- $\mathbf{1}_n$ — Vector $\mathbf{1}_n = [1, ..., 1]^T \in \mathbb{R}^n$
- $\lambda_{min}([\cdot])$ — Smallest eigenvalue of square matrix $[\cdot]$
- $\lambda_{max}([\cdot])$ — Largest eigenvalue of square matrix $[\cdot]$
- $\bar{\sigma}([\cdot])$ — Maximum singular value of $[\cdot]$
- $\underline{\sigma}([\cdot])$ — Minimum singular value of $[\cdot]$
- $P > 0 \in \mathbb{R}^{n\times n}$ — P is symmetric and positive-definite, $P = \begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$, and $*$ denotes symmetry
- $P \geqslant 0 \in \mathbb{R}^{n\times n}$ — P is symmetric and positive semi-definite
- A^T — Transpose of matrix A
- $rank(A)$ — Rank of matrix A
- $ker(A)$ — Kernel of matrix A
- A^* — Conjugate of A
- $\mathbf{He}(A)$ — Corresponds to $\mathbf{He}(A) = A + A^*$
- $\text{tr}(A)$ — Trace of A
- $\text{diag}(a_1, a_2, ..., a_n)$ — Diagonal matrix containing $a_1, a_2, ..., a_n$ on the diagonal
- $\text{Blkdiag}(A_1, A_2, ..., A_n)$ — Block diagonal matrix with matrices $A_1, A_2,$ $..., A_n$ on the diagonal
- $\text{Col}(A_1, A_2, ..., A_n)$ — Column block matrix $(A_1^T, A_2^T, ..., A_n^T)^T$

List of Acronyms

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Declaration

I declare that the work contained in this thesis has only been submitted for the cotutelle dual award under the cotutelle agreement between Northumbria University and Université Polytechnique Hauts-de-France and has not been submitted for any other award and that it is all my own work. I also confirm that this work fully acknowledges opinions, ideas and contributions from the work of others. Any ethical clearance for the research presented in this thesis has been approved. Approval has been sought and granted by the University Ethics Committee on 05/11/2019.

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General Introduction

A MAS consists of multiple autonomous subsystems that interact between themselves and with their immediate environment in order achieve certain tasks in a cooperative manner. Since the last decade, MASs have become an important subject of research and their study is mostly inspired by cooperative behaviours observed in various animal species, such as bird flocking and bee swarming. As such, in robotics for instance, researchers have developed various cooperative control protocols and applied them to various areas such as the military, automation in industry, transportation of heavy loads, search and rescue missions in hazardous areas, etc. In these recent years, MASs have become more and more complex as their size, communication requirements and their need for adaptation to different situations they might encounter, increase. This translates into the necessity to develop efficient distributed algorithms in terms of robustness and safety. Indeed, the problem of safety in cooperative MASs has arisen as a consequence of of their complexity and the nature of their operations and wireless communication exchanges, which renders them vulnerable to not only physical faults, but also to cyber-attacks. The safety of MASs has thus become one of the focal points of the control scientific community for the past decade, and is the main focus of this dissertation. There are several uncontrolled external factors that could decrease safety, reliability and performance in cooperative MASs. One of these factors are faults. A fault is generally defined as an unexpected deviation of at least one system variable, property or feature from its acceptable, usual, standard and expected state.

On the other hand, communication between agents in a MAS, also called the communication topology, can be represented using a graph which describes agents as nodes and their communication links as edges. A communication topology could be either undirected, if all information exchanges in the network are bidirectional, or directed, if there are at least two given communicating agents in the network where information flow is unidirectional. Furthermore, the communication topology could either remain fixed or change with time. The first is called a fixed topology and the second, a switching topology. As such, it is evident that graph theory plays an important role in the analysis of MASs. Indeed, the Laplacian matix associated with a MAS's graph provides information about the network topology which dictates the behaviour of the MAS. Hence, an unwanted deviation from the nominal Laplacian matrix or the nominal behaviour of a node, might indicate the occurrence of an anomaly in the network. Indeed, when it comes to connected MASs, two categories of faults could be distinguished in this dissertation, depending on which element of the graph they affect

- *Node faults:* or component faults, are faults that affect a node/agent's components, i.e., its actuator(s), sensor(s) or the general behaviour of its dynamics.

- *Link faults:* or communication faults, are faults that affect the communication topology of the MAS, i.e., the information flow between two given agents. It can be an unexpected change in the topology's parameters, a broken link, an attack etc.

The main objective of this thesis is the study of distributed fault detection and isolation in connected MASs. It is observed from the literature that the existing FDI schemes in MASs suffer from various limitations with respect to different challenges cited below

- *FDI for MASs with temporal constraints:* Most FDI observer methods proposed do not take convergence time performance specifications into account and their convergence time can be negatively impacted by different factors such as the initials conditions, unknown perturbations, structure of the graph topologies, switching graph topologies, etc. Fast convergence is a critical index that is can be pursued in practice in order to have an idea on the expected convergence time and thus, allows the designer to achieve a better robustness and performance of the FDI modules. The fixed-time stability property allows to avoid the problem of transient behaviours. What is more, this property can help to significantly simplify the residual generation and fault detection process, whereby the convergence time is estimated a-priori and is independent of the structure of the communication topology and the initial conditions. This property has not been sufficiently explored in literature in its application to fault and attack detection and identification in MASs. Indeed, as pointed out in [Yang *et al.* 2019], the "fixed-time/prescribed-time fault detection for MASs has not received adequate attention, which constitutes a promising topic" and "existing finite-time techniques may provide a basis for the development of novel efficient fault detection and Fault-Tolerant Control (FTC) approaches for MASs that seek a fast convergence rate and desirable fault tolerance performance".

- *FDI for MASs with switching topologies:* Indeed, it is usually considered that the communication topology of a MAS stays fixed during the detection process. In many applications however, this can be impractical and communication topologies can be switching rather than fixed. In such situations, the predefined-time principle can be useful as observers can be designed such that faults are detected in a timely manner. As such, contrary to finite-time schemes, the estimation of the settling time does not require the knowledge of the initial conditions, thus allowing for a step by step design basis of the detection scheme. FDI design for MASs under switching topologies is a challenging issue, especially when cyber-attacks are concerned. Hence, this problem merges into the previous point, where before each switching instant, the fixed-time property can be used to achieve a fast convergence.

- *FDI for MASs with heterogeneous agents and directed topologies:* In practice, MASs can sometimes be composed of heterogeneous agents, for economical reasons or otherwise. This heterogeneity property can present some challenges in the design of a distributed fault or attack detection scheme. The same can be said for MASs with directed communication topologies. In fact, in such cases, information is even more restrained, hence further complicating the detection and isolation process. Indeed, distributed fault detection in MASs with heterogeneous agents and/or directed topologies, namely when cyber-attacks are concerned, is still an open problem.

- *FDI for MASs subject to cyber-attacks and physical faults:* In practice, MASs can be subject to either physical faults (actuator/sensor faults), cyber-attacks (attacks on communication links) or both. The proposed algorithms should thus be able to detect at least one anomaly type, and in the case where both occur, an cyber-attack identification scheme must be put in place in order to discern physical faults from cyber-attacks.

Given the objectives of this thesis, the proposed detection modules also need to satisfy the following

- *Distributed algorithm:* MASs in most practical cases are limited in the information that they are able to sense given that not all measurements are available to each agent. This along with the fact that FDI decisions can only be trusted locally, and not exchanged, namely in the presence of communication noise and possible malicious attacks, motivates the need for a distributed algorithm. Indeed, distributed schemes imply that each agent can run its fault and/or attack diagnosis module based on its own states and the information exchanged between its neighbouring agents.

- *Robustness:* MASs can be subject to all sorts of perturbations whose effects on the system are unknown and sometimes unpredictable. Consequently, the algorithms proposed in this thesis should be robust with respect to these perturbations, whether they are dynamic disturbances or uncertainties, measurement noise or communication noise etc.

Motivated by the above constraints, the following fault detection algorithms and schemes have been proposed

- First, a distributed methodology for the detection of actuator faults in a class of linear MASs with unknown disturbances is proposed in [Taoufik *et al.* 2020c], whose main features are highlighted in the following. The formulation of the distributed actuator FDI problem for a class of linear MASs with disturbances is performed through the use of a cascade of fixed-time SMOs, where each agent having access to their state, and neighbouring information exchanges, can give an exact estimate of the state of the overall MAS. An LMI-based approach is applied to design distributed global residual signals at each agent, based on mixed $\mathcal{H}\_/\mathcal{H}_\infty$ norms. The above combined approaches allow treating the actuator fault detection problem while keeping a distributed design approach, as information obtained by each interacting agent only comes from its neighbours.

- Then, the previous point is extended to nonholonomic systems, where a new distributed robust fault detection scheme for MASs composed of agents with nonlinear nonholonomic dynamics is proposed in [Taoufik *et al.* 2021b]. This considered model is relevant for many applications (for instance, ground mobile robots, surface ships, underwater vehicles etc., see [Kolmanovsky & McClamroch 1995] for an extended survey). In this proposed scheme, the use of predefined-time stability techniques to derive adequate distributed SMOs is investigated, which enable to reconstruct the global system state in a predefined-time and generate proper residual signals. The proposed scheme ensures global fault detection, where each agent is capable of detecting its own faults and those occurring elsewhere in the system using only local information (contrary to most of the existing works).

- The results obtained in the first point are then extended to the case of MASs with higher order integrator dynamics [Taoufik *et al.* 2020b], where only the first state variable is measurable. Here, a new approach to identify faults and deception attacks in a cooperating MASs with a switching topology is introduced. The proposed protocol makes an agent act as a central node monitoring the whole system activities in a distributed fashion whereby a bank of distributed predefined-time SMOs for global state estimation are designed, which are then used to generate residual signals capable of identifying cyber-attacks despite the switching topology.

- In this thesis, the problem of distributed fault detection for a team of heterogeneous MASs with linear dynamics is also investigated in [Taoufik *et al.* 2020a], where a new output observer scheme is proposed which is effective for both directed and undirected topologies. The main advantage of this approach is that the design, being dependant only on the input-output relations, renders the computational cost, information exchange and scalability very effective compared to other FDI approaches that employ the whole state estimation of the agents and their neighbours as a basis for their design.

- A more general model is studied in [Taoufik *et al.* 2021a], where actuator, sensor and communication faults/attacks are considered in the robust detection and isolation process for nonlinear heterogeneous MASs with disturbances and communication parameter uncertainties, where the topology is not required to be undirected. This is done using a distributed finite-frequency mixed $\mathcal{H}\_/\mathcal{H}_\infty$ nonlinear UIO-based approach.

The rest of this thesis is organised as follows:

- **Chapter 1:** Basic introductory concepts with regards to MASs, attack and FDI in MASs and their applications are given. Basic algebraic graph theory concepts and some useful mathematical tools are presented for describing the communication topology among agents in a MAS. Additionally, a brief literature review and state of the art of recent works on attack and FDI in the context of connected MASs is conducted.

- **Chapter 2:** The problem of distributed fault detection is investigated, for MASs with linear dynamics, MASs with nonholonomic dynamics and finally for MASs with higher order integrator nonlinear dynamics under switching topologies subject to deception attacks. This is done using the fixed-time property by designing a bank of adequate distributed observers and a residual based approach.
- **Chapter 3:** This chapter is concerned with heterogeneous MASs and is composed of two parts. In the first part, a distributed output observer approach is designed for linear MASs with actuator faults, whereby the aim is to reduce the dimensions of the FDI units. In the second part, a distributed nonlinear UIO-based approach is used for MASs with quasi nonlinear dynamics subject not only to actuator faults, but also to sensor and communication faults.
- **Chapter 4:** In this chapter, the obtained results are summarised and possible future research directions are given.

The works presented in this thesis are published in the following

Peer Reviewed Journal Articles

- Taoufik, A., Defoort, M., Djemai, M., Busawon, K. (2021). A distributed fault detection scheme in disturbed heterogeneous networked systems. *Nonlinear Dynamics*, 1-20.
- Taoufik, A., Defoort, M., Djemai, M., Busawon, K., & Diego Sánchez-Torres, J. (2021). Distributed global fault detection scheme in multi-agent systems with chained-form dynamics. *International Journal of Robust and Nonlinear Control*, 31(9), 3859-3877.
- Taoufik, A., Defoort, M., Busawon, K., Dala, L., & Djemai, M. (2020). A Distributed Observer-Based Cyber-Attack Identification Scheme in Cooperative Networked Systems under Switching Communication Topologies. *Electronics*, 9(11), 1912.

International Conferences

- Taoufik, A., Busawon, K., Defoort, M., & Djemai, M. (2020, September). An output observer approach to actuator fault detection in multi-agent systems with linear dynamics. *28th Mediterranean Conference on Control and Automation* (pp. 562-567). IEEE.
- Taoufik, A., Defoort, M., Djemai, M., Busawon, K., & Sánchez-Torres, J. D. (2020). Distributed global actuator fault-detection scheme for a class of linear multi-agent systems with disturbances. *IFAC-PapersOnLine*, 53(2), 4202–4207.
- Taoufik, A., Defoort, M., Busawon & Djemai, M. (2021). A global approach to fault detection in multi-agent systems with switching topologies subject to cyber-attacks. *5th IEEE-International Conference on Control, Automation and Diagnosis*, Grenoble, France.

CHAPTER 1

Background and State of the Art

Content

1.1 Introduction

In recent years, MASs have become more and more complex as their size, communication requirements and their need for adaptation to different situations they might encounter, increase. This translates into the necessity to develop efficient distributed algorithms in terms of robustness and safety. Indeed, the problem of safety in cooperative MASs has been one of the focal points of the control scientific community for the past decade, and is the main focus of this dissertation.

In this chapter, some concepts and mathematical tools required to introduce our results presented in Chapters 2 and 3, are defined. The Chapter starts by giving a brief literature review on MASs and defines their main concepts and characteristics. This is followed by a general overview on graph theory and some mathematical tools associated with it. A general overview on cooperative MASs is then provided. Then, a literature review on existing FDI, Fault Estimation (FE) and Attack Detection techniques and approaches in MASs are presented followed by a brief introduction to convergence rate analysis and related mathematical background. Finally, the research motivation behind this thesis is given to highlight the points that are lacking in the literature, followed by the contributions of this dissertation that tackled these issues/points.

1.2 Literature Review

1.2.1 Multi-Agent Systems (MASs)

The term "agent" as found in literature can refer to different systems with subtle differences, e.g., "intelligent agents" [Sycara *et al.* 1996, Wong & Sycara 1999], "autonomous agents" [Jennings *et al.* 1998], etc. Throughout this thesis, the adopted definition of the term agent is the one used in the control community as required by the context of our work. [Reza Davoodi *et al.* 2016] has described it as

Definition 1.1 *[Reza Davoodi et al. 2016] An agent is a dynamical system with a state vector that evolves through time, based on its past values and a control input vector.*

Hence, an agent can refer to a Wheeled Mobile Robot (WMR), a drone, an Unmanned Aerial Vehicle (UAV), an Autonomous Underwater Vehicle (AUV), etc, examples of these systems with different architectures are shown in Figure 1.1. A MAS composed of multiple mobile robot systems is called a Multi-Robot System (MRS). Throughout this thesis, the terms agent and mobile robot, for instance, could be used interchangeably. On the other hand, the notion of a MAS in the control community's point of view, can be described by the following definition

Definition 1.2 *A multi-agent system is a set of agents that exchange information and collaborate with each other based on a common control strategy to achieve an objective* *as a single entity. An objective that otherwise could not be achieved by each agent alone.*

The interaction and coordination described in Definition 1.2 with a common group objective, is in essence, widely inspired by the inherent natural behaviours observed in many biological species. Natural social behaviours are fish schoolings (Fig. 1.2), bird flockings (Fig. 1.3), bee swarming (Fig. 1.4), stampedes (Fig. 1.5), to name a few. In all of these examples, through their collective animal behaviour, the animal groups perform complex tasks by perceiving their neighbours' behaviours and acting accordingly without an external supervising entity. In a stampede situation for instance, a group of mammals suddenly start running in the same direction in fear of a predator, where each mammal perceives the position of those around him in order to define its direction and speed by trying to achieve the the same thing at the same time.

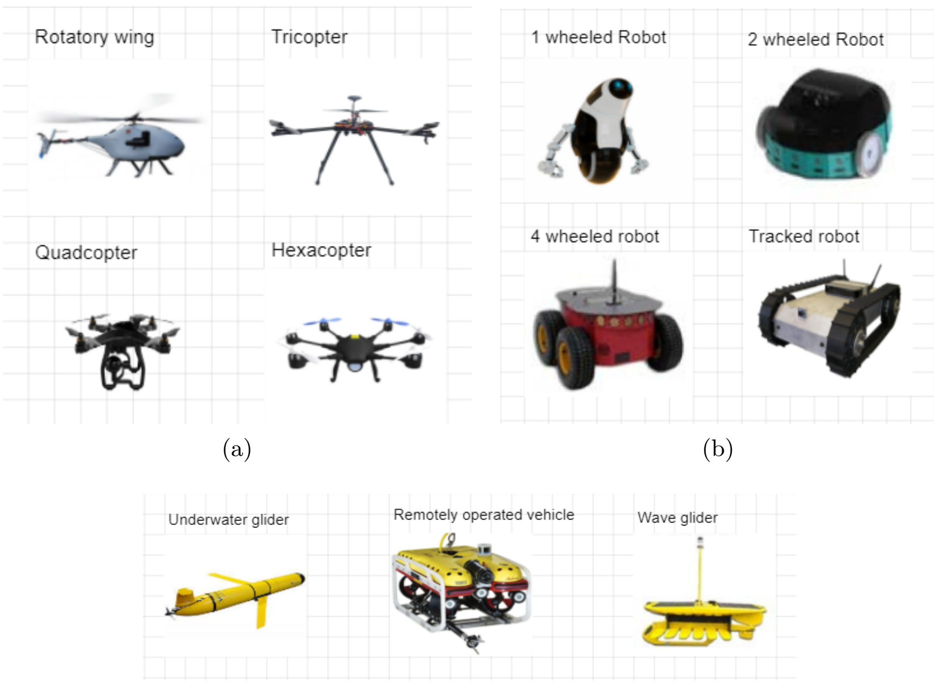


Figure 1.1: Some classical examples of agents, **(a)** UAVs, **(b)** WMRs and **(c)** AUVs.

These different types of observed natural phenomena have become a driving force for the development of algorithms for cooperative control and fault diagnosis in MASs by researchers and scientists in a multitude of fields. Indeed, when looking through literature, one could see that MASs have drawn a great amount of interest from researchers in a wide range of application settings, such as multi-robot systems [Anggraeni *et al.* 2019, Taoufik *et al.* 2021b, Arrichiello *et al.* 2015, Hernandez-Martinez *et al.* 2013], UAVs [Ghamry & Zhang 2016, Kamel *et al.* 2015], ground and underwater vehicles [Millán *et al.* 2013], transportation systems [Li *et al.* 2015], microgrids

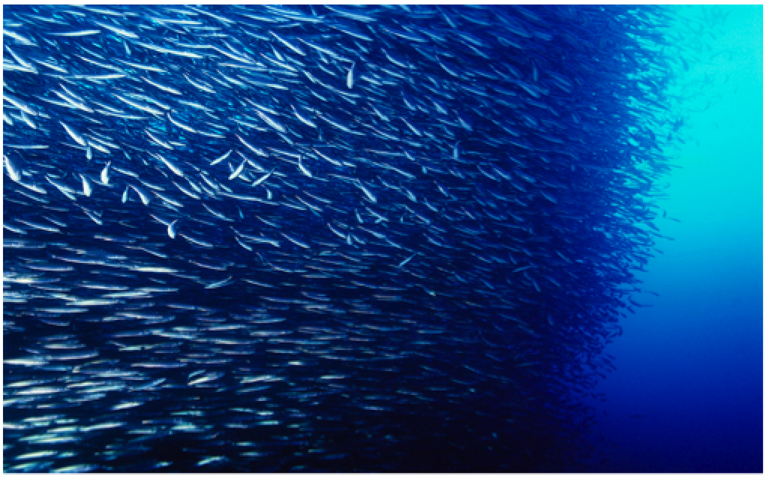


Figure 1.2: An example of fish schooling.

Figure 1.3: An example of birds flocking in a V-shape form.

[Bidram *et al.* 2013, Kantamneni *et al.* 2015], formation flying of satellites [Barua & Khorasani 2007], etc.

This is largely thanks to the sheer number of advantages they provide. Indeed, not only they achieve missions or objectives that could not be achieved by single agents, but they also allow for more safety features. For instance, when there is a faulty or malfunctioning agent in a cooperative MAS, the detection of such anomaly can lead to it being removed and its place taken by another agent, hence allowing for the mission to carry on. Even though each one of the aforementioned applications in the last paragraph has its own complexity, one of the underlying challenges is this safety feature, namely the detection of such anomalies.

Figure 1.4: An example of a swarm of bees.

Figure 1.5: An example of a stampede situation of a herd of mammals.

Some of the main features of MASs are given and detailed below.

Agent dynamics

Each agent in the MAS is characterized by a dynamical equation that could be either linear or nonlinear. These models have a degree and could thus be of first-order, second-order or high-order dynamics.

MAS's homogeneity

A MAS can either be composed of agents having different dynamics or similar dynamics. This is referred to as the heterogeneity/homogeneity of a MAS. In this thesis, the following definition is adopted when referring to a *homogeneous* or a *heterogeneous* MAS.

Definition 1.3 *A MAS is said to be a homogeneous one if both the dynamics and the exchanged information of all the agents are the same, otherwise, it is called a heterogeneous or a non-homogeneous MAS.*

MAS's communication topology

Information exchange amongst agents is a key characteristic in MASs. Indeed, each agent is required to have the capability to communicate with other agents in its vicinity, which are called its *neighbours*. The agents transmit, receive and perceive information. This can either be through a wired network (different types of cables, electric...), a wireless network (Wi-Fi modules, radio...) or through a sensor-based network, where agents are equipped with on-board relative information sensors, in the latter case, information is not really transmitted, but rather measured.

A *topology* or *communication topology* refers to the structure of these interactions, and describes the direction of information flow between all of the agents comprising the MAS. It can be divided into two types based on whether or not the network remains the same or changes with time. These types are given below

- *Fixed topology:* A MAS topology is said to be fixed when the communication network among the agents stays fixed and does not change with time.
- *Switching topology:* A MAS topology is said to be switching when the interaction between agents changes with time, and so does the communication network. Indeed, in some practical situations, it could be impossible for the agents to keep a fixed network either willingly or unwillingly.

A topology can also be classified into two categories depending on the direction of information flow between the agents, these categories are defined below

- *Undirected topology:* A communication topology is undirected if all the agents in the MAS are able to both, send information to their neighbours and receive information from them.
- *Directed topology:* A communication topology is called directed if for two given agents in the MAS, one is able to send its information to the other but is unable to receive the other's information.

1.2.2 Algebraic Graph Theory

The communication topology described in subsection 1.2.1 is modelled using algebraic graph theory, making graph theory one of the fundamental tools in the study of MASs. Indeed, a MAS relies particularly on the communication network that connects the agents and algebraic graphs can be used as a representation of a network of agents. This characterization of the MAS topology using graph theory can be exploited in order to analyse problems in MASs such as controllability and control, observability and observer design, stability, etc.

In rest of this subsection, some important results, definitions and properties of algebraic graph theory which will be needed in the next chapter are given. The reader is referred to the books [Bondy *et al.* 1976, Biggs *et al.* 1993, Mesbahi & Egerstedt 2010] for a more comprehensive treatment on algebraic graph theory.

1.2.2.1 Graph Theory Basics

Suppose that there are N agents in a network of a MAS which interact between themselves through a communication or sensor based network as described in the previous subsection. They could also be considered to communicate amongst themselves through a network that is both communication based and sensor based. The interaction pattern between agents can be modelled by describing the communication topology as detailed above in the form of a graph by using graph theory and linear algebra.

A graph, denoted $\mathcal{Q}$ is naturally composed of a finite set, such that each set has a finite number of elements. Each element is called a *vertex* or a *node* and the set of nodes is denoted in this dissertation by $\mathcal{N}$ where each node represents an agent in the MAS, hence one could define $\mathcal{N} \triangleq \{1, ..., N\}$. On the other hand, the interaction between two nodes in the graph is called an *edge* or *link* and represents the information flow between two given agents. The set of edges in the graph $\mathcal{Q}$ is denoted $\mathcal{F} \subseteq \mathcal{N} \times \mathcal{N}$. Accordingly, the corresponding communication graph can be defined as $\mathcal{Q} \triangleq (\mathcal{N}, \mathcal{F})$, where the cardinality $|\mathcal{N}|$ represents the order of the graph $\mathcal{Q}$ and the cardinality $|\mathcal{F}|$ represents its size. Consider two nodes i and j in the graph $\mathcal{Q}$, an edge (i, j) contained in set $\mathcal{F}$ denotes that agent j can obtain information from agent i, but the opposite is not necessarily true. Indeed, if agent i receives information from agent j, then the edge (j, i) is also contained in $\mathcal{F}$. It is considered that "self-edges", i.e., (i, i) are not allowed, except if otherwise specified.

Undirected and Directed graphs

An *undirected* graph is composed of a set of nodes $\mathcal{N}$ and a set of edges $\mathcal{F}$ such that, for each pair of nodes (i, j), $i \neq j \in \mathcal{N}$, if there exists an edge (i, j) such that $(i, j) \in \mathcal{F}$, then $(j, i) \in \mathcal{F}$. Otherwise, the graph is called a *digraph* or a *directed graph*. An undirected graph can be viewed as a special case of a digraph. An undirected or a directed graph is said to be a weighted graph, if each edge (i, j), $i \neq j \in \mathcal{N}$ in the graph is associated with a weighing coefficient denoted a_{ij}. In the case of an undirected weighted graph, one has $a_{ij} = a_{ji}$.

For visualization and illustrative purposes, nodes are represented as circles containing the number/label of the agent (see Figure 1.6), while if the edge (i, j), $i \neq j$ linking j to i exists, it is represented by an arrow directed from the circle labelled j to the one labelled i, an example of a directed graph is shown in Figure 1.7a, which is a graph representation of a MAS composed of $N = 4$ agents. In the case of undirected graphs, only one headless arrow is represented for simplicity, see Figure 1.7b that shows an example of a graph representation of a MAS composed of $N = 5$ agents.

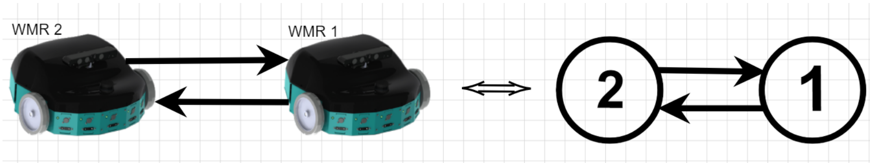
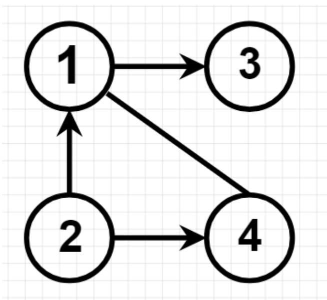
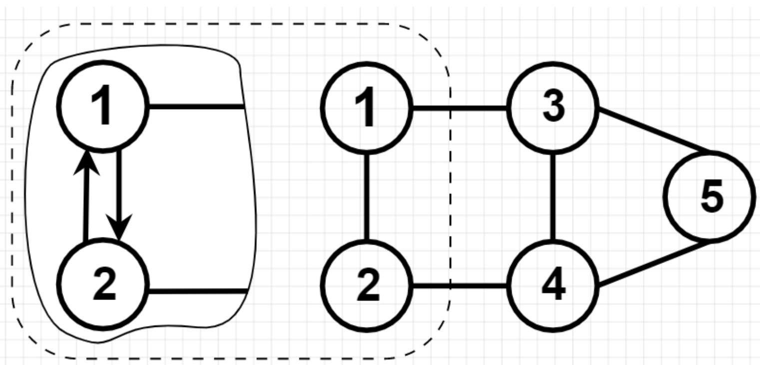


Figure 1.6: A visual illustration of interaction modelling of two WMRs using graph theory.



(a)



(b)

Figure 1.7: Examples of graph representations of **(a)** a directed MAS topology, **(b)** an undirected MAS topology.

Agent's neighbours

If an edge $(i, j) \in \mathcal{F}$, then node i is said to be a *neighbour* of node j. The set of neighbours of the node i is defined as $\mathcal{N}_i \triangleq \{i \neq j \;: (i, j) \in \mathcal{F}\}$ and the cardinality $|\mathcal{N}_i|$ of $\mathcal{N}_i$ denotes the number of agent i's neighbours. For the edge (i, j), node i is called the parent node and j the child node.

Example 1.1 *Consider the graph represented in Figure 1.7a composed of $N = 4$ agents. In this example, the set of nodes corresponding to the agents is $\mathcal{N} = \{1, 2, 3, 4\}$ and the set of edges is given as $\mathcal{F} = \{(1, 2), (1, 4), (3, 1), (4, 1), (4, 2)\}$. The sets of the agents' neighbours are given as $\mathcal{N}_1 = \{2, 4\}$, $\mathcal{N}_2 = \emptyset$, $\mathcal{N}_3 = \{1\}$, and $\mathcal{N}_4 = \{1, 2\}$.*

Connected graphs

Definition 1.4 *A directed path going from a node j to a node i in the graph is a set composed of a sequence of edges, starting from node j and ending at node i connecting distinct and consecutively adjacent nodes in-between j and i.*

Definition 1.5 *A directed graph is said to be strongly connected if for every two distinct nodes in the graph there exists a directed path from one to the other. An undirected graph is fully connected if there is an undirected path between every pair of distinct nodes.*

This means that for each node in the graph, there exists at least an edge connecting it to another node, in other words, each node always has at least one neighbour in a connected graph.

Directed trees and spanning trees

Definition 1.6 *A directed tree is a directed graph in which all nodes in the graph have exactly one parent node except for one node. The node with no parent nodes is called the root of the graph.*

A subgraph $\mathcal{Q}^1 = (\mathcal{N}^1, \mathcal{F}^1)$ of $\mathcal{Q} = (\mathcal{N}, \mathcal{F})$ is a graph such that the subset of nodes $\mathcal{N}^1 \subseteq \mathcal{N}$ and the subset of edges $\mathcal{F}^1 \subseteq \mathcal{F} \cap (\mathcal{N}^1 \times \mathcal{N}^1)$.

Definition 1.7 *A directed spanning tree is a subgraph $\mathcal{Q}^1 = (\mathcal{N}^1, \mathcal{F}^1)$ of the directed graph $\mathcal{Q} = (\mathcal{N}, \mathcal{F})$ such that the subgraph $\mathcal{Q}^1$ is a directed tree and $\mathcal{N}^1 = \mathcal{N}$.*

The directed graph $\mathcal{Q}$ is said to contain a directed spanning tree if a directed spanning tree is a subgraph of $\mathcal{Q}$. In undirected graphs, if there is an undirected spanning tree, it is equivalent to the graph being connected. Conversely, in directed graphs, the existence of a directed spanning tree is a weaker condition than the graph being strongly connected.

1.2.2.2 Adjacency and Degree Matrices

The structure of a given graph can be represented by a matrix called the *connectivity* or *adjacency matrix*, which can be defined as follows

Definition 1.8 *The adjacency matrix of the graph $\mathcal{Q}$ composed of N agents/nodes, is the $N \times N$ matrix defined as $\mathcal{A}(\mathcal{Q}) = [a_{ij}] \in \mathbb{R}^{N \times N}$, $\forall i \neq j \in \mathcal{N}$ such that*

$$a_{ij} = \begin{cases} 0, & \text{if } (i,j) \notin \mathcal{F} \\ 1 & \text{if } (i,j) \in \mathcal{F} \end{cases} \tag{1.1}$$

It is to be noted that the above definition of the adjacency matrix is used for the case of an unweighed graph where a_{ij} can either be 1 or 0, with 1 meaning that there is an edge between i and j and 0 signifying the absence of such an edge. In the case of weighed graph however, elements of the adjacency matrix are rather chosen as $a_{ij} > 0$ instead of 1 when $(i,j) \in \mathcal{F}$ and 0 otherwise.

For a directed graph, the in-degree of a node i is defined as $d_{ij} = \sum_{j=1}^{N} a_{ij}$ and its out-degree as $d_{ji} = \sum_{j=1}^{N} a_{ji}$. For an undirected graph, the in-degree of node i is the same as its out-degree, and is simply called the degree of node i.

Definition 1.9 *A graph i is said to be balanced if $\forall i$, $d_{ij} = d_{ji}$.*

For an undirected graph, $\mathcal{A}(\mathcal{Q})$ is symmetric and thus all directed graphs are balanced.

Definition 1.10 *The degree matrix of the graph $\mathcal{Q}$, denoted $\mathcal{D}(\mathcal{Q})$, is the diagonal $N \times N$ matrix defined as*

$$\mathcal{D}(\mathcal{Q}) = diag(d_1, ..., d_N) \tag{1.2}$$

where $\forall i \in \mathcal{N}, i \neq j,\ d_i = \sum_{j=1}^{N} a_{ij}$.

Hence, in the case of a directed graph, it is the in-degrees that are used as elements of the degree matrix.

1.2.2.3 Laplacian Matrix

Another key matrix that describes a graph and is crucial in the study of MAS, which includes information on both the adjacency and degree matrices, is called the Laplacian matrix and can be defined as follows

Definition 1.11 *The Laplacian matrix of the graph $\mathcal{Q}$, denoted $\mathcal{L}(\mathcal{Q})$, is the $N \times N$ matrix defined as $\mathcal{L}(\mathcal{Q}) \triangleq [l_{ij}]$, such that*

$$l_{ij} = \begin{cases} d_i, & \text{if } i = j \\ -a_{ij} & \text{if } i \neq j,\ j \in \mathcal{N}_i \\ 0 & \text{otherwise} \end{cases} \tag{1.3}$$

Alternatively, one has $\mathcal{L}(\mathcal{Q}) = \mathcal{D}(\mathcal{Q}) - \mathcal{A}(\mathcal{Q}) \in \mathbb{R}^{N\times N}$. It is clear that for the case of an undirected graph, $\mathcal{L}(\mathcal{Q})$ is always symmetric. On the other hand, when a directed graph is concerned, $\mathcal{L}(\mathcal{Q})$ is not symmetric and can be referred to in the literature as the nonsymmetric Laplacian matrix or the directed Laplacian matrix [Lafferriere *et al.* 2005]. For an undirected graph, $\mathcal{L}(\mathcal{Q})$ is positive semidefinite and all its nonzero eigenvalues are positive. For a directed graph, all of the nonzero eigenvalues of $\mathcal{L}(\mathcal{Q})$ have positive real parts, in other words, all of the nonzero eigenvalues of $-\mathcal{L}(\mathcal{Q})$ have negative real parts. For an undirected graph, 0 is a simple eigenvalue of $\mathcal{L}(\mathcal{Q})$ if and only if the undirected graph is connected. For a directed graph, 0 is a simple eigenvalue of $\mathcal{L}(\mathcal{Q})$ if the directed graph is strongly connected, this is to say that if the directed graph has at least a spanning tree, then 0 is a simple eigenvalue of $\mathcal{L}(\mathcal{Q})$. When undirected graphs are concerned, denote by $\lambda_i(\mathcal{L}(\mathcal{Q}))$ the *ith* smallest eigenvalue of $\mathcal{L}(\mathcal{Q})$, then

$$\lambda_2(\mathcal{L}(\mathcal{Q})) = \min_{x\perp\mathbf{1}_N, x\neq 0} \frac{x^T\mathcal{L}x}{x^Tx}$$

where $\mathbf{1}_N$ is the $N\times 1$ column vector of all entries equal to 1, referred to as the the *algebraic connectivity* of $\mathcal{Q}$ which is positive if and only if $\mathcal{Q}$ is connected [Wilson 2015]
.

The following is satisfied for $\mathcal{L}(\mathcal{Q})$ in both the undirected and directed case

- $l_{ij} < 0, \quad \forall i,j \in \{1,2,...,N\}, i\neq j$
- $\sum_{j=1}^{N} l_{ij} = 0, \quad \forall i \in \{1,2,...,N\}$
- $\mathcal{L}(\mathcal{Q})$ always has zero row sums and 0 is an eigenvalue of $\mathcal{L}(\mathcal{Q})$ with the associated eigenvector $\mathbf{1}_N$.

The argument $(\mathcal{Q})$ can be dropped for $\mathcal{A}(\mathcal{Q})$, $\mathcal{D}(\mathcal{Q})$ and $\mathcal{L}(\mathcal{Q})$ for the sake of simplicity. In this dissertation, $\mathcal{L}$ is going to be referred to as the Laplacian matrix without any prefix.

Example 1.2 *Consider the unweighed directed graph illustrated in Figure 1.7a, the adjacency, degree and Laplacian matrices are given as*

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \quad \mathcal{D} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix},$$

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 2 \end{bmatrix}$$

For the unweighed undirected graph illustrated in Figure 1.7b, the adjacency, degree *and Laplacian matrices are*

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}, \quad \mathcal{D} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix},$$
$$\mathcal{L} = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 3 & -1 & -1 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

1.2.3 Cooperative Control in MASs

Cooperative control in MASs studies the problem of controlling a MAS in the aim of fulfilling a common objective using local or global information. Indeed, the fundamental role in a MAS cooperative control is information sharing amongst agents, where each agent requires information sent to it through a communication network or sensed by it through relative information sensing devices. Based on this knowledge, agents make their own decisions in such a way that a specified behaviour is achieved and eventually maintained. One fundamental behaviour in MAS cooperative control is *consensus seeking*, which requires every agent to agree on certain common values. These common values of interest depend on the type and architecture of the agent as well as the application at hand. For example, common values for a team of WMRs could be either the positions, velocities or both. In the problem of consensus seeking in MAS, two main types could be distinguished, the *leaderless consensus* problem and *leader-follower consensus* problem. Both problems have attracted considerable research interest in the past couple of decades and early milestone results can be mentioned, [Olfati-Saber & Murray 2004, Ren & Beard 2005, Fax & Murray 2004, Olfati-Saber *et al.* 2007, Ren & Cao 2011]. The term *"leader"* appears in both problems and can be defined as follows

Definition 1.12 *A leader in the leader-follower consensus in a MAS refers to an agent which produces a reference state trajectory that constitutes a common value of interest for the MAS and thus its control objective. The leader could either be an actual physical agent or simply a virtual agent.*

1.2.3.1 Leaderless Consensus

The leaderless consensus problem, also referred to as the *consensus producing* or the *averaging consensus*, is the consensus problem where agents are not required to track any reference trajectory. In this case, the consensus state depends on the initial conditions of all agents in the MAS as well as the structure of the communication topology.

Consider a MAS composed of N agents, where each agent's dynamics are modelled by the following differential equation

$$\dot{x}_i(t) \quad = F_i(x_i(t), u_i(t)), \quad \forall i \in \{1, 2, ..., N\} \tag{1.4}$$

where $x_i(t) \in \mathbb{R}^{n_x}$, $u_i(t) \in \mathbb{R}^{n_u}$ are the state vector and the control input respectively of agent i and $F_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$. Then the leaderless consensus control problem then can be formulated as

Definition 1.13 *For a MAS, leaderless consensus is said to be achieved, if regardless of the initial conditions $x_i(0)$, $\forall i, j \in \{1, 2, ..., N\}$: $||x_i(t) - x_j(t)|| \to 0$ as $t \to \infty$.*

Hence, the objective is to design a distributed control algorithm aimed at driving the integrity or a handful of states of the agents to some common final value. This final value that is reached as $t \to \infty$ is referred to as the consensus value. Now consider a MAS where all agents have single-integrator dynamics

$$\dot{x}_i(t) \quad = u_i(t), \quad \forall i \in \{1, 2, ..., N\} \tag{1.5}$$

where $x_i(t) \in \mathbb{R}$ is the state variable of agent i and $u_i(t) \in \mathbb{R}$ its control input. A solution to the consensus problem as described in definition 1.13, can be given by the following control protocol [Jadbabaie *et al.* 2003, Olfati-Saber & Murray 2004, Lin *et al.* 2004]

$$u_i(t) \quad = -L \textstyle\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)), \quad \forall i \in \{1, 2, ..., N\} \tag{1.6}$$

where L is the controller gain. The coefficients a_{ij} are the adjacency matrix entries defined in the previous subsection.

Many results can be found with regards to the problem of leaderless consensus for single integrator MAS. In [Ren & Beard 2005], the problem has been studied for directed fixed topologies, where the MAS achieves consensus if the communication graph has a directed spanning tree. [Olfati-Saber & Murray 2004] studied the averaging consensus control problem in single integrator MAS with both fixed and dynamically changing topologies.

Example 1.3 *Consider a team of 5 mobile robots interacting according to the topology shown in Figure 1.8, with the single integrator dynamics (1.5) where $x_i(t)$ represents agent i's position. Figure 1.9a shows the leaderless consensus where the initial conditions of the agents are $x_1(0) = 0.5m$, $x_2(0) = 2.5m$, $x_3(0) = 2m$, $x_4(0) = 1.5m$ and $x_5(0) = 3m$. Figure 1.9b shows the leaderless consensus for the case where the initial conditions are $x_1(0) = 6m$, $x_2(0) = 4m$, $x_3(0) = 3m$, $x_4(0) = 1m$ and $x_5(0) = 0.5m$. One can clearly see that the final agreement value is dependent on the initial conditions and that this value is constant as $t \to \infty$.*

On another hand, when it comes to MAS, a broad class of agents can be modelled using double integrator dynamics [Shames *et al.* 2011, Mei *et al.* 2014, Guo *et al.* 2018].

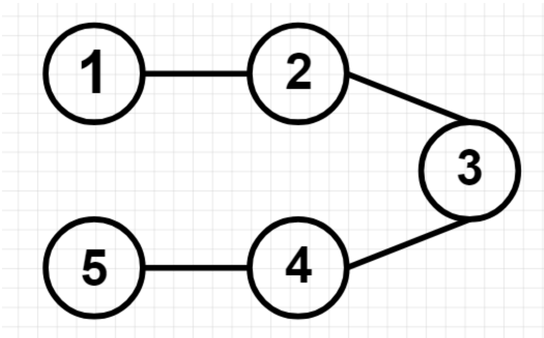
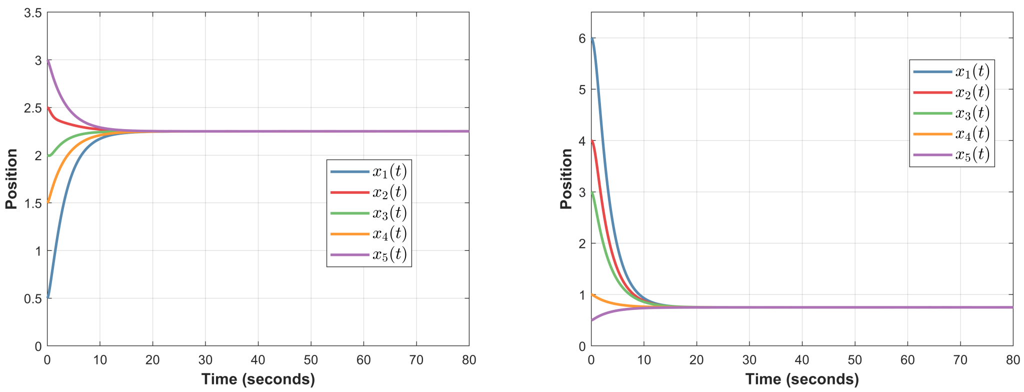


Figure 1.8: Example of a communication topology in the leaderless case.



(a) (b)

Figure 1.9: Leaderless consensus for different initial conditions.

Robotic vehicles for instance can be linearised and represented as double integrator dynamical systems such that the positions and velocities are the state information [Jadbabaie *et al.* 2003, Olfati-Saber *et al.* 2007]. In this case, the model of the *ith* agent $\forall i \in \{1, 2, ..., N\}$ is given as

$$\begin{cases} \dot{x}_{1i}(t) & = x_{2i}(t) \\ \dot{x}_{2i}(t) & = u_i(t) \end{cases} \tag{1.7}$$

where $x_{1i}(t) \in \mathbb{R}$ and $x_{2i}(t) \in \mathbb{R}$ are agent i's state variables and $u_i(t) \in \mathbb{R}$ is its control input. The variables $x_{1i}(t)$ and $x_{2i}(t)$ can be interpreted as agent i's position and velocity respectively, in the context of a mechanical mobile system such as a mobile robot, or as the phase and frequency respectively, when a power network is concerned.

As opposed to leaderless consensus case for agents with single integrator dynamics, those with double integrator dynamics can have a dynamic consensus value. A simple solution to the consensus problem, for agents with double integrator dynamics, as described in definition 1.13 where $x_i(t) = [x_{1i}(t), x_{2i}(t)]^T$, can be given by the following control protocol [Ren & Atkins 2007, Ren & Beard 2008]

$$u_i(t) \quad = -\sum_{j=1}^{N} a_{ij}\Big[L_1(x_{1i}(t) - x_{1j}(t)) + L_2(x_{2i}(t) - x_{2j}(t))\Big], \quad \forall i \in \{1, 2, ..., N\} \tag{1.8}$$

where L_1 and L_2 are the consensus gains. Thanks to the simplicity and broad range of applications of double integrator MASs, they have been widely studied in the literature under various constraints. For example, [Xie & Wang 2007] solved the the consensus problem for double integrator MASs with undirected communication topologies. In [Zhu *et al.* 2009, Yu *et al.* 2010], necessary and sufficient conditions are given with respect to the topology and Laplacian matrix for a general consensus problem in MASs with double integrator dynamics, where the importance of the eigenvalues of the Laplacian matrix in achieving consensus is studied. The necessity of the existence of a spanning tree for the case of the consensus problem in double integrator MASs is shown in [Ren & Atkins 2007]. In [Guo *et al.* 2018], a distributed consensus protocol for double integrator MASs is proposed where relative positions and absolute velocities of agents are used. Leaderless consensus for the case of switching or dynamically changing topologies was investigated in [Ren & Beard 2005].

[Ren *et al.* 2007] generalised the existing first-order and second-order consensus protocols in literature, to the higher-order integrator case. In [Davoodi *et al.* 2016] for instance, the consensus problem was solved for MASs with general higher order linear dynamics. Nonlinear Sliding Mode Control (SMC) based consensus protocols in second-order and higher-order MAS can be found in [Yu *et al.* 2016, Zuo *et al.* 2017, Su & Lin 2015].

1.2.3.2 Leader-Follower Consensus

In contrast to the leaderless consensus control protocols, where the agents' states, or common values of interest, converge towards a final value which is dependent on the initial conditions and the communication topology, in a multitude of applications, the agents are rather needed to follow a reference trajectory that could be either time-varying or constant. This situation is referred to in literature as the *leader-following consensus*, *leader-follower consensus*, *consensus tracking* or *model reference consensus* [Ren *et al.* 2007, Gallehdari *et al.* 2017]. This trajectory in the leader-follower consensus control is produced by the leader as described in definition 1.12, and the rest of the agents are referred to as the *followers*. This leader is generally labelled by 0 or r. Let us consider the leader's dynamics as follows

$$\dot{x}_r(t) \quad = F_r(x_r(t), u_r(t)) \tag{1.9}$$

where $x_r(t) \in \mathbb{R}^{n_x}$, $u_r(t) \in \mathbb{R}^{n_u}$ are the state vector and the control input respectively of the leader and $F_r : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_x}$. Then $u_r(t)$ can be designed in order to drive the rest of the agents' (the followers) common values of interest to any desired reference trajectory. Hence, in this case, the objective is to both, for all agents of the MAS to reach consensus with respect to a common state, and for this common state to converge towards the reference trajectory produced by the leader. Therefore, the leader-follower consensus objective can be defined as follows

Definition 1.14 *For a MAS composed of a leader and followers, the leader-follower consensus is said to be achieved regardless of the initial conditions if* $\forall i \in \{1, 2, ..., N\}$, $||x_i(t) - x_r(t)|| \to 0$ *as* $t \to \infty$.

In most applications, namely those requiring distributed leader-follower consensus protocols, the leader's information is only available to a selected number of followers. In [Ren & Atkins 2007] it is shown that for the leader-follower consensus to be achieved, the graph topology is required to have a directed spanning tree with the leader as its root. A simple distributed leader-follower control protocol design for $u_i(t)$ in the single-integrator MAS case (1.5), with a leader whose dynamics are given as $\dot{x}_r(t) = u_r(t)$, can be expressed as follows

$$u_i(t) \quad = -\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) - b_i(x_i(t) - x_r(t)), \quad \forall i \in \{1, 2, ..., N\} \tag{1.10}$$

where $b_i = 0$ when the ith follower does not have access to the leader's information and $b_i > 0$ otherwise, in the case of an unweighed graph. $b_i = 1$ when agent i can receive information from the leader.

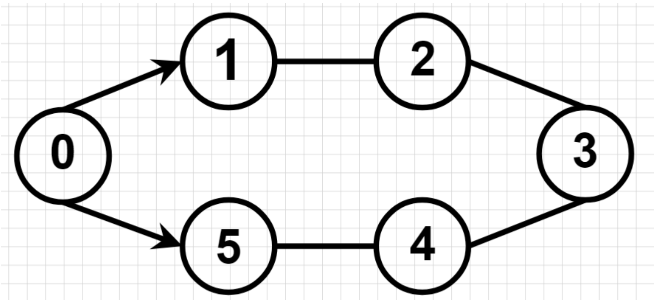


Figure 1.10: Example of a communication topology in the leader-follower case.

Example 1.4 *Consider a team of 5 mobile robots interacting according to the topology shown in Figure 1.10, with the single integrator dynamics* (1.5) *where* $x_i(t)$ *represents agent i's position and consider a virtual leader labelled 0 and whose state is* $x_r(t)$ *and is only connected to the 1st and 5th agents. The graph clearly has at least one spanning tree with 0 as the root. Figure 1.11a shows the leader-follower consensus where the initial conditions of the agents are* $x_1(0) = 0.5m$, $x_2(0) = 2.5m$, $x_3(0) = 2m$, $x_4(0) = 1.5m$ *and* $x_5(0) = 3m$. *Figure 1.11b shows the leader-follower consensus for the case where the initial conditions are* $x_1(0) = 6m$, $x_2(0) = 4m$, $x_3(0) = 3m$, $x_4(0) = 1m$ *and* $x_5(0) = 0.5m$. *One can clearly see that regardless of the initial conditions the final agreement value as* $t \to \infty$ *is that of the leader.*

The problem of leader-follower consensus control design has also been extensively investigated in the past decade for first-order, second-order, and higher-order MASs. In [Ren 2007], a consensus control protocol in MASs with single integrator dynamics

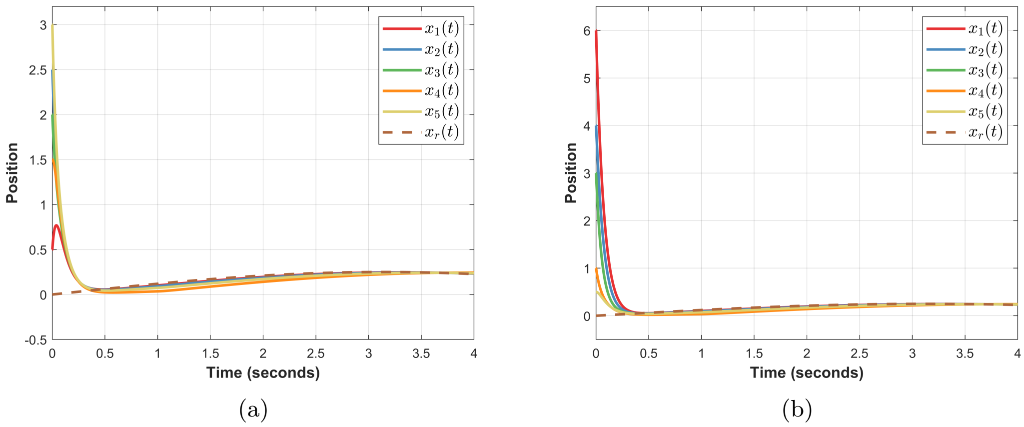


Figure 1.11: Leader-Follower consensus for different initial conditions.

was designed to track a time-varying trajectory produced by a leader which was extended to MASs with double integrator dynamics in [Hong *et al.* 2008]. In [Zhu & Cheng 2010], the problem of leader-follower consensus control for second-order MASs with switching topology and time-varying delays was investigated. The leader-follower consensus problem was solved for both switching and fixed communication topologies in MASs with higher-order dynamics, in [Ni & Cheng 2010]. Researchers have also proposed leader-follower consensus control algorithms for MRSs with nonholonomic constraints [Khoo *et al.* 2010, Defoort *et al.* 2016], general linear MASs under fixed and switching topologies [Davoodi *et al.* 2016, Gallehdari *et al.* 2017], etc.

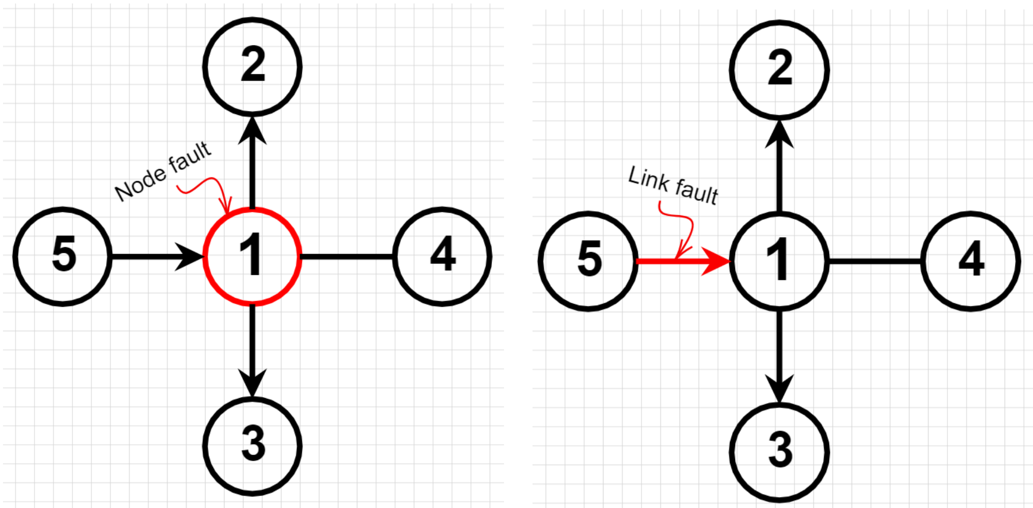
Formation Control

Another control problem in MASs worth mentioning, is *formation control*, in which *formation producing* and *formation tracking* could be distinguished, depending on the existence or absence of a reference trajectory [Ren & Cao 2011]. This type of control in MASs consists of producing a given geometric shape by controlling positions, velocities and depending on the application, orientations of the agents.

1.2.4 Fault Detection and Isolation (FDI) in MASs

There are several uncontrolled external factors that could decrease safety, reliability and performance in MASs. One of these factors are *faults*. A fault is generally defined as an unexpected deviation of at least one system variable, property or feature from its acceptable, usual, standard and expected state [van Schrick 1997]. FDI in MASs can be defined as the act of *detecting* these anomalies/faults in an agent, by setting up an alarm that determines whether or not the fault has occurred and the time at which it occurred, then *isolating* the faulty agent in the MAS, i.e., locating the source of the fault. *FE* can also be mentioned and refers to the act of reconstructing the shape and magnitude of the fault. Sometimes, a third action besides detection and isolation called *Fault Identification* [Ding 2008] can be beneficial as well. This action consists of, after detecting a fault, identifying its type and nature. Indeed, when it comes to MASs, two categories of faults could be distinguished in this dissertation, depending on which element of the graph they affect [Qin *et al.* 2014] (See Figure 1.12a-1.12b for a visual representation)

- *Node faults:* or component faults, are faults that affect a node/agent's components, i.e., its actuator(s), sensor(s) or the general behaviour of its dynamics.
- *Link faults:* or communication faults, are faults that affect the communication topology of the MAS, i.e., the information flow between two given agents. It can be an unexpected change in the topology's parameters, a broken link, etc.



(a) (b)

Figure 1.12: Example illustrating both fault categories in MASs: **(a)** Node faults, **(b)** Link faults.

Both fault types are called *physical faults*, as opposed to *cyber faults* which will be discussed later on in this subsection (see 1.2.4.2). Node faults in literature can be referred to as *actuator* faults, *sensor* faults, *process* faults, or any combination of these three terms (See Figure 1.13 for a visual representation)

- *Actuator faults*: are faults that cause a difference between the supposed control input of an actuator and its actual command.
- *Sensor faults*: are faults that cause deviations between the measured variables of the system with respect to their actual real values.

- *Process faults*: are faults that affect the general dynamics of the system, but do not exclusively appear in the actuator and/or sensor channels. They cause undesirable changes in the behaviour of the system even if the sensors and actuators are safe.

Given that a typical characteristic of MASs as opposed to single systems, is information sharing, it is more challenging to detect such faults in a distributed manner. Indeed, FDI in single systems can be analogous to centralised FDI schemes in MASs which will be discussed later on in this subsection. In MASs, because of their communicative and cooperative nature, if and when a fault occurs, its effects can propagate and spreads throughout the whole set of agents which is then bound to halt the MAS's mission and potentially lead to disastrous effects. One way to increase resilience of MASs with respect to these anomalies is to design a robust cooperative control algorithm that is resilient to the effects of certain faults. Another way, as pointed out in [Teixeira *et al.* 2014], is to develop monitoring schemes to detect failures in the MAS caused by attacks and faults. On the other hand, [Ding 2008] has provided an exhaustive literature review on model-based techniques in fault detection and isolation where observer-based techniques have been proven to be powerful software-based tools in *fault diagnosis* due to their efficiency and on-line implementation capabilities, these techniques are the ones concerned by this dissertation.

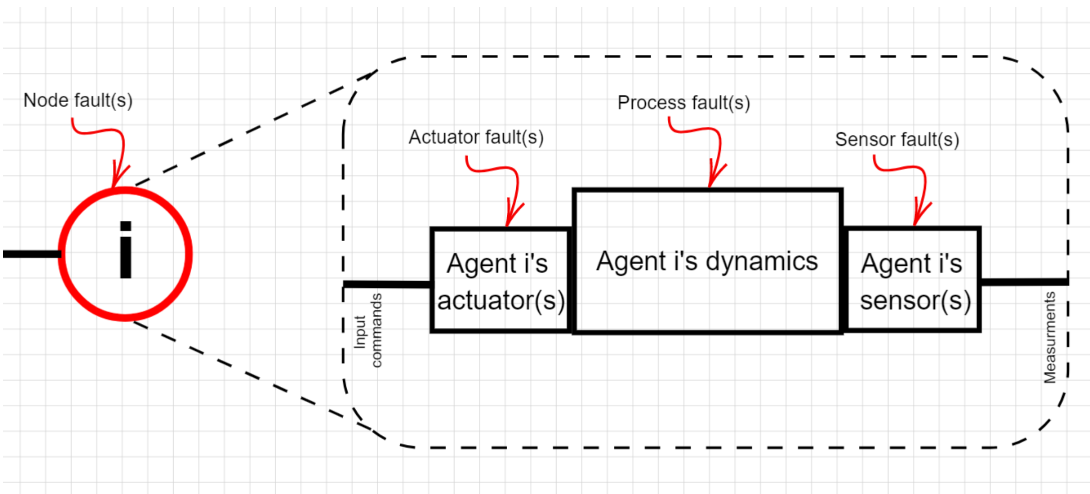


Figure 1.13: Node fault types.

Many criteria have to be taken into account when designing a FDI algorithm for MASs, depending on the application at hand. A very important criterion that should be considered is the architecture of the algorithm. This is defined in the following.

1.2.4.1 FDI Architectures in MASs

Many FDI solutions have been proposed in the literature, their architectures can be classified into three distinct categories, the *centralised architecture*, *decentralised architecture* and *distributed architecture*. These terms, in the context of FDI, are defined and described as follows

- *Centralised Architecture:*

  In centralised FDI architectures, the FDI algorithm is installed in only one agent or unit called the central agent/unit. This agent might be part of the system or an outside monitoring unit. The central unit receives information from other agents in the MAS and thus has global knowledge of the system's state, which is then used to run the FDI protocol. Therefore, other agents are not able to perform their own fault diagnosis.

  Centralised schemes require that each agent has access to the global measurements. Such architecture may lead to communicational, computational and especially, in regards to FDI, safety problems. They usually can only be used for very small sized MASs and if the central unit is compromised, so is the whole FDI protocol along with it. See Figure 1.14 for an illustration of a centralised FDI architecture for an example of a MAS composed of 3 agents.

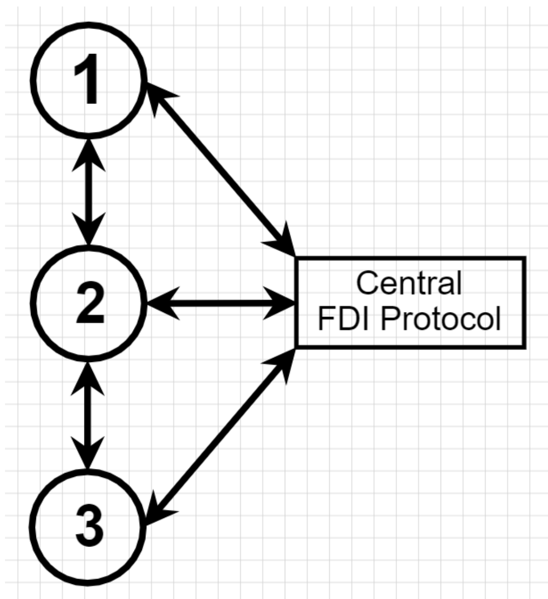


Figure 1.14: An example of the structure of centralised architectures.

- *Decentralised Architecture:*

  Decentralised FDI architectures are introduced to remedy some of the disadvantages presented by centralised ones, namely by enhancing scalability and reliability of the protocols. In these architectures, only local information is considered in the FDI modules and do not rely on neighbouring information, thus leading to simpler and less communicationally demanding fault detection schemes. This implies that each agent can locally detect or estimate its own faults based on its information. Once, the agent detects its own fault, it can share its fault information with other agents. Nevertheless, this structure is less adequate than the distributed architectures when measurement noise in the sensors, and perturbations in the communication links (uncertainties, noise, etc.) are involved. See Figure 1.15 for an illustration of a decentralised FDI architecture for an example of a MAS composed of 3 agents.

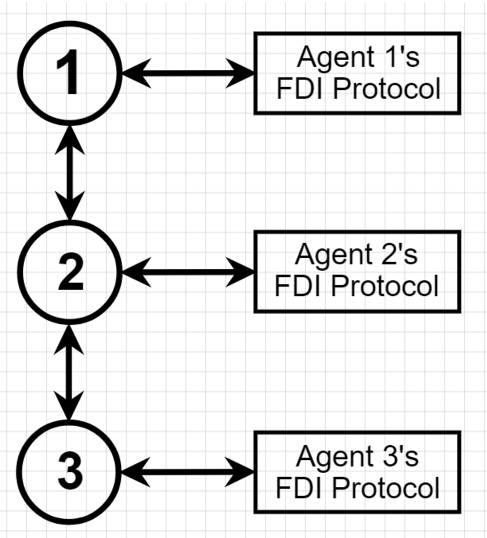


Figure 1.15: An example of the structure of decentralised architectures.

- *Distributed Architecture:*

  In distributed architectures, the FDI algorithm is installed in each agent in the MAS where the modules use local information as well as the neighbours' information. It is introduced in order to improve the reliability and scalability of centralized and decentralized architectures and is the most suitable for MASs especially in terms of *reliability*. In these schemes, each agent can detect or estimate faults based on its own information along with the information exchanged between neighbouring agents. In the context of FDI in MASs, this structure is the most adequate, namely when there are measurement noises in the sensors, and perturbations in the communication links. Recent surveys of different approaches to fault diagnosis in MASs [Qin *et al.* 2014, Song & He] highlight other advantages of distributed designs in contrast with centralised and decentralised ones. Given the above, all of the FDI schemes designed and proposed in this thesis are distributed algorithms.

  Figure 1.16 depicts an illustration of a distributed FDI architecture for an example of a MAS composed of 3 agents. Figure 1.17 shows a sketch of the illustrating distributed architectures where the typically used information is depicted as well as the faults that are to be detected.

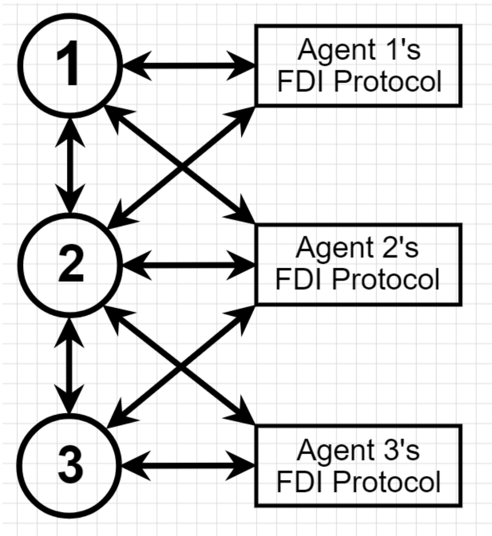


Figure 1.16: An example of the structure of distributed architectures.

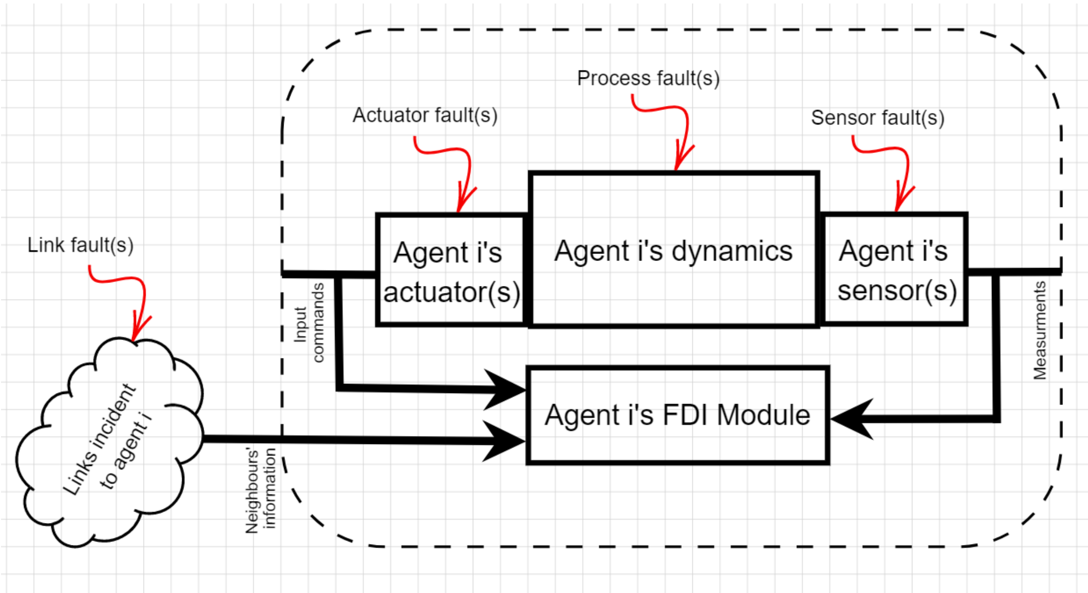


Figure 1.17: A sketch illustrating distributed FDI schemes in MASs subject to node and link anomalies.

Some other challenges and criteria that are to be taken into account before designing an FDI algorithm for MASs besides its architecture, which are defined and detailed in subsection 1.2.1, include

- *Undirected/Directed topologies*
- *Fixed/Switching topologies*
- *Heterogeneous/Homogeneous agents*

In the following, a literature review on recent model based FDI techniques and methods in MASs is provided.

FDI Techniques in MASs

Many early works in the literature have focused on designing centralised FDI and FE protocols for MAS, in which agents comprising the system communicate with a central unit which in turn performs detecting and/or isolating or estimating fault signals, e.g., [Meskin & Khorasani 2009a, Menon & Edwards 2013, Liu *et al.* 2016b, Meskin & Khorasani 2009b]. In [Menon & Edwards 2013] for example, a robust FE scheme based on sliding mode observers was proposed using relative measurements. The authors studied a linear model where orthogonal transformations were applied to the initial unobservable concatenated model to create observable sub-models used to estimate actuator faults. [Meskin & Khorasani 2009a] solved the FDI problem in a centralised manner for linear systems where structured residual sets were proposed with dependent fault signatures, the scheme was then applied to a network of unmanned vehicles. Early works have also studied FDI in MASs from a decentralised point of view [Li *et al.* 2009, Stanković *et al.* 2010, Boem *et al.* 2016, Arrichiello *et al.* 2015]. [Arrichiello *et al.* 2015] for instance, proposed a decentralised observer-based FDI strategy for a team of networked robots where each robot of the team is able to detect and isolate faults occurring in other robots using local observers, residual vectors are then computed to set up alarms when a fault occurs, thresholds are derived based on the dynamics of the residual vectors. Early works have also dabbled in distributed designs, e.g., [Franco *et al.* 2006, Yan & Edwards 2008, Meskin & Khorasani 2009a, Meskin *et al.* 2010]. Between the three aforementioned approaches, which have been detailed previously, the centralised FDI schemes seem to be the least favourite in the FDI community because of their weak reliability, which renders their usability counter intuitive in the context of fault detection as the main objective is to maintain a high reliability of the detection scheme. Decentralised schemes lead to a better autonomy of the design where each agent can handle its own FDI tasks using local measurements only. On the other hand distributed FDI schemes have proven to be the most reliable and as such have become popular and widespread. This is because, as opposed to the centralised and decentralised schemes, they lead to significantly more reliability as the failure of an FDI module does not compromise the FDI process. Indeed, in centralised schemes, the failure of the central unit leads to the failure of the entire FDI protocol, and in decentralised schemes, the failure of an FDI module installed in a given agent can lead to a fault happening in said agent going unnoticed [Ferrari *et al.* 2011].

[Meskin & Khorasani 2009a] and [Meskin *et al.* 2010] have both focused on the design of distributed FDI schemes for networked homogeneous unmanned vehicles subject to actuator faults based on geometric approaches. In [Shames *et al.* 2011] and [Shames *et al.* 2012], the authors have investigated the problem of distributed FDI design for homogeneous interconnected second-order MASs with undirected graph topologies, by constructing a bank of Unknown Input Observers (UIOs) to detect one fault at a time in the MAS. [Shi *et al.* 2014] proposes a distributed FDI methodology for discrete-time second-order MAS based on the optimal robust observers. [Davoodi *et al.* 2013] focused on heterogeneous MASs equipped with relative information sensors, in which agents are modelled with different dynamics and an agent can then detect its own and its neighbours' faults. More recently, [Quan *et al.* 2018] proposed a FDI protocol for first order MASs with undirected graphs in a leader-follower control set-up, using first order SMOs. However, most of these works only consider first, second order or undisturbed MASs.

[Liu *et al.* 2016a] proposed UIOs to detect actuator faults in higher-order homogeneous nonlinear MASs with undirected graphs, where residuals were generated for each fault which are robust with respect to dynamic disturbances using the $\mathcal{H}_\infty$ norm. A MASs with the same constraints was studied in [Gao *et al.* 2017] but without the nonlinear components using a reduced order UIO-based approach, however, in these works, measurement and communication noise is not considered. In [Chadli *et al.* 2017], the development and design of a distributed FDI scheme was carried out for MASs composed of agents with discrete-time Linear Parameter-Varying (LPV) dynamics using UIO-based filters and the mixed $\mathcal{H}_\infty/\mathcal{H}_-$ performance technique, which was achieved by solving a set of Linear Matrix Inequalities (LMIs). On top of detecting faults in-spite of the presence of external disturbances, each agent is able to estimate its own state and the state of its nearest neighbours. In [Davoodi *et al.* 2016], the problem of FDI in MASs with linear dynamics and undirected topologies, under both state consensus control and leader-follower control settings, is investigated where distributed Luenberger observers are designed based on locally exchanged information. Residuals are then designed based on the mixed $\mathcal{H}_\infty/\mathcal{H}_-$ performances to detect sensor faults in agents and their immediate neighbourhoods, in these work however, only sensor faults can be detected and the proposed schemes are not valid in the presence of only actuator faults. [Hajshirmohamadi *et al.* 2019] extended the work of [Davoodi *et al.* 2016] to accomplish both actuator fault and sensor fault detection. Both works constitute a multi-objective fault detection and passive fault-tolerant control strategy, hence creating a water bed effect where a compromise between the fault detection and the fault-tolerant control performances respectively has to be found. Both works however, only consider heterogeneous linear MASs with fixed topologies.

When it comes to distributed FE in MASs, [Han *et al.* 2019] has proposed a distributed intermediate robust observer-based scheme for homogeneous MASs with Lipschitz nonlinearities and no measurement noise, where new intermediary variables were introduced for FE purposes. [Liu *et al.* 2018] also proposed a distributed FE observer for homogeneous MASs with Lipschitz nonlinearities but with undirected communication topologies and only actuator faults. In [Zhang *et al.* 2016], a novel distributed adjustable parameter-based FE observer was designed to estimate actuator faults. [Xia *et al.* 2017] used a distributed robust dissipativity-based reduced-order observer for the purpose of FE in homogeneous MASs. An adaptive distributed FE full order observer was employed in [Zhang *et al.* 2015] for MASs with directed graphs, where the agents are modelled with undisturbed linear dynamics. The works mentioned above only consider homogeneous linear systems or systems with Lipschitz nonlinearities and can thus not be applicable for MASs with more complex dynamics such as MASs with chained form dynamics for instance.

On the other hand, in [Wu *et al.* 2019], a fixed-time SMO was presented in order to successfully fulfil the FDI tasks of a class of nonlinear MASs subject to actuator faults and temporal constraints through the generation of some auxiliary variables received from, and sent to neighbouring agents. It is shown therein, that the use of fixed-time observers presents many advantages with respect to availability of the state information and the problem of transient behaviours, making convergence time a key factor in FDI for MASs. However, most works mentioned to this point have proposed algorithms which only guarantee asymptotic convergence (i.e., the bound of the settling time depends on the initial conditions). [Arrichiello *et al.* 2015] and [Quan *et al.* 2018], for instance, have both used SMOs to achieve FDI but the convergence time of the state estimation errors is heavily impacted by the initial conditions and the graph topology. It is shown that, when a fault occurs before the convergence of the FDI observers, the fault might not be detected and one can obtain possible erroneous FDI results. This convergence time is discussed in details in Subsection 1.2.5, before that, attacks in MASs are discussed in the following.

1.2.4.2 Attacks in MASs

In the context of cooperative networked MASs or simply Networked Control Systems (NCS), where computational resources and communication networks are integrated, agents constantly exchange information amongst themselves, or with a computational unit through a wireless network. Consequently, these systems which include computational resources, communicational capabilities and hardware, also referred to as Cyber-Physical Systems (CPS) in the literature, introduce a high degree of connectivity which exposes them to a new kind of malicious threats, known as *cyber-attacks*, *cyber faults* or simply *attacks*. Since CPSs use open communication platform architectures, they are vulnerable to suffering not only physical faults/malfunctions but also these adversarial attacks. Detecting these latter threats has thus also become a central focus for system security and control in MASs for the FDI community, along with physical faults. Figure 1.18 depicts an illustration of the environment of a cooperative networked MAS, its basic components, the cyber/physical layers and the possible threats it is exposed to. Local malfunctions here, refer to physical anomalies/faults that impact the physical space of the MAS.

Recent real-world situations where cyber-attacks have occurred were recorded. Prominent examples include: multiple power blackouts in many countries in the world like Brazil [Conti 2010], the attack on the water distribution system in Queensland, Australia [Slay & Miller 2007], the Stuxnet attack that took control of actuators and sensors in a Iranian nuclear facility prompted the Iranians to replace thousands of failed centrifuges [Lindsay 2013], the cyber-attack against a Ukrainian power grid that

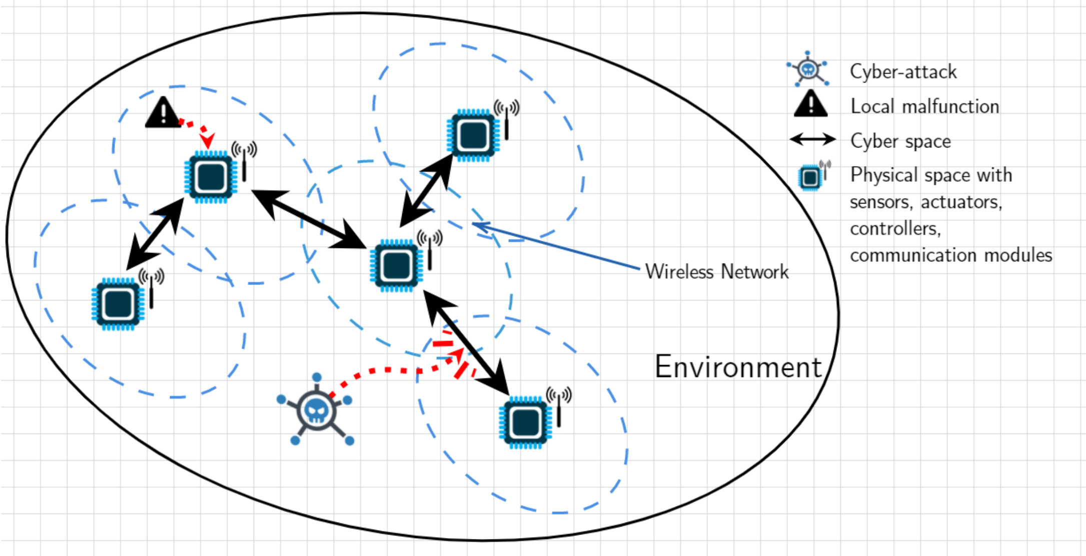


Figure 1.18: An illustration of a cooperative networked MAS, basic components, cyber/physical layers and possible threats.

led to power being cut for many hours [Sullivan & Kamensky 2017], etc. Clearly, these types of malicious attacks aimed at degrading or interrupting the operation of connected systems, exploit their aforementioned vulnerabilities and can have extremely destructive effects, not only from a process point of view but also from environmental and financial ones.

As in any connected system, the well known Confidentiality, Integrity, and Availability (CIA) triad of a CPS can be threatened and fall prey to two types of attacks, *non-targeted attacks* and *targeted attacks*. In the latter case, the attacker/hacker has an actual idea of the model and structure of the system that he targets, he knows that he is targeting a control system, and has a perceived knowledge, full or partial, on its dynamics. In non-targeted attacks, the attacker does not have any knowledge on the system, and thus targets and corrupts general information that he can get his hands on. Evidently, the degree of destructiveness and influence the attack induces on the system depends on the attacker's knowledge of the system.

The three main security elements in connected cooperative MASs mentioned in the previous paragraph, that need to be maintained are: Confidentiality, Integrity, and Availability. This means that in the context of connected MASs:

- *Confidentiality:* Unauthorized users have to be kept away from the exchanged information. The lack of confidentiality is called disclosure.
- *Integrity:* The trustworthiness of data has to be guaranteed. Attacks that compromise integrity are called *deception attacks*.
- *Availability:* The needed information has to be accessible and usable to agents

that need it upon demand. Attacks that compromise availability are called *Denial of Service (DoS)* attacks.

A specific kind of deception attack is called a *replay attack* [Khazraei *et al.* 2017, Gallo *et al.* 2018a]. In these types of attacks, the attacker hijacks, the transmitted information, records the readings for a certain time and then replays it with a delay instead of the actual information, possibly while injecting it with exogenous signal. Another type of deception attack is called *False Data Injection Attack (FDIA)* [Boem *et al.* 2017]. It is a situation where the attacker targets the transmitted information and makes sure that the received information is fake/invalid by injecting with malicious false data. Other kinds of deception attacks classified as targeted attacks could be mentioned, where the system dynamics are known to the attacker, include *covert attacks* [Smith 2011] or *stealth attacks* [Hashemi *et al.* 2018]. In these situations, the attacker modifies some information readings by physically tampering with the individual sensory units or by getting access to one or multiple communication channels.

A DoS attack [Rezaee *et al.* 2021], sometimes called a *jamming attack* in literature [Guan & Ge 2017] compromises accessibility of information to agents. These attacks consist of assaulting data availability through blocking information flows between different agents in networked MASs or CPSs. The attacker jams the communication channels and prevent data exchange.

A radical difference worth mentioning when comparing faults and attacks in MASs, is that one could talk about attacks only when a networked system is concerned. Hence, in MASs where agents measure their neighbours' information using sensors, i.e., "sensor based" networks where no information is effectively exchanged over a wireless network, one cannot talk about attacks, as access to these information by an external attacker is not possible.

It is shown in [Cardenas *et al.* 2008] that information security techniques such as adding encryption and authentication schemes can help make some attacks more difficult to succeed, but that they are far from being sufficient for protecting systems against cyber-attacks, as it is well-known that just because some of the components of a system are secure, it does not mean that the overall system is. On the other hand, in the context of networked control systems, the problem of attack diagnosis is getting closer to the problem of fault diagnosis, where model based observer techniques have proven to be efficient in attack detection, as seen in the literature [Teixeira *et al.* 2010, Tan *et al.* 2020, Song & He , Zhang *et al.* 2021a]. It is shown that even though the difference, mathematically speaking, between fault detection and attack detection in the FDI community, can sometimes seem semantic [Teixeira *et al.* 2010] in certain scenarios, the attack diagnosis/detection problem in networked MASs is a challenging one, as it addresses a broader spectrum of feared scenarios and the attack model varies depending on the nature of these scenarios. Indeed, FDI techniques are not always effective in detecting attacks and should thus be tailored specifically for this purpose as the attack should be properly modelled and can vary from a situation to another

[Zhang *et al.* 2021a] .

Throughout this thesis, attacks in CPSs as defined in the context of cooperative networked MASs with distributed FDI schemes are taken into account. Given that in these systems, only agents' exchanged information is exposed, i.e., agents do not expose their physical components to the attacker, the latter can only target exchanges between different agents, whatever that exchanged information might be. Another one of the points this thesis focuses on is the *attack identification* which consists of securely determining whether a detected anomaly is, in fact, an attack caused by an outsider entity. The term FDI in connected MASs can sometimes be used as a general umbrella term encompassing all types of anomalies that can affect an agent or a communication link.

In the following, a brief literature review on recent model based attack detection techniques and methods in MASs is provided.

Attack Detection Techniques in MASs

Given the discussion above, the detecting and isolating of cyber-attacks in connected MASs have received immense attention in recent years, e.g. the reader is referred to [Tan *et al.* 2020, Zhang *et al.* 2021a] for recently established surveys on attack detection methods. Many approaches to cyber-attack detection were studied, where it is highlighted in [Tan *et al.* 2020] that most attack detection schemes in literature can be divided into either model-based or data-driven methods. In most data-driven approaches, heuristic or deep learning algorithms are used to construct a "model" where an attack is detected if the system measurement data does not match the constructed "model", where certain assumptions could be made with respect to the measurements. As for model-based approaches, thanks to the development of FDI algorithms and their advantages mentioned above for physical faults, observer-based fault diagnosis techniques are widely accepted as powerful tools used to solve attack detection problems in connected MASs [Gallo *et al.* 2018b, Tan *et al.* 2020, Song & He]. Hence, this thesis is interested in FDI techniques for attack detection, isolation and identification in MASs. On the other hand, due to their complexity, distributed attack detection and identification for MASs is premature [Song & He]. Indeed, some of the challenges encountered in the literature in attack diagnosis include modelling of the attack, attacks in nonlinear MASs, attacks in MASs with switching communication topologies, attacks in MASs with uncertainties and noise, etc.

[Fawzi *et al.* 2014] for instance, has focussed on the problem of state estimation for linear systems in the case where some of the sensors/actuators are non-simultaneously targeted by deception attacks. In [Corradini & Cristofaro 2017], a SMO was proposed to detect scalar attacks for a linearised model of an electric power network, where sensor and actuator attacks are considered not to occur simultaneously. Deception attack detection for linear MASs with first-order agent dynamics was studied in [Pasqualetti *et al.* 2011] such that the attack is treated as an unknown input signal injected into the system and the global model information is available throughout the attack detection process. A threshold-based distributed scheme similar to one used in physical fault detection was given in [Ferrari *et al.* 2011] to detect attacks on the communication channels. In [Gallo *et al.* 2018a], the observer-detector scheme designed in [Boem *et al.* 2017] was improved, where a distributed UIO based methodology was proposed to estimate the state and detect a FDIA in the communication network, specifically to identify whether output measurements received from an agent's neighbours corrupted by FDIA or not, detection thresholds are then designed and the results were verified on direct current microgrids. DoS and replay attack detection for MASs have also received increased attention in the past few years. The problem of replay attack detection in connected MASs composed of vehicles with double integrator dynamics under a cooperative cruise control was solved in [Merco *et al.* 2018] using a residual based method. In [Biron *et al.* 2018] , the problem of DoS attack detection was solved using decentralised SMOs for MASs comprised of a network of vehicles modelled with double integrator dynamics, where an augmented system was used which also includes the derivative of the control input. As for attack estimation/reconstruction, a SMO based design was used in [Taha *et al.* 2016] in order to approximately reconstruct the attacks using pseudo-inverse techniques. In [Nateghi *et al.* 2018], a SMO based on Higher-Order Sliding Mode (HOSM) differentiators along with a sparse recovery algorithm for a class of nonlinear systems was proposed, where the finite-time convergence of the observers was shown and a reconstruction of a class of cyber-attacks was achieved. A mode estimator was proposed in [Kim *et al.* 2017] for switched nonlinear CPSs where the state and attack vectors' estimates were achieved. In [Yu & Yuan 2020] an extended state observer was used to estimate both the state vector and attack signals in nonlinear NCS where the nonlinear part was modelled as a disturbance which was then decoupled from the system given that certain assumptions are satisfied. Most of the works mentioned above however, do not consider the case where both attacks and physical faults are taken into account. Additionally, most works in the literature do not consider the challenges of the existence of switching topologies, communication noise and/or measurement noise, under both faults and cyber-attacks.

1.2.4.3 Fault-Tolerant and Attack Resilient Control in MASs

Both *fault-tolerant control* and *attack resilient control* problems in connected MAS deal with the adjustment of the control mechanisms in order to deal with faults and attacks respectively. This entails that in spite of the presence of faults or attacks, their effects are reduced such that the desired control objectives are still achieved. In MAS, fault-tolerant control or *fault-tolerant consensus* for consensus-type control protocols are deployed when physical faults are concerned. Conversely when attacks are concerned, attack resilient control or *secure consensus* are used [Zhang *et al.* 2021a]. Fault-tolerant controllers [Gallehdari *et al.* 2016, Khalili *et al.* 2019] are designed based on FDI information through the use of locally exchanged information (see [Yang *et al.* 2019] for a survey).

In literature [Blanke *et al.* 2006], two main types of fault-tolerant/attack resilient control can be defined

- *Passive*: the controller parameters do not vary with respect to time, and are designed off-line such that the system performance is guaranteed in the presence of all predefined anomalies.

- *Active*: the controller parameters are adaptive and the controller is reconfigured according to information about the type and shape of the anomaly, which are received from a FDI module, such that the desired performance of the system is recovered.

Fault-tolerant control with respect to physical threats in MASs has been widely studied in the literature [Chen *et al.* 2016, Wang & Yang 2019, Liu *et al.* 2019]. On the other hand, the attack resilient control problem has been investigated in parallel with the attack detection problem in the past few years and is yet to be as widely explored as the fault-tolerant control problem. When it comes to attack resilient control, a cooperative control algorithm that is resilient to certain deception attacks was designed in [Pasqualetti *et al.* 2011] for MASs with unperturbed linear dynamics and perfect measurements using an UIO approach. [Lu & Yang 2018] solved the problem of distributed consensus control for linear MASs under a general DoS attack, where only a set of communication channels were jammed. In [Zhang *et al.* 2018], secure consensus of MASs with Lipschitz nonlinearities subject to a DoS attack was addressed, where the attacker jams a set of communication links such that the connectivity is broken and the directed spanning tree is lost. Some more recent works on secure consensus of MASs under deception and DoS attacks can also be found in [Mustafa & Modares 2019, Modares *et al.* 2019, Deng & Wen 2020, Shi & Yan 2020].

Figure 1.19 depicts a chart listing all of the typical factors impacting FDI and attack detection protocol designs as well as fault-tolerant control and attack resilient control in MASs, as discussed thus far.

As mentioned in page 31, for the study of FDI for cooperative MASs, the convergence rate is a critical topic [Yang *et al.* 2019, Wu *et al.* 2019]. Many existing algorithms only guarantee asymptotic or finite-time convergence (i.e., the bound of the settling time depends on the initial condition of the agents). However, many applications require a uniformly bounded convergence time. As such, in the following Subsection, convergence rate analysis is discussed, where the concept of fixed-time and predefined-time stability are introduced.

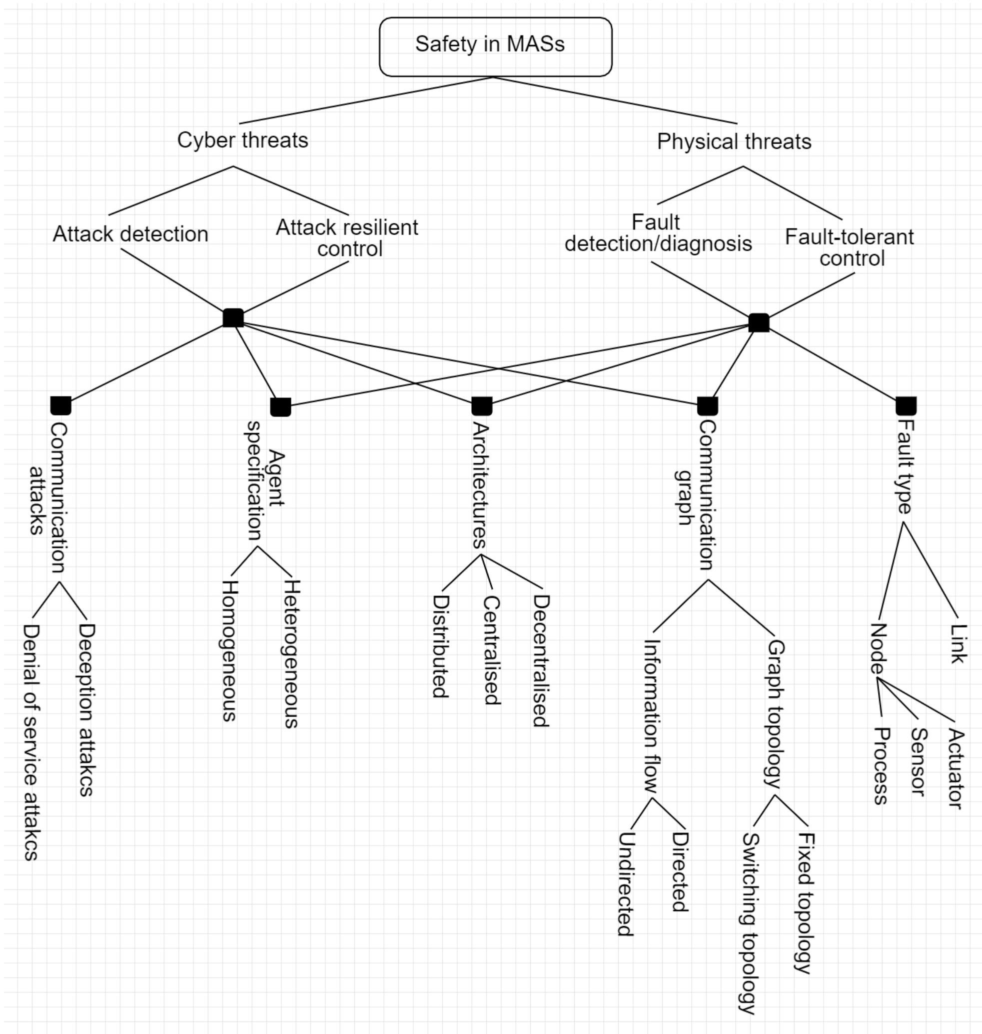


Figure 1.19: A chart summarising factors impacting: attack detection, FDI, fault-tolerant control and attack resilient control designs in connected MASs.

1.2.5 Convergence Rate Analysis

As mentioned in the previous subsection, an important topic in the study of the fault and/or attack detection problems in MASs, is the *convergence rate*, which characterises the *settling time* of the fault/attack detection observers. Fast convergence is a critical index that is typically pursued in practice in order to have an idea on the expected convergence time and thus, allows the designer to achieve a better robustness and performance of the FDI modules. This knowledge presents many advantages with respect to the transient behaviours and leads to the fault being detected in a timely manner. As discussed earlier in this chapter, most FDI observer methods proposed in literature do not take this performance specification into account and most of the proposed observers' convergence time can be negatively impacted by different factors such as the initials conditions, unknown perturbations, structure of the graph topologies, etc.

One could distinguish four types of convergence rates: *asymptotic stability*, *finite-time stability*, *fixed-time stability* and *predefined-time stability*. Each representing a stronger notion of settling-time than the previous.

1.2.5.1 Asymptotic Stability

Most of the aforementioned observer-based fault detection algorithms discussed in the previous subsection only consider asymptotic convergence, e.g., [Davoodi *et al.* 2016, Quan *et al.* 2018, Hajshirmohamadi *et al.* 2019]. This is to say that the convergence rate is exponential and the settling time is infinite. As such the state estimates cannot reach their real values in a finite time.

1.2.5.2 Finite-time Stability

Consider the following nonlinear system

$$\begin{cases} \dot{\xi}(t) &= \Phi(t, \xi(t), \phi) \\ \xi(0) &= \xi_0 \end{cases} \tag{1.11}$$

where $\xi(t) \in \mathbb{R}^n$ is the state vector and $\phi \in \mathbb{R}^g$ where $g \in \mathbb{N}$, is the vector containing the system's parameters, which are considered to be constants ($\dot{\phi} = 0$). $\Phi : \mathbb{R}_+ \times \mathbb{R}^n$ is assumed to be a nonlinear function with its origin as an equilibrium point, i.e., $\Phi(t, 0, \phi) = 0$. $\xi(0) = \xi_0 \in \mathbb{R}^n$ are the system's initial conditions. Then, the finite-time stability can be defined as follows

Definition 1.15 *[Bhat & Bernstein 2000] The origin of* (1.11) *is said to be globally finite-time stable if it is globally asymptotically stable and any solution* $\xi(t, \xi_0)$ *of* (1.11) *reaches the equilibrium point at some finite time moment, i.e.,* $\forall t \geqslant \Gamma(\xi_0)$, $\xi(t, \xi_0) = 0$, *where* $\Gamma : \mathbb{R}^n \longrightarrow \mathbb{R}_+ \cup \{0\}$ *is called the settling-time function.*

The asymptotic stability of the origin is implied by the finite-time stability of the origin and as such, finite-time stability is a stronger condition than asymptotic stability.

Lemma 1.1 *[Bhat & Bernstein 2000] If there exists a continuously differentiable positive definite and radially unbounded function* $V : \mathbb{R}^n \to \mathbb{R}$ *such that*

$$\begin{aligned} &V(0) = 0 \\ &V(\xi) > 0, \quad \forall \xi \in \mathbb{R}^n \backslash \{0\}, \end{aligned}$$

and the derivative of V *along the trajectories of* (1.11) *satisfies*

$$\dot{V}(\xi) \leqslant -\alpha V^p(\xi) \tag{1.12}$$

where $\alpha > 0$ *and* $p \in (0,1)$*. Then, the origin of the system* (1.11) *is said to be globally finite-time stable and the settling time is estimated as*

$$\Gamma(\xi_0) \leqslant \frac{1}{\alpha(1-p)}(V(\xi_0))^{1-p} \tag{1.13}$$

In finite-time stability, the power exponent is always less than one. Additionally, it should be noted that the estimated bound of the settling time in finite-time stability, depends on the initial conditions of the agents. Consequently, this bound cannot be a priori estimated in a distributed fashion.

1.2.5.3 Fixed-time Stability

Despite the fact that finite-time stability presents many advantages when compared with asymptotic stability, the estimation of convergence time bounds depends on initial states, the knowledge of which is usually unavailable. This has given introduction to fixed-time stability, in which the convergence information is provided in advance. This crucial information yields more exploitable options for the observer designer.

Definition 1.16 *[Polyakov 2011] The origin of* (1.11) *is said to be a globally fixed-time equilibrium if it is globally finite-time stable and there exists a strictly positive number* T_{max} *such that for all* $\xi_0 \in \mathbb{R}^n$ *the settling-time function* $\Gamma : \mathbb{R}^n \to \mathbb{R}_+$ *is bounded, i.e.* $\Gamma(\xi_0) \leqslant T_{max}$ *for all* $\xi_0 \in \mathbb{R}^n$*, the solution* $\xi(t,\xi_0)$ *of system* (1.11) *is defined and* $\xi(t,\xi_0) \in \mathbb{R}^n$ *for* $t \in [0, T_{max}) : \lim_{t\to T_{max}} \xi(t,\xi_0) = 0$.

Lemma 1.2 *[Polyakov 2011] If there exists a continuously differentiable positive definite and radially unbounded function* $V : \mathbb{R}^n \to \mathbb{R}$ *such that*

$$\begin{array}{l} V(0) = 0 \\ V(\xi) > 0, \quad \forall \xi \in \mathbb{R}^n \backslash \{0\}, \end{array}$$

and the derivative of V *along the trajectories of* (1.11) *satisfies*

$$\dot{V}(\xi) \leqslant -\alpha(V(\xi))^p - (\beta V(\xi))^q \tag{1.14}$$

where $\alpha > 0$*,* $\beta > 0$*,* $q > 0$ *and* $p \in (0,1)$*. Then, the origin of the system* (1.11) *is said to be globally finite-time stable and the settling time is estimated as*

$$T_{max} = \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)} \tag{1.15}$$

Hence, in the case where $p = 1 - \frac{1}{\mu}$, $q = 1 + \frac{1}{\mu}$ and $\mu > 1$, the settling time can be estimated by a less conservative bound [Parsegov *et al.* 2012]

$$T_{max} \quad = \frac{\pi\mu}{2\sqrt{\alpha\beta}} \tag{1.16}$$

The concept of fixed-time stability can be introduced to design observers such that the upper bound of the convergence time is independent of the initial conditions of the system. To discern the case where the settling-time bound is set in advance as a function of the system's parameters, the concept of predefined-time stability is introduced. To derive a simple relationship between the control parameters and the upper bound of the convergence time, the concept of predefined-time stability has recently been proposed [Jimenez-Rodriguez *et al.* 2020b]. Based on this concept, [Sánchez-Torres *et al.* 2018] has proposed a class of robust algorithms that ensures a predefined upper bound of the settling time where the convergence time is set a-priori as a parameter of the protocol and does not depend on the initial conditions.

1.2.5.4 Predefined-time Stability

It is indeed interesting, in many applications, to define a settling time $T_p \in \mathcal{T}$ in advance, where $\mathcal{T} = \{T_{max} \in \mathbb{R}_+ : \Gamma(\xi_0) \leqq T_{max}, \forall \xi_0 \in \mathbb{R}^n\}$. Consider the system

$$\dot{\xi}(t) = -\left(\alpha|\xi(t)|^p + \eta|\xi(t)|^q\right)^r \operatorname{sign}(\xi(t)), \quad \xi(0) = \xi_0 \tag{1.17}$$

where $\xi \in \mathbb{R}$. The real numbers $\alpha, \eta, p, q, r > 0$ are the system's parameters which satisfy the constraints $rp < 1$ and $rq > 1$. The concept of predefined-time stability is defined as follows

Definition 1.17 *[Jimenez-Rodriguez et al. 2020b] The origin of* (1.11) *is said to be predefined-time stable if it is fixed-time stable for $T_{max} \in \mathbb{R}_+$, there exists some $\phi \in \mathbb{R}^l$ such that the settling-time function of* (1.11) *satisfies*

$$T_p = \sup_{\xi_0 \in \mathbb{R}^n} \Gamma(\xi_0) \leqslant T_{max}, \quad \forall \xi_0 \in \mathbb{R}^n$$

The following lemma concerning predefined-time stability can also be recalled

Lemma 1.3 *[Aldana-López et al. 2019] For the parameter vector ϕ containing the real numbers $\alpha, \eta, p, q, r > 0$ satisfying the constraints $rp < 1$ and $rq > 1$, the origin $\xi = 0$ of system* (1.17) *is fixed-time stable and the settling time function satisfies $T(\xi_0) \leq T_f = \gamma(\phi)$, where*

$$\gamma(\phi) = \left(\frac{\Theta(\frac{1-rp}{q-p})\Theta(\frac{rq-1}{q-p})}{\Theta(r)(q-p)\alpha^r}\right)\left(\frac{\alpha}{\eta}\right)^{\frac{1-rp}{q-p}} \tag{1.18}$$

and $\Theta(\cdot)$ is the gamma function defined as

$$\Theta(z) = \int_0^{+\infty} e^{-t} t^{z-1} dt$$

The lemma presented below is used to derive a Lyapunov-like condition for characterizing predefined-time stability of a system, where a settling time bound T_p is set in advance as a function of system parameters ϕ, i.e. $T_p = T_p(\phi)$, and a strong notion of this class of stability is given when $T_p = T_f$, i.e., T_p is the least upper bound of the settling time.

Lemma 1.4 *[Aldana-López et al. 2019] Let us consider system* (1.11). *Suppose there exists a continuous radially unbounded candidate Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}$ such that*

$$\begin{aligned} &V(0) = 0 \\ &V(\xi) > 0, \quad \forall \xi \in \mathbb{R}^n \backslash \{0\}, \end{aligned}$$

and the derivative of V along the trajectories of (1.11) *satisfies*

$$\dot{V}(\xi) \leqslant -\tfrac{\gamma(\phi)}{T_p} \left(\alpha V^p + \eta V^q\right)^r, \quad \forall \xi \in \mathbb{R}^n \backslash \{0\}, \tag{1.19}$$

where $\alpha, \eta, p, q, r > 0$, $rp < 1$, $rq > 1$, $\gamma(\phi)$ is given in (1.18). *Then, the origin of* (1.11) *is predefined-time-stable with predefined time T_p.*

1.3 Research Motivation and Thesis Contribution

Given the relevant overviews and state of the art presented in the previous section, the motivation behind this thesis is detailed herein, followed by a summary of its contributions.

1.3.1 Research Motivation

The main objective of this thesis is the study of distributed fault detection and isolation in MASs. It is evident from the literature and state of the art investigated in Section 1.2 that the existing FDI schemes suffer from various limitations with respect to different challenges which have been detailed in the General Introduction part of the thesis. They are briefly restated in the following

- *FDI for MASs with temporal constraints:* As expressed in the previous section, the fixed-time stability property allows to avoid the problem of transient behaviours. What is more, this property can help to significantly simplify the residual generation and fault detection process, whereby the convergence time is estimated a-priori and is independent of the structure of the communication topology and the initial conditions. This property has not been sufficiently explored in literature in its application to fault and attack detection and identification in MASs.
- *FDI for MASs with switching topologies:* This problem merges into the previous point. Indeed, it is usually considered that the communication topology of a MAS stays fixed during the detection process. In many applications however, this can be impractical and communication topologies can be switching rather

than fixed. In such situations, the predefined-time principle can be useful as observers can be designed such that faults are detected in a timely manner. As such, contrary to finite-time schemes, the estimation of the settling time does not require the knowledge of the initial conditions, thus allowing for a step by step design basis of the detection scheme. Some works in the literature have tackled the problem of FDI in MASs with switching topologies [Gallehdari *et al.* 2017, Wang & Yang 2019, Saboori & Khorasani 2015, Zhang *et al.* 2017], these works however, do not consider the possibility of the occurrence of a cyber-attack.

- *FDI for MASs with heterogeneous agents and directed topologies:* In practice, MASs can sometimes be composed of heterogeneous agents, for economical reasons or otherwise. This heterogeneity property can present some challenges in the design of a distributed fault or attack detection scheme. The same can be said for MASs with directed communication topologies. In fact, in such cases, information is even more restrained, hence further complicating the detection and isolation process. Indeed, distributed fault detection in MASs with heterogeneous agents and/or directed topologies, namely when cyber-attacks are concerned, is still an open problem.

- *FDI for MASs subject to cyber-attacks and physical faults:* In practice, MASs can be subject to either physical faults (actuator/sensor faults), cyber-attacks (attacks on communication links) or both. The proposed algorithms should thus be able to detect at least one anomaly type, and in the case where both occur, an cyber-attack identification scheme must be put in place in order to discern physical faults from cyber-attacks.

Motivated by the above, the contributions of this thesis with respect to existing literature are detailed in the next subsection.

1.3.2 Thesis Contribution

Given the previously discussed constraints, the contributions of the thesis are as follows

- First, a distributed methodology for the detection of actuator faults in a class of linear MASs with unknown disturbances is proposed in [Taoufik *et al.* 2020c], whose main features are highlighted in the following. The formulation of the distributed actuator FDI problem for a class of linear MASs with disturbances is performed through the use of a cascade of fixed-time SMOs, where each agent having access to their state, and neighbouring information exchanges, can give an exact estimate of the state of the overall MAS. An LMI-based approach is applied to design distributed global residual signals at each agent, based on mixed $\mathcal{H}_{-}/\mathcal{H}_{\infty}$ norms. The above combined approaches allow treating the actuator fault detection problem while keeping a distributed design approach, as information obtained by each interacting agent only comes from its neighbours. The previous point is then extended to nonholonomic systems, where a new distributed robust fault detection scheme for MASs composed of agents with nonlinear nonholonomic dynamics is proposed in [Taoufik *et al.* 2021b]. This considered model is relevant for many applications (for instance, ground mobile robots, surface ships, underwater vehicles etc., see [Kolmanovsky & McClamroch 1995] for an extended survey). In this proposed scheme, the use of predefined-time stability techniques to derive adequate distributed SMOs is investigated, which enable to reconstruct the global system state in a predefined-time and generate proper residual signals. The proposed scheme ensures global fault detection, where each agent is capable of detecting its own faults and those occurring elsewhere in the system using only local information (contrary to most of the existing works). The results obtained in the first point are then extended to the case of MASs with higher order integrator dynamics [Taoufik *et al.* 2020b], where only the first state variable is measurable. Here, a new approach to identify faults and deception attacks in a cooperating MASs with a switching topology is introduced. The proposed protocol makes an agent act as a central node monitoring the whole system activities in a distributed fashion whereby a bank of distributed predefined-time SMOs for global state estimation are designed, which are then used to generate residual signals capable of identifying cyber-attacks despite the switching topology.

- In this thesis, the problem of distributed fault detection for a team of heterogeneous MASs with linear dynamics is also investigated in [Taoufik *et al.* 2020a], where a new output observer scheme was proposed which is effective for both directed and undirected topologies. The main advantage of this approach is that the design, being dependant only on the input-output relations, renders the computational cost, information exchange and scalability very effective compared to other FDI approaches that employ the whole state estimation of the agents and their neighbours as a basis for their design. A more general model is studied in [Taoufik *et al.* 2021a], where actuator, sensor and communication faults/attacks are considered in the robust detection and isolation process for nonlinear heterogeneous MASs with disturbances and communication parameter uncertainties, where the topology is not required to be undirected. This was done using a distributed finite-frequency mixed $\mathcal{H}_{-}/\mathcal{H}_{\infty}$ nonlinear UIO-based approach.

The remainder of this thesis is organised as follows:

- **Chapter 2:** The problem of distributed fault detection is investigated, for MASs with linear dynamics, MASs with nonholonomic dynamics and finally for MASs with higher order integrator nonlinear dynamics under switching topologies subject to deception attacks. This is done using the fixed-time property by designing a bank of adequate distributed observers and a residual based approach.

- **Chapter 3:** This chapter is concerned with heterogeneous MASs and is composed of two parts. In the first part, a distributed output observer approach is designed for linear MASs with actuator faults, whereby the aim is to reduce the dimensions of the FDI units. In the second part, a distributed nonlinear UIO-based approach is used for MASs with quasi nonlinear dynamics subject not only to actuator faults, but also to sensor and communication faults.

- **Chapter 4:** In this chapter, the obtained results are summarised and possible future research directions are given.

CHAPTER 2

Distributed FDI in MASs Subject to Temporal Constraints

Content

2.1 Introduction

In this chapter, the problem of distributed FDI in connected MASs is investigated using the fixed-time property whereby an agent can detect faults occurring in the MAS. Some works have addressed the distributed fault detection problem for MASs using relative information or using communication and sensing based frameworks [Chadli *et al.* 2016, Davoodi *et al.* 2016, Quan *et al.* 2018, Barboni *et al.* 2020, Teixeira *et al.* 2014]. In these works, only the faults of the agent and its neighbours can be detected by an agent. On the other hand, in [Chadli *et al.* 2016] for instance, inputs are assumed to be exchangeable, which is not always feasible, namely in sensitive cooperative operations. Conversely, most of the works on observer-based fault and attack detection in cooperative MASs consider second order systems, linear systems or do not consider the case of a possibly dynamic communication topologies subject to attacks. Additionally, in the study of FDI and fault-tolerant cooperative control for MASs, the convergence rate is a critical topic [Yang *et al.* 2019]. Many existing algorithms only guarantee infinite convergence (such as asymptotic and exponential convergence) or finite-time convergence (i.e., the bound of the settling time depends on the initial condition of the agents). However, many applications require an uniformly bounded convergence time. For instance, in MASs where the topology is switching and agents are required to exactly estimate the state before each of the topology's switching instants. This information can also be useful in the context of the development of efficient FTC algorithms for MASs that seek a fast convergence rate [Yang *et al.* 2019]. Therefore, the concept of fixed-time stability has been introduced in this Chapter in order to derive algorithms with an uniformly bounded convergence time [Polyakov 2011], see Fig. 2.1 for a visual representation and evolution of convergence rates. As it will be shown in subsequent Sections, FDI observer design can largely benefit from a-priori estimated convergence time. Indeed, not only does this information facilitate the design and convergence analysis of the observers, but it also avoids erroneous FDI results by estimating the time at which the transient behaviours dissipate. The convergence time of the algorithms proposed in this Chapter are independent of the graph topology's structure and the initial conditions. Taking into consideration all of the above, this Chapter is organised as follows

- In Section 2.2: Some preliminaries are provided which present the description of the graph topology and gives some important Lemmas.
- In Section 2.3 [Taoufik *et al.* 2020c]: A distributed methodology for the detection of actuator faults in a class of higher order linear MASs with unknown disturbances is proposed whose main features are highlighted in the following: (i) The formulation of the distributed actuator FDI problem is performed through the use of a cascade of fixed-time (see 1.2.5.3) SMOs, where each agent having access to their state, can give an estimate of the state of the overall MAS. (ii) An LMI-based approach is then applied to generate robust residual signals at each agent capable of detecting a fault anywhere in the MAS, based on mixed

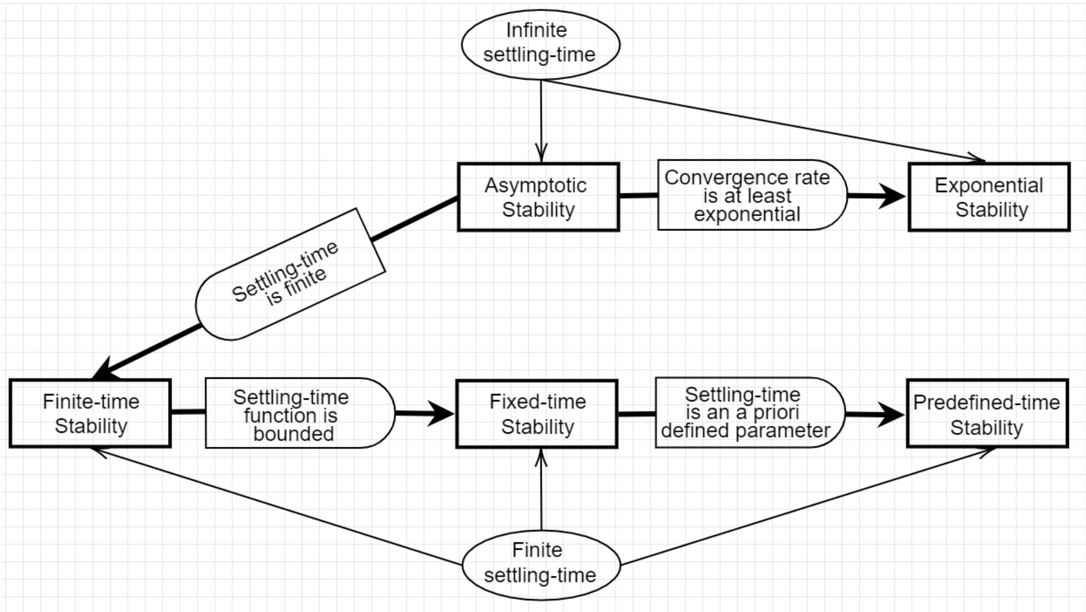


Figure 2.1: Evolution of convergence rates.

$\mathcal{H}\_/\mathcal{H}_\infty$ norms.

- In Section 2.4 [Taoufik *et al.* 2021b]: The idea behind the previous Section is extended and a novel distributed FDI scheme for nonholonomic MASs where each agent is capable of detecting its faults and those occurring elsewhere in the system using only local information. In order to deal with possible non-cooperative and malicious activities, the objective is to make each agent act as a central node monitoring the whole system's activities. The main contributions of this Section are as follows: (i) Distributed predefined-time (see 1.2.5.4) observers for global state estimation are introduced for a MASs with chained form dynamics whereby the convergence time is set a-priori; (ii) The equivalent control concept is used to estimate the disturbances and generate adequate residuals for actuator fault detection.

- In Section 2.5 [Taoufik *et al.* 2020b]: The previous results are extended to the case of switching communication topologies subject to cyber-attacks. Indeed, a novel fault and attack detection scheme is proposed, where in the occurrence of one anomaly type, the latter is identified. It is worth noting that here, each agent requires the exchange of the output and their global estimates which is equivalent to N information. This significantly reduces the communication burden when compared to [Taoufik *et al.* 2020c] for instance. The main contributions compared to the existing works in the literature are: (i) The design of a bank of distributed predefined-time SMOs for global state estimation for a multi-agent system with integrator dynamics whereby only the position is available for measurement and the convergence time is an a priori user defined parameter, in order to overcome the problem of attack detection and identification despite the switching topology.

(ii) A residual based approach is proposed where the equivalent control concept is used to detect different faults and attacks that might occur anywhere in the system (i.e., an intrusion attack reflective of a local fault in agent or a cyber-attack affecting a communication link between two given agents) in a distributed way based on the topological properties of the network. This allows detection and identification of multiple simultaneous attacks and faults.

A qualitative comparison between the features of the proposed methods in this Chapter and some of the existing results in the literature, is illustrated in Table 2.1.

Reference	LFDI	NFDI	GFDI	MFFDI	AD	CR
[Teixeira *et al.* 2010]	Yes	Yes	No	No	Yes	Asymptotic
[Shames *et al.* 2011]	Yes	Yes	No	No	No	Asymptotic
[Teixeira *et al.* 2014]	Yes	Yes	No	No	Yes	Asymptotic
[Chadli *et al.* 2016]	Yes	Yes	No	Yes	No	Asymptotic
[Davoodi *et al.* 2016]	Yes	Yes	No	Yes	No	Asymptotic
[Liu *et al.* 2016b]	Yes	Yes	No	No	No	Asymptotic
[Gao *et al.* 2017]	Yes	Yes	No	No	No	Asymptotic
[Quan *et al.* 2018]	Yes	Yes	Yes	No	No	Asymptotic
[Wu *et al.* 2019]	Yes	Yes	Yes	Yes	No	Fixed-time
[Taoufik *et al.* 2020c]	Yes	Yes	Yes	Yes	No	Fixed-time
[Taoufik *et al.* 2021b]	Yes	Yes	Yes	Yes	No	Predefined-time
[Taoufik *et al.* 2020b]	Yes	Yes	Yes	Yes	Yes	Predefined-time

Table 2.1: Qualitative comparison between some existing works, where the following acronyms are used, LFDI: Local FDI, NFDI: FDI in neighbouring agents, GFDI: FDI in other agents beyond the 1-hop neighbourhood in the MAS, MFFDI: FDI of multiple faults at a time, AD: Attack and fault detection and CR: Convergence rate of the observers.

- In Section 2.6 the main conclusions of this chapter are drawn.

Sections 2.3, 2.4 and 2.5 are, in turn, divided into three Subsection each: A problem formulation Subsection which lays out the system description as well as defines the information exchange. A main result Subsection, where the proposed methodology is detailed and finally a simulation example Subsection where an illustrative simulation is carried out to show the effectiveness of the proposed algorithms. Fig. 2.2 depicts a summary of the contribution of each Section.

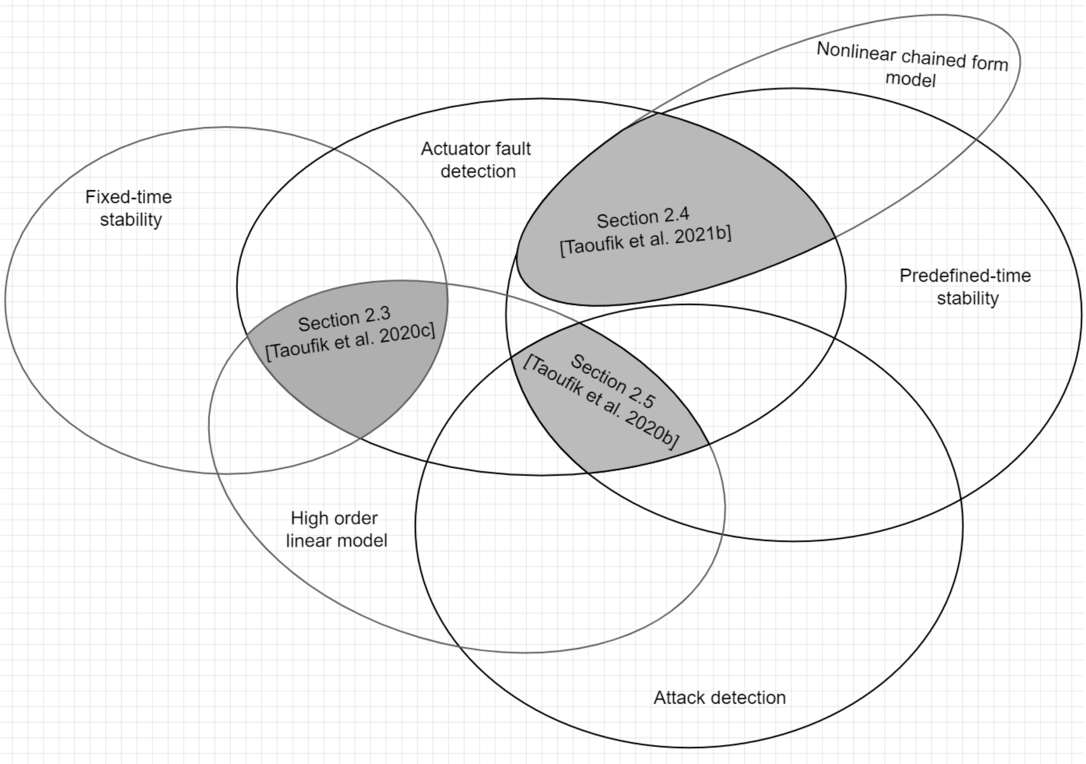


Figure 2.2: Summary of the contributions of each Section.

2.2 Preliminaries

Notations

Let us recall some of the notations employed in this Chapter. For any non-negative real number α the function $x \to \lceil x \rfloor^{\alpha}$ is defined as $\lceil x \rfloor^{\alpha} = |x|^{\alpha}\text{sign}(x)$ for any $x \in \mathbb{R}$. One can define

$$\lceil x \rfloor^{\alpha} = [\text{sign}(x_1)\lceil x_1 \rfloor^{\alpha}, \text{sign}(x_1)\lceil x_2 \rfloor^{\alpha}, ..., \text{sign}(x_N)\lceil x_N \rfloor^{\alpha}]^T$$

where $x = [x_1, x_2, ..., x_N]^T \in \mathbb{R}^N$. The set of real-valued $m \times n$ matrices is given by $\mathbb{R}^{m \times n}$. $\lambda_{min}([\cdot])$ represents the smallest eigenvalue of square matrix $[\cdot]$, and $\lambda_{max}([\cdot])$ the largest one. $||(\cdot)||_n$ denotes the $n-$norm of vector $(\cdot)$. Throughout this Chapter, for a real matrix $P \in \mathbb{R}^{n \times n}$, $P > 0$ denotes that P is symmetric and positive-definite. For an arbitrarily real matrices X, Y and Z, the matrix:

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$$

is a real symmetric matrix, where $*$ implies symmetry. The superscript T stands for the matrix transpose and we denote by I the identity matrix and by $\mathbf{1}$ the vector with all elements one, both with appropriate dimensions. For the sake of simplicity, the time argument is omitted when it is not required for clarity.

Topology Description

Consider the graph as described in Subsection 1.2.2.1, denoted $\mathcal{Q} = (\mathcal{N}, \mathcal{F})$ and composed of N systems, $\mathcal{N} = \{1, ..., N\}$ is the node set consisting of N nodes, and $\mathcal{F} \subseteq \mathcal{N} \times \mathcal{N}$ is the fixed edge set. It is considered that $\mathcal{Q}$ is connected and $\mathcal{N}_i \subset \{1, ..., N\} \backslash \{i\}$ is the non-empty subset of agents that agent i can sense and interact with. The Laplacian matrix $\mathcal{L}$ is defined as:

$$\mathcal{L} = \mathcal{D} - \mathcal{A} \quad \in \mathbb{R}^{N \times N} \tag{2.1}$$

where $\mathcal{D} \in \mathbb{R}^{N \times N}$ is the degree diagonal matrix and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the adjacency matrix defined by $a_{ij} > 0$ when the i^{th} agent can receive information from the j^{th} agent and $a_{ij} = 0$ otherwise.

Let us denote by $\mathcal{L}_i \in \mathbb{R}^{(N-1)\times(N-1)}$ the Laplacian matrix $\mathcal{L}$ defined without agent i, and by:

$$\mathcal{L}^i = \mathrm{diag}(\ell_1^i, \ldots, \ell_{i-1}^i, \ell_{i+1}^i, \ldots, \ell_N^i) \in \mathbb{R}^{(N-1)\times(N-1)}$$

the associated diagonal matrix defining the interconnections between agent i and the remaining agents, $\ell_k^i > 0$ if the information of agent i is accessible by the k^{th} agent, otherwise $\ell_k^i = 0$.

Example 2.1 *Let us consider a team of 4 agents. The communication among each agent is given according to the topology in Fig. 2.4, the Laplacian matrix is as follows*

$$\mathcal{L}^1 = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

The associated Laplacian sub-matrices as specified in the above are given as

$$\begin{cases} \mathcal{L}^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathcal{L}^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathcal{L}^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathcal{L}^4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ \mathcal{L}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \mathcal{L}_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \mathcal{L}_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ \mathcal{L}_4 = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}. \end{cases} \tag{2.2}$$

The following assumption in considered throughout this Chapter.

Assumption 2.2.1 *In this work, communication links are assumed to be bidirectional. This means that the considered graph is undirected [Fax & Murray 2004], i.e., the adjacency matrix $\mathcal{A}$ is symmetric.*

Useful lemmas

Let us recall some complementary key lemmas that will be used throughout this Chapter.

Lemma 2.1 *[Aldana-López et al. 2019] Let $n \in N$. If $a = (a_1, \ldots, a_n)$ is a sequence of positive numbers, then the following inequality is satisfied for $\alpha, \eta, p, q, k > 0$ with $pk < 1$ and $qk > 1$*

$$\frac{1}{n}\sum_{i=1}^{N} a_i \left(\alpha a_i^p + \eta a_i^q\right)^k \geqslant \left(\frac{1}{n}\sum_{i=1}^{N} a_i\right)\left(\alpha\left(\frac{1}{n}\sum_{i=1}^{N} a_i\right)^p + \eta\left(\frac{1}{n}\sum_{i=1}^{N} a_i\right)^q\right)^k$$

Lemma 2.2 *[Basile & Marro 1992] Let $z = [z_1, \ldots, z_N]^T \in \mathbb{R}^N$ and*

$$\|z\|_p = \left(\sum_{i=1}^{N} |z_i|^p\right)^{\frac{1}{p}}.$$

Then, for all $l \geqslant r$

$$\|z\|_l \leqslant \|z\|_r.$$

Lemma 2.3 *[Aldana-López et al. 2019] Let*

$$f(z) = z\left(\alpha z^p + \eta z^q\right)^k$$

for $\alpha, \eta, p, q, k > 0$ with $pk < 1$ and $qk > 1$. Then $f(z)$ is monotonically increasing for all $z > 0$.

2.3 Actuator FDI in MASs with Linear Dynamics

2.3.1 Problem Formulation

Consider N agents interacting to achieve a common objective, indexed $i = 1, 2, \ldots, N$. The dynamics of the i^{th} agent are given by:

$$\begin{cases} \dot{\xi}_i(t) = A\xi_i(t) + B_u(u_i(x) + f_i(t)) + B_e d_i(t) \\ y_i(t) = C\xi_i(t) \end{cases} \tag{2.3}$$

where $\xi_i = [\xi_{i,1}, \xi_{i,2}, ..., \xi_{i,n}]^T \in \mathbb{R}^n$ and $u_i(x) \in \mathbb{R}$ are the state vector and the control input respectively of the i^{th} agent. Note that the control and FDI problems are independent and $u_i(x) = P_i(\xi_i, \{\xi_j\}_{j \in \mathcal{N}_i})$, where P_i is the control protocol of agent i and is a function of its state along with information received by their neighbours. $d_i \in \mathbb{R}$ represents the disturbances and is an unknown bounded scalar, with B_e as its distribution matrix and $B_e d_i = [\zeta_{i,1}, \zeta_{i,2}, ..., \zeta_{i,n}]^T$. $f_i \in \mathbb{R}$ represents the fault signal where $f_i = 0$ is equivalent to a fault-free situation and

$f_i \neq 0$ indicates the occurrence of an actuator fault in agent i. Here, it is assumed that agent i can measure all of its state. The state matrices A and B_u are expressed as:

$$A = \begin{bmatrix} 0 & \bar{a}_{1,2} & \bar{a}_{1,3} & \dots & \bar{a}_{1,n} \\ 0 & 0 & \bar{a}_{2,3} & \dots & \bar{a}_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \bar{a}_{n-1,n} \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \in \mathbb{R}^{n\times n}, \quad B_u = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^n$$

$y_i(t) \in \mathbb{R}^n$ is the measured output of agent i, with $C = I \in \mathbb{R}^{n\times n}$. Each agent receives its neighbours' estimates of other agents as well as its own measurements. Many systems can be modelled using the upper triangular form described above, such as omnidirectional mobile robots, UAV, power network systems, etc.

Assumption 2.3.1 *It is assumed that $B_e \in \mathbb{R}^n$ is a known matrix with rank(B_e, B_u) $\neq 0$.*

Given the graph described in the introduction Section, the objective in this Section is the design of a FDI scheme such that each agent is able to detect an actuator fault in the entire network of agents with disturbances solely by using information exchanged between neighbouring agents. Figure 2.3 depicts the proposed scheme.

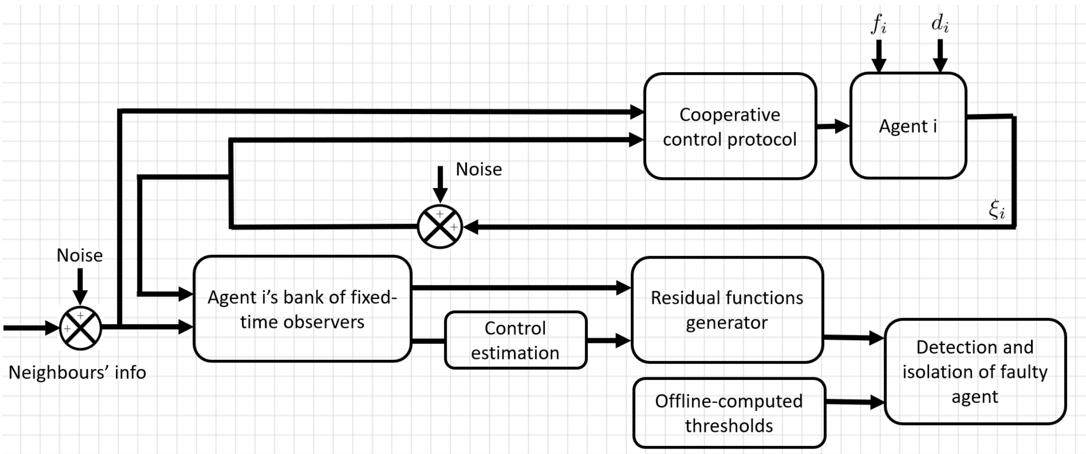


Figure 2.3: Proposed FDI scheme.

2.3.2 Main Result

The distributed FDI scheme is carried out through the use of a cascade of fixed-time observers to reconstruct the global state of the overall system.

2.3.2.1 Global State Reconstruction

The observer proposed here, for each agent i is able to obtain an estimate of all other agents in the network. Denoting by $\hat{\xi}^k_{i,m}$, agent k's estimate of the m^{th} state variable of agent i and by $\xi_{i,m}$ the m^{th} measured state variable of agent i. For all $i = 1, \ldots, N$, $k = 1, \ldots, N$, $k \neq i$ the proposed distributed fixed-time observer has the following structure:

$$\begin{aligned}\dot{\hat{\xi}}^k_{i,m} &= \bar{a}_{m,m+1}\hat{\xi}^k_{i,m+1} + \ldots + \bar{a}_{m,n}\hat{\xi}^k_{i,n} + \alpha_{i,m}\mathrm{sign}\Big(\sum_{j=1}^{N} a_{ij}(\hat{\xi}^j_{i,m} - \hat{\xi}^k_{i,m}) + \ell^i_k(\xi_{i,m} - \hat{\xi}^k_{i,m})\Big)\\ &+ \eta_{i,m}\Big\lceil \sum_{j=1}^{N} a_{ij}(\hat{\xi}^j_{i,m} - \hat{\xi}^k_{i,m}) + \ell^i_k(\xi_{i,m} - \hat{\xi}^k_{i,m})\Big\rfloor^2,\\ & m = \{1, 2, ..., n-1\}\end{aligned} \tag{2.4}$$

$$\begin{aligned}\dot{\hat{\xi}}^k_{i,n} &= \alpha_{i,n}\mathrm{sign}\Big(\sum_{j=1}^{N} a_{ij}(\hat{\xi}^j_{i,n} - \hat{\xi}^k_{i,n}) + \ell^i_k(\xi_{i,n} - \hat{\xi}^k_{i,n})\Big)\\ &+ \eta_{i,n}\Big\lceil \sum_{j=1}^{N} a_{ij}(\hat{\xi}^j_{i,n} - \hat{\xi}^k_{i,n}) + \ell^i_k(\xi_{i,n} - \hat{\xi}^k_{i,n})\Big\rfloor^2\end{aligned}$$

The error is defined by:

$$\varepsilon^k_{i,m} = \hat{\xi}^k_{i,m} - \xi_{i,m} \tag{2.5}$$

Differentiating equation (2.5) yields the dynamics of the observation error:

$$\begin{aligned}\dot{\varepsilon}^k_{i,m} &= \bar{a}_{m,m+1}\varepsilon^k_{i,m+1} + \ldots + \bar{a}_{m,n}\varepsilon^k_{i,n} + \alpha_{i,m}\mathrm{sign}\Big(\sum_{j=1}^{N} a_{ij}(\varepsilon^j_{i,m} - \varepsilon^k_{i,m}) - \ell^i_k\varepsilon^k_{i,m}\Big)\\ &+ \eta_{i,m}\Big\lceil \sum_{j=1}^{N} a_{ij}(\varepsilon^j_{i,m} - \varepsilon^k_{i,m}) - \ell^i_k\varepsilon^k_{i,m}\Big\rfloor^2 - \zeta_{i,m},\\ & m = \{1, 2, ..., n-1\}\end{aligned} \tag{2.6}$$

$$\begin{aligned}\dot{\varepsilon}^k_{i,n} &= \alpha_{i,n}\mathrm{sign}\Big(\sum_{j=1}^{N} a_{ij}(\varepsilon^j_{i,n} - \varepsilon^k_{i,n}) - \ell^i_k\varepsilon^k_{i,n}\Big) + \eta_{i,n}\Big\lceil \sum_{j=1}^{N} a_{ij}(\varepsilon^j_{i,n} - \varepsilon^k_{i,n}) - \ell^i_k\varepsilon^k_{i,n}\Big\rfloor^2\\ &- U_i\end{aligned}$$

with $U_i = u_i + \zeta_{i,n} + f_i$.

Assumption 2.3.2 *It is assumed that inputs u_i of each agent, disturbances $\zeta_{i,m}$ with $m = 1, 2, ..., n$ and faults f_i are bounded by known constants such that $\forall i \in \{1, 2, ..., N\}$, $|u_i| \leqslant u_{max}$, $|\zeta_{i,m}| \leqslant \zeta_{max}$ and $|f_i| \leqslant f_{max}$ respectively with $u_{max} \in \mathbb{R}^+$, $\zeta_{max} \in \mathbb{R}^+$, $f_{max} \in \mathbb{R}^+$, and $U_{max} = u_{max} + \zeta_{max} + f_{max}$.*

Equations (2.6) can be rewritten in a more compact form:

$$\begin{aligned}\dot{\mathcal{E}}_{i,m} = {} & \bar{a}_{m,m+1}\mathcal{E}_{i,m+1} + \ldots + \bar{a}_{m,n}\mathcal{E}_{i,n} \\ & - \alpha_{i,m}\text{sign}((\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,m}) - \eta_{i,m}\lceil(\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,m}\rfloor^2 \\ & - \mathbf{1}\zeta_{i,m}, \qquad m = \{1, 2, ..., n-1\}\end{aligned} \tag{2.7}$$

$$\dot{\mathcal{E}}_{i,n} = -\alpha_{i,n}\text{sign}((\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,n}) - \eta_{i,n}\lceil(\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,n}\rfloor^2 - \mathbf{1}U_i$$

Each agent i concatenates the estimation errors in the vector $\mathcal{E}_{i,m}$ i.e. $\mathcal{E}_{i,m} = [\varepsilon^1_{i,m}, \varepsilon^2_{i,m}, ..., \varepsilon^N_{i,m}]^T$. The fixed-time convergence property of the estimation errors is summarized in the following theorem:

Theorem 2.1: [Taoufik *et al.* 2020c]

Considering Assumptions 2.3.1-2.3.2 are satisfied with $i = 1, 2, ..., N$, the observer gains are expressed as:

$$\eta_{i,m} = \frac{\sigma_i\sqrt{N}}{(2\lambda_{min}(\mathcal{L}_i + \mathcal{L}^i))^{\frac{3}{2}}} \qquad \forall m = 1, 2, ..., n \tag{2.8}$$

$$\alpha_{i,m} = \zeta_{max} + \sigma_i\sqrt{\frac{\lambda_{max}(\mathcal{L}_i + \mathcal{L}^i)}{2\lambda_{min}(\mathcal{L}_i + \mathcal{L}^i)}} \qquad \forall m = 1, 2, ..., n-1 \tag{2.9}$$

$$\alpha_{i,n} = U_{max} + \sigma_i\sqrt{\frac{\lambda_{max}(\mathcal{L}_i + \mathcal{L}^i)}{2\lambda_{min}(\mathcal{L}_i + \mathcal{L}^i)}} \tag{2.10}$$

The distributed observer described by Eq. (2.4) achieves the convergence of the observation errors to zero in a finite time, where $\sigma_i > 0$, and this time is bounded by:

$$T^i_o := \frac{n\pi}{\sigma_i} \tag{2.11}$$

Proof The proof consists of showing the fixed-time stability of (2.7) in a recursive manner. For that, consider the following Lyapunov function associated with the n^{th} dynamics of the agents:

$$V^i_n = \frac{1}{2}(\mathcal{E}_{i,n})^T(\mathcal{L}_i + \mathcal{L}^i)(\mathcal{E}_{i,n}) \tag{2.12}$$

Differentiating (2.12) results in:

$$\begin{aligned}\dot{V}^i_n = {} & (\mathcal{E}_{i,n})^T(\mathcal{L}_i + \mathcal{L}^i) - (\mathcal{E}_{i,n})^T(\mathcal{L}_i + \mathcal{L}^i)\mathbf{1}U_i \\ & \times(-\alpha_{i,n}\text{sign}((\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,n}) - \eta_{i,n}\lceil(\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,n}\rfloor^2) \\ & \leqslant -(\alpha_{i,n} - U_{max})||(\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,n}||_1 \\ & - \eta_{i,n}N^{-\frac{1}{2}}(2\lambda_{min}(\mathcal{L}_i + \mathcal{L}^i))^{\frac{3}{2}}(V^i_n)^{\frac{3}{2}} \\ & \leqslant -\sigma_i(V^i_n)^{\frac{1}{2}} - \sigma_i(V^i_n)^{\frac{3}{2}}\end{aligned} \tag{2.13}$$

This guarantees that $\mathcal{E}_{i,n}$ containing the n^{th} state estimation errors is fixed-time stable at the origin with the settling time bounded by $T_1^i = \frac{\pi}{\sigma_i}$, the dynamics of $\mathcal{E}_{i,n-1}$ are written in the form (2.14) after the convergence of $\mathcal{E}_{i,n}$.

$$\begin{aligned}\dot{\mathcal{E}}_{i,n-1} &= -\alpha_{i,n-1}\text{sign}((\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,n-1}) \\ &\quad - \eta_{i,n-1}\lceil(\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,n-1}\rfloor^2 - \zeta_{i,n-1}\end{aligned} \tag{2.14}$$

Similarly, we have :

$$\begin{aligned}\dot{V}_{n-1}^i &\leqslant -(\alpha_{i,n-1} - \zeta_{max})||(\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,n-1}||_1 \\ &\quad - \eta_{i,n-1}N^{-\frac{1}{2}}(2\lambda_{min}(\mathcal{L}_i + \mathcal{L}^i))^{\frac{3}{2}}(V_{n-1}^i)^{\frac{3}{2}} \\ &\leqslant -\sigma_i(V_{n-1}^i)^{\frac{1}{2}} - \sigma_i(V_{n-1}^i)^{\frac{3}{2}}\end{aligned}$$

$\mathcal{E}_{i,n-1}$ converges to 0 in a fixed time bounded by $T_2^i = 2T_1^i$. Recursively, the dynamics of $\mathcal{E}_{i,1}$ reduce to:

$$\dot{\mathcal{E}}_{i,1} = -\alpha_{i,1}\text{sign}((\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,1}) - \eta_{i,1}\lceil(\mathcal{L}_i + \mathcal{L}^i)\mathcal{E}_{i,1}\rfloor^2 - \zeta_{i,1}$$

Following the same reasoning, $\mathcal{E}_{i,1}$ converges to 0 within a fixed-time horizon bounded by $T_o^i := T_n^i = nT_1^i$. This concludes the proof of Theorem 1.

Theorem 2.1 guarantees that this observer could recover the agent i's state within a fixed time by the remaining agents if Assumptions 2.3.1-2.3.2 are satisfied. Therefore $\forall m \in 1, 2, ..., n, \forall i \in 1, 2, ..., N,\ \forall k \in 1, 2, ..., N$, it is safe to use $\hat{x}_{i,m}^k$ in the residual generation and evaluation step after $t = T_o^i$. One could note that the settling estimation time T_o^i, obtained in Theorem 2.1 is independent of the initial observation errors for each corresponding agent.

Remark 2.3.1 *According to Theorem 2.1, the fixed-time observer (2.4) guarantees a perfect estimation of the global system state despite the presence of bounded control inputs, uncertainties, and faults in a fixed-time. Furthermore, it ensures the viability of the residual signals defined in the next Sub-subsection, at a given prescribed-time while avoiding the problem of transient behaviours and avoiding an over-tuning of the switching times. As for the digital implementation of the proposed distributed observers, it is possible to apply some discretisation algorithms [Polyakov et al. 2019, Jiménez-Rodríguez et al. 2020a]. Concerning the effect of measurement noise, one may consider similar reasoning as in [Ménard et al. 2017], where it is shown that a good trade-off between robustness properties to measurement noise and fast convergence is obtained by appropriately tuning the observer gains. Indeed, it can be shown that in the presence of measurement noise, the estimation errors (2.5) due to the disturbances decrease as $\alpha_{i,m}$, $\alpha_{i,n}$ and $\eta_{i,m}$ increase, but at the same time increase due to noise as these parameters increase. Hence, through a reasonable choice of the settling time a good compromise between robustness and a sufficiently fast estimation can be achieved.*

Remark 2.3.2 *It is worth mentioning that conditions (2.8)-(2.10) in Theorem 2.1 provide an explicit way to tune the gains in order to achieve a prescribed settling time, regardless of the initial conditions, the disturbance signals affecting each agent and the input signals. Additionally, physical bounds of perturbations, states, and inputs can be estimated a-priori for any MAS.*

2.3.2.2 Residual Generation and Fault Detection

In the following, an LMI-based approach is used to design a distributed actuator FDI scheme residual generator based on mixed $\mathcal{H}\_/\mathcal{H}_\infty$ norms. The use of this approach rather than the one that involves the reconstruction of the disturbances and faults for this system is supported by the fact that the fault detection scheme is less computationally demanding and the gains are only computed once. Using the proposed distributed observers, each agent can complete the global state by concatenating the estimated states obtained from the observer. For an agent k, the corresponding global state vector can be expressed as $X_{i,g} = [(\hat{\xi}_i^1)^T, (\hat{\xi}_i^2)^T, \ldots, (\hat{\xi}_i^N)^T]^T \in \mathbb{R}^{(n \cdot N)}$, where $\hat{\xi}_i^k$ is the agent i's estimate of the state of agent k previously obtained, where $\hat{\xi}_i^k = [(\hat{\xi}_{k,1}^i)^T, (\hat{\xi}_{k,2}^i)^T, ..., (\hat{\xi}_{k,n}^i)^T]^T$. Using (2.3), the associated global state space representation can be written in the following form:

$$\begin{cases} \dot{X}_{i,g}(t) = \bar{A}_i X_{i,g}(t) + \bar{B}_{u,i}(U_{i,g}(t) + F_{i,g}(t)) + \bar{B}_{e,i} E_{i,g}(t) \\ Y_i(t) = X_{i,g}(t) \end{cases} \tag{2.15}$$

where $E_G^i = [d_1, d_2, ..., d_N]^T$ and $F_{i,g} = [f_1, f_2, ..., f_N]^T$. $U_{i,g}(t) = [u_1, u_2, ..., u_N]^T$ is the concatenated input vector. Matrices $\bar{A}_i$, $\bar{B}_{u,i}$ and $\bar{B}_{e,i}$ are defined by block diagonal notation $\bar{Z}_i = diag(Z, Z, \ldots, Z)$. Differentiating $Y_i(t)$ in (2.15) yields to:

$$\dot{Y}_i(t) = \bar{A}_i X_{i,g}(t) + \bar{B}_{u,i}(U_{i,g}(t) + F_{i,g}(t)) + \bar{B}_{e,i} E_{i,g}(t) \tag{2.16}$$

Let us denote by $Y_{i,g}(t)$ the new output described as:

$$Y_{i,g}(t) = \dot{Y}_i(t) - \bar{B}_{u,i} \hat{U}_{i,g}(X_{i,g}(t)) \tag{2.17}$$

$\hat{U}_{i,g}(X_{i,g}(t)) = [\hat{u}_{i,1}, \hat{u}_{i,2}, ..., \hat{u}_{i,N}]^T$ is the reconstructed input, where $\hat{u}_i = u_i$ since each agent has access to its own control input vector. Equations (2.15) can be re-written as:

$$\begin{cases} \dot{X}_{i,g}(t) = \bar{A}_i X_{i,g}(t) + \bar{B}_{u,i}(\hat{U}_{i,g}(t) + F_{i,g}(t)) + \bar{B}_{e,i} E_{i,g}(t) \\ Y_{i,g}(t) = \bar{A}_i X_{i,g}(t) + \bar{B}_{u,i} F_{i,g}(t) + \bar{B}_{e,i} E_{i,g}(t) \end{cases} \tag{2.18}$$

Remark 2.3.3 *It should be also highlighted that $\dot{Y}_i(t)$ and thus $Y_{i,g}(t)$, can easily be obtained using a robust fixed-time differentiator [Moreno 2021, Aldana-López et al. 2021].*

From system (2.18) and Remark 2.3.3, it is clear that one can apply a mixed $\mathcal{H}\_/\mathcal{H}_\infty$ norm optimization technique. A classical FDI scheme is comprised of a residual generator and a residual evaluation process. To design the residual generator, the following fault detection observer is used:

$$\dot{\hat{X}}_{i,g}(t) = \bar{A}_i \hat{X}_{i,g}(t) + \bar{B}_{u,i} \hat{U}_{i,g}(X_{i,g}(t)) + H_i(Y_{i,g}(t) - \hat{Y}_{i,g}(t)) \tag{2.19}$$

$$\hat{Y}_{i,g}(t) = \bar{A}_i \hat{X}_{i,g}(t) \tag{2.20}$$

$$r_i(t) = V_i(Y_{i,g}(t) - \hat{Y}_{i,g}(t)) \tag{2.21}$$

where for each agent i, $\hat{X}_{i,g} \in \mathbb{R}^{(n\cdot N)}$ and $\hat{Y}_{i,g} \in \mathbb{R}^{(n\cdot N)}$ represent, respectively, the state and output estimation vectors. $r_i \in \mathbb{R}^{(n\cdot N)}$ is the residual signal vector. $H_i \in \mathbb{R}^{(n\cdot N)\times(n\cdot N)}$ and $V_i \in \mathbb{R}^{(n\cdot N)\times(n\cdot N)}$ are the gain matrices representing the observer gain and the residual post filter weight, respectively. Considering the residual error $e_i(t) = X_{i,g} - \hat{X}_{i,g}$, one can write:

$$\dot{e}_i(t) = \underline{A}_i e_i(t) + \underline{B}_{e,i} E_{i,g}(t) + \underline{B}_{u,i} F_{i,g}(t) \tag{2.22}$$

$$r_i(t) = V_i(\bar{A}_i e_i(t) + \bar{B}_{e,i} E_{i,g}(t) + \bar{B}_{u,i} F_{i,g}(t)) \tag{2.23}$$

with $\underline{A}_i = (\bar{A}_i - H_i\bar{A}_i)$, $\underline{B}_{e,i} = (\bar{B}_{e,i} - H_i\bar{B}_{e,i})$ and $\underline{B}_{u,i} = (\bar{B}_{u,i} - H_i\bar{B}_{u,i})$. In the frequency domain, (2.23) can be written as:

$$r_i(s) = T_d^i(s)E_{i,g}(s) + T_f^i(s)F_{i,g}(s) \tag{2.24}$$

with $T_d^i(s) = V_i\bar{A}_i(sI - \underline{A}_i)^{-1}\underline{B}_{e,i} + V_i\bar{B}_{e,i}$ and $T_f^i(s) = V_i\bar{A}_i(sI - \underline{A}_i)^{-1}\underline{B}_{u,i} + V_i\bar{B}_{u,i}$. Here, the $\mathcal{H}_\infty$ norm is used to measure the maximum effect of the disturbances on the residual and the $\mathcal{H}\_$ index to measure the minimum effect of the fault on the residual. For system (2.18), designing the fault detection observer (2.19)-(2.21) is equivalent to find matrices H_i and V_i, using the combined $\mathcal{H}\_/\mathcal{H}_\infty$ strategy to guarantee sensitivity of the residuals to the faults and robustness against perturbation. Therefore, the objective is to minimize the following:

$$\min J_i = \min \frac{||T_d^i(s)||_\infty}{||T_f^i(s)||_-} \tag{2.25}$$

where J_i represents a trade-off between sensitivity and robustness. In this section, the approach in [Wang *et al.* 2007] is used. For given $\gamma_i > 0$ and $\beta_i > 0$, the error system (2.21)-(2.22) is asymptotically stable and the following are satisfied:

$$||T_d^i(s)||_\infty < \gamma_i \tag{2.26}$$

$$||T_f^i(s)||_- > \beta_i \tag{2.27}$$

if there exist matrices $Q_i > 0$, $M_i > 0$ and H_i such that the following LMIs hold:

$$\begin{bmatrix} Q_i\underline{A}_i + (\underline{A}_i)^T Q_i + (\bar{A}_i)^T M_i \bar{A}_i & Q_i\underline{B}_{e,i} + (\bar{A}_i)^T M_i \bar{B}_{e,i} \\ * & -\gamma_i^2 I + (\bar{B}_{e,i})^T M_i \bar{B}_{e,i} \end{bmatrix} < 0 \tag{2.28}$$

$$\begin{bmatrix} Q_i\underline{A}_i + (\underline{A}_i)^T Q_i - (\bar{A}_i)^T M_i \bar{A}_i & (\bar{A}_i)^T M_i \bar{B}_{u,i} - P^i\underline{B}_{u,i} \\ * & \beta_i^2 I - (\bar{B}_{u,i})^T M_i \bar{B}_{u,i} \end{bmatrix} < 0 \tag{2.29}$$

with $M_i = (V_i)^T V_i$.

2.3.2.3 Residual Evaluation

The remaining task for fault detection and isolation (identifying the faulty agent in the fleet) is to evaluate the obtained residuals, when the convergence of the estimation errors is obtained (which depends on the parameters of the cascade of fixed-time observers as well as those of the fixed-time differentiator). The residuals are evaluated after this settling time, by comparing the generated residuals with a threshold defined hereafter.

Given Remark 2.3.1, when discrete-time implementation and measurement noise are considered, the residual r_i does not exactly equal to zero in the absence of fault. Hence, once the residual signal is generated, it is important to define an evaluation function. Amongst some standard evaluation functions presented in [Ding 2008], the following Root Mean Square (RMS) evaluation function is chosen. The selected evaluation function $J_i^k(t)$ generated by agent i is expressed as follows:

$$J_i^k(t) = \|r_i^k(t)\|_{\mathrm{RMS}} = \left(\frac{1}{T_w} \int_t^{t+T_w} ||r_i^k(\tau)||^2 d\tau \right)^{\frac{1}{2}} \tag{2.30}$$

where $r_i = [(r_i^1)^T, (r_i^2)^T, ..., (r_i^N)^T]^T$ and T_w is the finite-time evaluation window. Let us denote by $J_i^{k_{th}} = \sup_{faultfree} \|J_i^k(t)\|_{\mathrm{RMS}}$ the threshold. The following is used as a decision logic:

$$J_i^k(t) > J_i^{k_{th}} \Rightarrow \text{A fault has occured at agent k,} \tag{2.31}$$

$$J_i^k(t) \leqslant J_i^{k_{th}} \Rightarrow \text{No fault has occured.} \tag{2.32}$$

Remark 2.3.4 *It should be mentioned that the distributed structure of the proposed scheme implies that each agent can estimate faults based on its states and the information exchanged between neighbouring agents, making it more adequate when there are measurement noises in the sensors, or perturbations in the communication links. Indeed, since the fault detection scheme is based not only on the state of the agent but also on the mergers' information from neighbouring agents:* (i) *a consensus on the occurrence of a fault is implicitly required for a fault to be detected, and* (ii) *the proposed scheme acts as a filter, which enables the reduction of the effect of measurement noises due to sensors while providing an accurate estimate of the global system state. Since only information between neighbouring agents is shared, the effect of perturbations due to the communication links is also reduced.*

2.3.3 Simulation Example

An illustrative numerical example is provided herein to show the effectiveness of the proposed scheme in this section. Consider a team of $N = 4$ agents governed by double integrator dynamics with unknown disturbances ($n = 2$), a special case of system (2.3) such that $\xi_{i,1}$ represents the position of the i^{th} robot, and $\xi_{i,2}$ its velocity, where:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_e = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, B_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The communication among each agent is given according to the topology in Fig. 2.4.

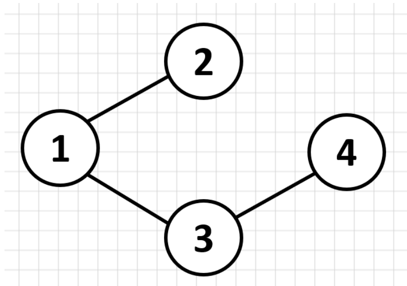


Figure 2.4: Communication topology.

The associated interconnection matrices as specified by the given topology are given by (2.2).

In this example, a consensus algorithm in the presence of a group reference velocity is used for each agent (with the velocity reference set to $v_d = 10m/s$ for each agent).

$$u_i = \dot{v}_d - \kappa(v_d - \xi_{i,2}) - \sum_{j=1}^{4} a_{ij}[(\xi_{i,1} - \xi_{j,1}) - \nu(\xi_{i,2} - \xi_{j,2})]$$

where κ and ν are the consensus gains set to 5 and 2.5 respectively. To check the robustness of the proposed scheme, two types of perturbations are considered: band-limited Gaussian white noise of power 0.005 for agents 2 and 4, and high frequency noise in the form of $d_2(t)$ and $d_4(t)$, for agents 1 and 3 that are modelled as follows: $d_1(t) = 3.5\sin(150t)$ and $d_3(t) = (2.5\sin(350t))^{0.2}$. Additionally, additive band-limited white measurement noise of power 10^{-8} has been added to all incoming and outgoing edges and measurements. This means that transmitted information between neighbouring agents has been affected by these communication noises. In order to show the effectiveness of our approach, multiple types of abrupt faults are considered, a ramp $f_1(t) = t - 5$ for $t \in [5, 15]$, two rectangular faults $f_2(t)$ and $f_4(t)$, and an exponential fault $f_3(t) = -e^{1.3-0.1/(t-10)}$ are added at various instants, and with different amplitudes. They are assumed to occur in agents 1, 2, 3 and 4 respectively as shown in Fig. 2.5. Using these matrices and, without loss of generality, by choosing $T_o^1 = T_o^2 = T_o^3 = T_o^4 = 0.5\,s$, from Theorem 2.1, the resulting σ_1, σ_2, σ_3 and σ_4 allow us to define the gains of the cascade observers expressed in Eq. (2.8)-(2.10):

$$\begin{cases} \alpha_{1,1} = 23.45, & \alpha_{1,2} = 38.45, \quad \eta_{1,1} = \eta_{1,2} = 14.01. \\ \alpha_{2,1} = 35.16, & \alpha_{2,2} = 50.16, \quad \eta_{2,1} = \eta_{2,2} = 45.30. \\ \alpha_{3,1} = 23.45, & \alpha_{3,2} = 38.45, \quad \eta_{3,1} = \eta_{3,2} = 14.01. \\ \alpha_{4,1} = 35.16, & \alpha_{4,2} = 50.16, \quad \eta_{4,1} = \eta_{4,2} = 45.30. \end{cases}$$

It is worth mentioning that the choice $T_o^1 = T_o^2 = T_o^3 = T_o^4$ allows the observers to

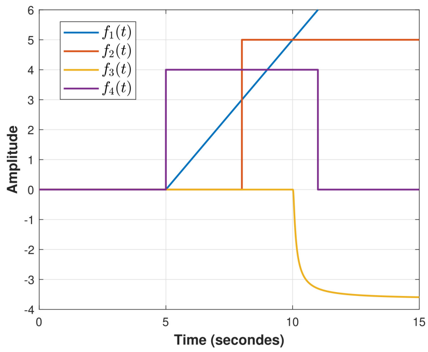


Figure 2.5: Fault signals in agents 1, 2, 3 and 4.

converge at the same time, hence simplifying the residual evaluation process.

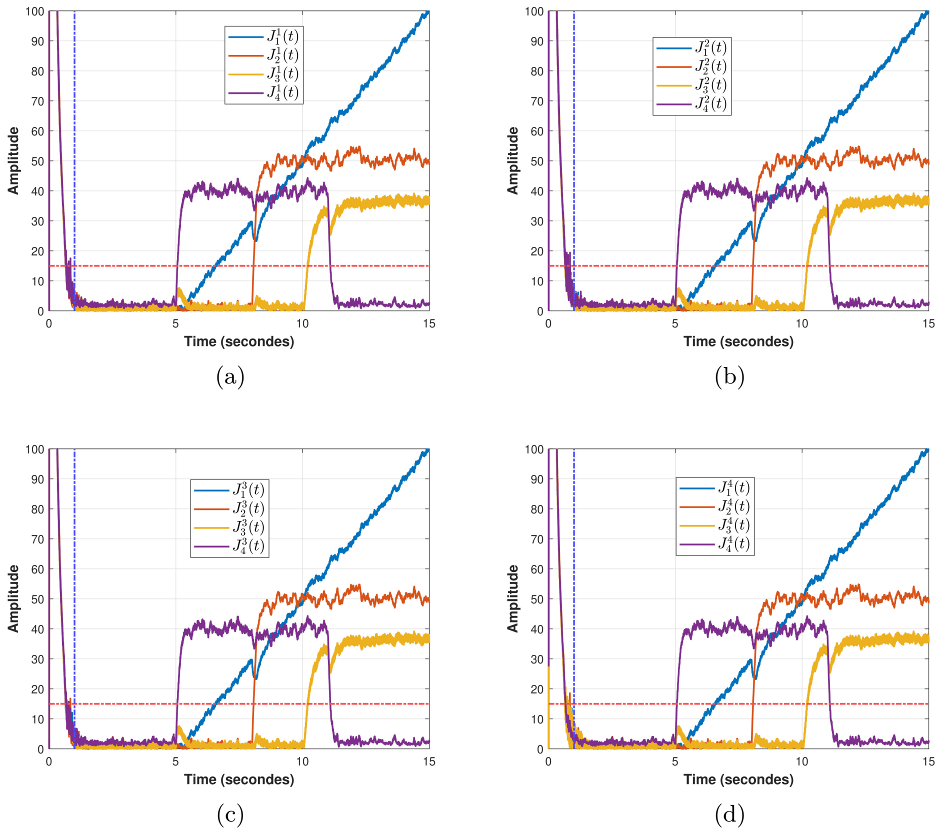


Figure 2.6: Residual evaluation functions: **(a)** $J_i^1(t)$ of agent 1 **(b)** $J_i^2(t)$ of agent 2 **(c)** $J_i^3(t)$ of agent 3 **(d)** $J_i^4(t)$ of agent 4. The dashed blue lines represent the convergence time after which the functions should be considered. The dashed red lines represent the thresholds.

Figure 2.6 shows the evaluation functions $J_i^1(t)$, $J_i^2(t)$, $J_i^3(t)$ and $J_i^4(t)$ of the residual vectors generated by agent 1, 2, 3 and 4 respectively. It is worth noting that these functions make sense only after the observers and differentiators converge. This is achieved after about 1.5 seconds (dashed blue vertical lines are added as delimiters in order to illustrate this). Each agent generates four evaluation functions, one from its own output and three from the estimates of the state of the three other agents. Each of these functions is capable of detecting the corresponding agent's fault from the point of view of the agent that generates it. Fig. 2.6d for example displays the functions $J_1^2(t)$, $J_2^2(t)$, $J_3^2(t)$ and $J_4^2(t)$ that detect faults occurring in agents 1, 2, 3 and 4 respectively from the point of view of agent 2. It is shown that, even though agent 2 does not directly communicate with neither agent 3 and 4, it uses their state estimates to synchronise fault detection. The functions are robust with respect to disturbances and faults can be easily distinguished. In accordance with this logic the same can be said for the other agents as the same pattern for fault detection is followed. These results show that our approach is useful for the problem of synchronised distributed actuator FDI in a network of communicating MASs, where each agent can detect faults occurring across the entirety of the system and isolate the faulty agent.

Remark 2.3.5 *It is worth mentioning that the proposed scheme in this Section requires each node in the network to exchange a large amount of information equivalent to $N \times n$ in order to feed its bank of observers to monitor each of the other agents, resulting in a distributed but potentially computationally heavy scheme. This means that the proposed scheme is more suitable for small scale MASs as the exchange load increases linearly with the number of nodes. However, the proposed approaches in this Chapter are general, and the amount of shared information could be reduced if fewer agents should be monitored. Furthermore, since each agent monitors the activity of the whole network, it can be noticed that a certain amount of redundancy exists, hence, if only a subset of agents act as monitoring units, the computational and exchange burdens could be reduced. It should also be noted that many control applications may benefit from this observer structure, and using it for both control and fault detection can considerably reduce the computational cost.*

2.4 FDI in MASs with Chained Form Dynamics

In this Section, fixed-time observers are used for robust state reconstruction and FDI in homogeneous cooperative MASs with chained form dynamics, where the predefined-time stability notion is used.

2.4.1 Problem Formulation

Consider a homogeneous MAS comprised of N unicycle-type mobile robots (see Fig. 2.7) labelled by $i \in 1, \ldots, N$ with nonholonomic constraints on the linear velocity. These robots have two fixed front wheels with one axis but are independently controlled by two separate DC motors and a rear caster wheel, which prevents the robot from tipping over as it moves on a plane.

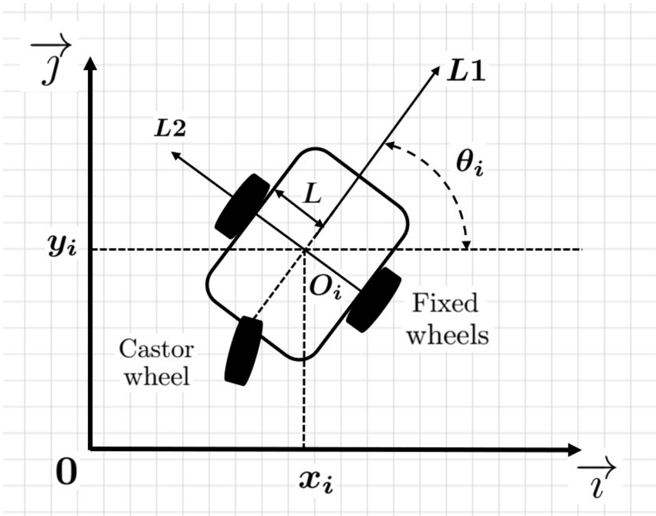


Figure 2.7: Unicycle-type mobile robot.

The nonholonomic constraint is defined by

$$\dot{x}_i(t)\sin(\theta_i(t)) - \dot{y}_i(t)\cos(\theta_i(t)) = 0 \tag{2.33}$$

where (x_i, y_i) denotes the position of the center of mass O_i of the body located in the middle of the two driving wheels, θ_i is the orientation of the robot, i.e., the angle separating the axes $\overrightarrow{i}$ and $L1$. The two wheels are separated by $d = 2L$. Using constraint (2.33), the kinematics of the robot in the presence of uncertainties can be modeled by the following differential equations [Laumond *et al.* 1998]

$$\begin{cases} \dot{x}_i(t) & = v_i(t)\cos(\theta_i(t)) + \varsigma_1\zeta_i(t) \\ \dot{y}_i(t) & = v_i(t)\sin(\theta_i(t)) + \varsigma_2\zeta_i(t) \\ \dot{\theta}_i(t) & = \omega_i(t) + \varsigma_3\zeta_i(t) \end{cases} \tag{2.34}$$

where v_i, ω_i, and ζ_i are the forward velocity, the angular velocity, and the unknown disturbances, respectively, of the i^{th} robot. ς_1, ς_2 and ς_3 are assumed to be known constants.

Let us denote by u_i^1 and u_i^2 the velocity commands of the left and right wheels, respectively, actuated by DC motors and let $f_{i,l}$ and $f_{i,r}$ be additive faults occurring on the left and right wheels respectively, and reflecting abnormal changes in their velocities [Skoundrianos & Tzafestas 2004]. One can express the inputs in terms of the velocities as follows:

$$\begin{cases} u_i^1 = v_{i,l} + f_{i,l} \\ u_i^2 = v_{i,r} + f_{i,r} \end{cases} . \tag{2.35}$$

$v_{i,l}$ and $v_{i,r}$ are the driving velocities of the left and right wheels respectively, which are measured through on-board odometric sensors. For a nonholonomic mobile robot, when slippage occurs between the ground and the wheels, ω_i and v_i are expressed as follows:

$$\begin{cases} \omega_i &= \dfrac{(1-s_1)u_i^1-(1-s_2)u_i^2}{d} = u_i^{1\star}+f_{i,1} \\ v_i &= \dfrac{(1-s_1)u_i^1+(1-s_2)u_i^2}{2} = u_i^{2\star}+f_{i,2} \end{cases} \tag{2.36}$$

where

$$\begin{cases} f_{i,1} &= \dfrac{(1-s_1)f_{i,l}-(1-s_2)f_{i,r}}{d} \\ f_{i,2} &= \dfrac{(1-s_1)f_{i,l}+(1-s_2)f_{i,r}}{2} \\ u_i^{1\star} &= \dfrac{(1-s_1)v_{i,l}-(1-s_2)v_{i,r}}{d} \\ u_i^{2\star} &= \dfrac{(1-s_1)v_{i,l}+(1-s_2)v_{i,r}}{2}, \end{cases} \tag{2.37}$$

s_1 and s_2 are the slip ratios of the right and left wheels respectively. $f_{i,1}$ and $f_{i,2}$ are the fault signals to be detected. It is worth noting that a fault on either wheel manifests itself in both signals.

Remark 2.4.1 *Note that the slip ratios [Lu et al. 2016, Gao et al. 2014] s_1 and s_2, are assumed to be known by agents since the left and right current velocities can be estimated through an adequate localization method. The design of a distributed SLAM method is however not in the scope of this thesis.*

Now consider the following [Kolmanovsky & McClamroch 1995] nonsingular state and input transformation

$$\begin{cases} \xi_{i,1} &= \theta_i \\ \xi_{i,2} &= x_i\sin\theta_i - y_i\cos\theta_i \\ \xi_{i,3} &= x_i\cos\theta_i + y_i\sin\theta_i \\ u_i^{1\star} &= \omega_i - f_{i,1} \\ u_i^{2\star} &= v_i - f_{i,2} \end{cases} \tag{2.38}$$

where $u_i^{1\star}$ and $u_i^{2\star}$ are defined in (2.36). With this change of coordinates, choosing the following feedback

$$\begin{cases} \mathcal{U}_{i,1} = \dot{u}_i^{1\star} \\ \mathcal{U}_{i,2} = u_i^{2\star} - \xi_{i,2}\xi_{i,3} \end{cases} \tag{2.39}$$

and adding a dynamical extension, (2.34) can be transformed into the following uncertain chained form system

$$\begin{cases} \dot{\xi}_{i,1} = \xi_{i,2} + \varsigma_3\zeta_i \\ \dot{\xi}_{i,2} = \mathcal{U}_{i,1} + \dot{f}_{i,1} \\ \dot{\xi}_{i,3} = \xi_{i,4}\xi_{i,2} + \zeta_i\left(\varsigma_1\sin\left(\xi_{i,1}\right) - \varsigma_2\cos\left(\xi_{i,1}\right) + \varsigma_3\xi_{i,4}\right) \\ \dot{\xi}_{i,4} = \mathcal{U}_{i,2} + f_{i,2} + \zeta_i\left(\varsigma_1\sin\left(\xi_{i,1}\right) - \varsigma_2\cos\left(\xi_{i,1}\right) + \varsigma_3\xi_{i,3}\right) \end{cases} \tag{2.40}$$

Note that for each agent i, $\mathcal{U}_{i,1}$ and $\mathcal{U}_{i,2}$ represent any cooperative feedback controller which uses exchanged information between neighbouring agents (i.e. [Sánchez-Torres *et al.* 2019, Defoort *et al.* 2016, Anggraeni *et al.* 2019]), i.e., they depend on $\Psi_i = [\xi_{i,1}, \xi_{i,2}, \xi_{i,3}, \xi_{i,4}]^T \in \mathbb{R}^4$ and $\Psi_j = [\xi_{j,1}, \xi_{j,2}, \xi_{j,3}, \xi_{j,4}]^T \in \mathbb{R}^4$, where $j \in \mathcal{N}_i = \{k : a_{ik} > 0\}$.

Remark 2.4.2 *The use of a dynamical extension is justified by the fact that almost all mobile robots can compute the angular velocity and acceleration through odometric sensors, which can be used in the residual generation scheme elaborated in this Section, as well as in many control strategies [Anggraeni et al. 2019].*

Assumption 2.4.1 *The inputs $\mathcal{U}_{i,1}$, $\mathcal{U}_{i,2}$ and uncertainties ζ_i of each agent, are assumed to be bounded by known constants $u_1^{max}, u_2^{max}, \zeta^{max} \in \mathbb{R}^+$, where $|\mathcal{U}_{i,1}| \leqslant u_1^{max}$, $|\mathcal{U}_{i,2}| \leqslant u_2^{max}$ and $|\zeta_i| \leqslant \zeta^{max}$. Additionally, for all agents, the left and right wheel faults $f_{i,l}$, $f_{i,r}$ and their first derivatives have known constant bounds such that $|f_{i,l}| \leqslant f_l^{max}$, $|f_{i,r}| \leqslant f_r^{max}$ and $|\dot{f}_{i,1}| \leqslant \dot{f}_1^{max}$, with $f_l^{max}, f_r^{max}, \dot{f}_1^{max} \in \mathbb{R}^+$.*

The purpose here is to show that a monitoring agent can detect an actuator fault in the entire network of agents despite the presence of unknown bounded disturbances, using a distributed scheme, i.e., solely by using its state and information exchanged with its neighbouring agents.

Hereafter, the proposed distributed global actuator fault detection design is laid out through the use of a cascade of predefined-time observers for the sake of residual generation. Indeed, the problem of global residual generation is to design a filter for each agent $i = \{1, 2, \ldots, N\}$, of the form

$$\begin{cases} \dot{\hat{\chi}}_i^k(t) &= f_r\left(t, \hat{\mathcal{U}}_{i,1}^k, \hat{\mathcal{U}}_{i,2}^k\right) + g_r\left(t, \hat{\Psi}_i^k, \hat{\zeta}_i^k\right) \\ r_i^k(t) &= h_r\left(t, \chi_i^k\right) \end{cases} \tag{2.41}$$

where

- $\hat{\chi}_i^k \in \mathbb{R}$ is agent k's estimate of agent i's filter variable, affected by both faults,
- $\hat{\Psi}_i^k = \left[\hat{\xi}_{i,1}^k(t), \hat{\xi}_{i,2}^k(t), \hat{\xi}_{i,3}^k(t), \hat{\xi}_{i,4}^k(t)\right]^T \in \mathbb{R}^4$ represents agent k's estimate of agent i's state,
- $\hat{\zeta}_i^k \in \mathbb{R}$ represents agent k's estimate of agent i's disturbances,
- $\hat{\mathcal{U}}_{i,1}^k$, $\hat{\mathcal{U}}_{i,2}^k \in \mathbb{R}$ represent agent k's estimate of agent i's control inputs,
- $r_i^k(t) \in \mathbb{R}$ represents agent k's estimate of agent i's residual signal containing information on the time and location of the fault occurrence. It should be close to zero in the absence of fault and become distinctly different from zero in the presence of a fault.

The proposed FDI scheme is described in Fig 2.8. First, the distributed predefined-time observers, described in Subsection 2.4.2, are used to estimate the state for each other agent in the fleet (i.e. $\hat{\chi}_i^k$). Then, the equivalent control concept is used

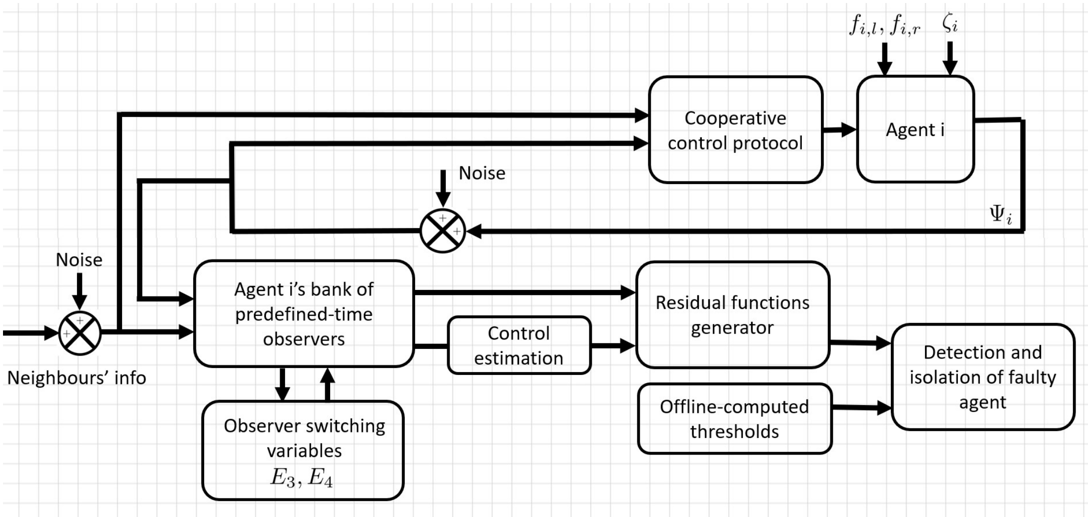


Figure 2.8: Proposed FDI scheme.

to estimate the disturbances $\hat{\zeta}_i^k$. From these estimated values, the monitoring agent is able to reconstruct the applied control input. At last, using the estimated values, a residual signal (2.41) can be derived in Subsection 2.4.2.2.

2.4.2 Main Result

2.4.2.1 State Reconstruction and Disturbance Estimation

The residual signals (2.41) are dependant on the estimates of the state variables, the disturbances, and the control inputs. They need to indicate the faults occurring not only on the robot and its neighbours but also on the MRS as a whole. To achieve this objective, the observer proposed herein, for each agent, i can obtain an estimate of all other agents' state variables, disturbances, and control inputs.

For this aim, the dynamics of the i^{th} robot (2.40) are divided into two subsystems, a disturbed double integrator subsystem $\Sigma_{i,1}$ and a second-order subsystem $\Sigma_{i,2}$:

$$\Sigma_{i,1} : \begin{cases} \dot{\xi}_{i,1} = \xi_{i,2} + \varsigma_3 \zeta_i \\ \dot{\xi}_{i,2} = \mathcal{U}_{i,1} + \dot{f}_{i,1} \end{cases}$$

$$\Sigma_{i,2} : \begin{cases} \dot{\xi}_{i,3} = \xi_{i,4}\xi_{i,2} + \zeta_i \left(\varsigma_1 \sin\left(\xi_{i,1}\right) - \varsigma_2 \cos\left(\xi_{i,1}\right) + \varsigma_3 \xi_{i,4}\right) \\ \dot{\xi}_{i,4} = \mathcal{U}_{i,2} + f_{i,2} + \zeta_i \left(\varsigma_1 \sin\left(\xi_{i,1}\right) - \varsigma_2 \cos\left(\xi_{i,1}\right) + \varsigma_3 \xi_{i,3}\right) \end{cases} . \tag{2.42}$$

Denote by $\hat{\xi}_{i,m}^k$, agent k's estimate of the m^{th} state variable of agent i and by $\xi_{i,m}$ the m^{th} measured state variable of agent i, and by $\hat{\zeta}_i^k$ agent k's estimate of agent i's disturbance signal, for all $m = \{1, 2, 3, 4\}$, $i = \{1, \ldots, N\}$ and $k = \{1, \ldots, N\}$. The proposed distributed predefined-time observer has the following structure:

$$\hat{\Sigma}_{i,1}^k : \begin{cases} \dot{\hat{\xi}}_{i,1}^k = \hat{\xi}_{i,2}^k + \kappa_1^k \left(\left(\alpha |\mathcal{I}_{i,1}^k|^p + \eta |\mathcal{I}_{i,1}^k|^q \right)^r + \delta_1 \right) \operatorname{sign}(\mathcal{I}_{i,1}^k) \\ \dot{\hat{\xi}}_{i,2}^k = \kappa_2^k \left(\left(\alpha |\mathcal{I}_{i,2}^k|^p + \eta |\mathcal{I}_{i,2}^k|^q \right)^r + \delta_2 \right) \operatorname{sign}(\mathcal{I}_{i,2}^k) \end{cases}$$

$$\hat{\Sigma}_{i,2}^k : \begin{cases} \dot{\hat{\xi}}_{i,3}^k & = E_3 \left[\hat{\xi}_{i,2}^k \hat{\xi}_{i,4}^k + \hat{\zeta}_i^k (\varsigma_1 \sin \hat{\xi}_{i,1}^k - \varsigma_2 \cos \hat{\xi}_{i,1}^k + \varsigma_3 \hat{\xi}_{i,4}^k) \right. \\ & \quad \left. + \kappa_3^k \left(\left(\alpha |\mathcal{I}_{i,3}^k|^p + \eta |\mathcal{I}_{i,3}^k|^q \right)^r + \delta_3 \right) \operatorname{sign}\left(\mathcal{I}_{i,3}^k \right) \right] \\ \dot{\hat{\xi}}_{i,4}^k & = E_4 \left[\hat{\zeta}_i^k (\varsigma_1 \sin \hat{\xi}_{i,1}^k - \varsigma_2 \cos \hat{\xi}_{i,1}^k + \varsigma_3 \hat{\xi}_{i,3}^k) \right. \\ & \quad \left. + \kappa_4^k \left(\left(\alpha |\mathcal{I}_{i,4}^k|^p + \eta |\mathcal{I}_{i,4}^k|^q \right)^r + \delta_4 \right) \operatorname{sign}\left(\mathcal{I}_{i,4}^k \right) \right] \end{cases} \tag{2.43}$$

with $\mathcal{I}_{i,m}^k = \sum_{j=1}^N a_{kj} \left(\hat{\xi}_{i,m}^j - \hat{\xi}_{i,m}^k \right) + \ell_k^i \left(\xi_{i,m} - \hat{\xi}_{i,m}^k \right)$. The constants κ_1^k, κ_2^k, κ_3^k, κ_4^k, α, η, δ_1, δ_2, δ_3, δ_4, p, q, r will be defined later. The components E_3 and E_4, will be defined later and represent observer switches.

Assumption 2.4.2 *The robot is considered to be bounded input bounded state stable.*

The error is defined as

$$\varepsilon_{i,m}^k = \xi_{i,m} - \hat{\xi}_{i,m}^k. \tag{2.44}$$

Differentiating (2.44) gives the observation error dynamics for the two subsystems $\Sigma_{i,1}^k$ and $\Sigma_{i,2}^k$ as follows:

$$\Sigma_{i,1}^k : \begin{cases} \dot{\varepsilon}_{i,1}^k = \varepsilon_{i,2}^k - \kappa_1^k \left(\left(\alpha |\mathcal{C}_{i,1}^k|^p + \eta |\mathcal{C}_{i,1}^k|^q \right)^r + \delta_1 \right) \operatorname{sign}\left(\mathcal{C}_{i,1}^k \right) + \varsigma_3 \zeta_i \\ \dot{\varepsilon}_{i,2}^k = -\kappa_2^k \left(\left(\alpha |\mathcal{C}_{i,2}^k|^p + \eta |\mathcal{C}_{i,2}^k|^q \right)^r + \delta_2 \right) \operatorname{sign}\left(\mathcal{C}_{i,2}^k \right) + \mathcal{U}_{i,1} + \dot{f}_{i,1} \end{cases}$$

$$\Sigma_{i,2}^k : \begin{cases} \dot{\varepsilon}_{i,3}^k & = -E_3 \hat{\xi}_{i,2}^k \hat{\xi}_{i,4}^k + \xi_{i,2}^k \xi_{i,4}^k - E_3 \hat{\zeta}_i^k (\varsigma_1 \sin\left(\hat{\xi}_{i,1}^k \right) - \varsigma_2 \cos\left(\hat{\xi}_{i,1}^k \right) + \varsigma_3 \hat{\xi}_{i,4}^k) \\ & \quad + \zeta_i^k \left(\varsigma_1 \sin\left(\xi_{i,1}^k \right) - \varsigma_2 \cos\left(\xi_{i,1}^k \right) + \varsigma_3 \xi_{i,4}^k \right) \\ & \quad - E_3 \kappa_3^k \left(\left(\alpha |\mathcal{C}_{i,3}^k|^p + \eta |\mathcal{C}_{i,3}^k|^q \right)^r + \delta_3 \right) \operatorname{sign}\left(\mathcal{C}_{i,3}^k \right) \\ \dot{\varepsilon}_{i,4}^k & = -E_4 \hat{\zeta}_i^k \left(\varsigma_1 \sin\left(\hat{\xi}_{i,1}^k \right) - \varsigma_2 \cos\left(\hat{\xi}_{i,1}^k \right) + \varsigma_3 \hat{\xi}_{i,3}^k \right) \\ & \quad + \zeta_i^k \left(\varsigma_1 \sin\left(\xi_{i,1}^k \right) - \varsigma_2 \cos\left(\xi_{i,1}^k \right) + \varsigma_3 \xi_{i,3}^k \right) \\ & \quad - E_4 \kappa_4^k \left(\left(\alpha |\mathcal{C}_{i,4}^k|^p + \eta |\mathcal{C}_{i,4}^k|^q \right)^r + \delta_4 \right) \operatorname{sign}\left(\mathcal{C}_{i,4}^k \right) + \mathcal{U}_{i,2} + f_{i,2} \end{cases} \tag{2.45}$$

with $\mathcal{C}_{i,m}^k = \sum_{j=1}^N a_{kj} \left(\varepsilon_{i,m}^k - \varepsilon_{i,m}^j \right) + \ell_k^i \varepsilon_{i,m}^k$.

Putting (2.45) in compact form, one can write

$$\mathcal{Z}_{i,1} : \begin{cases} \dot{\mathcal{E}}_{i,1} & = \mathcal{E}_{i,2} - \mathcal{H}\left(\left(\mathcal{L}_i + \mathcal{L}^i \right) \mathcal{E}_{i,1} \right) + \mathbf{1} \varsigma_3 \zeta_i \\ \dot{\mathcal{E}}_{i,2} & = -\mathcal{H}\left(\left(\mathcal{L}_i + \mathcal{L}^i \right) \mathcal{E}_{i,2} \right) + \mathbf{1} \left(\mathcal{U}_{i,1} + \dot{f}_{i,1} \right) \end{cases}$$

$$\mathcal{Z}_{i,2}:\left\{\begin{array}{ll}\dot{\mathcal{E}}_{i,3} & = -E_3\left(\hat{X}_{i,2}\circ\hat{X}_{i,4}\right)+\mathbf{1}\xi_{i,2}\circ\mathbf{1}\xi_{i,4}-E_3\hat{\Upsilon}_i\circ\left(\varsigma_1\sin\hat{X}_{i,1}-\varsigma_2\cos\hat{X}_{i,1}\right.\\ & \left.+\varsigma_3\hat{X}_{i,4}\right)+\zeta_i\left(\varsigma_1\sin\left(\mathbf{1}\xi_{i,1}\right)-\varsigma_2\cos\left(\mathbf{1}\xi_{i,1}\right)+\varsigma_3\mathbf{1}\xi_{i,4}\right)\\ & -E_3\mathcal{H}\left(\left(\mathcal{L}_i+\mathcal{L}^i\right)\mathcal{E}_{i,3}\right)\\ \dot{\mathcal{E}}_{i,4} & = -E_4\hat{\Upsilon}_i\circ\left(\varsigma_1\sin\left(\hat{X}_{i,1}\right)-\varsigma_2\cos\left(\hat{X}_{i,1}\right)+\varsigma_3\hat{X}_{i,3}\right)+\mathbf{1}\left(\mathcal{U}_{i,2}+f_{i,2}\right)\\ & +\zeta_i\left(\varsigma_1\sin\left(\mathbf{1}\xi_{i,1}\right)-\varsigma_2\cos\left(\mathbf{1}\xi_{i,1}\right)+\varsigma_3\mathbf{1}\xi_{i,3}\right)-E_4\mathcal{H}\left(\left(\mathcal{L}_i+\mathcal{L}^i\right)\mathcal{E}_{i,4}\right)\end{array}\right. \tag{2.46}$$

where $\circ$ represents the Hadamard-Schur product, $\hat{\Upsilon}_i=\left[\hat{\zeta}_i^1,\hat{\zeta}_i^2,\ldots,\hat{\zeta}_i^N\right]^T$ and $\hat{X}_{i,m}=\left[\hat{\xi}_{i,m}^1,\hat{\xi}_{i,m}^2,...,\hat{\xi}_{i,m}^N\right]^T$.

For $m=\{1,2,3,4\}$, the term $\mathcal{H}\left(\left(\mathcal{L}_i+\mathcal{L}^i\right)\mathcal{E}_{i,m}\right)$ is expressed as

$$\begin{aligned}\mathcal{H}\left(\left(\mathcal{L}_i+\mathcal{L}^i\right)\mathcal{E}_{i,m}\right) &= \kappa_m^i\left(\left(\alpha|\left(\mathcal{L}_i+\mathcal{L}^i\right)\mathcal{E}_{i,m}|^p+\eta|\left(\mathcal{L}_i+\mathcal{L}^i\right)\mathcal{E}_{i,m}|^q\right)^r+\delta_m\right)\\ &\quad\times\operatorname{sign}\left(\left(\mathcal{L}_i+\mathcal{L}^i\right)\mathcal{E}_{i,m}\right)\end{aligned}$$

For this case, each agent i concatenates the estimation errors in vector $\mathcal{E}_{i,m}=\left[\varepsilon_{i,m}^1,\varepsilon_{i,m}^2,...,\varepsilon_{i,m}^N\right]^T$.

Theorem 2.2: [Taoufik *et al.* 2021b]

For constant positive definite parameters α, η, p,q and r, which satisfy the constraints $rp<1$ and $rq>1$, and given that assumptions 2.2.1, 2.4.1 and 2.4.2 are satisfied, the predefined-time observer gains are expressed as

$$\begin{array}{ll}\delta_1=\dfrac{\varsigma_3\zeta^{max}}{\kappa_1}, & \delta_2=\dfrac{u_1^{max}+\dot{f}_1^{max}}{\kappa_2},\\ \delta_3=\dfrac{1}{\kappa_3}, & \delta_4=\dfrac{\varsigma_3\xi_3^{max}\zeta^{max}+u_2^{max}+f_2^{max}}{\kappa_4}\end{array} \tag{2.47}$$

with $\kappa_m^i=\dfrac{N\gamma(\phi)}{\lambda_{min}(\mathcal{L}^i+\mathcal{L}_i)T_p^m}$, $\kappa_m=\min\{\kappa_m^1,\ldots,\kappa_m^N\}$ for $m=\{1,2,3,4\}$, and

$$\begin{aligned}E_3 &= \left\{\begin{array}{ll}1 & \text{when} \quad t\geqslant T_p^1+T_p^2+T_p^4\\ 0 & \text{otherwise}\end{array}\right.\\ E_4 &= \left\{\begin{array}{ll}1 & \text{when} \quad t\geqslant T_p^1+T_p^2\\ 0 & \text{otherwise}\end{array}\right.\end{aligned} \tag{2.48}$$

where $\xi_3^{max}\in\mathbb{R}^+$ is the physical maximum value of the state variable $\xi_{i,3}$ arising from Assumption 2.4.2 such that $|\xi_{i,3}|\leqslant\xi_3^{max}$. $\gamma(\phi)$ is defined in equation (1.18) and T_p^m is the settling time which is an user-defined parameter. The m^{th} dynamics of the agents' observation errors (2.46) converge to zero in a predefined time $T=\sum_{m=1}^4 T_p^m$.

Remark 2.4.3 *Note that without loss of generality, the settling time is considered to be the same for all of the m^{th} dynamics of agents for notational convenience.*

Proof Taking advantage of the structure of the error dynamics (2.46), which their coupling terms and its input are in the regular form [Loukyanov & Utkin 1981], the proof of the predefined time-convergence is made in two steps. First, the convergence of trajectories of subsystem $\mathcal{Z}_{i,1}$ is achieved in a predefined-time $T_p^1+T_p^2$. Hence, following the block control principle [Drakunov *et al.* 1990, Loukianov 2002], the convergence of subsystem $\mathcal{Z}_{i,2}$ is studied after time $T_p^1+T_p^2$ to circumvent the coupling problems between the two subsystems.

1. Convergence of $\mathcal{Z}_{i,1}$ This part of the proof consists of showing the predefined-time stability of $\mathcal{Z}_{i,1}$ in a recursive manner. For that, consider the following Lyapunov function associated with the concatenated second error dynamics of the agents

$$V_2^i = \frac{1}{N}\sqrt{\lambda_{\min}\left(\mathcal{L}^i+\mathcal{L}_i\right)\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)\left(\mathcal{E}_{i,2}\right)}\ . \tag{2.49}$$

Differentiating (2.49), it follows:

$$\begin{aligned}
\dot{V}_2^i &= \frac{1}{N}\sqrt{\frac{\lambda_{\min}\left(\mathcal{L}^i+\mathcal{L}_i\right)}{\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)\left(\mathcal{E}_{i,2}\right)}}\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)\left(\dot{\mathcal{E}}_{i,2}\right)\\
&= \frac{1}{N}\sqrt{\frac{\lambda_{\min}\left(\mathcal{L}^i+\mathcal{L}_i\right)}{\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)\left(\mathcal{E}_{i,2}\right)}}\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)\\
&\times\left(-\mathcal{H}\left(\left(\mathcal{L}_i+\mathcal{L}^i\right)\mathcal{E}_{i,2}\right)+\mathbf{1}\left(\mathcal{U}_{i,1}+\dot{f}_{i,1}\right)\right).
\end{aligned} \tag{2.50}$$

Let the set $\mathcal{S}_m = \left[s_m^1,\dots,s_m^N\right]^T = \left(\mathcal{L}_i+\mathcal{L}^i\right)\mathcal{E}_{i,m}$. Then, it follows that

$$\begin{aligned}
\dot{V}_2^i &= \frac{\sqrt{\lambda_{\min}\left(\mathcal{L}^i+\mathcal{L}_i\right)}}{N}\left(-\frac{\left(\mathcal{S}_2\right)^T\mathcal{H}\left(\mathcal{S}_2\right)}{\sqrt{\left(\mathcal{E}_{i,1}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)\left(\mathcal{E}_{i,2}\right)}}+\frac{\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)}{\sqrt{\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)\left(\mathcal{E}_{i,2}\right)}}\right.\\
&\left.\times\mathbf{1}\left(\mathcal{U}_{i,1}+\dot{f}_{i,1}\right)\right)\\
&= \frac{\sqrt{\lambda_{\min}\left(\mathcal{L}^i+\mathcal{L}_i\right)}}{N}\left(-\frac{1}{\sqrt{\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)\left(\mathcal{E}_{i,2}\right)}}\sum_{i=1}^N\kappa_2^i|s_2^i|\left(\alpha|s_2^i|^p+\eta|s_2^i|^q\right)^r\right.\\
&\left.-\frac{\delta_2}{\sqrt{\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)\left(\mathcal{E}_{i,2}\right)}}\sum_{i=1}^N\kappa_2^i|s_2^i|+\frac{\left(\mathcal{E}_{i,2}\right)^T\left(\mathcal{L}^i+\mathcal{L}_i\right)}{\sqrt{\left(\mathcal{S}_2\right)^T\mathcal{E}_{i,2}}}\mathbf{1}\left(\mathcal{U}_{i,1}+\dot{f}_{i,1}\right)\right)\\
&= \frac{\sqrt{\lambda_{\min}\left(\mathcal{L}^i+\mathcal{L}_i\right)}}{N}\left(-\Delta_1\left(\mathcal{S}_2\right)+\Delta_2\left(\mathcal{S}_2\right)\right).
\end{aligned} \tag{2.51}$$

Using Lemma 2.1, and given that $\sum_{i=1}^N\kappa_2^i \geqslant \kappa_2$ and $||\mathcal{S}_2||_1 = \sum_{i=1}^N|s_2^i|$, the following inequality from the first term in (2.51) can be deduced:

$$\begin{aligned}\Delta_1(\mathcal{S}_2) &= \frac{1}{\sqrt{(\mathcal{E}_{i,2})^T(\mathcal{L}^i+\mathcal{L}_i)(\mathcal{E}_{i,2})}}\sum_{i=1}^N \kappa_2^i|s_2^i|\left(\alpha|s_2^i|^p+\eta|s_2^i|^q\right)^r\\ &\geqslant \frac{\kappa_2 N}{\sqrt{(\mathcal{E}_{i,2})^T(\mathcal{L}^i+\mathcal{L}_i)(\mathcal{E}_{i,2})}}\sum_{i=1}^N \frac{1}{N}|s_2^i|\left(\alpha|s_2^i|^p+\eta|s_2^i|^q\right)^r\\ &\geqslant \frac{\kappa_2 N}{\sqrt{(\mathcal{E}_{i,2})^T(\mathcal{L}^i+\mathcal{L}_i)(\mathcal{E}_{i,2})}}\\ &\quad\times\left(\frac{1}{N}\|\mathcal{S}_2\|_1\right)\left(\alpha\left(\frac{1}{N}\sum_{i=1}^N\|\mathcal{S}_2\|_1\right)^p+\eta\left(\frac{1}{N}\sum_{i=1}^N\|\mathcal{S}_2\|_1\right)^q\right)^r\end{aligned} \tag{2.52}$$

Using Lemma 2.2, $\|\mathcal{S}_2\|_1 \geqslant \|\mathcal{S}_2\|_2 = \sqrt{\mathcal{S}_2^T\mathcal{S}_2} = \sqrt{(\mathcal{E}_{i,2})^T(\mathcal{L}^i+\mathcal{L}_i)^2(\mathcal{E}_{i,2})}$. Thus, expressing $\mathcal{E}_{i,2}$ as a linear combination of the eigenvectors of $(\mathcal{L}^i+\mathcal{L}_i)$, it follows that $(\mathcal{E}_{i,2})^T(\mathcal{L}^i+\mathcal{L}_i)^2(\mathcal{E}_{i,2}) \geqslant \lambda_{\min}(\mathcal{L}^i+\mathcal{L}_i)(\mathcal{E}_{i,2})^T(\mathcal{L}^i+\mathcal{L}_i)(\mathcal{E}_{i,2})$.

Now, using Lemma 2.3, it yields

$$-\Delta_1(\mathcal{S}_2) \leqslant -\kappa_2\sqrt{\lambda_{\min}(\mathcal{L}^i+\mathcal{L}_i)}\left(\alpha(V_2^i)^p+\eta(V_2^i)^q\right)^r. \tag{2.53}$$

On the other hand, from the second terms of (2.51) one can conclude that

$$\begin{aligned}\Delta_2(\mathcal{S}_2) &\leqslant \lambda_{\min}(\mathcal{L}^i+\mathcal{L}_i)\left(-\frac{\delta_2}{\|\mathcal{S}_2\|_1}\sum_{i=1}^N\kappa_2^i|s_2^i|+\frac{\mathcal{S}_2^T}{\|\mathcal{S}_2\|_1}\mathbf{1}\left(\mathcal{U}_{i,1}+\dot{f}_{i,1}\right)\right)\\ &\leqslant \lambda_{\min}(\mathcal{L}^i+\mathcal{L}_i)\left(-\kappa_2\delta_2+u_1^{\max}+\dot{f}_1^{\max}\right)\leqslant 0\end{aligned}. \tag{2.54}$$

By combining inequalities (2.53) and (2.54), the following inequality is obtained from (2.51):

$$\dot{V}_2^i \leqslant -\frac{\kappa_2}{N}\lambda_{\min}(\mathcal{L}^i+\mathcal{L}_i)\left(\alpha(V_2^i)^p+\eta(V_2^i)^q\right)^r \leqslant -\frac{\gamma(\phi)}{T_p^2}\left(\alpha(V_2^i)^p+\eta(V_2^i)^q\right)^r. \tag{2.55}$$

Consequently, according to Lemma (1.4), $\mathcal{E}_{i,2}$ converges towards the origin with the settling time T_p^2. For $t > T_p^2$, the dynamics of $\mathcal{E}_{i,1}$ become

$$\dot{\mathcal{E}}_{i,1} = -\mathcal{H}((\mathcal{L}_i+\mathcal{L}^i)\mathcal{E}_{i,1}) + \mathbf{1}\varsigma_3\zeta_i.$$

Setting $V_1^i = \frac{1}{N}\sqrt{\lambda_{\min}(\mathcal{L}^i+\mathcal{L}_i)(\mathcal{E}_{i,1})^T(\mathcal{L}^i+\mathcal{L}_i)(\mathcal{E}_{i,1})}$ and following the same steps of the proof for V_2^i,

$$-\Delta_1(\mathcal{S}_1) \leqslant -\kappa_1\sqrt{\lambda_{\min}(\mathcal{L}^i+\mathcal{L}_i)}\left(\alpha(V_1^i)^p+\eta(V_1^i)^q\right)^r. \tag{2.56}$$

Thus $\Delta_2(\mathcal{S}_1) \leqslant \lambda_{\min}(\mathcal{L}^i+\mathcal{L}_i)(-\kappa_1\delta_1+\varsigma_3\zeta^{\max}) \leqslant 0$. Hence, as a result, the dynamics of V_1^i have the form

$$\dot{V}_1^i \leqslant -\frac{\kappa_1}{N}\lambda_{\min}(\mathcal{L}^i+\mathcal{L}_i)\left(\alpha(V_1^i)^p+\eta(V_1^i)^q\right)^r \leqslant -\frac{\gamma(\phi)}{T_p^1}\left(\alpha(V_1^i)^p+\eta(V_1^i)^q\right)^r. \tag{2.57}$$

Similarly to $\mathcal{E}_{i,2}$, $\mathcal{E}_{i,1}$ converges to the origin with the settling time $T_p^1 + T_p^2$.

2. Convergence of $\mathcal{Z}_{i,2}$ In order to show the predefined-time stability of $\mathcal{Z}_{i,2}$, let us consider the following two sub-steps.

- For $t > T_p^1 + T_p^2$, since the dynamics of $\mathcal{E}_{i,1}$ have converged to 0, i.e., $\mathcal{E}_{i,1}$ has reached and remains on the sliding manifold ($\dot{\mathcal{E}}_{i,1} = \mathcal{E}_{i,1} = 0$) and $E_4 = 1$, the chosen gains allow us to get an online estimation of the disturbance expressed as

$$\hat{\Upsilon}_i \quad = \frac{E_4 \kappa_1^i \delta_1}{\varsigma_3} \operatorname{sign}\left(\left(\mathcal{L}_i + \mathcal{L}^i\right)\mathcal{E}_{i,1}\right)_{\text{eq}} \tag{2.58}$$

where $\hat{\zeta}_i^k = \zeta_i$ when $t > T_p^1 + T_p^2$, the term $\operatorname{sign}\left(\left(\mathcal{L}_i + \mathcal{L}^i\right)\mathcal{E}_{i,1}\right)_{\text{eq}}$, is a continuous function which denotes the equivalent value sign function. In practice, this sign function is usually implemented by a high frequency component with a certain sampling period which can be cut-off using a low pass filter[Utkin 2013]. The recovered signal is called the equivalent information injection and equation (2.58) represents the global disturbance estimation.

The dynamics of $\mathcal{E}_{i,4}$ are then reduced to

$$\dot{\mathcal{E}}_{i,4} \quad = -\varsigma_3\left(\hat{\Upsilon}_i \circ \mathcal{E}_{i,3}\right) - \mathcal{H}\left(\left(\mathcal{L}_i + \mathcal{L}^i\right)\mathcal{E}_{i,4}\right) + \mathbf{1}\left(\mathcal{U}_{i,2} + f_{i,2}\right). \tag{2.59}$$

Considering Assumption 2.4.2, since the input and the fault are physically bounded, $\mathcal{E}_{i,3}$ remains bounded for all $t \in \left[T_p^1 + T_p^2, T_p^1 + T_p^2 + T_p^4\right]$. Indeed, $\mathcal{E}_{i,3}$ only depends on the measured states since $\hat{X}_{i,3} = 0$ during this time interval. Hence, $|\mathcal{E}_{i,3}| \leqslant \mathbf{1}\xi_3^{\max}$. Similarly to the previous step, by setting $V_4^i = \frac{1}{N}\sqrt{\lambda_{\min}\left(\mathcal{L}^i + \mathcal{L}_i\right)\left(\mathcal{E}_{i,4}\right)^T\left(\mathcal{L}^i + \mathcal{L}_i\right)\left(\mathcal{E}_{i,4}\right)}$, one can get

$$-\Delta_1\left(\mathcal{S}_4\right) \leqslant -\kappa_4\sqrt{\lambda_{\min}\left(\mathcal{L}^i + \mathcal{L}_i\right)}\left(\alpha\left(V_4^i\right)^p + \eta\left(V_4^i\right)^q\right)^r \tag{2.60}$$

$$\Delta_2(\mathcal{S}_4) \quad \leqslant \lambda_{\min}\left(\mathcal{L}^i + \mathcal{L}_i\right)\left(-\kappa_4\delta_4 + \varsigma_3\xi_3^{\max}\zeta^{\max} + u_2^{\max} + f_2^{\max}\right) \leqslant 0. \tag{2.61}$$

As a result

$$\begin{aligned} \dot{V}_4^i \quad &\leqslant -\frac{\kappa_4}{N}\lambda_{\min}\left(\mathcal{L}^i + \mathcal{L}_i\right)\left(\alpha\left(V_4^i\right)^p + \eta\left(V_4^i\right)^q\right)^r \\ &\leqslant -\frac{\gamma\left(\phi\right)}{T_p^4}\left(\alpha\left(V_4^i\right)^p + \eta\left(V_4^i\right)^q\right)^r. \end{aligned} \tag{2.62}$$

Hence, $\mathcal{E}_{i,4}$ converges to zero in a predefined-time $T_p^1 + T_p^2 + T_p^4$.

- Finally, for $t > T_p^1 + T_p^2 + T_p^4$, given that $\dot{\mathcal{E}}_{i,1} = \dot{\mathcal{E}}_{i,2} = \dot{\mathcal{E}}_{i,4} = 0$ and $E_3 = 1$, the dynamics of $\mathcal{E}_{i,3}$ become

$$\dot{\mathcal{E}}_{i,3} \quad = -\mathcal{H}\left(\left(\mathcal{L}_i + \mathcal{L}^i\right)\mathcal{E}_{i,3}\right). \tag{2.63}$$

By setting $V_3^i = \frac{1}{N}\sqrt{\lambda_{\min}\left(\mathcal{L}^i + \mathcal{L}_i\right)\left(\mathcal{E}_{i,3}\right)^T\left(\mathcal{L}^i + \mathcal{L}_i\right)\left(\mathcal{E}_{i,3}\right)}$ and following the same procedure

$$-\Delta_1\left(\mathcal{S}_3\right) \leqslant -\kappa_3\sqrt{\lambda_{\min}\left(\mathcal{L}^i + \mathcal{L}_i\right)}\left(\alpha\left(V_3^i\right)^p + \eta\left(V_3^i\right)^q\right)^r \tag{2.64}$$

$$\Delta_2\left(\mathcal{S}_3\right) \quad \leqslant -\kappa_3\delta_3\lambda_{\min}\left(\mathcal{L}^i+\mathcal{L}_i\right)\leqslant 0 \tag{2.65}$$

Consequently,

$$\dot{V}_3^i \quad \leqslant -\frac{\gamma\left(\phi\right)}{T_p^3}\left(\alpha\left(V_3^i\right)^p+\eta\left(V_3^i\right)^q\right)^r\ . \tag{2.66}$$

Therefore, $\mathcal{E}_{i,3}$ converges to the origin with the settling time $T=T_p^1+T_p^2+T_p^3+T_p^4$. This concludes the proof.

Remark 2.4.4 *Similarly to remark 2.3.1 in the previous Section, according to Theorem 2.2, the predefined-time observer (2.43) guarantees a perfect estimation of the global system state in the presence of bounded control inputs, uncertainties, and faults in a predefined-time T. Additionally, the concept of predefined-time stability is instrumental in tuning the switching times defined in Eq. (2.48) adequately, independently of the initial conditions of the system. Furthermore, it ensures the viability of the residual signals defined in 2.4.2.2, at a given prescribed-time while avoiding the problem of transient behaviours and avoiding an over-tuning of the switching times.*

Remark 2.4.5 *Similar remarks as 2.3.2 can be made in this Section for conditions (2.47) in Theorem 2.2 and observers (2.43). Moreover, since global knowledge on the fixed communication topology is known to all agents, the information $\lambda_{min}\left(\mathcal{L}^i+\mathcal{L}_i\right)$ defined in Theorem 2.2, and therefore, $\kappa_m = min\{\kappa_m^1,\ldots,\kappa_m^N\}$ can both be computed a-priori. Moreover, if all T_p^m are the same, $\kappa_m = \frac{N\gamma(\phi)}{gT_p}$ with $g=\max\{\lambda_{min}\left(\mathcal{L}^1+\mathcal{L}_1\right),\ldots,\lambda_{min}\left(\mathcal{L}^N+\mathcal{L}_N\right)\}$.*

Remark 2.4.6 *Note that the design of observers for the considered class of MASs, i.e. nonholonomic systems, due to the dynamics $\dot{\xi}_{i,3}$ in Eq. (2.40), is far from being straightforward. Hence, conventional methods developed for linear or quasi linear systems cannot be easily applied. Here, the idea is to design a switching observer as introduced in Eq. (2.43) with the observer switching components E_3 and E_4. Indeed, the dynamics of the ith robot are divided into two coupled subsystems. To solve the coupling between the two subsystems in Eq. (2.42), we use a switching scheme and fixed-time stability concepts. The concept of fixed-time stability is very useful to give the switching times used in Eq. (2.48). Note that due to fixed-time stability, the switching times are independent of the initial conditions of the system. The proposed distributed observers give an exact estimate of the global system state in a predefined-time. This ensures the viability of the residual signals at a given prescribed-time while avoiding the problem of transient behaviours. At last, it should also be noted that contrary to existing fixed-time observers such as the one proposed in the previous Section ([Taoufik et al. 2020c]), the proposed distributed predefined-time observer ensures an estimation of the global state of the system with a good estimate of the settling time. This enables to avoid an over tuning of the switching times given in Eq. (2.48) and to reduce the transient behaviour.*

2.4.2.2 Residual Generation and Fault Detection

Since an online estimation of the disturbance has been obtained in (2.58), an observer generating the residual in the form of (2.41) can be designed by the means of a new variable. Let us define a new variable χ_i whose dynamics are explicitly affected by both faults of the system such that $\chi_i = E_r(\xi_{i,2} + \xi_{i,4})$, where E_r is the residual switch introduced since the residuals can only be designed when all of the global state estimates have converged toward zero i.e.,

$$E_r = \begin{cases} 1 & \text{when} \quad t \geqslant T \\ 0 & \text{otherwise} \end{cases}$$

with $T = T_p^1 + T_p^2 + T_p^3 + T_p^4$. One can write

$$\dot{\chi}_i = \dot{\xi}_{i,2} + \dot{\xi}_{i,4} = \mathcal{U}_{i,1} + \mathcal{U}_{i,2} + \zeta_i\left(\varsigma_1 \sin\left(\xi_{i,1}\right) - \varsigma_2 \cos\left(\xi_{i,1}\right) + \varsigma_3 \xi_{i,3}\right) + \left(\dot{f}_{i,1} + f_{i,2}\right). \tag{2.67}$$

The fault detection filter is expressed as

$$\dot{\hat{\chi}}_i^k = \hat{\mathcal{U}}_{i,1}^k + \hat{\mathcal{U}}_{i,2}^k + \hat{\zeta}_i^k\left(\varsigma_1 \sin\left(\hat{\xi}_{i,1}^k\right) - \varsigma_2 \cos\left(\hat{\xi}_{i,1}^k\right) + \varsigma_3 \hat{\xi}_{i,3}^k\right) + \varrho\left(\chi_i - \hat{\chi}_i^k\right) \tag{2.68}$$

and the associated residual signal is defined as

$$r_i^k = E_r\left(\chi_i - \hat{\chi}_i^k\right) \tag{2.69}$$

where

- $\hat{\mathcal{U}}_{i,1}^k$ and $\hat{\mathcal{U}}_{i,2}^k$ are the estimates of the applied control inputs obtained using the estimate of the global system state,
- ϱ is the residual filter gain.

Proposition 2.1: [Taoufik *et al.* 2021b]

For all $\varrho > 0$ and $E_r = 1$, r_i^k is null if $f_{i,r} = f_{i,l} = 0$ and different from zero if $f_{i,r} \neq 0$ or $f_{i,l} \neq 0$.

Proof The observation error, $e_i^k = \chi_i - \hat{\chi}_i^k$, dynamics when $\hat{\chi}_i^k$ has converged, i.e., $E_r = 1$ are expressed as

$$\begin{aligned} \dot{e}_i^k = \dot{\chi}_i - \dot{\hat{\chi}}_i^k &= \mathcal{U}_{i,1} + \dot{f}_{i,1} + \mathcal{U}_{i,2} + f_{i,2} + \zeta_i\left(\varsigma_1 \sin\left(\xi_{i,1}\right) - \varsigma_2 \cos\left(\xi_{i,1}\right) + \varsigma_3 \xi_{i,3}\right) \\ &\quad - \mathcal{U}_{i,1}^k\left(\hat{\Psi}_i^k\right) - \mathcal{U}_{i,2}^k\left(\hat{\Psi}_i^k\right) - \hat{\zeta}_i^k\left(\varsigma_1 \sin\left(\hat{\xi}_{i,1}^k\right) - \varsigma_2 \cos\left(\hat{\xi}_{i,1}^k\right) + \varsigma_3 \hat{\xi}_{i,3}^k\right) \\ &\quad - \varrho\left(\chi_i - \hat{\chi}_i^k\right) \\ &= \dot{f}_{i,1} + f_{i,2} - \varrho e_i^k \end{aligned}$$

Thus, the chosen residual $r_i^k = E_r e_i^k$ is close to zero if there is no fault, i.e., $\lim_{t\to\infty} \exp(-\varrho t) = 0$ and different from zero when a fault occurs. The signal r_i^k is used to detect a fault on either wheels of the i^{th} robot by the k^{th} robot.

Similarly to the previous Section, given Remark 2.4.4, is it obvious that the residual r_i^k does not exactly equal to zero in the absence of fault. Hence, once the residual signal is generated, the following RMS evaluation function is chosen

$$J_i^k = \|r_i^k(t)\|_{\text{RMS}} = \left(\frac{1}{T_w}\int_t^{t+T_w} ||r_i^k(\tau)||^2 d\tau\right)^{\frac{1}{2}} \tag{2.70}$$

where T_w is the finite-time evaluation window. The inaccuracies due to measurement noise and sampling, denoted Π_i^k, are considered as unstructured unknown inputs. The observation error dynamics $e_i^k = \chi_i - \hat{\chi}_i^k$ and residuals in a fault-free scenario are expressed as

$$\begin{array}{rcl} \dot{e}_i^k &=& \Pi_i^k - \varrho e_i^k \\ r_i^k &=& e_i^k \end{array} \tag{2.71}$$

Given Assumption 2.2.1-2.4.2, Π_i^k remains bounded. Let us denote its RMS norm bound ϖ_i^k estimated off-line in the absence of a fault ($||\Pi_i^k||_{\text{RMS}} \leqslant \varpi_i^k$). The following RMS threshold, denoted $J_i^{k_{th}}$ is then defined

$$J_i^{k_{th}} = \sup_{\substack{f_{i,r}=f_{i,l}=0 \\ ||\Pi_i^k||_{\text{RMS}} \leqslant \varpi_i^k}} ||r_i^k(t))||_{\text{RMS}} \tag{2.72}$$

This corresponds to the tolerant limit for uncertainties, measurement errors, etc.. Based on the evaluation function, one can detect a fault through the following decision logic

$$\begin{array}{l} J_i^k > J^{k_{th}} \Longrightarrow \text{A fault is detected} \\ J_i^k \leqslant J^{k_{th}} \Longrightarrow \text{No fault is detected} \end{array} \tag{2.73}$$

with $J^{k_{th}} = \max\{J_1^{k_{th}}, \dots, J_N^{k_{th}}\}$. The isolation of a faulty agent in the team is thus achieved when the residual is larger than the defined threshold. In this Section, one could make the same remarks as 2.3.4.

2.4.3 Simulation Example

Consider a MAS composed of $N = 6$ robots governed by unicycle-type dynamics (2.34) labelled by numbers 1 through 6 with $\varsigma_1 = \varsigma_2 = \varsigma_3 = 2$ and $d = 0.2$ m. The communicating among each agent is given according to the fixed topology shown in Fig. 2.9, and is characterised by the following Laplacian matrix

$$\mathcal{L}(\mathcal{Q}) = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 & 0 \\ -1 & 0 & 2 & 0 & 0 & -1 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Hence, Assumption 2.2.1 is fulfilled.

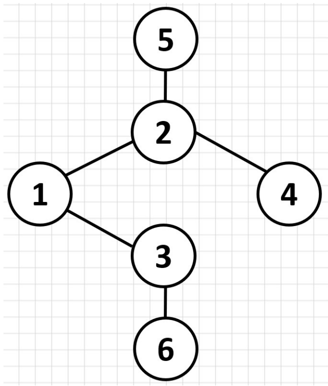


Figure 2.9: Communication topology.

The initial positions $x_i(0), y_i(0)$ and $\theta_i(0)$ of the agents are given as

$$\begin{cases} [x_1(0), y_1(0), \theta_1(0)]^T & = [0, 0, 0]^T \\ [x_2(0), y_2(0), \theta_2(0)]^T & = [-0.5, 0, \pi]^T \\ [x_3(0), y_3(0), \theta_3(0)]^T & = [0.5, 0, \pi/4]^T \\ [x_4(0), y_4(0), \theta_4(0)]^T & = [-1, 0, 0]^T \\ [x_5(0), y_5(0), \theta_5(0)]^T & = [1, 0, \pi/12]^T \\ [x_6(0), y_6(0), \theta_6(0)]^T & = [0, -1, \pi/2]^T \end{cases}.$$

For each agent, distributed observers are designed to estimate the global state in a desired prescribed time $T = \sum_{m=1}^{4} T_p^m = 4s$, with $T_p^1 = T_p^2 = T_p^3 = T_p^4 = 1s$. Given Remark 2.4.3, this choice allows the observers to converge at the same time, hence simplifying the residual evaluation process. The sampling period is set as $T_s = 10^{-5}s$.

In order to check the robustness of the proposed scheme, band-limited white measurement noise of power 10^{-8} has been added to all agents. Moreover, additive band-limited white communication noises of random powers between 10^{-8} and 10^{-7} have been added to all edges in the network. This means that transmitted information between neighbouring agents has been affected by these communication noises. For the unknown disturbance ζ_i in (2.34), two types of dynamic perturbations are considered: a band-limited white noise of power 10^{-6} for agents 3 and 4, high frequency noise modelled by $\zeta_1(t) = \zeta_5(t) = 2\sin(2000t)$ for agents 1 and 5, and a combination of both noises for agents 2 and 6.

In the following, two typical fault situations are considered: i) an exponential signal representing a progressive loss of efficiency in one of the wheels, ii) an out of control situation simulated by a sine function [De Loza *et al.* 2015]. They occur in agents 6 and 5 as follows

$$\begin{aligned} f_{6,l}(t) &= \begin{cases} 0 & t < 11s, \\ 0.2(e^{-10(t-11)} - 1) & t \geqslant 11s. \end{cases} \\ f_{5,r}(t) &= \begin{cases} 0 & t < 10s, \\ -0.2\sin(t-10) & t \geqslant 10s; \end{cases} \end{aligned}.$$

Here, in order to show the effectiveness of the proposed FDI scheme, the following consensus control protocol is used [Maghenem *et al.* 2018], where $\mathcal{U}_{i,1}$ and $\mathcal{U}_{i,2}$ are defined as

$$\begin{aligned}\mathcal{U}_{i,1} &= -k_1^w \xi_{i,2} - k_2^w(\xi_{i,1} - d_{\xi_{i,1}}) - \frac{p(t)}{2}\Big((\bar{\varphi}(\xi_{i,1})\textstyle\sum_{j=1}^N a_{ij}(z_i - z_j))^2 \\ &+(\varphi(\xi_{i,1})\textstyle\sum_{j=1}^N a_{ij}(z_i - z_j))^2\Big)\end{aligned} \tag{2.74}$$

$$\mathcal{U}_{i,2} = \int_0^\infty \Big[- k_1^v v_i - k_2^v \varphi(\xi_{i,1})^T \sum_{j=1}^N a_{ij}(z_i - z_j)\Big] dt - \xi_{i,2}\xi_{i,3} \tag{2.75}$$

with $\varphi(\xi_{i,1}) = \begin{bmatrix}\cos(\xi_{i,1}) & \sin(\xi_{i,1})\end{bmatrix}^T$, $\bar{\varphi}(\xi_{i,1}) = \begin{bmatrix}\sin(\xi_{i,1}) & -\cos(\xi_{i,1})\end{bmatrix}^T$ and $z_i = \begin{bmatrix}x_i - d_{x_i} & y_i - d_{y_i}\end{bmatrix}^T$, where $d_{x_i} = 0.5i$, $d_{y_i} = i$ and $d_{\xi_{i,1}} = 0.5$ form the desired coordinates. The consensus errors are defined as $e_i = \varphi(\xi_{i,1})\sum_{j=1}^N a_{ij}(z_i - z_j)$. k_1^v, k_2^v, k_1^w and k_2^w are the consensus gains chosen as 3, 5, 4 and 6 respectively. $p(t)$ is a persistently exciting signal chosen as $p(t) = 0.1\sin(0.1t)$, and

$$y_i = \int(\dot{\xi}_{i,4} + \xi_{i,2}\xi_{i,3})\sin(\xi_{i,1})dt, \quad x_i = \int(\dot{\xi}_{i,4} + \xi_{i,2}\xi_{i,3})\cos(\xi_{i,1})dt$$

From the proposed predefined-time observer, the inputs (2.74)-(2.75) can be robustly estimated at each node as follows

$$\begin{cases}\hat{\mathcal{U}}_{i,1}^k &= -k_1^w \hat{\xi}_{i,2}^k - k_2^w(\hat{\xi}_{i,1}^k - d_{\xi_{i,1}}) - \frac{p(t)}{2}\Big((\bar{\varphi}(\hat{\xi}_{i,1}^k)\sum_{j=1}^N a_{ij}(\hat{z}_i^k - \hat{z}_j^k))^2 \\ &+(\varphi(\hat{\xi}_{i,1}^k)\sum_{j=1}^N a_{ij}(\hat{z}_i^k - \hat{z}_j^k))^2\Big) \\ \hat{\mathcal{U}}_{i,2}^k &= \int_0^\infty \Big[- k_1^v(\dot{\hat{\xi}}_{i,4}^k + \hat{\xi}_{i,2}^k\hat{\xi}_{i,3}^k) - k_2^v\varphi(\hat{\xi}_{i,1}^k)^T\sum_{j=1}^N a_{ij}(\hat{z}_i^k - \hat{z}_j^k)\Big]dt - \hat{\xi}_{i,2}^k\hat{\xi}_{i,3}^k \\ \hat{z}_i^k &= \begin{bmatrix}(\dot{\hat{\xi}}_{i,4}^k + \hat{\xi}_{i,2}^k\hat{\xi}_{i,3}^k)\cos(\hat{\xi}_{i,1}^k) - d_{x_i} \\ (\dot{\hat{\xi}}_{i,4}^k + \hat{\xi}_{i,2}^k\hat{\xi}_{i,3}^k)\sin(\hat{\xi}_{i,1}^k) - d_{y_i}\end{bmatrix}\end{cases}$$

For the estimation of the perturbations, the first-order low pass filters are chosen with cut-off frequencies of $\omega_c = 1000$ r/s. The parameter vector of the observers ϕ verifying the conditions in Theorem 2.2 is given as

$$\phi = [\alpha, \eta, p, q, r]^T = [1.5, 3, 0.5, 4, 0.25]^T.$$

Additionally, one can easily check that Assumptions 2.4.1 and 2.4.2 are verified with $u_1^{max} = 2.15$, $u_2^{max} = 4.75$, $\zeta^{max} = 2.4$, $\dot{f}_1^{max} = 1$, $f_2^{max} = 1$ and $\xi_3^{max} = 4.43$. Provided that in this example $T_p^1 = T_p^2 = T_p^3 = T_p^4$, and from Remark 2.4.5, $g = max\{\lambda_{min}(\mathcal{L}^1 + \mathcal{L}_1), \ldots, \lambda_{min}(\mathcal{L}^6 + \mathcal{L}_6)\} = 0.438$, the gains are selected as $\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = 60.92$ and

$$\begin{cases}\delta_1 = 0.079 \\ \delta_2 = 0.052 \\ \delta_3 = 0.016 \\ \delta_4 = 0.443\end{cases}$$

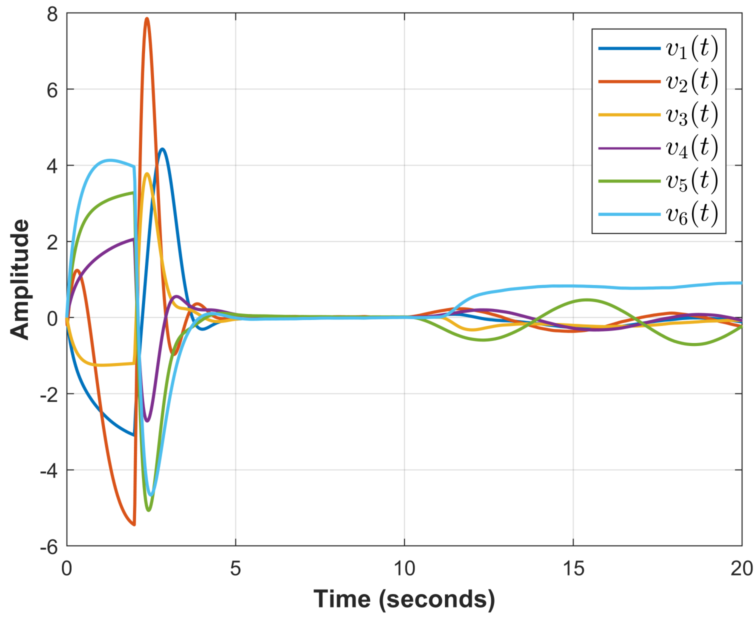


Figure 2.10: Linear velocities $v_i(t)$.

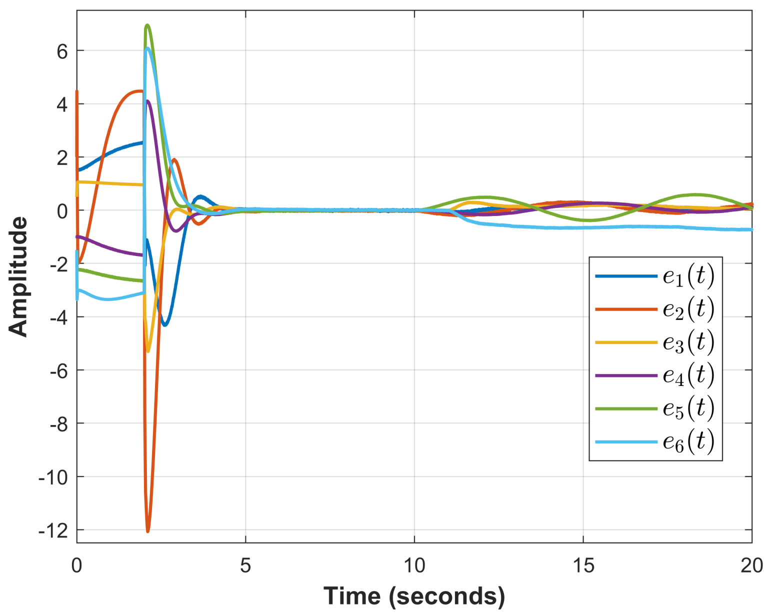


Figure 2.11: Consensus errors.

Figures 2.10 and 2.11 represent the linear velocity and consensus errors respectively, for each agent. The residual filter gains are chosen as 4. Figures 2.12a-2.12c show the global residual signals generated by the monitoring agents 1, 3 and 6 respectively with respect to the rest of the agents. It is worth noting that these signals are

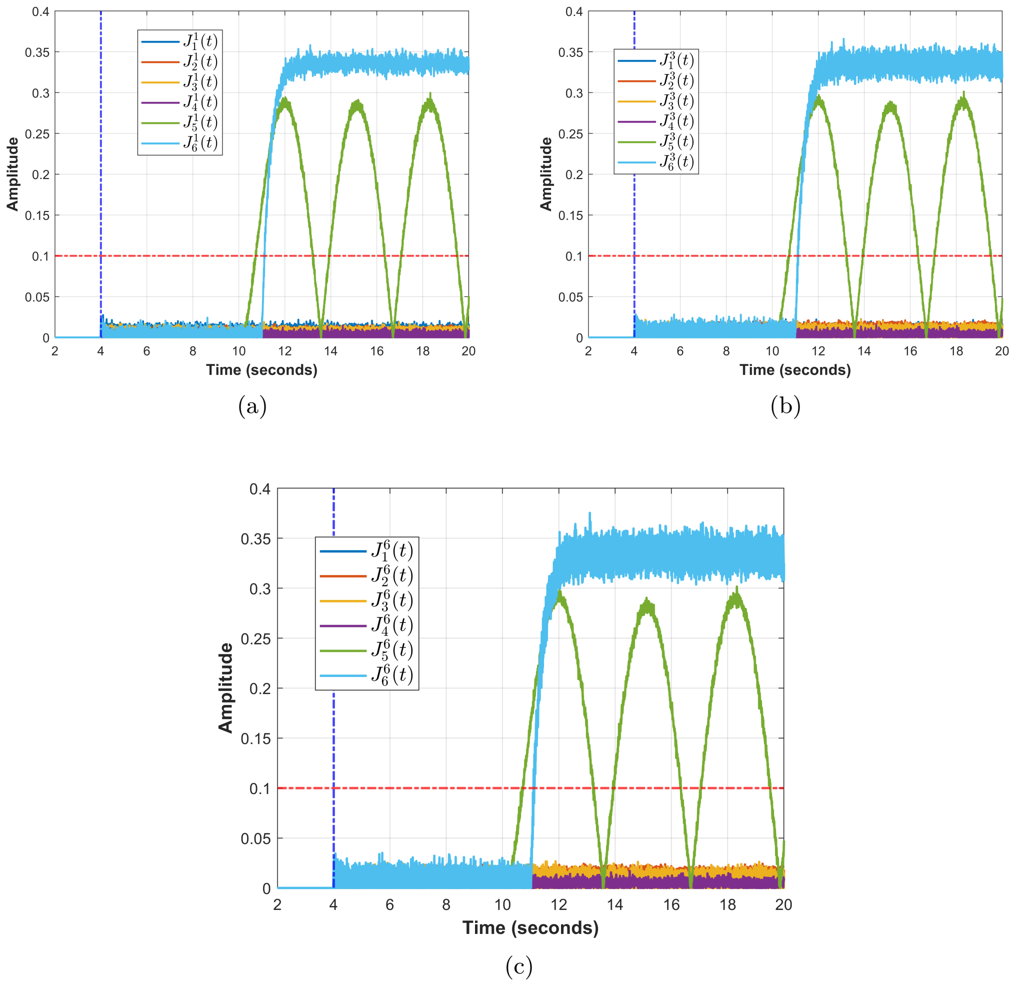


Figure 2.12: Global residual signals generated by (**a**) Agent 1, (**b**) Agent 3, and (**c**) Agent 6.

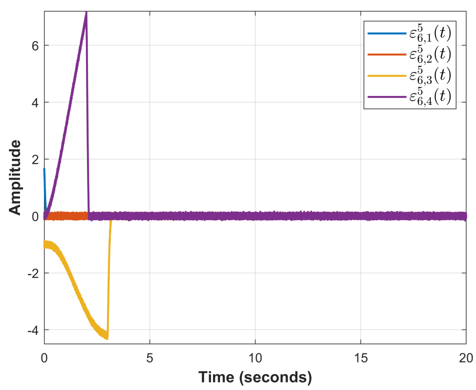


Figure 2.13: Agent 5's state estimation errors of agent 6 (Ex. 2.4.3).

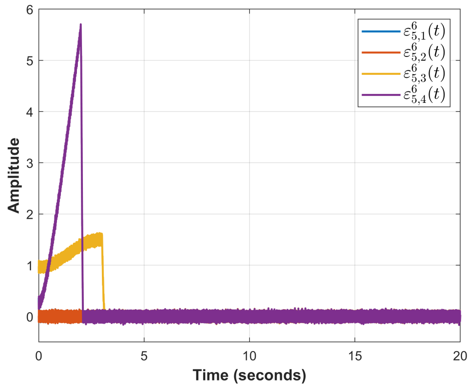


Figure 2.14: Agent 6's state estimation errors of agent 5.

read after the observers converge, the dashed blue vertical lines are added as the corresponding delimiters while dashed red lines are added to show the chosen thresholds. The convergence is achieved after $T = 4s$ as defined previously. One can easily notice that the residual signals are robust with respect to disturbances and measurement/edge noises and faults can be distinguished. Figs. 2.13-2.14 show agent 5's estimation errors of agent 6 and agent 6's estimation errors of agent 5.

Remark 2.4.7 *It is worthy of mention that the proposed scheme in this Section presents similar disadvantages as the one proposed in the previous Section, namely an information exchange at each agent that is equivalent to $4N$. As such, similar remarks as in 2.3.5 could be made here.*

2.5 Attack Detection in MASs with Switching Topologies

2.5.1 Preliminaries

In Section 2.3 and 2.4, the considered topology is fixed and safe from any attacks. In this Section, both switching topologies and cyber-attacks are considered. Before formulating the problem, let us first modify the topology description as follow:

Topology description: In this Section, it is further assumed from the description of the graph topology in Section 2.2, that the communication topology is time-varying. As a result, we denote by $\tilde{T} = \{\tau_1, \tau_2, ..., \tau_M\}$ the set of all possible known topologies and by $\mathcal{M} := \{1, ..., M\}$ the set of indices corresponding to these topologies. More precisely, the communication topology is characterised by a switching graph $\mathcal{Q}^{\sigma(t)} = \mathcal{Q}(t)$ where $\sigma(t) : [0, \infty) \longrightarrow \mathcal{M}$ is a piecewise constant switching signal and determines the communication topology with $0 = t_0 < t_1 < t_2...$ being the switching instants of $\sigma(t)$. Furthermore, it is assumed that $\sigma(t)$ satisfies the minimum dwell time condition [Jadbabaie *et al.* 2003]. That is, the time interval between any consecutive switching instants is larger than or equal to a minimum dwell time, and $t_{w+1} - t_w = \tau_w < T_w$ with T_w a known constant. Therefore, when $\sigma(t) = \mathsf{s} \in \mathcal{M}$, the topology $\mathcal{Q}(t) = \mathcal{Q}^{\sigma(t)} = \mathcal{Q}^{\mathsf{s}}$ is activated. In this Section, all modes of $\mathcal{Q}^{\mathsf{s}}$ satisfy Assumption 2.2.1.

For the remainder of this Section, the active mode is referred to using the superscript s. The adjacency matrix $\mathcal{A}^{\mathsf{s}} = [a^{\mathsf{s}}_{ij}] \in \mathbb{R}^{N\times N}$ is defined by $a^{\mathsf{s}}_{ij} > 0$ when the i^{th} agent can receive information from the j^{th} agent and $a^{\mathsf{s}}_{ij} = 0$ otherwise. Let $\mathcal{D}^{\mathsf{s}}$ be the in-degree diagonal matrix with entries $d^{\mathsf{s}}_i = \sum_{j=1}^{N} a^{\mathsf{s}}_{ij}$. Hence, the Laplacian matrix $\mathcal{L}^{\mathsf{s}}$ is defined as:

$$\mathcal{L}^{\mathsf{s}} = \mathcal{D}^{\mathsf{s}} - \mathcal{A}^{\mathsf{s}} \in \mathbb{R}^{N\times N}$$

Similarly to Section 2.2, for a given active mode, let us denote by $\mathcal{L}^{\mathsf{s}}_i \in \mathbb{R}^{(N-1)\times(N-1)}$ the Laplacian matrix $\mathcal{L}^{\mathsf{s}}$ defined without agent i, and by:

$$\mathcal{L}^{i,\mathsf{s}} = diag(\ell^{i,\mathsf{s}}_1, \ldots, \ell^{i,\mathsf{s}}_{i-1}, \ell^{i,\mathsf{s}}_{i+1}, \ldots, \ell^{i,\mathsf{s}}_N) \in \mathbb{R}^{(N-1)\times(N-1)}$$

the associated diagonal matrix defining the interconnections between agent i and the remaining agents under the active topology s, $\ell^{i,\mathsf{s}}_k > 0$ if information of agent i is accessible by the k^{th} agent; otherwise $\ell^{i,\mathsf{s}}_k = 0$.

2.5.2 Problem Formulation

Consider a homogeneous MAS composed of N agents labelled by $i \in \{1, ..., N\}$, and described by the following nth-order dynamics

$$\begin{cases} \dot{\xi}_{i,1}(t) & = \xi_{i,2}(t) \\ \dot{\xi}_{i,2}(t) & = \xi_{i,3}(t) \\ \vdots & \\ \dot{\xi}_{i,n-1}(t) & = \xi_{i,n}(t) \\ \dot{\xi}_{i,n}(t) & = u_i(t) + f^a_i(t) \\ z_i(t) & = \xi_{i,1}(t) \end{cases} \tag{2.76}$$

where $\xi_{i,m}(t) \in \mathbb{R}$ is agent i's m^{th} state variable with $\xi_i(t) = [\xi_{i,1}(t), \xi_{i,2}(t), ..., \xi_{i,n}(t)]^T \in \mathbb{R}^n$, $f^a_i(t) \in \mathbb{R}$ is an actuator fault affecting the dynamics of the agent which could be exogenous, $u_i(t) \in \mathbb{R}$ is the control input and $z_i(t) \in \mathbb{R}$ is the agent i's internal measurement. Note that there is a multitude of practical applications of such systems, namely robotic systems, power systems, etc. Research on cyber-attack identification for such systems is of both practical and theoretical significance.

Furthermore, it is considered that agents have access to their own control inputs, but they do not receive their neighbours' inputs. If needed, they have to reconstruct them using state estimates from exchanged information which are possibly corrupted. The exchanged information is expressed as

$$\begin{cases} z_{ki}(t) = \ell^{i,\mathsf{s}}_k(z_i(t) + \check{f}^e_{ki}(t)), \\ \hat{z}^{kj}_i(t) = a^{\mathsf{s}}_{kj}(\hat{z}^j_i(t) + f^e_{kj}(t)) \end{cases} \tag{2.77}$$

where $z_{ki}(t) \in \mathbb{R}$ is the agent i's output signal sent to the agent k with $z_{kk}(t) = z_k(t)$, and $\hat{z}_i^{kj}(t) \in \mathbb{R}$ is agent j's estimate of agent i's output which is sent to agent k, the term $\hat{z}_i^j(t)$ will be defined in the Subsection 2.5.3. Both pieces of information are subject to an edge attack denoted $\check{f}^e_{ki}(t) \in \mathbb{R}$ and $f^e_{kj}(t) \in \mathbb{R}$, respectively. Note that, these attacks affect all broadcasted information of an agent to another. In this section, a solution to the following questions is investigated:

- How can one detect a cyber-attack anywhere in the MAS, where each agent can only measure the first state variable, while keeping a distributed approach of the FDI scheme?
- How can one distinguish said attacks from actuator fault? i.e., how can one identify a cyber-attack?

The conceptual idea in this work is that information locally produced by the sensors is considered to be secure, while the one sent over the communication network/cyber layer of the system is vulnerable to external attacks. Fig. 2.15 depicts the structure of the scheme proposed in this Section. The main result is laid out in the Subsection 2.5.3.

Remark 2.5.1 *It is worth noting that the proposed scheme here only requires the exchange of the output and their global estimates which is equivalent to N for each agent, hence resulting in less information exchange as [Wu et al. 2019] or [Taoufik et al. 2020c] for instance.*

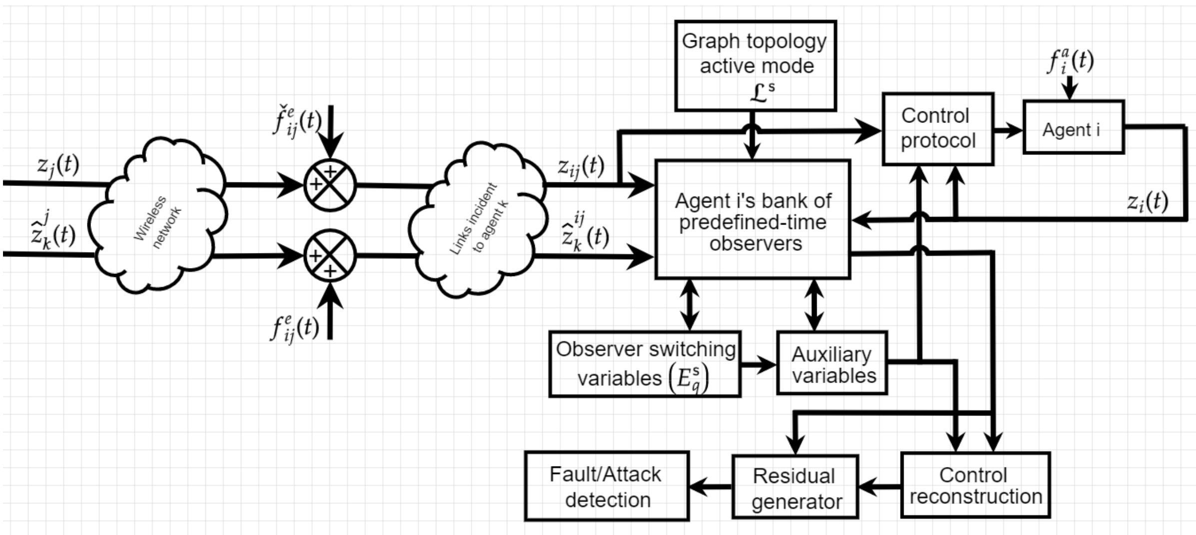


Figure 2.15: Proposed fault/attack detection scheme, where $\mathcal{L}^s$ refers to the switching topology described in Subsection 2.5.1 and E_q^s is defined in 2.3.

2.5.3 Main Result

Herein, the proposed distributed bank of predefined-time observers for output, state estimation and global cyber-attack detection scheme is laid out.

2.5.3.1 Global Output and State Estimation

Let us define the 'monitored' agent i as the agent to be diagnosed by a 'monitoring' agent k. First, let us consider the case of a fixed communication topology, where no cyber-attack exists in the system (i.e., $\check{f}^e_{ki} = f^e_{kj} = 0$). Denote by $\hat{\xi}^k_{i,m}$, agent k's estimate of the m^{th} state variable of agent i and by $\hat{z}^k_i$, agent k's estimate of agent i's output. The proposed distributed switched observer takes the following structure:

$$\begin{cases} \dot{\hat{\xi}}^k_{i,1} & = \hat{\xi}^k_{i,2} + \mathcal{V}(\mathcal{I}^k_{i,1}) = \hat{z}^k_i \\ \vdots & \\ \dot{\hat{\xi}}^k_{i,n-1} & = \hat{\xi}^k_{i,n} + E^{\mathsf{s}}_{n-2}\mathcal{V}(\mathcal{I}^k_{i,n-1}) \\ \dot{\hat{\xi}}^k_{i,n} & = E^{\mathsf{s}}_{n-1}\mathcal{V}(\mathcal{I}^k_{i,n}) \end{cases} \tag{2.78}$$

with $\mathcal{V}(\mathcal{I}^k_{i,l}) = \kappa^{k,\mathsf{s}}_l((\alpha|\mathcal{I}^k_{i,l}|^p + \eta|\mathcal{I}^k_{i,l}|^q)^r + \delta^{\mathsf{s}}_l)\text{sign}(\mathcal{I}^k_{i,l})$,

$$\begin{cases} \mathcal{I}^k_{i,1} = \sum_{j=1}^N a^{\mathsf{s}}_{kj}(\hat{z}^{kj}_i - \hat{z}^k_i) + \ell^{i,\mathsf{s}}_k(z_{ki} - \hat{z}^k_i) \\ \mathcal{I}^k_{i,m} = \tilde{\xi}^k_{i,m} - \hat{\xi}^k_{i,m}, \quad m \in \{2, ..., n\} \end{cases} \tag{2.79}$$

and the variables E^{s}_q, $\forall q \in \{1, ..., n-1\}$ are defined later.

The auxiliary state variables $\tilde{\xi}^k_{i,m}$, $\forall m \in \{2, ..., n\}$ are defined as

$$\begin{cases} \tilde{\xi}^k_{i,2} & = \hat{\xi}^k_{i,2} + E^{\mathsf{s}}_1\kappa^{k,\mathsf{s}}_1\delta^{\mathsf{s}}_1\text{sign}(\mathcal{I}^k_{i,1})_{eq} \\ \vdots & \\ \tilde{\xi}^k_{i,n-1} & = \hat{\xi}^k_{i,n-1} + E^{\mathsf{s}}_{n-2}\kappa^{k,\mathsf{s}}_{n-2}\delta^{\mathsf{s}}_{n-2}\text{sign}(\mathcal{I}^k_{i,n-2})_{eq} \\ \tilde{\xi}^k_{i,n} & = \hat{\xi}^k_{i,n} + E^{\mathsf{s}}_{n-1}\kappa^{k,\mathsf{s}}_{n-1}\delta^{\mathsf{s}}_{n-1}\text{sign}(\mathcal{I}^k_{i,n-1})_{eq} \end{cases} \tag{2.80}$$

where the subscript eq denotes the equivalent value of sign function. In the following, it is assumed that the effect of the filter dynamics is negligible w.r.t. those of the observer. Let us define the errors as

$$\begin{cases} \varepsilon^k_{i,1} & = z_{ki} - \hat{z}^k_i \\ \varepsilon^k_{i,m} & = \xi_{i,m} - \hat{\xi}^k_{i,m}, \quad \forall m \in \{2, ..., n\} \end{cases}$$

Differentiating them yields the following error dynamics:

$$\begin{cases} \dot{\varepsilon}^k_{i,1} & = \varepsilon^k_{i,2} - \kappa^{k,\mathsf{s}}_1((\alpha|\mathcal{I}^k_{i,1}|^p + \eta|\mathcal{I}^k_{i,1}|^q)^r + \delta^{\mathsf{s}}_1)\text{sign}(\mathcal{I}^k_{i,1}) \\ \vdots & \\ \dot{\varepsilon}^k_{i,n-1} & = \varepsilon^k_{i,n} - E^{\mathsf{s}}_{n-2}\kappa^{k,\mathsf{s}}_{n-1}((\alpha|\mathcal{I}^k_{i,n-1}|^p + \eta|\mathcal{I}^k_{i,n-1}|^q)^r \\ & \quad +\delta^{\mathsf{s}}_{n-1})\text{sign}(\mathcal{I}^k_{i,n-1}) \\ \dot{\varepsilon}^k_{i,n} & = u_i + f^a_i - E^{\mathsf{s}}_{n-1}\kappa^{k,\mathsf{s}}_n((\alpha|\mathcal{I}^k_{i,n}|^p + \eta|\mathcal{I}^k_{i,n}|^q)^r + \delta^{\mathsf{s}}_n)\text{sign}(\mathcal{I}^k_{i,n}) \end{cases} \tag{2.81}$$

where $\mathcal{I}_{i,1}^{k}$ can be expressed in terms of the output estimation errors as $\mathcal{I}_{i,1}^{k} = \sum_{j=1}^{N} a_{kj}^{\mathsf{s}}(\varepsilon_{i,1}^{j} - \varepsilon_{i,1}^{k}) + \ell_{k}^{i,\mathsf{s}}\varepsilon_{i,1}^{k}$. Putting (2.81) in compact form, the following is obtained:

$$\begin{cases} \dot{\mathcal{E}}_{i,1} & = \mathcal{E}_{i,2} - \mathcal{H}(\mathcal{E}_{i,1}) \\ \vdots & \\ \dot{\mathcal{E}}_{i,n-1} & = \mathcal{E}_{i,n} - E_{n-2}^{\mathsf{s}}\mathcal{H}(\mathcal{E}_{i,n-1}) \\ \dot{\mathcal{E}}_{i,n} & = \mathbf{1}(u_i + f_i^a) - E_{n-1}^{\mathsf{s}}\mathcal{H}(\mathcal{E}_{i,n}) \end{cases} \tag{2.82}$$

where for each agent $\forall i \in \{1, ..., N\}$ and $\forall l \in \{1, ..., n\}$, the estimation errors, the state estimates and the auxiliary variables are concatenated in the vectors: $\mathcal{E}_{i,l} = [\varepsilon_{i,l}^{1}, ..., \varepsilon_{i,l}^{N}]^T$, $\hat{X}_{i,l} = [\hat{\xi}_{i,l}^{1}, ..., \hat{\xi}_{i,l}^{N}]^T$, $\tilde{X}_{i,l} = [\tilde{\xi}_{i,l}^{1}, ..., \tilde{\xi}_{i,l}^{N}]^T$. Let us denote $L_i^{\mathsf{s}} = \mathcal{L}^{i,\mathsf{s}} + \mathcal{L}_i^{\mathsf{s}}$. The terms $\mathcal{H}(\mathcal{E}_{i,l}), \forall l \in \{1, ..., n\}$ are expressed as

$$\begin{cases} \mathcal{H}(\mathcal{E}_{i,1}) & = \kappa_1^{i,\mathsf{s}}((\alpha|L_i^{\mathsf{s}}\mathcal{E}_{i,1}|^p + \eta|L_i^{\mathsf{s}}\mathcal{E}_{i,1}|^q)^k + \delta_1^{\mathsf{s}})\mathrm{sign}(L_i^{\mathsf{s}}\mathcal{E}_{i,1}) \\ \mathcal{H}(\mathcal{E}_{i,m}) & = \kappa_m^{i,\mathsf{s}}((\alpha|\mathcal{E}_{i,m}|^p + \eta|\mathcal{E}_{i,m}|^q)^k + \delta_m^{\mathsf{s}})\mathrm{sign}(\mathcal{E}_{i,m}), \quad \forall m \in \{2, ..., n\} \end{cases}$$

Assumption 2.5.1 *For every agent, the state variables, the control and fault signals are bounded, and their maximum values are known, i.e., for* $\bar{\xi}_{i,l}, \bar{u}, \bar{f}^a \in \mathbb{R}_+$, $i \in \{1, ..., N\}$ *and* $l \in \{1, ..., n\}$*:* $|\xi_{i,l}(t)| \leqslant \bar{\xi}_{i,l}$, $|u_i(t)| \leqslant \bar{u}$, $|f_i^a(t)| \leqslant \bar{f}^a$.

Theorem 2.3: [Taoufik *et al.* 2020b]

For constant positive definite parameters α, η, p, q and r, which satisfy the constraints $rp < 1$ and $rq > 1$, and given Assumption 2.5.1, for a fixed communication topology and in the absence of cyber-attack, for each agent, the observation errors (2.82) converge towards zero in a predefined time $T^{\mathsf{s}} = \sum_{j=1}^{n-1} T_p^{j,\mathsf{s}}$ independently of initial conditions, with the observer gains:

$$\begin{cases} \delta_q^{\mathsf{s}} & = \dfrac{\bar{\xi}_{i,q+1}}{\kappa_q^{\mathsf{s}}}, \quad \forall q \in \{1, ..., n-1\} \\ \delta_n^{\mathsf{s}} & = \dfrac{\bar{u} + \bar{f}^a}{\kappa_n} \end{cases} \tag{2.83}$$

with

$$\begin{cases} \kappa_1^{i,\mathsf{s}} & = \dfrac{N\gamma(\phi)}{\lambda_i^{\mathsf{s}} T_p^{1,\mathsf{s}}} \\ \kappa_m^{i,\mathsf{s}} & = \dfrac{N\gamma(\phi)}{T_p^{m,\mathsf{s}}}, \quad \forall m \in \{2, ..., n\} \end{cases}$$

and

$$E_q^{\mathsf{s}} = \begin{cases} 1 & \text{when} \quad t \geqslant \sum_{j=1}^{q} T_p^{j,\mathsf{s}} \\ 0 & \text{otherwise} \end{cases}, \quad \forall q \in \{1, ..., n-1\}$$

where $\kappa_m^{\mathsf{s}} = min\{\kappa_m^{1,\mathsf{s}}, \ldots, \kappa_m^{N,\mathsf{s}}\}$ and $\lambda_{min}(L_i^{\mathsf{s}}) = \lambda_i^{\mathsf{s}}$. $\gamma(\phi)$ is defined in Equation (1.18), E_m^{s} represents the observer switches and $T_p^{m,\mathsf{s}}$ is the settling-time for each dynamic which is an user-defined parameter, considered to be the same for all of the m^{th} dynamics of the agents for notational convenience.

Proof The proof is done step by step by taking advantage of the switching conditions. Indeed, due to this, at each step, only a one-dimensional, corresponding sub-dynamical system is studied.

Step 1: Initially, $E_1^{\mathsf{s}} = E_2^{\mathsf{s}} = ... = 0$, the error dynamics are expressed as

$$\begin{cases} \dot{\mathcal{E}}_{i,1} & = \mathcal{E}_{i,2} - \mathcal{H}(\mathcal{E}_{i,1}) \\ \vdots & \\ \dot{\mathcal{E}}_{i,n-1} & = \mathcal{E}_{i,n} \\ \dot{\mathcal{E}}_{i,n} & = \mathbf{1}(f_i^a + u_i) \end{cases} \tag{2.84}$$

Consider the following Lyapunov function associated with the concatenated first error dynamics of the agents

$$V_1^i = \frac{1}{N}\sqrt{\lambda_i^{\mathsf{s}} \mathcal{E}_{i,1}^T L_i^{\mathsf{s}} \mathcal{E}_{i,1}}$$

Differentiating it results in

$$\dot{V}_1^i = \frac{1}{N}\sqrt{\frac{\lambda_i^{\mathsf{s}}}{\mathcal{E}_{i,1}^T L_i^{\mathsf{s}} \mathcal{E}_{\ \ i,1}}}\, \mathcal{E}_{i,1}^T L_i^{\mathsf{s}} (\mathcal{E}_{i,2} - \mathcal{H}(\mathcal{E}_{i,1})) \tag{2.85}$$

By setting $\mathcal{S}_1 = [s_1^1, \ldots, s_1^N]^T = L_i^{\mathsf{s}} \mathcal{E}_{i,1}$, one obtains

$$\begin{aligned} \dot{V}_1^i \ &= \frac{\sqrt{\lambda_i^{\mathsf{s}}}}{N}\Big(- \frac{1}{\sqrt{\mathcal{E}_{i,1}^T L_i^{\mathsf{s}} \mathcal{E}_{i,1}}} \textstyle\sum_{i=1}^N \kappa_1^{i,\mathsf{s}} |s_1^i| (\alpha |s_1^i|^p + \eta |s_1^i|^q)^r \\ & - \frac{\delta_1^{\mathsf{s}}}{\sqrt{\mathcal{E}_{i,1}^T L_i^{\mathsf{s}} \mathcal{E}_{i,1}}} \textstyle\sum_{i=1}^N \kappa_1^{i,\mathsf{s}} |s_1^i| + \frac{\mathcal{E}_{i,1}^T L_i^{\mathsf{s}} \mathcal{E}_{i,2}}{\sqrt{\mathcal{S}_1^T \mathcal{E}_{i,1}}}\Big) \end{aligned} \tag{2.86}$$

Then, it follows that

$$\dot{V}_1^i = \frac{\sqrt{\lambda_i^{\mathsf{s}}}}{N}(-\Delta_1(\mathcal{S}_1) + \Delta_2(\mathcal{S}_1))$$

with

$$\begin{cases} \Delta_1(\mathcal{S}_1) & = (\mathcal{E}_{i,1}^T L_i^{\mathsf{s}} \mathcal{E}_{i,1})^{-\frac{1}{2}} \sum_{i=1}^N \kappa_1^{i,\mathsf{s}} |s_1^i| (\alpha |s_1^i|^p + \eta |s_1^i|^q)^r \\ \Delta_2(\mathcal{S}_1) & = -\delta_1^{\mathsf{s}} (\mathcal{E}_{i,1}^T L_i^{\mathsf{s}} \mathcal{E}_{i,1})^{-\frac{1}{2}} \sum_{i=1}^N \kappa_1^{i,\mathsf{s}} |s_1^i| + \mathcal{E}_{i,1}^T L_i^{\mathsf{s}} \mathcal{E}_{i,2} (\mathcal{S}_1^T \mathcal{E}_{i,1})^{-\frac{1}{2}} \end{cases}$$

Considering Lemma 2.1, and taking into account the fact that $\sum_{i=1}^N \kappa_1^{i,\mathsf{s}} \geqslant \kappa_1^{\mathsf{s}}$ and $||\mathcal{S}_1||_1 = \sum_{i=1}^N |s_1^i|$, the term $\Delta_1(\mathcal{S}_1)$ can be expressed as

$$\Delta_1(\mathcal{S}_1) \geqslant \frac{\kappa_1^{\mathsf{s}} ||\mathcal{S}_1||_1}{\sqrt{\mathcal{E}_{i,1}^T L_i^{\mathsf{s}} \mathcal{E}_{i,1}}} (\alpha (\tfrac{1}{N} \textstyle\sum_{i=1}^N ||\mathcal{S}_1||_1)^p + \eta (\tfrac{1}{N} \sum_{i=1}^N ||\mathcal{S}_1||_1)^q)^r \tag{2.87}$$

Using Lemma 2.2, it can be shown that

$$||\mathcal{S}_1||_1 \geqslant ||\mathcal{S}_1||_2 = (\mathcal{S}_1)^T(\mathcal{S}_1) = \sqrt{(\mathcal{E}_{i,1})^T(L_i^{\mathsf{s}})^2(\mathcal{E}_{i,1})} \tag{2.88}$$

By expressing $\mathcal{E}_{i,1}$ as a linear combination of the eigenvectors of L_i^{s}, the term $\mathcal{E}_{i,1}^T(L_i^{\mathsf{s}})^2\mathcal{E}_{i,1}$ can be bounded as

$$\mathcal{E}_{i,1}^T(L_i^{\mathsf{s}})^2\mathcal{E}_{i,1} \geqslant \lambda_i^{\mathsf{s}}\mathcal{E}_{i,1}^T L_i^{\mathsf{s}}\mathcal{E}_{i,1}$$

Thus, using Lemma 2.3, one has

$$-\Delta_1(\mathcal{S}_1) \leqslant -\kappa_1^{\mathsf{s}}\sqrt{\lambda_i^{\mathsf{s}}}\big(\alpha(V_1^i)^p + \eta(V_1^i)^q\big)^r \tag{2.89}$$

On the other hand, from the second term $\Delta_2(\mathcal{S}_1)$, the following can be deduced

$$\begin{aligned} \Delta_2(\mathcal{S}_1) \ &\leqslant \lambda_i^{\mathsf{s}}\Big(- \frac{\delta_1^{\mathsf{s}}}{||\mathcal{S}_1||}\textstyle\sum_{i=1}^N \kappa_1^{i,\mathsf{s}}|s_1^i| + \dfrac{(\mathcal{S}_1)^T}{||\mathcal{S}_1||}(\mathcal{E}_{i,2})\Big) \\ &\leqslant \lambda_i^{\mathsf{s}}(-\kappa_1^{\mathsf{s}}\delta_1^{\mathsf{s}} + \bar{\xi}_{i,2}) \\ &\leqslant 0 \end{aligned} \tag{2.90}$$

By combining (2.89) and (2.90), the following is obtained from (2.86)

$$\begin{aligned} \dot{V}_1^i \ &\leqslant -\frac{\kappa_1^{\mathsf{s}}}{N}\lambda_i^{\mathsf{s}}\big(\alpha(V_1^i)^p + \eta(V_1^i)^q\big)^r \\ &\leqslant -\frac{\gamma(\phi)}{T_p^{1,\mathsf{s}}}\big(\alpha(V_1^i)^p + \eta(V_1^i)^q\big)^r \end{aligned} \tag{2.91}$$

Therefore, in accordance with Lemma 1.19, $\mathcal{E}_{i,1}$ converges towards the origin with the settling time $T_p^{1,\mathsf{s}}$ (i.e., $\mathcal{E}_{i,1} = \dot{\mathcal{E}}_{i,1} = 0$). As a result, at $t = T_p^{1,\mathsf{s}}$ ($E_1^{\mathsf{s}} = 1$), we have

$$\begin{aligned} \mathcal{E}_{i,2} - \mathcal{H}(\mathcal{E}_{i,1})_{eq} \ &= X_{i,2} - \hat{X}_{i,2} - \mathcal{H}(\mathcal{E}_{i,1})_{eq} \\ &= 0 \end{aligned} \tag{2.92}$$

Hence, one gets $\tilde{X}_{i,2} = X_{i,2}$. At this point, one can go to the next step.

Step 2: At $t = T_p^{1,\mathsf{s}}$, the error dynamics become

$$\begin{cases} \dot{\mathcal{E}}_{i,2} &= \mathcal{E}_{i,3} - \mathcal{H}(\mathcal{E}_{i,2}) \\ \vdots & \\ \dot{\mathcal{E}}_{i,n-1} &= \mathcal{E}_{i,n} \\ \dot{\mathcal{E}}_{i,n} &= \mathbf{1}(f_i^a + u_i) \end{cases} \tag{2.93}$$

Selecting the Lyapunov function $V_2^i = \frac{1}{N}\sqrt{\mathcal{E}_{i,2}^T\mathcal{E}_{i,2}}$ and by following the same reasoning as before, one gets

$$\begin{aligned} -\Delta_1(\mathcal{S}_2) &\leqslant -\kappa_2^{\mathsf{s}}\big(\alpha(V_2^i)^p + \eta(V_2^i)^q\big)^r \\ \Delta_2(\mathcal{S}_2) &\leqslant -\frac{\delta_2^{\mathsf{s}}}{||\mathcal{S}_2||}\textstyle\sum_{i=1}^N \kappa_2^{i,\mathsf{s}}|s_2^i| + \frac{(\mathcal{S}_2)^T(\mathcal{E}_{i,3})}{||\mathcal{S}_2||} \\ &\leqslant -\kappa_2^{\mathsf{s}}\delta_2^{\mathsf{s}} + \bar{\xi}_{i,3} \\ &\leqslant 0 \end{aligned} \tag{2.94}$$

with $\mathcal{S}_2 = [s_2^1, \ldots, s_2^N]^T = \mathcal{E}_{i,2}$. Then, it is straightforward to show that

$$\dot{V}_2^i \leqslant -\frac{\gamma(\phi)}{T_p^{2,\mathsf{s}}}\big(\alpha(V_2^i)^p + \eta(V_2^i)^q\big)^r$$

Consequently, $\mathcal{E}_{i,2}$ converges towards the origin with the settling time $T_p^{1,\mathsf{s}} + T_p^{2,\mathsf{s}}$ (i.e., $\mathcal{E}_{i,2} = \dot{\mathcal{E}}_{i,2} = 0$). Therefore, at $t = T_p^{1,\mathsf{s}} + T_p^{2,\mathsf{s}}$ and $E_2^{\mathsf{s}} = 1$.

Step n: Now, fast forward to the nth step, at $t = \sum_{j=1}^{n-1} T_p^{j,\mathsf{s}}$, the error dynamics become

$$\dot{\mathcal{E}}_{i,n} = \mathbf{1}(f_i^a + u_i) - \mathcal{H}(\mathcal{E}_{i,n}) \tag{2.95}$$

Taking as the Lyapunov function $V_n^i = \frac{1}{N}\sqrt{(\mathcal{E}_{i,n})^T(\mathcal{E}_{i,n})}$ and by setting $\mathcal{S}_n = [s_n^1, \ldots, s_n^N]^T = \mathcal{E}_{i,n}$, and following the same procedure as before, the following inequalities are obtained for the terms $\Delta_1(\mathcal{S}_n)$ and $\Delta_2(\mathcal{S}_n)$

$$\begin{aligned} -\Delta_1(\mathcal{S}_n) &\leqslant -\kappa_n^{\mathsf{s}}\big(\alpha(V_n^i)^p + \eta(V_n^i)^q\big)^r \\ \Delta_2(\mathcal{S}_2) &\leqslant -\frac{\delta_n^{\mathsf{s}}}{||\mathcal{S}_n||}\textstyle\sum_{i=1}^N \kappa_n^{i,\mathsf{s}}|s_n^i| + \frac{(\mathcal{S}_n)^T\mathbf{1}(\bar{u} + \bar{f}^a)}{||\mathcal{S}_n||} \\ &\leqslant -\kappa_n^{\mathsf{s}}\delta_n^{\mathsf{s}} + \bar{u} + \bar{f}^a \\ &\leqslant 0 \end{aligned} \tag{2.96}$$

The proof is thus concluded at the nth step.

Now, let us consider the presence of a possible cyber-attack in the network. Due to the presence of these attacks, the output estimation errors is expressed as

$$\varepsilon_{i,1}^k = z_i - \hat{z}_i^k + \ell_k^{i,\mathsf{s}}\check{f}_{ki}^e + \textstyle\sum_{j=1}^N a_{kj}^{\mathsf{s}} f_{kj}^e \tag{2.97}$$

Assumption 2.5.2 *It is assumed that* $\forall i, j \in \{1, .., N\}, i \neq j$, $\check{f}_{ki}^e(t)$, $f_{kj}^e(t)$ *and their derivatives are bounded.*

In this case, the following theorem can be stated.

Theorem 2.4: [Taoufik *et al.* 2020b]

For constant positive definite parameters α, η, p, q and r, which satisfy the constraints $rp < 1$ and $rq > 1$, given Assumptions 2.5.1-2.5.2 and in the presence of one or multiple cyber attacks incident to agent k, in the case of fixed communication topology, the observation errors converge towards zero in a predefined time $T^{\mathsf{s}} = \sum_{j=1}^{n-1} T_p^{j,\mathsf{s}}$ independently of initial conditions, and the gains are given as

$$\begin{cases} \delta_q^{\mathsf{s}} & = \dfrac{\bar{\xi}_{i,q+1} + \bar{F}_k^{\mathsf{s}(q)}}{\kappa_q^{\mathsf{s}}}, \quad \forall q \in \{1, ..., n-1\} \\ \delta_n^{\mathsf{s}} & = \dfrac{\bar{u} + \bar{f}^a + \bar{F}_k^{\mathsf{s}(n)}}{\kappa_n^{\mathsf{s}}} \end{cases} \tag{2.98}$$

with

$$F_k^{\mathsf{s}(l)} = \frac{d^l}{dt^l}(\ell_k^{i,\mathsf{s}} \check{f}_{ki}^e + \sum_{j=1}^{N} a_{kj}^{\mathsf{s}} f_{kj}^e), \quad \forall l \in \{1, ..., n\}$$

where $F_k^{\mathsf{s}(l)}$ corresponds to the l^{th} time derivative of $F_k^{\mathsf{s}} = \ell_k^{i,\mathsf{s}} \check{f}_{ki}^e + \sum_{j=1}^{N} a_{kj}^{\mathsf{s}} f_{kj}^e$ and $\bar{F}_k^{\mathsf{s}(l)}$ is the corresponding upper bound. The gains $\kappa_l^{i,\mathsf{s}}$ and the observer switches E_m^{s} remain the same as in Theorem 2.3.

Proof When cyber-attacks are considered, (2.79) becomes

$$\begin{cases} \mathcal{I}_{i,1}^k = \sum_{j=1}^{N} a_{kj}^{\mathsf{s}}(\hat{z}_i^j - \hat{z}_i^k) + \ell_k^{i,\mathsf{s}}(z_i - \hat{z}_i^k) \\ + \sum_{j=1}^{N} a_{kj}^{\mathsf{s}} f_{kj}^e + \ell_k^{i,\mathsf{s}} \check{f}_{ki}^e, \\ \mathcal{I}_{i,m}^k = \tilde{\xi}_{i,m}^k - \hat{\xi}_{i,m}^k, \quad m \in \{2, ..., n\} \end{cases} \tag{2.99}$$

Furthermore, the auxiliary variables (2.80) become

$$\begin{cases} \tilde{\xi}_{i,2}^k & = \hat{\xi}_{i,2}^k + E_1^{\mathsf{s}} \mathcal{V}(\mathcal{I}_{i,1}^k)_{eq} = \hat{\xi}_{i,2}^k + \varepsilon_{i,2}^k - \ell_k^{i,\mathsf{s}} \dot{\check{f}}_{ki}^e \\ & - \sum_{j=1}^{N} a_{kj}^{\mathsf{s}} \dot{f}_{kj}^e \\ \vdots & \\ \tilde{\xi}_{i,n}^k & = \hat{\xi}_{i,n}^k + E_{n-1}^{\mathsf{s}} \mathcal{V}(\mathcal{I}_{i,n-1}^k)_{eq} = \hat{\xi}_{i,n}^k + \varepsilon_{i,n}^k \\ & - \ell_k^{i,\mathsf{s}} \check{f}_{ki}^{e(n-1)} - \sum_{j=1}^{N} a_{kj}^{\mathsf{s}} f_{kj}^{e(n-1)} \end{cases} \tag{2.100}$$

and the concatenated errors are expressed as

$$\begin{cases} \dot{\mathcal{E}}_{i,1} & = \mathcal{E}_{i,2} + \mathbf{1} F_k^{\mathsf{s}(1)} - \mathcal{H}(\mathcal{E}_{i,1}) \\ \vdots & \\ \dot{\mathcal{E}}_{i,n-1} & = \mathcal{E}_{i,n} + \mathbf{1} F_k^{\mathsf{s}(n-1)} - E_{n-2}^{\mathsf{s}} \mathcal{H}(\mathcal{E}_{i,n-1}) \\ \dot{\mathcal{E}}_{i,n} & = \mathbf{1}(u_i + f_i^a + F_k^{\mathsf{s}(n)}) - E_{n-1}^{\mathsf{s}} \mathcal{H}(\mathcal{E}_{i,n}) \end{cases} \tag{2.101}$$

The rest of the proof straightforwardly follows the same reasoning as Theorem 2.3 and is thus omitted for brevity.

Note that the use of the predefined-time concept is very useful when dealing with switching topologies. Indeed, using our proposed scheme, one can immediately derive the following proposition:

Proposition 2.2: [Taoufik *et al.* 2020b]

Consider the switching topologies described in Subsection 2.5.2. Selecting T^{s} such that $T^{\mathsf{s}} < T_w, \ \ \forall \mathsf{s} \in \mathcal{M}$ and observer parameters (2.98), the distributed switched observers guarantee the predefined-time stability of the estimation errors regardless of initial conditions at each switching instant.

Similarly to Section 2.5.2, the following remark could be made:

Remark 2.5.2 *The global fault estimation protocol proposed in this Section, is a distributed one. Each neighbouring agent can only exchange local information during the fault estimation process. Furthermore, provided that all of the possible topologies are known to all agents, constants* λ_i^{s} *and therefore* $\kappa_1^{\mathsf{s}} = min\{\kappa_1^{1,\mathsf{s}}, \ldots, \kappa_1^{N,\mathsf{s}}\}$ *can be computed a priori. If all* $T_p^{m,\mathsf{s}}$ *are the same (i.e.,* $T_p^{1,\mathsf{s}} = \ldots = T_p^{n,\mathsf{s}} = T_p$*),* $\kappa_1^{\mathsf{s}} = \frac{N\gamma(\phi)}{gT_p}$ *with* $g = \max\{\lambda_1^{\mathsf{s}}, \ldots, \lambda_N^{\mathsf{s}}\}$*.*

2.5.3.2 Residual Generation and Cyber-Attack Identification

The idea is to compute the difference between the actual input of an agent and the estimated input. The difference should indeed be null in the case of no attacks or faults. The next step is to identify the source and type of faults, specifically deception attacks and thus trigger the appropriate alarms. Note that, for Theorems 2.3 and 2.4, the upper bounds of the control inputs are used in Assumption 2.5.1 to design the predefined-time distributed observers. It is shown herein, through a residual based approach how one can detect actuator faults or cyber-attacks with a global approach using input estimates if the control structure is known. In the following, let us consider the following typical linear higher-order consensus control algorithm [Ren *et al.* 2007, Ren & Atkins 2007], used with the available information

$$u_i = -\sum_{j \in \mathcal{N}_i} a_{ij}^{\mathsf{s}} \left[\gamma_1^{\mathsf{s}}(z_i - z_{ij}) + \sum_{m=2}^{n} \gamma_m^{\mathsf{s}} (\tilde{\xi}_{i,m}^i - \tilde{\xi}_{j,m}^i) \right] + \mu_i^{\mathsf{s}} \tilde{\xi}_{i,n}^i \tag{2.102}$$

where $\forall l \in \{1, ..., n\}$, $\forall i \in \{1, ..., N\}$, γ_l^{s} and μ_i^{s} are the consensus gains. It can be noticed that communication faults spread in the MAS through u_i, and thus need to be detected as they occur. In the absence of edge faults, consensus is achieved provided a suitable selection of μ_i^{s}, γ_1^{s} and γ_m^{s} due to the fixed-time stability property of the proposed distributed observers [Jiang & Wang 2010].

Proposition 2.3: [Taoufik *et al.* 2020b]

Define agent k as the monitoring agent, agent i as the monitored agent, agents $p \in \mathcal{N}_k$ as agent k's neighbours and agents $j \in \mathcal{N}_i$ as agent i's neighbours, where agent i may or may not be a direct neighbour of k and $j \neq i$. Using protocol (2.102), an agent k can detect a cyber-attack on a communication link incident to agent k or i and local faults f_i^a anywhere in the fleet, provided one anomaly type happens, using the following residual signal:

$$r_i^k(t) = \mathcal{V}(\mathcal{I}_{i,n}^k)_{eq} - \hat{u}_i^k \tag{2.103}$$

where

$$\hat{u}_i^k = -\sum_{j\in\mathcal{N}_i} a_{ij}^{\mathsf{s}}[\gamma_1^{\mathsf{s}}(\hat{z}_i^k - \hat{z}_j^k) + \sum_{m=2}^{n} \gamma_m^{\mathsf{s}}(\tilde{\xi}_{i,m}^k - \tilde{\xi}_{j,m}^k)] + \mu_i^{\mathsf{s}}\tilde{\xi}_{i,n}^k$$

is agent i's reconstructed input by agent k with $\hat{u}_k^k = u_k$.

Proof After the convergence of errors, the actual applied control input for each agent becomes

$$\begin{aligned} u_i = \; & -\textstyle\sum_{j\in\mathcal{N}_i} a_{ij}^{\mathsf{s}}\left[\gamma_1^{\mathsf{s}}(z_i - z_j) + \sum_{m=2}^{n}\gamma_m^{\mathsf{s}}(\xi_{i,m} - \xi_{j,m})\right] \\ & + \mu_i^{\mathsf{s}}\xi_{i,n} - \textstyle\sum_{j\in\mathcal{N}_i} a_{ij}^{\mathsf{s}}(\gamma_1^{\mathsf{s}}\check{f}_{ij}^e + \sum_{m=2}^{n}\gamma_m^{\mathsf{s}}\check{f}_{ij}^{e(m-1)}) \end{aligned}$$

Furthermore, the reconstructed input generated by the monitoring agent k is expressed as

$$\begin{aligned} \hat{u}_i^k = \; & -\textstyle\sum_{j\in\mathcal{N}_i} a_{ij}^{\mathsf{s}}\left[\gamma_1^{\mathsf{s}}(z_i - z_j) + \sum_{m=2}^{n}\gamma_m^{\mathsf{s}}(\xi_{i,m} - \xi_{j,m})\right] \\ & + \mu_i^{\mathsf{s}}\xi_{i,n} - \textstyle\sum_{j\in\mathcal{N}_i}\gamma_1^{\mathsf{s}}a_{ij}^{\mathsf{s}}\Big[\ell_k^{i,\mathsf{s}}\check{f}_{ki}^e - \ell_k^{j,\mathsf{s}}\check{f}_{kj}^e + \sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}}f_{kp}^e \\ & - \textstyle\sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}}f_{kp}^e\Big] + \sum_{j\in\mathcal{N}_i}\sum_{m=2}^{n} a_{ij}^{\mathsf{s}}\gamma_m^{\mathsf{s}}\Big[\ell_k^{i,\mathsf{s}}\check{f}_{ki}^{e(m-1)} \\ & + \ell_k^{j,\mathsf{s}}\check{f}_{kj}^{e(m-1)}\Big] + \mu_i^{\mathsf{s}}\Big[\ell_k^{i,\mathsf{s}}\check{f}_{ki}^{e(n-1)} + \textstyle\sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}}f_{kp}^{e(n-1)}\Big] \end{aligned}$$

Therefore, the residual signals (2.103) become

$$\begin{aligned} r_i^k(t) & = (u_i - \hat{u}_i^k) + f_i^a - \ell_k^{i,\mathsf{s}}\check{f}_{ki}^{e(n)} - \textstyle\sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}}f_{kp}^{e(n)} \\ & = \Theta_{f^e}^k + f_i^a \end{aligned} \tag{2.104}$$

where $\Theta_{f^e}^k$ is

$$\begin{aligned} \Theta_{f^e}^k & = \textstyle\sum_{j\in\mathcal{N}_i}\gamma_1^{\mathsf{s}}a_{ij}^{\mathsf{s}}\Big[\ell_k^{i,\mathsf{s}}\check{f}_{ki}^e - \ell_k^{j,\mathsf{s}}\check{f}_{kj}^e + \sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}}f_{kp}^e - \sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}}f_{kp}^e\Big] \\ & - \textstyle\sum_{j\in\mathcal{N}_i}\sum_{m=2}^{n} a_{ij}^{\mathsf{s}}\gamma_m^{\mathsf{s}}\Big[\ell_k^{i,\mathsf{s}}\check{f}_{ki}^{e(m-1)} + \ell_k^{j,\mathsf{s}}\check{f}_{kj}^{e(m-1)}\Big] - \mu_i^{\mathsf{s}}\Big[\ell_k^{i,\mathsf{s}}\check{f}_{ki}^{e(n-1)} \\ & + \textstyle\sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}}f_{kp}^{e(n-1)}\Big] - \sum_{j\in\mathcal{N}_i} a_{ij}^{\mathsf{s}}(\gamma_1^{\mathsf{s}}\check{f}_{ij}^e + \sum_{m=2}^{n}\gamma_m^{\mathsf{s}}\check{f}_{ij}^{e(m-1)}) \\ & - \ell_k^{i,\mathsf{s}}\check{f}_{ki}^{e(n)} - \textstyle\sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}}f_{kp}^{e(n)} \end{aligned} \tag{2.105}$$

Note that, when the control efforts u_i are known to other agents in the network, the term $(u_i - \hat{u}_i^k)$ in Equation (2.104) disappears. In this case, the residual signals become

$$\begin{aligned} r_i^k(t) &= f_i^a - \ell_k^{i,\mathsf{s}} \check{f}_{ki}^{e(n)} - \textstyle\sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}} f_{kp}^{e(n)} \\ &= \Theta_{f^e}^k + f_i^a \end{aligned} \tag{2.106}$$

where $\Theta_{f^e}^k = -\ell_k^{i,\mathsf{s}} \check{f}_{ki}^{e(n)} - \sum_{p\in\mathcal{N}_k} a_{kp}^{\mathsf{s}} f_{kp}^{e(n)}$. As a result, the defined residual signals (2.103) generated by the monitoring agent k are able to detect the presence of a cyber-attack or an actuator fault and distinguish as per Proposition 2.3.

Residual Evaluation and Decision Logic:

Once the residual signals are generated, it is important to be able to interpret them in order to find the root of the fault and thus make corrective measures accordingly. Indeed, from Equation (2.103), it can be noticed that, when a cyber-attack incident to agent k or i occurs while there is no local malfunction, agent k's generated residual signal for itself is $r_k^k = 0$ and $r_i^k \neq 0$ for all $k \neq i$ regardless of whether or not agent i is a neighbour of k. On the other hand, when there is no cyber-attack, the residuals provide explicit estimations of the local faults, with $r_k^k = f_k^a$ and $r_i^k = f_i^a$. r_k^k is thus used to identify a cyber-attack in the system as it is only sensitive to local faults. The proposed cyber-attack identification scheme is thus summarized in Algorithm 1.

Algorithm 1 Observer Design and Decision Logic

Result: Distributed Cyber-attack Identification
while *communication topology* s *is active* **do**
  Choose observer convergence time T^{s} in accordance with Proposition 2.2
  Define Laplacian sub-matrices $\mathcal{L}_i^{\mathsf{s}}$ and $\mathcal{L}^{i,\mathsf{s}}$
  Compute observer gains from Theorems 2.3–2.4
  Define a monitoring agent k
  for $q \in \{1, 2, ..., N\}$ **do**
    Generate residual signals r_q^k from Equation (2.103)
  end
  if $r_k^k = 0$ *and* $r_i^k = 0$ **then**
    No cyber-attack or local faults exist in the network
    else if $r_k^k = 0$ *and* $r_i^k \neq 0,\ \forall i \neq k$ **then**
      A cyber-attack has occurred in the network
    else if $r_k^k \neq 0$ *and* $r_i^k = 0$ **then**
      An actuator fault has occurred in agent k
    else if $r_k^k = 0$ *and* $\exists! i \neq k$ *such that* $r_i^k \neq 0$ **then**
      An actuator fault has occurred in agent i
  end
end

Remark 2.5.3 *Note that our approach does not present limitation with respect to the number of detectable attacks in the system, contrary to some existing works, for in-*

stance in [Teixeira et al. 2010]. Indeed, Proposition 2.3 can be used to detect simultaneous actuator faults and cyber-attacks, and discern them from each other thus achieving the cyber-attack identification objective. Moreover, the predefined-time stability principle is useful to design fast converging switched observers to solve the problem of switching communication topologies as pointed out in Proposition 2.2. This allows for avoiding false alarms and achieving fast convergence of the estimation errors before the next topology switching instant. Furthermore, it is worth mentioning that the proposed approach in this Section, can also be used when communication attacks on the communication weights a_{ij}^{s} *or sudden abrupt quality drops in the exchanged information occur. These attacks manifest themselves in the generated residuals as exponentially decaying signals.*

Remark 2.5.4 *It is worth noting that the proposed scheme here only requires the exchange of the output and their global estimates which is equivalent to* N *for each agent, hence resulting in less information exchange as [Wu et al. 2019] or [Taoufik et al. 2020c] for instance.*

2.5.4 Simulation Example

Cyber-attack identification in cooperative MRSs.

Here, an illustrative numerical example is given in order to show the effectiveness of the proposed global cyber-attack identification protocol. For this, let us consider a team of $N = 5$ WMRs that are labelled with numbers 1 through 5 and are moving in a two-dimensional plane (see Figure 2.16). In this example, the robots have to cooperate in order to render the steady state axial jerk null and thus achieve constant linear position, velocity and acceleration synchronization of the network of WMRs.

Here, we assume non-slipping and pure rolling conditions and since our aim is to achieve linear acceleration synchronization, only the dynamics along the x-direction are considered. In this case, each robot can be modelled with the following simplified triple integrator dynamics which is a special case of system (2.76):

$$\begin{cases} \dot{x}_i(t) &= \dot{\xi}_{i,1}(t) &= \xi_{i,2}(t) \\ \dot{v}_i(t) &= \dot{\xi}_{i,2}(t) &= \xi_{i,3}(t) \\ \dot{a}_i(t) &= \dot{\xi}_{i,3}(t) &= u_i(t) + f_i^a(t) \\ z_i(t) &= \xi_{i,1}(t) & \end{cases}$$

where $\xi_{i,1}(t)$, $\xi_{i,2}(t)$, $\xi_{i,3}(t)$ and $f_i^a(t)$ are the x-position, the linear velocity on the x-axis, the linear acceleration on the x-axis and an internal fault affecting the local jerk of a robot. The proposed residual observer-based cyber-attack identification algorithm can be implemented on the on-board micro-controllers as depicted in Figure 2.16. Furthermore, the robots are assumed to be equipped with WiFi modules and broadcast their information through a wireless network described by the graph topologies illustrated in Figure 2.17, which are characterised by the Laplacian matrices:

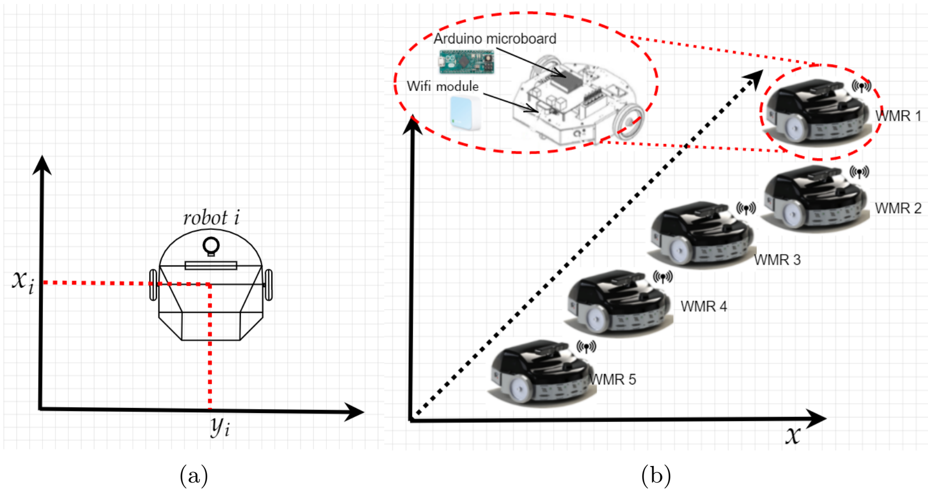


Figure 2.16: The setup of the studied problem where: (**a**) represents the upper perspective view of a WMR on the x-y 2D plane and (**b**) represents an illustration of the set-up of the five mobile robots.

$$\mathcal{L}^1 = \begin{bmatrix} 3 & -1 & -1 & 0 & -1 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathcal{L}^2 = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The communication topology is assumed to switch from $\mathcal{L}^1$ to $\mathcal{L}^2$ at $t_1 = 12$ s. In this example, in order to achieve state consensus (i.e., position, velocity and acceleration consensus). The following cooperative control is used for each robot

$$u_i = a_i^r(t) + \mu_i^{\mathsf{s}}\tilde{\xi}_{i,3}^i - \sum_{j=1}^5 a_{ij}^{\mathsf{s}}\left[\gamma_1^{\mathsf{s}}(z_1 - z_{ij}) - \gamma_2^{\mathsf{s}}(\tilde{\xi}_{i,2}^i - \tilde{\xi}_{j,2}^i) - \gamma_3^{\mathsf{s}}(\tilde{\xi}_{i,3}^i - \tilde{\xi}_{j,3}^i)\right]$$

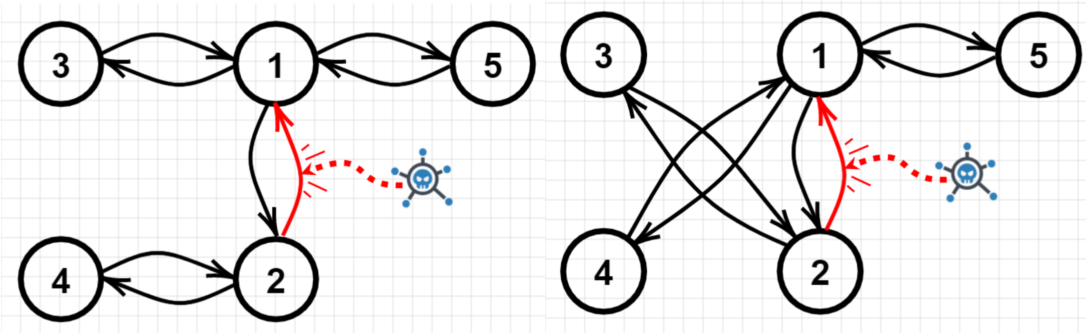
where $\forall i \in \{1, ..., N\}$, μ_i^{s}, γ_1^{s}, γ_2^{s} and γ_3^{s} are the consensus gains set to 5, 4, 3, and, 2.5, respectively, for both possible communication topology modes $\mathsf{s} \in \{1, 2\}$, and $a_i^r(t) = 1ms^{-2}$ is the reference acceleration. Hence, $\forall \mathsf{s} \in \{1, 2\}$, the exchanged signals between agents are given as

$$z_{ki}(t) = \ell_k^{i,\mathsf{s}}(z_i(t) + \check{f}_{ki}^e(t) + \Delta z_{ki}(t))$$

and

$$\hat{z}_i^{kj}(t) = a_{kj}^{\mathsf{s}}(\hat{z}_i^j(t) + f_{kj}^e(t) + \Delta\hat{z}_i^{kj}(t))$$

where $\Delta z_{ki}(t) = 0.1\sin(z_{ki}(t))$, and $\Delta\hat{z}_i^{kj}(t) = 0.01\sin(\hat{z}_i^{kj}(t))$ are noise due to some communication uncertainties.



(a) (b)

Figure 2.17: The corresponding topology graphs, where: (**a**) corresponds to $\mathcal{L}^1$ for $t < 12$ s and (**b**) to $\mathcal{L}^2$ for $t \geqslant 12$ s.

The initial positions of the five agents on the x-axis are given as $\xi_{1,1}(0) = 0$ m, $\xi_{2,1}(0) = 1.5$ m, $\xi_{3,1}(0) = 3$ m, $\xi_{4,1}(0) = 4.5$ m and $\xi_{5,1}(0) = 0.5$ m respectively. The initial velocities and acceleration are set to 0. For each of the mobile robots, the distributed observers are designed to estimate the global state in the desired predefined time $T^1 = T^2 = 3$ s with $T_p^{1,1} = T_p^{2,1} = T_p^{3,1} = T_p^{1,2} = T_p^{2,2} = T_p^{3,2} = 1$ s which satisfies the conditions set out in Proposition 2.2. It is worth mentioning that this choice allows the observers to converge at the same time, hence simplifying the residual evaluation process. The observer parameters are chosen as

$$\phi = [\alpha, \eta, p, q, r]^T = [1, 2, 1.5, 3, 0.5]^T$$

used for each corresponding topology satisfying the conditions in Theorems 2.3–2.4. On the other hand, to obtain the equivalent values, first-order low pass filters are used with cut-off frequency of 100 s^{-1} for the first, second and third dynamics. In order to verify the performance of the proposed scheme, the following two simulation scenarios are carried out on MATLAB. The sampling period is set as $T_s = 10^{-5}s$.

First Scenario: In the 1st scenario, a fault occurs in robot 3 causing an out of control situation that affects its local jerk simulated by the following function $f_3^a(t)$:

$$f_3^a(t) = \begin{cases} 0 & t < 4s \\ 0.5\sin(5t) + 15 & 5s \leqslant t \leqslant\ 8.5s \\ 0 & t > 8.5s \end{cases}$$

This fault only represents a local malfunction in the robot 3 and thus needs to be distinguished from a cyber-attack. It can be clearly seen from Figure 2.18 corresponding to the $1st$ scenario that the residuals generated by the monitoring agents for the monitored agent 3, i.e., r_3^1, r_3^2, r_3^4 and r_3^4 respectively, provide an explicit estimation of f_3^a.

Second Scenario: In the 2nd scenario, a FDIA occurs in information exchanges

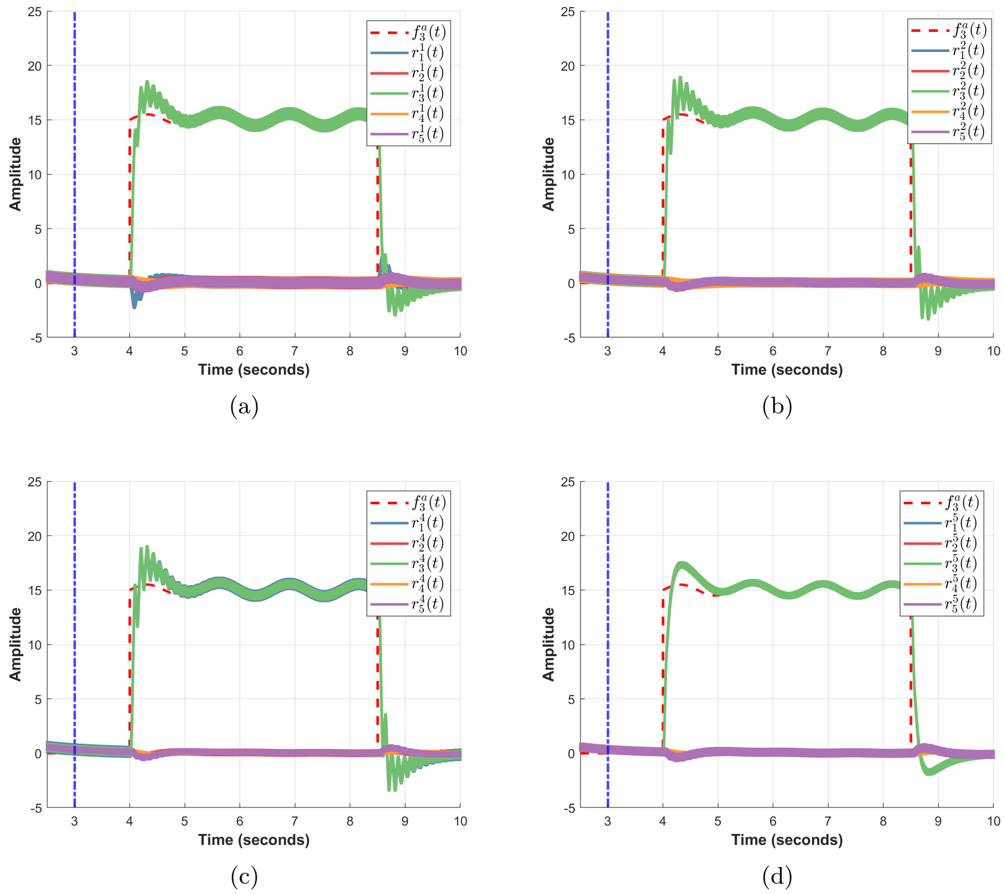


Figure 2.18: Residuals in scenario 1 by agents 1, 2, 4, and 5, shown in sub-figures (**a**), (**b**), (**c**) and (**d**) respectively. The vertical dashed blue line represents the convergence time.

flowing from robots 1 to 2 at $t = T^e = 10$ s, for the first topology such that

$$\check{f}_{12}^{e}(t) = f_{12}^{e}(t) = \begin{cases} 0 & t < 10 \text{ s} \\ 100(1 - e^{1-0.1t}) & t \geqslant 10 \text{ s} \end{cases}$$

Note that the topology switches at $t_1 = 13$ s and $\check{f}_{12}^{e}(t) = f_{12}^{e}(t)$ remains throughout the topology change (see Figure 2.17). Therefore, the gains are computed from Theorem 2.4 and Remark 2.5.2 as

$$\begin{cases} \kappa_1^1 &= 56.98 \\ \kappa_2^1 &= 24.98 \\ \kappa_3^1 &= 24.98 \\ \delta_1^1 &= 0.35 \\ \delta_2^1 &= 1.2 \\ \delta_3^1 &= 1.5 \end{cases} \quad \text{and} \quad \begin{matrix} \kappa_1^2 &= 58.92 \\ \kappa_2^2 &= 24.98 \\ \kappa_3^2 &= 19.98 \\ \delta_1^2 &= 0.33 \\ \delta_2^2 &= 1.2 \\ \delta_3^2 &= 1.5 \end{matrix}$$

It should be recalled that these gains are valid for both scenarios. Figure 2.19 corresponding to the 2nd scenario shows that a cyber-attack in the form of the simulated functions $\check{f}^e_{12}(t)$ and $f^e_{12}(t)$, incident to agent 1 in both topologies, can be distinguished even in the presence of some reasonable communication noise. Indeed, the residual signals r^1_1, r^2_2, r^3_3, r^4_4 and r^5_5 stay around 0 after the cyber-attack appears in the system and throughout the topology change.

Consequently, according to Proposition 2.3, one can distinguish and identify a cyber-attack in the networked MAS.

Remark 2.5.5 *It should be noted that the proposed scheme in this Section is able to detect multiple occurring actuator faults and isolate them. It can also detect the occurrence of at least one attack in the MAS but is not capable of isolating it. Furthermore, as mentioned in Proposition 2.3, the identification process can only be carried out if one type of anomaly (either fault or attack) occurs at a time. As for the number of monitoring agents, similar remarks as 2.3.5 can be observed.*

2.6 Conclusions

In this Chapter, the problem of robust global distributed FDI is solved for: higher order linear connected MASs with unknown disturbances, connected MASs with chained form dynamics and higher order integrator connected MASs subject to cyber-attacks under switching topologies in Sections 2.3, 2.4 and 2.5 respectively. The proposed schemes use cascades of distributed SMOs and the fixed-time stability properties, whereby each agent gives an exact estimate of the global system state and the observer gains are designed through a prescribed time without any a-priori information on the initial conditions of the system. As such, through residual based approaches, any agent/node acts as a monitoring unit to the whole system behaviour and can detect simultaneous faults anywhere in the MAS. In Section 2.5, the previous results are extended to connected MASs with switching topologies subject to communication attacks where a novel distributed cyber-attack identification scheme was proposed. The proposed algorithms act as filters and are thus robust to both measurement noise, communication noise and dynamics disturbances. Illustrative numerical examples are given in Sections 2.3-2.5 to show the efficacy of the proposed schemes.

On the other hand, one could note that the schemes proposed in this Chapter only consider the case of homogeneous MASs and undirected topologies. In the next Chapter, distributed scalable algorithms are developed for heterogeneous MASs where the graph topologies are not required to be undirected.

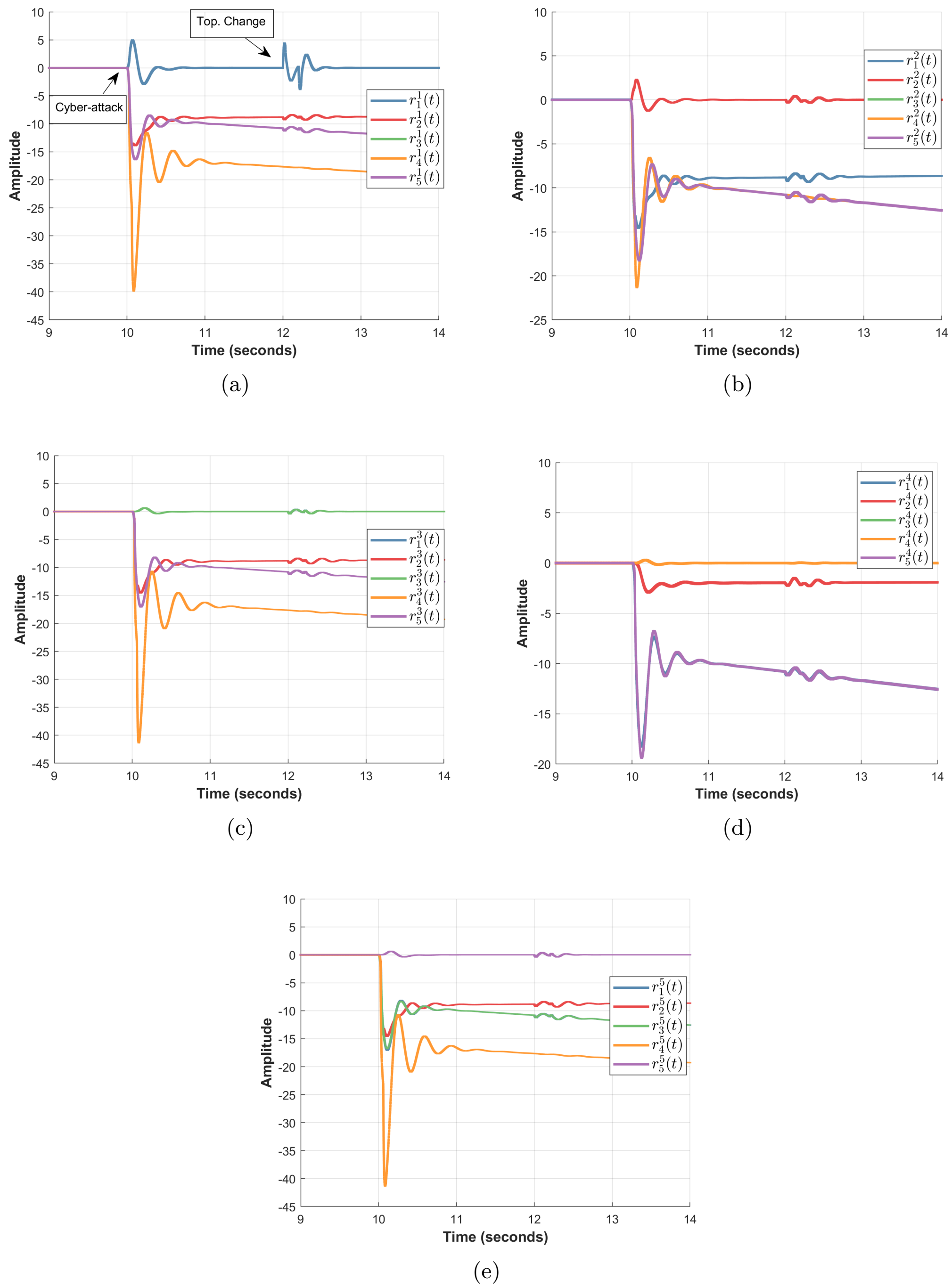


Figure 2.19: Residuals generated by all agents in scenario 2, where: (**a**) Agent 1's residual signals, (**b**) Agent 2's residual signals, (**c**) Agent 3's residual signals, (**d**) Agent 4's residual signals, (**e**) Agent 5's residual signals.

CHAPTER 3

Distributed FDI in Heterogeneous MASs

Content

3.1 Introduction

This Chapter is concerned with distributed FDI in heterogeneous MASs where communication topologies are not required to be undirected. In the previous Chapter, FDI schemes were designed for homogeneous MASs. However, due to the growing size, complexity and heterogeneity of MASs, the design of fault and attack detection observers is rendered more intricate. Indeed, multiple factors are at play when FDI design in MASs is concerned, namely, the dimensions of the FDI module and the communication/implementation costs. This Chapter takes concerns into account.

One can make the observation that in most works, FDI observers are mostly based on full state observers (e.g., [Liu *et al.* 2016a, Chadli *et al.* 2017]). However, it could be noted that any type of observer could be employed for FDI purposes, as long as an estimate of the output is obtained. It could thus be argued that it is not necessary to estimate all of the state variables as far as FDI is concerned. Hence, Section 3.2 of this Chapter concerns distributed actuator FDI in heterogeneous MASs using output observers, i.e., observer which are based solely on the input-output relationships of the system.

On the other hand, the issue of FDIs in heterogeneous MASs subject to unknown disturbances, faults and attacks is still a challenging issue. Indeed, on top of actuator and sensor faults, MASs can be subjected to multiple types of cyber-attacks [Zhang *et al.* 2021b, Khalaf *et al.* 2018, Gallo *et al.* 2018a, Lu & Yang 2018, Boem *et al.* 2017, Smith 2015], which are thoroughly described in Chapter 1. Thus, Section 3.3 of this Chapter is concerned with the issue of distributed FDI in heterogeneous MASs under unknown disturbances, actuator/sensor faults and communication attacks. There is a multitude of ways to detect and isolate faults and cyber-attacks in MASs (see [Song & He]). Some works proposed centralised architectures to detect faults or attacks [Pasqualetti *et al.* 2013, Jan *et al.* 2015], due to their simplicity, whereby the analysis of all data is done by a central unit. However, in order to avoid long distance data transmissions, reduce complexity and improve scalability namely in larger systems, the detection and isolation process should be distributed.

A great deal of existing works in the literature either focus on linear and/or homogeneous MASs [Khan *et al.* 2020, Quan *et al.* 2018, Menon & Edwards 2013, Davoodi *et al.* 2013, Chadli *et al.* 2017, Davoodi *et al.* 2016, Li *et al.* 2021, Liang *et al.* 2021], do not consider the effect of disturbances [Khan *et al.* 2020, Teixeira *et al.* 2014], or do not consider the effect of measurement and communication noise [Quan *et al.* 2018, Liu *et al.* 2016a, Han *et al.* 2019]. However, it is a well known fact that disturbances and noise are practically inevitable. Furthermore, some works only consider actuator faults [Quan *et al.* 2018, Liang *et al.* 2021, Liu *et al.* 2016a, Wu *et al.* 2019] or only sensor faults [Davoodi *et al.* 2013, Chadli *et al.* 2017, Davoodi *et al.* 2016]. In [Chadli *et al.* 2017, Teixeira *et al.* 2014, Liu *et al.* 2016a, Liu *et al.* 2016b], UIOs were used in fault detection. Nevertheless, most of the existing works on fault detection using UIOs consider that the generated residual signals are completely decoupled from the unknown input. Indeed, they usually require a strict rank condition to decouple the unknown input vector, which can be infeasible. In [Liu *et al.* 2016a] for instance, an UIO residual based scheme for nonlinear homogeneous MASs with actuator faults was proposed, where faults and disturbances were decoupled from the error dynamics assuming some rank conditions. In [Chadli *et al.* 2017], UIOs were combined with the mixed $\mathcal{H}_-/\mathcal{H}_\infty$ method for fault detection purpose where only sensor faults were considered. Furthermore, the $\mathcal{H}_-$ performance index method proposed therein as well as in [Davoodi *et al.* 2013, Davoodi *et al.* 2016] for instance, is only applicable when the distribution matrix of the sensor faults is of full column rank. In Section 3.3, one contribution is to relax such condition using the finite frequency approach introduced in [Iwasaki *et al.* 2005]. Furthermore, in [Davoodi *et al.* 2016, **?**] for instance, multiple faults cannot occur in the MAS, which is a drawback, especially in large-sized MASs.

In [Quan *et al.* 2018, Davoodi *et al.* 2016, Li *et al.* 2021, Liang *et al.* 2021, Liu *et al.* 2016a], information from neighbouring FDI filters was transmitted among agents, which may weaken the distributed property of the detection scheme. Indeed, if and when an observer fails to accurately give an estimate at a given instant for an agent, all surrounding observers in its neighbourhood are compromised, which in turn compromises their respective neighbours' observers, thus creating a destructive snowball effect that might lead to confusing results, instigate false alarms, etc. In Section 3.3, such drawback is removed since observers do not communicate between themselves. Unlike [Quan *et al.* 2018, Li *et al.* 2021, Liang *et al.* 2021, Liu *et al.* 2016a], where the topology is assumed to be undirected, a directed communication graph is considered in this Chapter. Additionally, the proposed scheme in Section 3.3, does not require knowledge beyond its 1-hop neighbourhood and is independent on the graph topology of the overall MAS, making it more scalable. Furthermore, as opposed to the detection filters proposed in [Wu *et al.* 2019, Liu *et al.* 2016a, Quan *et al.* 2018, Liang *et al.* 2021] where their size increases as the graph topology grows, in the proposed scheme, the size of the filter is only limited to the size of the neighbourhood of each agent independently, hence improving the scalability and reducing the computational burdens. Given the limitations discussed above with respect to the existing studies, the main contributions of this Chapter are summarised as follows

- In Section 3.2 [Taoufik *et al.* 2020a]: An output observer design methodology for fault detection and isolation is proposed for MASs with linear dynamics. As mentioned before, the main advantage of this approach is that the design, being dependant only on the input-output relations, renders the computational cost and scalability very effective compared to other FDI approaches that employ the whole state estimation of the agent and its neighbours as a basis for their design.
- In Section 3.3 [Taoufik *et al.* 2021a]: A more general problem is studied where actuator, sensor and communication faults are considered in the robust detection and isolation process for Lipschitz nonlinear heterogeneous MASs with disturbances and communication parameter uncertainties, without global knowledge about the communication graph and under directed graphs. A distributed finite-

frequency mixed $\mathcal{H}_-/\mathcal{H}_\infty$ nonlinear UIO based FDI scheme is designed, such that actuator and sensor faults along with the communication faults are treated separately. Hence, the rank condition on the measurement fault distribution matrix as required by [Davoodi *et al.* 2016, Li *et al.* 2021] for instance, is relaxed. Additionally, the scheme is capable of detecting and distinguishing multiple faults and attacks at a given time instant. Sufficient conditions in terms of a set of LMIs are provided for the proposed finite-frequency $\mathcal{H}_-/\mathcal{H}_\infty$ UIO based method, where the coupling between Lyapunov matrices and the observer matrices is avoided. This LMI characterisation enables to reduce conservatism by introducing additional design variables.

- In Section 3.4, the main conclusions of this Chapter are drawn.

Each of the Sections 3.2 and 3.3, are divided into three Subsection each: A problem formulation Subsection which lays out the system description and defines the information exchange, a main result Subsection, where the proposed methodology is detailed and finally a simulation example Subsection where an illustrative simulation is carried out to show the effectiveness of the proposed schemes. Fig. 2.2 depicts a summary of the contribution of each Section. Table 3.1 shows a brief comparison with the contributions of this Chapter and some existing works.

Topology Description

In this Chapter, the topology is represented by a directed graph $\mathcal{Q} = (\mathcal{N}, \mathcal{F})$, where $\mathcal{V} = \{1, \ldots, N\}$ is the node set and $\mathcal{F} \subseteq \mathcal{N} \times \mathcal{N}$ is the edge set. It is described by an adjacency matrix $\mathcal{A} \in \mathbb{R}^{N\times N}$ that contains positive weight entries. If information flows from node j to i, then $a_{ij} > 0$, otherwise $a_{ij} = 0$. The neighbouring set of node i, denoted by $\mathcal{N}_i \subseteq \mathcal{V} \subset \{1, ..., N\}\backslash\{i\},\ N_i = |\mathcal{N}_i|$, is the subset of nodes that node i can sense and interact with. Alternatively, one could note $\mathcal{N}_i = \{i_1, i_2, ..., i_{N_i}\} \subseteq \{1, ..., N\}$. It is considered that $\mathcal{Q}$ is strongly connected.

Notations

Let us recall some of the notations employed in this Chapter. The superscript T stands for the matrix transpose. I is the identity matrix with appropriate dimensions. For a given $k \geq 0$, we will denote by $\int_0^t \overbrace{\cdots}^{k \text{ times}} \int_0^t f(\tau)d\tau...d\tau = \mathcal{I}_k\{f(t)\}$, the k integrations of the function $f(t)$ with respect to time, i.e., $\mathcal{I}_0\{f(t)\} = f(t)$. Given a transfer function $T_{xy}(s)$ linking y to x, its $\mathcal{H}_\infty$ norm is defined as

$$||T_{xy}||_\infty = \sup_\omega \bar{\sigma}(T_{xy}(j\omega)),$$

where $\bar{\sigma}$ is the maximum singular value of $T_{xy}(s)$. Its $\mathcal{H}_-$ index is defined as

$$||T_{xy}||_- = \inf_\omega \underline{\sigma}(T_{xy}(j\omega)),$$

where $\underline{\sigma}$ is the minimum singular value of $T_{xy}(s)$.

For a square matrix A, $\mathbf{He}(A) = A + A^*$ where the superscript A^* corresponds to the conjugate of A. $\mathrm{tr}(A)$ is the trace of A. $\mathbf{1}_n$ and I_n refer to a column of all entries 1 and an identity matrix respectively and of dimensions n. $0_{m\times n}$ denotes a null matrix of dimension $m\times n$. j refers to the imaginary unit. $\mathrm{diag}(a_1, a_2, ..., a_n)$ denotes the diagonal matrix containing $a_1, a_2, ..., a_n$ on the diagonal. $\mathrm{Blkdiag}(A_1, A_2, ..., A_n)$ denotes the block diagonal matrix with matrices $A_1, A_2, ..., A_n$ on the diagonal. $\mathrm{Col}(A_1, A_2, ..., A_n)$ denotes the column block matrix $(A_1^T, A_2^T, ..., A_n^T)^T$. For the sake of simplicity the time argument is omitted when it's not required for clarity.

	Linear	*D&N*	*Hetero.*	*A&S Faults*	*Attacks*	*UTR*	*RISR*	*ACIR*	*GK*
[Khan *et al.* 2020]	Yes	No	No	No	Yes	Yes	No	No	Yes
[Quan *et al.* 2018]	Yes	No	No	No	No	Yes	No	No	Yes
[Davoodi *et al.* 2016]	Yes	Yes	No	No	No	Yes	Yes	No	Yes
[Teixeira *et al.* 2014]	Yes	No	No	No	Yes	Yes	No	No	Yes
[Chadli *et al.* 2017]	Yes	Yes	Yes	No	No	Yes	Yes	No	No
[Liu *et al.* 2016b]	No	No	No	No	No	Yes	No	Yes	Yes
[Teixeira *et al.* 2010]	Yes	No	No	No	Yes	Yes	No	No	Yes
[Taoufik *et al.* 2020b]	Yes	No	No	No	Yes	Yes	No	No	No
[Taoufik *et al.* 2020a]	Yes	No	Yes	No	No	No	Yes	No	No
[Taoufik *et al.* 2021a]	No	Yes	Yes	Yes	Yes	No	No	No	No

Table 3.1: Brief comparison with some existing works, where the following acronyms are used: D&N: Both Disturbances and Noise; Hetero.: Heterogeneous; A&S Faults: Both Actuator and Sensor Faults; UTR: Undirected Topology Required; RISR: Relative Information Sensors Required; AGIR: Access to the Collective Input Required; GK: Global Knowledge.

3.2 Actuator FDI in heterogeneous MASs with Linear Dynamics

3.2.1 Problem Formulation

Consider a team of N heterogeneous agents indexed $i = 1, 2, \ldots, N$. The dynamics of the i^{th} agent are given by the following linear dynamic model

$$\begin{cases} \dot{\xi}_i(t) = A_i\xi_i(t) + B_i(u_i(t) + f_i(t)) \\ y_i(t) = C_i\xi_i(t) \end{cases} \tag{3.1}$$

where $\xi_i = [\xi_{i,1}, \xi_{i,2}, ..., \xi_{i,n}]^T \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}$ and $y_i(t) \in \mathbb{R}$ are the state vector, the control input and the output signal respectively of the i^{th} agent. $f_i(t) \in \mathbb{R}$ represents the fault signal where $f_i(t) = 0$ is equivalent to a fault-free system while $f_i(t) \neq 0$ indicates the occurrence of an actuator fault in agent i. $\forall i \in \{1, 2, ..., N\}$, A_i, B_i and C_i are constant matrices with appropriate dimensions. The following Assumption is considered in this Section.

Assumption 3.2.1 *Each agent measures its output and the relative output with its neighbours.*

The relative measured outputs are defined as $y_{ij}(t) = y_i(t) - y_j(t)$ where $j \in \mathcal{N}_i = \{i_1, i_2, ..., i_{N_i}\} \subseteq [1, N]$ denotes the set of agents that agent i can sense (i^{th} agent's immediate neighbours). The vector $X_i = [(\xi_i)^T, (\xi_{i_1})^T, ..., (\xi_{i_{N_i}})^T]^T \in \mathbb{R}^{n \times (N_i+1)}$ includes the i^{th} agent states and the states of its neighbours. According to the definition of the relative information $y_{ij}(t)$, the following virtual model for the i^{th} agent can be laid out as follows

$$\begin{cases} \dot{X}_i(t) = \mathbf{A}_i X_i(t) + \mathbf{B}_i \mathbf{U}_i(t) \\ Z_i(t) = \mathbf{C}_i X_i(t) \end{cases} \tag{3.2}$$

where $Z_i(t) = [y_i, y_{ii_1}, y_{ii_2}, ..., y_{ii_{N_i}}]^T \in \mathbb{R}^{N_i+1}$ is all the information that agent i can sense, $\mathbf{U}_i(t) = [(u_i + f_i), (u_{i_1} + f_{i_1}), (u_{i_2} + f_{i_2}), ..., (u_{i_{N_i}} + f_{i_{N_i}})]^T \in \mathbb{R}^{N_i}$, $\mathbf{A}_i = \text{Blkdiag}(A_i, A_{i_1}, ..., A_{i_{N_i}})$, $\mathbf{B}_i = \text{Blkdiag}(B_i, B_{i_1}, ..., B_{i_{N_i}})$, while $\mathbf{C}_i$ is defined as

$$\mathbf{C}_i = \begin{bmatrix} C_i & 0 & \dots & 0 \\ C_i & -C_{i_1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_i & 0 & \dots & -C_{i_{N_i}} \end{bmatrix} \tag{3.3}$$

Using the virtual model (3.2), the agent i's model includes its information as well as its neighbours' relative information. This is the basis for elaborating the observer with the aim that an agent can detect not only its own faults but also those that occur in its neighbours using relative outputs. This Section studies the design of a distributed residual generator capable of achieving this objective through the use of output observers.

3.2.2 Main Result

In this Subsection, an output observer design is first provided in Subsubsection 3.2.2.1 followed by an application to actuator fault detection in MASs in Subsubsection 3.2.2.2.

3.2.2.1 An Output Observer Design

In this section, an output observer is proposed. For that, consider the following single input linear system

$$\begin{aligned} & y^{(n)}(t) + a_{n-1}y^{(n-1)}(t) + \cdots + a_1\dot{y}(t) + a_0y(t) \\ & \quad = b_0u(t) + b_1\dot{u}(t) + \cdots + b_mu^{(m)}(t) \end{aligned} \tag{3.4}$$

where $y^{(k)}$ denotes the k^{th} time derivative of y and $m < n$.. Let us first define some notations to be used throughout this Section, in the form of the following Remark.

Remark 3.2.1 *It can be noted that*

$$\mathcal{I}_1\{y^{(k)}(t)\} = \int_0^t y^{(k)}(\tau)d\tau = y^{(k-1)}(t) - y^{(k-1)}(0) \tag{3.5}$$

In a similar manner

$$\mathcal{I}_2\{y^{(k)}(t)\} = \int_0^t \int_0^\lambda y^{(k)}(\tau)d\tau d\lambda = y^{(k-2)}(t) - y^{(k-2)}(0) - y^{(k-1)}(0)t \tag{3.6}$$

Generalising this notation, one gets

- *for* $p \leqslant k$
$$\mathcal{I}_p\{y^{(k)}(t)\} = y^{(k-p)}(t) - \sum_{i=0}^{p-1} y^{(k-p+i)}(0)\frac{t^i}{i!}, \tag{3.7}$$

- *for* $p > k$
$$\mathcal{I}_p\{y^{(k)}(t)\} = \mathcal{I}_{p-k}\{y(t)\} - \sum_{i=0}^{k-1} y^{(i)}(0)\frac{t^{p-k+i}}{(p-k+i)!}, \tag{3.8}$$

Let us integrate each side of system (3.4) $(n-1)$ times. It can be shown that the $(n-1)\,th$ derivative of the Left-Hand-Side (LHS) of Equation (3.4) is given as

$$\begin{aligned} \mathrm{LHS}^{(n-1)} = {} & \dot{y}(t) + \sum_{k=0}^{n-1} a_k\mathcal{I}_{(n-1-k)}\{y(t)\} - y(0)P_0(t) - \sum_{k=1}^{n-1} y^{(k)}(0)P_k(t) \\ & - \sum_{i=1}^{n-1} y^{(i)}(0)\frac{t^{i-1}}{(i-1)!} \end{aligned} \tag{3.9}$$

where

$$P_k(t) = a_{n-1}\frac{t^k}{k!} + a_{n-2}\frac{t^{k+1}}{(k+1)!} + \cdots + a_{k+1}\frac{t^{n-2}}{(n-2)!} = \sum_{i=0}^{n-2-k} a_{(n-1-i)}\frac{t^{k+i}}{(k+i)!} \tag{3.10}$$

and that the $(n-1)\,th$ derivative of its Right-Hand-Side (RHS) is given as

$$\mathrm{RHS}^{(n-1)} = \sum_{k=0}^{m} b_k\mathcal{I}_{(n-1-k)}\{u(t)\} - \sum_{k=0}^{m} u^{(k)}(0)Q_k(t) \tag{3.11}$$

where
$$Q_k(t) = \sum_{i=0}^{m-2-k} b_{(m-1-i)} \frac{t^{k+i}}{(k+i)!}. \tag{3.12}$$

Combining (3.9) and (3.11), the following is obtained

$$\begin{aligned}\dot{y}(t) - y(0)P_0(t) = &-\sum_{k=0}^{n-1} a_k \mathcal{I}_{(n-1-k)}\{y(t)\} + \sum_{k=1}^{n-1} y^{(k)}(0)P_k(t) + \sum_{i=1}^{n-1} y^{(i)}(0)\frac{t^{i-1}}{(i-1)!} \\ &+ \sum_{k=0}^{m} b_k \mathcal{I}_{(n-1-k)}\{u(t)\} - \sum_{k=0}^{m} u^{(k)}(0)Q_k(t)\end{aligned} \tag{3.13}$$

In the above expression only the initial conditions $y^{(k)}(0); k = 1, ..., (n-1)$ are unknown except $y(0)$, which is known. The following observer for system (3.13) is proposed

$$\begin{aligned}\dot{\hat{y}}(t) - y(0)P_0(t) = &-\sum_{k=0}^{n-1} a_k \mathcal{I}_{(n-1-k)}\{\hat{y}(t)\} + \sum_{k=1}^{n-1} \hat{y}^{(k)}(0)P_k(t) + \sum_{i=1}^{n-1} \hat{y}^{(i)}(0)\frac{t^{i-1}}{(i-1)!} \\ &+ \sum_{k=0}^{m} b_k \mathcal{I}_{(n-1-k)}\{u(t)\} - \sum_{k=0}^{m} u^{(k)}(0)Q_k(t) \\ &+ \sum_{i=0}^{n-1} k_{n-1-i}\mathcal{I}_i\{y(t) - \hat{y}(t)\}\end{aligned} \tag{3.14}$$

where k_i, $\forall i \in \{0, 1, ..., n-1\}$ are observer gains. By setting the estimation error as $\varepsilon(t) = y(t) - \hat{y}(t)$, its dynamics are

$$\begin{aligned}\dot{\varepsilon}(t) = &-\sum_{k=0}^{n-1} a_k \mathcal{I}_{(n-1-k)}\{\varepsilon(t)\} + \sum_{k=1}^{n-1} \varepsilon^{(k)}(0)P_k(t) + \sum_{i=1}^{n-1} \varepsilon^{(i)}(0)\frac{t^{i-1}}{(i-1)!} \\ &- \sum_{i=0}^{n-1} k_{n-1-i}\mathcal{I}_i\{\varepsilon(t)\}\end{aligned} \tag{3.15}$$

The stability of (3.15) is shown through Theorem 3.1.

Theorem 3.1: [Taoufik *et al.* 2020a]

The output estimation error dynamics (3.15) are asymptotically stable for any arbitrary initial conditions $\hat{y}^{(i)}(0)$, $\forall i \in \{1, ..., (n-1)\}$, if the polynomial $s^n + \sum_{i=0}^{n-1}(a_i + k_i)s^i$ is Hurwitz and $a_0 + k_0 \neq 0$.

Proof Applying the Laplace transform on equation (3.15) yields

$$\begin{aligned}sE(s) - E(0) = &-\sum_{k=0}^{n-1} a_k \frac{1}{s^{(n-1-k)}}E(s) + \sum_{i=1}^{n-1} \varepsilon^{(i)}(0)\frac{1}{s^i} + \sum_{k=1}^{n-1} \varepsilon^{(k)}(0)\mathcal{L}\{P_k(t)\} \\ &- (k_{n-1} + \frac{k_{n-2}}{s} + ... + \frac{k_0}{s^{n-1}})E(s)\end{aligned} \tag{3.16}$$

where $E(s) = \mathcal{L}\{\varepsilon(t)\}$. Since

$$\mathcal{L}\{P_k(t)\} = \sum_{i=k}^{n-2} a_{(n-1+k-i)} \mathcal{L}\{\frac{t^i}{i!}\},$$

one has

$$\begin{aligned} sE(s) = &-\sum_{k=0}^{n-1} a_k \frac{1}{s^{(n-1-k)}} E(s) + \sum_{k=1}^{n-1} \varepsilon^{(k)}(0) \sum_{i=k}^{n-2} a_{(n-1+k-i)} \frac{1}{s^{i+1}} \\ &+ \sum_{i=1}^{n-1} \varepsilon^{(i)}(0) \frac{1}{s^i} - (k_{n-1} + \frac{k_{n-2}}{s} + ... + \frac{k_0}{s^{n-1}}) E(s) \end{aligned} \tag{3.17}$$

In other words

$$\begin{aligned} &\left(s + k_{n-1} + a_{n-1} + \frac{k_{n-2} + a_{n-2}}{s} + ... + \frac{k_0 + a_0}{s^{n-1}}\right) E(s) \\ &= (s + \sum_{i=0}^{n-1} \frac{k_i + a_i}{s^{n-1-i}}) E(s) \\ &= \sum_{k=1}^{n-1} \varepsilon^{(k)}(0) \sum_{i=k}^{n-2} a_{(n-1+k-i)} \frac{1}{s^{i+1}} + \sum_{i=1}^{n-1} \varepsilon^{(i)}(0) \frac{1}{s^i} \end{aligned} \tag{3.18}$$

That is

$$E(s) = \frac{\sum_{k=1}^{n-1} \varepsilon^{(k)}(0) \sum_{i=k}^{n-2} a_{(n-1+k-i)} \dfrac{1}{s^{i+1}} + \sum_{i=1}^{n-1} \varepsilon^{(i)}(0) \frac{1}{s^i}}{s + \sum_{i=0}^{n-1} \dfrac{a_i + k_i}{s^{(n-1-i)}}} \tag{3.19}$$

By multiplying the denominator and nominator in equation (3.19) by $s^{(n-1)}$ one gets

$$E(s) = \frac{N(s)}{D(s)} \tag{3.20}$$

where

$$N(s) = \sum_{k=1}^{n-1} \varepsilon^{(k)}(0) \sum_{i=k}^{n-2} a_{(n-1+k-i)} s^{n-2-i} + \sum_{i=1}^{n-1} \varepsilon^{(i)}(0) s^{n-1-i} \tag{3.21}$$

and

$$D(s) = s^n + \sum_{i=0}^{n-1} (a_i + k_i) s^i \tag{3.22}$$

Hence, observer gains k_i, $\forall i \in \{0, 1, ..., n-1\}$ are chosen such that the polynomial (3.22) is stable and $a_0 + k_0 \neq 0$. Therefore, applying the final value theorem, where $\varepsilon_\infty = \lim_{t \to +\infty} \varepsilon(t) = \lim_{s \to 0} sE(s)$, the following can be concluded

$$\varepsilon_\infty = \lim_{s \to 0} \frac{s \left(\sum_{k=1}^{n-1} \varepsilon^{(k)}(0) a_{(k+1)} s^0 + \varepsilon^{(n-1)}(0) s^0 \right)}{a_0 + k_0} = 0 \tag{3.23}$$

It can thus be deduced that the error dynamics are asymptotically stable. As such, the estimate $\hat{y}(t)$ converges to the output $y(t)$ regardless of the initial conditions. This concludes the proof.

3.2.2.2 Application to Actuator FDI in MASs

In this section, the previously defined output observer is used to generate residual signals capable of detecting actuator faults occurring in the concerned agent as well as its neighbours in a distributed manner, by using relative information. The virtual model (3.2) can be rewritten in the following transfer function form

$$\begin{cases} \dfrac{Y_i(s)}{U_i(s)+F_i(s)} = \dfrac{b_{i,0}+b_{i,1}s+...+b_{i,m}s^m}{s^n+a_{i,n-1}s^{n-1}+...+a_{i,1}s+a_{i,0}} \\ \dfrac{Z_{ii_1}(s)}{U_{i_1}(s)+F_{i_1}(s)} = \dfrac{b_{i_1,0}+b_{i_1,1}s+...+b_{i_1,m}s^m}{s^n+a_{i_1,n-1}s^{n-1}+...+a_{i_1,1}s+a_{i_1,0}} \\ \vdots \\ \dfrac{Z_{ii_{N_i}}(s)}{U_{i_{N_i}}(s)+F_{i_{N_i}}(s)} = \dfrac{b_{i_{N_i},0}+b_{i_{N_i},1}s+...+b_{i_{N_i},m}s^m}{s^n+a_{i_{N_i},n-1}s^{n-1}+...+a_{i_{N_i},1}s+a_{i_{N_i},0}} \end{cases} \tag{3.24}$$

with $Y_i(s)=\mathcal{L}\{y_i(t)\}$, $Z_{ij}(s)=\mathcal{L}\{y_{ij}(t)\}$, $U_i(s)=\mathcal{L}\{u_i(t)\}$ and $F_i(s)=\mathcal{L}\{f_i(t)\}$, $\forall i\in\{1,2,...,N\}$ and corresponding $j\in\mathcal{N}_i=\{i_1,i_2,...,i_{N_i}\}\subseteq\{1,...,N\}$. Parameters $b_{i,p}$, $\forall p\in\{0,...,m\}$ and $a_{i,p}$, $\forall p\in\{0,...,n-1\}$ are the transfer function coefficients.

Let us set

$$\begin{cases} \mathcal{A}_i^T = [a_{i,n-1},a_{i,n-2},...,a_{i,1},a_{i,0}] \\ \mathcal{B}_i^T = [b_{i,m},b_{i,m-1},...,b_{i,1},b_{i,0}] \end{cases} \tag{3.25}$$

and

$$\begin{cases} \mathcal{Y}_i = [y_i^{(n-1)}(t),y_i^{(n-2)}(t),...,\dot{y}_i(t),y_i(t)]^T \\ \mathcal{U}_i = [u_i^{(m)}(t),u_i^{(m-1)}(t),...,\dot{u}_i(t),u_i(t)]^T \\ \mathcal{F}_i = [f_i^{(m)}(t),f_i^{(m-1)}(t),...,\dot{f}_i(t),f_i(t)]^T \end{cases} \tag{3.26}$$

In the time domain, the first term in (3.24) can be written as

$$y_i^{(n)}(t) = -\mathcal{A}_i^T\mathcal{Y}_i + \mathcal{B}_i^T(\mathcal{U}_i+\mathcal{F}_i) \tag{3.27}$$

By integrating it $(n-1)$ times, one gets

$$\dot{y}_i(t) = -\mathcal{A}_i^T\mathbb{Y}_i + \mathcal{B}_i^T(\mathbb{U}_i+\mathbb{F}_i) + \delta_i^0(t) + \phi_i^0(t) \tag{3.28}$$

where the polynomial $\delta_i^0(t)$ is a known function which depends on the known initial conditions $y_i(0)$ and $u_i(0)$, of the i^{th} agent, while the polynomial $\phi_i^0(t)$ is an unknown function that is dependent on the unknown initial conditions of the agent, i.e., $\phi_i^0(t) =$

$f(\dot{y}(0), \ddot{y}_i(0), y_i^{(3)}(0), ...)$. In particular, these vectors are defined as

$$\begin{cases} \delta_i^0(t) & = \mathcal{A}_i^T \begin{pmatrix} y_i(0) \\ \vdots \\ \mathcal{I}_{n-2}(y_i(0)) \\ 0 \end{pmatrix} - \mathcal{B}_i^T \begin{pmatrix} u_i(0) \\ \vdots \\ \mathcal{I}_{n-2}(u_i(0)) \\ 0 \end{pmatrix} \\ \phi_i^0(t) & = \mathcal{A}_i^T \Phi_i - \mathcal{B}_i^T \Psi_i + \sum_{j=0}^{n-2} y^{j+1}(0) \dfrac{t^j}{j!} \end{cases} \tag{3.29}$$

where

$$\Phi_i = \begin{pmatrix} \mathcal{R}_{n-2}^i(t) \\ \vdots \\ \mathcal{I}_{n-3}(\mathcal{R}_1^i(t))) \\ 0 \\ 0 \end{pmatrix}, \Psi_i = \begin{pmatrix} \mathcal{S}_{m-1}^i(t) \\ \vdots \\ \mathcal{I}_{n-3}(\mathcal{S}_1^i(t))) \\ 0 \\ 0 \end{pmatrix}.$$

with

$$\begin{cases} \mathcal{R}_k^i = \sum_{j=0}^{k} y^j(0) \dfrac{t^j}{j!} - y_i(0) \\ \mathcal{S}_k^i = \sum_{j=1}^{k} u^j(0) \dfrac{t^j}{j!} - u_i(0) \end{cases}$$

The matrices $\mathbb{Y}_i$, $\mathbb{U}_i$ and $\mathbb{F}_i$ are given by

$$\begin{cases} \mathbb{Y}_i = [y_i(t), ..., \mathcal{I}_{n-2}(y_i(t)), \mathcal{I}_{n-1}(y_i(t))]^T \\ \mathbb{U}_i = [\mathcal{I}_{n-1-m}(u_i(t)), ..., \mathcal{I}_{n-2}(u_i(t)), \mathcal{I}_{n-1}(u_i(t))]^T \\ \mathbb{F}_i = [\mathcal{I}_{n-1-m}(f_i(t)), ..., \mathcal{I}_{n-2}(f_i(t)), \mathcal{I}_{n-1}(f_i(t))]^T \end{cases} \tag{3.30}$$

It is reasonable to assume that the system is initially fault free and thus the initial conditions of fault signals $f_i(0), \forall i$ are null.

Similarly to Eq. (3.28), one can obtain, for the other terms in (3.24)

$$\begin{cases} \dot{y}_{ii_1}(t) & = -\mathcal{A}_{i_1}^T \mathbb{Y}_{ii_1} + \mathcal{B}_{i_1}^T (\mathbb{U}_i + \mathbb{F}_i) + \delta_{i_1}^0(t) + \phi_{i_1}^0(t) \\ \vdots & \\ \dot{y}_{ii_{N_i}}(t) & = -\mathcal{A}_{i_{N_i}}^T \mathbb{Y}_{ii_{N_i}} + \mathcal{B}_{i_{N_i}}^T (\mathbb{U}_{i_{N_i}} + \mathbb{F}_{i_{N_i}}) + \delta_{i_{N_i}}^0(t) + \phi_{i_{N_i}}^0(t) \end{cases} \tag{3.31}$$

From equations (3.30) and (3.31), the following compact form is obtained

$$\begin{aligned} \dot{Z}_i(t) &= \begin{pmatrix} \dot{y}_i(t) \\ \dot{y}_{ii_1}(t) \\ \vdots \\ \dot{y}_{ii_{N_i}}(t) \end{pmatrix} \\ &= -\bar{\mathcal{A}}_i \begin{pmatrix} \mathbb{Y}_i \\ \mathbb{Y}_{ii_1} \\ \vdots \\ \mathbb{Y}_{ii_{N_i}} \end{pmatrix} + \bar{\mathcal{B}}_i \begin{pmatrix} \mathbb{U}_i + \mathbb{F}_i \\ \mathbb{U}_{i_1} + \mathbb{F}_{i_1} \\ \vdots \\ \mathbb{U}_{i_{N_i}} + \mathbb{F}_{i_{N_i}} \end{pmatrix} + \begin{pmatrix} \delta_i^0(t) + \phi_i^0(t) \\ \delta_{i_1}^0(t) + \phi_{i_1}^0(t) \\ \vdots \\ \delta_{i_{N_i}}^0(t) + \phi_{i_{N_i}}^0(t) \end{pmatrix} \end{aligned} \tag{3.32}$$

with $\bar{\mathcal{A}}_i = \text{Blkdiag}(\mathcal{A}_i^T, \mathcal{A}_{i_1}^T, ..., \mathcal{A}_{i_{N_i}}^T)$ and $\bar{\mathcal{B}}_i = \text{Blkdiag}(\mathcal{B}_i^T, \mathcal{B}_{i_1}^T, ..., \mathcal{B}_{i_{N_i}}^T)$.

Applying the observer structure (3.14) on system (3.32) gives

$$\begin{aligned}\dot{\hat{Z}}_i(t) = \begin{pmatrix} \dot{\hat{y}}(t) \\ \dot{\hat{y}}_{ii_1}(t) \\ \vdots \\ \dot{\hat{y}}_{ii_{N_i}}(t) \end{pmatrix} = -\bar{\mathcal{A}}_i \begin{pmatrix} \hat{\mathbb{Y}}_i \\ \hat{\mathbb{Y}}_{ii_1} \\ \vdots \\ \hat{\mathbb{Y}}_{ii_{N_i}} \end{pmatrix} + \bar{\mathcal{B}}_i \\ \times \begin{pmatrix} \mathbb{U}_i \\ \mathbb{U}_{i_1} \\ \vdots \\ \mathbb{U}_{i_{N_i}} \end{pmatrix} + \begin{pmatrix} \delta_i^0(t) \\ \delta_{i_1}^0(t) \\ \vdots \\ \delta_{i_{N_i}}^0(t) \end{pmatrix} + K_i \begin{pmatrix} \mathbb{Y}_i - \hat{\mathbb{Y}}_i \\ \mathbb{Y}_{ii_1} - \hat{\mathbb{Y}}_{ii_1} \\ \vdots \\ \mathbb{Y}_{ii_{N_i}} - \hat{\mathbb{Y}}_{ii_{N_i}} \end{pmatrix}\end{aligned} \tag{3.33}$$

where $\hat{Z}_i(t)$ is the estimate of $Z_i(t)$ and K_i contains the observer gains and is defined as

$$\begin{aligned} K_i &= \text{Blkdiag}\Big([k^i_{i,n-1}, k^i_{i,1}, ..., k^i_{i,0}], [k^i_{i_1,n-1}, k^i_{i_1,1}, ..., k^i_{i_1,0}], \\ &\quad ..., [k^i_{i_{N_i},n-1}, k^i_{i_{N_i},1}, ..., k^i_{i_{N_i},0}]\Big) \end{aligned} \tag{3.34}$$

The following Theorem can be presented.

Theorem 3.2: [Taoufik *et al.* 2020a]

The observer (3.33) is an asymptotic observer for system (3.32) in the absence of faults, for any arbitrary functions $\delta_p^0(t)$ and $\phi_p^0(t)$, $\forall p \in \{1, ..., (n-1)\}$, if the polynomials $s^n + \sum_{j=0}^{n-1}(a_{i,j} + k^i_{i,j})s^j, s^n + \sum_{j=0}^{n-1}(a_{i_1,j} + k^i_{i_1,j})s^j, ..., s^n + \sum_{j=0}^{n-1}(a_{i_{N_i},j} + k^i_{i_{N_i},j})s^j$ are Hurwitz and $a_{i,0} + k^i_{p,0} \neq 0$, $\forall i \in \{1, 2, ..., N\}, \forall p \in \mathcal{N}_i \cup i$.

Proof Define $\mathcal{E}_i = Z_i - \hat{Z}_i$. The error dynamics are written as

$$\dot{\mathcal{E}}_i = -(\bar{\mathcal{A}}_i + K_i) \begin{pmatrix} \mathbb{Y}_i - \hat{\mathbb{Y}}_i \\ \mathbb{Y}_{ii_1} - \hat{\mathbb{Y}}_{ii_1} \\ \vdots \\ \mathbb{Y}_{ii_{N_i}} - \hat{\mathbb{Y}}_{ii_{N_i}} \end{pmatrix} + \bar{\mathcal{B}}_i \begin{pmatrix} \mathbb{F}_i \\ \mathbb{F}_{i_1} \\ \vdots \\ \mathbb{F}_{i_{N_i}} \end{pmatrix} + \begin{pmatrix} \phi_i^0(t) \\ \phi_{i_1}^0(t) \\ \vdots \\ \phi_{i_{N_i}}^0(t) \end{pmatrix} \tag{3.35}$$

Applying the Laplace transform yields

$$\Big(s\mathcal{E}_i(s) - \mathcal{E}_i(0)\Big) = -(\bar{\mathcal{A}}_i + K_i)\bar{\Delta}(s)\mathcal{E}_i(s) + \bar{\mathcal{B}}_i\bar{\Upsilon}(s) \begin{pmatrix} F_i(s) \\ F_{i_1}(s) \\ \vdots \\ F_{i_{N_i}}(s) \end{pmatrix} + \begin{pmatrix} \phi_i^0(s) \\ \phi_{i_1}^0(s) \\ \vdots \\ \phi_{i_{N_i}}^0(s) \end{pmatrix} \tag{3.36}$$

where

$$\begin{cases} \Delta(s)^T = [1, ..., \frac{1}{s^{n-2}}, \frac{1}{s^{n-1}}] \\ \Upsilon(s)^T = [\frac{1}{s^{n-m-1}}, ..., \frac{1}{s^{n-2}}, \frac{1}{s^{n-1}}] \\ \bar{\Delta}(s) = \text{Blkdiag}(\underbrace{\Delta(s), ..., \Delta(s)}_{(N_i+1) \text{ times}}) \\ \bar{\Upsilon}(s) = \text{Blkdiag}(\underbrace{\Upsilon(s), ..., \Upsilon(s)}_{(N_i+1) \text{ times}}) \end{cases} \tag{3.37}$$

Similarly to the previous Subsection, (3.36) can be equivalently expressed as

$$\mathcal{E}_i(s) = (sI + (\bar{\mathcal{A}}_i + K_i)\bar{\Delta}(s))^{-1}\left(\bar{\mathcal{B}}_i\bar{\Upsilon}(s)\begin{pmatrix} F_i(s) \\ F_{i_1}(s) \\ \vdots \\ F_{i_{N_i}}(s) \end{pmatrix} + \begin{pmatrix} \phi_i^0(s) \\ \phi_{i_1}^0(s) \\ \vdots \\ \phi_{i_{N_i}}^0(s) \end{pmatrix} + \mathcal{E}_i(0)\right) \tag{3.38}$$

where I is the identity matrix with appropriate dimensions. Hence, observer gains are chosen such that the polynomials $sI + (\bar{\mathcal{A}}_i + K_i)\bar{\Delta}(s)$ are stable. Using the final value theorem, it can be shown that

$$\lim_{s\to 0} s\mathcal{E}_i(s) \quad = -sI(sI + (\bar{\mathcal{A}}_i + K_i)\bar{\Delta}(s))^{-1}\bar{\mathcal{B}}_i\bar{\Upsilon}(s)\begin{pmatrix} F_i(s) \\ F_{i_1}(s) \\ \vdots \\ F_{i_{N_i}}(s) \end{pmatrix} \tag{3.39}$$

Hence in the absence of a fault, the error dynamics are asymptotically stable.

Remark 3.2.2 *It can be noticed that the fault detectability condition will be dependent on the system parameters, if for instance for non null* $b_{p,0} \neq 0, \forall p \in \{i, i_1, ..., i_{N_i}\}$, *one could note that*

$$\lim_{s\to 0} s\mathcal{E}_i(s) = Blkdiag\begin{pmatrix} \frac{sb_{i,0}}{a_{i,0} + k_{i,0}^i} & \frac{sb_{i_1,0}}{a_{i_1,0} + k_{i_1,0}^i} & \cdots & \frac{sb_{i_{N_i},0}}{a_{i_{N_i},0} + k_{i_{N_i},0}^i} \end{pmatrix}\begin{pmatrix} F_i(s) \\ F_{i_1}(s) \\ \vdots \\ F_{i_{N_i}}(s) \end{pmatrix} \tag{3.40}$$

hence, in the case of a constant fault for example, the error dynamics are different from 0.

Residual evaluation: Given (3.40), the following residual vector can thus be defined at agent i

$$\mathcal{R}_i = \begin{pmatrix} r_i^i \\ r_{i_1}^i \\ \vdots \\ r_{i_{N_i}}^i \end{pmatrix} = Q_i \begin{pmatrix} y_i - \hat{y}_i \\ y_{ii_1} - \hat{y}_{ii_1} \\ \vdots \\ y_{ii_{N_i}} - \hat{y}_{ii_{N_i}} \end{pmatrix}$$

where Q_i is the post residual gain matrix used to highlight the effect of faults. It can be concluded from Theorem 3.2 that the residual signals are null when there is no fault, and different from 0 otherwise.

Fault isolation: For isolation purpose, let us define a detection flag τ_i for each agent, where if $\tau_i = 1$, $r_i^i \neq 0$ and $\tau_i = 0$ otherwise. It is assumed that an agent i can request its neighbour's detection flag $\tau_j, j \in \mathcal{N}_i$. Hence, the following decision logic can be deduced: if $r_i^i > 0$, then agent i is faulty, if $r_j^i > 0, j \in \mathcal{N}_i$ and $\tau_j = 1$, then agent j is faulty.

Remark 3.2.3 *It is worth noting that the scheme proposed in this Section does not require for the FDI module to have the same dimensions as the system. Indeed, the proposed output observer relies solely on the input-output relations and does not need to reconstruct the entire state vector as opposed to most works in the literature (e.g. [Teixeira et al. 2014, Liu et al. 2016a, Chadli et al. 2017]). However, a major drawback of the proposed scheme is that it does not take into account the effect of uncertainties, disturbances in its design.*

3.2.3 Simulation Example

In this section, an illustrative numerical example is provided to show the effectiveness of the proposed output observer residual generator scheme. Consider a team of four heterogeneous agents governed by linear dynamics and communicating amongst each other according to the topology in Fig. 3.1, where the matrices are defined as $A_i = \begin{bmatrix} -2 & -0.5i \\ 1 & 0 \end{bmatrix}$, $B_i = \begin{bmatrix} 1+0.1i \\ 0 \end{bmatrix}$ and $C_i = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

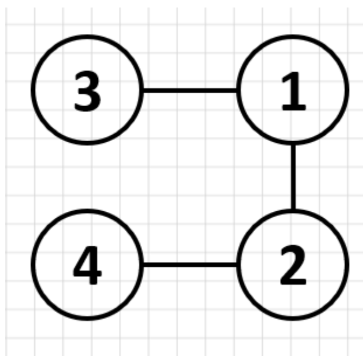


Figure 3.1: Communication topology.

Without loss of generality, the control commands are chosen as $u_1(t) = u_2(t) = u_3(t) = u_4(t) = 1$. Since the measurement matrix is the same for each agent, the observer gains satisfying Theorem 3.2 are chosen as

$$\begin{cases} k_{3,0}^{3} &= -5 \\ k_{1,0}^{3} &= -2 \\ k_{3,1}^{3} &= -20 \\ k_{1,1}^{3} &= -15 \\ k_{3,2}^{3} &= -17 \\ k_{1,2}^{3} &= -10 \end{cases}, \quad \begin{cases} k_{4,0}^{4} &= -7 \\ k_{2,0}^{4} &= -3 \\ k_{4,1}^{3} &= -18 \\ k_{2,1}^{4} &= -17 \\ k_{4,2}^{4} &= -14 \\ k_{2,2}^{4} &= -12 \end{cases}.$$

$$\begin{cases} k_{1,0}^{1} &= -2 \\ k_{2,0}^{1} &= -3 \\ k_{3,0}^{1} &= -5 \\ k_{1,1}^{1} &= -15 \\ k_{2,1}^{1} &= 17 \\ k_{3,1}^{1} &= -20 \\ k_{1,2}^{1} &= -10 \\ k_{2,2}^{1} &= -12 \\ k_{3,2}^{1} &= -17 \end{cases}, \quad \begin{cases} k_{2,0}^{2} &= -3 \\ k_{1,0}^{2} &= -2 \\ k_{4,0}^{2} &= -7 \\ k_{2,1}^{2} &= -17 \\ k_{1,1}^{2} &= 15 \\ k_{4,1}^{2} &= -18 \\ k_{2,2}^{2} &= -12 \\ k_{1,2}^{2} &= -10 \\ k_{4,2}^{2} &= -14 \end{cases},$$

In order to showcase the effectiveness of our methodology, two abrupt faults $f_2(t)$ and $f_4(t)$ are assumed to happen in agents 2 and 4, respectively. The fault $f_2(t)$ is a simulated rectangular pulsed signal with amplitude of 100 with $20 \leqslant t \leqslant 50$. The fault $f_4(t)$ is simulated as a rectangular signal that appears at $t = 70s$ and has an amplitude of 90. It is assumed that no fault occurs in agents 1 and 3. Noises are added to all agents' outputs to test the robustness of our approach. By using the isolation logic discussed in the previous Subsection, one can notice that agent 1 detects the fault at agent 2, which is confirmed by the flag $\tau_2 = 1$ at $t = 70$ s. Agent 2 detects agent 4's fault as confirmed by the flag $\tau_4 = 1$ between $t = 20$ s and $t = 50$ s, on the other hand, it is seen that all of agent 2's residual signals react at $t = 70$ s, this could mean that any of the agents 1,2 and 4 could be faulty. It is confirmed however from the flags $\tau_1 = \tau_4 = 0$ that the source of the fault is agent 2 itself. Agent 3's residual signals stay around 0 for the whole duration of the simulation since no faults occur either in agent 3 itself or in its neighbouring agent 1. Similarly to agent 2, both agent 4's residual signals react between $t = 20$ s and $t = 50$ s, where $\tau_2 = 1$ confirms that the faulty agent is agent 4 itself.

From Fig. 3.2, it can be concluded that the fault detection protocol proposed for a fleet of communicating heterogeneous systems is effective. It is robust against the output noise and sensitive to the faults. Furthermore, every agent is capable of detecting not only its own faults, but the faults occurring in its neighbours as well.

Considering Remark 3.2.3, the next Section proposes a FDI scheme encompassing not only actuator faults, but also sensor and communication faults, while taking into account the effects of noise, dynamic disturbances and communication uncertainties.

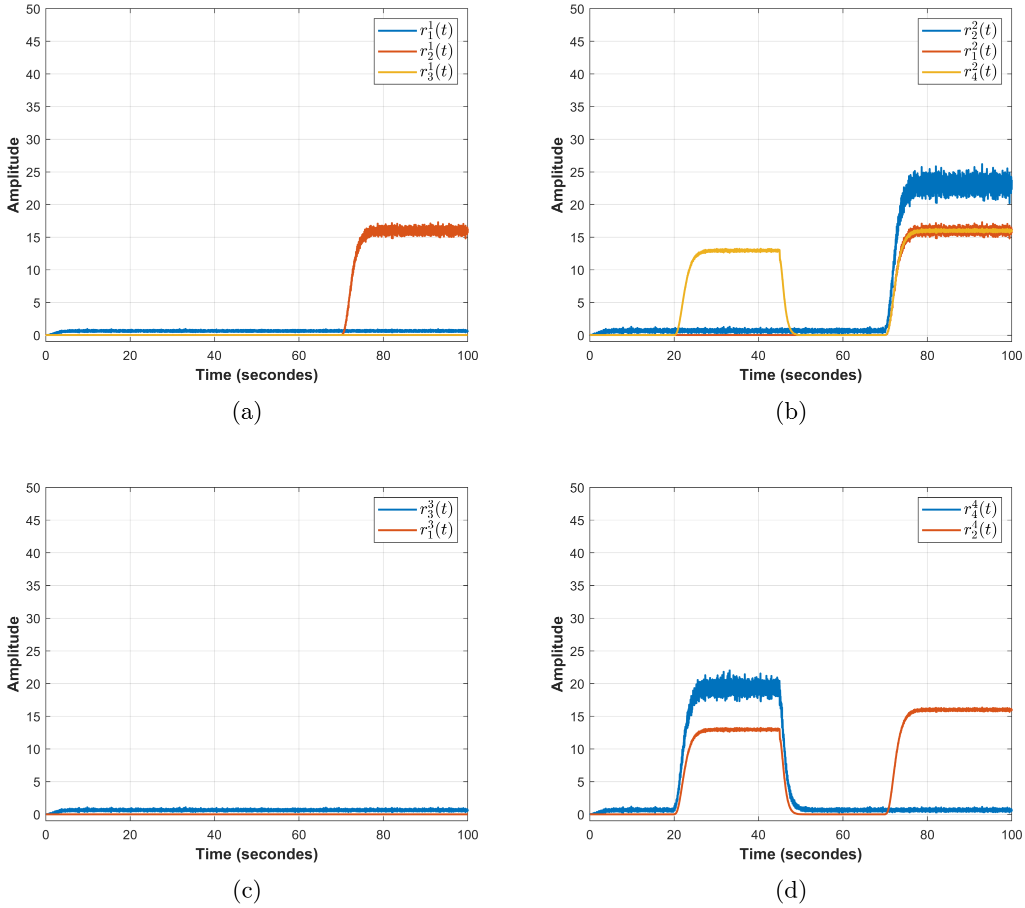


Figure 3.2: Fault residual signals generated by agents (a) 1, (b) 2, (c) 3 and (d) 4.

3.3 Attack and FDI in heterogeneous MASs with Nonlinear Dynamics

3.3.1 Problem Formulation

Consider a heterogeneous MAS composed of N agents labelled by $i \in \{1, ..., N\}$, and described by the following uncertain dynamics

$$\begin{cases} \dot{\xi}_i(t) & = A_i\xi_i(t) + B_{u_i}u_i(t) + B_{d_i}d_i(t) + B_{f_i}f_{a_i}(t) + \varphi_i(\xi_i(t)) \\ y_i(t) & = C_i\xi_i(t) + D_{d_i}d_i(t) + D_{f_i}f_{s_i}(t) \end{cases}, \tag{3.41}$$

where $\xi_i \in \mathbb{R}^{n_x}$, $u_i \in \mathbb{R}^{n_u}$, $y_i \in \mathbb{R}^{n_y}$, $d_i \in \mathbb{R}^{n_d}$, $f_{a_i} \in \mathbb{R}^{n_{f_a}}$, $f_{s_i} \in \mathbb{R}^{n_{f_s}}$ are the state vector, the control input, the output, the L_2-norm bounded disturbances and noise, the actuator fault and the sensor fault signals respectively. Matrices $A_i \in \mathbb{R}^{n_x \times n_x}$, $B_{u_i} \in \mathbb{R}^{n_x \times n_u}$, $B_{d_i} \in \mathbb{R}^{n_x \times n_d}$, $B_{f_i} \in \mathbb{R}^{n_x \times n_{f_a}}$, $C_i \in \mathbb{R}^{n_y \times n_x}$, $D_{d_i} \in \mathbb{R}^{n_y \times n_d}$, $D_{f_i} \in \mathbb{R}^{n_y \times n_{f_s}}$ are known constant matrices. $\varphi_i(\xi_i(t)) \in \mathbb{R}^{n_x}$ is a known function representing the nonlinearity of agent i.

3.3.1.1 Communication Faults

In this Section, the measured outputs are exchanged between neighbouring agents. Hence, an agent i receives from each neighbour $j \in \mathcal{N}_i$ its output (resp. input), corrupted by parameter uncertainties associated with the communication link between i and j, $\Delta a_{ij}(t) \in \mathbb{R}$ and by faults due to link faults, packet losses or potential cyber-attacks denoted $f_{ij}^z(t) \in \mathbb{R}^{n_{f_{z_{ij}}}}$ (resp. $f_{ij}^u(t) \in \mathbb{R}^{n_{f_u}}$), i.e.

$$\begin{aligned} z_{ij}(t) &= a_{ij}(1 + \Delta a_{ij}(t))y_j(t) + D_{z_{ij}} f_{ij}^z(t), \\ u_{ij}(t) &= a_{ij}(1 + \Delta a_{ij}(t))u_j(t) + D_{u_{ij}} f_{ij}^u(t), \end{aligned} \tag{3.42}$$

with $z_{ii}(t) = y_i(t)$ and $u_{ii}(t) = u_i(t)$. $D_{z_{ij}} \in \mathbb{R}^{n_y \times n_{f_{z_{ij}}}}$ and $D_{u_{ij}} \in \mathbb{R}^{n_u \times n_{f_u}}$ are known constant matrices. It is also assumed that the parameter uncertainties $\Delta a_{ij}(t)$ satisfy $|\Delta a_{ij}(t)| < a_{ij}$.

Remark 3.3.1 *It is worth noting that the considered faults cover a wide range of cyber-attacks that have been studied in the literature. For instance, assume that* $\Delta a_{ij} = 0$ *for the sake of clarity,*

- *In the case of a communication parametric fault [Teixeira et al. 2014] for i, affecting all its incoming information from agent* j*, one has*

  $$\begin{aligned} z_{ij}(t) &= (a_{ij} + f_{a_{ij}(t)}(t))y_j(t) \\ &= a_{ij}y_j(t) + f_{a_{ij}}(t)y_j(t), \end{aligned}$$

  where analogously to (3.42), one could note that $f_{ij}^z(t) = f_{a_{ij}}(t)y_j(t)$ *and* $D_{z_{ij}} = I_{n_y}$. $f_{a_{ij}}(t)$ *represents a parametric fault affecting the communication parameter* a_{ij}.

- *In a denial of service attack situation affecting all incoming information from agent* j*, one has* $f_{ij}^z(t) = -a_{ij}\delta(t - t_{ij})y_j(t)$ *and* $D_{z_{ij}} = I_{n_y}$ *[Rezaee et al. 2021], where*

  $$\delta(t - t_{ij}) = \begin{cases} 1, & t \geqslant t_{ij} \\ 0, & else \end{cases},$$

  and t_{ij} *is the instant at which the attack occurs.*

- *Conversely, in a false data injection situation in the transmitted information, agent* j *transmits or agent* i *receives fake/invalid information, that is,* $f_{ij}^z(t)$ *contains the injected malicious information [Boem et al. 2017]. In the case where the malicious information* $f_{ij}^z(t) \in \mathbb{R}$ *affects all incoming transmitted data equally, then one could set* $D_{z_{ij}} = \mathbf{1}_{n_y}$.

- *Under replay attacks, the attacker intercepts the transmitted information and replays it with a delay instead of the actual information. In this case, one could* *write [Gallo et al. 2018a], $f_{ij}^z(t) = \delta_{ij}(t-t_{ij})(-a_{ij}y_j(t) + y_j(t-\mathcal{T}_{ij}))$ and $D_{z_{ij}} = I_{n_y}$, where*

$$\delta_{ij}(t-t_{ij}) = \begin{cases} 1, & t \geqslant t_{ij} \\ 0, & else \end{cases},$$

and $t_{ij} > 0$ is the instant at which the attack occurs and $\mathcal{T}_{ij} \in \mathbb{R}$ is the time delay.

The same remarks could be made w.r.t. $u_{ij}(t)$. Contrary to agent/node attacks or faults in the form of the signals $f_{a_i}(t)$, $f_{s_i}(t)$, edge/communication attacks cannot be detected locally by an emitting agent j, and thus need its neighbours to detect them. It is worth mentioning that the introduced problem can represent many potential practical applications to FDI in networked MASs. As discussed in the introduction Section, such applications include electric power networks and micro-grids, multi-robot and multi-vehicle systems, etc. [Teixeira et al. 2010, Taoufik et al. 2020b, Ren & Atkins 2007].

3.3.1.2 Concatenated local model

Let us first denote

$$\begin{cases} x_{v_i} &= [\xi_i^T, \xi_{i_1}^T, ..., \xi_{i_{N_i}}^T]^T \in \mathbb{R}^{n_x^i}, \\ d_{v_i} &= [d_i^T, d_{i_1}^T, ..., d_{i_{N_i}}^T]^T \in \mathbb{R}^{n_d^i}, \\ f_{vs_i} &= [f_{s_i}^T, f_{s_{i_1}}^T, ..., f_{s_{i_{N_i}}}^T]^T \in \mathbb{R}^{n_{f_s}^i}, \\ f_{va_i} &= [f_{a_i}^T, f_{a_{i_1}}^T, ..., f_{a_{i_{N_i}}}^T]^T \in \mathbb{R}^{n_{f_a}^i}, \\ z_i &= [(y_i - y_{i_1})^T, ..., (y_i - y_{i_{N_i}})^T]^T \in \mathbb{R}^{\underline{n}_z^i}, \\ y_{v_i} &= [y_{i_1}^T, ..., y_{i_{N_i}}^T]^T \in \mathbb{R}^{\underline{n}_z^i}, \\ u_{v_i} &= [u_{i_1}^T, ..., u_{i_{N_i}}^T]^T \in \mathbb{R}^{\underline{n}_u^i}, \end{cases} \tag{3.43}$$

the concatenated state, disturbance, fault signals, relative information, output and input of agent i ($i_j \in \mathcal{N}_i$), where $n_x^i = n_x(N_i+1)$, $n_d^i = n_d(N_i+1)$, $n_{f_a}^i = n_{f_a}(N_i+1)$, $n_{f_s}^i = n_{f_s}(N_i+1)$, $\underline{n}_z^i = n_y N_i$ and $\underline{n}_u^i = n_u N_i$. A virtual output is given as

$$z_{v_i} = Z^i \begin{pmatrix} y_i \\ z_i \end{pmatrix} + \Delta Z^i \begin{pmatrix} y_i \\ y_{v_i} \end{pmatrix} + D_{vz_i} f_i^z \in \mathbb{R}^{n_z^i}, \tag{3.44}$$

where

$$\begin{cases} Z^i & = \begin{pmatrix} I_{n_y} & 0_{n_y \times \underline{n}_z^i} \\ 0_{\underline{n}_z^i \times n_y} & \mathcal{A}_i \end{pmatrix} \in \mathbb{R}^{n_z^i \times n_z^i}, \\ \Delta Z^i & = \begin{pmatrix} 0_{n_y \times n_y} & 0_{n_y \times \underline{n}_z^i} \\ 0_{\underline{n}_z^i \times n_y} & \mathcal{A}_i \Delta \mathcal{A}_i \end{pmatrix} \in \mathbb{R}^{n_z^i \times n_z^i}, \\ \Delta \mathcal{A}_i & = \text{diag}(\underbrace{\Delta a_{ii_1}, ..., \Delta a_{ii_1}}_{n_y \text{ times}}, ..., \Delta a_{ii_{N_i}}, ..., \Delta a_{ii_{N_i}}) \\ & \in \mathbb{R}^{\underline{n}_z^i \times \underline{n}_z^i}, \\ \mathcal{A}_i & = \text{diag}(\underbrace{a_{ii_1}, ..., a_{ii_1}}_{n_y \text{ times}}, ..., a_{ii_{N_i}}, ..., a_{ii_{N_i}}) \\ & \in \mathbb{R}^{\underline{n}_z^i \times \underline{n}_z^i}, \\ D_{vz_i} & = \begin{pmatrix} 0_{n_y \times \underline{n}_{fz}^i} \\ -\text{Blkdiag}[D_{z_{ii_1}}, D_{z_{ii_2}}, \ldots, D_{z_{ii_{N_i}}}] \end{pmatrix} \\ & \in \mathbb{R}^{n_z^i \times \underline{n}_{fz}^i}, \\ z_{v_i} & = [y_i^T, z_{ii_1}^T, ..., z_{ii_{N_i}}^T]^T \in \mathbb{R}^{n_z^i}, \\ f_i^z & = [f_{ii_1}^z, f_{ii_2}^z, ..., (f_{ii_{N_i}}^z)^T]^T \in \mathbb{R}^{\underline{n}_{fz}^i}, \end{cases}$$

with $\underline{n}_{fz}^i = \sum_{j \in \mathcal{N}_i} n_{f_{z_{ij}}} \neq 0$, $n_z^i = n_y(N_i+1)$. z_{v_i} and f_i^z are the concatenated measured vector available for agent i and the associated communication fault signals, respectively. $\bar{\mathcal{A}}_i = \mathcal{A}_i + \Delta\mathcal{A}_i \in \mathbb{R}^{\underline{n}_z^i \times \underline{n}_z^i}$ is the actual local adjacency matrix of agent i which takes into account the parametric uncertainty associated with the communication links. Replacing outputs and inputs with their respective values from (3.41) yields

$$\begin{cases} \dot{x}_{v_i}(t) & = \widetilde{A}_i x_{v_i}(t) + \widetilde{B}_{u_i} u_{v_i}(t) + \widetilde{\underline{B}}_{u_i} u_i(t) + \widetilde{B}_{d_i} d_{v_i}(t) + \widetilde{B}_{f_i} f_{va_i}(t) + \varphi_{v_i}(x_{v_i}(t)) \\ z_{v_i}(t) & = Z^i(\widetilde{C}_i x_{v_i}(t) + \widetilde{D}_{d_i} d_{v_i}(t) + \widetilde{D}_{f_i} f_{vs_i}(t)) + D_{vz_i} f_i^z(t) + \Delta Z^i \begin{pmatrix} y_i(t) \\ y_{v_i}(t) \end{pmatrix} \end{cases}, \tag{3.45}$$

where

$$\begin{cases} \varphi_{v_i}(x_{v_i}(t)) & = \text{Col}(\varphi_i(\xi_i(t)), ..., \varphi_{i_{N_i}}(x_{i_{N_i}}(t))), \\ \widetilde{A}_i & = \text{Blkdiag}(A_i, A_{i_1}, ..., A_{i_{N_i}}), \\ \widetilde{\underline{B}}_{u_i} & = \text{Col}(B_{u_i}, 0_{n_x \times n_u}, \ldots, 0_{n_x \times n_u}), \\ \widetilde{B}_{u_i} & = \text{Col}(0_{n_x \times \underline{n}_u^i}, \text{Blkdiag}(B_{u_{i_1}}, ..., B_{u_{i_{N_i}}})), \\ \widetilde{B}_{d_i} & = \text{Blkdiag}(B_{d_i}, B_{d_{i_1}}, ..., B_{d_{i_{N_i}}}), \\ \widetilde{B}_{f_i} & = \text{Blkdiag}(B_{f_i}, B_{f_{i_1}}, ..., B_{f_{i_{N_i}}}), \end{cases}$$

$\widetilde{C}_i$, $\widetilde{D}_{d_i}$ and $\widetilde{D}_{f_i}$ correspond to the following tilde notation

$$\widetilde{\Theta}_i = \begin{bmatrix} \Theta_i & 0 & \ldots & 0 \\ \Theta_i & -\Theta_{i_1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \Theta_i & 0 & \ldots & -\Theta_{i_{N_i}} \end{bmatrix},$$

with $\widetilde{A}_i \in \mathbb{R}^{n_x^i \times n_x^i}$, $\widetilde{B}_{u_i} \in \mathbb{R}^{n_x^i \times n_u^i}$, $\widetilde{B}_{f_i} \in \mathbb{R}^{n_x^i \times n_{fa}^i}$, $\widetilde{C}_i \in \mathbb{R}^{n_z^i \times n_x^i}$, $\widetilde{D}_{d_i} \in \mathbb{R}^{n_z^i \times n_d^i}$, $\widetilde{D}_{f_i} \in \mathbb{R}^{n_z^i \times n_{fs}^i}$. Let us make the following assumption on the parametric uncertainties

Assumption 3.3.1 *There exist a time-varying matrix* $\delta_i(t) \in \mathbb{R}^{\underline{n}_z^i \times \underline{n}_z^i}$ *and known matrices* $\boldsymbol{X}_i$ *and* M_i *with appropriate dimensions such that*

$$\Delta Z^i = \boldsymbol{X}_i \delta_i(t) M_i, \tag{3.46}$$

with $\bar{\sigma}(\delta_i) \leq \delta_M$.

Remark 3.3.2 *It is worth noting that this assumption stems from the definition of the graph topology in this Section, and is standard for bounded uncertainties [Ding 2008].*

Under this assumption, one could rewrite system (3.45) as

$$\begin{cases} \dot{x}_{v_i}(t) &= \widetilde{A}_i x_{v_i}(t) + \widetilde{B}_{u_i} u_{v_i}(t) + \widetilde{\underline{B}}_{u_i} u_i(t) + \widetilde{B}_{d_i} d_{v_i}(t) + \widetilde{B}_{f_i} f_{va_i}(t) + \varphi_{v_i}(x_{v_i}(t)), \\ z_{v_i}(t) &= Z^i \widetilde{C}_i x_{v_i}(t) + Z^i \widetilde{D}_{d_i} d_{v_i}(t) + D_{\mathcal{F}_i} \mathcal{F}_i(t) - \mathbf{X}_i \phi_i(t), \end{cases}, \tag{3.47}$$

where $\mathcal{F}_i(t) = \begin{pmatrix} f_{vs_i}(t) \\ f_i^z(t) \end{pmatrix}$, $D_{\mathcal{F}_i} = \begin{pmatrix} Z^i \widetilde{D}_{f_i} & D_{vz_i} \end{pmatrix}$, $\phi_i(t) = -\delta_i(t) D_{\phi_i} \begin{pmatrix} x_{v_i}(t) \\ d_{v_i}(t) \\ f_{vs_i}(t) \end{pmatrix}$, $D_{\phi_i} = M_i \begin{pmatrix} \text{Blkdiag}(C_i^T, ..., C_{i_{N_i}}^T) \\ \text{Blkdiag}(D_{d_i}^T, ..., D_{d_{i_{N_i}}}^T) \\ \text{Blkdiag}(D_{f_i}^T, ..., D_{f_{i_{N_i}}}) \end{pmatrix}^T$.

Note that, in the case where $\widetilde{D}_{f_i} = 0$, $D_{\mathcal{F}_i}$ is selected as $D_{\mathcal{F}_i} = D_{vz_i}$. The robust distributed FDI objective is the design of residual generators for each agent using locally exchanged information capable of detecting and isolating not only the agent's own faults but also the faults of its neighbours as well as attacks targeting incoming communication links.

The following Assumption and Lemma are going to be used in the next section.

Assumption 3.3.2 *The nonlinear functions* $\varphi_i(\xi_i(t))$ *are Lipschitz, with Lipschitz constant* θ_i, $\forall i = \{1, 2, ..., N\}$, *i.e.,* $\forall \xi_i, \hat{\xi}_i \in \mathbb{R}^{n_x}$

$$||\varphi_i(\xi_i) - \varphi_i(\hat{\xi}_i)|| \leqslant \theta_i ||\xi_i - \hat{x}_i||.$$

Remark 3.3.3 *It is worth noting that Assumption 3.3.2 restricts the class of considered nonlinearities in Eq. (3.41) and has been considered in many works [Raghavan & Hedrick 1994].*

Lemma 3.1 *[Boyd et al. 1994] Given real matrices* F_i *and* J_i *of appropriate dimensions, then the following inequality holds for any strictly positive scalar* ε_i:

$$F_i J_i^T + J_i F_i^T \leqslant \varepsilon_i J_i J_i^T + \varepsilon_i^{-1} F_i F_i^T.$$

3.3.2 Main Result

3.3.2.1 Proposed Distributed Fault Detection and Isolation Scheme

The aim here is to design robust residual generators which are sensitive to all types of faults in spite of the presence of uncertainties using UIOs. Consider the following observer

$$\begin{cases} \dot{q}_{vi}(t) &= N_i q_{vi}(t) + G_{1i}u_i(t) + G_{2i}U_i(t) + L_i z_{v_i}(t) + T_i\varphi_{v_i}(\hat{x}_{v_i}(t)) \\ \hat{x}_{v_i}(t) &= q_{vi}(t) - H_i z_{v_i}(t) \\ \hat{z}_{vi}(t) &= Z^i\widetilde{C}_i\hat{x}_{v_i}(t) \end{cases}, \tag{3.48}$$

where $U_i(t) = \mathrm{Col}(u_{ii_1}(t), ..., u_{ii_{N_i}}(t))$. The matrices N_i, G_{1i}, G_{2i}, L_i, T_i and H_i will be described hereafter. Define the state estimation error as $e_{v_i}(t) = x_{v_i}(t) - \hat{x}_{v_i}(t)$. Then

$$e_{v_i}(t) = (I + H_i Z^i \widetilde{C}_i)x_{v_i}(t) - q_{vi}(t) + H_i V_{v_i} v_i(t),$$

where $\mathcal{D}_i(t) = \begin{pmatrix} d_{v_i}(t) \\ \phi(t) \end{pmatrix}$, $V_{v_i} = \begin{pmatrix} Z^i\widetilde{D}_{d_i} & -\mathbf{X}_i & D_{\mathcal{F}_i}\end{pmatrix}$ and $v_i(t) = \begin{pmatrix} \mathcal{D}_i(t) \\ \mathcal{F}_i(t)\end{pmatrix}$. Therefore, its dynamics is expressed as

$$\begin{aligned} \dot{e}_{v_i}(t) &= N_i e_{v_i}(t) + (T_i\widetilde{A}_i - S_i Z^i\widetilde{C}_i - N_i)x_{v_i}(t) + T_i\varphi_{v_i}^{e_{v_i}} + T_i\widetilde{B}_{f_i} f_{va_i}(t) \\ &+ (T_i\underline{\widetilde{B}}_{u_i} - G_{1i})u_i(t) + S_i\mathbf{X}_i\phi_i(t) - S_i D_{\mathcal{F}_i}\mathcal{F}_i(t) + (T_i\widetilde{B}_{d_i} - S_i Z^i\widetilde{D}_{d_i})d_{v_i}(t) \\ &+ T_i\widetilde{B}_{u_i}u_{v_i}(t) - G_{2i}((\mathcal{A}_{u,i}\Delta\mathcal{A}_{u,i} + \mathcal{A}_{u,i})u_{v_i}(t) + D_{u_i}f_{u_i}(t)) + H_i V_{v_i}\dot{v}_i(t) \end{aligned} \tag{3.49}$$

where

$$T_i = I + H_i Z^i \widetilde{C}_i, \tag{3.50a}$$
$$S_i = L_i + N_i H_i, \tag{3.50b}$$

and

$$\begin{cases} \varphi_{v_i}^{e_{v_i}}(t) &= \varphi_{v_i}(x_{v_i}(t)) - \varphi_{v_i}(\hat{x}_{v_i}(t)) \\ f_{u_i}(t) &= \mathrm{Col}(f^u_{ii_1}(t), ..., f^u_{ii_{N_i}}(t)), \\ D_{u_i} &= \mathrm{Blkdiag}(D_{u_{ii_1}}, ..., D_{u_{ii_{N_i}}}), \\ \Delta\mathcal{A}_{u,i} &= \mathrm{diag}(\underbrace{\Delta a_{ii_1}, ..., \Delta a_{ii_1}}_{n_u \text{ times}}, ..., \Delta a_{ii_{N_i}}, ..., \Delta a_{ii_{N_i}}), \\ \mathcal{A}_{u,i} &= \mathrm{diag}(\underbrace{a_{ii_1}, ..., a_{ii_1}}_{n_u \text{ times}}, ..., a_{ii_{N_i}}, ..., a_{ii_{N_i}}). \end{cases}$$

By imposing the following

$$H_i V_{v_i} = 0, \tag{3.51a}$$
$$T_i\widetilde{A}_i - S_i Z^i \widetilde{C}_i = N_i, \tag{3.51b}$$
$$T_i\underline{\widetilde{B}}_{u_i} - G_{1i} = 0, \tag{3.51c}$$
$$T_i\widetilde{B}_{u_i} - G_{2i}\mathcal{A}_{u,i} = 0, \tag{3.51d}$$

(3.49) becomes

$$\begin{aligned}\dot{e}_{v_i}(t) &= N_i e_{v_i}(t) + (T_i\widetilde{B}_{d_i} - S_i Z^i \widetilde{D}_{d_i})d_{v_i}(t) + T_i\widetilde{B}_{f_i} f_{va_i}(t) - S_i D_{\mathcal{F}_i}\mathcal{F}_i(t) \\ &\quad - T_i\widetilde{B}_{u_i}(\mathcal{A}_{u,i}^{-1}\mathcal{A}_{u,i})\Delta\mathcal{A}_{u,i}u_{v_i}(t) - T_i\widetilde{B}_{u_i}\mathcal{A}_{u,i}^{-1}D_{u_i}f_{u_i}(t) + T_i\varphi_{vi}^{e_{v_i}}(t) + S_i\mathbf{X}_i\phi_i(t).\end{aligned} \tag{3.52}$$

By setting new concatenated uncertainties vector as $\underline{\phi}_i(t) = \begin{pmatrix}\phi_i(t) \\ \Delta\mathcal{A}_{u,i}u_{v_i}(t)\end{pmatrix}$, the error dynamics becomes

$$\begin{aligned}\dot{e}_{v_i}(t) &= N_i e_{v_i}(t) + (T_i\widetilde{B}_{d_i} - S_i Z^i \widetilde{D}_{d_i})d_{v_i}(t) + T_i\varphi_{vi}^{e_{v_i}}(t) \\ &\quad - S_i D_{\mathcal{F}_i}\mathcal{F}_i(t) + (S_i\underline{\mathbf{X}}_i - T_i\bar{\mathbf{X}}_i)\underline{\phi}_i(t) - T_i\mathcal{B}_i\underline{\mathcal{F}}_i(t),\end{aligned} \tag{3.53}$$

where $\mathcal{B}_i = \begin{pmatrix}-\widetilde{B}_{f_i} & \widetilde{B}_{u_i}\mathcal{A}_{u,i}^{-1}D_{u_i}\end{pmatrix}$, $\underline{\mathcal{F}}_i(t) = \begin{pmatrix}f_{a_i}(t) \\ f_{u_i}(t)\end{pmatrix}$, $\underline{\mathbf{X}}_i = \begin{pmatrix}\mathbf{X}_i & 0_{n_z^i\times(n_u\cdot N_i)}\end{pmatrix}$, $\bar{\mathbf{X}}_i = \begin{pmatrix}0_{n_x^i\times\underline{n}_z^i} & -\widetilde{B}_{u_i}\end{pmatrix}$.

On the other hand, define the following residual vector

$$r_i(t) = W_i(z_{v_i}(t) - \hat{z}_{vi}(t)), \tag{3.54}$$

where W_i is a pre-set post residual gain matrix used to highlight the effects of the faults on the residual signals. In this Section, since it does not directly affect the residual signals, it is considered that $\underline{\mathcal{F}}_i(t)$ affects the residual signals over a finite frequency domain, which can be uniformly expressed as [Li & Yang 2013]

$$\Omega_{\underline{\mathcal{F}}_i} := \{\omega_f \in \mathbb{R} \mid \kappa(\omega_f - \omega_{f_1})(\omega_f - \omega_{f_2}) \leqslant 0\}, \tag{3.55}$$

where $\kappa \in \{1, -1\}$, ω_{f_1} and ω_{f_2} are given positive scalars characterizing the frequency range of the fault vector $\underline{\mathcal{F}}_i$. Indeed, if one selects

- $\kappa = 1$ and $\omega_{f_1} < \omega_{f_2}$, then the set $\Omega_{\underline{\mathcal{F}}_i}$ corresponds to the middle frequency range
$$\Omega_{\underline{\mathcal{F}}_i} := \{\omega_f \in \mathbb{R} \mid \ \omega_{f_1} \leqslant \omega_f \leqslant \omega_{f_2}\}.$$
- $\kappa = 1$ and $-\omega_{f_1} = \omega_{f_2} = \omega_{f_l}$, then the set $\Omega_{\underline{\mathcal{F}}_i}$ corresponds to the low frequency range
$$\Omega_{\underline{\mathcal{F}}_i} := \{\omega_f \in \mathbb{R} \mid \ |\omega_f| \leqslant \omega_{f_l}\}.$$
- $\kappa = -1$ and $-\omega_{f_1} = \omega_{f_2} = \omega_{f_h}$, then the set $\Omega_{\underline{\mathcal{F}}_i}$ corresponds to the high frequency range
$$\Omega_{\underline{\mathcal{F}}_i} := \{\omega_f \in \mathbb{R} \mid \ |\omega_f| \geqslant \omega_{f_l}\}.$$

The objective here is to simultaneously achieve local state estimation (asymptotic stability of the error dynamics) and fault/attack detection. Theorems 3.3 and 3.4 are proposed in this section to solve this problem through a set of matrix inequalities using the $\mathcal{H}_\infty$, $\mathcal{H}_-$ performance indexes. Hence, to summarise, the proposed fault/attack detection scheme is obtained through simultaneously satisfying the following, for some performance scalar variables γ_i, $\varrho_i\beta_i$ and η_i $\forall i \in \{1, ..., N\}$.

1. To guarantee asymptotic stability of the error dynamics (3.53).

2. To ensure a reasonable sensitivity of the residuals to the possible output attacks/faults over all frequency ranges, by satisfying

$$||T_{r_{\mathcal{F}_i}\mathcal{F}_i}||_- > \gamma_i, \tag{3.56}$$

where $r_{\mathcal{F}_i}$ is the residual signal defined for the case with no disturbance $d_{v_i} = 0$, no uncertainty $\underline{\phi}_i = 0$ and no fault $\underline{\mathcal{F}}_i = 0$.

3. To ensure a reasonable sensitivity of the residuals to the possible input attacks/faults over a finite frequency range defined in the set $\Omega_{\underline{\mathcal{F}}_i}$, by satisfying

$$||T_{r_{\underline{\mathcal{F}}_i}\underline{\mathcal{F}}_i}||_- > \varrho_i, \tag{3.57}$$

for all solutions of (3.53) such that,

$$\begin{aligned}&\int_0^\infty \Big(\kappa(\omega_{f_1}e_{v_i}(t) + j\dot{e}_{v_i}(t))(\omega_{f_2}e_{v_i}(t) - j\dot{e}_{v_i}(t))^T\Big)dt \\ &\leqslant 0,\end{aligned} \tag{3.58}$$

where κ, ω_{f_1}, ω_{f_2} are as defined in $\Omega_{\underline{\mathcal{F}}_i}$, and $r_{\underline{\mathcal{F}}_i}$ is the residual signal defined for the case with no disturbance $d_{v_i} = 0$, no uncertainty $\underline{\phi}_i = 0$ and no fault $\mathcal{F}_i = 0$.

4. To guarantee a good disturbances and uncertainties rejection performance w.r.t. to the residual signals over all frequency ranges, i.e.

$$||T_{r_{\mathcal{D}_i}d_{v_i}}||_\infty < \eta_i, \quad ||T_{r_{\mathcal{D}_i}\underline{\phi}_i}||_\infty < \beta_i, \tag{3.59}$$

where $r_{\mathcal{D}_i}$ is the residual signal defined without fault $\mathcal{F}_i = 0$ and $\underline{\mathcal{F}}_i = 0$.

For the rest of the manuscript, the time argument is omitted where it is not needed for clarity.

Theorem 3.3: [Taoufik *et al.* 2021a]

For $d_{v_i} = 0$, $\underline{\phi}_i = 0$, $\underline{\mathcal{F}}_i = 0$, $\mathcal{F}_i \neq 0$, let γ_i, θ_{m_i}, σ_{1i} and ε_i be strictly positive scalars, the error dynamics (3.53) is asymptotically stable and the performance index (3.56) is guaranteed if $\forall i \in \{1, ..., N\}$, there exist symmetric positive definite matrices P_i, matrices U_i, R_i and unstructured nonsingular matrices Y_i such that the following optimisation problem is solved

$$\max_{P_i, Y_i, U_i, R_i} \gamma_i$$

subject to

$$\begin{pmatrix} \Psi_i^1 & \Psi_i^2 & \Psi_i^3 & \Psi_i^4 \\ * & \Psi_i^5 & 0 & \Psi_i^6 \\ * & * & -\varepsilon_i I & \Psi_i^7 \\ * & * & * & \Psi_i^8 \end{pmatrix} < 0, \tag{3.60}$$

$$U_i V_{v_i} = 0, \tag{3.61}$$

where

$$\begin{aligned}
\Psi_i^1 &= Y_i\widetilde{A}_i + U_i Z^i \widetilde{C}_i \widetilde{A}_i - R_i Z^i \widetilde{C}_i + \widetilde{A}_i^T Y_i^T + \widetilde{A}_i^T (Z^i\widetilde{C}_i)^T U_i^T - (Z^i\widetilde{C}_i)^T R_i^T \\
&+\varepsilon_i \theta_{m_i} I - (Z^i\widetilde{C}_i)^T W_i^T W_i Z^i \widetilde{C}_i, \\
\Psi_i^2 &= -R_i D_{\mathcal{F}_i} - (Z^i\widetilde{C}_i)^T W_i^T W_i D_{\mathcal{F}_i}, \\
\Psi_i^3 &= Y_i + U_i Z^i \widetilde{C}_i, \\
\Psi_i^4 &= -Y_i + P_i + \sigma_{1i}\widetilde{A}_i^T Y_i^T + \sigma_{1i}\widetilde{A}_i^T (Z^i\widetilde{C}_i)^T U_i^T - \sigma_{1i}(Z^i\widetilde{C}_i)^T R_i^T, \\
\Psi_i^5 &= -D_{\mathcal{F}_i}^T W_i^T W_i D_{\mathcal{F}_i} + \gamma_i^2 I, \\
\Psi_i^6 &= -\sigma_{1i} D_{\mathcal{F}_i}^T R_i^T, \\
\Psi_i^7 &= \sigma_{1i} Y_i^T + \sigma_{1i}(Z^i\widetilde{C}_i)^T U_i^T, \\
\Psi_i^8 &= -\sigma_{1i}(Y_i + Y_i^T),
\end{aligned}$$

and the observer gains are specified as

$$\begin{aligned}
S_i &= Y_i^{-1} R_i, \\
H_i &= Y_i^{-1} U_i, \\
N_i &= (I + Y_i^{-1} U_i Z^i \widetilde{C}_i)\widetilde{A}_i - Y_i^{-1} R_i Z^i \widetilde{C}_i, \\
G_{1i} &= (I + Y_i^{-1} U_i Z^i \widetilde{C}_i)\underline{\widetilde{B}}_{u_i}, \\
G_{2i} &= (I + Y_i^{-1} U_i Z^i \widetilde{C}_i)\widetilde{B}_{u_i}\mathcal{A}_{u,i}^{-1}, \\
L_i &= Y_i^{-1} R_i - N_i Y_i^{-1} U_i.
\end{aligned} \tag{3.62}$$

Proof The performance index (3.56) corresponds to the following function

$$\mathcal{J}_{\mathcal{F}_i} = \int_0^\infty \Big(r_{\mathcal{F}_i}^T r_{\mathcal{F}_i} - \gamma_i^2 \mathcal{F}_i^T \mathcal{F}_i \Big) dt > 0. \tag{3.63}$$

Let us select the candidate Lyapunov function
$V_i(e_{v_i}) = e_{v_i}^T P_i e_{v_i}$, then

$$\begin{aligned}
\dot{V}(e_{v_i}) &= e_{v_i}^T (N_i^T P_i + P_i N_i) e_{v_i} + (\varphi_{v_i}^{e_{v_i}})^T T_i^T P_i e_{v_i} \\
&+ e_{v_i}^T P_i T_i \varphi_{v_i}^{e_{v_i}} + \mathcal{F}_i^T (-S_i D_{\mathcal{F}_i})^T P_i e_{v_i} \\
&+ e_{v_i}^T P_i (-S_i D_{\mathcal{F}_i}) \mathcal{F}_i.
\end{aligned} \tag{3.64}$$

On the other hand, (3.63) can be expressed as

$$\begin{aligned}
\mathcal{J}_{\mathcal{F}_i} &= \textstyle\int_0^\infty \Big([e_{v_i}^T(t)(Z^i\widetilde{C}_i)^T + \mathcal{F}_i^T(t) D_{\mathcal{F}_i}^T)] W_i^T W_i \\
&\times (Z^i \widetilde{C}_i e_{v_i}(t) + D_{\mathcal{F}_i}\mathcal{F}_i(t)) - \gamma_i^2 \mathcal{F}_i^T \mathcal{F}_i - \dot{V}(e_{v_i}) \Big) dt \\
&+ \textstyle\int_0^\infty \Big(\dot{V}(e_{v_i}) \Big) dt > 0.
\end{aligned} \tag{3.65}$$

According to Assumption 3.3.2, it can be shown that

$$\begin{aligned}
(\varphi_{v_i}^{e_{v_i}})^T \varphi_{v_i}^{e_{v_i}} = ||\varphi_{v_i}^{e_{v_i}}||^2 \quad &\leqslant \theta_i^2 ||x_i(t) - \hat{x}_i(t)||^2 \\
&+ \theta_{i_1}^2 ||x_{i_1}(t) - \hat{x}_{i_1}(t)||^2 + \ldots \\
&+ \theta_{i_{N_i}}^2 ||x_{i_{N_i}}(t) - \hat{x}_{i_{N_i}}(t)||^2 \\
&\leqslant \theta_{M_i} e_{v_i}^T e_{v_i},
\end{aligned} \tag{3.66}$$

where $\theta_{M_i} = \max(\theta_i^2, \theta_{i_1}^2, ..., \theta_{i_{N_i}}^2)$.

Since $V(e_{v_i}) = e_{v_i}^T P_i e_{v_i} \geq 0$ and using Lemma 3.1 and equation (3.66), (3.65) can be shown to be equivalent to

$$\begin{pmatrix} \Upsilon_i & -P_i S_i D_{\mathcal{F}_i} - (Z^i \widetilde{C}_i)^T W_i^T W_i D_{\mathcal{F}_i} \\ \star & -D_{\mathcal{F}_i}^T W_i^T W_i D_{\mathcal{F}_i} + \gamma_i^2 I \end{pmatrix} < 0, \tag{3.67}$$

where $\Upsilon_i = N_i^T P_i + P_i N_i + \varepsilon_i \theta_{M_i} I + \varepsilon_i^{-1} P_i T_i T_i^T P_i - (Z^i \widetilde{C}_i)^T W_i^T W_i Z^i \widetilde{C}_i$. Using the Schur complement, (3.67) can be re-written as

$$\mathcal{T}_{1i} + \mathcal{V}_{1i} \mathcal{S}_{1i} + \mathcal{S}_{1i}^T \mathcal{V}_{1i}^T < 0, \tag{3.68}$$

with

$$\mathcal{T}_{1i} = \begin{pmatrix} \varepsilon_i \theta_{M_i} I - (Z^i \widetilde{C}_i)^T W_i^T W_i Z^i \widetilde{C}_i & -(Z^i \widetilde{C}_i)^T W_i^T W_i D_{\mathcal{F}_i} & 0 \\ * & -D_{\mathcal{F}_i}^T W_i^T W_i D_{\mathcal{F}_i} + \gamma_i^2 I & 0 \\ * & * & -\varepsilon_i I \end{pmatrix},$$

$$\mathcal{S}_{1i} = \begin{pmatrix} N_i & -S_i D_{\mathcal{F}_i} & T_i \end{pmatrix}, \ \mathcal{V}_{1i} = \begin{pmatrix} P_i \\ 0 \\ 0 \end{pmatrix}.$$

Using the congruence transformation $\begin{pmatrix} I & \mathcal{T}_{1i}^T \end{pmatrix}$, (3.68) is equivalent to

$$\begin{pmatrix} \mathcal{T}_{1i} + \mathcal{K}_{1i} \mathcal{S}_{1i} + \mathcal{S}_{1i}^T \mathcal{K}_{1i}^T & -\mathcal{K}_{1i} + \mathcal{V}_{1i} + \mathcal{S}_{1i}^T \mathcal{Y}_{1i}^T \\ * & -(\mathcal{Y}_{1i} + \mathcal{Y}_{1i}^T) \end{pmatrix} < 0, \tag{3.69}$$

for new general matrices $\mathcal{K}_{1i}$ and $\mathcal{Y}_{1i}$. Hence, by selecting

$$\mathcal{K}_{1i}^T = \begin{pmatrix} Y_i^T & 0 & 0 \end{pmatrix}, \ \ \mathcal{Y}_{1i} = \sigma_{1i} Y_i,$$

for a scalar σ_{1i} and a nonsingular general matrix Y_i, one can obtain the following sufficient condition

$$\begin{pmatrix} \Pi_i^1 & \Pi_i^2 & Y_i T_i & \Pi_i^3 \\ * & \Pi_i^4 & 0 & \Pi_i^5 \\ * & * & -\varepsilon_i I & \sigma_{1i} T_i^T Y_i^T \\ * & * & * & -\sigma_{1i}(Y_i + Y_i^T) \end{pmatrix} < 0,$$

with

$$\begin{aligned} \Pi_i^1 &= Y_i N_i + N_i^T Y_i^T + \varepsilon_i \theta_{M_i} I - (Z^i \widetilde{C}_i)^T W_i^T W_i Z^i \widetilde{C}_i, \\ \Pi_i^2 &= -Y_i S_i D_{\mathcal{F}_i} - (Z^i \widetilde{C}_i)^T W_i^T W_i D_{\mathcal{F}_i}, \\ \Pi_i^3 &= -Y_i + P_i + \sigma_{1i} N_i^T Y_i^T, \\ \Pi_i^4 &= -D_{\mathcal{F}_i}^T W_i^T W_i D_{\mathcal{F}_i} + \gamma_i^2 I, \\ \Pi_i^5 &= -\sigma_{1i} D_{\mathcal{F}_i}^T S_i^T Y_i^T. \end{aligned}$$

Replacing N_i and T_i with their respective values, and applying the linearising change of variables $U_i = Y_i H_i$, $R_i = Y_i S_i$, (3.60) is obtained. Furthermore, pre-multiplying (3.51a) with Y_i yields (3.61). Therefore, solving (3.60) under the imposed constraints (3.61), and using the observer gains (3.62) guarantees the residual performance index (3.56) and the asymptotic stability of the error dynamics (3.49).

Theorem 3.4: [Taoufik *et al.* 2021a]

For $d_{v_i} = 0$, $\underline{\phi}_i = 0$, $\mathcal{F}_i = 0$, $\underline{\mathcal{F}}_i \neq 0$, let ϱ_i, θ_{M_i}, σ_{2i} and ε_i be strictly positive scalars, an arbitrary design matrix K_i, the error dynamics (3.53) is asymptotically stable and the performance index (3.57) is guaranteed if $\forall i \in \{1, ..., N\}$ over a finite frequency domain defined in (3.55), there exist symmetric positive definite matrices X_i, symmetric matrices $\mathcal{X}_i$, matrices U_i, R_i and unstructured nonsingular matrices Y_i such that the following optimisation problem is solved

$$\max_{X_i, \mathcal{X}_i, Y_i, U_i, R_i} \varrho_i$$

subject to

$$\begin{pmatrix} \Sigma_i^1 & \Sigma_i^2 & \Sigma_i^3 & \Sigma_i^4 \\ * & \Sigma_i^5 & \Sigma_i^6 & \Sigma_i^7 \\ * & * & -\varepsilon_i I & \Sigma_i^8 \\ * & * & * & \Sigma_i^9 \end{pmatrix} < 0, \tag{3.70}$$

$$\kappa \mathcal{X}_i \geqslant 0,$$

where

$$\begin{aligned}
\Sigma_i^1 &= Y_i \widetilde{A}_i + U_i Z^i \widetilde{C}_i \widetilde{A}_i - R_i Z^i \widetilde{C}_i + \widetilde{A}_i^T Y_i^T - (Z^i \widetilde{C}_i)^T W_i^T W_i Z^i \widetilde{C}_i \\
&\quad + (Z^i \widetilde{C}_i \widetilde{A}_i)^T U_i^T - (Z^i \widetilde{C}_i)^T R_i^T - \omega_{f_1} \omega_{f_2} \mathcal{X}_i + \varepsilon_i \theta_{M_i} I, \\
\Sigma_i^2 &= -U_i Z^i \widetilde{C}_i \mathcal{B}_i + \widetilde{A}_i^T Y_i^T K_i^T + (Z^i \widetilde{C}_i \widetilde{A}_i)^T U_i^T K_i^T \\
&\quad - (Z^i \widetilde{C}_i)^T R_i^T K_i^T, \\
\Sigma_i^3 &= Y_i + U_i Z^i \widetilde{C}_i, \\
\Sigma_i^4 &= -Y_i + X_i - j\omega_{f_a} \mathcal{X}_i + \sigma_{2i} \widetilde{A}_i^T Y_i^T + \sigma_{2i} (Z^i \widetilde{C}_i \widetilde{A}_i)^T U_i^T \\
&\quad - \sigma_{2i} (Z^i \widetilde{C}_i)^T R_i^T, \\
\Sigma_i^5 &= \varrho_i^2 I - K_i Y_i \mathcal{B}_i - K_i U_i Z^i \widetilde{C}_i \mathcal{B}_i - \mathcal{B}_i^T Y_i^T K_i^T - \mathcal{B}_i^T (Z^i \widetilde{C}_i)^T U_i^T K_i^T, \\
\Sigma_i^6 &= K_i Y_i + K_i U_i Z^i \widetilde{C}_i, \\
\Sigma_i^7 &= -K_i Y_i - \sigma_{2i} \mathcal{B}_i^T Y_i^T - \sigma_{2i} \mathcal{B}_i^T (Z^i \widetilde{C}_i)^T U_i^T, \\
\Sigma_i^8 &= \sigma_{2i} Y_i^T + \sigma_{2i} (Z^i \widetilde{C}_i)^T U_i^T, \\
\Sigma_i^9 &= -(\mathcal{X}_i + \sigma_{2i} Y_i + \sigma_{2i} Y_i^T),
\end{aligned}$$

and $\mathcal{B}_i = \begin{pmatrix} -\widetilde{B}_{f_i} & \widetilde{B}_{u_i} \mathcal{A}_{u,i}^{-1} D_{u_i} \end{pmatrix}$. The observer gains are then computed as in (3.62).

Proof Let us select the candidate Lyapunov function $V_i(e_{v_i}) = e_{v_i}^T X_i e_{v_i}$, then

$$\begin{aligned}
\dot{V}(e_{v_i}) &= e_{v_i}^T (N_i^T X_i + X_i N_i) e_{v_i} + (\varphi_{v_i}^{e_{v_i}})^T T_i^T X_i e_{v_i} \\
&\quad + e_{v_i}^T X_i T_i \varphi_{v_i}^{e_{v_i}} - \underline{\mathcal{F}}_i^T (T_i \mathcal{B}_i)^T X_i e_{v_i} - e_{v_i}^T X_i (T_i \mathcal{B}_i) \underline{\mathcal{F}}_i.
\end{aligned} \tag{3.71}$$

To solve (3.57) over a finite frequency domain as defined in (3.55), one could define the following function

$$\mathcal{J}_{\underline{\mathcal{F}}_i} = \int_0^\infty \Big(\varrho_i^2 \underline{\mathcal{F}}_i^T \underline{\mathcal{F}}_i - r_{\underline{\mathcal{F}}_i}^T r_{\underline{\mathcal{F}}_i} - \mathrm{tr}(\mathbf{He}(\mathcal{W}_i) \mathcal{X}_i) + \dot{V}(e_{v_i}) \Big) dt < 0, \tag{3.72}$$

where $\mathcal{W}_i = (\omega_{f_1} e_{v_i} + j\dot{e}_{v_i})(\omega_{f_2} e_{v_i} + j\dot{e}_{v_i})^*$ and $\mathcal{X}_i$ is a symmetric matrix. From (3.58), one gets

$$\int_0^\infty \kappa \mathcal{W}_i dt \leqslant 0.$$

Moreover, it can be shown through the Parseval's theorem [Zhou & Doyle 1998] that

$$\int_0^\infty \mathcal{W}_i dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Big((\omega_{f_1} - \omega)(\omega_{f_2} - \omega) \check{\mathbf{e}}_i(\omega) \check{\mathbf{e}}_i^T(\omega) \Big) d\omega,$$

where $\check{\mathbf{e}}_i(\omega)$ is the Fourier transform of $e_{v_i}(t)$. Choosing $\mathcal{X}_i$ such that $\kappa \mathcal{X}_i \geqslant 0$, it yields

$$\mathrm{tr}((\textstyle\int_0^\infty \mathcal{W}_i dt)^* \mathcal{X}_i) + \mathrm{tr}((\textstyle\int_0^\infty \mathcal{W}_i dt) \mathcal{X}_i) \leqslant 0,$$

or equivalently, $\mathrm{tr}(\mathbf{He}(\mathcal{W}_i)\mathcal{X}_i) \leqslant 0$. Therefore, (3.57) is guaranteed for all solutions of (3.53) satisfying (3.58), if

$$\varrho_i^2 \underline{\mathcal{F}}_i^T \underline{\mathcal{F}}_i - r_{\underline{\mathcal{F}}_i}^T r_{\underline{\mathcal{F}}_i} + \dot{V}(e_{v_i}) - \mathrm{tr}(\mathbf{He}(\mathcal{W}_i)\mathcal{X}_i) < 0. \tag{3.73}$$

By setting $\omega_{f_a} = \frac{\omega_{f_1} + \omega_{f_2}}{2}$, then

$$\begin{array}{l}
-\mathrm{tr}(\mathbf{He}(\mathcal{W}_i)\mathcal{X}_i) \\
= -e_{v_i}^T \omega_{f_1}\omega_{f_2} \mathcal{X}_i e_{v_i} - \dot{e}_{v_i}^T \mathcal{X}_i \dot{e}_{v_i} - e_{v_i}^T j\omega_{f_a} \mathcal{X}_i \dot{e}_{v_i} + \dot{e}_{v_i}^T j\omega_{f_a} \mathcal{X}_i e_{v_i} \\
= -e_{v_i}^T \omega_{f_1}\omega_{f_2} \mathcal{X}_i e_{v_i} - e_{v_i}^T N_i^T \mathcal{X}_i N_i e_{v_i} - (\varphi_{v_i}^{e_{v_i}})^T T_i^T \mathcal{X}_i N_i e_{v_i} \\
+ \underline{\mathcal{F}}_i^T \mathcal{B}_i^T T_i^T \mathcal{X}_i N_i e_{v_i} - e_{v_i}^T N_i^T \mathcal{X}_i T_i \varphi_{v_i}^{e_{v_i}} - (\varphi_{v_i}^{e_{v_i}})^T T_i^T \mathcal{X}_i T_i \varphi_{v_i}^{e_{v_i}} \\
+ \underline{\mathcal{F}}_i^T \mathcal{B}_i^T T_i^T \mathcal{X}_i T_i \varphi_{v_i}^{e_{v_i}} + e_{v_i}^T N_i^T \mathcal{X}_i T_i \mathcal{B}_i \underline{\mathcal{F}}_i + (\varphi_{v_i}^{e_{v_i}})^T T_i^T \mathcal{X}_i T_i \mathcal{B}_i \underline{\mathcal{F}}_i \\
- e_{v_i}^T j\omega_{f_a} \mathcal{X}_i T_i \varphi_{v_i}^{e_{v_i}} + e_{v_i}^T j\omega_{f_a} \mathcal{X}_i T_i \mathcal{B}_i \underline{\mathcal{F}}_i - e_{v_i}^T j\omega_{f_a} N_i e_{v_i} \\
+ e_{v_i}^T N_i^T j\omega_{f_a} \mathcal{X}_i e_{v_i} + (\varphi_{v_i}^{e_{v_i}})^T T_i^T j\omega_{f_a} \mathcal{X}_i e_{v_i} - \underline{\mathcal{F}}_i^T \mathcal{B}_i^T T_i^T j\omega_{f_a} \mathcal{X}_i e_{v_i} \\
- \underline{\mathcal{F}}_i^T \mathcal{B}_i^T T_i^T \mathcal{X}_i T_i \mathcal{B}_i \underline{\mathcal{F}}_i.
\end{array} \tag{3.74}$$

On the other hand, using Lemma 3.1 and (3.66), one has

$$\begin{array}{ll}
\dot{V}(e_{v_i}) & < e_{v_i}^T (N_i^T X_i + X_i N_i + \varepsilon_i \theta_{M_i} I \\
& + \varepsilon_i^{-1} X_i T_i T_i^T X_i) e_{v_i} - \underline{\mathcal{F}}_i^T (T_i \mathcal{B}_i)^T X_i e_{v_i} \\
& - e_{v_i}^T X_i (T_i \mathcal{B}_i) \underline{\mathcal{F}}_i.
\end{array} \tag{3.75}$$

Replacing (3.74) and (3.75) into (3.73) gives

$$\begin{pmatrix} \Xi_{1i}^1 & \Xi_{1i}^2 & \Xi_{1i}^3 \\ * & \Xi_{1i}^4 & \Xi_{1i}^5 \\ * & * & \Xi_{1i}^6 \end{pmatrix} < 0, \tag{3.76}$$

where

$$\begin{array}{ll}
\Xi_{1i}^1 & = -\omega_{f_1}\omega_{f_2} \mathcal{X}_i - N_i^T \mathcal{X}_i N_i - j\omega_{f_a} \mathcal{X}_i N_i + j\omega_{f_a} N_i^T \mathcal{X}_i + N_i^T X_i + X_i N_i \\
& + \varepsilon_i \theta_{M_i} I - (Z^i \widetilde{C}_i)^T W_i^T W_i Z^i \widetilde{C}_i, \\
\Xi_{1i}^2 & = N_i^T \mathcal{X}_i T_i \mathcal{B}_i + j\omega_{f_a} \mathcal{X}_i T_i \mathcal{B}_i - X_i T_i \mathcal{B}_i, \\
\Xi_{1i}^3 & = -N_i^T \mathcal{X}_i T_i - j\omega_{f_a} \mathcal{X}_i T_i + X_i T_i, \\
\Xi_{1i}^4 & = -\mathcal{B}_i^T T_i^T \mathcal{X}_i T_i \mathcal{B}_i + \varrho_i^2 I, \\
\Xi_{1i}^5 & = \mathcal{B}_i^T T_i^T \mathcal{X}_i T_i, \\
\Xi_{1i}^6 & = -T_i^T \mathcal{X}_i T_i - \varepsilon_i I.
\end{array}$$

It can be re-written as

$$\mathcal{T}_{2i} + \mathcal{V}_{2i}\mathcal{S}_{2i} + \mathcal{S}_{2i}^T\mathcal{V}_{2i}^T - \mathcal{S}_{2i}^T\mathcal{X}_i\mathcal{S}_{2i} < 0, \tag{3.77}$$

with

$$\mathcal{T}_{2i} = \begin{pmatrix} -\omega_{f_1}\omega_{f_2}\mathcal{X}_i + \varepsilon_i\theta_{M_i}I - (Z^i\widetilde{C}_i)^T W_i^T W_i Z^i\widetilde{C}_i & 0 & 0 \\ * & \varrho_i^2 I & 0 \\ * & * & -\varepsilon_i I \end{pmatrix},$$

$$\mathcal{S}_{2i} = \begin{pmatrix} N_i & -T_i\mathcal{B}_i & T_i \end{pmatrix}, \; \mathcal{V}_{2i} = \begin{pmatrix} X_i - j\omega_{f_a}\mathcal{X}_i \\ 0 \\ 0 \end{pmatrix}.$$

Similarly to Theorem 1, (3.77) can be shown to be equivalent to

$$\begin{pmatrix} \mathcal{T}_{2i} + \mathcal{K}_{2i}\mathcal{S}_{2i} + \mathcal{S}_{2i}^T\mathcal{K}_{2i}^T & -\mathcal{K}_{2i} + \mathcal{V}_{2i} + \mathcal{S}_{2i}^T\mathcal{Y}_{2i}^T \\ * & -(\mathcal{X}_i + \mathcal{Y}_{2i} + \mathcal{Y}_{2i}^T) \end{pmatrix} < 0, \tag{3.78}$$

for new general matrices $\mathcal{K}_{2i}$ and $\mathcal{Y}_{2i}$. Hence, by selecting

$$\mathcal{K}_{2i}^T = \begin{pmatrix} Y_i^T & Y_i^T K_i^T & 0 \end{pmatrix}, \quad \mathcal{Y}_{2i} = \sigma_{2i}Y_i,$$

for a scalar σ_{2i}, an arbitrary matrix K_i and a nonsingular general matrix Y_i, one can obtain the following sufficient condition

$$\begin{pmatrix} \Xi_{2i}^1 & \Xi_{2i}^2 & Y_iT_i & \Xi_{2i}^3 \\ * & \Xi_{2i}^4 & K_iY_iT_i & \Xi_{2i}^5 \\ * & * & -\varepsilon_i I & \sigma_{2i}T_i^TY_i^T \\ * & * & * & \Xi_{2i}^6 \end{pmatrix} < 0,$$

with

$$\begin{aligned}
\Xi_{2i}^1 &= Y_iN_i + N_i^TY_i^T - \omega_{f_1}\omega_{f_2}\mathcal{X}_i + \varepsilon_i\theta_{M_i}I - (Z^i\widetilde{C}_i)^T W_i^T W_i Z^i\widetilde{C}_i, \\
\Xi_{2i}^2 &= -Y_iT_i\mathcal{B}_i + N_i^TY_i^TK_i^T, \\
\Xi_{2i}^3 &= -Y_i + X_i - j\omega_{f_a}\mathcal{X}_i + \sigma_{2i}N_i^TY_i^T, \\
\Xi_{2i}^4 &= \varrho_i^2 I - K_iY_iT_i\mathcal{B}_i - \mathcal{B}_i^TT_i^TY_i^TK_i^T, \\
\Xi_{2i}^5 &= -K_iY_i - \sigma_{2i}\mathcal{B}_i^TT_i^TY_i^T, \\
\Xi_{2i}^6 &= -(\mathcal{X}_i + \sigma_{2i}Y_i + \sigma_{2i}Y_i^T).
\end{aligned}$$

By replacing N_i and T_i with their respective values, and applying the linearising change of variables $U_i = Y_iH_i$, $R_i = Y_iS_i$, (3.70) is obtained. This guarantees the residual performance index (3.57) and the asymptotic stability of the error dynamics (3.49).

Remark 3.3.4 *Given that the LMIs (3.70) $\forall i$ are in the complex domain, most solvers cannot directly handle them. Hence, the following equivalent statements are used for a complex Hermitian matrix $L(x)$*

1. $L(x) < 0$.

2. $\begin{pmatrix} Re(L(x)) & Im(L(x)) \\ -Im(L(x)) & Re(L(x)) \end{pmatrix} < 0.$

where $Re(L(x))$ represents the real part of $L(x)$ and $Im(L(x))$ its imaginary part. More details can be found in [Gahinet et al. 1996].

Theorem 3.5: [Taoufik *et al.* 2021a]

For $\mathcal{F}_i = 0$, $\underline{\mathcal{F}}_i = 0$, $d_{v_i} \neq 0$, $\underline{\phi}_i \neq 0$, let β_i, η_i, θ_{M_i}, σ_{3i} and ε_i be strictly positive scalars, the error dynamics (3.53) is asymptotically stable and the performance indexes (3.59) are guaranteed if $\forall i \in \{1, ..., N\}$, there exist symmetric positive definite matrices Q_i, matrices U_i, R_i and unstructured nonsingular matrices Y_i such that for all possible uncertainties, under the imposed constraint (3.61)

$$\min_{Q_i,Y_i,U_i,R_i} \beta_i + \eta_i$$

subject to

$$\begin{pmatrix} \Phi_i^1 & \Phi_i^2 & \Phi_i^3 & \Phi_i^4 & \Phi_i^5 \\ * & \Phi_i^6 & \Phi_i^7 & 0 & \Phi_i^8 \\ \star & * & \Phi_i^9 & 0 & \Phi_i^{10} \\ * & * & * & -\varepsilon_i I & \Phi_i^{11} \\ * & * & * & * & \Phi_i^{12} \end{pmatrix} < 0, \tag{3.79}$$

where

$$\begin{aligned}
\Phi_i^1 &= Y_i\widetilde{A}_i + U_iZ^i\widetilde{C}_i\widetilde{A}_i - R_iZ^i\widetilde{C}_i + \widetilde{A}_i^TY_i^T + \varepsilon_i\theta_{M_i}I \\
&\quad +(Z^i\widetilde{C}_i\widetilde{A}_i)^TU_i^T - (Z^i\widetilde{C}_i)^TR_i^T + (Z^i\widetilde{C}_i)^TW_i^TW_iZ^i\widetilde{C}_i, \\
\Phi_i^2 &= Y_i\widetilde{B}_{d_i} + U_iZ^i\widetilde{C}_i\widetilde{B}_{d_i} - R_iZ^i\widetilde{D}_{d_i} + Z^i\widetilde{C}_iW_i^TW_iZ^i\widetilde{D}_{d_i}, \\
\Phi_i^3 &= R_i\underline{\mathbf{X}}_i - Y_i\bar{\mathbf{X}}_i - U_iZ^i\widetilde{C}_i\bar{\mathbf{X}}_i - (Z^i\widetilde{C}_i)^TW_i^TW_i\underline{\mathbf{X}}_i, \\
\Phi_i^4 &= Y_i + Y_iH_iZ^i\widetilde{C}_i, \\
\Phi_i^5 &= -Y_i + Q_i + \sigma_{3i}\widetilde{A}_i^TY_i^T + \sigma_{3i}(Z^i\widetilde{C}_i\widetilde{A}_i)^TU_i^T - \sigma_{3i}(Z^i\widetilde{C}_i)^TR_i^T, \\
\Phi_i^6 &= (Z^i\widetilde{D}_{d_i})^TW_i^TW_iZ^i\widetilde{D}_{d_i} - \eta_i^2I, \\
\Phi_i^7 &= -\underline{\mathbf{X}}_i^TW_i^TW_iZ^i\widetilde{D}_{d_i}, \\
\Phi_i^8 &= \sigma_{3i}\widetilde{B}_{d_i}^TY_i^T + \sigma_{3i}\widetilde{B}_{d_i}^T(Z^i\widetilde{C}_i)^TU_i^T - \sigma_{3i}Z^i\widetilde{D}_{d_i}^TR_i^T, \\
\Phi_i^9 &= \underline{\mathbf{X}}_i^TW_i^TW_i\underline{\mathbf{X}}_i - \beta_i^2I, \\
\Phi_i^{10} &= \sigma_{3i}\underline{\mathbf{X}}_i^TR_i^T - \sigma_{3i}\bar{\mathbf{X}}_i^TY_i^T - \sigma_{3i}\bar{\mathbf{X}}_i^T(Z^i\widetilde{C}_i)^TU_i^T, \\
\Phi_i^{11} &= \sigma_{3i}Y_i^T + \sigma_{3i}(Z^i\widetilde{C}_i)^TU_i^T, \\
\Phi_i^{12} &= -\sigma_{3i}(Y_i + Y_i^T).
\end{aligned}$$

The observer gains are then computed as in (3.62).

Proof Let us select the candidate Lyapunov function $V_i(e_{v_i}) = e_{v_i}^TQ_ie_{v_i}$, then

$$\begin{aligned}
\dot{V}(e_{v_i}) &= e_{v_i}^T(N_i^TQ_i + Q_iN_i)e_{v_i} + (\varphi_{v_i}^{e_{v_i}})^TT_i^TQ_ie_{v_i} \\
&\quad +e_{v_i}^TQ_iT_i\varphi_{v_i}^{e_{v_i}} + \underline{\phi}_i^T(t)(S_i\underline{\mathbf{X}}_i - T_i\bar{\mathbf{X}}_i)^TQ_ie_{v_i} + e_{v_i}^TQ_i(S_i\underline{\mathbf{X}}_i - T_i\bar{\mathbf{X}}_i)\underline{\phi}_i(t) \\
&\quad +d_{v_i}^T(T_i\widetilde{B}_{d_i} - S_iZ^i\widetilde{D}_{d_i})^TQ_ie_{v_i} + e_{v_i}^TQ_i(T_i\widetilde{B}_{d_i} - S_iZ^i\widetilde{D}_{d_i})d_{v_i}(t).
\end{aligned} \tag{3.80}$$

The performance index is equivalent to

$$\mathcal{J}_{\mathcal{D}_i} = \int_0^{\infty} \Big(r_{\mathcal{D}_i}^T r_{\mathcal{D}_i} - \beta_i^2 \underline{\phi}_i^T \underline{\phi}_i - \eta_i^2 d_{v_i}^T d_{v_i} \Big) dt < 0. \tag{3.81}$$

Combining the two yields

$$\begin{aligned}
\mathcal{J}_{\mathcal{D}_i} &= \textstyle\int_0^{\infty} \Big([e_{v_i}^T (Z^i \widetilde{C}_i)^T + d_{v_i}^T (Z^i \widetilde{D}_{d_i})^T] W_i^T W_i [Z^i \widetilde{C}_i e_{v_i}(t) + Z^i \widetilde{D}_{d_i} d_{v_i}(t)] \\
&\quad - \eta_i^2 d_{v_i}^T d_{v_i} - e_{v_i}^T (Z^i \widetilde{C}_i)^T W_i^T W_i \underline{\mathbf{X}}_i \underline{\phi}_i(t) - d_{v_i}^T (Z^i \widetilde{D}_{d_i})^T W_i^T W_i \underline{\mathbf{X}}_i \underline{\phi}_i(t) \\
&\quad - \beta_i^2 \underline{\phi}_i^T \underline{\phi}_i + \underline{\phi}_i^T \underline{\mathbf{X}}_i^T W_i^T W_i \underline{\mathbf{X}}_i \underline{\phi}_i - \underline{\phi}_i^T \underline{\mathbf{X}}_i^T W_i W_i^T Z^i \widetilde{C}_i e_{v_i} \\
&\quad - \underline{\phi}_i^T \underline{\mathbf{X}}_i^T W_i^T W_i Z^i \widetilde{D}_{d_i} d_{v_i} + \dot{V}(e_{v_i}) \Big) dt - \textstyle\int_0^{\infty} \dot{V}(e_{v_i}) dt \\
&< 0.
\end{aligned} \tag{3.82}$$

The above inequality can be expressed as

$$\begin{pmatrix} \Gamma_{i1} & \Gamma_{i2} + \Upsilon_i^{de} \\ \star & \Upsilon_i^{dd} \end{pmatrix} < 0,$$

where

$$\begin{cases}
\Gamma_{i1} &= N_i^T Q_i + Q_i N_i + (Z^i \widetilde{C}_i)^T W_i^T W_i Z^i \widetilde{C}_i + \varepsilon_i \theta_{M_i} I + \varepsilon_i^{-1} Q_i T_i T_i^T Q_i \\
\Gamma_{i2} &= Q_i \Big(T_i \widetilde{B}_{d_i} - S_i Z^i \widetilde{D}_{d_i} \quad (S_i \underline{\mathbf{X}}_i - T_i \bar{\mathbf{X}}_i) \Big) \\
\Upsilon_i^{de} &= \Big(Z^i \widetilde{C}_i W_i^T W_i Z^i \widetilde{D}_{d_i} \quad -(Z^i \widetilde{C}_i)^T W_i^T W_i \underline{\mathbf{X}}_i \Big) \\
\Upsilon_i^{dd} &= \begin{pmatrix} (Z^i \widetilde{D}_{d_i})^T W_i^T W_i Z^i \widetilde{D}_{d_i} - \eta_i^2 I & -\underline{\mathbf{X}}_i^T W_i^T W_i Z^i \widetilde{D}_{d_i} \\ \star & \underline{\mathbf{X}}_i^T W_i^T W_i \underline{\mathbf{X}}_i - \beta_i^2 I \end{pmatrix}
\end{cases}$$

Similarly to Theorem 3.3, the above is equivalent to

$$\mathcal{T}_{3i} + \mathcal{V}_{3i} \mathcal{S}_{3i} + \mathcal{S}_{3i}^T \mathcal{V}_{3i}^T < 0, \tag{3.83}$$

where

$$\mathcal{T}_{3i} = \begin{pmatrix}
(Z^i \widetilde{C}_i)^T W_i^T W_i Z^i \widetilde{C}_i + \varepsilon_i \theta_{M_i} I & Z^i \widetilde{C}_i W_i^T W_i Z^i \widetilde{D}_{d_i} & -(Z^i \widetilde{C}_i)^T W_i^T W_i \underline{\mathbf{X}}_i & 0 \\
* & (Z^i \widetilde{D}_{d_i})^T W_i^T W_i Z^i \widetilde{D}_{d_i} - \eta_i^2 I & -\underline{\mathbf{X}}_i^T W_i^T W_i Z^i \widetilde{D}_{d_i} & 0 \\
* & * & \underline{\mathbf{X}}_i^T W_i^T W_i \underline{\mathbf{X}}_i - \beta_i^2 I & 0 \\
* & * & * & -\varepsilon_i I
\end{pmatrix},$$

$$\mathcal{S}_{3i} = \Big(N_i \quad T_i \widetilde{B}_{d_i} - S_i Z^i \widetilde{D}_{d_i} \quad S_i \underline{\mathbf{X}}_i - T_i \bar{\mathbf{X}}_i \quad T_i \Big), \; \mathcal{V}_{3i} = \begin{pmatrix} Q_i \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

By selecting

$$\mathcal{K}_{3i}^T = \begin{pmatrix} Y_i^T & 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{Y}_{3i} = \sigma_{3i} Y_i,$$

for a scalar σ_{3i} and a nonsingular general matrix Y_i, one can obtain the following sufficient condition

$$\begin{pmatrix} \Lambda_i^1 & \Lambda_i^2 & \Lambda_i^3 & Y_i T_i & \Lambda_i^4 \\ * & \Lambda_i^5 & -\underline{\mathbf{X}}_i^T W_i^T W_i Z^i \widetilde{D}_{d_i} & 0 & \Lambda_i^6 \\ \star & * & \Lambda_i^7 & 0 & \Lambda_i^8 \\ * & * & * & -\varepsilon_i I & \sigma_{3i} T_i^T Y_i^T \\ * & * & * & * & -\sigma_{3i}(Y_i + Y_i^T) \end{pmatrix} < 0,$$

where

$$\begin{aligned}
\Lambda_i^1 &= Y_i N_i + N_i^T Y_i^T + (Z^i \widetilde{C}_i)^T W_i^T W_i Z^i \widetilde{C}_i + \varepsilon_i \theta_{M_i} I, \\
\Lambda_i^2 &= Y_i T_i \widetilde{B}_{d_i} - Y_i S_i Z^i \widetilde{D}_{d_i} + Z^i \widetilde{C}_i W_i^T W_i Z^i \widetilde{D}_{d_i}, \\
\Lambda_i^3 &= Y_i S_i \underline{\mathbf{X}}_i - Y_i T_i \bar{\mathbf{X}}_i - (Z^i \widetilde{C}_i)^T W_i^T W_i \underline{\mathbf{X}}_i, \\
\Lambda_i^4 &= -Y_i + Q_i + \sigma_{3i} N_i^T Y_i^T, \\
\Lambda_i^5 &= (Z^i \widetilde{D}_{d_i})^T W_i^T W_i Z^i \widetilde{D}_{d_i} - \eta_i^2 I, \\
\Lambda_i^6 &= \sigma_{3i} \widetilde{B}_{d_i}^T T_i^T Y_i^T - \sigma_{3i} Z^i \widetilde{D}_{d_i}^T S_i^T Y_i^T, \\
\Lambda_i^7 &= \underline{\mathbf{X}}_i^T W_i^T W_i \underline{\mathbf{X}}_i - \beta_i^2 I, \\
\Lambda_i^8 &= \sigma_{3i} \underline{\mathbf{X}}_i^T S_i^T Y_i^T - \sigma_{3i} \bar{\mathbf{X}}_i^T T_i^T Y_i^T.
\end{aligned}$$

Replacing N_i and T_i with their respective values, and applying the linearising change of variables $U_i = Y_i H_i$, $R_i = Y_i S_i$, (3.79) is obtained. This guarantees the residual performance index (3.59) and the asymptotic stability of the error dynamics (3.49).

Remark 3.3.5 *One could note that it is possible to relax constraint (3.61). Indeed, this equality constraint implies that the span of the rows of U_i is included in ker(V_{v_i}). Hence, one could turn this into a minimisation of its maximum singular value which could be minimised, i.e., for a scalar $\vartheta_i > 0$*

$$\min_{U_i} \quad \vartheta_i$$

subject to

$$-\vartheta_i I + U_i V_{v_i} \vartheta_i^{-1} (U_i V_{v_i})^T < 0. \tag{3.84}$$

Applying the Schur complement to (3.84) yields the following LMI

$$\begin{pmatrix} \vartheta_i I & U_i V_{v_i} \\ * & \vartheta_i I \end{pmatrix} < 0. \tag{3.85}$$

Remark 3.3.6 *Note that here, as opposed to what is typically done in literature, we do not impose that $T_i \widetilde{B}_{d_i} - S_i Z^i \widetilde{D}_{d_i} = S_i \boldsymbol{X}_i = 0$. Indeed, maintaining this constraint while solving the proposed inequalities can be unfeasable for some systems. Contrary to other works using unknown input observer, our approach does not require invertibility conditions except on Y_i which is inherently required by the proposed LMIs. Thus, no rank condition is required for the existence of the unknown input observer to solve the LMIs.*

3.3.2.2 Residual Evaluation:

In order to isolate the faulty element (the specific faulty agent and/or faulty link), the residuals are evaluated by comparing them with an off-line computed threshold defined hereafter. For this purpose, let us select the following RMS evaluation functions [Ding 2008], $\forall p \in \mathcal{N}_i \cup i$

$$\begin{aligned} J^e_{i,p}(t) &= ||r^p_i(t)||_{\text{RMS}} \\ &= \Big(\frac{1}{T_w}\int_t^{t+T_w}(r^p_i(\tau))^T r^p_i(\tau)d\tau\Big)^{\frac{1}{2}}, \end{aligned} \tag{3.86}$$

where T_w is a finite evaluation window with

$$r^T_i(t) = [(r^i_i(t))^T, (r^{i_1}_i(t))^T, ..., (r^{i_{N_i}}_i(t))^T],$$

and $r^p_i(t) \in \mathbb{R}^{n_y}, \forall p \in \mathcal{N}_i \cup i$. Noise, disturbances, communication uncertainties (etc.) are treated as unstructured unknown inputs and the RMS threshold is computed as

$$J^e_{ip_{th}} = \sup_{\text{attack/fault free}} ||r^p_i(t)||_{\text{RMS}}, \tag{3.87}$$

where one could set $J^e_{ith} = \max\{J^e_{ii_{th}}, ..., J^e_{ii_{N_{i\,th}}}\}$. For isolation purpose, let us define the secure detection flags π_i, such that if $J^e_{i,i}(t) \leqslant J^e_{ith}$ then $\pi_i = 0$ and $\pi_i = 1$ when $J^e_{i,i}(t) > J^e_{ith}$. An agent i is assumed to request the secure detection flag of its neighbours when a fault or an attack has been detected through the generated residual functions $J^e_{i,j}(t)$, $j \in \mathcal{N}_i$.

In order to summarise the proposed scheme, two algorithms are proposed hereafter. The optimisation Algorithm 2 is ran offline and proposes steps to compute the observer matrix gains using a finite-frequency mixed $\mathcal{H}_\infty/\mathcal{H}_-$ approach by simultaneously combining Theorems 3.3-3.5 and Remark 3.3.5. Define the multi-objective cost function

$$s_i = \frac{\lambda_{i1}\eta_i + \lambda_{i2}\beta_i + \lambda_{i3}\vartheta_i}{\lambda_{i4}\gamma_i + \lambda_{i5}\varrho_i}, \tag{3.88}$$

where $\lambda_{i1}, \lambda_{i2}, \lambda_{i3}, \lambda_{i4}, \lambda_{i5}$ are positive trade-off weighing constants.

Remark 3.3.7 *It should be noted that Algorithm* (3.88) *ensures that the best solution with respect to the cost function* (3.88) *is obtained. This renders the residual functions as sensible as possible to the fault and attack signals while guaranteeing the best possible attenuation performance of the disturbances and communication uncertainties with respect to the residual evaluation functions. It is also worth mentioning that the proposed method introduces additional design variables to the optimisation problem (e.g. matrix variables Y_i), and no products between Lyapunov matrices (P_i, Q_i or X_i) and the observer matrices N_i. It allows the use of different Lyapunov matrices for each Theorem, and solving Algorithm 1 with the common design variable Y_i which, unlike Lyapunov matrices, is only required to be nonsingular. This fact, along with the addition of variables σ_{1i}, σ_{2i}, σ_{3i} and matrix K_i, allows more degree of freedom and reduces the conservatism of the overall solution.*

Algorithm 2 Observer-Detector module parameter computation at agent i (offline)

1. Construct the local model (3.47)
2. Define $\Omega_{\underline{\mathcal{F}}_i}$ and choose the multi-objective weights $\lambda_{i1}, \lambda_{i2}, \lambda_{i3}, \lambda_{i4}$ and λ_{i5},
3. Set σ_{1i}, σ_{2i}, σ_{3i}, W_i, K_i and ε_i,
4. Minimise s_i by simultaneously solving Theorems 3.3-3.5 and (3.85) in Remark 3.3.5,
5. Compute the observer matrix gains S_i, H_i, N_i, G_{1i}, G_{2i} and L_i from (3.62) and T_i from (3.50a),
6. Compute the thresholds (3.87).

Algorithm 3 given in the following, is ran on-line and summarises the detection and isolation logic where an agent i is said to be faulty if $f_{a_i}(t) \neq 0$ and/or $f_{s_i}(t) \neq 0$.

Algorithm 3 Decision logic for agent i (online)

1. Apply the evaluation functions (3.86),
2. If $\exists j \in \mathcal{N}_i$ such that $J^e_{i,j}(t) > J^e_{ith}$, and $J^e_{i,i}(t) \leqslant J^e_{ith}$ then request π_j. If $\pi_j \neq 0$ then node j is faulty,
   else the link (i, j) incident to agent i is faulty,
3. If $J^e_{i,p}(t) > J^e_{ith}$, $\forall p \in \mathcal{N}_i \cup i$, then agent i is faulty. Request π_j, $j \in \mathcal{N}_i$, if $\pi_j \neq 0$ then agent j is also
   faulty, else the link $\{i, j\}$ incident to node i is faulty,
4. If $J^e_{i,p}(t) < J^e_{ith}$, $\forall p \in \mathcal{N}_i \cup i$, then no fault/attack
   has occurred.

3.3.3 Simulation Example

To show the effectiveness of the proposed algorithm, let us consider a heterogeneous MAS composed of one-link flexible joint manipulator robots. In the following, there are three followers labelled 1 to $N = 3$ and one virtual leader labelled 0. They are connected according to the directed graph topology represented in Fig. 3.3. The associated adjacency matrix is given as

$$\mathcal{A} = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

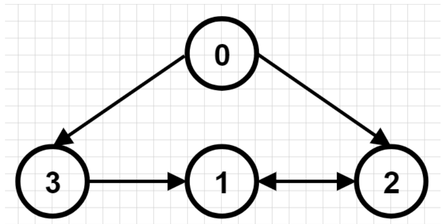


Figure 3.3: Communication topology.

Their dynamics is expressed as [Raghavan & Hedrick 1994]

$$\begin{cases} \dot{\theta}_{m_i} &= \omega_{m_i}, \\ \dot{\omega}_{m_i} &= \dfrac{k_i}{J_{m_i}}(\theta_{l_i} - \theta_{M_i}) - \dfrac{B_i}{J_{m_i}}\omega_{m_i} + \dfrac{K_{\tau_i}}{J_{m_i}}u_i, \\ \dot{\theta}_{l_i} &= \omega_{l_i}, \\ \dot{\omega}_{l_i} &= -\dfrac{k_i}{J_{l_i}}(\theta_{l_i} - \theta_{M_i}) - \dfrac{m_i g h_i}{J_{l_i}}\sin(\theta_{l_i}), \end{cases}$$

where θ_{m_i} is the rotation angle of the motor, θ_{l_i} is the rotation angle of the link, ω_{m_i} and ω_{l_i} are their angular velocities. The following table summarises the parameters.

Parameter	Unit
Link inertia J_{l_i}	kgm^2
Motor inertia J_{m_i}	kgm^2
Viscous friction coefficient B_i	NmV^{-1}
Amplifier gain K_{τ_i}	NmV^{-1}
Torsional spring constant k_i	$Nm \cdot rad^{-1}$
Link length h_i	m
Mass m_i	kg
Gravitational acceleration g	ms^{-1}

Table 3.2: Parameters and units.

By setting, for all $i = 1, 2, 3$, $\xi_i^T = \begin{pmatrix} \theta_{m_i} & \omega_{m_i} & \theta_{l_i} & \omega_{l_i} \end{pmatrix} = \begin{pmatrix} x_{i1} & x_{i2} & x_{i3} & x_{i4} \end{pmatrix}$ and $x_0^T = \begin{pmatrix} x_{01} & x_{02} & x_{03} & x_{04} \end{pmatrix}$ where x_0 is the virtual leader state, the state space representation can be given as

$$A_i = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_i}{J_{m_i}} & -\dfrac{B_i}{J_{m_i}} & \dfrac{k_i}{J_{m_i}} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k_i}{J_{l_i}} & 0 & -\dfrac{k_i}{J_{l_i}} & 0 \end{pmatrix}, \ B_{u_i} = \begin{pmatrix} 0 \\ \dfrac{K_{\tau_i}}{J_{m_i}} \\ 0 \\ 0 \end{pmatrix},$$

$$B_{d_i} = \begin{pmatrix} 0 \\ 0.1 \\ 0 \\ 0.5 \end{pmatrix}, \ \varphi_i(x_i(t)) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\dfrac{m_i g b_i}{J_{l_i}} \sin(\theta_{l_i}) \end{pmatrix},$$

$$B_{f_i} = B_{u_i}, D_{f_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ D_{f_2} = D_{f_3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$D_{d_1} = \begin{pmatrix} 0.05 \\ 0.1 \end{pmatrix}, \ D_{d_2} = \begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix}, \ D_{d_3} = \begin{pmatrix} 0.5 \\ 0.7 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, C_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, C_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$D_{z_{13}} = \mathbf{1}$, $D_{u_{13}} = 1$, $D_{z_{31}} = \mathbf{1}$, $D_{u_{31}} = 1$, $D_{z_{12}} = I$, $D_{u_{12}} = 1$, $D_{z_{21}} = I$, $D_{u_{21}} = 1$.

In the following simulations, the parameter uncertainties are considered as $\Delta a_{ij}(t) = 0.1\sin(a_{ij}t)$ and the perturbations $d_i(t)$ as Gaussian white noise with values in $[-0.2, 0.2]$. For the followers, the parameters are chosen as $m_1 = m_2 = m_3 = 0.21kg$, $k_1 = 0.18Nm \cdot rad^{-1}$, $k_2 = 0.1Nm \cdot rad^{-1}$, $k_3 = 0.22Nm \cdot rad^{-1}$, $B_1 = 4.6 \times 10^{-2}NmV^{-1}$, $B_2 = 3.6 \times 10^{-2}NmV^{-1}$, $B_3 = 5.6 \times 10^{-2}NmV^{-1}$, $J_{m_1} = J_{m_2} = J_{m_3} = 3.7 \times 10^{-3}kgm^2$, $J_{l_1} = J_{l_2} = J_{l_3} = 9.3 \times 10^{-3}kgm^2$, $K_{\tau_1} = 0.08NmV^{-1}$, $K_{\tau_2} = 0.085NmV^{-1}$, $K_{\tau_3} = 0.09NmV^{-1}$, $g = 9,8m/s^2$, $h = 0.3m$. The leader parameters are given as $m_0 = 0.21kg$, $k_0 = 0.18Nm \cdot rad^{-1}$, $B_0 = 4.6 \times 10^{-2}NmV^{-1}$, $J_{m_0} = 3.7 \times 10^{-3}kgm^2$, $J_{l_0} = 9.3 \times 10^{-3}kgm^2$, $K_{\tau_0} = 0.08NmV^{-1}$.

It is thus easy to verify that $\theta_{M_1} = \theta_{M_2} = \theta_{M_3} = 3.3$. The initial conditions are given as $x_0(0) = (0, 0, 0, 0)$, $x_1(0) = (0.1, 0, 0.2, 0)$, $x_2(0) = (0.5, 0, 0.1, 0)$, $x_3(0) = (0.3, 0, 0.4, 0)$. In this example, a tweaked version of the leader-follower control algorithm proposed in [Ding & Zheng 2016] is used based on the estimated state:

$$u_i = -M_i[\textstyle\sum_{j=1}^3 a_{ij}(\hat{x}_i - \hat{x}_i^j) + g_{0i}(\hat{x}_i - \xi_0)],$$

where

$$\begin{aligned} \hat{x}_{v_i}^T &= \begin{pmatrix} \hat{x}_i^i & \hat{x}_i^{i_1} & \dots & \hat{x}_i^{i_{N_i}} \end{pmatrix} \\ e_{v_i}^T &= \begin{pmatrix} e_i^i & e_i^{i_1} & \dots & e_i^{i_{N_i}} \end{pmatrix} \\ &= \begin{pmatrix} e_{i1}^i & \dots & e_{i4}^i & e_{i1}^{i_{N_i}} & \dots & e_{i4}^{i_{N_i}} \end{pmatrix}, \end{aligned}$$

$\hat{x}_i^p \in \mathbb{R}^4$, $e_i^p = \xi_p - \hat{x}_i^p \in \mathbb{R}^4$, $\forall p \in \mathcal{N}_i \cup i$, M_i is a control gain matrix and g_{0i} defines the communication link between agent i and leader 0 ($g_{0i} = 1$ when 0 communicates with i and $g_{0i} = 0$ otherwise). The control gains are given as

$$\begin{cases} M_1 &= \begin{bmatrix} 1.6207 & 0.2210 & -0.5444 & 3.2570 \end{bmatrix}, \\ M_2 &= \begin{bmatrix} 1.6924 & 0.2308 & -0.5685 & 3.4011 \end{bmatrix}, \\ M_3 &= \begin{bmatrix} 1.7642 & 0.2405 & -0.5925 & 3.5452 \end{bmatrix}. \end{cases}$$

The multi-objective weights are chosen as $\lambda_{i1} = \lambda_{i2} = \lambda_{i3} = \lambda_{i4} = \lambda_{i5} = 1$, $\forall i$. The vector $\underline{\mathcal{F}}_i$ is assumed to belong to the finite-frequency domain $[0, 0.1)$. It is worth noting that inequalities (3.60), (3.70), (3.79) and (3.85) can be solved using an appropriate solver (YALMIP, etc. [Lofberg 2004]).

$\forall i \in \{1, 2, 3\}$, Algorithm 2 is applied for $\sigma_{1i} = 1$, $\sigma_{2i} = 0.2$, $\sigma_{3i} = 0.1$, $K_i = -2B_{u_i}$, $\varepsilon_i = 0.04$ and $W_i = I$, yielding $\eta_1 = 0.2, \beta_1 = 0.2, \vartheta_1 = 0.01, \gamma_1 = 0.1, \varrho_1 = 0.81$, $\eta_2 = 0.15, \beta_2 = 0.15, \vartheta_2 = 0.02, \gamma_2 = 0.1, \varrho_2 = 0.85$, $\eta_3 = 0.04, \beta_3 = 0.4, \vartheta_3 = 0.01, \gamma_3 = 0.7, \varrho_3 = 0.77$.

Remark 3.3.8 *It should be highlighted that the computation of the matrix gains is done offline and once. Based on Theorems 3.3-3.5, for each agent, the observer matrix gains are computed according to Algorithm 1. Therefore, a set of LMIs has to be solved offline and once. One can note that the dimension and number of LMIs linearly increase as the state and number of agents increase. Here, $4N$ LMIs (N is the number of agents) should be solved. For an agent i, their dimensions are: $(3n_x^i + n_{f_s}^i + \underline{n}_{f_z}^i) \times (3n_x^i + n_{f_s}^i + \underline{n}_{f_z}^i)$ for Theorem 3.3, $(3n_x^i + n_{f_a}^i + N_i n_{f_u}) \times (3n_x^i + n_{f_a}^i + N_i n_{f_u})$ for Theorem 3.4, $(3n_x^i + n_d^i + \underline{n}_z^i + \underline{n}_u^i) \times (3n_x^i + n_d^i + \underline{n}_z^i + \underline{n}_u^i)$ for Theorem 3.5 and $n_x^i \times n_x^i$ for Remark 3.3.5. These dimensions are given in Table 3.3 for the illustrative example. Additionally, for each agent, the size of the FDI modules (i.e. Eq. (3.48)) is only dependent on the number of neighbouring agents regardless of the agents' control inputs, which makes the proposed scheme highly scalable.*

Agent	LMIST1	LMIST2	LMIST3	LMISR5
1	40×40	39×39	46×46	12×12
2	27×27	26×26	30×30	8×8
3	13×13	13×13	14×14	4×4

Table 3.3: LMI dimensions for each agent, where: LMIST1: LMI Size in Theorem 3.3, LMIST2: LMI Size in Theorem 3.4, LMIST3: LMI Size in Theorem 3.5, LMIST1: LMI Size in Remark 3.3.5.

Remark 3.3.9 *It is interesting to note that for implementation of the method proposed in this Section, each agent sends its corrupted output and its corrupted control input (dimension $n_u + n_y$). This can increase the communication cost in contrast with [Davoodi et al. 2016] for instance, where the FDI modules only require estimated outputs to be broadcasted (dimension n_y). However, as opposed to [Davoodi et al. 2016], the proposed method does not require the agents to be equipped with relative information* *sensors. Indeed, requiring that the agents are equipped with both relative information sensors and wireless communication modules, can limit the cost effectiveness of the method proposed therein.*

Let us consider hereafter two scenarios. In the first one, two faults occur in the network: a sensor fault $f_{s_1}(t)$ at agent 1 and an actuator fault $f_{a_3}(t)$ at agent 3, as represented in Fig. 3.4. Figs. 3.5-3.7 show the generated residual evaluation functions by agents 1, 2 and 3 respectively. The worst case analysis of the evaluation functions corresponding to the non faulty operation of the network under disturbances and uncertainties leads to the following thresholds $J^e_{1th} = 0.048$, $J^e_{2th} = 0.03$ and $J^e_{3th} = 0.027$ under the evaluation window $T_w = 10s$. It is usually not easy to accurately compute the value of the supremum of the RMS function in (3.87) to simultaneously prevent false alarms and avoid missed detections.

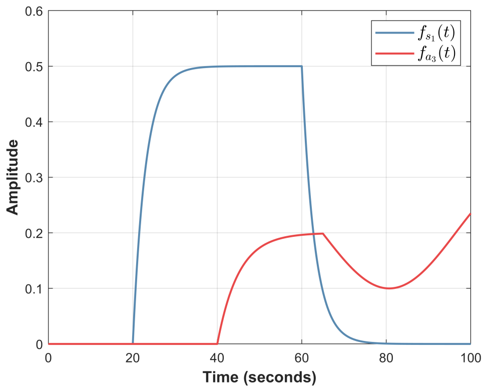


Figure 3.4: Faults signal in scenario 1.

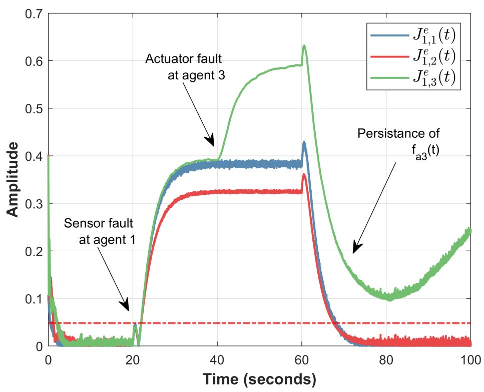


Figure 3.5: Residual evaluation functions at agent 1 in scenario 1. The dashed red lines represent the threshold.

Remark 3.3.10 *As such, a series of Monte-Carlo simulations have been conducted where the supremum of the RMS function in (3.87) is calculated under the healthy operation of the MAS, with different noises, disturbances and uncertainties. The corresponding maximum value has been taken as an appropriate threshold. The sampling*

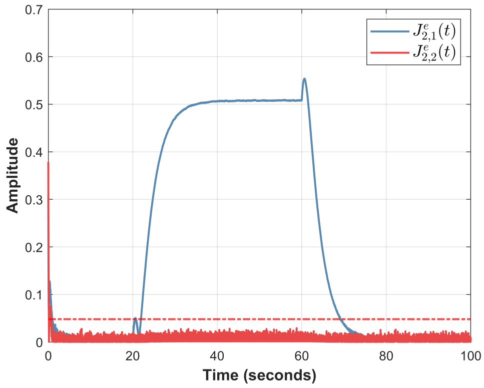


Figure 3.6: Residual evaluation functions at agent 2 in scenario 1.

period is set as $T_s = 10^{-1}$ *s. One could see from Figs. 3.5-3.7 that the faults could be clearly distinguished. Additionally, according to Algorithm 3, one can see from Fig. 3.5 that all generated functions* $J^e_{1,1}(t)$, $J^e_{1,2}(t)$ *and* $J^e_{1,3}(t)$ *increase at around* $t = 20s$ *and exceed the defined threshold due to the sensor fault* $f_{s_1}(t)$ *occurring at agent* 1. *This confirms that a fault has occurred at agent* 1. *Fig. 3.6 further confirms this, since only* $J^e_{2,1}(t)$ *increases due to this fault. At* $t = 40s$, *the actuator fault* $f_{a_3}(t)$ *occurs at agent* 3, *where one can see in Fig. 3.5 that agent* 1 *detects it (its residual evaluation function for agent* 3, *i.e.* $J^e_{1,3}(t)$, *is greater than* J^e_{1th} *even though both* $J^e_{1,1}(t)$ *and* $J^e_{1,2}(t)$ *are lower that* J^e_{1th}*). Hence, according to Algorithm 3, agent* 1 *can distinguish that the fault* $f_{s_1}(t)$ *has disappeared and that agent* 3 *is now faulty. This is confirmed for agent* 3 *in Fig. 3.7.*

It is worth mentioning that the sensor fault matrices D_{f_2} *and* D_{f_3} *are not full column rank. Hence, the methods proposed in [Davoodi et al. 2016, Li et al. 2021] for instance, cannot be applied. Moreover, the effectiveness of the proposed method has been shown for heterogeneous MASs under directed topologies. Besides, compared with the decentralised observer proposed in [Chen & Lin 2014] for example, in which faults occurring at agent* i *can only be detected by the agent itself, our distributed observer can detect both the agent's faults and its neighbours' faults. At last, it can be noticed that the matching condition, i.e.* $rank(C_iB_{f_i}) = n_{f_a}$, *required in many existing works (e.g. [Zhang et al. 2015]), is not needed in our methodology. Indeed, this condition is not satisfied for agents* 2 *and* 3.

In the second scenario, two types of faults are considered: a data injection attack incident to agent 1 targeting the link going from agent 3 to 1, i.e. $f^z_{13}(t) = f^u_{13}(t)$ occurring at $15s \leqslant t \leqslant 40s$, and a replay attack incident to agent 2 at the link going from agent 1 to 2 at $t = 70s$, i.e. $f^z_{21}(t)$ and $f^u_{21}(t)$ with a delay of $\mathcal{T}_{12} = 70s$. $f^z_{13}(t)$, $f^u_{13}(t)$, $f^z_{21}(t)$ and $f^u_{21}(t)$ are represented in Fig. 3.8. Figs. 3.9-3.11 show the generated evaluation functions by agents 1, 2 and 3 respectively in the second scenario. The worst case analysis of the evaluation functions corresponding to the attack-less operation of the network under disturbances and uncertainties leads to the following thresholds

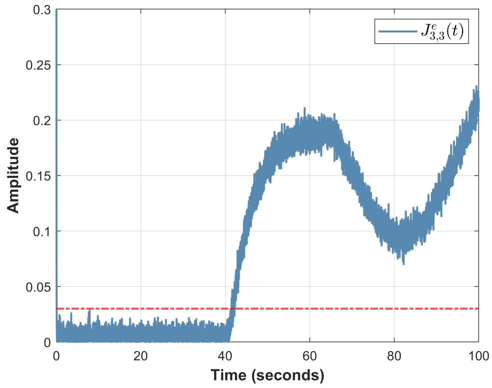


Figure 3.7: Residual evaluation functions at agent 3 in scenario 1.

$J^e_{1th} = 0.016, J^e_{2th} = 0.017, J^e_{3th} = 0.02$. It is clear from the evaluation functions that

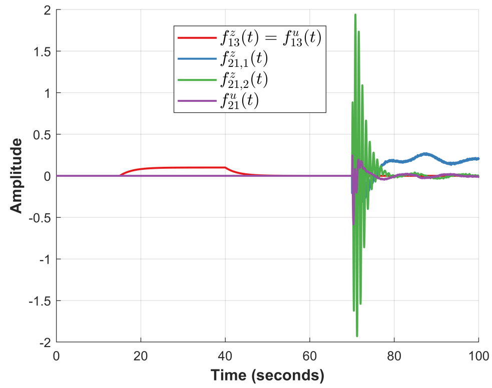


Figure 3.8: Simulated attack signals in scenario 2, where $f^z_{21}(t) = [f^z_{21,1}(t), f^z_{21,2}(t)]^T$.

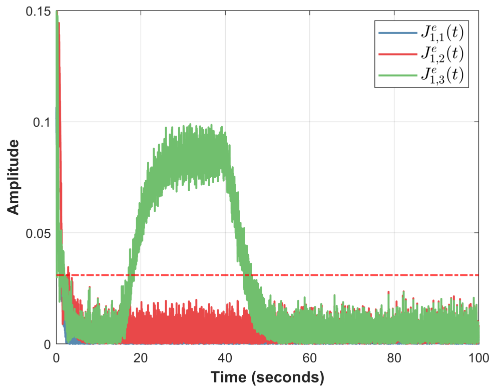


Figure 3.9: Residual evaluation functions at agent 1 in scenario 2.

the attacks can be distinguished when surpassing the computed thresholds. Indeed, from Fig. 3.9, one can see that the data injection attack in the link from 3 to 1 has been detected according to Algorithm 3. It is confirmed that this fault is an edge fault upon requesting agent 3's detection flag, as $J^e_{3,3}$ stays below the defined threshold throughout the duration of the attack. From Fig. 3.10, the replay attack in the link from agent 1 to 2 has been detected by $J^e_{2,1}(t)$ at $t = 70s$ which is confirmed by the fact that $J^e_{1,1}$ does not react to the attack.

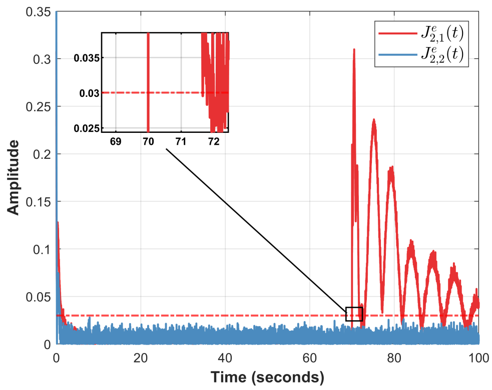


Figure 3.10: Residual evaluation functions at agent 2 in scenario 2.

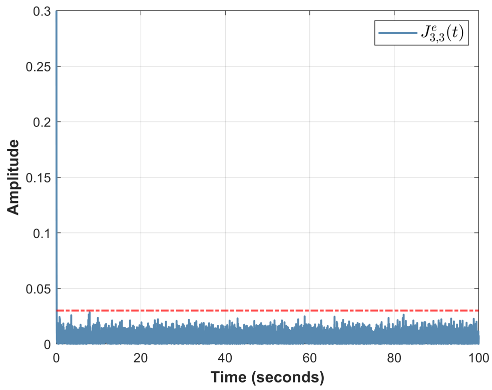


Figure 3.11: Residual evaluation functions at agent 3 in scenario 2.

The control efforts corresponding to the faultless case and scenario 1 and 2 are depicted in Fig. 3.12. Figs. 3.13-3.15 shows the estimation errors generated by the FDI modules for agents 1, 2 and 3 respectively. It can clearly be seen that the estimation errors converge to zero in the absence of any fault or attack.

From these simulations, it can be seen that the proposed FDI scheme is able to detect and isolate attacks, actuator faults and sensor faults in the presence of disturbances, noise and communication uncertainties.

One can make the remark on the potential use of the proposed scheme for reconfiguration purposes. Hence, in order to avoid the leader follower tracking error from suffering from the attacks by preventing further catastrophic effects after detection,

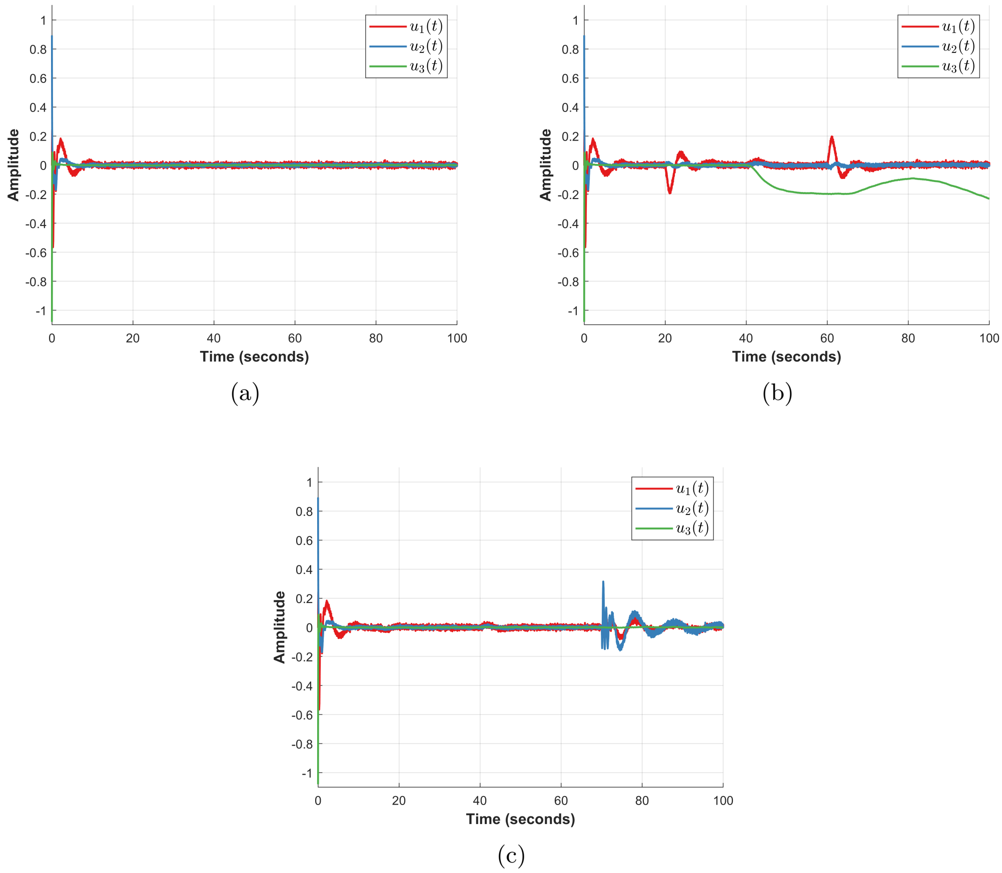


Figure 3.12: Control efforts in: (a) the faultless case, (b) scenario 1 and (c) scenario 2.

we propose removing the faulty link from the control algorithm. The new proposed algorithm can be rewritten as

$$u_i = -M_i[\sum_{j=1}^{3} a_{ij}E_{ij}(t)(\hat{x}_i - \hat{x}_i^j) + g_{0i}(\hat{x}_i - x_0)]$$

where

$$E_{ij}(t) = \begin{cases} 1 & J^e_{j,i}(t) < J^e_{ith}, \\ 0 & \text{otherwise} \end{cases}$$

For this, it is necessary to make the further assumption that after a link interruption, the topology graph stays connected, i.e., the leader is still the root of the graph. The difference between the tracking errors in case of the persistence of faulty edges and their removal is shown in Figures 3.16-3.18 and Figures 3.19-3.21 respectively. It can be seen that the leader follower consensus is reached when the edges under attack are removed, whereas the consensus cannot be reached in the occurrence of an attack.

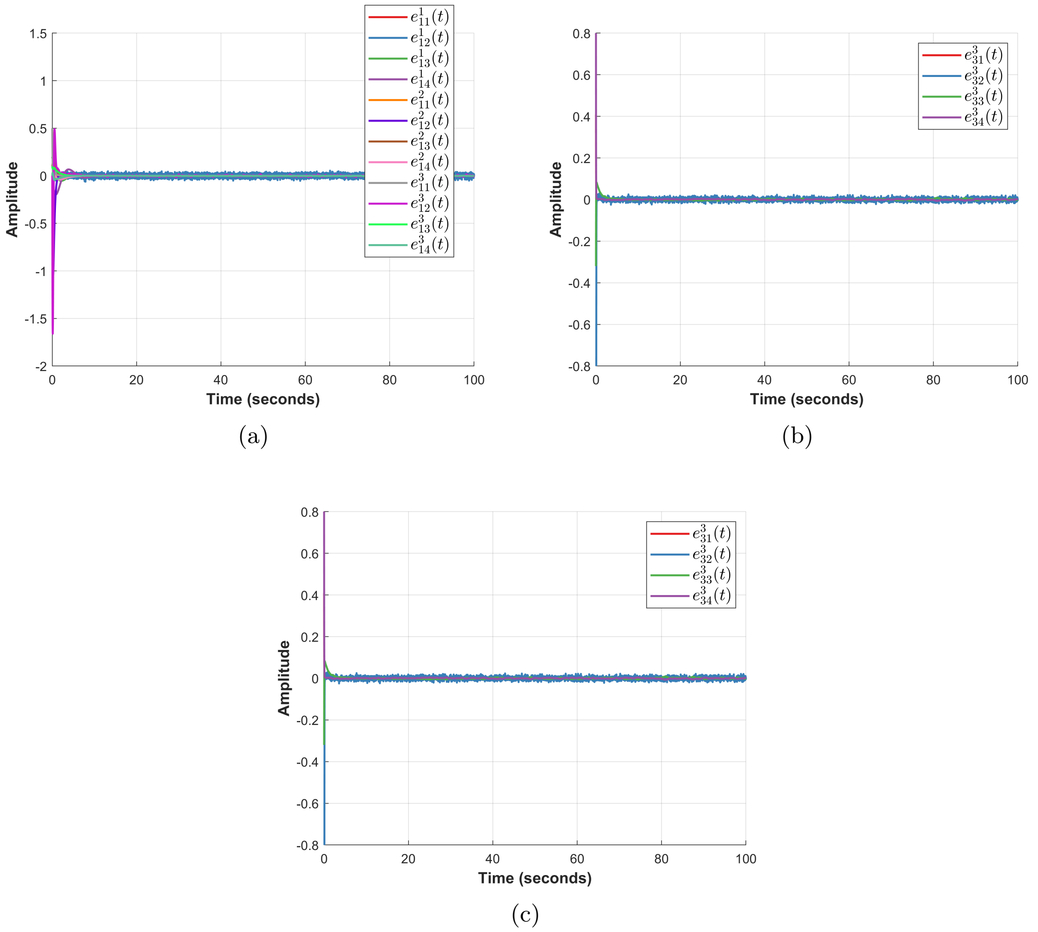
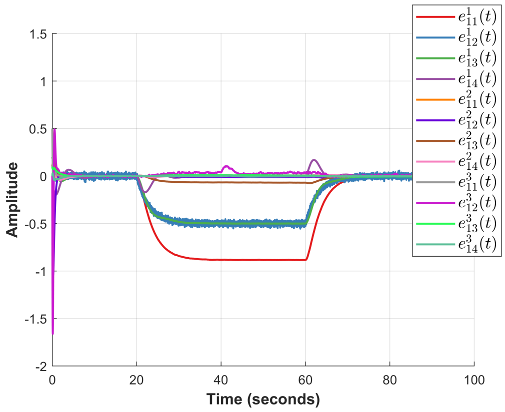


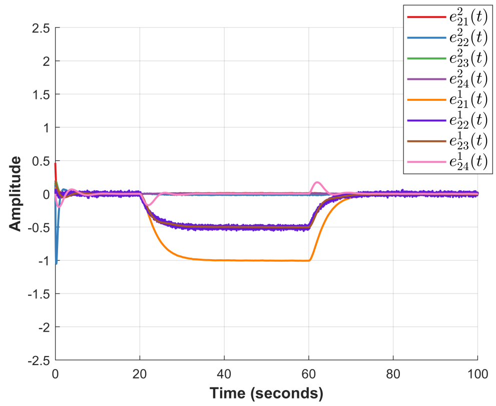
Figure 3.13: Estimation errors in the faultless and attackless case at: (a) agent 1, (b) agent 2 and (c) agent 3.

3.4 Conclusions

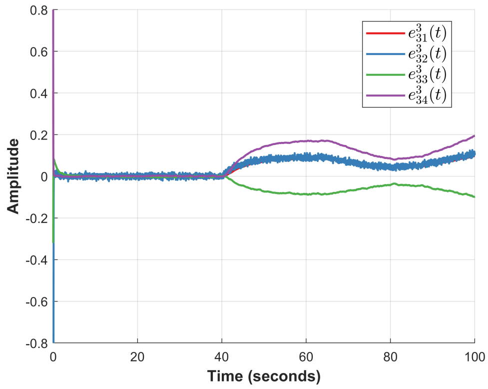
In this Chapter, a new approach for actuator FDI in MASs with linear dynamics using relative information has been proposed, where output observers has been developed in Section 3.2. It was shown that the proposed FDI scheme can detect and isolate faulty agents in every neighbourhood using knowledge on the input-output relations. In Section 3.3, the problem of FDI in Lipschitz nonlinear MASs with disturbances, subject to actuator, sensor and communication faults has been addressed. A multi-objective finite-frequency $\mathcal{H}_-/\mathcal{H}_\infty$ design along with nonlinear UIOs have been proposed. Sufficient conditions have been derived in terms of a set of LMIs. The combination of UIOs, removal of strict rank conditions and finite-frequency method has been shown to provide extra degrees of freedom in the FDI filter design. Additionally, the multi-objective method guarantees that the evaluation functions are robust with respect to all admissible disturbances and uncertainties and sensitive to all types of faults. Illustrative numerical examples have been studied in order to showcase the effectiveness of



(a)



(b)



(c)

Figure 3.14: Estimation errors in scenario 1 at: (a) agent 1, (b) agent 2 and (c) agent 3.

the proposed schemes.

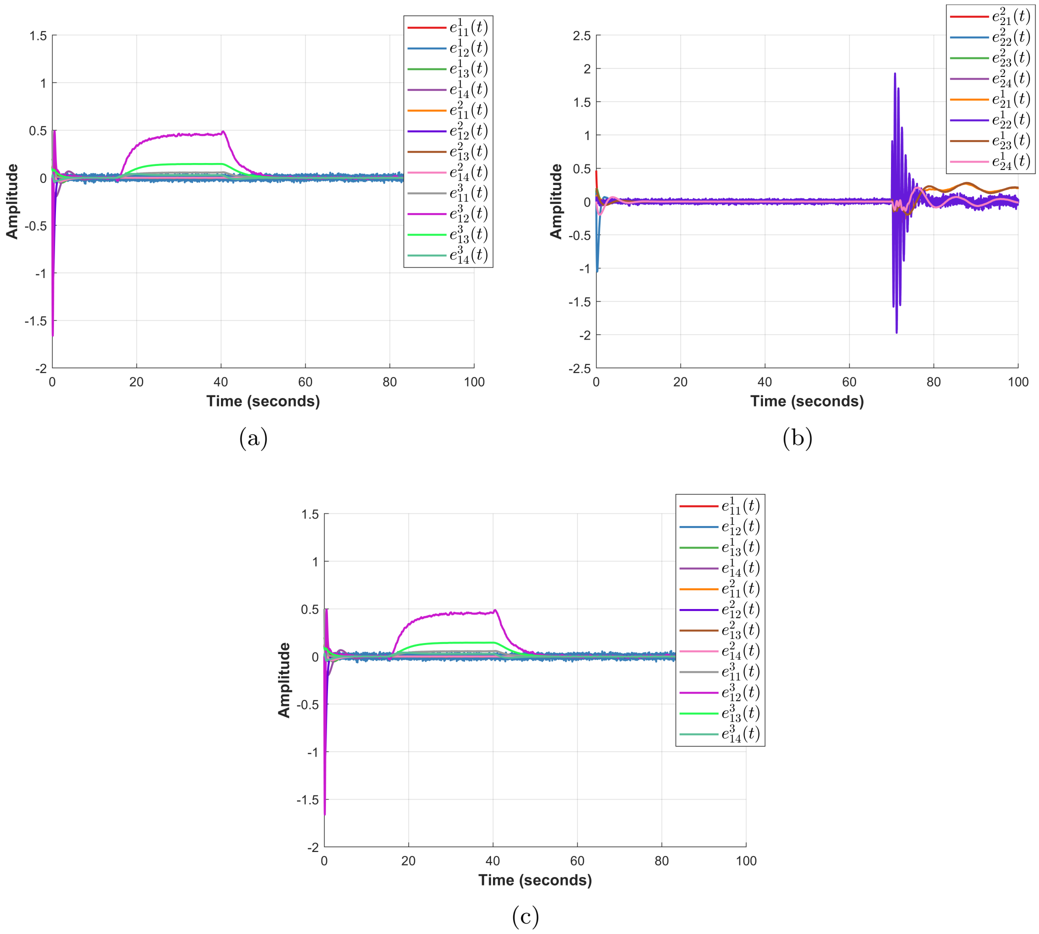


Figure 3.15: Estimation errors in scenario 2 at: (a) agent 1, (b) agent 2 and (c) agent 3.

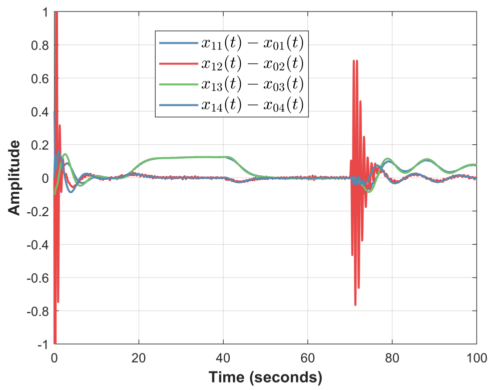


Figure 3.16: Tracking error at agent 1 in the presence of attacks.

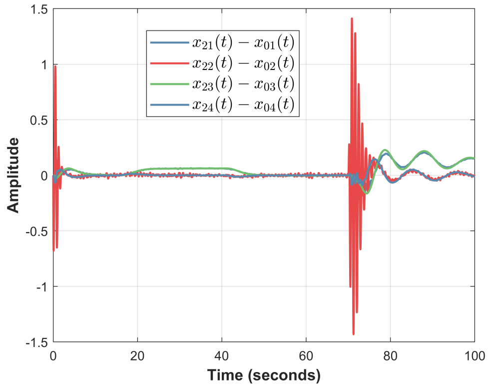


Figure 3.17: Tracking error at agent 2 in the presence of attacks.

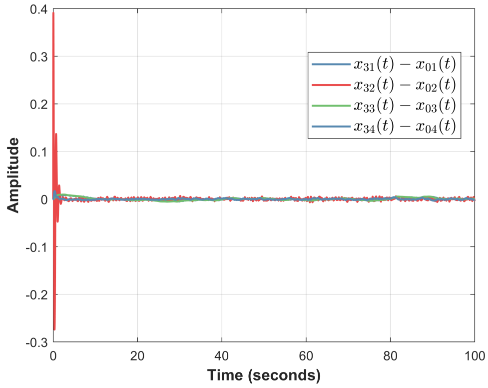


Figure 3.18: Tracking error at agent 3 in the presence of attacks.

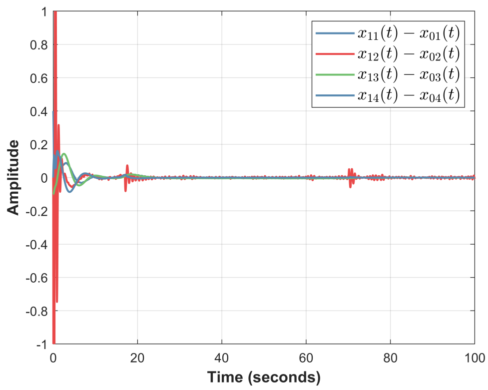


Figure 3.19: Tracking error at agent 1 in the presence of attacks when the faulty edge $\{1, 3\}$ is removed at $t = 17s$.

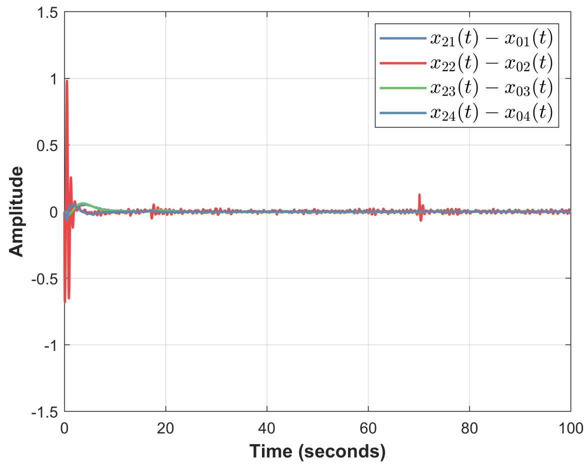


Figure 3.20: Tracking error at agent 2 in the presence of attacks when the faulty edge $\{2, 1\}$ is removed at $t = 70.1s$.

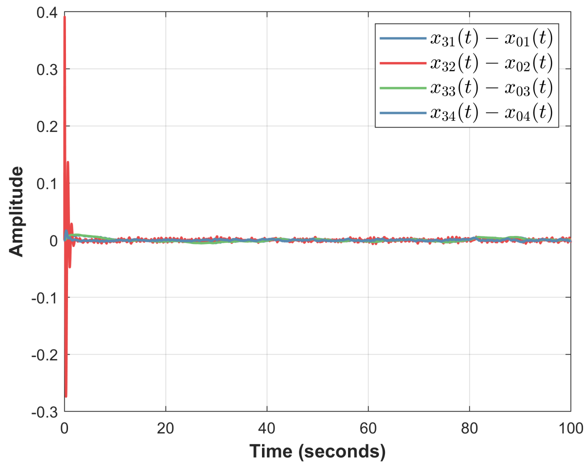


Figure 3.21: Tracking error at agent 3 in the presence of attacks.

CHAPTER 4

General Conclusions and Perspectives

4.1 General Conclusions

The main focus of this thesis was the study of distributed fault and attack detection and isolation in connected MASs. It was shown from the state of the art that most of the existing FDI schemes suffer from various limitations with respect to different challenges such as FDI for MASs with temporal constraints, FDI for MASs with switching topologies, FDI for MASs with heterogeneous agents and directed topologies and FDI for MASs subject to cyber-attacks and physical faults. As such different novel distributed and robust algorithms have been proposed in order to tackle these issues. The main results of this dissertation are summarised as follows

- Chapter 1 presented some basic introductory concepts with regards to MASs, attack and FDI in MASs and their applications were given. Basic algebraic graph theory concepts and some useful mathematical tools were presented for describing the communication topology among agents in a MAS. Additionally, a brief literature review and state of the art of recent works on attack and FDI in the context of connected MASs was conducted, various limitations of these works were identified, and the motivation of the thesis was thoroughly stated.
- Chapter 2 was concerned with global robust attack and FDI in connected MASs, using the fixed-time property in order to tackle the problem of transient behaviour, facilitate the residual generation process and allow for fast convergence in switching topology settings. First in Section 2.3 of the Chapter, a distributed methodology for the detection of actuator faults in a class of linear MASs with unknown disturbances was proposed, whose main features are highlighted in the following. The formulation of the distributed actuator FDI problem for a class of linear MASs with disturbances is performed through the use of a cascade of fixed-time SMOs, where each agent having access to their state, and neighbouring

information exchanges, can give an exact estimate of the state of the overall MAS. An LMI-based approach is applied to design distributed global residual signals at each agent, based on mixed $\mathcal{H}_{-}/\mathcal{H}_{\infty}$ norms. The above combined approaches allow treating the actuator fault detection problem while keeping a distributed design approach, as information obtained by each interacting agent only comes from its neighbours. Then, in Section 2.4 of the Chapter, the previous point is extended to nonholonomic systems, where a new distributed robust fault detection scheme for MASs composed of agents with nonlinear nonholonomic dynamics was proposed. In this proposed scheme, the use of predefined-time stability techniques to derive adequate distributed SMOs was investigated, which enable to reconstruct the global system state in a predefined-time and generate proper residual signals. The proposed scheme ensures global fault detection, where each agent is capable of detecting its own faults and those occurring elsewhere in the system using only local information (contrary to most of the existing works). Finally, in Section 2.5 of the Chapter, the results obtained in the first point are extended to the case of MASs with higher order integrator dynamics, where only the first state variable is measurable. Here, a new approach to identify faults and deception attacks in a cooperating MASs with a switching topology is introduced. The proposed protocol makes an agent act as a central node monitoring the whole system activities in a distributed fashion whereby a bank of distributed predefined-time SMOs for global state estimation are designed, which are then used to generate residual signals capable of identifying cyber-attacks despite the switching topology. In all of the proposed proposed algorithms in Chapter 2, the input command is assumed not to be transmitted.

- Chapter 3 was concerned with the problem of attack and FDI in connected heterogeneous MASs where the undirected topology restriction was removed. In Section 3.2 of the Chapter, the problem of distributed fault detection for a team of heterogeneous MASs with linear dynamics was solved, where a new output observer scheme was proposed which was effective for both directed and undirected topologies. The main advantage of this approach is that the design, being dependant only on the input-output relations, renders the computational cost, information exchange and scalability very effective compared to other FDI approaches that employ the whole state estimation of the agents and their neighbours as a basis for their design. In Section 3.3 of the Chapter, a more general model was studied, where actuator, sensor and communication faults/attacks are considered in the robust detection and isolation process for nonlinear heterogeneous MASs with disturbances and communication parameter uncertainties, where the topology was not required to be undirected. This was done using a distributed finite-frequency mixed $\mathcal{H}_{-}/\mathcal{H}_{\infty}$ nonlinear UIO-based approach.

- Simulation examples were given for each of the proposed algorithms to show their effectiveness and robustness.

4.2 Future Perspectives

It is evident that despite the results that have been obtained in this dissertation, there exists various aspects that could be further studied in future works. Some of these topics constituting future research are highlighted in the following

- In Chapter 2, the communication topologies are assumed to be undirected. This assumption that the methods proposed therein rely on, could be relaxed and extended to the case of MASs switching directed communication topologies.
- The fixed-time property has been used in developing FDI schemes for homogeneous MASs. This could be extended to heterogeneous MASs. Additionally, one interesting prospect, is the design of fault and/or attack tolerant control protocols based on information provided by the fixed-time observers.
- In Chapter 3, instead of considering Lipschitz nonlinear systems, one could investigate other classes of nonlinear uncertain systems including chained-form dynamics. Furthermore, the Chapter studies schemes for the case where the topology is fixed. The extension of these results to the case of detection and isolation under switching topologies could be envisaged.
- The potential of the proposed schemes in reconfiguration was shown in Chapter 3, hence, based on the proposed FDI schemes in this thesis, it would be possible to design some attack and fault accommodation and reconfiguration strategies in connected MASs.
- The algorithms proposed in this thesis consider the information exchanges to be received with synchronously and with no delays. Hence, an interesting scope is to study such schemes for the case where information is received with a certain delay. Moreover, discretisation of the proposed algorithms and experimental validation could be considered in future work.

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Bibliography

[Aldana-López *et al.* 2019] Rodrigo Aldana-López, David Gómez-Gutiérrez, Esteban Jiménez-Rodríguez, Juan Diego Sánchez-Torres and Michael Defoort. *Enhancing the settling time estimation of a class of fixed-time stable systems.* International Journal of Robust and Nonlinear Control, vol. 29, no. 12, pages 4135–4148, 2019. (Cited on pages 40 and 51.)

[Aldana-López *et al.* 2021] Rodrigo Aldana-López, Richard Seeber, David Gómez-Gutiérrez, Marco Tulio Angulo and Michael Defoort. *Designing predefined-time differentiators with bounded time-varying gains.* arXiv preprint arXiv:2106.06429, 2021. (Cited on page 56.)

[Anggraeni *et al.* 2019] Pipit Anggraeni, Michael Defoort, Mohamed Djemai and Zongyu Zuo. *Control strategy for fixed-time leader–follower consensus for multi-agent systems with chained-form dynamics.* Nonlinear Dynamics, vol. 96, no. 4, pages 2693–2705, 2019. (Cited on pages 9, 63 and 64.)

[Arrichiello *et al.* 2015] Filippo Arrichiello, Alessandro Marino and Francesco Pierri. *Observer-based decentralized fault detection and isolation strategy for networked multirobot systems.* IEEE Transactions on Control Systems Technology, vol. 23, no. 4, pages 1465–1476, 2015. (Cited on pages 9, 29 and 31.)

[Barboni *et al.* 2020] Angelo Barboni, Hamed Rezaee, Francesca Boem and Thomas Parisini. *Detection of Covert Cyber-Attacks in Interconnected Systems: A Distributed Model-Based Approach.* IEEE Transactions on Automatic Control, 2020. (Cited on page 46.)

[Barua & Khorasani 2007] Amitabh Barua and Khashayar Khorasani. *Intelligent model-based hierarchical fault diagnosis for satellite formations.* In 2007 IEEE International Conference on Systems, Man and Cybernetics, pages 3191–3196. IEEE, 2007. (Cited on page 10.)

[Basile & Marro 1992] Giuseppe Basile and Giovanni Marro. Controlled and conditioned invariants in linear system theory. Prentice Hall Englewood Cliffs, NJ, 1992. (Cited on page 51.)

[Bhat & Bernstein 2000] Sanjay P Bhat and Dennis S Bernstein. *Finite-time stability of continuous autonomous systems.* SIAM Journal on Control and Optimization, vol. 38, no. 3, pages 751–766, 2000. (Cited on page 38.)

[Bidram *et al.* 2013] Ali Bidram, Ali Davoudi, Frank L Lewis and Zhihua Qu. *Secondary control of microgrids based on distributed cooperative control of multi-*

agent systems. IET Generation, Transmission & Distribution, vol. 7, no. 8, pages 822–831, 2013. (Cited on page 9.)

[Biggs *et al.* 1993] Norman Biggs, Norman Linstead Biggs and Biggs Norman. Algebraic graph theory. Number 67. Cambridge university press, 1993. (Cited on page 13.)

[Biron *et al.* 2018] Zoleikha Abdollahi Biron, Satadru Dey and Pierluigi Pisu. *Real-time detection and estimation of denial of service attack in connected vehicle systems.* IEEE Transactions on Intelligent Transportation Systems, vol. 19, no. 12, pages 3893–3902, 2018. (Cited on page 35.)

[Blanke *et al.* 2006] Mogens Blanke, Michel Kinnaert, Jan Lunze, Marcel Staroswiecki and Jochen Schröder. Diagnosis and fault-tolerant control, volume 2. Springer, 2006. (Cited on page 35.)

[Boem *et al.* 2016] Francesca Boem, Lorenzo Sabattini and Cristian Secchi. *Decentralized fault diagnosis for heterogeneous multi-agent systems.* In 2016 3rd Conference on Control and Fault-Tolerant Systems (SysTol), pages 771–776. IEEE, 2016. (Cited on page 29.)

[Boem *et al.* 2017] Francesca Boem, Alexander J Gallo, Giancarlo Ferrari-Trecate and Thomas Parisini. *A distributed attack detection method for multi-agent systems governed by consensus-based control.* In 2017 IEEE 56th Annual Conference on Decision and Control (CDC), pages 5961–5966. IEEE, 2017. (Cited on pages 33, 34, 98 and 113.)

[Bondy *et al.* 1976] John Adrian Bondy, Uppaluri Siva Ramachandra Murty*et al.* Graph theory with applications, volume 290. Macmillan London, 1976. (Cited on page 13.)

[Boyd *et al.* 1994] Stephen Boyd, Laurent El Ghaoui, Eric Feron and Venkataramanan Balakrishnan. Linear matrix inequalities in system and control theory. SIAM, 1994. (Cited on page 116.)

[Cardenas *et al.* 2008] Alvaro A Cardenas, Saurabh Amin and Shankar Sastry. *Secure control: Towards survivable cyber-physical systems.* In 2008 The 28th International Conference on Distributed Computing Systems Workshops, pages 495–500. IEEE, 2008. (Cited on page 33.)

[Chadli *et al.* 2016] Mohammed Chadli, Mohammadreza Davoodi and Nader Meskin. *Distributed state estimation, fault detection and isolation filter design for heterogeneous multi-agent linear parameter-varying systems.* IET Control Theory & Applications, vol. 11, no. 2, pages 254–262, 2016. (Cited on pages 46 and 48.)

[Chadli *et al.* 2017] Mohammed Chadli, Mohammadreza Davoodi and Nader Meskin. *Distributed state estimation, fault detection and isolation filter design for heterogeneous multi-agent linear parameter-varying systems.* IET Control Theory

& Applications, vol. 11, no. 2, pages 254–262, 2017. (Cited on pages 30, 98, 99, 101 and 109.)

[Chen & Lin 2014] Gang Chen and Qing Lin. *Observer-based consensus control and fault detection for multi-agent systems*. Control Theory & Applications, vol. 31, no. 5, pages 584–591, 2014. (Cited on page 133.)

[Chen *et al.* 2016] Jianliang Chen, Weidong Zhang, Yong-Yan Cao and Hongjun Chu. *Observer-based consensus control against actuator faults for linear parameter-varying multiagent systems*. IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 47, no. 7, pages 1336–1347, 2016. (Cited on page 36.)

[Conti 2010] Juan Pablo Conti. *The day the samba stopped [power blackouts]*. Engineering & Technology, vol. 5, no. 4, pages 46–47, 2010. (Cited on page 31.)

[Corradini & Cristofaro 2017] Maria Letizia Corradini and Andrea Cristofaro. *Robust detection and reconstruction of state and sensor attacks for cyber-physical systems using sliding modes*. IET Control Theory & Applications, vol. 11, no. 11, pages 1756–1766, 2017. (Cited on page 34.)

[Davoodi *et al.* 2013] Mohammad Reza Davoodi, Khashayar Khorasani, Heidar Ali Talebi and Hamid Reza Momeni. *Distributed fault detection and isolation filter design for a network of heterogeneous multiagent systems*. IEEE Transactions on Control Systems Technology, vol. 22, no. 3, pages 1061–1069, 2013. (Cited on pages 29, 98 and 99.)

[Davoodi *et al.* 2016] Mohammadreza Davoodi, Nader Meskin and Khashayar Khorasani. *Simultaneous fault detection and consensus control design for a network of multi-agent systems*. Automatica, vol. 66, pages 185–194, 2016. (Cited on pages 21, 23, 30, 37, 46, 48, 98, 99, 100, 101, 131 and 133.)

[De Loza *et al.* 2015] Alejandra Ferreira De Loza, Jérôme Cieslak, David Henry, Ali Zolghadri and Leonid M Fridman. *Output tracking of systems subjected to perturbations and a class of actuator faults based on HOSM observation and identification*. Automatica, vol. 59, pages 200–205, 2015. (Cited on page 74.)

[Defoort *et al.* 2016] Michael Defoort, Guillaume Demesure, Zongyu Zuo, Andrey Polyakov and Mohamed Djemai. *Fixed-time stabilisation and consensus of non-holonomic systems*. IET Control Theory & Applications, vol. 10, no. 18, pages 2497–2505, 2016. (Cited on pages 23 and 63.)

[Deng & Wen 2020] Chao Deng and Changyun Wen. *Distributed resilient observer-based fault-tolerant control for heterogeneous multiagent systems under actuator faults and DoS attacks*. IEEE Transactions on Control of Network Systems, vol. 7, no. 3, pages 1308–1318, 2020. (Cited on page 36.)

[Ding & Zheng 2016] Lei Ding and Wei Xing Zheng. *Network-based practical consensus of heterogeneous nonlinear multiagent systems*. IEEE Transactions on Cybernetics, vol. 47, no. 8, pages 1841–1851, 2016. (Cited on page 130.)

[Ding 2008] Steven X Ding. Model-based fault diagnosis techniques: design schemes, algorithms, and tools. Springer Science & Business Media, 2008. (Cited on pages 24, 25, 58, 115 and 127.)

[Drakunov *et al.* 1990] S. V. Drakunov, D. B. Izosimov, A. G. Lukyanov, V. A. Utkin and V. I. Utkin. *The Block Control Principle I.* Automation and Remote Control, vol. 51, pages 601–608, 1990. (Cited on page 67.)

[Fawzi *et al.* 2014] Hamza Fawzi, Paulo Tabuada and Suhas Diggavi. *Secure estimation and control for cyber-physical systems under adversarial attacks.* IEEE Transactions on Automatic control, vol. 59, no. 6, pages 1454–1467, 2014. (Cited on page 34.)

[Fax & Murray 2004] J Alexander Fax and Richard M Murray. *Information flow and cooperative control of vehicle formations.* IEEE Transactions on Automatic Control, vol. 49, no. 9, pages 1465–1476, 2004. (Cited on pages 18 and 50.)

[Ferrari *et al.* 2011] Riccardo MG Ferrari, Thomas Parisini and Marios M Polycarpou. *Distributed fault detection and isolation of large-scale discrete-time nonlinear systems: An adaptive approximation approach.* IEEE Transactions on Automatic Control, vol. 57, no. 2, pages 275–290, 2011. (Cited on pages 29 and 34.)

[Franco *et al.* 2006] Elisa Franco, Reza Olfati-Saber, Thomas Parisini and Marios M Polycarpou. *Distributed fault diagnosis using sensor networks and consensus-based filters.* In Proceedings of the 45th IEEE Conference on Decision and Control, pages 386–391. IEEE, 2006. (Cited on page 29.)

[Gahinet *et al.* 1996] Pascal Gahinet, Arkadi Nemirovski, Alan J Laub and Mahmoud Chilali. *LMI control toolbox.* The Math Works Inc, 1996. (Cited on page 124.)

[Gallehdari *et al.* 2016] Zahra Gallehdari, Nader Meskin and Khashayar Khorasani. *A distributed control reconfiguration and accommodation for consensus achievement of multiagent systems subject to actuator faults.* IEEE Transactions on Control Systems Technology, vol. 24, no. 6, pages 2031–2047, 2016. (Cited on page 35.)

[Gallehdari *et al.* 2017] Z Gallehdari, N Meskin and K Khorasani. *Distributed reconfigurable control strategies for switching topology networked multi-agent systems.* ISA Transactions, vol. 71, pages 51–67, 2017. (Cited on pages 21, 23 and 41.)

[Gallo *et al.* 2018a] Alexander J Gallo, Mustafa S Turan, Francesca Boem, Giancarlo Ferrari-Trecate and Thomas Parisini. *Distributed watermarking for secure control of microgrids under replay attacks.* IFAC-PapersOnLine, vol. 51, no. 23, pages 182–187, 2018. (Cited on pages 32, 34, 98 and 113.)

[Gallo *et al.* 2018b] Alexander J Gallo, Mustafa S Turan, Pulkit Nahata, Francesca Boem, Thomas Parisini and Giancarlo Ferrari-Trecate. *Distributed cyber-attack detection in the secondary control of DC microgrids.* In 2018 European Control Conference (ECC), pages 344–349. IEEE, 2018. (Cited on page 34.)

[Gao *et al.* 2014] Haibo Gao, Xingguo Song, Liang Ding, Kerui Xia, Nan Li and Zongquan Deng. *Adaptive motion control of wheeled mobile robot with unknown slippage*. International Journal of Control, vol. 87, no. 8, pages 1513–1522, 2014. (Cited on page 63.)

[Gao *et al.* 2017] Xianwen Gao, Xiuhua Liu and Jian Han. *Reduced order unknown input observer based distributed fault detection for multi-agent systems*. Journal of the Franklin Institute, vol. 354, no. 3, pages 1464–1483, 2017. (Cited on pages 30 and 48.)

[Ghamry & Zhang 2016] Khaled A Ghamry and Youmin Zhang. *Fault-tolerant cooperative control of multiple UAVs for forest fire detection and tracking mission*. In 2016 3rd Conference on Control and Fault-Tolerant Systems (SysTol), pages 133–138. IEEE, 2016. (Cited on page 9.)

[Guan & Ge 2017] Yanpeng Guan and Xiaohua Ge. *Distributed attack detection and secure estimation of networked cyber-physical systems against false data injection attacks and jamming attacks*. IEEE Transactions on Signal and Information Processing over Networks, vol. 4, no. 1, pages 48–59, 2017. (Cited on page 33.)

[Guo *et al.* 2018] Zhenhong Guo, Chunjing Jiang, Jie Mei and Guangfu Ma. *Fully Distributed Consensus for Second-order Uncertain Multi-agent Systems under a Directed Graph*. In 2018 15th International Conference on Control, Automation, Robotics and Vision (ICARCV), pages 492–496. IEEE, 2018. (Cited on pages 20 and 21.)

[Hajshirmohamadi *et al.* 2019] Shahram Hajshirmohamadi, Farid Sheikholeslam and Nader Meskin. *Distributed simultaneous fault detection and leader-following consensus control for multi-agent systems*. ISA Transactions, vol. 87, pages 129–142, 2019. (Cited on pages 30 and 37.)

[Han *et al.* 2019] Jian Han, Xiuhua Liu, Xianwen Gao and Xinjiang Wei. *Intermediate observer-based robust distributed fault estimation for nonlinear multiagent systems with directed graphs*. IEEE Transactions on Industrial Informatics, vol. 16, no. 12, pages 7426–7436, 2019. (Cited on pages 30 and 98.)

[Hashemi *et al.* 2018] Navid Hashemi, Carlos Murguia and Justin Ruths. *A comparison of stealthy sensor attacks on control systems*. In 2018 Annual American Control Conference (ACC), pages 973–979. IEEE, 2018. (Cited on page 33.)

[Hernandez-Martinez *et al.* 2013] Eduardo Gamaliel Hernandez-Martinez, José Job Flores-Godoy and Guillermo Fernandez-Anaya. *Decentralized discrete-time formation control for multirobot systems*. Discrete Dynamics in Nature and Society, vol. 2013, 2013. (Cited on page 9.)

[Hong *et al.* 2008] Yiguang Hong, Guanrong Chen and Linda Bushnell. *Distributed observers design for leader-following control of multi-agent networks*. Automatica, vol. 44, no. 3, pages 846–850, 2008. (Cited on page 23.)

[Iwasaki *et al.* 2005] Tetsuya Iwasaki, Shinji Hara and Alexander L Fradkov. *Time domain interpretations of frequency domain inequalities on (semi) finite ranges.* Systems & Control Letters, vol. 54, no. 7, pages 681–691, 2005. (Cited on page 99.)

[Jadbabaie *et al.* 2003] Ali Jadbabaie, Jie Lin and A Stephen Morse. *Coordination of groups of mobile autonomous agents using nearest neighbor rules.* IEEE Transactions on Automatic Control, vol. 48, no. 6, pages 988–1001, 2003. (Cited on pages 19, 20 and 78.)

[Jan *et al.* 2015] Mian Ahmad Jan, Priyadarsi Nanda, Xiangjian He and Ren Ping Liu. *A sybil attack detection scheme for a centralized clustering-based hierarchical network.* In 2015 IEEE Trustcom/BigDataSE/ISPA, volume 1, pages 318–325. IEEE, 2015. (Cited on page 98.)

[Jennings *et al.* 1998] Nicholas R Jennings, Katia Sycara and Michael Wooldridge. *A roadmap of agent research and development.* Autonomous Agents and Multi-Agent Systems, vol. 1, no. 1, pages 7–38, 1998. (Cited on page 8.)

[Jiang & Wang 2010] Fangcui Jiang and Long Wang. *Consensus seeking of high-order dynamic multi-agent systems with fixed and switching topologies.* International Journal of Control, vol. 83, no. 2, pages 404–420, 2010. (Cited on page 87.)

[Jiménez-Rodríguez *et al.* 2020a] Esteban Jiménez-Rodríguez, Rodrigo Aldana-López, Juan Diego Sánchez-Torres, David Gómez-Gutiérrez and Alexander G. Loukianov. *Consistent Discretization of a Class of Predefined-Time Stable Systems.* In 21st IFAC World Congress 2020 - 1st Virtual IFAC World Congress (IFAC-V 2020), 2020. (Cited on page 55.)

[Jimenez-Rodriguez *et al.* 2020b] Esteban Jimenez-Rodriguez, Aldo Jonathan Vázquez-Muñoz, Juan Diego Sánchez-Torres, Michael Defoort and Alexander G Loukianov. *A Lyapunov-like Characterization of Predefined-Time Stability.* IEEE Transactions on Automatic Control, 2020. (Cited on pages 39 and 40.)

[Kamel *et al.* 2015] Mohamed A Kamel, Khaled A Ghamry and Youmin Zhang. *Fault tolerant cooperative control of multiple UAVs-UGVs under actuator faults.* In 2015 International Conference on Unmanned Aircraft Systems (ICUAS), pages 644–649. IEEE, 2015. (Cited on page 9.)

[Kantamneni *et al.* 2015] Abhilash Kantamneni, Laura E Brown, Gordon Parker and Wayne W Weaver. *Survey of multi-agent systems for microgrid control.* Engineering Applications of Artificial Intelligence, vol. 45, pages 192–203, 2015. (Cited on page 9.)

[Khalaf *et al.* 2018] Mohsen Khalaf, Amr Youssef and Ehab El-Saadany. *Joint detection and mitigation of false data injection attacks in AGC systems.* IEEE Transactions on Smart Grid, vol. 10, no. 5, pages 4985–4995, 2018. (Cited on page 98.)

[Khalili *et al.* 2019] Mohsen Khalili, Xiaodong Zhang, Yongcan Cao, Marios M Polycarpou and Thomas Parisini. *Distributed Fault-Tolerant Control of Multiagent Systems: An Adaptive Learning Approach.* IEEE Transactions on Neural Networks and Learning Systems, vol. 31, no. 2, pages 420–432, 2019. (Cited on page 35.)

[Khan *et al.* 2020] Aadil Sarwar Khan, Abdul Qayyum Khan, Naeem Iqbal, Muhammad Sarwar, Atif Mahmood and M Asim Shoaib. *Distributed fault detection and isolation in second order networked systems in a cyber-physical environment.* ISA Transactions, 2020. (Cited on pages 98 and 101.)

[Khazraei *et al.* 2017] Amir Khazraei, Hamed Kebriaei and Farzad Rajaei Salmasi. *Replay attack detection in a multi agent system using stability analysis and loss effective watermarking.* In 2017 American Control Conference (ACC), pages 4778–4783. IEEE, 2017. (Cited on page 32.)

[Khoo *et al.* 2010] Suiyang Khoo, Lihua Xie and Zhihong Man. *Leader-follower consensus control of a class of nonholonomic systems.* In 2010 11th International Conference on Control Automation Robotics & Vision, pages 1381–1386. IEEE, 2010. (Cited on page 23.)

[Kim *et al.* 2017] Hunmin Kim, Pinyao Guo, Minghui Zhu and Peng Liu. *Attack-resilient estimation of switched nonlinear cyber-physical systems.* In 2017 American Control Conference (ACC), pages 4328–4333. IEEE, 2017. (Cited on page 35.)

[Kolmanovsky & McClamroch 1995] Ilya Kolmanovsky and N Harris McClamroch. *Developments in nonholonomic control problems.* IEEE Control Systems Magazine, vol. 15, no. 6, pages 20–36, 1995. (Cited on pages 3, 42 and 63.)

[Lafferriere *et al.* 2005] Gerardo Lafferriere, Alan Williams, J Caughman and JJP Veerman. *Decentralized control of vehicle formations.* Systems & Control Letters, vol. 54, no. 9, pages 899–910, 2005. (Cited on page 17.)

[Laumond *et al.* 1998] Jean-Paul Laumond, S Sekhavat and F Lamiraux. *Guidelines in nonholonomic motion planning for mobile robots.* In Robot Motion Planning and Control, pages 1–53. Springer, 1998. (Cited on page 62.)

[Li & Yang 2013] Xiao-Jian Li and Guang-Hong Yang. *Fault detection in finite frequency domain for Takagi-Sugeno fuzzy systems with sensor faults.* IEEE Transactions on Cybernetics, vol. 44, no. 8, pages 1446–1458, 2013. (Cited on page 117.)

[Li *et al.* 2009] Wei Li, WH Gui, YF Xie and Steven X Ding. *Decentralized fault detection system design for large-scale interconnected system.* IFAC Proceedings Volumes, vol. 42, no. 8, pages 816–821, 2009. (Cited on page 29.)

[Li *et al.* 2015] Shukai Li, Lixing Yang and Ziyou Gao. *Coordinated cruise control for high-speed train movements based on a multi-agent model.* Transportation

Research Part C: Emerging Technologies, vol. 56, pages 281–292, 2015. (Cited on page 9.)

[Li *et al.* 2021] Shanglin Li, Yangzhou Chen and Jingyuan Zhan. *Simultaneous observer-based fault detection and event-triggered consensus control for multi-agent systems*. Journal of the Franklin Institute, vol. 358, no. 6, pages 3276–3301, 2021. (Cited on pages 98, 99, 100 and 133.)

[Liang *et al.* 2021] Dingguo Liang, Ying Yang, Rongchang Li and Ruijie Liu. *Finite-frequency H_-/H_∞ unknown input observer-based distributed fault detection for multi-agent systems*. Journal of the Franklin Institute, vol. 358, no. 6, pages 3258–3275, 2021. (Cited on pages 98 and 99.)

[Lin *et al.* 2004] Zhiyun Lin, Mireille Broucke and Bruce Francis. *Local control strategies for groups of mobile autonomous agents*. IEEE Transactions on Automatic Control, vol. 49, no. 4, pages 622–629, 2004. (Cited on page 19.)

[Lindsay 2013] Jon R Lindsay. *Stuxnet and the limits of cyber warfare*. Security Studies, vol. 22, no. 3, pages 365–404, 2013. (Cited on page 31.)

[Liu *et al.* 2016a] Xiuhua Liu, Xianwen Gao and Jian Han. *Observer-based fault detection for high-order nonlinear multi-agent systems*. Journal of the Franklin Institute, vol. 353, no. 1, pages 72–94, 2016. (Cited on pages 30, 98, 99 and 109.)

[Liu *et al.* 2016b] Xiuhua Liu, Xianwen Gao and Jian Han. *Robust unknown input observer based fault detection for high-order multi-agent systems with disturbances*. ISA transactions, vol. 61, pages 15–28, 2016. (Cited on pages 29, 48, 98 and 101.)

[Liu *et al.* 2018] Xiuhua Liu, Xianwen Gao and Jian Han. *Distributed fault estimation for a class of nonlinear multiagent systems*. IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 50, no. 9, pages 3382–3390, 2018. (Cited on page 30.)

[Liu *et al.* 2019] Chun Liu, Bin Jiang, Ron J Patton and Ke Zhang. *Decentralized output sliding-mode fault-tolerant control for heterogeneous multiagent systems*. IEEE Transactions on Cybernetics, vol. 50, no. 12, pages 4934–4945, 2019. (Cited on page 36.)

[Lofberg 2004] Johan Lofberg. *YALMIP: A toolbox for modeling and optimization in MATLAB*. In 2004 IEEE international Conference on Robotics and Automation (ICRA), pages 284–289. IEEE, 2004. (Cited on page 131.)

[Loukianov 2002] A. G. Loukianov. *Robust Block Decomposition Sliding Mode Control Design*. Mathematical Problems in Engineering, vol. 8, no. 4-5, pages 349–365, 2002. (Cited on page 67.)

[Loukyanov & Utkin 1981] A. G. Loukyanov and V. I. Utkin. *Method of reducing equations for dynamic systems to a regular form*. Automation and Remote Control, vol. 42, no. 4, pages 413–420, 1981. (Cited on page 67.)

[Lu & Yang 2018] An-Yang Lu and Guang-Hong Yang. *Distributed consensus control for multi-agent systems under denial-of-service*. Information Sciences, vol. 439, pages 95–107, 2018. (Cited on pages 36 and 98.)

[Lu *et al.* 2016] Hao Lu, Guangming Xiong and Konghui Guo. *Motion Predicting of Autonomous Tracked Vehicles with Online Slip Model Identification*. Mathematical Problems in Engineering, vol. 2016, 2016. (Cited on page 63.)

[Maghenem *et al.* 2018] Mohamed Maghenem, Abraham Bautista, Emmanuel Nuno, Antonio Loría and Elena Panteley. *Consensus of multi-agent systems with nonholonomic restrictions via Lyapunov's direct method*. IEEE Control Systems Letters, vol. 3, no. 2, pages 344–349, 2018. (Cited on page 74.)

[Mei *et al.* 2014] Jie Mei, Wei Ren and Jie Chen. *Consensus of second-order heterogeneous multi-agent systems under a directed graph*. In 2014 American Control Conference, pages 802–807. IEEE, 2014. (Cited on page 20.)

[Ménard *et al.* 2017] Tomas Ménard, Emmanuel Moulay and Wilfrid Perruquetti. *Fixed-time observer with simple gains for uncertain systems*. Automatica, vol. 81, pages 438–446, 2017. (Cited on page 55.)

[Menon & Edwards 2013] Prathyush P Menon and Christopher Edwards. *Robust fault estimation using relative information in linear multi-agent networks*. IEEE Transactions on Automatic Control, vol. 59, no. 2, pages 477–482, 2013. (Cited on pages 29 and 98.)

[Merco *et al.* 2018] Roberto Merco, Zoleikha Abdollahi Biron and Pierluigi Pisu. *Replay attack detection in a platoon of connected vehicles with cooperative adaptive cruise control*. In 2018 Annual American Control Conference (ACC), pages 5582–5587. IEEE, 2018. (Cited on page 35.)

[Mesbahi & Egerstedt 2010] Mehran Mesbahi and Magnus Egerstedt. Graph theoretic methods in multiagent networks. Princeton University Press, 2010. (Cited on page 13.)

[Meskin & Khorasani 2009a] Nader Meskin and Khashayar Khorasani. *Actuator fault detection and isolation for a network of unmanned vehicles*. IEEE Transactions on Automatic Control, vol. 54, no. 4, pages 835–840, 2009. (Cited on page 29.)

[Meskin & Khorasani 2009b] Nader Meskin and Khashayar Khorasani. *Fault detection and isolation of discrete-time Markovian jump linear systems with application to a network of multi-agent systems having imperfect communication channels*. Automatica, vol. 45, no. 9, pages 2032–2040, 2009. (Cited on page 29.)

[Meskin *et al.* 2010] Nader Meskin, Khashayar Khorasani and Camille Alain Rabbath. *A hybrid fault detection and isolation strategy for a network of unmanned vehicles in presence of large environmental disturbances*. IEEE Transactions on Control Systems Technology, vol. 18, no. 6, pages 1422–1429, 2010. (Cited on page 29.)

[Millán *et al.* 2013] Pablo Millán, Luis Orihuela, Isabel Jurado and Francisco Rodriguez Rubio. *Formation control of autonomous underwater vehicles subject to communication delays.* IEEE Transactions on Control Systems Technology, vol. 22, no. 2, pages 770–777, 2013. (Cited on page 9.)

[Modares *et al.* 2019] Hamidreza Modares, Bahare Kiumarsi, Frank L Lewis, Frank Ferrese and Ali Davoudi. *Resilient and robust synchronization of multiagent systems under attacks on sensors and actuators.* IEEE transactions on cybernetics, vol. 50, no. 3, pages 1240–1250, 2019. (Cited on page 36.)

[Moreno 2021] Jaime A Moreno. *Arbitrary order fixed-time differentiators.* IEEE Transactions on Automatic Control, 2021. (Cited on page 56.)

[Mustafa & Modares 2019] Aquib Mustafa and Hamidreza Modares. *Attack analysis and resilient control design for discrete-time distributed multi-agent systems.* IEEE Robotics and Automation Letters, vol. 5, no. 2, pages 369–376, 2019. (Cited on page 36.)

[Nateghi *et al.* 2018] Shamila Nateghi, Yuri Shtessel, Jean-Pierre Barbot and Christopher Edwards. *Cyber attack reconstruction of nonlinear systems via higher-order sliding-mode observer and sparse recovery algorithm.* In 2018 IEEE Conference on Decision and Control (CDC), pages 5963–5968. IEEE, 2018. (Cited on page 35.)

[Ni & Cheng 2010] Wei Ni and Daizhan Cheng. *Leader-following consensus of multi-agent systems under fixed and switching topologies.* Systems & Control Letters, vol. 59, no. 3-4, pages 209–217, 2010. (Cited on page 23.)

[Olfati-Saber & Murray 2004] Reza Olfati-Saber and Richard M Murray. *Consensus problems in networks of agents with switching topology and time-delays.* IEEE Transactions on Automatic Control, vol. 49, no. 9, pages 1520–1533, 2004. (Cited on pages 18 and 19.)

[Olfati-Saber *et al.* 2007] Reza Olfati-Saber, J Alex Fax and Richard M Murray. *Consensus and cooperation in networked multi-agent systems.* Proceedings of the IEEE, vol. 95, no. 1, pages 215–233, 2007. (Cited on pages 18 and 20.)

[Parsegov *et al.* 2012] Sergey Parsegov, Andrey Polyakov and Pavel Shcherbakov. *Nonlinear fixed-time control protocol for uniform allocation of agents on a segment.* In 2012 IEEE 51st IEEE conference on decision and control (CDC), pages 7732–7737. IEEE, 2012. (Cited on page 39.)

[Pasqualetti *et al.* 2011] Fabio Pasqualetti, Antonio Bicchi and Francesco Bullo. *Consensus computation in unreliable networks: A system theoretic approach.* IEEE Transactions on Automatic Control, vol. 57, no. 1, pages 90–104, 2011. (Cited on pages 34 and 36.)

[Pasqualetti *et al.* 2013] Fabio Pasqualetti, Florian Dörfler and Francesco Bullo. *Attack detection and identification in cyber-physical systems.* IEEE Transactions on Automatic Control, vol. 58, no. 11, pages 2715–2729, 2013. (Cited on page 98.)

[Polyakov *et al.* 2019] Andrey Polyakov, Denis Efimov and Bernard Brogliato. *Consistent discretization of finite-time and fixed-time stable systems.* SIAM Journal on Control and Optimization, vol. 57, no. 1, pages 78–103, 2019. (Cited on page 55.)

[Polyakov 2011] Andrey Polyakov. *Nonlinear feedback design for fixed-time stabilization of linear control systems.* IEEE Transactions on Automatic Control, vol. 57, no. 8, pages 2106–2110, 2011. (Cited on pages 39 and 46.)

[Qin *et al.* 2014] Liguo Qin, Xiao He and DH Zhou. *A survey of fault diagnosis for swarm systems.* Systems Science & Control Engineering: An Open Access Journal, vol. 2, no. 1, pages 13–23, 2014. (Cited on pages 24 and 27.)

[Quan *et al.* 2018] Yue Quan, Wen Chen, Zhihai Wu and Li Peng. *Distributed fault detection and isolation for leader–follower multi-agent systems with disturbances using observer techniques.* Nonlinear Dynamics, vol. 93, no. 2, pages 863–871, 2018. (Cited on pages 30, 31, 37, 46, 48, 98, 99 and 101.)

[Raghavan & Hedrick 1994] Sekhar Raghavan and J Karl Hedrick. *Observer design for a class of nonlinear systems.* International Journal of Control, vol. 59, no. 2, pages 515–528, 1994. (Cited on pages 115 and 129.)

[Ren & Atkins 2007] Wei Ren and Ella Atkins. *Distributed multi-vehicle coordinated control via local information exchange.* International Journal of Robust and Nonlinear Control, vol. 17, no. 10-11, pages 1002–1033, 2007. (Cited on pages 21, 22, 87 and 113.)

[Ren & Beard 2005] Wei Ren and Randal W Beard. *Consensus seeking in multiagent systems under dynamically changing interaction topologies.* IEEE Transactions on Automatic Control, vol. 50, no. 5, pages 655–661, 2005. (Cited on pages 18, 19 and 21.)

[Ren & Beard 2008] Wei Ren and Randal W Beard. *Consensus algorithms for double-integrator dynamics.* Distributed Consensus in Multi-vehicle Cooperative Control: Theory and Applications, pages 77–104, 2008. (Cited on page 21.)

[Ren & Cao 2011] Wei Ren and Yongcan Cao. Distributed coordination of multi-agent networks: emergent problems, models, and issues, volume 1. Springer, 2011. (Cited on pages 18 and 23.)

[Ren *et al.* 2007] Wei Ren, Kevin L Moore and Yangquan Chen. *High-order and model reference consensus algorithms in cooperative control of multivehicle systems.* 2007. (Cited on pages 21 and 87.)

[Ren 2007] Wei Ren. *Multi-vehicle consensus with a time-varying reference state.* Systems & Control Letters, vol. 56, no. 7-8, pages 474–483, 2007. (Cited on page 23.)

[Reza Davoodi *et al.* 2016] Mohammad Reza Davoodi, Zahra Gallehdari, Iman Saboori, Hamed Rezaee, Elham Semsar-kazerooni, Nader Meskin, Farzaneh Abdollahi and Khashayar Khorasani. *An Overview of Cooperative and Consensus Control of Multiagent Systems*, Aug 2016. (Cited on page 8.)

[Rezaee *et al.* 2021] Hamed Rezaee, Thomas Parisini and Marios M Polycarpou. *Resiliency in dynamic leader–follower multiagent systems*. Automatica, vol. 125, page 109384, 2021. (Cited on pages 33 and 113.)

[Saboori & Khorasani 2015] Iman Saboori and Khashayar Khorasani. *Actuator fault accommodation strategy for a team of multi-agent systems subject to switching topology*. Automatica, vol. 62, pages 200–207, 2015. (Cited on page 41.)

[Sánchez-Torres *et al.* 2018] Juan Diego Sánchez-Torres, David Gómez-Gutiérrez, Esteban López and Alexander G Loukianov. *A class of predefined-time stable dynamical systems*. IMA Journal of Mathematical Control and Information, vol. 35, pages i1–i29, 2018. (Cited on page 39.)

[Sánchez-Torres *et al.* 2019] Juan Diego Sánchez-Torres, Michael Defoort and Aldo Jonathan Muñoz-Vázquez. *Predefined-time stabilisation of a class of nonholonomic systems*. International Journal of Control, pages 1–8, 2019. (Cited on page 63.)

[Shames *et al.* 2011] Iman Shames, André MH Teixeira, Henrik Sandberg and Karl H Johansson. *Distributed fault detection for interconnected second-order systems*. Automatica, vol. 47, no. 12, pages 2757–2764, 2011. (Cited on pages 20, 29 and 48.)

[Shames *et al.* 2012] Iman Shames, André MH Teixeira, Henrik Sandberg and Karl H Johansson. *Distributed fault detection and isolation with imprecise network models*. In 2012 American Control Conference (ACC), pages 5906–5911. IEEE, 2012. (Cited on page 29.)

[Shi & Yan 2020] Peng Shi and Bing Yan. *A survey on intelligent control for multiagent systems*. IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 51, no. 1, pages 161–175, 2020. (Cited on page 36.)

[Shi *et al.* 2014] Jiantao Shi, Xiao He, Zidong Wang and Donghua Zhou. *Distributed fault detection for a class of second-order multi-agent systems: an optimal robust observer approach*. IET Control Theory & Applications, vol. 8, no. 12, pages 1032–1044, 2014. (Cited on page 29.)

[Skoundrianos & Tzafestas 2004] Elias N Skoundrianos and Spyros G Tzafestas. *Finding fault-fault diagnosis on the wheels of a mobile robot using local model neural networks*. IEEE Robotics & Automation Magazine, vol. 11, no. 3, pages 83–90, 2004. (Cited on page 62.)

[Slay & Miller 2007] Jill Slay and Michael Miller. *Lessons learned from the maroochy water breach*. In International Conference on Critical Infrastructure Protection, pages 73–82. Springer, 2007. (Cited on page 31.)

[Smith 2011] Roy S Smith. *A decoupled feedback structure for covertly appropriating networked control systems.* IFAC Proceedings Volumes, vol. 44, no. 1, pages 90–95, 2011. (Cited on page 33.)

[Smith 2015] Roy S Smith. *Covert misappropriation of networked control systems: Presenting a feedback structure.* IEEE Control Systems Magazine, vol. 35, no. 1, pages 82–92, 2015. (Cited on page 98.)

[Song & He] Jiahao Song and Xiao He. *Model-based fault diagnosis of networked systems: A survey.* Asian Journal of Control. (Cited on pages 27, 33, 34 and 98.)

[Stanković *et al.* 2010] Srdjan Stanković, Nemanja Ilić, Željko Djurović, Miloš Stanković and Karl Henrik Johansson. *Consensus based overlapping decentralized fault detection and isolation.* In 2010 Conference on Control and Fault-Tolerant Systems (SysTol), pages 570–575. IEEE, 2010. (Cited on page 29.)

[Su & Lin 2015] Shize Su and Zongli Lin. *Distributed consensus control of multi-agent systems with higher order agent dynamics and dynamically changing directed interaction topologies.* IEEE Transactions on Automatic Control, vol. 61, no. 2, pages 515–519, 2015. (Cited on page 21.)

[Sullivan & Kamensky 2017] Julia E Sullivan and Dmitriy Kamensky. *How cyber-attacks in Ukraine show the vulnerability of the US power grid.* The Electricity Journal, vol. 30, no. 3, pages 30–35, 2017. (Cited on page 31.)

[Sycara *et al.* 1996] Katia Sycara, Anandeep Pannu, M Willamson, Dajun Zeng and Keith Decker. *Distributed intelligent agents.* IEEE Expert, vol. 11, no. 6, pages 36–46, 1996. (Cited on page 8.)

[Taha *et al.* 2016] Ahmad F Taha, Junjian Qi, Jianhui Wang and Jitesh H Panchal. *Risk mitigation for dynamic state estimation against cyber attacks and unknown inputs.* IEEE Transactions on Smart Grid, vol. 9, no. 2, pages 886–899, 2016. (Cited on page 35.)

[Tan *et al.* 2020] Sen Tan, Josep M Guerrero, Peilin Xie, Renke Han and Juan C Vasquez. *Brief survey on attack detection methods for cyber-physical systems.* IEEE Systems Journal, vol. 14, no. 4, pages 5329–5339, 2020. (Cited on pages 33 and 34.)

[Taoufik *et al.* 2020a] Anass Taoufik, Krishna Busawon, Michael Defoort and Mohamed Djemai. *An output observer approach to actuator fault detection in multi-agent systems with linear dynamics.* In 2020 28th Mediterranean Conference on Control and Automation (MED), pages 562–567. IEEE, 2020. (Cited on pages 4, 42, 99, 101, 104 and 108.)

[Taoufik *et al.* 2020b] Anass Taoufik, Michael Defoort, Krishna Busawon, Laurent Dala and Mohamed Djemai. *A Distributed Observer-Based Cyber-Attack Identification Scheme in Cooperative Networked Systems under Switching Commu-*

nication Topologies. Electronics, vol. 9, no. 11, page 1912, 2020. (Cited on pages 4, 42, 47, 48, 82, 85, 87, 101 and 113.)

[Taoufik *et al.* 2020c] Anass Taoufik, Michael Defoort, Mohamed Djemai, Krishna Busawon and Juan Diego Sánchez-Torres. *Distributed global actuator fault-detection scheme for a class of linear multi-agent systems with disturbances*. IFAC-PapersOnLine, vol. 53, no. 2, pages 4202–4207, 2020. (Cited on pages 3, 42, 46, 47, 48, 54, 71, 80 and 90.)

[Taoufik *et al.* 2021a] Anass Taoufik, Michael Defoort, Krishna Busawon and Mohamed Djemai. *An output observer approach to actuator fault detection in multi-agent systems with linear dynamics*. Nonlinear Dynamics, pages 1-20, 2021. (Cited on pages 4, 43, 99, 101, 118, 121 and 124.)

[Taoufik *et al.* 2021b] Anass Taoufik, Michael Defoort, Mohamed Djemai, Krishna Busawon and Juan Diego Sánchez-Torres. *Distributed global fault detection scheme in multi-agent systems with chained-form dynamics*. International Journal of Robust and Nonlinear Control, vol. 31, no. 9, pages 3859–3877, 2021. (Cited on pages 3, 9, 42, 46, 48, 67 and 72.)

[Teixeira *et al.* 2010] André Teixeira, Henrik Sandberg and Karl H Johansson. *Networked control systems under cyber attacks with applications to power networks*. In Proceedings of the 2010 American Control Conference, pages 3690–3696. IEEE, 2010. (Cited on pages 33, 48, 89, 101 and 113.)

[Teixeira *et al.* 2014] André Teixeira, Iman Shames, Henrik Sandberg and Karl H Johansson. *Distributed fault detection and isolation resilient to network model uncertainties*. IEEE Transactions on Cybernetics, vol. 44, no. 11, pages 2024–2037, 2014. (Cited on pages 25, 46, 48, 98, 101, 109 and 112.)

[Utkin 2013] Vadim I Utkin. Sliding modes in control and optimization. Springer Science & Business Media, 2013. (Cited on page 70.)

[van Schrick 1997] Dirk van Schrick. *Remarks on terminology in the field of supervision, fault detection and diagnosis*. IFAC Proceedings Volumes, vol. 30, no. 18, pages 959–964, 1997. (Cited on page 23.)

[Wang & Yang 2019] Xin Wang and Guang-Hong Yang. *Fault-tolerant consensus tracking control for linear multiagent systems under switching directed network*. IEEE Transactions on Cybernetics, vol. 50, no. 5, pages 1921–1930, 2019. (Cited on pages 36 and 41.)

[Wang *et al.* 2007] Jian Liang Wang, Guang-Hong Yang and Jian Liu. *An LMI approach to $\mathcal{H}\_$ index and mixed $\mathcal{H}\_/\mathcal{H}_\infty$ fault detection observer design*. Automatica, vol. 43, no. 9, pages 1656–1665, 2007. (Cited on page 57.)

[Wilson 2015] Robin J Wilson. Introduction to graph theory. Pearson Higher Ed, 2015. (Cited on page 17.)

[Wong & Sycara 1999] HC Wong and K Sycara. *Adding Security and Trust to Multi-Agent Systems Proc. of Autonomous Agents99 (Work. on Deception, Fraud and Trust in Agent Societies)*. 1999. (Cited on page 8.)

[Wu *et al.* 2019] Yanming Wu, Zhanshan Wang and Zhanjun Huang. *Distributed fault detection for nonlinear multi-agent systems under fixed-time observer*. Journal of the Franklin Institute, vol. 356, no. 13, pages 7515–7532, 2019. (Cited on pages 30, 36, 48, 80, 90, 98 and 99.)

[Xia *et al.* 2017] Jingping Xia, Bin Jiang and Ke Zhang. *Dissipativity-based robust reduced-order fault estimation observer design of multi-agent systems*. International Journal of Control, Automation and Systems, vol. 15, no. 6, pages 2619–2627, 2017. (Cited on page 30.)

[Xie & Wang 2007] Guangming Xie and Long Wang. *Consensus control for a class of networks of dynamic agents*. International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal, vol. 17, no. 10-11, pages 941–959, 2007. (Cited on page 21.)

[Yan & Edwards 2008] Xing-Gang Yan and Christopher Edwards. *Robust decentralized actuator fault detection and estimation for large-scale systems using a sliding mode observer*. International Journal of control, vol. 81, no. 4, pages 591–606, 2008. (Cited on page 29.)

[Yang *et al.* 2019] Hao Yang, Qing-Long Han, Xiaohua Ge, Lei Ding, Yuhang Xu, Bin Jiang and Donghua Zhou. *Fault-tolerant cooperative control of multiagent systems: A survey of trends and methodologies*. IEEE Transactions on Industrial Informatics, vol. 16, no. 1, pages 4–17, 2019. (Cited on pages 2, 35, 36 and 46.)

[Yu & Yuan 2020] Yang Yu and Yuan Yuan. *Event-triggered active disturbance rejection control for nonlinear network control systems subject to dos and physical attacks*. ISA Transactions, vol. 104, pages 73–83, 2020. (Cited on page 35.)

[Yu *et al.* 2010] Wenwu Yu, Guanrong Chen and Ming Cao. *Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems*. Automatica, vol. 46, no. 6, pages 1089–1095, 2010. (Cited on page 21.)

[Yu *et al.* 2016] Wenwu Yu, He Wang, Fei Cheng, Xinghuo Yu and Guanghui Wen. *Second-order consensus in multiagent systems via distributed sliding mode control*. IEEE Transactions on Cybernetics, vol. 47, no. 8, pages 1872–1881, 2016. (Cited on page 21.)

[Zhang *et al.* 2015] Ke Zhang, Bin Jiang and Vincent Cocquempot. *Adaptive technique-based distributed fault estimation observer design for multi-agent systems with directed graphs*. IET Control Theory & Applications, vol. 9, no. 18, pages 2619–2625, 2015. (Cited on pages 30 and 133.)

[Zhang *et al.* 2016] Ke Zhang, Bin Jiang and Peng Shi. *Adjustable parameter-based distributed fault estimation observer design for multiagent systems with directed* *graphs*. IEEE Transactions on Cybernetics, vol. 47, no. 2, pages 306–314, 2016. (Cited on page 30.)

[Zhang *et al.* 2017] Ke Zhang, Bin Jiang and Vincent Cocquempot. *Distributed fault estimation observer design for multi-agent systems with switching topologies*. IET Control Theory & Applications, vol. 11, no. 16, pages 2801–2807, 2017. (Cited on page 41.)

[Zhang *et al.* 2018] Wenbing Zhang, Zidong Wang, Yurong Liu, Derui Ding and Fuad E Alsaadi. *Sampled-data consensus of nonlinear multiagent systems subject to cyber attacks*. International Journal of Robust and Nonlinear Control, vol. 28, no. 1, pages 53–67, 2018. (Cited on page 36.)

[Zhang *et al.* 2021a] Dan Zhang, Gang Feng, Yang Shi and Dipti Srinivasan. *Physical safety and cyber security analysis of multi-agent systems: A survey of recent advances*. IEEE/CAA Journal of Automatica Sinica, vol. 8, no. 2, pages 319–333, 2021. (Cited on pages 33, 34 and 35.)

[Zhang *et al.* 2021b] Jingqiu Zhang, Subham Sahoo, Jimmy Chih-Hsien Peng and Frede Blaabjerg. *Mitigating Concurrent False Data Injection Attacks in Cooperative DC Microgrids*. IEEE Transactions on Power Electronics, vol. 36, no. 8, pages 9637–9647, 2021. (Cited on page 98.)

[Zhou & Doyle 1998] Kemin Zhou and John Comstock Doyle. Essentials of robust control, volume 104. Prentice hall Upper Saddle River, NJ, 1998. (Cited on page 122.)

[Zhu & Cheng 2010] Wei Zhu and Daizhan Cheng. *Leader-following consensus of second-order agents with multiple time-varying delays*. Automatica, vol. 46, no. 12, pages 1994–1999, 2010. (Cited on page 23.)

[Zhu *et al.* 2009] Jiandong Zhu, Yu-Ping Tian and Jing Kuang. *On the general consensus protocol of multi-agent systems with double-integrator dynamics*. Linear Algebra and its Applications, vol. 431, no. 5-7, pages 701–715, 2009. (Cited on page 21.)

[Zuo *et al.* 2017] Zongyu Zuo, Bailing Tian, Michael Defoort and Zhengtao Ding. *Fixed-time consensus tracking for multiagent systems with high-order integrator dynamics*. IEEE Transactions on Automatic Control, vol. 63, no. 2, pages 563–570, 2017. (Cited on page 21.)