RESEARCH PAPERS
FACULTY OF MATERIALS SCIENCE AND TECHNOLOGY IN TRNAVA
SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

10.2478/rput-2013-0006 2013, Volume 21, Special Number

SIMULATION OF PHOTON PROPAGATION IN TISSUE USING MATLAB

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ABSTRACT

This paper deals with the light transport, photon trajectory and its radiation in tissue. A model based on Monte Carlo simulation has been implemented in Matlab to get inside into photon interaction with tissue. The project is aimed to non-invasive pulse oximetry measurement of fetal oxygen saturation in the maternal abdomen. One of the fundamental challenges is to ensure a sufficient penetration depth which covers maternal and fetal tissue. This contribution investigates the photon trajectories and analyse the number of photons which stayed in tissue and their radiation distribution. The principle and photon propagation rules, needed for simulation, are presented in this article. Finally the results are compared with literature.

KEY WORDS

Monte Carlo simulation, photon propagation, tissue, light transport

INTRODUCTION

Optical technique is a useful analysis method in biomedical diagnosis. The simulation of light transport provides statements about the photon interaction with tissue. In the case of fetal pulse oximetry, it will be possible to evaluate the light distribution and the penetration depth under different conditions without the need of suitable patients. These parameters are important for further investigations, for instance simulating pulse curve shapes or determining the oxygenation of arterial blood. For this reason this paper deals with the fundamental photon propagation rules and the radiation in tissue.

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THE PHOTON PROPAGATION RULES

The implemented algorithm is based on Wang and Jacques steady-state light transport model which was written a in a standard language C [WaJa92]. The single steps of the implementation are described in the following.

Figure 1 shows the schematic of the Cartesian coordinate system which describes the model. The z-coordinate represents the depth of the tissue, where the x- and y-direction are assumed as infinity wide.

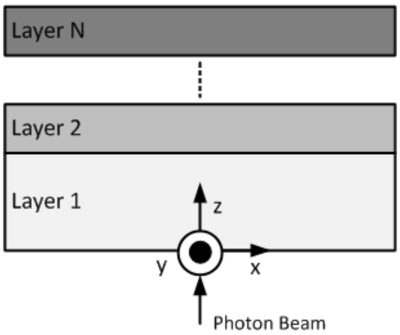

Fig. 1 *Schematic of the Cartesian coordinate system, which describes the implemented simulation model, the y axis points outwards [WaJa92]*

Launching a photon

The start position of each photon is determined by the coordinates $x, y, z = 0,0,0$ and the initial direction is orthogonal to the tissue surface, which is given by $\mu_x, \mu_y, \mu_z = (0,0,1)$. When the photons penetrate into the tissue, some specular reflectance at the surface will occur. The specular reflectance R_{sp} can be described by

$$R_{sp} = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

and the photon weight will be decreased by R_{sp}

$$W = 1 - R_{sp}$$

Moving the photon

After photon injection the step size s will be calculated by using the equation:

$$s = \frac{-\ln(\xi)}{\mu_t},$$

where μ_t is an interaction coefficient equals the sum of the absorption coefficient μ_a and scatter coefficient μ_s. The parameter ξ is a random variable, which is uniformly distributed over the interval $(0,1)$. A decision has to be made, which distinguishes whether the step size s is long enough to reach a boundary or not.

If the photon didn't reach a boundary the position of the photon will be updated by:

$$x \leftarrow x + \mu_x \cdot s$$

$$y \leftarrow y + \mu_y \cdot s$$
$$z \leftarrow z + \mu_z \cdot s$$

Absorption and scattering of the photon

By moving a photon inside the tissue the photon weight is decreasing due to absorption. The amount of photon weight loss is defined by:

$$\Delta W = W \cdot \frac{\mu_a}{\mu_t}$$

The photon weight is then updated by:

$$W \leftarrow W - \Delta W$$

For scattering the photon, the azimuth $\psi \in [0, 2\pi)$ and deflection angle $\theta \in [0, \pi)$ have to be taken into account. The final photon directions are computed by the following equation:

$$\mu_x' = \frac{\sin\theta}{\sqrt{1 - \mu_z^2}} \quad \mu_x\, \mu_z \cos\psi - \mu_y \sin\psi \quad + \mu_x \cos\theta$$
$$\mu_y' = \frac{\sin\theta}{\sqrt{1 - \mu_z^2}} \quad \mu_y\, \mu_z \cos\psi - \mu_x \sin\psi \quad + \mu_y \cos\theta$$
$$\mu_z' = -\sin\theta \cos\psi \quad \sqrt{1 - \mu_z^2} + \mu_z \cos\theta$$

For the special case, that the incident angle is orthogonal to the surface of the tissue, the photon direction is following the formulas:

$$\mu_x' = \sin\theta \cos\psi$$
$$\mu_y' = \sin\theta \sin\psi$$
$$\mu_z' = \mathrm{SIGN}(\mu_z) \cos\theta$$

$$\mathrm{SIGN}\ x = \begin{array}{r} -1 \text{ if } x < 0 \\ 0 \text{ if } x = 0 \\ 1 \text{ if } x > 0 \end{array}$$

Finally the photon direction is updated:

$$\mu_x \leftarrow \mu_x'$$
$$\mu_y \leftarrow \mu_y'$$
$$\mu_z \leftarrow \mu_z'$$

Reflection and transmission at a boundary

If the step size is long enough to hit the boundary, then the photon moves to the boundary. Subsequently the program decides whether the photon escapes the tissue or is internally reflected. This depends on the angle of incidence α_i and the angle of transmission α_t. The internal reflectance $R(\alpha_i)$ is then calculated by Fresnel's formula:

$$R\ \alpha_i = \frac{1}{2} \frac{\sin^2\ a_i - a_t}{\sin^2\ a_i + a_t} + \frac{\tan^2\ a_i - a_t}{\tan^2\ a_i + a_t}.$$

The finally decision is realized by comparing the internal reflectance with a random number. After this step the absorption and scattering will computed correspondingly (see [WaJa92] for more details).

Photon Termination

A photon is terminated if it escapes the tissue or if the photon weight decreases below a defined threshold inside of the tissue. In the case that the photon weight is lower than the threshold, the current photon gets a further chance in m (e.g., $m = 10$) for surviving with a weight of mW [WaJa92]. The photon is terminated if it does not survive the so called roulette:

$$W = \begin{matrix} mW & \text{if } \xi \leq 1/m \\ 0 & \text{if } \xi > 1/m \end{matrix}$$

VERIFICATION OF RESULTS

Simulations were done for three different cases to verify the implemented algorithm. The results of these scenarios were compared with other results from literature [Giov55, Huls80, PKJW89 and WaJa92].

The first simulation includes ten runs with 50,000 photon packets and the investigated slab has the following optical properties:

OPTICAL PROPERTIES OF THE SLAB
OF THE FIRST SIMULATION Table 1

	n	μ_a	μ_s	g	d
medium	1	10 cm^{-1}	90 cm^{-1}	0.75	0.02 cm

Table 2 shows the results of the computed total diffuse reflectance R_d and total transmittance T_t. The simulation results are consistent with the data from literature.

Furthermore we determined the average number of photons which goes through the tissue. Overall 41,848 of 50,000 photon packets escaped the tissue by transmittance in this simulation and the average absorption was 0.2418.

COMPARISON OF THE FIRST SIMULATION RESULTS Table 2

Source	R_d Average	T_t Average
Van de Hulst [Huls80]	0.09739	0.66096
Wang and Jacques [WaJa92]	0.09734	0.66096
Prahl et al. [PKJW89]	0.09711	0.66159
Matlab	0.09734	0.66100

For the second simulation, we investigate ten simulations with 5,000 photon packets. The optical properties were determined as follows:

OPTICAL PROPERTIES OF THE SLAB
OF THE SECOND SIMULATION Table 3

	n	μ_a	μ_s	g
medium	1.5	10 cm^{-1}	90 cm^{-1}	0

We computed the average of the total diffuse reflectance in the semi-infinite turbid medium. The comparison with literature is shown in Table **4**. The relative error is 0.76 % between the conducted simulation and the investigations of Giovanelli [Giov55].

In this simulation, 2,599 photons were absorbed by tissue and the value of average absorption was 0.4466.

COMPARISON OF THE SECOND SIMULATION RESULTS
Table 4

Source	Reflectance R_d Average
Giovanelli [Giov55]	0,26000
Wang and Jaques [WaJa92]	0,25907
Prahl et al. [PKJW89]	0,26079
Matlab	0,25803

The final verification was realized with one simulation run of multi-layered tissues. There were 100,000 photon packets and the specific optical properties of three layers are shown in Optical properties of three layers for last simulation Table 5. The refractive indices of the top and bottom ambient media are both set to 1. Average absorption was 0.4310 and number of absorbed photons was 42,415.

OPTICAL PROPERTIES OF THREE LAYERS
FOR LAST SIMULATION Table 5

	n	μ_a	μ_s	g	d
medium above	1				
layer 1	1.37	1	100	0.90	0.1
layer 2	1.37	1	10	0	0.1
layer 3	1.37	2	10	0.70	0.2
medium below	1				

The total diffuse reflectance and total transmittance of a slab of turbid medium and the comparison is shown in Comparison of the third simulation results Table **6**. The standard error is 0.0012.

COMPARISON OF THE THIRD SIMULATION RESULTS
Table 6

Source	Reflectance R_d	Transmittance T_t
Gardner [GaWe92]	0,2381	0,0974
Wang and Jacques [WaJa92]	0,2375	0,0965
Matlab	0,2341	0,0934

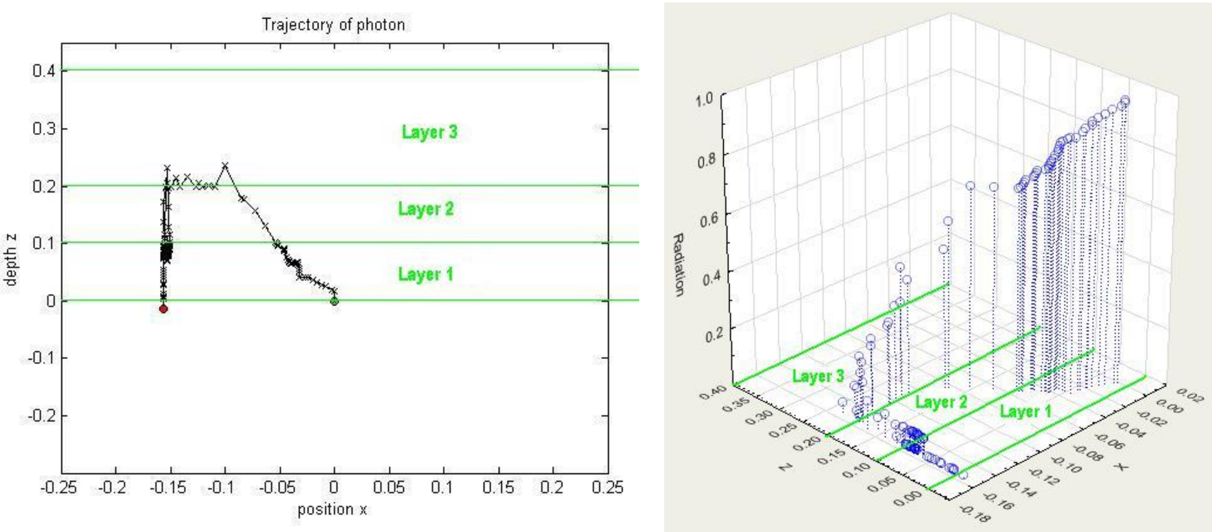

Fig. 2 *Trajectory of a single photon in cm (left) and radiation distribution of the single photon (right)*

Fig. *2* shows the photon interaction with the tissue for a single photon. These results are based on the multi-layered tissue simulation where the corresponding parameters are given by Optical properties of three layers for last simulation Table 5. The starting point of the photon is indicated with a green point at coordinates (0; 0). Termination occurred when the last significant fraction of remaining photon weight escaped from tissue at the position indicated by the red point at the coordinates (-0.156; -0.013) with a residual weight of 0.011143.

CONCLUSION

We investigated the reflectance and transmittance of photons in tissue for verification our results. The calculated values were compared with the available experimental data which shows an overall agreement. In addition, we looked into photon track inside the turbid medium and its radiation. The results reported in this paper provide fundamental knowledge for further investigations in view of non-invasive pulse oximetry.

ACKNOWLEDGMENT

This publication is the result of implementation of the project: “Increase of Power Safety of the Slovak Republic” (ITMS: 26220220077) supported by the Research & Development Operational Programme funded by the ERDF. This work was also supported by the grant 50749917 of the program IPID - “International Promovieren in Deutschland”.

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