PAPER • OPEN ACCESS

Control Design of an Inverted Pendulum on a Moving Base Using State Feedback

View the article online for updates and enhancements.

Control Design of an Inverted Pendulum on a Moving Base Using State Feedback

[1]**Victoria Oguntosin**, [1]**Samson Ogheneovo Oruma**, [1]**Ayoola Akindele**, [1]**Ademola Abdulkareem**

[1]*Department of Electrical & Information Engineering*, Covenant University, Ota, Ogun State

Corresponding author: victoria.oguntosin@covenantuniversity.edu.ng

Abstract. A fascinating and essential control problem for researchers is the inverted pendulum. It remains a generally accepted standard used in control and robotics for validation of emerging control technologies. A mathematical model of the inverted pendulum on a moving base system was derived, an expression of the linearised state space representation of the system was also presented, a controller to achieve specific criterion of the steady state error, rise and settling time, and was obtained, an implementation of this control on MATLAB was carried out together with the simulated animation of the system using Simulink and Simscape.

1.   Introduction

The inverted pendulum is one in which the mass center is beyond the fulcrum position. The structure is inverted 180° from its equilibrium position, with a solid rod and held firmly at its pivot. It is generally unstable without control. It is the most popular standard used in nonlinear control to authenticate the effectiveness of developing control technologies and substantiate their execution [1]. To achieve inverted pendulum control, a feedback system for control is adopted, which records angle of the pendulum and changes the pivot point sideways to effect corrective action. The inverted pendulum remains a conventional problem in control. It has many variations such as the rotational inverted pendulum [2], a moving base inverted pendulum [3], triple inverted pendulum [4], the parallel type dual system [5], the quadruple type [6], and the spherical pendulum-type [7]. Other examples of the inverted pendulum comprise the human body, the central component in the design of early seismometers. In the recent past, it is used in robotics, personal transporters such as scooters that self-balance and electric unicycles [7].

In this paper, a mathematical model using a representation in state space of inverted pendulum on a moving base was achieved, and controller design using state-space was implemented with MATLAB. The simulated open-loop and closed loop simulation of inverted pendulum using Simulink and Simscape were obtained. The significance of this work is to provide a comprehensive study of the full state feedback control design for an inverted pendulum system on a moving base. Control design has

been used in several mechatronics and robotics [8] applications and is, therefore, an integral component in the design of any structure, be it electrical, mechanical or structural [9].

The rest of this article is as follows; materials and methods which contained mathematical modelling and state-space representation, controller design and implementation on MATLAB and simulation of an inverted pendulum on Simulink and Simscape. Next section is the results and discussion and conclusion.

Fig1: Physical setup of inverted pendulum
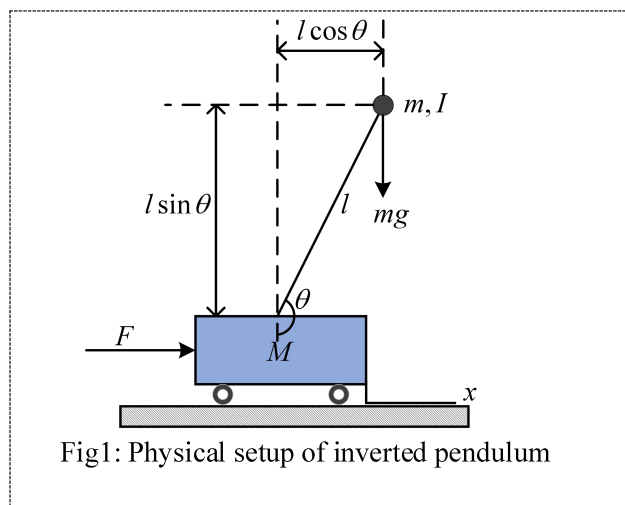
2.   Methods

2.1. Mathematical model and State-space Representation of Inverted Pendulum Structure on a Moving Base:

Figure 1 illustrates the basic structure of an inverted pendulum structure on a moving base. Where $M = 0.5\,Kg, m = 0.2Kg, \mu = 0.1Nm^{-1}s^{-1}, l = 0.3\,m, I = 6 \times 10^{-3}\,Kgm^2$. M is the base mass, m is the pendulum mass, μ the frictional coefficient of the moving base, l is the pendulum length with respect to mass center, I represents the pendulum's moment-of-Inertia. A Force, F is given to the base which causes the structure to move by a distance x. The angle of the pendulum measured from the vertical is given by θ

The summation of all the horizontal forces in the FBD of Fig. 1 will result in equation (1)

$$M\ddot{x} + b\dot{x} + N = F \;\;-------------------\; (1)$$

The reaction force, N is obtained as shown in equation 2.

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 sin\theta \;-------------\; (2)$$

Replacing eqn (2) into eqn (1), produces eqn 3.

$$(M + m)\ddot{x} + b\dot{x} + ml\,\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \text{----------} \textbf{(3)}$$

The perpendicular forces to the pendulum are added up to yield equation (4).

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta \text{---------} \textbf{(4)}$$

P and N terms of equation 4 are eliminated to give rise to equation (5).

$$-Pl\sin\theta - Nl\cos\theta = I\,\ddot{\theta} \text{-------------------} \textbf{(5)}$$

The combination of equations 4 and 5, gives equation 6

$$\ddot{\theta}(I + ml^2) + \sin\theta(mgl) = -ml\,\ddot{x}\cos\theta \text{..} \textbf{(6)}$$

Eqns 3 and 6 are linearised about the vertical equilibrium position, $\pi = \theta$. The approximations of the equations are obtained:

$$\cos\theta = \cos(\phi + \pi) \approx -1 \text{..} \textbf{(7)}$$

$$\sin\theta = \sin(\phi + \pi) \approx -\emptyset \text{..} \textbf{(8)}$$

$$\dot{\phi}^2 = \dot{\theta}^2 \approx 0 \text{..} \textbf{(9)}$$

The substitutions of eqs $7 - 9$ in the nonlinear equations gives the linearised equations of motion with u replacing F are derived

$$\ddot{\phi}(I + ml^2) - \phi\,mgl = \ddot{x}ml \text{------------------} \textbf{(10)}$$

$$\ddot{x}(M + m) + b\ddot{x} - ml\,\ddot{\phi} = u \text{------------------} \textbf{(11)}$$

2.2 Transfer Function:

The Laplace transformations of equations 10 and 11 give:

$$\phi(s)(ml^2 + I)s^2 - mgl\phi(s) = ml\,X(s)s^2 \text{-------------} \textbf{(12)}$$

$$X(s)(M + m)s^2 + X(s)bs - lm\phi(s)s^2 = U(s) \text{-------------} \textbf{(13)}$$

The output is $\Phi(s)$ while the input is $U(s)$. $X(s)$ is obtained as:

$$X(s) = \left[\frac{I + ml^2}{ml} - \frac{g}{s^2}\right]\phi(s) \text{------------------} \textbf{(14)}$$

Equation 14 is substituted into equation 13 to give:

$$(M + m)\left[\frac{ml^2 + I}{lm} - \frac{g}{s^2}\right]\phi(s)s^2 + b\left[\frac{ml^2 + I}{lm} - \frac{g}{s^2}\right]s\phi(s) - lm\phi(s)s^2 = U(s) \,\text{--}\, \textbf{(15)}$$

The transfer function is obtained as:

$$\frac{\phi(s)}{U(s)} = \frac{s^2\dfrac{lm}{q}}{s^4 + \dfrac{b(ml^2 + I)}{q}s^3 - \dfrac{(m + M)mlg}{q}s^2 - \dfrac{bmgl}{q}s} \,\text{----------}\, \textbf{(16)}$$

$$q = \left[(m + M)(ml^2 + I) - (lm)^2\right] \,\text{----------------}\, \textbf{(17)}$$

From eqn 16, a pole, P and a zero, Z is at the origin which cancels out, and a different transfer function of the system is given in eqn 18.

$$P_{cart}(s) = \frac{\phi(s)}{U(s)} = \frac{s\frac{lm}{q}}{s^3 + \frac{b(ml^2+I)}{q}s^2 - \frac{(m+M)mlg}{q}s - \frac{bmgl}{q}}\left[\frac{rad}{N}\right] \,\text{----------}\, \textbf{(18)}$$

The transfer function with the position as the output can be obtained in a comparable fashion to give rise to equation 19.

$$P_{cart}(s) = \frac{X(s)}{U(s)} = \frac{\dfrac{(ml^2 + I)s^2 - mgl}{q}}{s^4 + \dfrac{b(ml^2 + I)}{q}s^3 - \dfrac{(m + M)mgl}{q}s^2 - \dfrac{bgml}{q}}\left[\frac{m}{N}\right] \,\text{------}\, \textbf{(19)}$$

2.3   Representation in State-Space:

The equations describing the inverted pendulum are nonlinear. A linearised state space representation is derived as shown in the following subsections.

2.3.1   Derivation of State – Space and Output Equations

Chosen states are

- $x_1 = x = $ *Position of the moving base in the horizontal direction,*
- $x_2 = \dot{x} = $ *Base velocity in the horizontal direction*
- $x_3 = \phi = $ *Base angular position*
- $x_4 = \dot{\phi} = $ *Base angular velocity*

The derivatives of the states are:

- $\dot{x}_1 = x_2$
- $\dot{x}_2 = \ddot{x}$

- $\dot{x}_3 = \dot{\emptyset} = x_4$
- $\dot{x}_4 = \ddot{\emptyset}$

Chosen Outputs are

- $y_1 = x_1 = $ *Position of the base*
- $y_2 = x_3 = $ *Pendulum angle*

Chosen input is the force $F = u$.

The two governing equations are:

$$F = (M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \cdots\cdots(20a)$$

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \cdots\cdots(20b)$$

$$\cos\theta = \cos(\pi + \emptyset) \sim -1 \cdots\cdots(20d)$$

$$\sin\theta = \sin(\pi + \emptyset) \sim -\emptyset \cdots\cdots(20e)$$

$$\dot{\theta}^2 = \dot{\emptyset}^2 \sim 0 \cdots\cdots(20d)$$

$$(M + m)\dot{x}_2 + bx_2 - ml\dot{x}_4 = u \quad \cdots\cdots(21)$$

$$(I + ml^2)\dot{x}_4 - mglx_3 = ml\dot{x}_2 \quad \cdots\cdots(22)$$

From (22):

$$\dot{x}_4 = \frac{mglx_3}{(I + ml^2)} + \frac{ml\dot{x}_2}{(I + ml^2)} \quad \cdots\cdots(23)$$

Substitute (23) into (21)

$$u = (M + m)\dot{x}_2 + bx_2 - ml\left[\frac{mglx_3}{(I + ml^2)} + \frac{ml\dot{x}_2}{(I + ml^2)}\right]\cdots\cdots(23a)$$

$$u = (M + m)\dot{x}_2 - \frac{m^2l^2\dot{x}_2}{(I + ml^2)} + bx_2 - \frac{m^2l^2gx_3}{(I + ml^2)}\cdots\cdots(23b)$$

$$u = \left[M + m - \frac{m^2l^2}{(I + ml^2)}\right]\dot{x}_2 + bx_2 - \frac{m^2l^2gx_3}{(I + ml^2)}\cdots\cdots(23c)$$

$$\dot{x}_2 = \frac{u(I + ml^2)}{(I + ml^2)(M + m) - m^2l^2} - \frac{bx_2(I + ml^2)}{(I + ml^2)(M + m) - m^2l^2}$$
$$+ \frac{m^2l^2gx_3}{(I + ml^2)(M + m) - m^2l^2} \cdots \cdots (23d)$$

$$(I + ml^2)(M + m) - m^2l^2 = MI + mI + mMl^2 + m^2l^2 - m^2l^2 = I(M + m) + mMl^2 \cdots \cdots (23e)$$

$$\dot{x}_2 = \frac{(I + ml^2)}{I(M + m) + mMl^2}u - \frac{b(I + ml^2)}{I(M + m) + mMl^2}x_2 + \frac{m^2l^2g}{I(M + m) + mMl^2}x_3 \cdots \cdots (23f)$$

From (22):

$$\dot{x}_2 = \frac{(I + ml^2)}{ml}\dot{x}_4 - gx_3 \qquad \cdots \cdots (24)$$

Substitute (24) into (21):

$$u = (M + m)\left[\frac{(I + ml^2)}{ml}\dot{x}_4 - gx_3\right] + bx_2 - ml\dot{x}_4 \cdots \cdots (25)$$

$$u = \frac{(M + m)(I + ml^2)}{ml}\dot{x}_4 - ml\dot{x}_4 - g(M + m)x_3 + bx_2 \cdots \cdots (26)$$

$$u = \left[\frac{(M + m)(I + ml^2)}{ml} - ml\right]\dot{x}_4 - g(M + m)x_3 + bx_2 \cdots \cdots (27)$$

$$\frac{(I + ml^2)(M + m) - m^2l^2}{ml} = \frac{MI + mI + mMl^2 + m^2l^2 - m^2l^2}{ml} = \frac{I(M + m) + mMl^2}{ml} \cdots \cdots (28)$$

$$u = \left[\frac{I(M + m) + mMl^2}{ml}\right]\dot{x}_4 - g(M + m)x_3 + bx_2 \cdots \cdots (29)$$

$$\dot{x}_4 = \frac{ml}{I(M + m) + mMl^2}u + \frac{g(M + m)ml}{I(M + m) + mMl^2}x_3 - \frac{bml}{I(M + m) + mMl^2}x_2 \cdots \cdots (30)$$

Since $\dot{x}_1 = x_2$

$$\dot{x}_2 = \frac{(I + ml^2)}{I(M + m) + mMl^2}u - \frac{b(I + ml^2)}{I(M + m) + mMl^2}x_2 + \frac{m^2l^2g}{I(M + m) + mMl^2}x_3 \cdots \cdots (31)$$

Since $\dot{x}_3 = x_4$

$$\dot{x}_4 = \frac{ml}{I(M + m) + mMl^2}u + \frac{g(M + m)ml}{I(M + m) + mMl^2}x_3 - \frac{bml}{I(M + m) + mMl^2}x_2 \cdots \cdots (32)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-b(I+ml^2)}{I(M+m)+mMl^2} & \dfrac{m^2l^2g}{I(M+m)+mMl^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-bml}{I(M+m)+mMl^2} & \dfrac{g(M+m)ml}{I(M+m)+mMl^2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{(I+ml^2)}{I(M+m)+mMl^2} \\ 0 \\ \dfrac{ml}{I(M+m)+mMl^2} \end{bmatrix} u \cdots (33)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \cdots\cdots (34)$$

2.4   State-Space Controller Design and MATLAB Implementation:

2.4.1   Design Assumptions

The following design specifications are given for the inverted pendulum:

- t_s for the base position, x and pendulum angle, θ should be less than 5s
- t_r for the base position, x must be less than 0.5s
- θ must be less than 20^0 (or 0.35rad)
- e_{ss} should not be greater than 2% for base position and pendulum angle

### 2.4.2   State-space controller design dynamic equations

The state space equations of the inverted pendulum system with operating parameters substituted into equations (19) and (20) is shown in equation 21;

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.1818 & 2.6727 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.1818 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 1.1812 \\ 0 \\ 4.5455 \end{bmatrix} U \cdots\cdots (35)$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdots\cdots (36)$$

2.4.2.1   State feedback Design

The control system with k being the matrix of control gain is shown in Fig 2.

Fig 2: Schematic diagram of full state-feedback
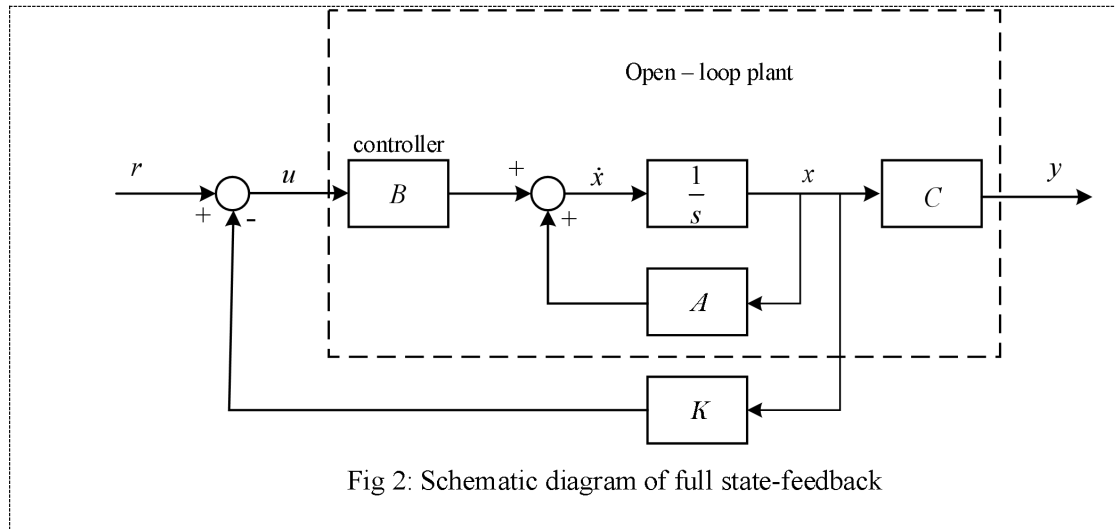
The system's open-loop poles were determined. From the results, there was a positive pole which confirms that the open loop response is unstable. Next, a check of the controllability of the system was carried out. Results showed that the system is controllable (Controllability of 4 which is equivalent to the number of control states).

2.4.2.2   The Linear Quadratic Regulator (LQR)

The LQR was utilised to find the gain (k matrix) and plot its step response. Different values of Q were tested and plotted to give the resulting step response.

2.4.2.3   Adding a Pre-Compensator

The schematic diagram of the inverted pendulum with pre-compensator added is as shown in Fig 3.
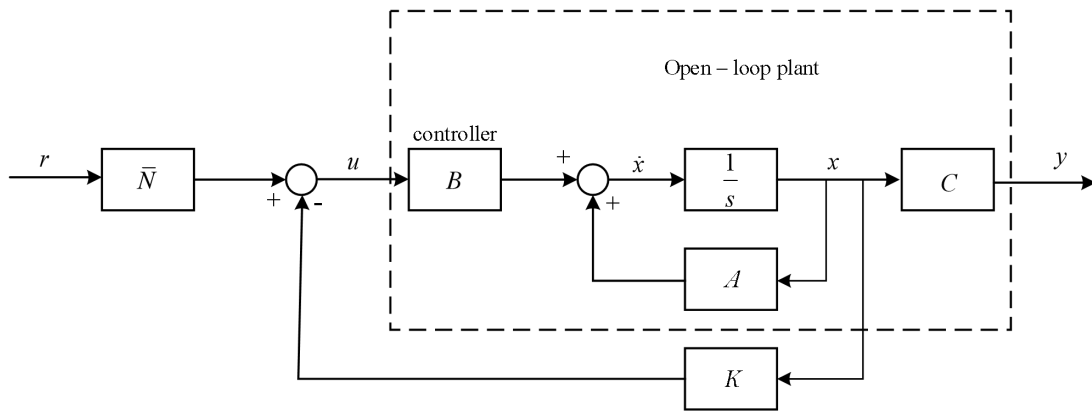
Figure 3: Schematic Diagram with Pre-Compensator added

The value of $\bar{N}$ was computed. The step response plot of the inverted pendulum with compensator and LQR was obtained.

2.4.2.4　Observer-Based Control

The observability of the system was checked for with selected observer poles at -40, -41, -43, -44 (about 4 times higher values). The state-feedback of the controller is then combined with the state estimator to get full compensation as shown by the matrix equations 14 and 15;

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B\bar{N} \\ 0 \end{bmatrix} r \cdots\cdots\cdots(37)$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + [0] r \cdots\cdots\cdots(38)$$

This was implemented in MATLAB with the step response plot.

2.5 Simulation with Simulink and Simscape

2.5.1　Simulink Modelling

The arrangement is indicated in Figure 4. Here the input force $\boldsymbol{F}$ makes the base to move in the horizontal direction and y (angular position $\boldsymbol{\theta}$ and the base horizontal position $\boldsymbol{x}$).
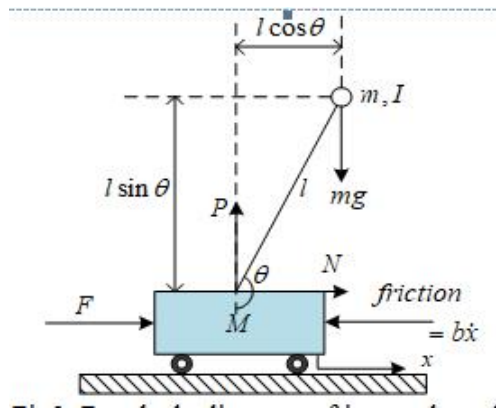
Figure 4: Schematic of the system

From Fig 4, it can be seen that the base and the pendulum possess a single-degree-of-freedom (x and θ). Using the second Newton equation of motion $F = ma$, the following are gotten:

$$\ddot{x} = \frac{1}{M}\sum_{cart} F_x = \frac{1}{M}(F - N - b\dot{x})\cdots\cdots\cdots(39)$$

$$\ddot{\theta} = \frac{1}{I}\sum_{pend} \tau = \frac{1}{I}(-Nl\cos\theta - Pl\sin\theta)\cdots\cdots\cdots(40)$$

Furthermore, the extra $x-$ and $y-$ components equations of 26 and 27 for the pendulum are modelled as shown in equations 41 to 44.

$$m\ddot{x}_p = \sum_{pend} F_x = N \cdots\cdots\cdots(41)$$

$$\Rightarrow N = m\ddot{x}_p\cdots\cdots\cdots(42)$$

$$m\ddot{y}_p = \sum_{pend} F_y = P - mg\cdots\cdots\cdots(43)$$

$$\Rightarrow P = m(\ddot{y}_p + g)\cdots\cdots\cdots(44)$$

The position coordinates of x_p & y_p are the exact function of θ. Similarly, the differential form can be represented in terms of θ. Addressing the $x-$ component equations:

$$x_p = x + l\sin\theta \cdots\cdots\cdots(45)$$

$$\dot{x}_p = \dot{x} + l\dot{\theta}\cos\theta \cdots\cdots\cdots(46)$$

$$\ddot{x}_p =+ l\ddot{\theta} \cos \theta - l\dot{\theta}^2 \sin \theta + \ddot{x} \cdots\cdots\cdots (47)$$

Similarly, for the vertical component equations:

$$y_p =- l \cos \theta \cdots\cdots\cdots (48)$$

$$\dot{y}_p = \dot{\theta} l \sin \theta \cdots\cdots\cdots (49)$$

$$\ddot{y}_p = l\theta \sin \theta + \dot{\theta}^2 l \cos \theta \cdots\cdots\cdots (50)$$

Putting into equations (29) and (31):

$$N = m\big(- \dot{\theta}^2 l \sin \theta + \ddot{x} + \ddot{\theta} l \cos \theta\big) \cdots\cdots\cdots (51)$$

$$P = m\big(\theta l \sin \theta + g + \dot{\theta}^2 l \cos \theta\big) \cdots\cdots\cdots (52)$$

Next, the above equations were modelled with Simulink, as shown in section 2.5.3.

2.5.2   Building the Nonlinear Model with Simulink

Using Simulink, the above equations were modelled into four function blocks (Figure 5).
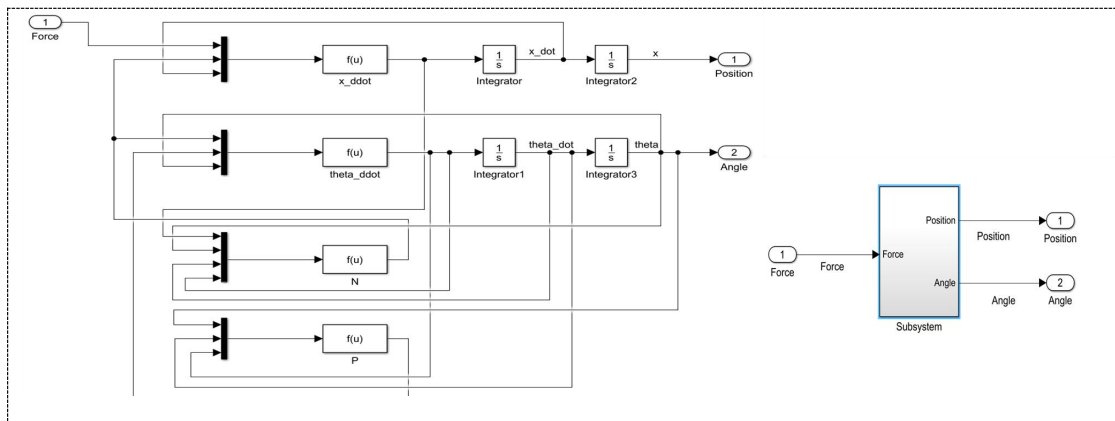
Figure 5: The Non-linear model with Simulink

The initial condition of the inverted pendulum (θ) was set to $pi = \pi$ to signify the inverted pendulum starts directed upwards. The mux block has its number of inputs increased from 2 to 3 to accommodate the set requirements. Finally, a subsystem of the first model shown in Figure 5b was created from that of Figure 5a

2.5.3   Building the nonlinear model with Simscape

The non-linear model was also designed in Simscape, thereby enabling viewing of the animation. The Simscape model was constructed after updating Simscape multibody first Generation 1G add-on in MATLAB. The resulting model and its equivalent subsystem model are as shown in Figure 6.
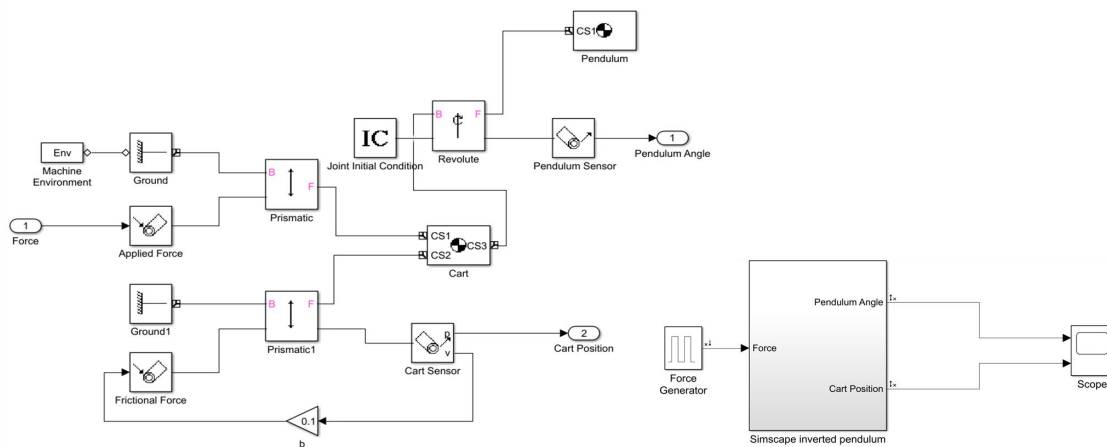

Figure 6: Non-linear model with Simscape

The base block was given the following parameters; mass = 0.5kg, Inertia = eye(3) and 3 ports were enabled. The pendulum block was given the following, mass = 0.2kg, Inertia =0.006. eye(3) with one port allowed. Other blocks were connected as shown in Figure 6a.

2.5.4   *Generating the open-loop response*

An impulse function was generated, and a scope was used to observe the output. The pulse generator block was configured to Amplitude of 1000, pulse width set to 0.01% of the period and period of 10s. Figure 7 shows the connection.
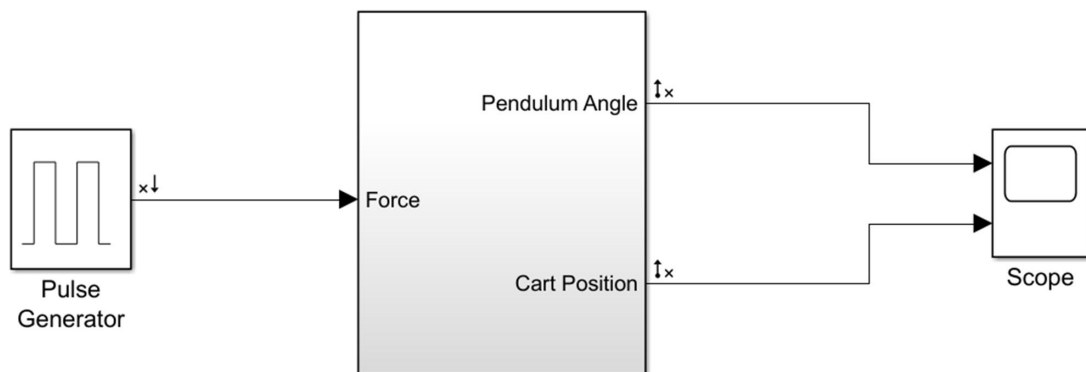
Figure 7: Simscape block diagram

3.   Results /Discussions

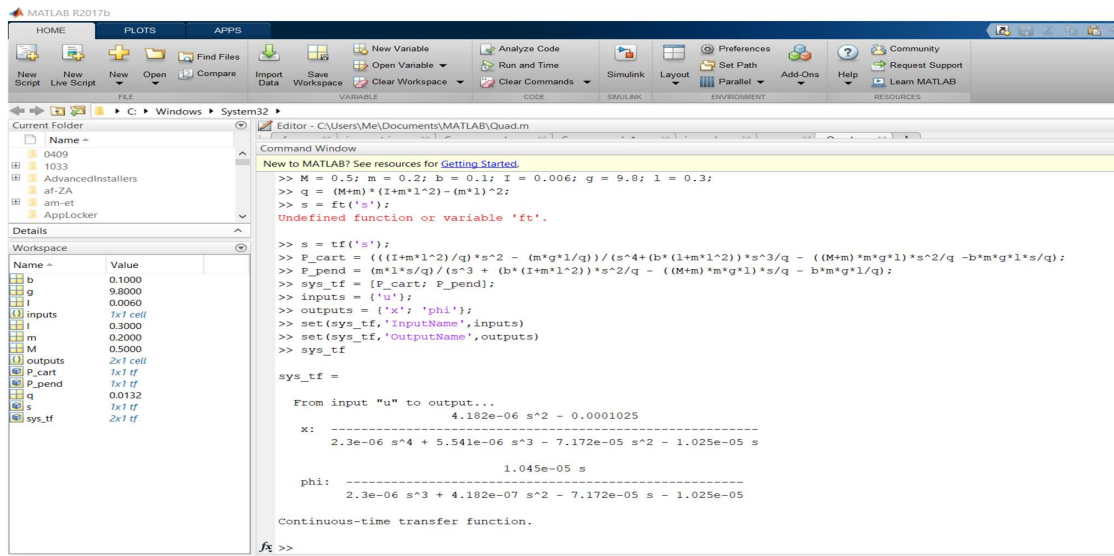The MATLAB code for the system is represented in Figure 8.

Figure 8: MATLAB code for the system

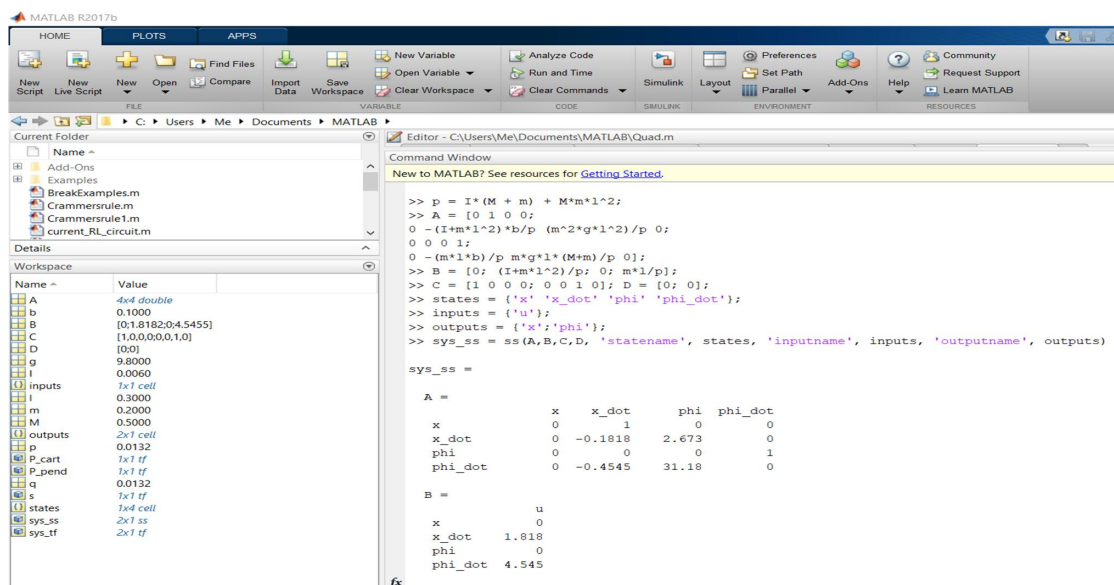The representation of the system in state space is seen in Figure 9

Figure 9: MATLAB code for state-space representation

Figure 10 shows the impulse and step response of the state space model and its corresponding poles, P and zeros, Z.
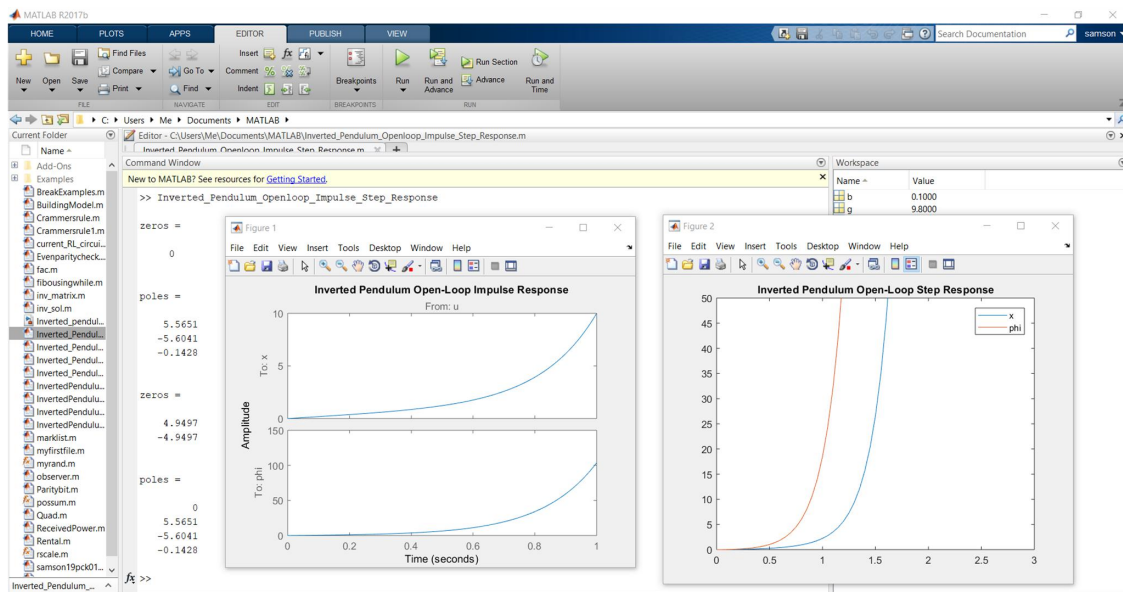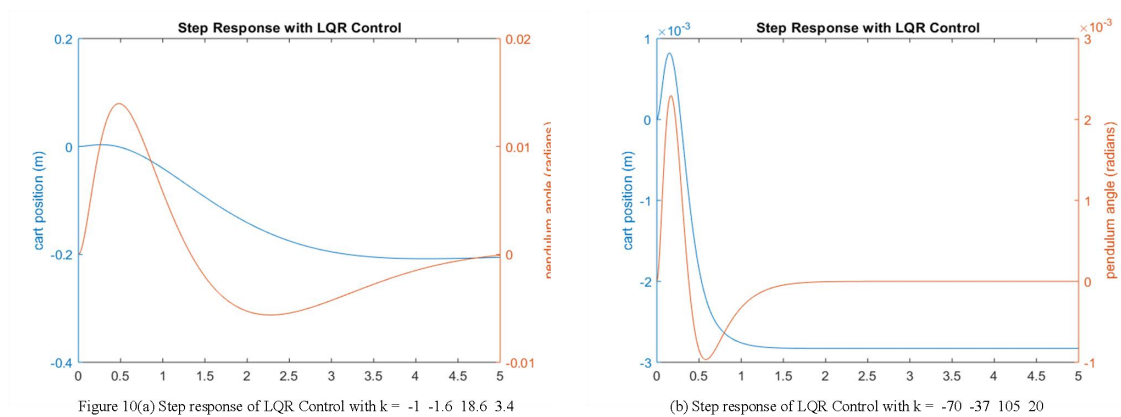

Figure 10: Impulse and step response of the state space model

From Figure 10, one of the poles is positive meaning that the system is naturally unstable. The check for controllability of the system results in a value of 4, indicating that the system is CC. Figure 11 shows the response of the system to a step input.

Figure 10(a) Step response of LQR Control with k = -1 -1.6 18.6 3.4

(b) Step response of LQR Control with k = -70 -37 105 20

(c) Step Response with Precompensation and LQR Control

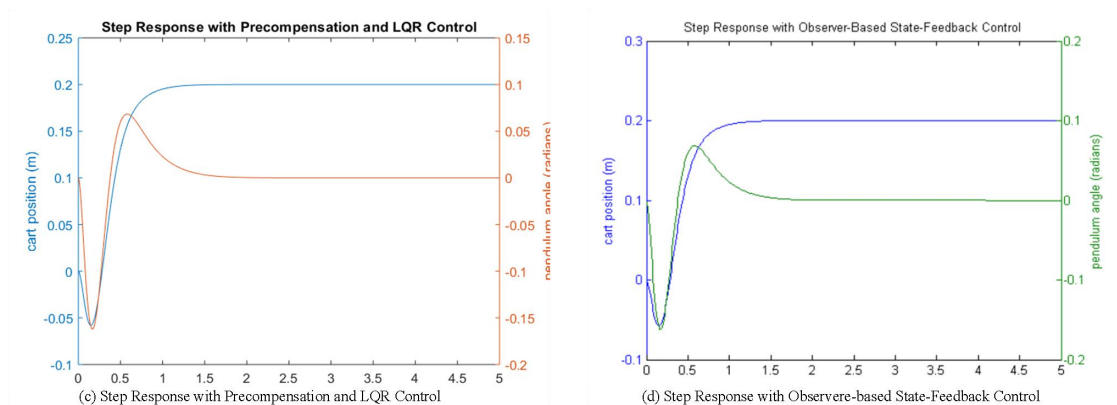(d) Step Response with Observere-based State-Feedback Control

Figure 11: Step Response

The open loop response of the simulation on Simscape and Simulink is shown in Figure 11. As expected, the system is unstable and therefore requires control action in order to achieve stability.
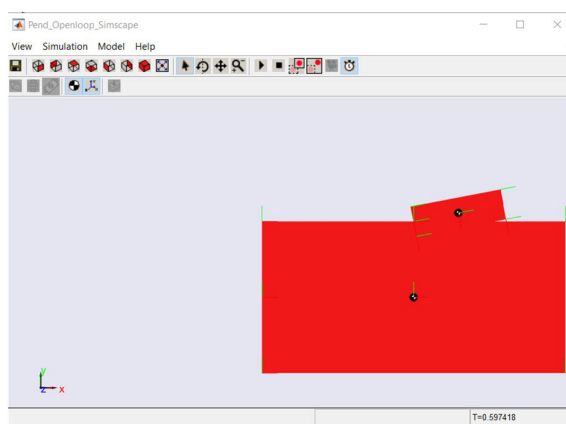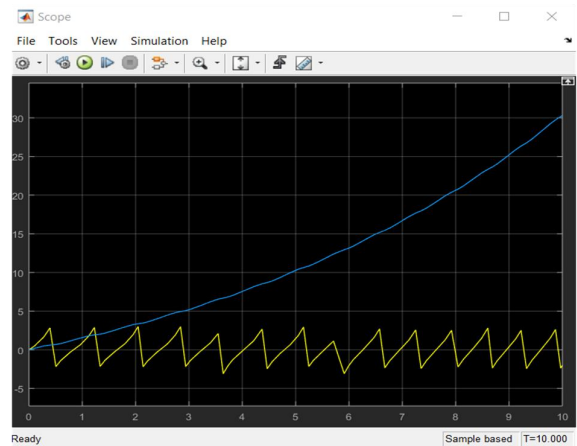

Fig.11(a): Inverted Pendulum Open – loop animation screenshot

(b): Output of Oscilloscope for inverted pendulum open – loop model

The animation of the closed loop simulation is given in Figure 12. The system was stable after the application of the state-space control design.
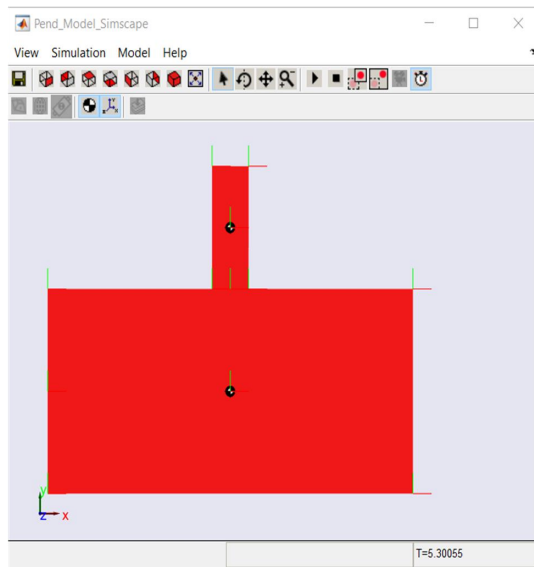
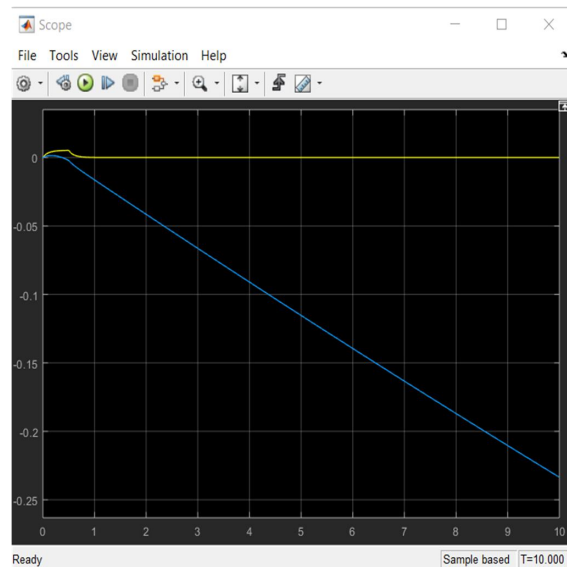Fig 12(a): animation screenshot for inverted pendulum simulation on simulink

Fig 12(b): Oscilloscope output for gains of P = 200, I = 1 and D = 20

4.   Conclusion

In this work, a mathematical model for an inverted pendulum on a moving base was established, which is naturally unstable without control and obtained its transfer function in both time and frequency domain and its state-space representation and implementation in MATLAB. A state-space controller for this generally accepted benchmark of dynamics and control was designed. Using Simulink and Simscape the simulation and animation of the system on a moving base with both open-loop and closed-loop state-space controlled system was carried out. The necessary mathematical relationships were analysed. The design specifications of reducing the time t_s for the base parameter and the pendulum angle to $< 5s$; reducing the rise time for the base position to less than 0.5s; setting the angle of the pendulum to be less than 20^0 and minimising the error, e_{ss} of base parameter and pendulum angle to $< 2\%$ was satisfied. Future work will include control design of the same system on a rotating base.

Acknowledgements

Reference

[1] Boubaker, O., (2013). The inverted pendulum benchmark in nonlinear control theory: A survey. International Journal of Advanced Robotic Systems, vol. 10.

[2] Mathew, N. J., Rao, K. K. & Sivakumaran, N. (2013). Swing up and stabilisation control of a rotary inverted pendulum. IFAC. 10(1).

[3] Bogdanov, A. (2004). Optimal control of a double inverted pendulum on a cart," CSEE, OGI School of Science and Engineering. 1–14.

[4] Chen, W., and Theodomile, N. (2016). Simulation of a Triple Inverted Pendulum Based on Fuzzy Control. World Journal of Engineering and Technology. 04(02). 267–272.

[5] Yi, J., Yubazaki, N., & Hirota, K. (2002). A new fuzzy controller for stabilisation of parallel-type double inverted pendulum system. Fuzzy Sets and Systems. 126(1). 105–119.

[6] Li, H., Miao, Z., & Wang, J. (2002). Variable universe adaptive fuzzy control on the quadruple inverted pendulum. Science in China, Series E: Technological Sciences. 45(2). 213–224.

[7] Chaturvedi, N. A., McClamroch, N. H., & Bernstein, D. S. (2008). Stabilisation of a 3D axially symmetric pendulum. Automatica, 44(9). 2258–2265.

[8] Oguntosin, V., & Akindele, A. (2019). Design of a joint angle measurement system for the rotary joint of a robotic arm using an Incremental Rotary Encoder. Journal of Physics: Conference Series. 1299(1).

[9] Oguntosin, V., Abdulkareem, A. (2020). Hand gesture control and design of a rotary pneumatic soft actuator using leap motion sensor. Int J Intell Robot Appl https://doi.org/10.1007/s41315-020-00140-5