SCR-Apriori for Mining 'Sets of Contrasting Rules'

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Abstract—In this paper, we propose an efficient algorithm for mining novel 'Set of Contrasting Rules'-pattern (SCR-pattern), which consists of several association rules. This pattern is of high interest due to the guaranteed quality of the rules forming it and its ability to discover useful knowledge. However, SCRpattern has no efficient mining algorithm. We propose SCR-Apriori algorithm, which results in the same set of SCR-patterns as the state-of-the-art approache, but is less computationally expensive. We also show experimentally that by incorporating the knowledge about the pattern structure into Apriori algorithm, SCR-Apriori can significantly prune the search space of frequent itemsets to be analysed.

I. INTRODUCTION

Association rules learning is a popular technique in data mining [1]. However, it is known that finding rules of high quality is not always an easy task [2]. This issue is even more significant in domains where the reliability of the obtained knowledge is required to be high (for example, in medicine). Also, association rules mining techniques usually generate a huge number of rules that have to be analysed by a human in order to choose meaningful and useful ones [3].

Recently a novel pattern called 'Sets of Contrasting Rules' was proposed [4]. This pattern was proven to produce rules of high-quality [5]. It was also shown that its structure allows identifying meaningful knowledge, in particular, *local differences* between subgroups of the dataset and *trigger factors*. However, no efficient algorithm for mining SCR-patterns was proposed so far. The state-of-the-art procedure of constructing SCR-patterns performs filtering of association rules obtained by other techniques and chooses those that can form this pattern [6]. Although this approach allows discovering all SCR-patterns in the dataset, it has high level of redundancy, as a vast majority of mined rules is filtered out.

In this paper, we address a question of efficient mining of SCR-patterns. We propose a novel SCR-Apriori algorithm that prunes the search space via incorporating knowledge about the structure of the pattern. The proposed algorithm is a modified version of existing Apriori [7] and CAR-Apriori [8] algorithms and incorporates advantages of both of them.

II. ASSOCIATION PATTERNS

A. Association Rules

The core notion of association patterns mining is *item*. Item is defined as a value of a particular attribute in a given record. A set of items X is referred to as an *itemset* and an *association*

rule is defined as an induction rule of the form $X \to Y$, where X and Y are itemsets and $X \cap Y = \emptyset$ [7]. The left-hand side of the association rule X is called *antecedent*, and its right-hand side Y is called *consequent*.

The support number of an itemset X on the dataset D is defined as a number of records in D that contain X. Similarly, the support number of an association rule $X \to Y$ is defined as a support number of the union $X \cup Y$. Often an alternative measure called support is used. It is defined as the ratio of the support number to the total number of records in the dataset. The ratio of the support of the rule to the support of its antecedent defines the confidence of the rule on a particular dataset, that is $conf_D(X \to Y) =$ $supp_D(X \cup Y) / supp_D(X)$. Support and confidence are two basic quality measures of association rules [2]. Most of association rules mining algorithms search for frequent and confident rules, that is for rules whose support and confidence are above or equal to a user-defined thresholds $(supp_D(X \to Y) \ge$ minSupp and $conf_D(X \to Y) \ge minConf)$.

B. Classification Rules and Contrast Patterns

In many applications, the elements of the dataset are distributed among multiple classes. We restrict ourselves to the case of only two classes Cl_1 and Cl_2 in this work for the sake of simplicity. We also assume that the belonging of an element to a class is specified by the value of a particular attribute, which we refer to as *class attribute* $Att_{Cl} = Cl_k$. If the consequent of an association rule is restricted to be composed of only the class attribute, then such rules are called *classification rules* [9]. These rules can be used to address a very important task in class-labeled data: the task of identification of patterns that explain differences between classes of the dataset, or *classification*.

A group of patterns called *contrast patterns* [10] was proposed as an alternative to classification rules. These patterns were also designed with the aim to understand differences between classes, however they are defined in a different way. An itemset is said to be a contrast pattern if it changes significantly its support from one class to another [11]. This change can be measured in terms of, for example, growth rate: $GrowthRate(X, Cl_1, Cl_2) = supp_{Cl_1}(X)/supp_{Cl_2}(X)$, where $supp_{Cl_k}(X)$ stands for support of itemset X in class Cl_k . Given ρ (with $\rho > 1$) a growth rate threshold, if

 $GrowthRate(X, Cl_1, Cl_2) \ge \rho$, then the itemset X is said to be a ρ -emerging pattern from class Cl_2 to class Cl_1 [12].

Some works suggested that contrast patterns differ meaningfully from itemsets and thus association and classification rules [10]. However, it was shown in [13] that a straightforward application of an existing association rules discovery algorithm can be successfully used to mine contrast patterns. It was also proven analytically in [5] that classification rules and contrast patterns can be considered as equivalent. To be more precise, there exists a functional dependence between the value of the growth rate and the value of the confidence. For every given confidence threshold α , it is possible to fix a growth rate threshold ρ in such a way that all ρ -emerging patterns will also form confident classification rules with $minConf = \alpha$.

C. 'Sets of Contrasting Rules'-Pattern

Contrary to many state-of-the-art patterns, 'Sets of Contrasting Rules'-pattern is composed of multiple rules and is based on notions of invariant and variable attributes [4], [14]. An attribute is said to be *varying* if its value can be changed externally to the system within the specified application task, and *invariant* otherwise. For example, assume that we need to analyse medical data with an attribute drug that specifies medicines taken by a patient, and another attribute age corresponding to his age. If it is possible to change the medication then the attribute drug is varying. At the same time, the value of the parameter age cannot be changed. That is it belongs to the group of invariant attributes. The SCR-pattern is defined as follows.

Definition 1. For a parameter α , a pair of classification rules R1 and R2 is called a pair of α -contrasting rules if:

- 1) $conf(R1) \ge \alpha \& conf(R2) \ge \alpha;$
- 2) R1 and R2 correspond to different classes;
- 3) antecedents of the rules are made up of the same attributes;
- 4) the number of attributes is at least two and at least one of the attributes is varying;
- 5) values of all invariant attributes are the same for both rules;
- 6) if there are no invariant attributes, then the values of at least one varying attribute is the same for both rules;
- at least one varying attribute has different values in the pair.

We say that the rule R2 is a *contrast pair* for the rule R1 and vice versa. An example of a pair of contrasting rules forming an SCR-pattern is given in Figure 1. The antecedents of these rules contain one invariant attribute (A) and two varying attributes (B and C). Sign '/' in the figure is used to visually separate invariant part of the pattern (attributes with same values) and its varying part (attributes with different values).

It was shown that, depending upon application, the varying part of the rule can be interpreted as *local differences* between classes of the dataset [5] or *trigger factors* [4]. The term *trigger factors* is used to reffer to the factors that can stimulate

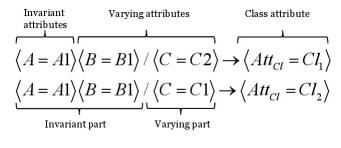


Fig. 1. Example of an SCR-pattern

transfer of data elements from one class to another. It was also analytically proven that rules forming SCR-pattern are guaranteed to be of high-quality [5]. This characteristic can be crucial for certain domains that require only highly confident knowledge (for example, in medicine).

For discovering SCR-patterns authors proposed to perform post-processing of association rules mined by classical algorithms [4]. However, only a very small fraction of discovered rules have contrast pair and thus can be used. Thereby, despite its advantages 'Sets of Contrasting Rules'-pattern has no efficient mining algorithm. In this paper we aim to solve this problem and to propose an efficient solution.

III. MINING ASSOCIATION PATTERNS

Agrawal et al. in [7] introduced the problem of association rules mining as the problem of mining all association rules that satisfy user-specified minimum support minSupp and minimum confidence minConf conditions. This task is usually divided into two steps [1]:

- 1) mining all frequent itemsets,
- 2) forming confident rules of these itemsets.

Forming confident rules of frequent itemsets is straightforward and most of the algorithms concentrate on the first task. In contrast, the task of constructing all frequent itemsets is considered to be computationally expensive [15], [16]. It is also known that its complexity grows exponentially with the number of attributes in the dataset. However, the search space of all possible itemsets can be effectively pruned basing on the downward-closure property: *all subsets of a frequent itemset are themselves frequent itemsets* [7]. It means that every nonfrequent itemset cannot be a part of any larger frequent itemset and it is possible to find a border that separates frequent and non-frequent itemsets. Thereby, the first task simply comes to searching for this border [17], [16].

The search through the set of items can be done in depthfirst or in width-first manners [18]. Both approaches result in the same set of frequent itemsets [19]. The width-first strategy forms the basis of multiple algorithms. Among them, there is Apriori algorithm proposed by Agrawal and Srikant [9]. The latter one is very popular and proved its efficiency in a great variety of applications.

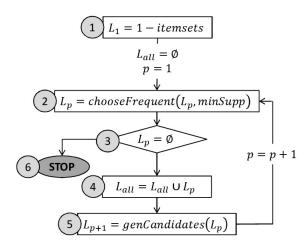


Fig. 2. Apriori: mining frequent itemsets

A. Apriori: Mining Frequent Itemsets

Let us examine the procedure of constructing frequent itemsets used in Apriori algorithm (see Figure 2). An itemset containing exactly p attributes is referred to as p-itemset and set L_p stands for a set of all *p*-itemsets. The procedure starts with the initialisation of L_1 with the set of all possible 1itemsets (all values of each attribute), see step 1. After that, all non-frequent itemsets are excluded from L_1 . This is done via comparing their support with a defined minSupp value (corresponds to step 2). If L_1 contains frequent itemsets (step 3), then these itemsets are added to the list of all frequent itemsets L_{all} (step 4). Finally, the set of all 2-itemsets L_2 is constructed on the step 5. From the downward-closure property, it follows that a 2-itemset is not frequent if at least one its subsets does not belong to the set of frequent 1-itemsets (L_1) . Thereby, the set L_2 can be constructed from the set L_1 through the operation of self-joining (see [9] for more details). This allows reducing the number of 2-itemset to be considered by eliminating all those, that are known to be non-frequent. Afterwards, the procedure restarts from the step 2 processing the set L_2 in a similar way. The procedure finishes when a newly generated on step 5 and filtered on step 2 set L_p is empty. The resulting set L_{all} consists of all frequent itemsets of different sizes.

Finally, all possible confident association rules are constructed by exploiting the set of generated frequent itemsets. Suppose we have a frequent itemset c and its non-empty subset β . Then an association rule is constructed as follows: $(c - \beta) \rightarrow \beta$. If the resulting rule meets the *minConf* condition it is added to the list of confident rules.

B. CAR-Apriori: Mining Frequent Ruleitems

Apriori algorithm can be used for the identification of classification rules through post-processing of the obtained association rules: we choose only those rules, whose consequents are composed of only class attribute. However, computational costs can be reduced if the search of classification rules is incorporated into Apriori algorithm. Such an algorithm, CAR-Apriori, was proposed in [8]. This algorithm is based on a new concept: *ruleitem*. A ruleitem is a construction of the form $condset < Att_{Cl} = Cl_k >$. A *condset* is essentially an itemset, and expression $< Att_{Cl} = Cl_k >$ specifies the value of the class attribute. The difference between a condset and an itemset is that the latter one can contain class attributes Att_{Cl} while the former one no.

The support of a ruleitem is defined as a ratio of the number of elements containing the specified condset and belonging to the class Cl_k to the number of all elements in the dataset. Each ruleitem essentially represents a rule of the form $condset \rightarrow$ $Att_{Cl} = Cl_k$ with the support equal to the support of the ruleitem and the confidence calculated as a fraction of the support of the ruleitem to the support of the corresponding condset.

The algorithm of mining classification rules is the same as Apriori with the difference that instead of frequent itemsets frequent ruleitems are mined. Also, classification rules are formed directly from ruleitems. Both Apriori and CAR-Apriori result in the same set of classification rules. However, the incorporation of class-specific information that is done in CAR-Apriori allows to speed-up the process.

IV. SCR-Apriori: Mining Frequent and Contrast SCR-Ruleitems

A. Intuition

Similar to using Apriori for mining classification rules, it is possible to use CAR-Apriori for mining 'Sets of Contrasting Rules'-pattern via filtering out all classification rules that have no contrast pair. However, this solution is not efficient.

We propose to further extend the idea of CAR-Apriori algorithm for direct mining of ruleitems that can potentially form SCR-pattern. Apart of excluding all non-frequent itemsets (as it is done in Apriori) and all itemsets that cannot form classification rules (as it is done in CAR-Apriori), we propose to further exclude those itemsets that cannot be used for constructing SCR-patterns, even if they can form frequent and confident classification rules on their own.

Recall that an SCR-pattern consists of two contrasting rules. Thereby, if for a certain frequent ruleitem there is no other frequent ruleitem that can potentially form a pair of contrast classification rules with the first ruleitem, then it can be excluded from the search space regardless of its frequency. In order to fulfil this task, we propose a new algorithm SCR-Apriori. This algorithm aims at mining all frequent and contrast *SCR-ruleitems* and forms SCR-patterns of them.

B. SCR-Apriori Algorithm

We define a *SCR-ruleitem* as follows: $condset < supp_{Cl_1}, supp_{Cl_2} >$, where $supp_{Cl_k}$ stands for support of the condset on the class Cl_k . Essentially an *SCR-ruleitem* is a condset accompanied with its support values in all classes defined on the dataset *D*. We also define a *contrast pair* for an *SCR-ruleitem* in a similar way as we

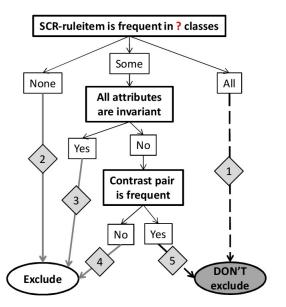


Fig. 3. Function for choosing frequent and contrast SCR-ruleitems

defined a contrast pair for a rule (see Definition 1) with the following differences:

- SCR-ruleitems in the pair can be composed of only one attribute that can be either invariant or varying (compare with condition 4 in Definition 1);
- all varying attributes can have different values even if there are no invariant attributes in the condsets of the considered SCR-ruleitems (compare with condition 6).

The procedure of mining frequent and contrast SCRruleitems is very similar to the procedure of mining frequent itemsets and frequent ruleitems presented in Figure 2. The only difference is in **step 2**. On this step SCR-ruleitems are filtered out from the set L_p not only depending on the value of their support on different classes but also on the fact if a given SCR-ruleitem has a *contrast pair* frequent on the opposite class. Considering this, the function *chooseFrequent* in the algorithm in Figure 2 should be replaced with the function *chooseFrequentAndContrast*. This function decides whether to exclude or not an SCR-ruleitem from L_p according to the algorithm given in Figure 3.

We analyse the structure and visualise different branches of the proposed algorithm using two examples. We utilize 2 sets of records consisting of 16 and 14 elements respectively (see Figure 4). All records are defined on 3 attributes A, B, and C. Each of these attributes has 2 possible values which we distinguish by numbers, e.g. the possible values of the first attribute are A1 and A2. Among these 3 attributes the first one (A) is invariant and the rest of the attributes (B and C) are varying. There are 2 classes Cl_1 and Cl_2 defined on each set of records. Records belonging to the first class are in **bold** while records belonging to the second class are wine-red.

Figures 5a and 5b show all SCR-ruleitems for the first (a) and the second (b) examples respectively. Each square

Example 1							
1	A1,B1,C1	5	A1,B1,C2	9	A1,B2,C1	13	A2,B1,C2
2	A1,B1,C1	6	A1,B2,C1	10	A1,B2,C1	14	A2,B1,C2
3	A1,B1,C1	7	A1,B2,C1	11	A1,B2,C2	15	A2,B2,C1
4	A1,B1,C2	8	A1,B2,C1	12	A2,B1,C2	16	A2,B2,C1
Example 2							
1	A1,B1,C1	5	A1,B2,C1	9	A2,B1,C1	13	A2,B2,C2
2	A1,B1,C1	6	A1,B2,C1	10	A2,B2,C2	14	A2,B2,C2
3	A1,B1,C1	7	A1,B2,C1	11	A2,B2,C2		
4	A1,B2,C1	8	A2,B1,C1	9	A2,B2,C2		

Fig. 4. List of records for Example 1 and Example 2

represents a particular SCR-ruleitem. For the sake of simplicity we present not the support of a condset on each class, but its support number. For both examples, we fix the minimum support number threshold equal to 2.

The fact that a particular condset is frequent in Cl_1 or Cl_2 is visualised with a think border or a wine-red background color respectively. A particular SCR-ruleitem having both a thick border and a wine-red background is frequent in both classes. A dashed border means that a particular SCR-ruleitem was not constructed by our algorithm due to exclusion of one or more of its subsets by by our algorithm.

The shape of links between SCR-ruleitems also reflects their frequency. In particular, a link up-coming from an SCRruleitem is thick or/and of wine-red color, if this SCR-ruleitem is frequent on the first or/and on the second class. Additionally, an up-coming link is dashed if the SCR-ruleitem was marked by our algorithm as the one that cannot form an SCR-pattern.

Let us now analyse the algorithm given in Figure 3. First, this algorithm examines the values of support of an SCRruleitem in all classes defined on the dataset.

Branch 1. If it is frequent on all classes then it can form an SCR-pattern even if it has no frequent contrast pair. Indeed, let us have a look on SCR-ruleitem A1C1<4, 5> in Figure 5-a. There is only one SCR-ruleitem that can form a contrast pair for it: A1C2<1, 1> and it is not frequent on any of 2 classes. However, we can form SCR-ruleitems A1B1C1<1, 2> and A1B2C1<3, 3> which will result in SCR-pattern $\{A1C1/B1 \rightarrow Att_{Cl} = Cl_2, conf = 0.67 :$ $A1C1/B2 \rightarrow Att_{Cl} = Cl_1, conf = 0.5\}$. Thereby, an SCRruleitem that is frequent on all classes should not be excluded.

Branch 2 of the algorithm corresponds to the case when the SCR-ruleitem is not frequent on any of the classes and suggests to exclude it. Indeed, if the condset of the SCRruleitem is not frequent, then no frequent rules can be formed neither from this condset itself, nor from its super-sets. And, thereby, no SCR-patterns can be produced (see for example SCR-ruleitems A1C2<1, 1> and B2C2<0, 0> in Figure 5a or A1C2<0, 0> and B1C2<0, 0> in Figure 5b).

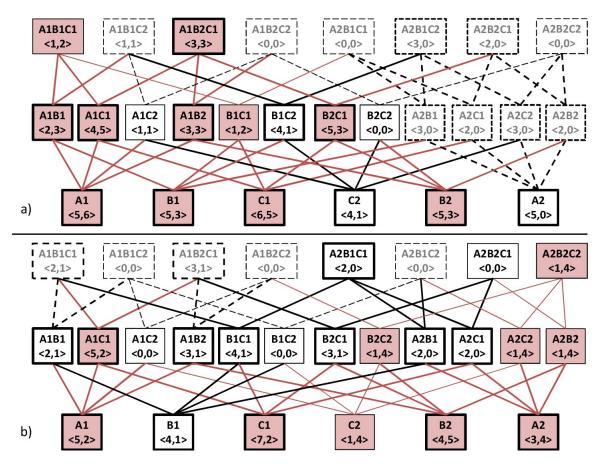


Fig. 5. Contrast ruleitems for Example 1 (a) and Example 2 (b)

If the SCR-ruleitem is frequent on only a fraction of classes, then the nature of its attributes should be considered.

Branch 3. Let us consider the case when all attributes forming SCR-ruleitem are invariant, for example SCR-ruleitem A2 < 5, 0 > in Figure 5a. Both rules from an SCR-pattern should have the same values of all invariant attributes. It means that it is impossible to construct an SCR-pattern in this case as some components of the rules forming this SCR-pattern will be frequent on only one class. In our example A2 is frequent only in the first class.

Finally, if the condset of the SCR-ruleitem consists of either varying or both varying and invariant attributes then the frequency of its contrast pairs on the opposite class is checked.

Branch 4 considers the case when SCR-ruleitem is frequent in one class but has no contrast pair frequent on another class. Let us consider as an example SCR-ruleitem A1B1<2, 1>in Figure 5b. The only contrast pair for this SCR-ruleitem is A1B2<3, 1>. No SCR-patterns can be produced of this pair as no frequent classification rule can be formed for the second class. From the downward-closure property it also follows that all supersets of A1B1<2, 1> will not be frequent on the second class as well, for example, A1B1C1<2, 1>. Thereby, supersets of the considered SCR-ruleitem can potentially form an SCR-pattern only with those SCR-ruleitems that have different values of at least one invariant attribute from the considered SCR-ruleitem. They have to be also frequent on the other class. However, each of such SCR-ruleitems has a subset that forms a contrast pair with the considered SCRruleitem. But we know that all of them are non-frequent on the other class. This means that no SCR-patterns can be formed from the supersets as well. In the considered example, SCR-ruleitem A1B1 < 2, 1 > has only one invariant attribute B with the value B1. Thereby, A1B1C1 < 2, 1 > as a superset of A1B1 < 2, 1 > can potentially form SCR-pattern with either A1B2C1 < 3, 1 > or A1B2C2 < 0, 0 >. However, as we know that A1B2 < 3, 1 > is not frequent on the second class, we can conclude the same about its supersets without calculating their support explicitly. The same reasoning is valid for another superset of A1B1 < 2, 1>, namely for A1B1C2 < 0, 0>.

Finally, **branch 5** of our algorithm considers the case of partially frequent SCR-ruleitems that have contrast pairs. We can show on an example that in this case it is possible to construct SCR-patterns and thus such SCR-ruleitems should not be excluded. Let us consider B1C1<4, 1> in Figure 5b which is frequent on the first class. Its contrast pair B2C2<1, 4> is frequent on the second class. Using the supersets of these

SCR-ruleitems we can construct SCR-pattern $\{A2/B1C1 \rightarrow Att_{Cl} = Cl_1, conf = 1 : A2/B2C2 \rightarrow Att_{Cl} = Cl_2, conf = 0.8\}$. Note, that SCR-ruleitem C2<4, 1> in Figure 5a is considered in a similar way. It has a contrast pair C1<6, 5> that is frequent on the second class. And it is also possible to construct SCR-pattern of their supersets B1C1<1, 2> and B1C2<4, 1>.

Using the algorithm given in Figure 3 it is possible to perform similar analysis for all SCR-ruleitems in Figure 5 and choose those that satisfy conditions of frequency and contrast. Note that, for example, SCR-ruleitem A2B1C2<3, 0> in Figure 5a is not considered by our algorithm even though it can form a frequent and confident association rule. No SCR-patterns can be formed of this SCR-ruleitem and it is excluded from the consideration because its subset A2 does not satisfy the conditions of our algorithm (branch 3).

When all frequent and contrast SCR-ruleitems are discovered, we can construct SCR-patterns directly using SCRruleitems and their contrast pairs. Note, however, that not all constructed SCR-ruleitems will form SCR-patterns. For example, if the parameter α is set to $\alpha = 0.6$, no SCR-patterns can be produced from a pair of SCR-ruleitems A1B1C1<1, 2>and A1B2C1<3, 3> in Figure 5a.

C. Performance evaluation

In order to quantify the gain in performance of SCR-Apriori as compared to CAR-Apriori we used the same dataset as in state-of-the-art works [4], [6], [14]: 5-percent sample of the California census dataset for the year 2000¹. We also performed the same preprocessing steps as in the cited papers. After that, we generated SCR-patterns with SCR-Apriori and via post-filtering of classification rules obtained with CAR-Apriori, as suggested in the state-of-the-art.

We mined SCR-patterns with minConf = 0.5 and minSupp = 0.07. The value of support is chosen to be so small in accordance with previous works [6], [14] and with the statement that contrast patterns with small support can be of particular interest [14].

We obtained the same set of SCR-patterns with both approaches. However, the number of mined SCR-ruleitems corresponds to 58% of the number of frequent ruleitems identified with CAR-Apriori. Also, the number of rules generated by SCR-Apriori corresponds to only 6.4% of the number of classification rules obtained with the state-of-the-art procedure. Thereby, the proposed algorithm allows obtaining the same results but with the substantial decrease of computation costs.

V. CONCLUSIONS

In this paper, we proposed a novel SCR-Appriori algorithm for direct mining of 'Sets of Contrasting Rules'-pattern. Through experimental results on real dataset, we showed that SCR-Apriori allows decreasing significantly computational costs, as compared to the state-of-the-art approach.

The proposed algorithm incorporates the knowledge about the structure of SCR-pattern into the mining process and is essentially a modified version of known Apriori and CAR-Apriori algorithms.

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¹https://www.census.gov/prod/cen2000/doc/pums.pdf