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ON THE CHOICE OF PRESTRESSING PARAMETERS

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Abstract: This paper focuses on the regulation of forces in a statically indeterminate system under the action of many loads. This regulation is realized by prestressing. The paper compares two proposed methods for selecting rational values of the prestressing parameters: maximizing the minimum bearing capacity margin for the elements of the system (1) and equalizing the margins for all elements (2). An illustrative example is provided.

Keywords: prestressing, bearing capacity margins, Chebyshev solution, margin equalization

О ВЫБОРЕ ПАРАМЕТРОВ ПРЕДНАПРЯЖЕНИЯ

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Аннотация: Рассматривается задача о регулировании усилий в статически неопределимой системе находящейся под воздействием многих нагружений. Регулирование выполняется путем создания предварительного напряжения. Сопоставляются два предлагаемых метода для выбора рациональных значений параметров преднапряжения: максимизация минимального по элементам системы запаса несущей способности (1) и выравнивание запасов по всем элементам (2). Приведен иллюстративный пример.

Ключевые слова: преднапряжение, запасы несущей способности, чебышевское решение, выравнивание запасов

1. INTRODUCTION

One of the effective ways to improve a design is prestressing, which regulates the internal forces in the system. Many works on structural optimization consider prestressing forces as design parameters, along with the cross-sectional dimensions of the structural members [6, 7, 8].
However, such a problem formulation is not the only possible one; there is often a problem of choosing prestressing parameters for a structure with known dimensions, which will not be changed unless absolutely necessary. For example, this situation is typical when analyzing existing structures under changed loading conditions (e.g. during reconstruction). There are many other cases when it is necessary to adjust internal forces in a structure (equalizing moments in continuous beams and stiffening girders of cable-stayed bridges [3, 6], adjustment of cable-stayed structures [9] and others [4]).

2. PROBLEM FORMULATION

We will assume that an internal force envelope diagram is obtained as a result of the analysis of an unstressed system for all the load cases.
With a known internal force envelope diagram we will determine the prestressing parameters, which make it possible, in a way, to improve the distribution of internal forces in the system (e.g. to expand the elastic deformation area, or to reveal the bearing capacity margins, improve the operating mode of the structure, etc.).
Let us show that this problem can be solved using optimization methods. To do this, consider the

expressions for true extreme forces (stresses) in an element (section) of the elastic system

$$S_i^+ = S_i^{prest} + S_i^{max};$$
$$S_i^- = S_i^{prest} + S_i^{min} \quad (i = 1, 2, ..., m) \tag{1}$$

Here S_i^{max}, S_i^{min} are the maximum and minimum internal forces in the *i*-th element, obtained as a result of a standard analysis of an elastic *n*-times statically indeterminate system, taking into account the deformation compatibility conditions and possible unfavorable load combinations. The calculated values $S_{i,0}^{max}$ and $S_{i,0}^{min}$ are corrected as follows (Fig. 1).

$$S_i^{\max} = \max\left(0, S_{i,0}^{\max}\right),$$
$$S_i^{\min} = \min\left(0, S_{i,0}^{\min}\right). \tag{2}$$

This ensures that values S_i^{max} are positive and S_i^{min}. are negative.

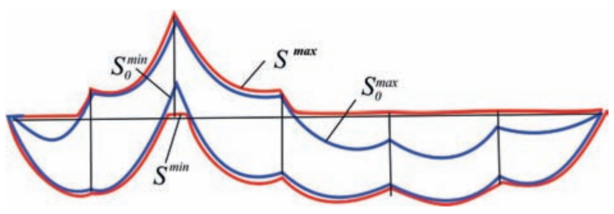

Figure 1.

The prestressing force is determined by the following expression

$$S_i^{prestr} = \sum_{j=1}^{n} s_{ij} x_j \quad (i = 1, 2, ..., m) \tag{3}$$

where s_{ij} is the force in the *i*-th element of the system from the action $x_j = 1$ of the *j*-th prestressing parameter (unknown of the force method).

If the values of tensile R_i^+ and compressive R_i^- bearing capacity are known, then the bearing capacity conditions are written as

$$S_i^+ \leq R_i^+, S_i^- \geq -R_i^+ \quad (i = 1, ..., m), \tag{4}$$

or as system of two-sided inequalities

$$-R_i^- - S_i^{min} \leq \sum_{j=1}^{n} s_{ij} x_j \leq R_i^+ - S_i^{max}$$
$$(i = 1, 2, ..., m). \tag{5}$$

System (5) determines the feasibility of the design. If it is consistent, i.e. there are values $x_j \left(j = 1, 2, ..., n \right)$ which satisfy inequalities (5), then they can be selected as prestressing parameters. If it is inconsistent, we have to set other values of R_i^+ and R_i^-.

It should be noted that if a system is made of an ideal elastoplastic material, and the inequalities (5) are consistent, it will exhibit purely elastic behavior after a certain number of plastic deformation cycles for all possible changes in the live load, i.e. the system will be adaptable. This conclusion follows directly from the Bleich-Melan adaptability theorem.

It means that in cases where the physical realization of an optimal elastic system can be achieved with the help of prestressing, it is always possible to design an optimal elastoplastic system that adapts to a given load program, and there is no need for the artificial regulation of forces.

Not to be bounded by the values of the bearing capacity of the truss members, assuming that the bearing capacity of each member is equal to the extreme force possible for it, i.e. consider the so-called fully stressed structure [5, p. 78], where every part is stressed to the maximum permissible stress at least under one of the possible load combinations, we will consider the following conditions instead of (5)

$$-S_i^{min} \leq \sum_{j=1}^{n} s_{ij} x_j \leq S_i^{max} \quad (i = 1, 2, ..., m), \tag{6}$$

that is, our goal is to find the prestress that should reduce the internal force variation range calculated without its effect.

When the system (6) is consistent, there is an infinite set of solutions which forms a domain Ω in the space of values $\mathbf{x} = \left(x_1, \ldots, x_n\right)$. You can choose from this set the values of the prestressing parameters which satisfy some predetermined conditions. Let us consider some of the possible options.

3. OPTIONS

Bearing capacity margin maximization.

We will assume that, all other things being equal, prestressing ensuring the maximum bearing capacity margin of the system will be the best. Since the values

$$
\begin{aligned}
f_i\left(\mathbf{x}\right) &= S_i^{max} - \sum_{j=1}^{n} s_{ij} x_j \quad \left(i = 1, 2, \ldots, m\right); \\
f_i\left(\mathbf{x}\right) &= S_i^{min} + \sum_{j=1}^{n} s_{ij} x_j \quad \left(i = m+1, \ldots, 2m\right)
\end{aligned} \tag{7}
$$

characterize these margins for all elements (sections) of the system, it is advisable to look for such a vector $\mathbf{x}^* = \left(x_1^*, \ldots, x_n^*\right)$, for which the following condition is satisfied

$$
L\left(\mathbf{x}^*\right) = \max_{x} \min_{1 \le i \le 2m} f_i\left(\mathbf{x}\right), \tag{8}
$$

i.e. the minimum bearing capacity margin for the elements of the system is maximized. Determining the value $L(\mathbf{x})$ from the condition (8) with limitations (6) is the problem of finding the Chebyshev point of a system of linear inequalities. This problem can also be solved as the following linear programming problem [2]:

find the maximum of a linear form

$$
z = x_{n+1} \tag{9}
$$

with limitations

$$
\begin{aligned}
f_i\left(\mathbf{x}\right) &= S_i^{max} - \sum_{j=1}^{n} s_{ij} x_j + x_{n+1} \\
&\left(i = 1, 2, \ldots, m\right); \\
f_i\left(\mathbf{x}\right) &= S_i^{min} + \sum_{j=1}^{n} s_{ij} x_j + x_{n+1} \\
&\left(i = m+1, \ldots, 2m\right)
\end{aligned} \tag{10}
$$

If we denote the Chebyshev solution of the inconsistent system as $X^* = \left(x_1^*, \ldots, x_n^*\right)$ then by creating the corresponding prestress in the system, we can obtain the following values of the required bearing capacity parameters:

$$
\begin{aligned}
\tilde{R}_i &= S_i^{max} - \sum_{j=1}^{n} s_{ij} x_j^*, \\
\tilde{R}_i &= S_i^{min} + \sum_{j=1}^{n} s_{ij} x_j^* \quad \left(i = 1, 2, \ldots, m\right)
\end{aligned} \tag{11}
$$

Strength margin equalization.

Strength margin of a complex multi-element system is often determined by forces in only a few design members, while other members have much larger bearing capacity margins. Therefore, you might want to use prestressing to obtain a system with uniform margins [1].

The consistency condition for the system of inequalities (3), which can be written as follows

$$
f_i\left(x_1, \ldots, x_n\right) \ge 0 \quad \left(i = 1, \ldots, 2m\right), \tag{12}
$$

indicates that there are interior points in the domain Ω defined by the inequalities (10) if at least one of these inequalities is strict. We will further proceed from this assumption.

Following [1], we consider an auxiliary function in the form of the product of the bearing capacity margins

$$
P = f_1 \cdot f_2 \cdots f_{2m}. \tag{13}
$$

The function P is smooth and takes positive values at all interior points of the domain Ω, and vanishes at the boundary of this domain, since here at least one of the functions (7) is equal to zero. Hence the smooth function P reaches its maximum at an interior point of the domain Ω.

To show that this maximum is realized at a single point M^* and, therefore, the local maximum coincides with the global one, consider the logarithm of P.

$$L = \ln P = \sum_{i=1}^{2m} \ln f_i . \tag{14}$$

The function L is negatively defined and concave in the domain Ω, as evidenced by the analysis of a matrix of the second partial derivatives

$$\frac{\partial^2 L}{\partial x_i \partial x_k} = \sum_{j=1}^{2m} \frac{1}{f_j} \cdot \frac{\partial^2 f_j}{\partial x_i \partial x_k} - \sum_{j=1}^{2m} \frac{\partial f_j}{\partial x_i} \cdot \frac{1}{f_j^2} \cdot \frac{\partial f_j}{\partial x_k} \quad (i,k = 1,\dots,n) \tag{15}$$

Taking into account that

$$f_j = \left(\sum_{s=1}^{n} a_{js} x_s + b_j \right), \tag{16}$$

we obtain

$$\frac{\partial^2 L}{\partial x_i \partial x_k} = -\sum_{j=1}^{2m} \cdot \frac{a_{ji} a_{jk}}{\left(\sum_{s=1}^{n} a_{js} x_s + b_j \right)^2} \quad (i,k = 1,\dots,n) \tag{17}$$

The negative definiteness of a matrix with such coefficients follows from the conditions of linear independence of functions (10).

The solution corresponding to the point M^*, located as far as possible from the boundaries of the permissible domain, and its deviations from the boundaries determine a balanced system of strength margins.

4. NUMERICAL EXAMPLE

As an illustrative example, consider a simple system of four bars with the same tension-compression stiffness shown in Fig. 2. It can be subjected to one of the three independent loads at a time: P_1=10 t, P_2=10 t and P_3=10 t.

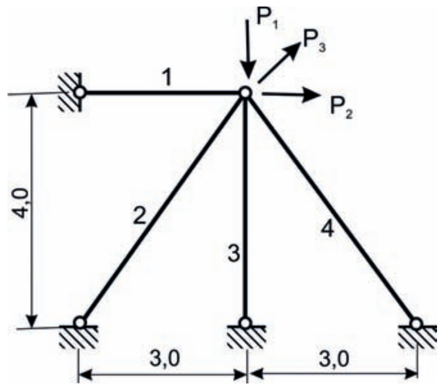

Figure 2.

Forces in the 1st and 3rd bars are used as the prestressing parameters of this twice statically indeterminate system. Force values obtained as a result of the static analysis are given in Table 1

Table 1

Bar	Forces from loads:			Extreme		Prestressing	
J	P_1	P_2	P_3	S^{max}	S^{min}	x_1=1	x_2=1
1	0.000	6.983	5.382	6.983	0.000	1.000	0.000
2	-3.162	2.514	4.375	4.375	3.808	-0.707	-0.707
3	-4.941	0.000	3.808	-3.162	-4.941	0.000	1.000
4	-3.162	-2.514	0.499	0.499	-3.162	0.707	-0.707

Inequalities of type (5) for this system have the form

$0 \le x_1 \le 6.983$,

$3.808 \le -0.707x_1 - 0.707x_2 \le 4.375$,

$-4.941 \le x_2 \le -3.162$,

$-3.162 \le 0.707x_1 - 0.707x_2 \le 0.499$.

Their graphical representation is shown in Fig. 3. As can be seen from Fig. 3, only four limitations shown with a bold line are active.

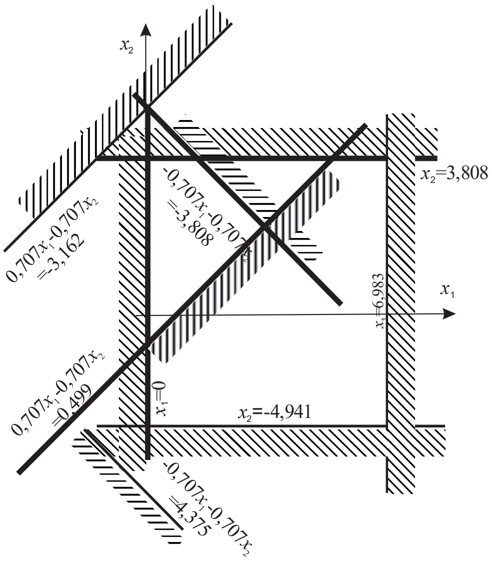

Figure 3

These limitations can be expressed in the form of the following inequalities:

$$x_1 \geq 0$$
$$-3.162 \leq -0.707x_1 - 0.707x_2$$
$$x_2 \leq 3.808$$
$$0.707x_1 - 0.707x_2 \leq 0.499$$

We will further consider only these limitations, although it should be noted that in practical problems it is impossible to discard inactive limitations in advance and, as a result, the amount of computation increases significantly.

It should also be noted, however, that using only active limitations has no effect on the calculation results, since the unaccounted values of the bearing capacity margins $f_j(\mathbf{x})$ a priori exceed the considered values.

Both solutions can be illustrated graphically for the considered problem with two unknowns. The solution to the Chebyshev point problem is shown in Fig. 4.*a*, where the lines of the function level are shown by the dotted line

$$L(\mathbf{x}) = \min[f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), f_3(\mathbf{x})].$$

The solution to the equal margin problem is shown in Fig. 4.*b*, where the lines of the function level are shown by the dotted line

$$P(\mathbf{x}) = f_1(\mathbf{x}) \times f_2(\mathbf{x}) \times f_3(\mathbf{x}) \times f_4(\mathbf{x}).$$

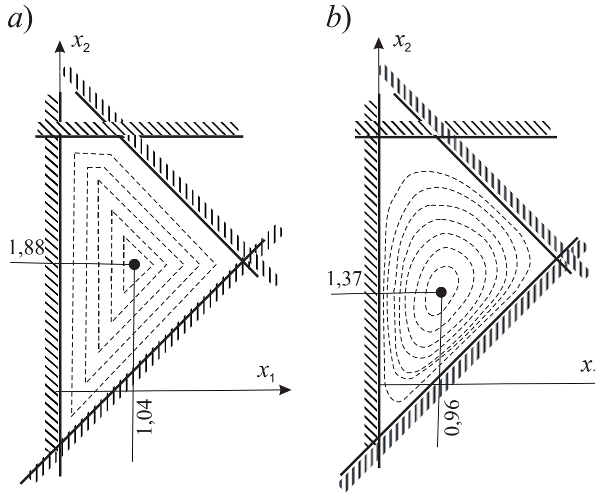

Figure 4

Table 2 provides forces in the bars adjusted by prestressing for the Chebyshev point problem. And while for a system without prestressing with bars of the same cross-section it was necessary at least to ensure the following values $R^+ = 6.983$ and $R^- = 4.941$, for a prestressed system we have $R^+ = 6.44$ and $R^- = 6.82$.

Table 2

j	P_1	P_2	P_3	S^{max}	S^{min}
1	-1.04	5.94	4.34	5.94	-1.04
2	-1.10	4.58	6.44	6.44	-1.10
3	-6.82	-1.88	1.93	1.93	-6.82
4	-2.57	-1.92	1.09	1.09	-2.57

Table 3 provides the adjusted force values for the equal margin problem. Here we have the minimum possible values of the bearing capacity parameters R+ = 6.02 and R– = 2.87.

Table 3

j	P_1	P_2	P_3	S^{max}	S^{min}
1	-0.96	6.02	4.42	6.02	-0.96
2	-1.51	4.16	6.02	6.02	-1.51
3	-6.31	-1.37	2.44	2.44	-1.37
4	-2.87	-2.22	0.79	0.79	-2.87

As you can see, in this example ensuring uniform bearing capacity margins is more advantageous in terms of the weight of the structure.

There can be other relationships between the considered solutions as well. The choice between them depends on many factors and is informal.

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