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1 Introduction

A substantial amount of research in macroeconomics focuses on the propagation of aggregate shocks via financial factors and their relation to individual firm characteristics. In a seminal work on this topic, Gertler & Gilchrist (1994) propose firm size as an effective proxy for financial constraints. Smaller firms are arguably more risky, less liquid and face an elevated external finance premium. Accordingly, smaller firms are more sensitive to aggregate shocks, as they tend to be in a weaker financial position. Standard heterogeneous firms models in the literature substantiate this presupposition, generating a strong correlation between firm size and its financial situation.

This paper provides new empirical evidence that casts doubt on a strong association between size and financial constraints. Using the Bank of Portugal's confidential credit registry database, matched with bank and firm balance sheet data between 2006 and 2017, we are able to construct detailed, firm-specific and credit-based measures of financial constraints. The credit registy database contains monthly information on actual, potential, short-term and long-term credit above 50 Euros extended to individuals and non-financial corporations by all financial institutions in Portugal. Using the substantial granularity of the data we provide three empirical facts that are counterfactual to the standard heterogeneous firms model with financial frictions.

Firstly, we show that financially constrained firms are found across the entire size distribution. In fact, going from the bottom 5% of the size distribution to the top 5% only reduces the probability of being constrained by about 10% for our preferred measure. Conversely, in the standard model with transitory productivity constrained firms are exclusively located at the bottom of the distribution. Secondly, we find that the share of assets held by constrained firms is substantially higher compared to the value that a standard model such as Khan & Thomas (2013) would suggest. For example, a calibrated version of the model that matches the share of constrained firms in the data produces only 1% of total assets in constrained firms, whilst the data implies a value of up to 12%. Thirdly, financially constrained firms are more sensitive to the business cycle, conditional on size. Using a number of different outcome variables we first replicate Crouzet & Mehrotra (2020)'s result, demonstrating that differences in firm cyclicality only emanate from very large firms. We then include our measures of a firm's financial position and show that these explain in part the heterogeneous elasticities across firms in support of the financial accelerator mechanism. Notably, this channel seems to be independent of Crouzet & Mehrotra (2020)'s size channel, since our measures as well as the top size dummy are statistically significant across all our measures of financial constraints. So in summary we show that (1) constrained firms exist across the entire size distribution, (2) have a higher asset share and (3) exhibit a larger cyclicality, conditional on size.

Our second contribution is to reconcile these empirical findings with the existing financial frictions theory. As outlined above, a standard heterogeneous firms model is unable to match these facts as it produces only small constrained and large unconstrained firms, in contrast to our empirical finding (1). This also implies an underestimation of the share of assets held by constrained firms, and size as a sufficient proxy for differential sensitivity to the business cycle, in contrast to empirical findings (2) and (3). In light of this, we explore how a richer structure of heterogeneity may enable the compatibility of empirical results and model. In particular, we present empirical evidence demonstrating that ex-ante heterogeneity across firms matters, persists over the firm's life cycle, and affects constrained and unconstrained firms differently. Firstly, the standard deviation of employment across firms is high and increasing with age, implying large size differences early in the life cycle and a wide range of optimal firm sizes. Secondly, the autocorrelation of employment remains high throughout a firm's life cycle, again pointing towards the importance of permanent firm differences. Thirdly, we also show that these statistics are more muted for constrained firms, in line with our theoretical predictions. Finally, we confirm the importance and differential incidence of ex-ante heterogeneity using the flexible statistical model developed by Pugsley et al. (2021).

This evidence of ex-ante heterogeneity then serves as motivation to refine the canonical theoretical model by including a permanent component to the firms' productivity process. The rather simple addition of ex-ante heterogeneity enables the model to match the stylized facts, as it introduces a large and persistent heterogeneity in optimal firm sizes and spells of financial constraints. We use the extended model to analyze how the existence of ex-ante heterogeneity shapes the aggregate responses following a perfect foresight shock to total factor productivity or to the collateral constraint parameter. Whilst aggregate differences are relatively muted for aggregate productivity innovations, financial shocks are amplified relative to the standard model with only a transitory productivity component. This is due to the cyclicality of large constrained firms in the top percentiles of the size distribution that need to cut investment when the borrowing constraint is tightened. This mechanism amplifies the output drop in response to a financial shock up to four times. Further, we demonstrate that matching the joint distribution of size and constrained firms gives rise to up to three times stronger output losses due to capital misallocation. Thus, this paper emphasizes the importance of targeting the joint distribution of size and financial constraints in order to correctly quantify the propagation and amplification of aggregate shocks in existing financial friction models. More generally, this paper reiterates the importance and relevance of richer productivity dynamics in heterogeneous agent models.

Literature Our work follows a large literature in macroeconomics that has analysed heterogeneous firms and financial frictions both theoretically and empirically.

Firstly, we relate to the empirical literature assessing the differences in cyclicality of constrained firms and the debate on how to identify these firms in the data. Gertler & Gilchrist (1994) find empirical evidence for the financial accelerator mechanism. They analyse the cyclical behaviour of small vs. large manufacturing firms and interpret this as evidence for the financial accelerator. Their main assumption is that size is a good proxy for financial constraints. Sharpe (1994) finds a statistically significant relationship between a firm's leverage ratio and the cyclicality of its labour force. Employment growth at highly leveraged firms is more sensitive as they are less likely to hoard labour. This cyclicality also holds for the size dimension, implicitly confirming Gertler & Gilchrist (1994)'s evidence. Related, Gilchrist & Himmelberg (1995) find that investment still responds to cash flow even after controlling for its role for forecasting future investment opportunities, with the effect being stronger for firms without full access to the capital market.

More recently, Crouzet & Mehrotra (2020), using firm level data underlying the Quarterly Financial Reports (QFR) provided the US Census Bureau, document that differences in size-related cyclicality only arise at the very top of the distribution, with the bottom 99.5% of firms having non-significant differences in cyclicality. Arguably, this evidence, together with the insignificance of standard financial proxies for financial constraints speaks against financial factors driving cyclicality differences.

These results are also related to Farre-Mensa & Ljungqvist (2016) findings, who suggest that typical measures of financial constraints are not associated with firms that behave as if they were constrained. Even indices that combine different firm characteristics such as the ones proposed by Kaplan & Zingales (1997), Whited & Wu (2006) and Hadlock & Pierce (2010) do not correlate well with firms that behave as financially constrained. These findings are also supported by Bodnaruk et al. (2015), who use text analysis of the 10-k financial reports to gauge if firms are constrained or not, and find a weak correlation with common constraint measures. Buehlmaier & Whited (2018) equally contribute to this literature by developing a new financial constraints measured based on text analysis. Finally, focusing on sensitivity of monetary policy Cloyne et al. (2018) find that age and dividend payments are an empirically relevant proxy for increased sensitivity to the funds rate.

Our paper, by making use of detailed firm level credit data, contributes to this literature by reiterating that size is indeed an insufficient proxy for financial constraints. Moreover, with information on credit lines available to the firm and overdue credit, we also provide evidence that supports a financial accelerator mechanism that is only weakly size dependent. Our measures of constraints statistically significantly increase cyclicality whilst the size group coefficients maintain magnitude and significance, in line with Crouzet & Mehrotra (2020).

Secondly, we contribute to the research on heterogeneous firm financial frictions models.

One of the early contributions in this literature by Cooley & Quadrini (2001) shares many features with our current model. They augment an otherwise standard Hopenhayn (1992) model of heterogeneous firms with financial frictions and persistent shocks. In doing so, they are able to match the empirical facts that both smaller firms, conditional on age, and younger firms, conditional on size, are more dynamic (ie. job creation and destruction, growth, volatility of growth and exit are all higher). In similar fashion, Pugsley et al. (2021) highlight the importance of ex-ante heterogeneity in explaining the firm size distribution and the recent decline in firm dynamism.

Another recent instance where permanent productivity differences plays a crucial role is Mehrotra & Sergeyev (2020). They argue that financial frictions played a relatively minor role in unemployment increases associated with the Great Recession and that employment was reduced due to shocks that affected unconstrained and constrained firms alike. Conversely, Khan & Thomas (2013) and Ottonello & Winberry (2018) argue for the importance of financial frictions in the propagation of financial and monetary policy shocks, respectively. Our theoretical contribution emphasizes the importance of permanent productivity differences for matching the observed distribution of constrained firms, conditional on size. We also highlight the importance of matching this distribution in amplifying both productivity and financial shocks, based on a model very similar to the literature above.

Finally, the paper also relates to the literature on misallocation of productive resources. While Buera et al. (2011) argue that financial frictions explain a large share of cross country differences in misallocation and consequently in aggregate productivity, Midrigan & Xu (2014) by parameterizing a firm dynamics model with financial frictions, consistent with producer level data, find small losses from misallocation. Other papers such as David et al. (2016), Restuccia & Rogerson (2017), Baqaee & Farhi (2020), Peters (2020) and Andreasen et al. (2021) highlight other sources of misallocation besides financial frictions, such as markups, imperfect information or even the tax code, regulation and capital controls. We contribute to this literature by illustrating that matching the distribution of constrained firms, conditional on size, amplifies the misallocation resulting from financial frictions up to three times, when compared to a standard firm dynamics and financial frictions model, in line with Pugsley et al. (2021).

Outlook The paper is structured as follows. Section 2 presents the data we use for the empirical analysis as well as to discipline our theoretical model. We proceed to present the three empirical findings outlined above in Section 3. Section 4 illustrates the importance of ex-ante heterogeneity in our data. In Section 5 we set out the model to incorporate and account for these facts and in Section 6 we discuss model predictions of aggregate effects. Finally, Section 7 concludes.

2 Data

We draw on a unique combination of datasets that cover the Portuguese economy between 2006 and 2017, all managed by the Bank of Portugal Microdata Research Laboratory.

The *Informação Empresarial Simplificada* (IES) Central Balance Sheet Database (CBSD) that is based on annual accounting data of individual firms. Portuguese firms have to fill out mandatory financial statements in order to comply with their statutory obligation. Consequently, this dataset covers the population of virtually all non-financial corporations in Portugal from 2006 onward. We combine this dataset with the Central Credit Register (CCR), that contains monthly information on the actual and potential credit above 50 euros extended to individuals and non-financial corporations, reported by all financial institutions in Portugal.[1] Actual credit includes loans that are truly taken up, such as mortgages, consumer loans, overdrafts and others. Potential credit encompasses all irrevocable commitments to the subject that have not materialised into actual credit, such as available credit on credit cards, credit lines, pledges granted by participants and other credit facilities.[2] We then merge these two databases on the firm level. Moreover, we also add the Monetary Financial Institutions Balance Sheet Database in order to

[1] Given that the firm balance sheet data is of yearly frequency, we consider the month in which the balance sheet data was reported. Results were robust to shifting and averaging the monthly credit data.

[2] Further details on the credit information used are documented in Appendix A.

gain information on the balance sheets of banks that extend credit to non-financial institutions. We merge this dataset on a firm level using the bank identifier and the share of loans extended by one firm to arrive at our detailed dataset.

Similar to Buera & Karmakar (2019), who use the same dataset, we restrict the set of firms in this panel dataset to those with at least five consecutive observations and to firms which are in business at the time of reporting. Furthermore, we only consider privately or publicly held firms and drop micro firms, i.e. those with overall credit amounts of less than 10,000 €. Descriptive statistics for the relevant variables can be found in Table B.3 and B.2 in Appendix C.

2.1 Measures of financial constraints

Based on the credit information in the data we construct several binary and continuous measures indicating whether a firm is financially constrained. Financial constraints are most commonly conceived as a supply side phenomenon. Firms that could potentially obtain credit in perfect credit markets are unable to do so due to asymmetric information considerations on the supply side. For example, a firm that has a profitable investment project that requires external financing cannot realise it as the bank is not satisfied with the creditworthiness of that firm. This may happen either via the price dimension - an interest rate that is too high - or on the quantity dimension - the credit is denied altogether. In this paper, we identify constrained firms along the quantity dimension, using the credit information for each firm.[3] Given that credit allowances are changing over time, this provides us with a time-varying and firm-specific measure for being financially constrained. It should be noted, however, that while credit information offers a far more detailed notion of a firm being constrained, relative to financial characteristics such as leverage or liquidity, it is still a proxy. Hence, taking this into account, we consider a wide range of binary and continuous measures for identifying whether a firm is constrained.

[3] See for example Custodio et al. (2021) who, using the same dataset, analyze how the price dimension affects firms' investment and employment.

Binary measures In our baseline definition a firm is credit constrained at time t, if it has no potential credit available at time t:

$$\text{Constrained I} := \mathbf{1}_{\text{Potential credit}_t=0}.$$

As outlined above, potential credit summarizes all the irrevocable commitments by credit institutions. Even though this measure enables an understanding of whether firms have drawn down their credit lines and are thus potentially constrained it also encompasses a lot of noise. One problem might be that specific banks may not grant credit lines. This would show up as potential credit = 0 in the data. But this does by no means make these firms credit constrained. They just simply do not have credit lines, which is quite common. In order to address this issue, we consider three approaches. First, we test the robustness of our results dropping all cases in which potential credit is zero throughout. Second, we estimate the panel regression using firm fixed effects, which will control for those cases by construction. Third, we introduce the following adjusted measure, demanding that potential credit was positive in the previous period $t-1$ and hence, the firm seems to have hit the credit line:

$$\text{Constrained II} := \mathbf{1}_{\text{Potential credit}_t=0\ \&\ \text{Potential credit}_{t-1}>0}.$$

Another issue when relying only on potential credit, might be that although, firms have exhausted their committed credit line, they could still successfully apply for a short- or long-term loan. In order to cope with this issue, we introduce two further measures. The third measure, our preferred measure of financial constraints, redefines those firms as unconstrained, which managed to secure short- or long-term credit, while potential credit was zero:

$$\text{Constrained III} := \mathbf{1}_{\text{Potential credit}_t=0\ \&\ \Delta\text{Effective credit}_{t+1}<0}.$$

The fourth measure augments the baseline definition by specifically considering those firms as being constrained for which overdue credit is growing:

$$\text{Constrained IV} := \mathbf{1}_{\text{Potential credit}_t = 0\ \&\ \Delta\text{Overdue credit}_{t+1} > 0}.$$

The rationale behind this definition is that growth in overdue credit is likely a signal for a firm in bad financial shape.

While the measures presented so far are conceptually in the spirit of a firm hitting the credit constraint and thus being strictly constrained, it might also be that a firm is granted credit, yet the amount is not sufficient to finance any planned investment. The following measure aims to capture this notion of potentially constrained firms:

$$\text{Constrained V} := \mathbf{1}_{\text{Total Credit}_{t+1} > \text{Total Credit}_t + \text{Potential Credit}_t}.$$

Continuous measures In order to account for different levels of severity of financial constraints, we also introduce two continuous measures. The first one is defined as the ratio of potential credit and cash to total liabilities of the firm, thereby accounting for the fact that a firm might have enough cash making it financially unconstrained, despite being credit constrained:

$$\text{Constrained VI} := \frac{\text{Potential credit}_t + \text{Cash}_t}{\text{Liabilities}_t}.$$

Our final measure can be interpreted as the continuous counterpart to the third measure taking into account potential credit and ex-post changes to short- and long-term credits:

$$\text{Constrained VII} := \frac{\text{Potential credit}_t + \Delta\text{Short- and long-term credit}_{t+1}}{\text{Liabilities}_t}.$$

Appendix A and B provide a more detailed description of the dataset and the constrained measures. Tables B.1, B.3 and B.2 report descriptive statistics. Figures D.1, D.2 and D.3 report the evolution of the share of constrained firms and credit over time.

3 Empirical analysis

The dataset covering the universe of Portuguese firms and their balance sheet and credit information allows us to generate novel evidence on the characteristics of constrained firms. This Section presents three empirical facts which form the basis for our critique and subsequent modification of the theoretical model. First, we illustrate that size, among other variables commonly used in the literature as proxies for financial constraints, is only weakly correlated to the firm's financial health. In fact, constrained firms can be found over the entire firm distribution. Second, we show that across all our constrained measures, these firms have a higher asset share compared to what a standard calibrated model would predict. Finally, we revisit the relationship between firm characteristics and the elasticity of firm outcomes with respect to the cycle. Similar to Crouzet & Mehrotra (2020), we find firms' cyclicality to differ only at the top 1% of the size distribution. But, more importantly, we find evidence that supports the financial accelerator mechanism theory, with constrained firms being more cyclical, and this to be orthogonal to the size cyclicality.

3.1 Constrained firms are found across the firm distribution

Our first stylized fact states that financially constrained firms populate the entire firm distribution along a number of common proxies for financial constraints. Figure 1 plots the share of firms that have zero potential credit and declining effective credit (measure III) over percentiles of age, total assets, liquidity ratio and leverage. Evidently, constrained firms can be found in every bin of the firm distribution. This finding is robust across all binary identifiers for being constrained, with only the overall fraction of constrained firms changing depending on the strictness of the specific measure, as documented in Figures D.4 - D.6 in Appendix D.1. While correlations are in line with the existing literature, they are not as strong as existing models would predict. In fact, when running a linear probability model, the probability of being constrained only reduces by about 10% for two standard deviation increase in total assets, which

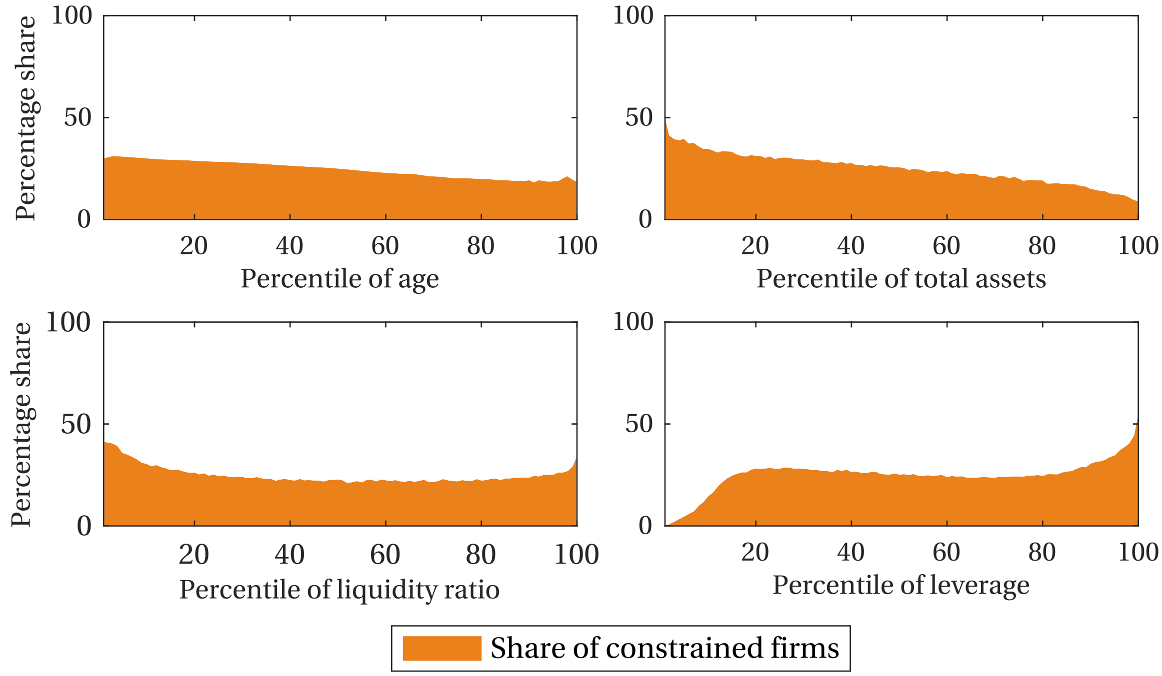

Figure 1: Decomposition of constrained and unconstrained firms
across percentiles of firm variables

Notes. Constrained firms are identified using measure constrained III which classifies firms as constrained if they have exhausted their potential credit and were not granted additional short- or long-term credit in that period as defined in Appendix B. Figure D.4 - D.6 in Appendix D.1 show the analogue plots for related measures.

is equivalent to going from the bottom 5% to the top 5% of the size distribution.[4] Even after accounting for potential attenuation bias, the main conclusion stands: standard firm models typically produce small constrained firms and large unconstrained firms, yet our data does not support this strong dichotomy. Moreover, even when controlling for a battery of financial variables the explanatory power to predict whether a firm is constrained is relatively low compared to the firms' fixed effects. Hence, existing proxies of financial constraints may be unable to capture this unobserved heterogeneity, which seems to play a substantial role in credit decisions.

3.2 Constrained firms account for a larger asset share

The first stylized fact establishes constrained firms across every bin of the firm size distribution. This finding contradicts a standard heterogeneous firms model, which generates only large un-

[4]See Table C.7 in Appendix C for the results of the linear probability model.

Table 1: Percentage of constrained firms and share of total assets in constrained firms

		Constrained measure				
	Model	I	II	III	IV	V
% Constrained	0.24	0.36	0.05	0.23	0.03	0.15
% Total assets in constrained	0.01	0.20	0.03	0.12	0.02	0.14

constrained and small constrained firms. As a consequence, these types of models will severely underestimate the fraction of total assets belonging to constrained firms compared to the data, which constitutes our second empirical fact.

Table 1 compares the empirical values for the fraction of constrained firms and the implied asset share in constrained firms, with the values implied by a standard Khan & Thomas (2013) model, calibrated to match 23% of constrained firms, in line with the Constrained measure III.[5] Depending on how strict the constrained measure is, the percentage of constrained firms in the data varies from 3% to 36% of total firms. Nonetheless, even for the strictest measure that only labels 3% of firms as constrained, the share of total assets in this group of firms is 2%. This value is still above what is implied by the calibrated model, which is only 1%, despite the model being calibrated to generate 23% of constrained firms. This low share of assets in constrained firms is a consequence of the implied distribution of firms. Whereas in the data, even for the more strict measures there are still constrained firms at the top of the distribution, in the model the constrained firms are all concentrated at the bottom of the distribution, causing an underestimation of the share of total assets in constrained firms.

3.3 Size and financial factors matter for cyclicality

A higher asset share held by constrained firms and a presence of constrained firms across the distribution do not necessarily warrant a reassessment of the cyclical properties of financial frictions models. These measures only imply relevance for aggregate cyclicality if these firms also exhibit a differential elasticity to aggregate shocks. This Section aims to test the financial

[5] For more details on the model and calibration see Section 5.

accelerator mechanism empirically. In particular, we test whether constrained firms are more cyclical than unconstrained firms. In order to make our results comparable to Crouzet & Mehrotra (2020), we first replicate their results for the universe of Portuguese firms and then augment their estimation strategy, using our set of firm-specific and time-varying measures of financial factors. Following Crouzet & Mehrotra (2020), the specification estimated is:

$$
\begin{aligned}
g_{i,t} = \Delta GDP_t + \sum_{j \in \mathscr{J}} (\alpha_j + \beta_j \Delta GDP_t) \mathbf{1}_{i \in \mathscr{S}_t^{(j)}} + (\zeta + \eta \Delta GDP_t) \text{Const.} n_{i,t} \\
+ \sum_{l \in \mathscr{L}} (\gamma_l + \delta_l \Delta GDP_t) \mathbf{1}_{i \in \mathscr{L}} + \epsilon_{i,t}, \qquad (1)
\end{aligned}
$$

where i identifies a firm and t identifies a year. The dependent variable $g_{i,t}$ is the year-on-year log change in turnover. The set $\mathscr{S}_t^{(j)}$ is a jth size group, e.g. all firms above the 90th but below the 99th percentile. Furthermore, ΔGDP_t is the year-on-year growth rate of GDP, and $\mathscr{L}$ is a set of industry dummies. Const.$n_{i,t}$ refers to the firm-specific variable measuring the strength of financial constraints, indexed by n.

Table 2 reports estimates of the semi-elasticity of firm-level growth in turnover to GDP growth. The first column reports estimates for the size groups $j \in \{[90, 99], [99, 99.5], [99.5, 100]\}$ with $[0, 90]$ as the reference group. On average, small firms have a semi-elasticity of roughly 2.5, meaning for any percent change in GDP growth, their turnover changes by 2.5%. Although, the coefficient for the size group $[90, 99]$ is insignificant, results are consistent with the view that larger firms are less sensitive to aggregate fluctuations. The difference is particularly notable at the very top of the firm distribution.

The second column reports results including the baseline binary measure for being constrained, Const.I. First, it is worth noting, that the estimation coefficients with respect to size hardly change. This is indicative that the mechanism going through size is somewhat independent to any financial accelerator mechanism and that size might not be a good proxy for the latter, as already pointed out by Crouzet & Mehrotra (2020). The coefficient for the interaction between the constrained measure and aggregate fluctuations is significant, offering support for

Table 2: Cyclicality in turnover conditional on size bins
and measures of financial constraints

	Turnover growth							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
GDP growth	2.512***	2.441***	2.440***	2.450***	2.470***	2.450***	2.522***	2.508***
	(0.023)	(0.027)	(0.023)	(0.028)	(0.025)	(0.030)	(0.023)	(0.025)
[90,99] × GDP growth	0.042	0.075	0.069	0.075	0.064	0.066	0.042	0.048
	(0.106)	(0.107)	(0.106)	(0.121)	(0.120)	(0.107)	(0.106)	(0.120)
[99,99.5] × GDP growth	-0.726**	-0.693**	-0.698**	-0.742**	-0.754**	-0.688**	-0.732**	-0.760**
	(0.300)	(0.300)	(0.300)	(0.342)	(0.341)	(0.300)	(0.300)	(0.342)
[99.5,100] × GDP growth	-1.426***	-1.382***	-1.391***	-1.607***	-1.610***	-1.384***	-1.428***	-1.650***
	(0.291)	(0.291)	(0.291)	(0.359)	(0.291)	(0.293)	(0.291)	(0.359)
Const.I × GDP growth		0.121**						
		(0.054)						
Const.II × GDP growth			0.561***					
			(0.135)					
Const.III × GDP growth				0.165**				
				(0.069)				
Const.IV × GDP growth					1.192***			
					(0.250)			
Const.V × GDP growth						0.161***		
						(0.048)		
Const.VI × GDP growth							-0.063**	
							(0.025)	
Const.VII × GDP growth								-0.168***
								(0.030)
Observations	1,326,344	1,326,344	1,326,344	1,161,187	1,161,187	1,326,344	1,324,992	1,160,194
R-squared	0.029	0.030	0.030	0.030	0.032	0.029	0.030	0.029
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm

Notes. Estimates report the semi-elasticity of turnover with respect to GDP. The first line is based on the equivalent regression without interaction fixed effects, as it is otherwise dropped due to multicollinearity, and included as a benchmark. Constrained measures are constructed as documented in Appendix B. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

the financial accelerator mechanism. Constrained firms have a 0.1 higher semi-elasticity relative to unconstrained firms according to the baseline measure.

However, as already pointed out when introducing the different measures for being constrained, the baseline measure might capture firms for which potential credit is zero, but in fact are unconstrained. Hence, the baseline measure offers a lower bound of the increased sensitivity of constrained firms. We therefore consider other binary measures trying to overcome those drawbacks which are reported in columns (3) to (6). Estimation results are supportive of the notion that the baseline measure acts as a lower bound and sensitivity might be up to 10 times higher for constrained firms as measured by Const.IV.

Estimates for the standardized continuous measures are also in line with the findings so far. The negative sign is due to their definition. The higher their value, the less constrained is a firm. Results for sales growth and growth in employees are closely in line, see Tables C.1 and C.2 in Appendix C. Besides using different measures, we considered a battery of robustness checks. First, we included time fixed effects to account for broader macroeconomic circumstances. Second, we estimated the specification using firm fixed effects. Third, we estimated the model excluding those firms for which potential credit is zero throughout. Fourth, we controlled for supply effects using aggregated bank data. Estimates were robust across all specifications. Results are reported in Tables C.3-C.6 in Appendix C.

4 Firm potential

In contrast to our stylized facts, the canonical firm financial frictions model à la Khan & Thomas (2013) predicts a very strong correlation between firm size and financial constraints, as firms require a relatively uniform minimum size to become unconstrained. Consequently, one factor that could potentially break this strong correlation are heterogeneous ex-ante conditions for firms, such as firm potential. Small firms may be unconstrained as they already reached their potential - i.e. optimal size - while large firms may still be growing and are still constrained. Equally, different potentials would create a dispersion of unconstrained firms across the entire firm size distribution, similar to our first stylized fact. Further, larger constrained firms may elevate the fraction of assets held by constrained firms closer to what we observe in the data. Accordingly, this Section investigates whether such ex-ante heterogeneity exists in our dataset.

Standard deviations and autocorrelations We follow Pugsley et al. (2021) by first computing both the standard deviation of log employment by age and the autocorrelation of log employ-

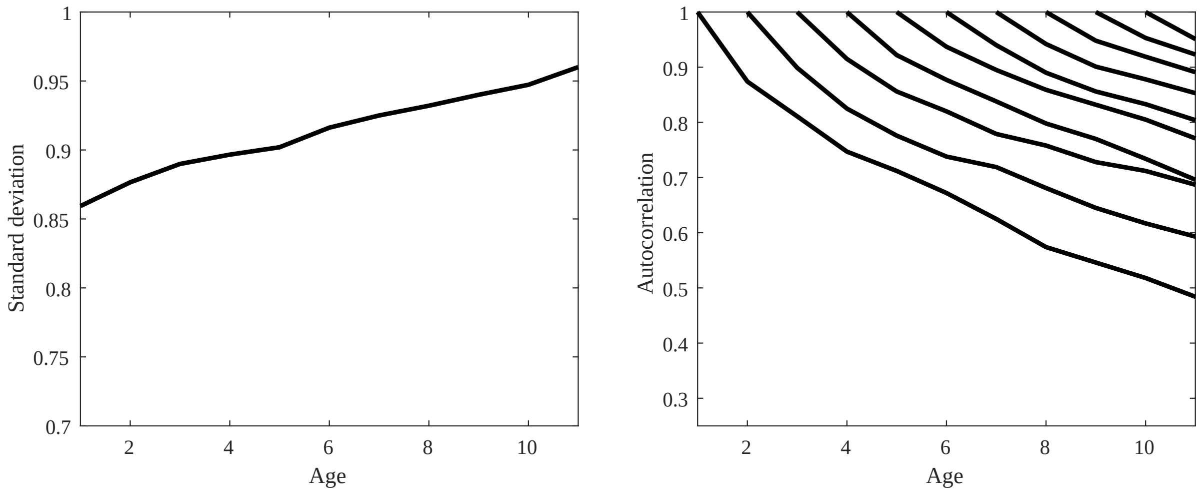

Figure 2: Standard deviation and autocorrelation of log employment by age

Notes. The left panel presents the standard deviation of log employment by age, after controlling for sector and year fixed effects. The right panel presents the autocorrelation of log employment between ages a and $h \leq a$. Across lines h changes, while a changes along the lines.

ment between age a and $h \leq a$.[6] The results are depicted in Figure 2. The left panel illustrates that even at early ages the standard deviation of log employment is already relatively high - above 0.8 - which implies substantial size differences even at early ages. Also, the standard deviation of log employment is increasing with age. This supports the idea that there exists ex-ante heterogeneity, as firms at birth are not all equal, and that firms will settle at different levels of employment in the long run, as the standard deviation is increasing with age. The right panel shows that the long run autocorrelations stabilize at relatively high levels. This suggests that ex-ante conditions are persistent and affect the firm even in the long run. These results are in line with the Pugsley et al. (2021) findings, who highlight the importance of ex-ante conditions in explaining the firm size distribution and firm dynamics.

One crucial implication of ex-ante heterogeneity for the standard financial frictions model is that unconstrained firms have reached their optimal size, determined by idiosyncratic ex-ante conditions, whilst constrained firms are still growing. Consequently, through the lens of the model, we would also expect both the standard deviation and autocorrelation to be higher for

[6] To prevent differences across sectors and business cycle conditions from explaining the majority of the standard deviation and autocorrelation, we first control for sector and year fixed effects and then use the residuals of log employment.

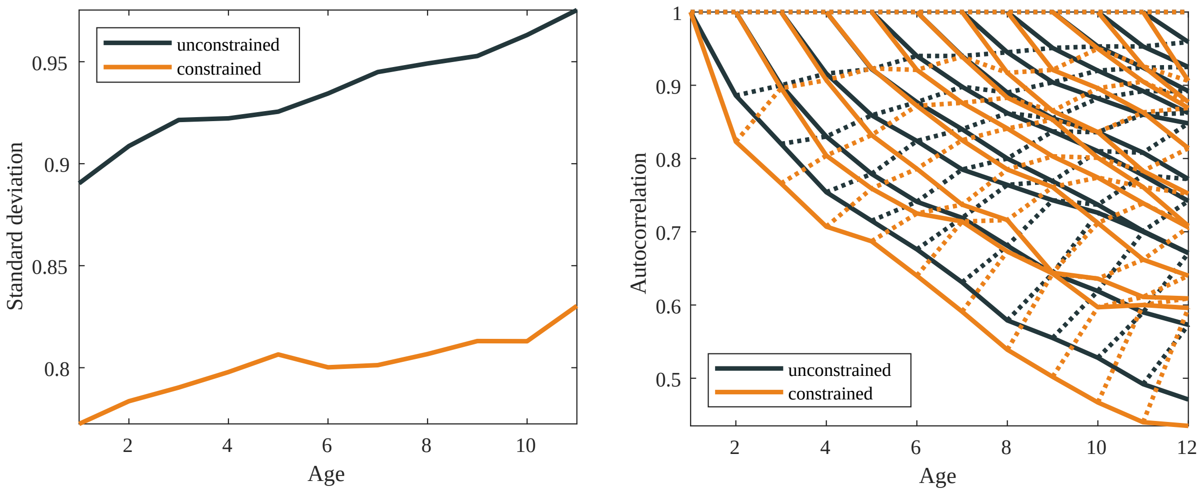

Figure 3: Standard deviation and autocorrelation of log employment by age and separated by constraint measure III

Notes. The left panel presents the standard deviation of log employment by age, after controlling for sector and year fixed effects. The right panel presents the autocorrelation of log employment between ages a and $h \leq a$. Across lines h changes, while a changes along the lines.

unconstrained compared to constrained firms. We plot these two statistics of log employment for both constrained and unconstrained firms plot in Figure 3. Here we are using the measure that exclusively takes the amount of potential credit available into account to determine which are the constrained firms. A firm is considered constrained if at age $a - h$ it has potential credit equal to zero. As expected, both of these measures are indeed lower for constrained firms than unconstrained ones. Of course, one may expect the opposite to be true, as constrained firms potentially have less resources to grow and so their employment tomorrow could have a stronger correlation with employment today. Yet, as pointed out above, the fact that the autocorrelation is higher across the life-cycle for unconstrained firms may be indicative that they are born closer to their optimal size, when compared to constrained firms. This may then explain why some young firms are constrained and others are not: the ones born closer to their optimal size have lower investments and do not become constrained, while firms that need to grow to reach the optimal size exhaust their credit lines.

Statistical model To gain even further understanding of the importance of ex-ante vs ex-post heterogeneity for constrained and unconstrained firms, we again follow Pugsley et al. (2021) and adopt the statistical model used therein. This model uses the information provided by the autocovariance structure of log employment to capture the importance of both types of heterogeneity.

Consider the following decomposition for employment n by firm i at age a

$$\underbrace{\ln n_{i,a}}_{\text{log employment}} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante component}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post component}}, \tag{2}$$

where

$$\begin{array}{llll}
u_{i,a} = \rho_u u_{i,a-1} + \theta_i, & u_{i,-1} \sim iid\left(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}\right), & \theta_i \sim iid\left(\mu_\theta, \sigma^2_\theta\right), & \left|\rho_u\right| \leq 1 \\
v_{i,a} = \rho_v v_{i,a-1}, & v_{i,-1} \sim iid\left(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}\right), & & \left|\rho_v\right| \leq 1 \\
w_{i,a} = \rho_w w_{i,a-1} + \varepsilon_{i,a}, & w_{i,-1} = 0, & \varepsilon_{i,a} \sim iid\left(0, \sigma^2_\varepsilon\right), & \left|\rho_w\right| \leq 1 \\
z_{i,a} \sim iid\left(0, \sigma^2_z\right) & & &
\end{array}$$

In this employment process, $u_{i,a} + v_{i,a}$ capture the ex-ante profile, while $w_{i,a} + z_{i,a}$ capture the ex-post one. The ex-ante component is determined by three shocks that are drawn just prior to the birth year, at $a = -1$. The shocks $v_{i,-1}$ and $u_{i,-1}$ represent the initial conditions of the firm, which allow for rich heterogeneity even at birth. θ_i is the permanent component, which will accumulate over the life-cycle at speed ρ_u. In particular, with $\rho_u < 1$, the long-run steady state level of employment will be given by $\frac{\theta_i}{1-\rho_u}$.

This specification will allow for rich heterogeneity not only in terms of optimal size of the firms, depending on the distribution of θ_i, but also in terms of the speed at which firms reach the steady state. As firms start at different points depending on $u_{i,-1}$ and $v_{i,-1}$, each shock with its own persistence parameter, the path from initial to steady state employment will highly differ across firms.

The ex-post component is formed of two different shocks, one i.i.d. shock with expected value of zero, and a persistent one that follows an AR(1) process with i.i.d. innovations $\epsilon_{i,a}$ and

Table 3: Static model parameters for constrained and unconstrained firms using measure III

	ρ_u	ρ_v	ρ_w	σ_θ	σ_u	σ_v	σ_ϵ	σ_z
Constrained	0.519	0.873	0.915	0.241	0.501	0.669	0.261	0.202
Unconstrained	0.432	0.765	0.883	0.399	0.755	0.743	0.311	0.175

persistence ρ_w. To abstract the ex-post component from affecting the ex-ante one, we set the initial conditions of the persistent shock to $w_{i,-1} = 0$.

To more clearly identify the ex-post and ex-ante contributions one can also derive the formula for the autocovariance, enabling a clear identification of the contribution of both components. The autocovariance formula is given by

$$Cov[\ln n_{i,a}, \ln n_{i,a-j}] = \underbrace{\left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right)\sigma_\theta^2 + \rho_u^{2(a+1)-j}\sigma_{\hat{u}}^2 + \rho_v^{2(a+1)-j}\sigma_{\hat{v}}^2}_{\text{Ex-ante component}}$$
$$\underbrace{+\sigma_\epsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \sigma_z^2 \mathbf{1}_{j=0}}_{\text{Ex-post component}}$$

The derivation of the autocovariance formula is presented in Appendix F. The autocovariance is a function of variance and persistence parameters of both ex-ante and ex-post shocks. We calibrate the model for constrained and unconstrained firms separately by minimizing the sum of squared differences between the model and empirical autocovariance. Here firms are split into constrained and unconstrained according to the measure Constrained III that takes into account whether firms that have zero potential credit received any credit independent of that. A firm is considered constrained if at age $a-h$ this measure is equal to zero.[7] Table 3 presents the parameters resulting from the calibration strategy.[8] Two key parameters of the model are ρ_u and σ_θ, as, together, they imply that steady state heterogeneity exists. The point estimates imply a standard deviation of steady state employment, σ_θ for constrained firms of

[7] In the Appendix D.2 we redo the exercise using measure Constrained I instead.
[8] Figure D.7 in Appendix D.2 plots the model fit to the data for both types of firms.

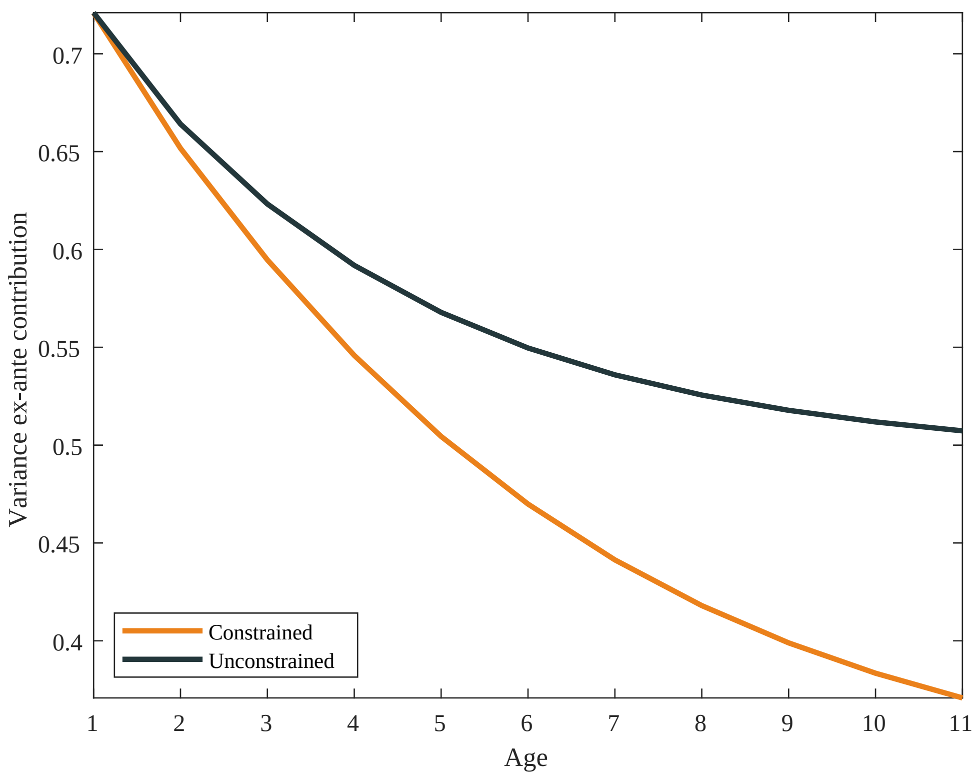

Figure 4: Variance ex-ante contribution.

Notes. Values for constrained firms presented in orange, while blue stands for the unconstrained firms.

0.241 and of 0.399 for unconstrained ones. This, again, highlights that there seem to be differences between both types of firms that originate from ex-ante conditions.

Figure 4 quantifies the importance of the ex-ante component for the variance of both constrained and unconstrained firms. For both types of firms, the ex-ante component contribution is above 80% at birth. Differences between both types of firms start to arise after year 3, with the ex-ante component explaining more than 50% of the standard deviation for unconstrained firms in the long run, while for constrained firms it is below 40%. As a robustness test we recalibrate the model using Constrained I measure, to guarantee results are not dependent on the selected measure. Table C.9 and Figure D.9 in the Appendix present the calibration of the statistical model and fit to data using the measure Constrained I. Figure D.10 presents the ex-ante contribution under the alternative calibration, supporting the previous results.

Again, one could have expected the opposite result, as constrained firms may have limited resources to grow and so initial conditions would be more prevalent in explaining the distribution. Instead, the fact that the ex-ante contribution is stronger for unconstrained firms is indicative that these firms are born closer to their optimal size. At the same time, constrained firms have not reached their optimal size yet, and so naturally less contribution to the employment dispersion is originating for permanent conditions.

All the empirical evidence in this Section suggests that ex-ante heterogeneity: 1) matters both in the short and in the long-run; 2) more strongly affects unconstrained than constrained firms. Moreover, the standard deviation of steady state level of employment is higher for unconstrained firms. All this evidence may be indicative that unconstrained firms start closer to their steady state level of employment, while firms that still need to grow exhaust their credit lines to reach their optimal size and so become constrained. This mechanism is mirrored in our general equilibrium firm dynamics model in the next section.

5 Model

In this Section we present a heterogeneous firms model with financial frictions which aims to reconcile the stylized facts above. We built on Khan & Thomas (2013) and introduce ex-ante heterogeneity through a permanent productivity component which can be interpreted as the firm's business potential. This will break up the strong correlation between size and being financially constrained. Firms with lower permanent productivity will reach their optimal amount of capital earlier and will be unconstrained from then on, while firms which draw a higher permanent component will be constrained much longer as they take longer to grow into their potential.

5.1 Households

Households choose consumption, savings and labor supply according to the following maximization problem:

$$V(k) = \max_{c,l,k'} \left\{U(c,l) + \beta \mathbb{E} V(k')\right\}$$

subject to:

$$k' + c = (1+r)k + \omega l + D$$

.

The first-order conditions for the household problem are standard:

$$U_l(c,l) = \omega U_c(c,l)$$

$$U_c(c,l) = \beta \mathbb{E}\left[(1+r')U_c(c',l')\right].$$

We use the following Greenwood-Hercowitz-Huffman (GHH) utility formulation:

$$U(C,N) = \log(C) + \psi(1-N)$$

Consequently, in the absence of aggregate risk, the first-order conditions are:

$$(1+r) = \frac{1}{\beta}$$

$$\omega = \psi C$$

5.2 Production

The production sector features a continuum of firms, indexed by i. Firms are either classified as entrants or incumbents, detailed below.

Incumbents Incumbent firm i produces according to the following production function:

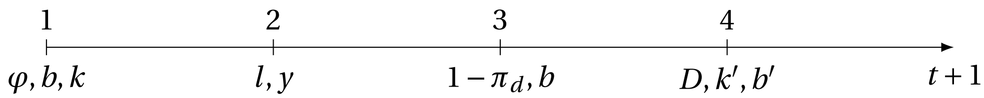

Figure 5: Within period timing of incumbent firm

$$y_i = \varphi_i k_i^{\alpha} l_i^{\nu}, \quad \alpha + \nu < 1.$$

where k and l are capital and labor inputs and φ denotes idiosyncratic productivity. Every firm's productivity comprises two components:

$$\ln \varphi_i = w_i + \theta_i,$$

where w_i is an idiosyncratic *transitory* productivity shock, which follows an AR(1) process with persistence ρ_w and variance of innovations σ_ϵ^2. θ_i is the *permanent* productivity component, drawn at birth from a normal distribution with mean μ_θ and variance σ_θ^2

$$\begin{aligned}
&\theta_i \sim iid\left(\mu_\theta, \sigma_\theta^2\right) \\
&w_i' = \rho_w w_i + \varepsilon_i \qquad \varepsilon_i \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \quad \left|\rho_w\right| \leq 1
\end{aligned}$$

The firm's total profits are revenue minus labour costs (in what follows we surppress i, the firm indicator, to ease on notation where possible):

$$\pi = y - \omega l,$$

where ω is the wage per unit of labor.

Figure 5 summarizes the within period timing of the incumbent. The firm enters the period with predetermined levels of debt b and capital k and immediately observes its idiosyncratic productivity φ composed of a permanent and transitory component. Next, the firm's labor decision is a static choice that can be found through the firm's first order condition:

$$l(k,\varphi;\omega) = \left(\frac{\upsilon\varphi}{\omega}k^{\alpha}\right)^{\frac{1}{1-\upsilon}}.$$

After the production stage, the firm may suffer an exogenous exit shock. The shock happens with probability π_d. Consequently, the value of the firm after the production stage is given by

$$V^1(x,\varphi) = \pi_d x + (1-\pi_d)V^2(x,\varphi)$$

If the firm survives the exit shock, at the end of the period it chooses debt b' and capital k' to take to the next period and dividends to distribute this period D to maximize its value

$$V^2(x,\varphi) = \max_{k',b',D}\left[D + \mathbb{E}_{\varphi'|\varphi}\Lambda V^1(x',\varphi')\right] \tag{3}$$

s.t.:

$$D \equiv x + qb' - k' \geq 0$$

$$b' \leq \xi x$$

$$x' \equiv x(k',b',\varphi') = y(l(k',z'),k',\varphi') - wl(k',\varphi') + (1-\delta)k' - b'$$

where ξ is the financial parameter that captures the financial frictions in the economy, x is the cash on hand that the firm starts the period today, which is given as the sum of profits plus the value of the non-depreciated capital minus the debt the firm has to pay back. q is the price of the bonds firms issue, with $\frac{1}{q}-1$ equal to the equilibrium interest rate, r. Λ is the firm discount factor. As the representative household is the owner of the firm, we assume $\Lambda = \beta$ in steady state.

We opt for a cash on hand collateral constraint following evidence from Kermani & Ma (2020) or Lian & Ma (2021), that illustrate firms' debt contracts and financial constraints not to depend solely on assets, but also on the firm's value and cash-flow. Our measure of cash on hand captures exactly these two sides, as it takes into account both the cash flow and the non-depreciated capital.

Entrants Entry in this model is exogenous. We assume there is a fixed measure, M_e, of entrants equal to the mass of firms exiting after receiving a death shock. The entrants are assumed to enter with zero debt ($b_0 = 0$) and are log normally distributed over their initial capital k_0 with the mean being anchored at a fraction of the mean of optimal capital levels. The choice of a log normal distribution is motivated by the right skewed distribution of entrants in the data. The initial productivity of each entrant, φ_0, follows the same process as the incumbents productivity. Note that firm entry takes place at the end of a period, and entrants start operating in the next period, given their initial state, (k_0, b_0, φ_0).

5.3 Firm Level Decisions

To characterize the firms' decisions we divide the firms into three groups, following Khan and Thomas (2013):

1. **Unconstrained firms.** Firms that can implement the optimal amount of capital and guarantee that in the future they will never be constrained again.

2. **Constrained firms, type 1.** Firms that can implement the optimal amount of capital but not the minimum savings policy that guarantees they will never be constrained again in the future.

3. **Constrained firms, type 2.** Firms that are constrained and cannot implement the optimal amount of capital nor the minimum savings policy.

Unconstrained Firms This group of firms can implement both the optimal amount of capital and the minimum savings policy that guarantees these firms will never be constrained in the future again. Given the absence of adjustment costs and the stochastic process for φ the optimal amount of capital is the solution to:

$$\max_{k'} -k' + \beta \mathbb{E}_{\varphi'|\varphi} \left[(\pi(k', \varphi') + (1-\delta)k' \right]$$

So the optimal amount of capital solves the following equation

$$\beta \mathbb{E}_{\varphi'|\varphi} \left[\frac{\partial \pi}{\partial k'}(k', \varphi') \right] = 1 + \beta\delta - \beta$$

which is when the expected marginal productivity of capital is equal to the marginal cost of an extra unit. The minimum savings policy these firms implement guarantees they will never be constrained again. It is given by

$$B^*(\varphi_i) = \min_{\varphi_j} \tilde{B}(k^*(\varphi_i), \varphi_j)$$

where $\tilde{B}(k^*(\varphi_i), \varphi_j)$ is the minimum savings that guarantees that going from state φ_i to φ_j the firm is still able to implement the optimal amount of capital. It is given by

$$\begin{aligned} \tilde{B}(k^*(\varphi_i), \varphi_j) =& \pi(k^*(\varphi_i), \varphi_j) + (1-\delta)k^*(\varphi_i) - k'^*(\varphi_j) + \\ & q \min \left\{ B^*(\varphi_j), \xi \left(\pi(k^*(\varphi_i), \varphi_j) + (1-\delta)k^*(\varphi_i) - \tilde{B}(k^*(\varphi_i), \varphi_j) \right) \right\} \end{aligned}$$

Given the optimal amount of capital and the minimum savings policy, the dividends distributed by the unconstrained firms are given by

$$D = x - k^* + qB^*$$

From the dividend constraint $D \geq 0$ we can extract the the minimum threshold for cash-on-hand that guarantees the firm is not constrained

$$\tilde{x} = k^* - qB^*$$

and the firms is constrained if $x \leq \tilde{x}$

Constrained Firms: Type 1 These firms can implement the optimal amount of capital, k^*, but not the optimal savings policy and are therefore partially constrained. As they may still be constrained in future states, they value internal financing more than households value dividends. As a results, for this type of firms, $D = 0$. The amount of debt is given by

$$b' = \frac{(k^* - x)}{q}$$

A firm is type 1 if it can adopt the above amount of debt and capital and at the same time guaranteeing that it does not default in the next period.

Constrained Firms: Type 2 Strictly constrained firms can not implement the optimal amount of capital. Those firms utilize all their borrowing capacity as their marginal value of net worth is greater than unity. Hence, their savings policy is simply

$$b' = \xi x,$$

and their maximum possible investment is consequently

$$k' = x + \xi x < k^*,$$

which is strictly smaller than their optimal level of capital k^*.

5.4 Simple model predictions

The way in which firms respond to different types of shocks will ultimately depend on whether they have reached their optimal amount of capital or whether they are still growing. Hence, in what follows, we refer to firms which can implement their optimal capital level as being unconstrained and otherwise as constrained. Consequently, type 1 constrained firms are considered unconstrained as they can implement the optimal amount of capital and their investment policy is the same as for unconstrained firms if shocks are relatively small.[9]

To gain more intuition on the respective investment elasticities to aggregate shocks and the role of ex-ante heterogeneity, we consider a slightly simplified version of the model as outlined in Appendix E. In this model we abstract from labour and assume there is no uncertainty except for a stochastic death shock. The main intuition about differential investment elasticities is captured in Proposition 1. Constrained firms will only respond more to an aggregate productivity shock if either their marginal product of capital is large enough, i.e. they are far from their potential, or the aggregate shock is quickly fading (ρ is close to 0) which gives unconstrained firms barely any incentive to adjust their capital amount. In fact, the elasticity of unconstrained firms is independent of their potential. The marginal product of capital of constrained firms is higher, the higher their potential and the farther they are from reaching their potential.

Proposition 1 *Constrained firms are more elastic to an aggregate TFP shock than unconstrained firms, absent any cyclicality in the constraint, if*

$$mpk > \rho \frac{\alpha}{1-\alpha} \frac{1}{1+q_t \xi}.$$

Proof: The proof is provided in Appendix E.

Hence, the overall aggregate response of output and capital depends on the distribution of constrained firms across the firms size distribution. Furthermore, the financial accelerator mechanism will only be present in the model economy, if Proposition 1 holds on average. However, as already pointed out by Crouzet & Mehrotra (2020), even for low values of ρ, constrained firms will respond less than unconstrained firms and a cyclical collateral constraint is necessary to increase the elasticity of constrained firms. In our discussion about aggregate implications further down, we consider the case of a temporary aggregate shock to total factor productivity (TFP) and a credit shock as a negative shock to borrowing conditions, separately.

[9] Large shocks could make the constraint bind again and they would become strictly constrained.

Table 4: Parameter values

Parameter	Description	Value		Source
β	Discount factor	0.96		Khan & Thomas (2013)
α	Returns on capital	0.30		Khan & Thomas (2013)
η	Returns on labor	0.60		Khan & Thomas (2013)
δ	Depreciation rate	0.065		Khan & Thomas (2013)
ψ	Labour preference	2.15		Khan & Thomas (2013)
π_d	Exogenous probability of exit	0.02		Data
μ_θ	Average of permanent productivity	0		Normalized
μ_w	Average of transitory shock	0		Normalized
		Trans + perm shock	Transitory shock	
ξ	Collateral constraint	0.57	0.57	Internally calibrated
σ_θ	Standard deviation of permanent productivity	0.16	-	Internally calibrated
ρ_w	Persistence of transitory shock	0.07	0.47	Internally calibrated
σ_w	Standard deviation of transitory shock	0.09	0.09	Internally calibrated
μ_{ke}	Relative size of entrants	0.01	0.09	Internally calibrated
σ_{ke}	Standard deviation of entrants	0.11	0.12	Internally calibrated

Notes. The calibration was done on the full model, i.e. including a transitory and a permanent component of productivity.

5.5 Solving and calibrating the model

Solution Method As outlined in Subsection 5.3, one can categorize firms into constrained, potentially constrained and unconstrained firms according to the two cash-on-hand thresholds together with current productivity. One can then directly solve for the capital and bond policy function numerically.

To solve for the general equilibrium, we approximate the firm distribution over a fixed grid of net worth using the histogram method proposed by Young (2010). This method has two advantages over a Monte Carlo simulation. For one, it is not prone to sampling bias which might arise from Monte Carlo simulations for specific conditional moments where the law of large numbers is not met. And as a consequence, it is considerably faster than a Monte Carlo approach. The steady state solution is then given at the wage which is leading to a clearance of the goods market.[10] Given the steady state wage, we also conduct a Monte Carlo simulation to study the firms' policy responses to aggregate shocks in partial equilibrium.

Calibration For most of the parameters, which are unrelated to distributions in the model, we follow Khan & Thomas (2013). The set of parameters chosen is documented in the upper

[10] Market clearing interest rates are given by $1/\beta$.

part of Table 4. The discount factor, β, is set to yield an average annual real interest rate of 4%. The production parameters, η and α, imply a labour share of 60% and capital share of 30%, respectively. Leisure preferences imply that households work one third of their available time.

Firm exit rates in the data are heterogeneous and tend to be lower for larger and older firms. In order to account for that without introducing a size based exit rate schedule, we compute a size weighted average exit rate. When not accounting for lower exit rates among performing firms, small firms with high potential are likely to drop out prior to reaching their optimal amount of capital.[11]

The mean productivity levels for the permanent and transitory component, μ_θ and μ_w, are normalized such that when transforming it to a log-normal distribution, the average productivity component equals one.[12] The rest of the distribution parameters, as well as the maximum borrowing capacity parameter, ξ, are calibrated using the simulated method of moments (SMM). The values presented in Table 4 minimize the distance between a set of empirical unconditional and conditional moments of the firm distribution, listed in Table 5, and their model counterparts.

Table 5 compares the fit of a model with just a transitory productivity component to a model including both, a transitory and a permanent productivity component. Both models were separately calibrated to find the best match to the data.

When calibrating the model with just a transitory shock to firms' productivity, we use the same ξ as the one internally calibrated in the two shock model and place extra weight on the unconditional share of constrained firms and size moments. Two reasons motivate us to restrict this calibration: 1) when comparing the aggregate responses across the two models we want the elasticity of constrained firms to be comparable and, as established in Proposition 1, the elasticity of these firms depends directly on ξ. Equally, this allows us to simulate the effect

[11]The model can still fit the data reasonably well for higher exit rates and far better than a model with just a transitory shock component, yet it gets harder to match the skewness of the firm size distribution as firms with high potential and a long growth path are proportionally more likely to exit before they reach their full size.

[12]Note that the mean of a log-normal distribution is affected not only by the location parameters but also the scale parameter. We adjust it accordingly, such that for any scale parameter, $\mu = 0$ yields an average productivity of 1, when transformed to a log-normal.

Table 5: Calibrated model fit

Moment	Data	Model trans. + perm. shock	Model transitory shock
Percentage of constrained firms	0.23	0.24	0.24
Share of constrained firms in bottom 20%	0.33	0.28	0.94
Size of 90th-percentile vs. median	9.44	9.65	9.75
Size of 90-th percentile vs. bottom 20%	30.24	38.17	51.75
Size of constrained firms 90th-percentile vs. median	7.35	7.57	2.43
Size of unconstrained firms 90th-percentile vs. median	9.67	9.27	4.90
Asset share of constrained firms	0.12	0.13	0.01
Share of constrained firms in top 10% vs. bottom 20%	0.36	0.35	0
Percentage of constrained firms in top 1%	0.09	0.06	0

Notes. All moment conditions were equally weighted when minimizing the percentage deviation from the empirical target values. All constrained firms moments are calculated using constrained measure III.

of an identical financial shock in both models; 2) the model with just a transitory shock, cannot match the conditional moments such as the relative size of constrained and unconstrained firms, or the share of constrained by size. [13] Given this, we specifically target the fraction of constrained firms and moments of the unconditional size distribution, giving less weight to those moments the model with just a transitory shock could not fit in the first place. This ensures that the underlying size distribution is the same across both models and different predictions are down to differences in the distribution of constrained firms over the firm size distribution.

As documented in the far right column of Table 5, the model with just transitory shocks to firms' productivity is not able to match the data well. While it matches the fraction of constrained firms and the unconditional size distribution, it is unable to generate large constrained firms and small unconstrained firms. Hence, with constrained firms concentrated at the bottom of the size distribution, a standard model with just a transitory shock drastically underestimates the asset share of constrained firms. In contrast, when accounting for ex-ante heterogeneity by including a permanent component, and thereby breaking the strong link between size and financial conditions, the model can match the data remarkably well, as documented in the second column of Table 5.

[13] In Table C.8 in Appendix D.3 we report the calibration of the one shock model without any restrictions, with a free ξ and equal weight for all moments. As can be seen, the model is not able to match the conditional moments and the share of constrained firms across the firm distribution cannot match the empirically observed one, as illustrated in Figure D.11.

6 Discussion

In this Section we start by discussing how the model fits the three stylized facts presented in Section 3 and then explore the impacts of accounting for different permanent productivity components across the distribution of firms.

First, we show that, while a model that incorporates both transitory and permanent components of the productivity process can generate constrained firms across the entire distribution, similar to what Figure 1 suggests, a model with only a transitory component fails to account for this. The implied distributions generated by both models will then cause the model with two productivity components to match the share of total assets in constrained firms, whereas the one shock model will severely underestimate this value.

Second, we illustrate that in a standard heterogeneous firms model with only a transitory productivity shock, the model cannot account for the independent effects of size and financial constraints on the investment response to a shock. When adding the permanent productivity shock to a standard model, we can now replicate the empirical findings from Section 3.3, with both size and financial frictions playing an important, yet orthogonal role in explaining the firms' response to an aggregate productivity shock.

Finally, we assess the implications of accounting for large constrained firms when faced with an aggregate productivity shock and a financial shock, respectively. Furthermore, we compare the degree of misallocation implied by the model including a permanent productivity component to the standard model with just a transitory component.

6.1 Replicating the stylized facts

Constrained firms across the distribution In Section 3.1 we highlight that constrained firms are found across the entire distribution of firms. As illustrated in Figure 1, even at the top of the distribution in terms of size, close to 10% of the firms are financially constrained.

Figure 6 compares the model generated share of constrained firms across the size distribu-

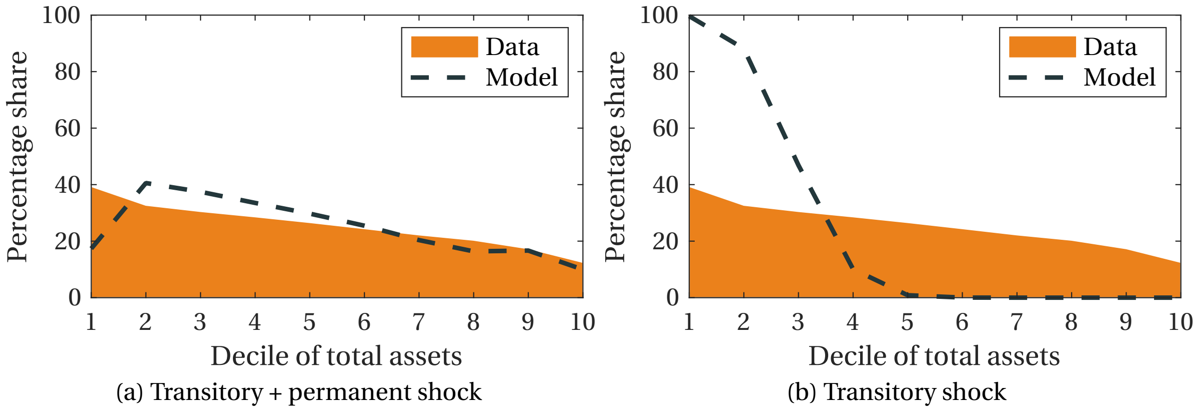

Figure 6: Share of constrained firms across the distribution.

Notes. Constrained firms are identified using measure constrained III which classifies firms as constrained if they have exhausted their potential credit and were not granted additional short- or long-term credit in that period.

tion with its empirical equivalent. When not allowing for ex-ante heterogeneity between firms, the model can still produce the same overall share of constrained firms, yet the distribution is completely off. Using only a transitory component, the model can neither generate small unconstrained firms nor large constrained firms as depicted in the right panel of Figure 6. On the other hand, the model with a transitory and a permanent productivity component generates small unconstrained firms, as well as, large constrained firms and is also able to match the untargeted deciles of the empirical distribution quite well.

The distribution generated by the model with two shocks is explained by the fact that some larger firms are still growing to reach their steady state capital and are still constrained. At the same time, the model with the two components, accounts for a larger share of small firms that are born at or close to their steady state level of capital.

Figure D.12 in Appendix D.3 offers a slightly different perspective, plotting the density distribution of constrained and unconstrained firms. It is possible to observe that while in the two shock model case the distributions overlap, in the one shock case they are completely separated, with the model only generating small constrained firms and large unconstrained ones.

Finally, The distributions generated by both models imply very different values for the share of total assets in constrained firms. Whereas the model with both a permanent and transitory shock generates a values for this moment in line with the data (13% in the model compared to 12% in the data), the model with just a transitory shock only generates 1% of total assets in constrained firms. Again, the permanent component model matches this key empirical moment whereas the simple model can not.

Size and financial constraints In Section 3.3 we showcase how size and financial frictions both affect firms' response to aggregate shocks and how the two channels appear to be orthogonal to each other. We now check how our benchmark model performs in replicating this stylized fact, and compare it to a similar model without the permanent component of the idiosyncratic productivity process.

To do this, we first run the firm simulation including aggregate productivity shocks. For now, aggregate shocks are assumed to have no persistence. This way, unconstrained firms are completely acyclical which follows from proposition 1 and hence constrained firms are more cyclical. For higher levels of persistence this is only true for a sufficient degree of cyclicality in the collateral constraint itself. Using the model generated data, we replicate the fixed effects regression from the empirical analysis as follows

$$\Delta \ln y_{i,t} = \Delta TFP_t + (\alpha_j + \beta_j \Delta TFP_t)\mathbf{1}_{i \in S_{[90,100]}} + (\zeta + \eta \Delta TFP_t)\mathbf{1}_{constrained} + \alpha_i + \epsilon_{i,t}$$

where $\Delta \ln y_{i,t}$ is firm i growth rate of output from period t to period $t+1$, ΔTFP_t is the aggregate TFP shock in period t, $\mathbf{1}_{i \in S_{[90,100]}}$ is a dummy for the firm's net worth being in the top decile of the firm distribution, $\mathbf{1}_{constrained}$ is a dummy variable equal to one if the firm is type 2 constrained firm in period t, α_i captures the firm's fixed effect due to heterogeneity in the permanent productivity component and $\epsilon_{i,t}$ is the residual.[14]

Results are presented in Table 6. As expected and outlined in proposition 1, large firms react less due to decreasing returns to scale and constrained firms are more elastic since aggregate

[14]We use the type two constrained firms in the model as these are the hard constrained firms and more comparable to the ones with zero potential credit in the data. Although, we equally test the regression accounting for both type 1 and type 2 constrained firms, and results are robust to it.

Table 6: Regression with model data.

	Output growth			
	Model trans. + perm. shock		Model transitory shock	
	(1)	(2)	(3)	(4)
ΔTFP_t	2.37	2.40	2.29	2.36
[90,100] × ΔTFP_t	-0.55	-0.55	0.25	0.15
Const. × ΔTFP_t		0.08		0.18

Notes. Each regression also includes firm fixed effects and all variables separately. Only interaction terms are reported to put focus on the orthogonality of the semi-elasticities w.r.t to size and being constrained in line with Table 2.

productivity shocks are i.i.d. However, when considering a standard model with just a transitory idiosyncratic productivity component, the coefficient estimating the semi-elasticity of large firms changes quite drastically when controlling for financial conditions. This is due to the high correlation of firm size and financial conditions in this model. When incorporating a permanent productivity component to the model the estimated semi-elasticity of the firms in the upper size decile hardly changes. This suggests that firm size and financial conditions are almost orthogonal[15] to each other, which is in line with the findings in Table 2.[16]

6.2 Aggregate effects

Aggregate productivity shock We now proceed to assess the aggregate implications of accounting for constrained firms across the entire firm size distribution. First, we consider an unexpected and temporary 1% increase in total factor productivity (TFP) as depicted in the upper left panel of Figure 7. In a direct response to the shock, firms employ more labour for any predetermined level of capital. While unconstrained firms do not increase their investment in capital due to the transitory nature of the shock, constrained firms leverage their increased net worth to borrow more. This explains why the lagged response in capital is much smaller than

[15]In fact, the correlation between firms size and being constrained is only -0.07, while it is roughly -0.90 in a model with only a transitory component.

[16]The positive size coefficient in the one shock model is opposite to the empirical evidence. This has to do with selection effects, as the largest firms in the one shock model are the unconstrained with more savings but not necessarily with the largest amount of capital, being the firms with the largest elasticity.

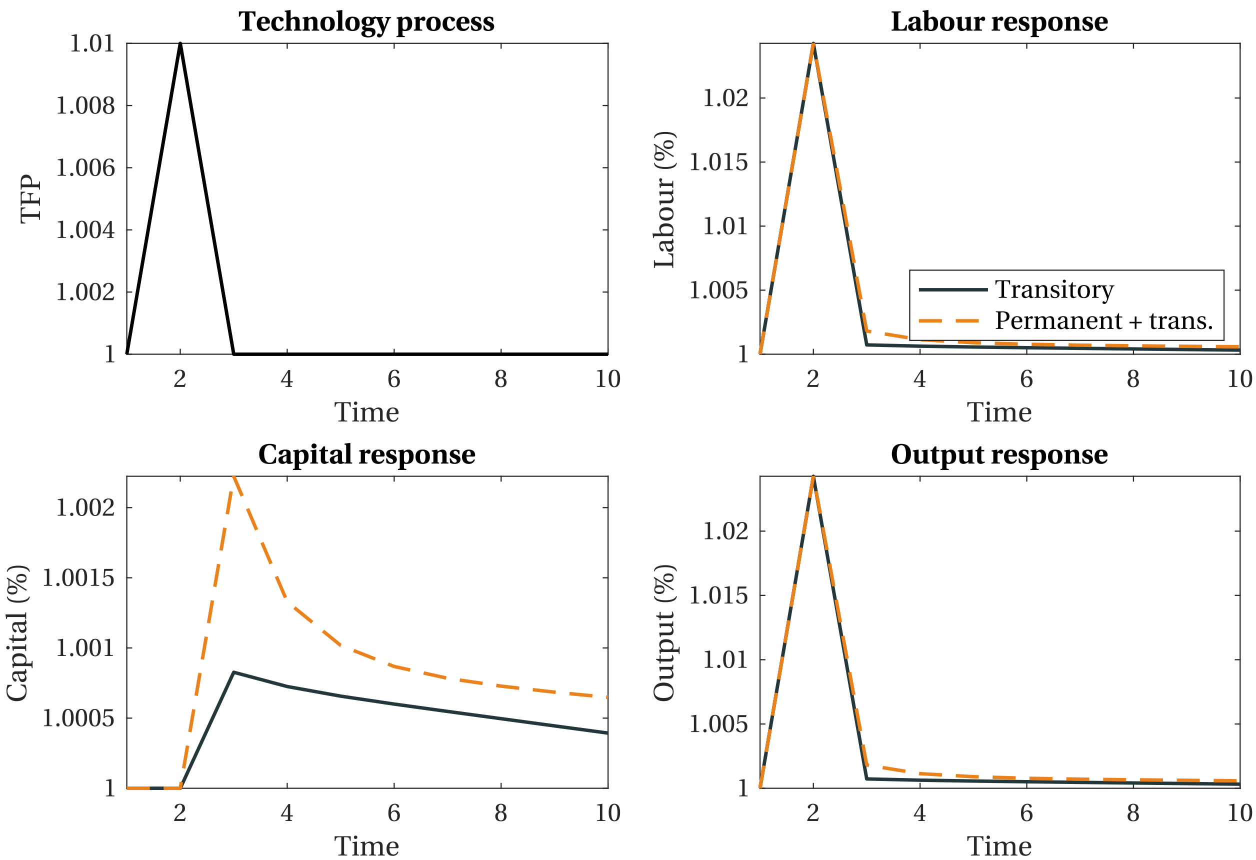

Figure 7: IRFs to the aggregate productivity shock.

Notes. Lines indicate the partial equilibrium response to a shock to overall TFP in the upper left panel, with wages fixed at their steady state level.

the response in labour, as only constrained firms react for the shown case of $\rho = 0$, which is only 23% of all firms in this calibration.

When comparing the two models, we can observe that the aggregate investment response is higher in a model with a permanent and a transitory productivity component. This is simply due to the fact that the asset share of constrained firms is substantially larger than in the transitory shock model. That is a direct consequence of the differences in the distribution of constrained firms, as highlighted in Figure 6.

In fact, Figure 8, which shows the capital elasticity over the size distribution, illustrates the key difference between both models quite well. For unconstrained firms, as already pointed out, the elasticity is zero and for constrained firms, the elasticity is decreasing with size due to decreasing returns to scale. The dashed line is indicating the unconditional average elasticity

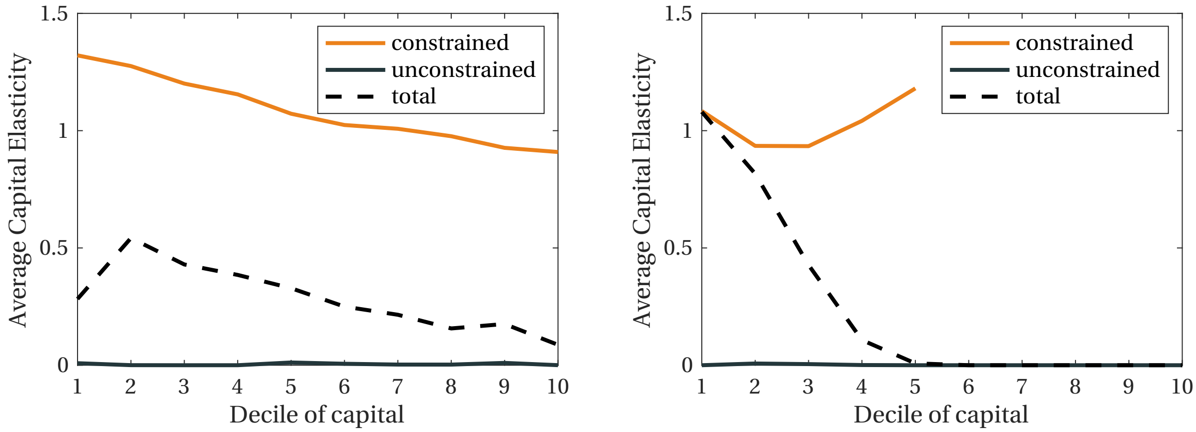

Figure 8: Conditional elasticity over the capital distribution

per decile bin. In a model with just a transitory shock, the overall elasticity is high for small constrained firms but drops to zero at some size cutoff after which all firms become unconstrained. When including a permanent component and thereby generating small unconstrained firms and large constrained firms, the average capital elasticity for small firms is lower than in the one component model but stays above zero for top quantiles of the size distribution. Hence, the capital weighted average elasticity is much higher in the model with a permanent and transitory component, and thus leading to a stronger aggregate capital response.

Further, Figure D.13 in Appendix D.3 reinforces that the mechanism comes from where the constrained firms are and not about the average elasticity of constrained firms. It depicts the elasticity density distribution across the two models. As these are not very different, the conclusion is that the large share of differential aggregate capital elasticity comes from the fact that the model features large constrained firms.

However, given the small magnitude of the capital response relative to the response in labour, the difference is barely showing up in aggregate output. Note, the effect would become stronger if the borrowing constraint was cyclical or the fraction of constrained firms in the economy was higher.[17] We proceed to investigate the former mechanism separately below.

[17] One should also note that the difference between the models would vanish and eventually flip if the TFP shock gets more persistent and unconstrained firms become more cyclical, as shown in proposition 1.

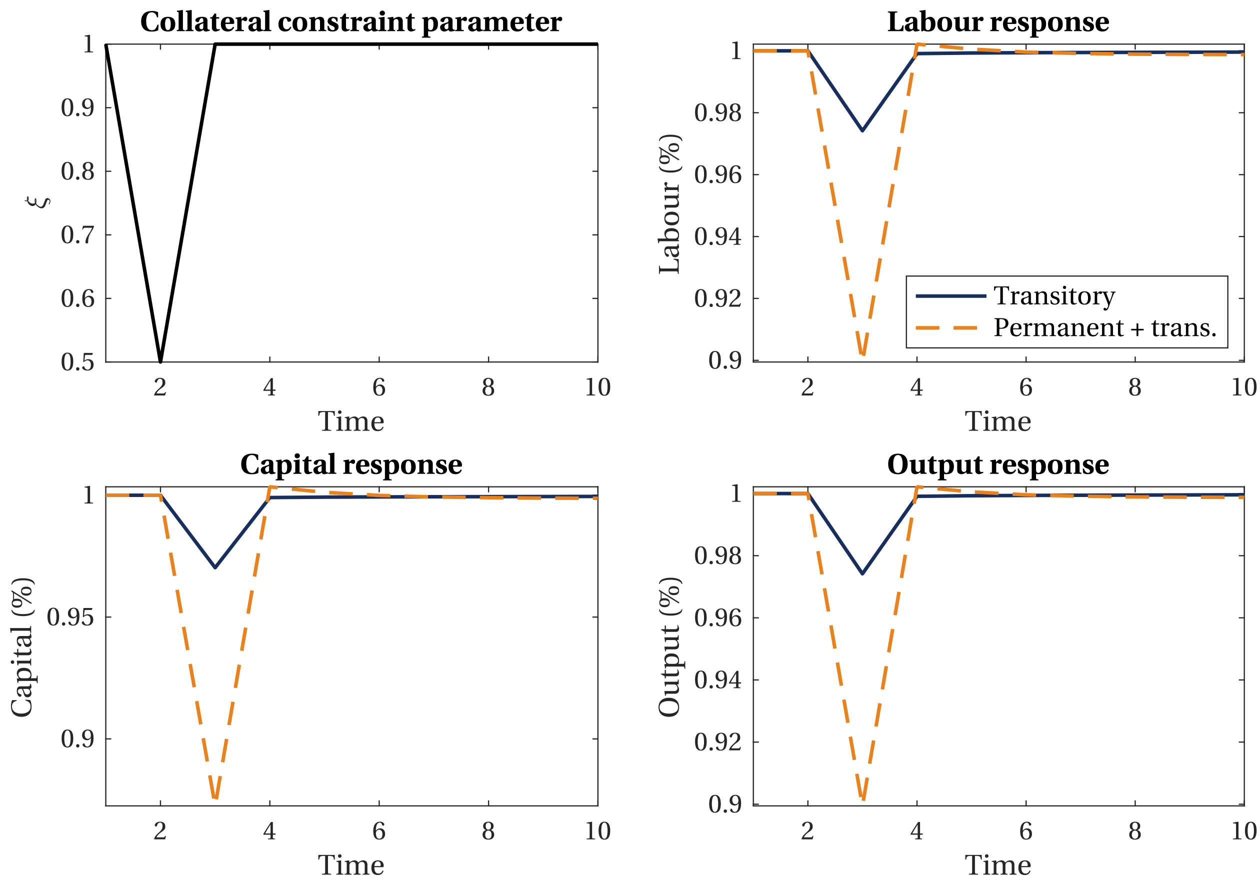

Figure 9: IRFs to a financial shock.

Notes. Lines indicate the partial equilibrium response to a shock to ξ in the upper left panel, with wages fixed at their steady state level.

Financial shock The muted aggregate amplification disappears when we examine a sudden unexpected increase in the severity of financial frictions. We assume a drop in the maximum borrowing capacity of 50%.[18] Again, given the sudden and transitory nature of the financial shock, we assume wages to be fixed at the general equilibrium level before the shock hits.[19]

Figure 9 shows the responses to the credit shock depicted in the upper left panel. Since the firm's capital stock is pre-determined, there is no direct impact in period $t = 2$, when the financial shock hits. However, the lower maximum borrowing capacity affects constrained firms in their investment decision, while unconstrained firms remain unaffected by the shock as they

[18]Khan & Thomas (2013) simulate a 88 percentage point drop in ξ. However, in their calibration the initial level of ξ is 1.38. In our calibration ξ is 0.56, hence a 50% drop equals a 28 percentage point drop in maximum borrowing allowances.

[19]General equilibrium results for this exercise lead to the same qualitative conclusions, but we prefer the partial equilibrium analysis to isolate the effect coming from the differences in the distribution of constrained firms.

finance their investment completely internally.

The resulting aggregate effect of constrained firms having to reduce their investment depends heavily on the distribution of these constrained firms along the firm size distribution. In a model with only transitory productivity shocks, all constrained firms will be concentrated at the lower end of the size distribution. When further accounting for the skewness in the firm size distribution, the capital and asset share of these constrained firms becomes marginal. Hence, despite the drastic shock to financing conditions, the aggregate responses in production factors and ultimately output is relatively minor.

However, when accounting for large constrained firms by introducing ex-ante heterogeneity via a permanent productivity component, aggregate effects get massively amplified simply due to the higher capital share of constrained firms. The quantitative magnitude of the effect clearly depends on the fraction of firms identified as being constrained by the different binary measures ranging from 36% (No potential credit) to as low as 4% (No potential credit and increasing overdue credit) of all firms. Yet, since all measures are suggestive of the notion that constrained firms exist along the entire firm size distribution, a model with just a transitory productivity component could drastically underestimate the aggregate effects of a credit shock.

Capital misallocation Besides the amplification of financial shocks, what does a more realistic distribution of financially constrained firms imply for the degree of misallocation in the economy? Pugsley et al. (2021) show that when accounting for ex-ante heterogeneity, the economy exhibits a stronger degree of misallocation. While our results in Table 7 are qualitatively in line, they suggest that the degree of misallocation can be substantially larger when accounting for the skewness in the firms' size and capital distribution, as well as, large constrained firms.

The stark result is mainly driven by firms with a high draw of permanent productivity but little initial capital. Two modelling assumptions lead to this. First, we assume independence of entry conditions and the firm's potential. Second, the firm's potential is manifested and observed when the firm enters which is quite a strong assumption and hence results should be

Table 7: Deviations from frictionless economy

	Deviations from 1st best	
	trans. + perm. shock	transitory shock
Consumption	0.949	0.982
Capital	0.894	0.952
Output	0.939	0.976
Employment	0.989	0.994
MPK deviation	0.016	0.010
MPK stdev	0.048	0.034

Notes. Reported values are relative to the models without any financial frictions, i.e. when setting the collateral constraint parameter ξ to a sufficiently large value that firms can directly implement their optimal amount of capital.

rather carefully interpreted as an upper bound of the level of misallocation.

Figures D.14 and D.15 in Appendix D.3 illustrate the mechanism explaining the larger capital misallocation. Figure D.14 showcases the density distribution of the MPKs in both models. It is possible to observe that in the two shock model there is a much higher dispersion of MPKs and average. Figure D.15 illustrates that the existence of large constrained firms also contributes to having larger MPK at the top of the distribution.

7 Conclusion

This paper documents three empirical facts that are counterfactual to the standard heterogeneous firm financial frictions model and subsequently analyses the importance of matching these facts in a quantitative financial frictions model with heterogeneous firms. Empirically we show that at any point of the firm distribution there are both constrained and unconstrained firms. These constrained firms also account for an elevated share of assets and are more cyclical, independent of size. We then proceed to demonstrate that heterogeneous ex-ante conditions, a potential explanatory factor for these facts, exist and affect constrained and unconstrained firms differently.

Next, we build a standard firm dynamic model, adding a permanent productivity component. We showcase that by adding this extra component to the productivity process helps us match the distribution of constrained firms across the size distribution, breaking the typical strong correlation between financial constraints and size, generating a large mass of small unconstrained and large constrained firms.

This mechanism has significant implications for aggregate responses to shocks. We find aggregate capital and output to respond slightly more to a productivity shock in a model that accounts for ex-ante firm heterogeneity than a model where idiosyncratic productivity is purely driven by a transitory component. More importantly, the effects of a financial shock are strongly affected with up to four times higher aggregate cyclicality compared to the standard model. This is due to large constrained firms which a model with just a transitory component is unable to generate. Finally, similar to Pugsley et al. (2021), we find a higher level of misallocation due to financial frictions when accounting for ex-ante heterogeneity driven by small firms with high potential.

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A Variable definitions

Central Credit Responsibility Database (Central de Responsabilidades de Crédito)

Identifier (tina) Anonymized tax identificaton number.

Global Credit (valor_global) is the sum of regular credit and potential credit, representing the total available credit that a firm accesses.

Regular Credit (valor_efectivo) is credit effectively used in a regular situation, i.e., without payment delays as defined in the respective contract. Examples of effective responsibilities are:

- Loans for the acquisition of financial instruments (shares, bonds, etc.);
- Discount and other credits secured by effects;
- Overdrafts on bank accounts;
- Leasing and factoring;
- Used amounts of credit cards.

Potential Credit (valor_potencial) represents irrevocable commitments of the participating entities. Banco de Portugal requires all credit-granting institutions to report to the CCR their outstanding loan exposure by instrument of all irrevocable credit obligations. Examples of potential responsibilities are:

- Unused amounts of credit cards;
- Lines of credit;
- Guarantees provided by participating entities;
- Guarantees and guarantees given in favor of the participating entities;
- Any other credit facilities likely to be converted into effective debts.

Overdue Credit (valor_vencido) All outstanding credit exposures recorded as non-performing (including overdue, written off, renegotiated credit, overdue credit in litigation, and written off credit in litigation) are aggregated to calculate overdue credits. It includes principal, interest and related fees.

Short-term Credit (valor_curto) Short-term credit is calculated using two different definitions. In the first place, short-term credit is defined based on the term-to-maturity as agreed in the credit contract, denoted by valor_curto_o. Specifically, short-term credit has an original maturity of equal to or less than one year. Before 2009, the CCR dataset did not streamline credit exposure based on the maturity structure. Therefore, for the data before 2009, the short-term credit is defined as the aggregation of commercial credit, discount funding, and other short-term funding, which are short-term funding by their nature. In the second place, short-term credit is defined based on residual maturity – the remaining time until the expiration or the repayment of the instrument, denoted by valor_curto_r. Specifically, it is credit with a residual maturity of equal to or less than one year. This variable is only available from 2009 onwards. Potential credit is excluded for both calculations.

Long-term Credit (valor_longo) Similar to short-term credit, long-term credit is defined based on original and residual maturities. More precisely, long-term credit is credit with an original or residual maturity of more than one year, denoted by valor_longo_o and valor_longo_r, respectively. Long-term credit defined on an original maturity basis (valor_longo_o) for the data before 2009 is the aggregation of total credit excluding commercial credit (type 1), discount funding (type 2), and other short-term funding (type 3). Potential credit is excluded for both calculations.

B Descriptive statistics

Table B.1 reports correlations between all measures. By construction, some correlation coefficients can be relatively low, as some measures represent subsets of others. In fact, correlation coefficients between those binary measures augmenting the baseline measure can be interpreted as the fraction of the subset relative to the baseline definition of being strictly constrained. The correlation coefficients between binary and continuous measures are close to zero, as the variance in the continuous measure is too high to be captured by a binary measure, underlining the importance of including continuous measures in the analysis. The negative sign is simply due to the definition of the continuous measures, as higher values are an indication for being less constrained.

In Table B.3 we report medians for different size related variables for the universe of firms in our panel dataset. Following Crouzet & Mehrotra (2020), we split firms in size bins based on their total asset amount. Furthermore, we document descriptive statistics for the mutually exclusive subsets of constrained and unconstrained firms, based on the baseline measure. We can observe that the median financially constrained firm has less assets, less turnover, is younger and employs less people compared to the median unconstrained firm. Table B.2 reports medians of financial variables for size bins and the subset of constrained and unconstrained firms. Financial variables are expressed in terms of ratios relative to the total amount of assets a firm owns. While the median of smaller firms tends to have a higher leverage ratio, yet also a higher liquidity ratio, there is hardly any difference between the median constrained and the median unconstrained firm. This is suggestive of the fact, that when controlling for size, there are constrained and unconstrained firms across the distribution of financial variables.

Table B.1: Correlation between different measures for being constrained

Constrained measure	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) $\mathbf{1}_{\text{Potential credit}_t=0}$	1						
(2) $\mathbf{1}_{\text{Pot. credit}_t=0\ \&\ \text{Pot. credit}_{t-1}>0}$	0.31***	1					
(3) $\mathbf{1}_{\text{Pot. credit}_t=0\ \&\ \Delta\text{Effective credit}_{t+1}<0}$	0.82***	0.24***	1				
(4) $\mathbf{1}_{\text{Pot. credit}_t=0\ \&\ \Delta\text{Overdue credit}_{t+1}>0}$	0.26***	0.12***	0.28***	1			
(5) $\mathbf{1}_{\text{Pot. credit}_t=0} + \dots$	0.73***	0.23***	0.60***	0.19***	1		
(6) $\frac{\text{Pot. credit}_t + \text{Cash}_t}{\text{Liabilities}_t}$	-0.00	-0.00	-0.00	-0.00	-0.00	1	
(7) $\frac{\text{Pot. credit}_t + \Delta\text{Short/Long term credit}_{t+1}}{\text{Liabilities}_t}$	-0.00*	-0.00	-0.00**	-0.00	-0.00	0.33***	1

Table B.2: Financial characteristics

Size group	0 - 90th	90th-99th	99-99.5th	>99.5th	constrained	unconstrained
Dividends (€ mio.)	0.00	0.04	0.34	1.28	0.00	0.00
Fixed tangible (€ mio.)	0.04	0.92	4.20	8.23	0.03	0.06
Investment (€ mio.)	-0.00	-0.01	-0.03	-0.02	-0.00	-0.00
Financial investments (€ mio.)	0.00	0.00	0.08	4.77	0.00	0.00
Equity (€ mio.)	0.06	1.43	7.42	26.18	0.04	0.10
Liabilities (€ mio.)	0.18	3.06	15.28	47.67	0.16	0.25
Total income (€ mio.)	0.19	2.63	10.93	25.18	0.12	0.33
EBIT (€ mio.)	0.01	0.12	0.65	2.30	0.00	0.01
Leverage	0.20	0.25	0.21	0.12	0.20	0.20
Liquidity ratio	0.06	0.02	0.01	0.01	0.04	0.05
Potential credit (€ mio.)	0.00	0.10	0.51	1.36	0.00	0.02
Effective credit (€ mio.)	0.04	1.12	4.90	9.98	0.03	0.06
Bank relationships	2	3	4	4	1	2

Table B.3: Size and age

Size group	0 - 90th	90th-99th	99-99.5th	>99.5th	constrained	unconstrained
Assets (€ mio.)	0.25	4.64	23.07	77.09	0.21	0.39
Sales (€ mio.)	0.02	1.01	2.06	0.11	0.00	0.07
Turnover (€ mio.)	0.02	2.50	9.81	15.91	0.12	0.31
Age	12	20	22	21	11	14
Employees	4	18	50	56	2	5

C Additional Tables

Table C.1: Cyclicality in sales (services) conditional on size bins and measures of financial constraints

	Sales growth							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
GDP growth	2.305***	2.230***	2.260***	2.261***	2.274***	2.249***	2.283***	2.290***
	(0.042)	(0.048)	(0.042)	(0.052)	(0.047)	(0.053)	(0.042)	(0.047)
[90,99] × GDP growth	0.212	0.257*	0.239	0.325*	0.306*	0.230	0.212	0.294*
	(0.150)	(0.151)	(0.150)	(0.171)	(0.170)	(0.151)	(0.150)	(0.170)
[99,99.5] × GDP growth	-1.042**	-1.000**	-1.016**	-1.389***	-1.412***	-1.015**	-1.047**	-1.399***
	(0.444)	(0.444)	(0.444)	(0.529)	(0.528)	(0.444)	(0.444)	(0.529)
[99.5,100] × GDP growth	-1.669***	-1.614***	-1.635***	-1.795***	-1.789***	-1.640***	-1.681***	-1.839***
	(0.470)	(0.471)	(0.470)	(0.571)	(0.571)	(0.471)	(0.470)	(0.571)
Const.I × GDP growth		0.176*						
		(0.098)						
Const.II × GDP growth			0.310					
			(0.222)					
Const.III × GDP growth				0.204				
				(0.127)				
Const.IV × GDP growth					0.915**			
					(0.430)			
Const.V × GDP growth						0.109		
						(0.085)		
Const.VI × GDP growth							-0.154***	
							(0.055)	
Const.VII × GDP growth								-0.183***
								(0.061)
Observations	805,053	805,053	805,053	709,948	709,948	805,053	804,639	709,661
R-squared	0.023	0.024	0.024	0.024	0.025	0.023	0.023	0.024
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm

Notes. Estimates report the semi-elasticity of sales with respect to GDP. The first line is based on the equivalent regression without interaction fixed effects, as it is otherwise dropped due to multicollinearity, and included as a benchmark. Constrained measures are constructed as documented in Appendix B. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table C.2: Cyclicality in employees conditional on size bins and measures of financial constraints

	Employee growth							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
GDP growth	1.036***	1.026***	1.010***	1.125***	1.105***	1.004***	1.040***	1.111***
	(0.013)	(0.016)	(0.013)	(0.017)	(0.015)	(0.017)	(0.013)	(0.015)
[90,99] × GDP growth	-0.004	-0.001	0.007	-0.121**	-0.118**	-0.006	-0.008	-0.121**
	(0.046)	(0.047)	(0.046)	(0.052)	(0.052)	(0.047)	(0.046)	(0.052)
[99,99.5] × GDP growth	-0.042	-0.044	-0.033	-0.196	-0.188	-0.034	-0.049	-0.185
	(0.132)	(0.132)	(0.132)	(0.153)	(0.153)	(0.132)	(0.132)	(0.153)
[99.5,100] × GDP growth	-0.330**	-0.330**	-0.318**	-0.398**	-0.385**	-0.316**	-0.341**	-0.401**
	(0.149)	(0.149)	(0.149)	(0.177)	(0.176)	(0.149)	(0.149)	(0.176)
Const.I × GDP growth		-0.026						
		(0.029)						
Const.II × GDP growth			0.129*					
			(0.067)					
Const.III × GDP growth				-0.032				
				(0.036)				
Const.IV × GDP growth					0.306**			
					(0.120)			
Const.V × GDP growth						0.063**		
						(0.026)		
Const.VI × GDP growth							-0.074***	
							(0.013)	
Const.VII × GDP growth								-0.087***
								(0.016)
Observations	1,289,884	1,289,884	1,289,884	1,128,989	1,128,989	1,289,884	1,288,586	1,128,066
R-squared	0.024	0.024	0.025	0.025	0.026	0.025	0.024	0.025
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm

Notes. Estimates report the semi-elasticity of employees with respect to GDP. The first line is based on the equivalent regression without interaction fixed effects, as it is otherwise dropped due to multicollinearity, and included as a benchmark. Constrained measures are constructed as documented in Appendix B. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table C.3: Cyclicality in turnover conditional on size bins and measures of financial constraints including time fixed effects

	Turnover growth							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
GDP growth	2.512***	2.441***	2.440***	2.450***	2.470***	2.450***	2.522***	2.508***
	(0.023)	(0.027)	(0.023)	(0.028)	(0.025)	(0.030)	(0.023)	(0.025)
[90,99] × GDP growth	0.146	0.176*	0.175*	0.168	0.157	0.161	0.125	0.139
	(0.101)	(0.102)	(0.101)	(0.116)	(0.116)	(0.102)	(0.101)	(0.116)
[99,99.5] × GDP growth	-0.732**	-0.702**	-0.700**	-0.714**	-0.725**	-0.709**	-0.764***	-0.734**
	(0.288)	(0.288)	(0.288)	(0.328)	(0.328)	(0.288)	(0.288)	(0.328)
[99.5,100] × GDP growth	-1.608***	-1.574***	-1.571***	-1.750***	-1.745***	-1.581***	-1.643***	-1.792***
	(0.253)	(0.253)	(0.253)	(0.317)	(0.316)	(0.253)	(0.253)	(0.316)
Const.I × GDP growth		0.088*						
		(0.053)						
Const.II × GDP growth			0.643***					
			(0.135)					
Const.III × GDP growth				0.148**				
				(0.067)				
Const.IV × GDP growth					1.516***			
					(0.249)			
Const.V × GDP growth						0.110***		
						(0.047)		
Const.VI × GDP growth							-0.164***	
							(0.024)	
Const.VII × GDP growth								-0.202***
								(0.030)
Observations	1,326,793	1,326,793	1,326,793	1,161,594	1,161,594	1,326,793	1,325,440	1,160,600
R-squared	0.017	0.017	0.018	0.017	0.020	0.017	0.017	0.017
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm

Notes. Estimates report the semi-elasticity of turnover with respect to GDP. The first line is based on the equivalent regression without interaction fixed effects, as it is otherwise dropped due to multicollinearity, and included as a benchmark. Constrained measures are constructed as documented in Appendix B. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table C.4: Cyclicality in turnover conditional on size bins and measures of financial constraints including firm fixed effects

	Turnover growth							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
GDP growth	2.512***	2.441***	2.440***	2.450***	2.470***	2.450***	2.522***	2.508***
	(0.023)	(0.027)	(0.023)	(0.028)	(0.025)	(0.030)	(0.023)	(0.025)
[90,99] × GDP growth	0.045	0.077	0.062	0.110	0.101	0.060	0.031	0.085
	(0.103)	(0.104)	(0.103)	(0.117)	(0.116)	(0.104)	(0.103)	(0.116)
[99,99.5] × GDP growth	-0.788***	-0.756**	-0.772***	-0.683**	-0.688**	-0.764***	-0.811***	-0.703**
	(0.296)	(0.296)	(0.296)	(0.339)	(0.338)	(0.296)	(0.296)	(0.339)
[99.5,100] × GDP growth	-1.493***	-1.456***	-1.474***	-1.744***	-1.739***	-1.468***	-1.516***	-1.779***
	(0.260)	(0.261)	(0.260)	(0.320)	(0.319)	(0.261)	(0.260)	(0.320)
Const.I × GDP growth		0.152***						
		(0.055)						
Const.II × GDP growth			0.363***					
			(0.139)					
Const.III × GDP growth				0.178**				
				(0.070)				
Const.IV × GDP growth					1.774***			
					(0.267)			
Const.V × GDP growth						0.100***		
						(0.049)		
Const.VI × GDP growth							-0.089***	
							(0.024)	
Const.VII × GDP growth								-0.143***
								(0.031)
Observations	1,324,676	1,324,676	1,324,676	1,158,814	1,158,814	1,324,676	1,323,310	1,157,805
R-squared	0.150	0.151	0.151	0.160	0.161	0.150	0.151	0.160
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm

Notes. Estimates report the semi-elasticity of turnover with respect to GDP. The first line is based on the equivalent regression without interaction fixed effects, as it is otherwise dropped due to multicollinearity, and included as a benchmark. Constrained measures are constructed as documented in Appendix B. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table C.5: Cyclicality in turnover conditional on size bins and measures of financial constraints excluding firms that have 0 potential credit in all periods

	Turnover growth							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
GDP growth	2.512***	2.441***	2.440***	2.450***	2.470***	2.450***	2.522***	2.508***
	(0.023)	(0.027)	(0.023)	(0.028)	(0.025)	(0.030)	(0.023)	(0.025)
[90,99] × GDP growth	0.076	0.081	0.113	0.035	0.035	0.075	0.079	0.025
	(0.106)	(0.106)	(0.106)	(0.120)	(0.120)	(0.106)	(0.106)	(0.120)
[99,99.5] × GDP growth	-0.734**	-0.738**	-0.693**	-0.726**	-0.722**	-0.725**	-0.736**	-0.722**
	(0.300)	(0.300)	(0.300)	(0.343)	(0.343)	(0.300)	(0.300)	(0.343)
[99.5,100] × GDP growth	-1.378***	-1.378***	-1.327***	-1.617***	-1.598***	-1.365***	-1.377***	-1.631***
	(0.283)	(0.283)	(0.283)	(0.345)	(0.344)	(0.283)	(0.283)	(0.344)
Const.I × GDP growth		-0.005						
		(0.066)						
Const.II × GDP growth			0.611***					
			(0.135)					
Const.III × GDP growth				0.127				
				(0.086)				
Const.IV × GDP growth					1.383***			
					(0.327)			
Const.V × GDP growth						0.046		
						(0.052)		
Const.VI × GDP growth							-0.051*	
							(0.028)	
Const.VII × GDP growth								-0.164***
								(0.032)
Observations	1,167,541	1,167,541	1,167,541	1,027,839	1,027,839	1,167,541	1,166,458	1,027,039
R-squared	0.031	0.031	0.032	0.031	0.033	0.031	0.031	0.031
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm

Notes. Estimates report the semi-elasticity of turnover with respect to GDP. The first line is based on the equivalent regression without interaction fixed effects, as it is otherwise dropped due to multicollinearity, and included as a benchmark. Constrained measures are constructed as documented in Appendix B. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table C.6: Cyclicality in turnover conditional on size bins and measures of financial constraints including bank controls

	Turnover growth							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
GDP growth	2.512***	2.441***	2.440***	2.450***	2.470***	2.450***	2.522***	2.508***
	(0.023)	(0.027)	(0.023)	(0.028)	(0.025)	(0.030)	(0.023)	(0.025)
[90,99] × GDP growth	0.095	0.092	0.098	0.118	0.083	0.088	0.064	0.070
	(0.108)	(0.107)	(0.122)	(0.107)	(0.121)	(0.108)	(0.107)	(0.121)
[99,99.5] × GDP growth	-0.685**	-0.688**	-0.744**	-0.675**	-0.758**	-0.677**	-0.723**	-0.764**
	(0.302)	(0.302)	(0.344)	(0.301)	(0.343)	(0.302)	(0.302)	(0.344)
[99.5,100] × GDP growth	-1.355***	-1.365***	-1.572***	-1.328***	-1.582***	-1.356***	-1.403***	-1.621***
	(0.295)	(0.294)	(0.363)	(0.294)	(0.361)	(0.295)	(0.294)	(0.362)
Const.I × GDP growth		0.140**						
		(0.059)						
Const.II × GDP growth			0.540***					
			(0.144)					
Const.III × GDP growth				0.195***				
				(0.075)				
Const.IV × GDP growth					1.274***			
					(0.263)			
Const.V × GDP growth						0.173***		
						(0.050)		
Const.VI × GDP growth							-0.055**	
							(0.026)	
Const.VII × GDP growth								-0.182***
								(0.032)
Observations	1,241,471	1,241,471	1,091,547	1,241,471	1,091,547	1,241,471	1,240,272	1,090,662
R-squared	0.031	0.031	0.030	0.038	0.033	0.030	0.030	0.030
Bank Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm	Firm	Firm

Notes. Estimates report the semi-elasticity of turnover with respect to GDP. The first line is based on the equivalent regression without interaction fixed effects, as it is otherwise dropped due to multicollinearity, and included as a benchmark. Constrained measures are constructed as documented in Appendix B. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table C.7: Linear probability regression: How age, total assets, leverage and liquidity ratio affect the probability of being constrained.

	Constrained binary			
	(1)	(2)	(3)	(4)
Age	-0.05***			
	(0.000)			
Total assets		-0.05***		
		(0.001)		
Leverage			0.03***	
			(0.000)	
Liquidity ratio				0.01***
				(0.000)
Constant	0.36***	0.36***	0.36***	0.36***
Observations	1,765,288	1,765,288	1,764,947	1,764,947
R-squared	0.011	0.000	0.000	0.000

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table C.8: Calibration fit for the 1 shock model with no restrictions

Moment	Data	Model transitory shock
Percentage of constrained firms	0.23	0.20
Share of constrained firms in bottom 20%	0.33	0.85
Size of 90th-percentile vs. median	9.44	9.08
Size of 90th percentile vs. bottom 20%	30.24	42.65
Size of constrained firms 90th-percentile vs. median	7.35	2.20
Size of unconstrained firms 90th-percentile vs. median	9.67	5.19
Asset share of constrained firms	0.12	0.01
Share of constrained firms in top 10% vs. bottom 20%	0.36	0
Percentage of constrained firms in top 1%	0.09	0

Notes. All moment conditions were equally weighted when minimizing the percentage deviation from the empirical target values.

Table C.9: Static model parameters for constrained and unconstrained firms using constrained measure I.

	ρ_u	ρ_v	ρ_w	σ_θ	σ_u	σ_v	σ_ϵ	σ_z
Constrained	0.471	0.867	0.901	0.276	0.741	0.642	0.277	0.184
Unconstrained	0.395	0.729	0.876	0.453	0.677	0.784	0.301	0.190

D Additional Figures

D.1 Descriptive Figures

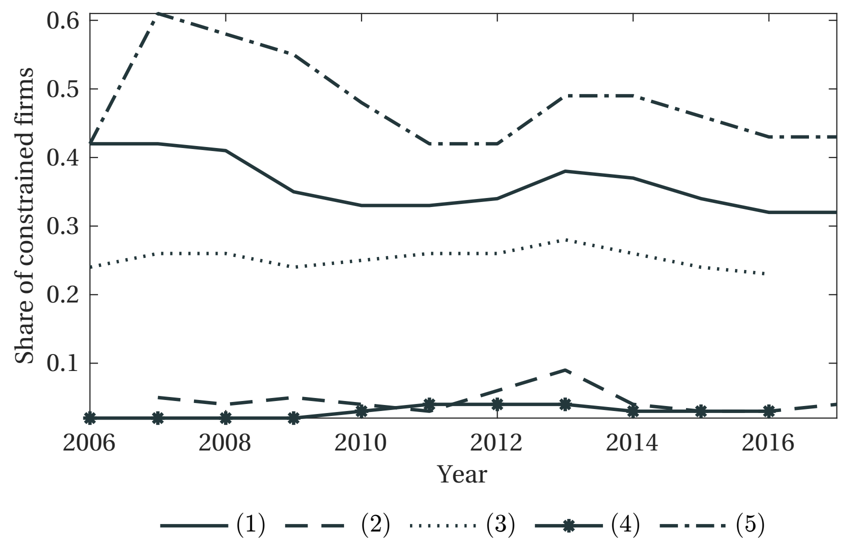


Figure D.1: Share of constrained firms over time. Measures 1 to 5 as defined in Section 2.1.

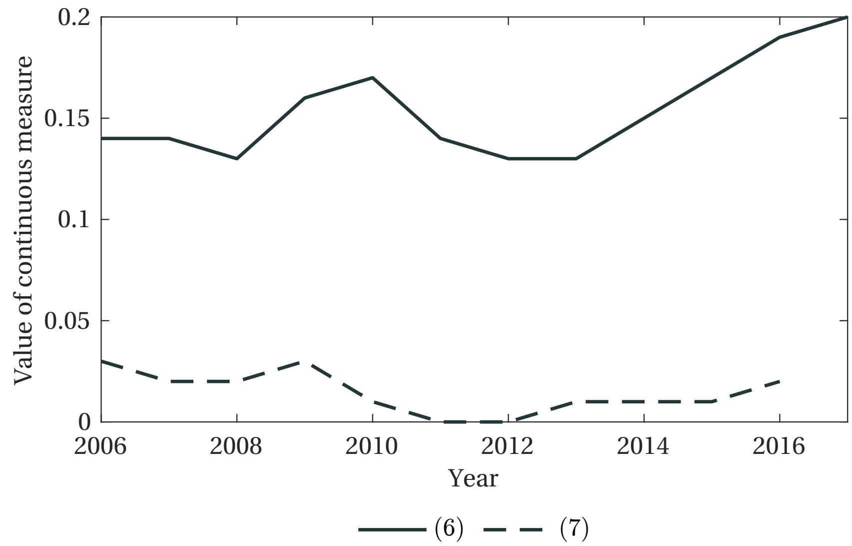


Figure D.2: Share of constrained firms over time. Measures 6 and 7 as defined in Section 2.1.

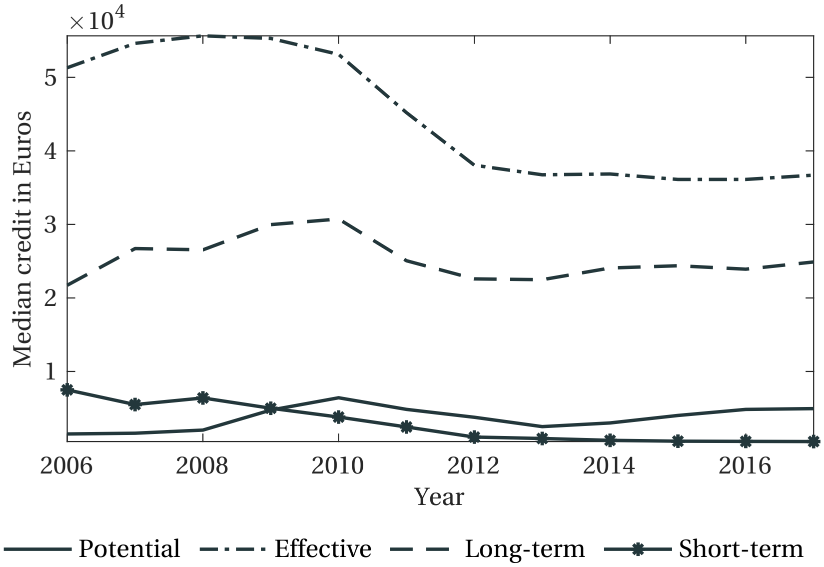


Figure D.3: Median values for Potential, Effective, Long-term and Short-term credit over time.

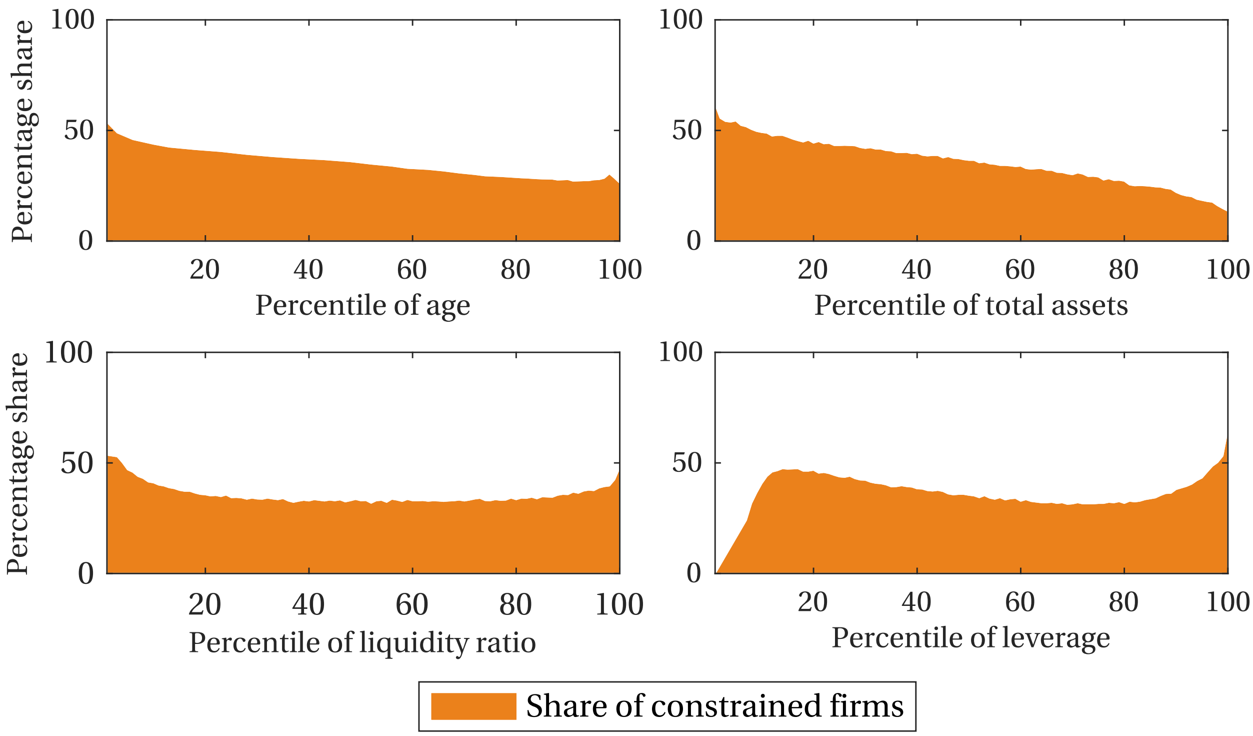


Figure D.4: Decomposition of constrained and unconstrained firms across percentiles of firm variables using measure Constrained I.

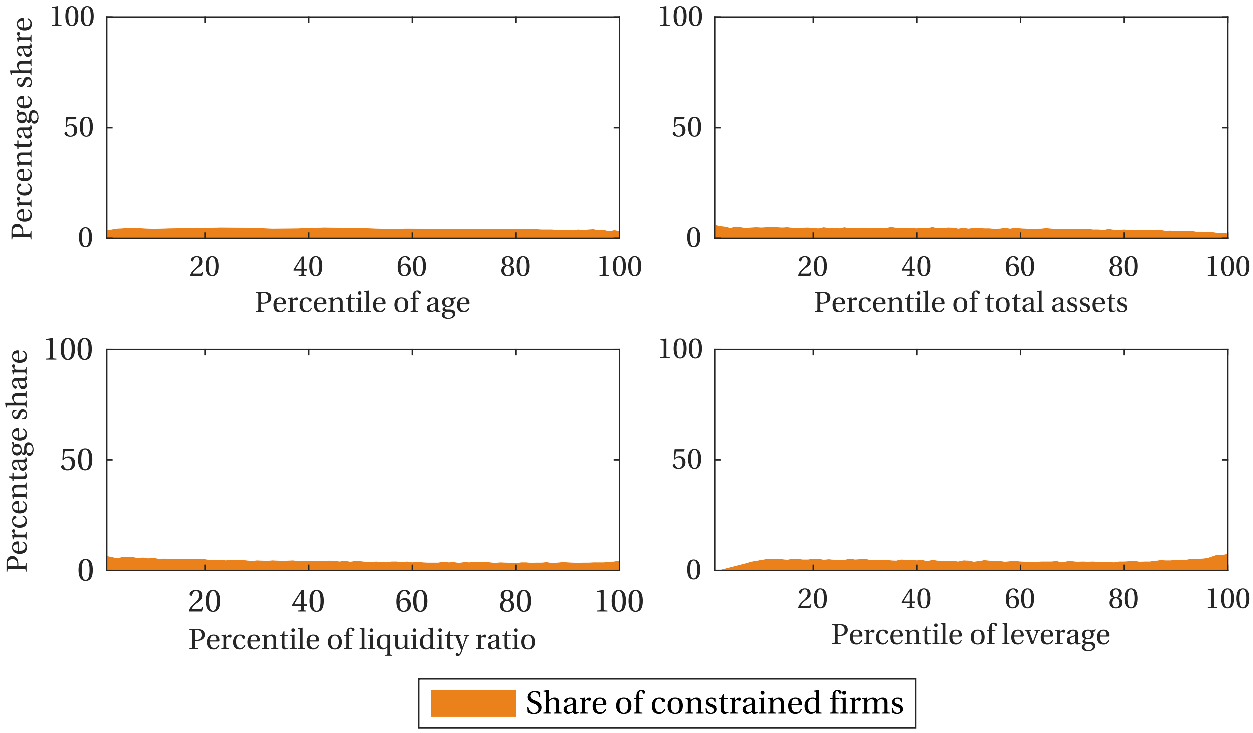


Figure D.5: Decomposition of constrained and unconstrained firms across percentiles of firm variables using measure Constrained II.

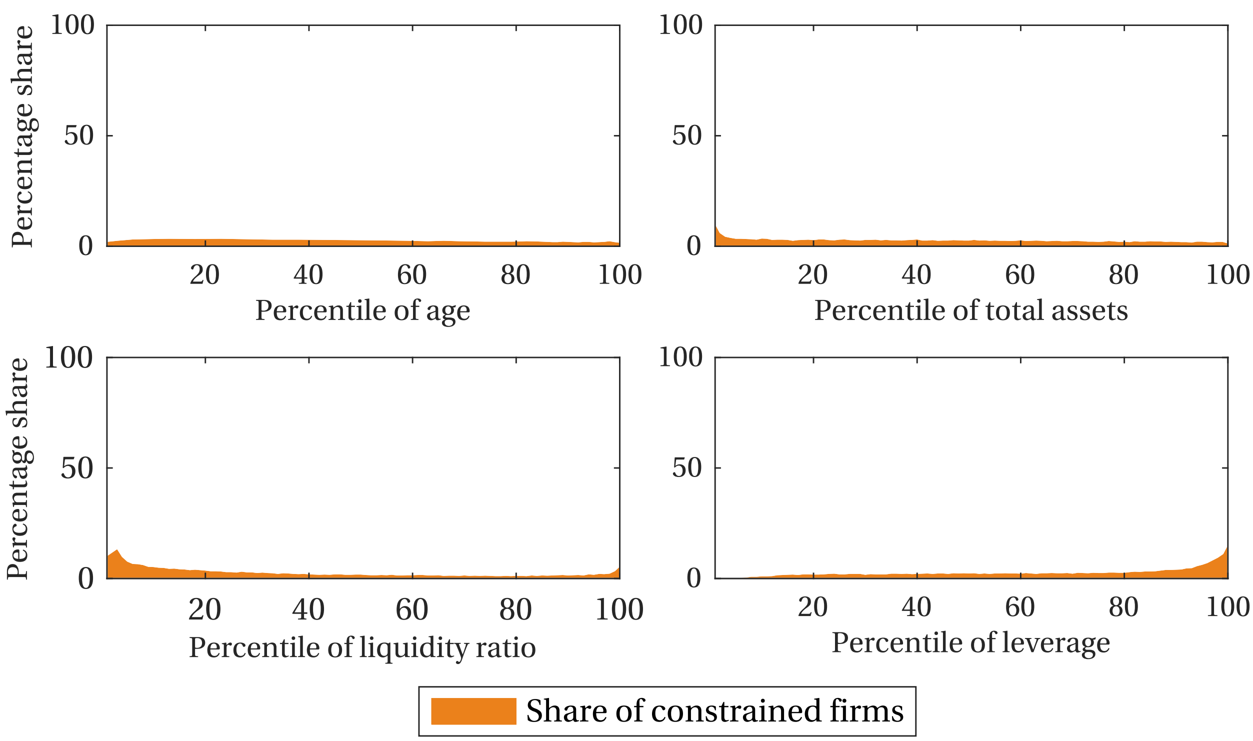


Figure D.6: Decomposition of constrained and unconstrained firms across percentiles of firm variables using measure Constrained IV

D.2 Statistical Model

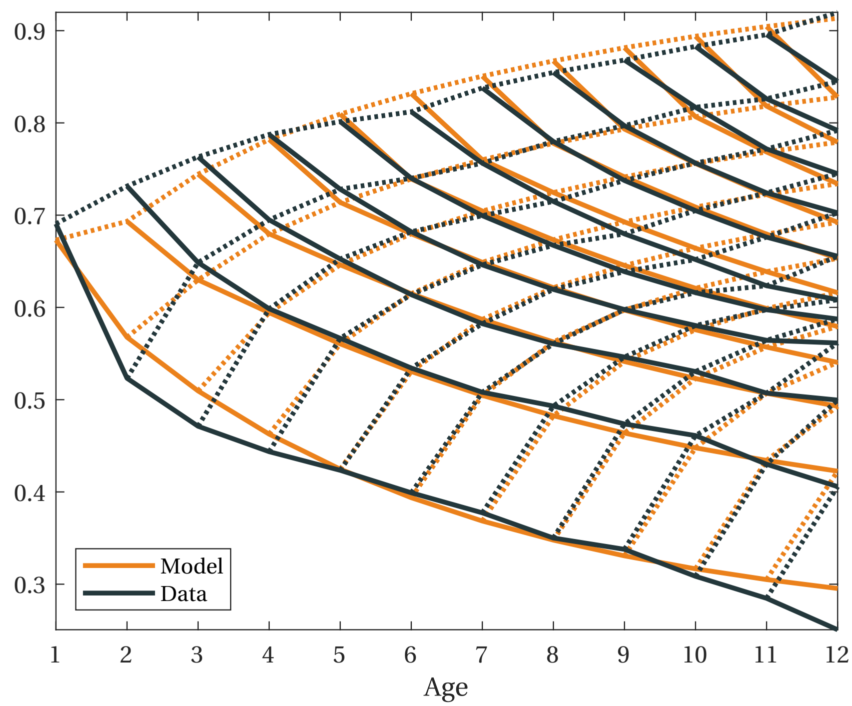
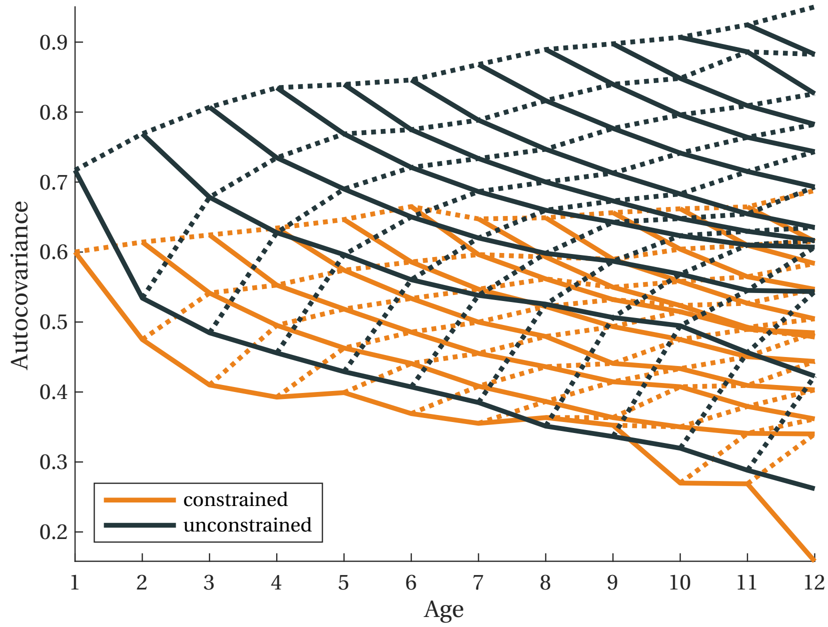
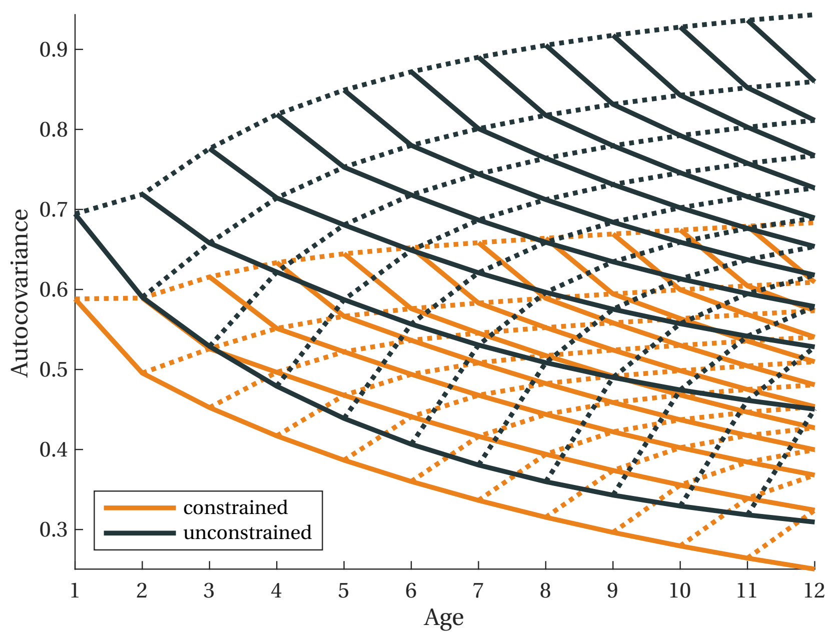


Figure D.7: Model fit of statistical model for employment process

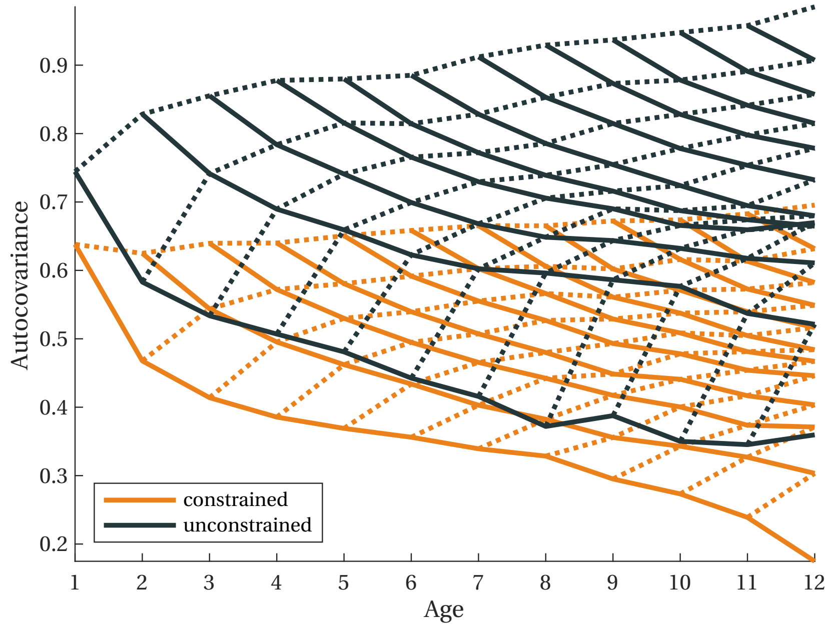


(a) Empirical conditional autocovariances

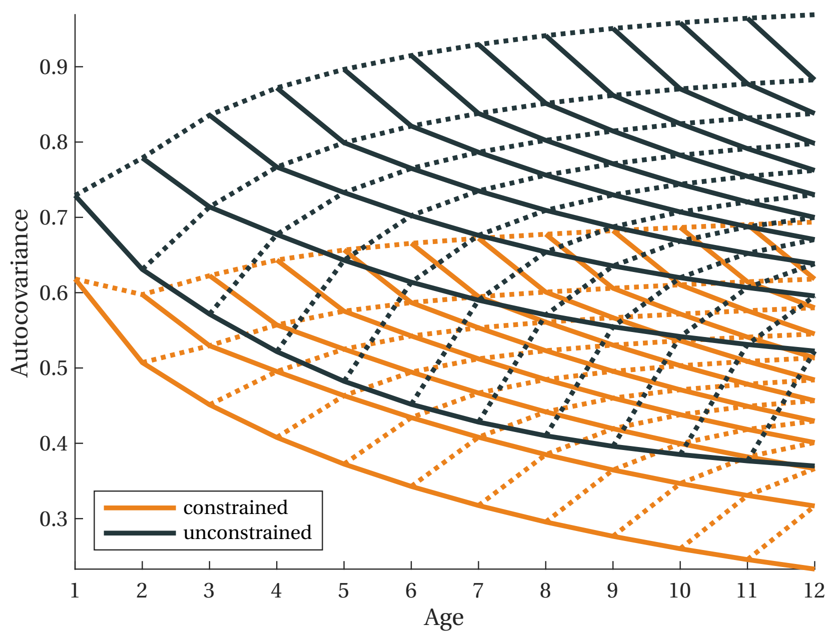


(b) Fitted conditional autocovariances

Figure D.8: Empirical and model autocovariance for constrained firms (orange) and unconstrained firms (blue) using the measure Constrained III.



(a) Empirical conditional autocovariances



(b) Fitted conditional autocovariances

Figure D.9: Empirical and model autocovariance for constrained firms (orange) and unconstrained firms (blue) using the measure Constrained I.

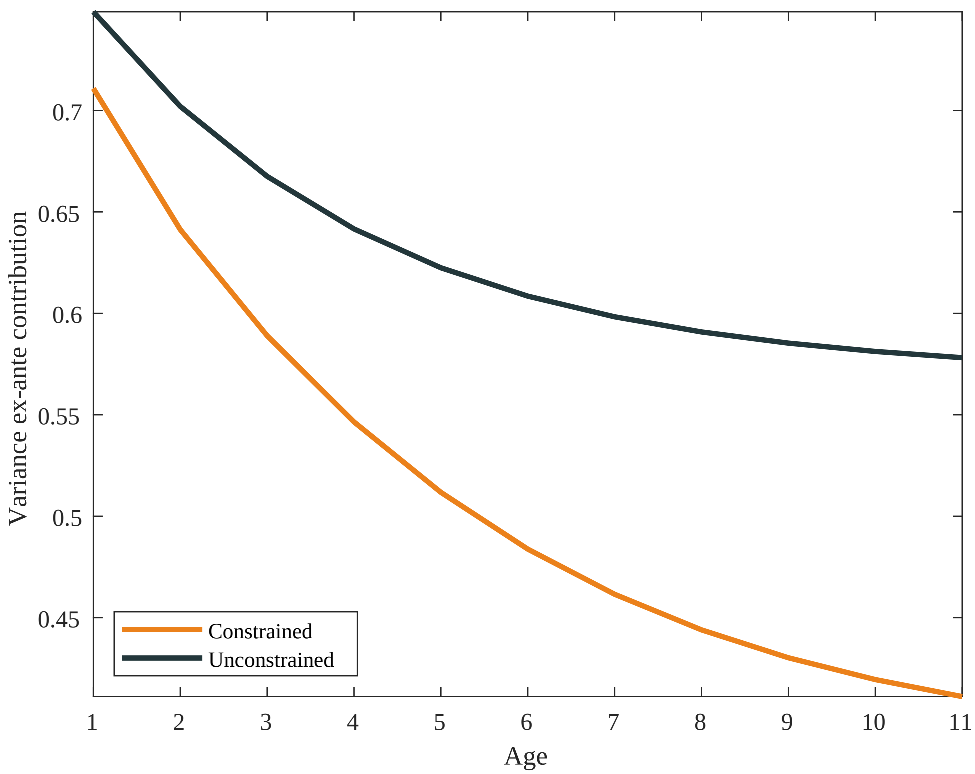
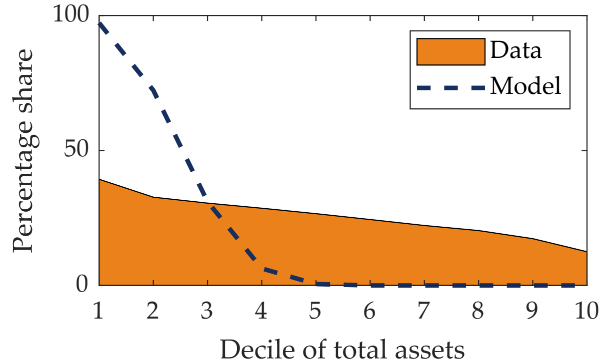


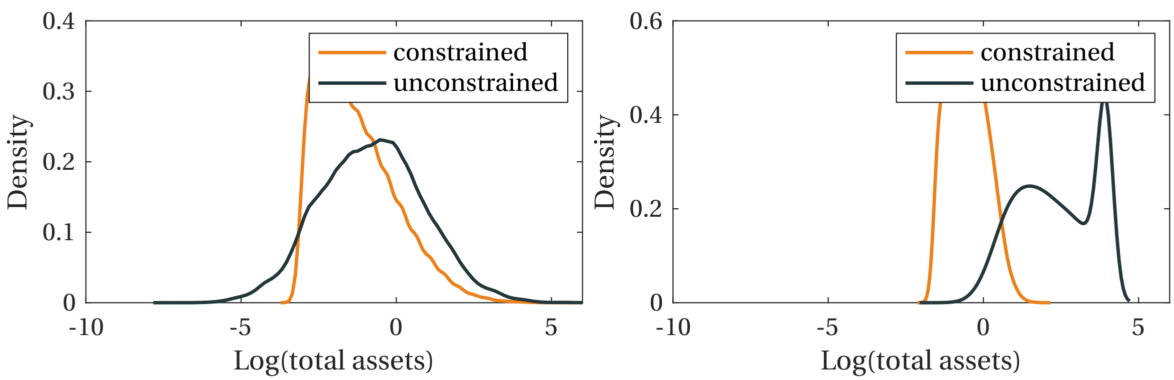
Figure D.10: Model implied contribution of the ex-ante component to the variance of employment for different ages. Constrained firms are identified using measure Constrained I.

D.3 Theoretical model



(a) Transitory shock

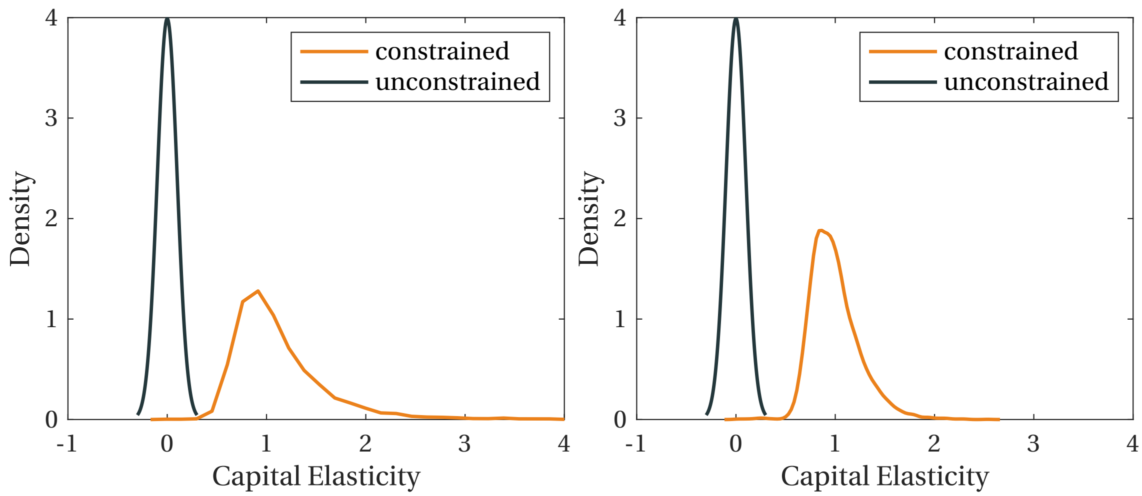
Figure D.11: Share of constrained firms across the distribution in the 1 shock model with calibration in Table C.8.



(a) Transitory + permanent shock

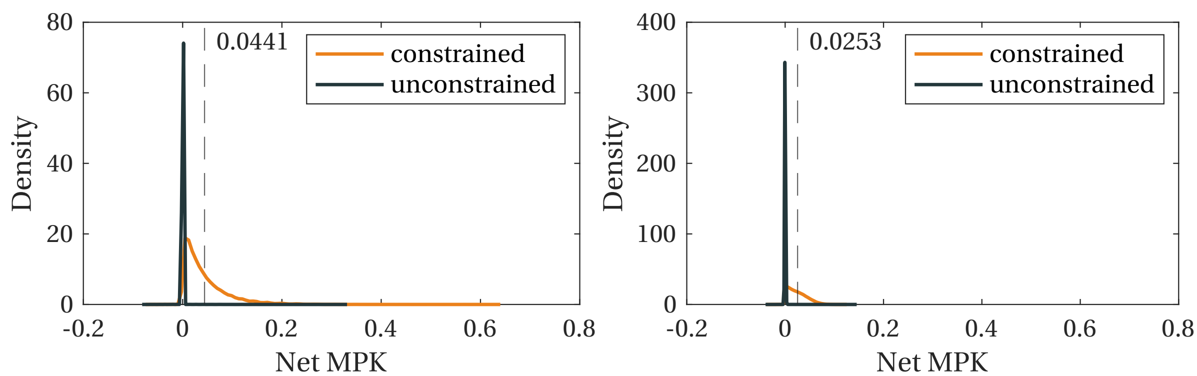
(b) Transitory shock

Figure D.12: Conditional distributions of log of total assets implied by the model



(a) Transitory + permanent shock (b) Transitory shock

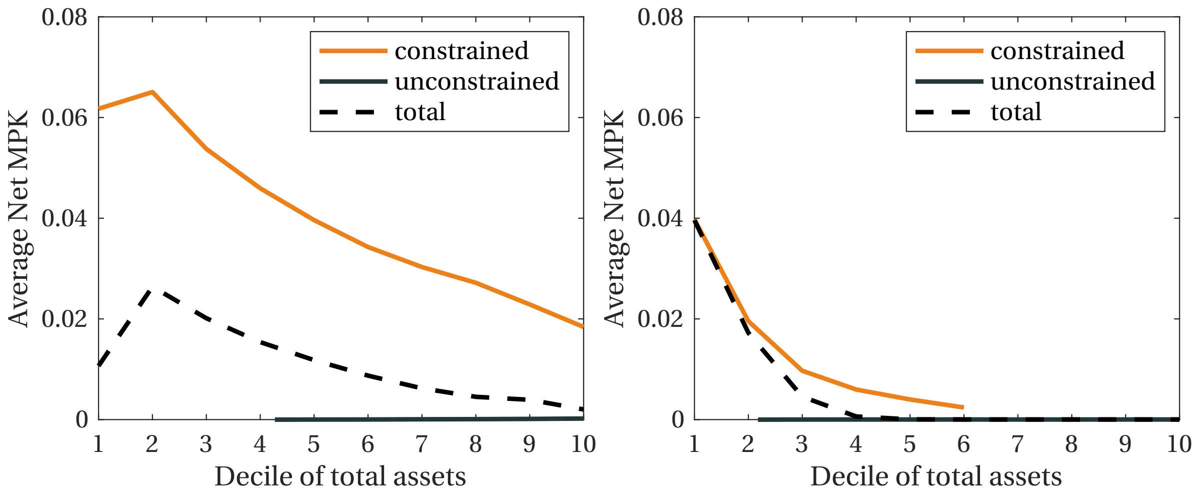
Figure D.13: Conditional distribution of capital elasticity



(a) Transitory + permanent shock (b) Transitory shock

Figure D.14: Conditional distributions of MPKs

Notes. The dashed line depicts the conditional mean of the marginal product of capital for constrained firms.



(a) Transitory + permanent shock

(b) Transitory shock

Figure D.15: Average MPK along total assets distribution

Notes. The line depicts the average MPK of constrained firms per decile bin of total assets along the entire size distribution.

E Simple model - results

Take a very simple model to analyse the impact of heterogeneous productivities on cyclicality, following Crouzet & Mehrotra (2020). Firms can only invest in physical capital, have permanent productivity and face no uncertainty, except for a stochastic death shock. The problem can be written as:

$$
\begin{aligned}
& V(k_{t,i}, b_{t,i}, \theta_i) = \pi_d x_{t,i} + (1-\pi_d)\left(x_{t,i} - k_{t+1,i} + q_t b_{t+1,i} + \beta V(k_{t+1,i}, b_{t+1,i}, \theta_i)\right) \\
& \text{subject to} \\
& x_{t,i} = z_t \theta_i k_{t,i}^{\alpha} + (1-\delta)k_{t,i} - b_{t,i} \\
& \xi x_{t,i} \geq b_{t+1,i} \\
& k_{t+1,i} \leq x_{t,i} + q_t b_{t+1,i}
\end{aligned}
$$

E.1 Unconstrained firms

Steady state growth. Unconstrained firms optimal capital $k^*_{t+1,i}$ is the solution to:

$$\beta^{-1} = (1-\delta) + \alpha z_t \theta_i k^{*\,\alpha-1}_{t+1,i}$$

Hence optimal capital $k^*_{t+1,i}$ is

$$k^*_{t+1,i} = \theta_i^{\frac{1}{1-\alpha}} \left(\frac{\alpha z_{t+1}}{\beta^{-1} - (1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

where we can choose $z := \left(\frac{\beta^{-1}-(1-\delta)}{\alpha}\right)$ such that, at steady state and for $\theta = 1$, we have that $k^*_{t+1,i} = 1$. In the absence of idiosyncratic shocks and constant total factor productivity z, unconstrained firms are not growing at steady state as they reached their optimal level of capital.

$$g_{uncons} = \frac{k^*_{t+1,i}(\theta_i)}{k^*_{t,i}(\theta_i)} = 1$$

Cyclicality. Now consider the following setup; at time $t = -1$, $z_t = z$. At time $t = 0$, firms learn the future path of z_t, for $t \geq 0$ will be

$$z_t = z \exp(\rho^t \epsilon)$$

The growth rate then becomes

$$g_{uncons} = \frac{k^*_{t+1,i}(\theta_i)}{k^*_{t,i}(\theta_i)} = \frac{\exp(\frac{\rho}{1-\alpha}\epsilon)\theta_i^{1/(1-\alpha)}}{\theta_i^{1/(1-\alpha)}} = \exp\left(\frac{\rho}{1-\alpha}\epsilon\right)$$

Hence, the elasticity of capital is the same across all unconstrained firms, independent of firm size and firm-specific productivity.

$$\frac{\Delta g_{uncons}}{\Delta\epsilon}|_{\epsilon\approx 0} = \frac{\rho}{1-\alpha}$$

E.2 Constrained firms

Steady state growth. Constrained firms invest according to their maximum investment capacity which is capped by the net worth constraint.

$$\begin{aligned} k_{t+1,i} &= n_{t,i} + q_t b_{t+1,i} \\ &= n_{t,i} + q_t \xi n_{t,i} \\ &= (1+q_t\xi)(z_t\theta_i k_{t,i}^{\alpha} + (1-\delta)k_{t,i} - b_{t,i}) \end{aligned}$$

Hence,

$$\begin{aligned} g_{cons} &= (1+q_t\xi)(z_t\theta_i k_{t,1}^{\alpha-1} + (1-\delta) - b_{t,i}/k_{t,i}) \\ &= (1+q_t\xi)(z_t\theta_i k_{t,1}^{\alpha-1} + (1-\delta) - \frac{\xi}{1+q_{t-1}\xi}) \end{aligned}$$

Due to decreasing returns to scale, the growth rate is affected by the size of the firm, with larger firms growing slower

$$\frac{\Delta g_{cons}}{\Delta k_{t,i}} = (1+q_t\xi)(\alpha-1)z_t\theta_i k_{t,1}^{\alpha-2} < 0$$

For firms of the same size, those with a higher permanent productivity component grow quicker

$$\frac{\Delta g_{cons}}{\Delta\theta_i} = (1+q_t\xi)z_t k_{t,1}^{\alpha-1} > 0$$

Cyclicality Now consider the same setup as for unconstrained firms; at time $t=-1$, $z_t = z$. At time $t=0$, firms learn the future path of z_t, for $t \geq 0$ will be

$$z_t = z\exp(\rho^t\epsilon)$$

The growth rate on impact then becomes

$$g_{cons} = (1+q_t\xi)(z\exp(\rho^0\epsilon)\theta_i k_{t,1}^{\alpha-1} + (1-\delta) - \frac{\xi}{1+q_{t-1}\xi})$$

So, the elasticity of capital with respect to the shock ϵ is decreasing on capital and increasing on the productivity of the firm

$$\frac{\Delta g_{cons}}{\Delta\epsilon}|_{\epsilon\approx 0} = (1+q_t\xi)(z\theta_i k_{t,1}^{\alpha-1}) = \frac{(1+q_t\xi)}{\alpha}mpk_i$$

With the derivative of the elasticity with respect to the size and productivity of the firm being negative and positive respectively

$$\frac{\Delta^2 g_{cons}}{\Delta\epsilon\Delta\theta_i}|_{\epsilon\approx 0} = (1+q_t\xi)(zk_{t,1}^{\alpha-1}) > 0$$

$$\frac{\Delta^2 g_{cons}}{\Delta\epsilon\Delta k_{t,1}}|_{\epsilon\approx 0} = (\alpha-1)(1+q_t\xi)(z\theta_i k_{t,1}^{\alpha-2}) < 0$$

When is the elasticity of constrained larger than unconstrained?

$$\frac{\Delta g_{cons}}{\Delta\epsilon}|_{\epsilon\approx 0} > \frac{\Delta g_{uncons}}{\Delta\epsilon}|_{\epsilon\approx 0}$$

This happens when the marginal product of capital of constrained firms is above a given threshold

$$mpk > \rho\frac{\alpha}{1-\alpha}\frac{1}{1+q_t\xi}$$

So, two factors will determine which elasticity is larger: (i) the marginal product of capital of constrained firms, which depends on the distribution in terms of both size and productivity. The smaller and the more productive constrained firms are, the higher their elasticity; (ii) the persistence of the aggregate shock. As ρ approaches zero, unconstrained firms will not react to the shock, while the elasticity of constrained firms on impact does not depend on the persistence of the shock.

F Statistical Model Derivation

This is reproduced from Pugsley et al. (2021) for reference. Write stochastic processes in MA representation:

$$\begin{aligned}
u_{i,t} &= \rho_u^{t+1} u_{i,-1} + \sum_{k=0}^{a} \rho_u^k \theta_i \\
v_{i,a} &= \rho_v^{a+1} v_{i,-1} \\
w_{i,a} &= \sum_{k=0}^{a} \rho_w^k \varepsilon_{i,a-k} = \sum_{k=0}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} \quad 0 \le j \le a
\end{aligned}$$

So the level of log employment of firm i at age a is:

$$\ln n_{i,a} = \rho_u^{a+1} u_{i,-1} + \sum^{a} \rho_u^k \theta_i + \rho_v^{a+1} v_{i,-1} + \sum_{i=1}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{i=1}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} + z_{i,a}$$

Then the autocovariance of log employment at age a and $a-j$ for $j \ge 0$ is:

$$\begin{aligned}
\mathrm{Cov}\left[\log n_{i,a}, \log n_{i,a-j}\right] &= \left(\sum_{k=0}^{a} \rho_u^k\right) \sigma_\theta^2 \left(\sum_{k=0}^{a-j} \rho_u^k\right) + \rho_u^{a+1} \sigma_{\tilde{u}}^2 \rho_u^{a-j+1} + \rho_v^{a+1} \sigma_{\tilde{v}}^2 \rho_v^{a-j+1} \\
&\quad + \mathrm{Cov}\left[\rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}, \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}\right] + \mathbf{1}_{\{j=0\}} \sigma_z^2 \\
&= \sigma_\theta^2 \left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right) + \sigma_{\tilde{u}}^2 \rho_u^{2(a+1)-j} + \sigma_{\tilde{v}}^2 \rho_v^{2(a+1)-j} + \sigma_\varepsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \mathbf{1}_{\{j=0\}} \sigma_z^2
\end{aligned}$$