Transmit Antenna Number Identification for MIMO Cognitive Radio Systems in the Presence of Alpha-Stable Noise

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Abstract—Identification of communication parameters, a major task of intelligent receivers, has important applications in intelligent systems, especially cognitive radio systems. Multiple antennas make the identification problem more challenging. In this paper, we focus on the problem of detecting the number of transmit antennas in multiple-input multiple-output (MIMO) cognitive radio systems. A novel identification algorithm is proposed to determine the number of transmit antennas for MIMO systems in the presence of alpha-stable noise. We first introduce the correlation matrix based on the fractional lower order statistics (FLOS) and provide a particular structure of FLOS-based correlation matrix. Then, the eigenvalues of the FLOS-based correlation matrix are employed to construct a test statistic and the central limit theorem is exploited to obtain the decision threshold. Finally, the transmit-antenna number is detected using a serial binary hypothesis test. Simulation results are demonstrated to evaluate the effectiveness of the proposed transmit-antenna number detection algorithm for MIMO systems in the presence of alpha-stable noise.

Index Terms—Alpha-stable noise, fractional lower order statistics, multiple-input multiple-output, parameter identification.

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) technology is a promising technique for high rate transmission over wireless channels [1], [2]. MIMO technology has been regarded as a key technique for intelligent systems, including cognitive radio, software defined radio and security monitoring [3], [4]. The utilization of MIMO technology in intelligent systems incurs several challenges for the identification of communication parameters, an important functional component of intelligent systems, such as identification of transmit-antenna

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number [5], [6], classification of space-time code [7], [8] and channel estimation [9], [10]. In particular, identification of transmit-antenna number is crucial for intelligent receivers [11], [12]. For example, in the cognitive radio systems, the knowledge of transmit-antenna number for primary users (PUs) is essential to enable secondary user intelligent transmission so that secondary users can adjust the transmit power and beamforming according to the transmit-antenna number of PUs to avoid interfering the PUs [13]. Moreover, detection of the transmit-antenna number is of interest for antenna selection, which is considered as a alternative method in hybrid MIMO system [14].

Several efficient identification algorithms have been devised to determine the transmit antenna number for MIMO systems in the literature. They fall into two main categories: the information-theoretic methods [15]–[17] and the feature-based (FB) algorithms [18]–[22]. In the fist method, detecting the number of transmit antennas is formulated as a model selection problem, which can determine the number of transmit antennas by choosing the minimum Kullback-Leibler length of every candidate models. O. Somekh *et al*. [15] have developed two algorithms based on the minimum description length (MDL) and Akaike information criterion (AIC) for detecting the number of transmitting antennas in MIMO systems. The MDL/AIC algorithms provide a robust estimator to determine the number of transmitting antennas for mild SNR conditions. However, these two works are sensitive to timing and frequency offsets. M. Shi *et al*. [16] presented a method based on the Schur complement test to adaptively estimate the number of transmit antennas in MIMO systems. The algorithm avoids the need for tracking the eigenvalues of the sample covariance matrix, and thus it has lower computational complexity. Additionally, the performance of the adaptive estimator does not depend on the number of receive antennas for MIMO systems. K. Hassan, *et al*. [17] investigated the problem of identification of transmit-antenna number for spatially-correlated MIMO and proposed two algorithms based on objective information theoretic criteria. These two algorithms are robust to the spatial correlation of MIMO channel, but their performance is sensitive to timing and frequency offsets.

Compared with the information-theoretic algorithms, the feature-based algorithms are usually simpler to implement. In [18], a novel algorithm was proposed to identify the number of base station antennas by employing the orthogonality of the pilot signals. This algorithm provides an acceptable detection

performance by using only one receiver antenna, but it require a priori knowledge about the pilot patterns. Mohammadkarimi *et al.* [19] presented a new feature-based algorithm that utilized the higher order statistics of the received signal to detect the number of transmit antennas. In [20], the authors developed two hypothesis testing based algorithm for identification of the number of transmitting antennas, namely Wishart-matrix's largest eigenvalue (WME) based algorithm and secondary moment based predicted eigenvalue threshold (SM-PET) algorithm. Li *et al.* [21] presented a hypothesis testing algorithm based on higher-order moments of eigenvalues to detect the number of transmit antennas for MIMO systems. In [22], a hypothesis testing algorithm based on the random matrix theory was developed to detect the transmit antenna number in frequency selective fading for non-cooperative MIMO-OFDM systems.

Most of these existing works assume that the received signal is impaired by additive white Gaussian noise (AWGN). However, in practice, this assumption is not always valid, as the received signal may be corrupted by non-Gaussian noise/interference, as pointed out in [23], [24]. Typical non-Gaussian noise impairments include human-made impulsive noise, co-channel interference from multiple access and atmospheric radio noise. Currently, several non-Gaussian models are adopted in the literature, such as the Gaussian mixture noise, generalized Gaussian noise and the alpha stable noise. The family of alpha-stable distributions has been proved as an accurate model for heavy-tailed noise [25], [26]. To the best of our knowledge, no work has yet considered the problem of identification of the number of transmit antennas in the presence of alpha-stable noise.

In this paper, a novel feature-based algorithm is proposed to estimate the number of transmitting antennas for MIMO systems in the presence of alpha-stable noise. We employ the eigenvalues of fractional lower-order statistics (FLOS) based correlation matrix of the received signal as features and exploit eigenvalues weighting to construct the test statistics to detect the number of transmit antennas. The proposed algorithm does not require a priori information about the pilot patterns and preamble sequences.

The main motivations and contributions of this paper are summarized as follows.

- It is the first work that detects the number of transmit antennas for MIMO system with unlicensed user in the presence of alpha-stable noise, which is scarcely mentioned.
- We present a novel feature-based identification algorithm, which employs the eigenvalues of the FLOS-based correlation matrix of the received signal as features and exploits hypothesis testing to determine the number of transmit antennas for MIMO system with unlicensed user. This is the first time FLOS is used in transmit antenna number detection
- The performance of the proposed algorithm is significantly improved over the existing algorithms for non-Gaussian noise. Furthermore, the proposed algorithm does not require prior knowledge about the pilot patterns and preamble sequence.
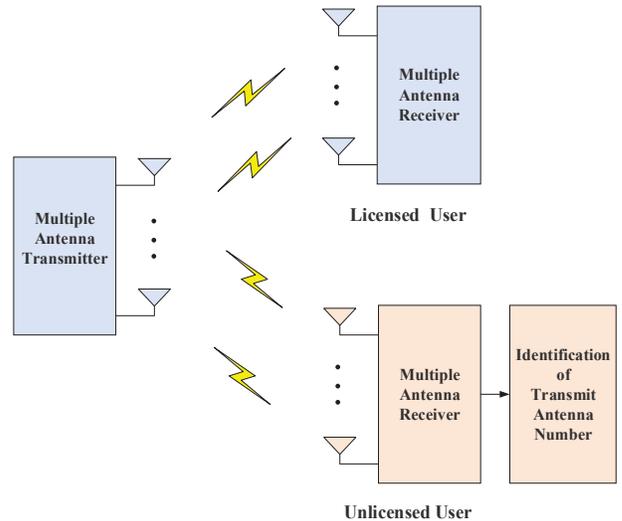

Fig. 1. MIMO system model with unlicensed user

The remainder of this paper is organized as follows. In Section II, the system model is presented. The fractional lower-order statistics based correlation matrix is introduced in Sections III. The proposed detection algorithm is derived and analyzed in Sections IV and V, respectively. The asymptotic consistency analysis is presented in Section VI. Simulation results are presented in Section VII, and some concluding remarks are provided in Section VIII.

II. SYSTEM MODEL

A. System Model

Fig. 1 illustrates a cognitive MIMO system model as a multi-antenna intelligent system. As shown in Fig. 1, the primary user is equipped with Q transmit antennas and the unlicensed user is equipped with K ($K > Q$) receive antennas. The unlicensed user has no prior information about the number of transmit antennas Q. The transmitted data symbols are MPSK (Phase-Shift-Keying) or MQAM (Quadrature Amplitude Modulation), $M \geq 4$. It is assumed that the receiver is completely synchronized with the transmitter. In this paper, the channel is assumed to be flat-fading and characterized by an $K \times Q$ matrix of flat fading coefficients. Under these assumptions, the received signal at the k-th antenna of unlicensed user can be expressed as

$$r_k(n) = \tilde{s}_k(n) + w_k(n) = \sum_{q=1}^{Q} h_{kq} s_q(n) + w_k(n), \quad (1)$$

where $s_q(n)$ is the transmitted samples at the q-th transmit antenna and $w_k(n)$ is the additive alpha-stable noise at the k-th receive antenna. h_{kq} represents the flat fading channel coefficient between the q-th transmit antenna and k-th receive antenna. The $K \times 1$ observation vector at the receiver, i.e., $\mathbf{r}(n) = [r_1(n), ..., r_K(n)]^T$, is expressed as

$$\mathbf{r}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{w}(n), \quad (2)$$

where $\mathbf{H}$ corresponds to the $K \times Q$ complex matrix of independent and identically distributed (i.i.d.) flat fading channels

and $\mathbf{w}(n) = [w_1(n), ..., w_K(n)]^T$ is the $K \times 1$ additive alpha-stable noise vector.

B. Noise Model

The noise samples are assumed to be independent and identically distributed (i.i.d.) following the symmetric alpha-stable ($S\alpha S$) distribution. The $S\alpha S$ noise does not have a closed-form and is most conveniently described by its characteristic function as

$$\varphi(u) = \exp\{jeu - \gamma|u|^{\alpha}\}. \tag{3}$$

The characteristic exponent α controls the heaviness of impulsiveness of the noise, and smaller α leads to the more frequent occurrence of impulses. The dispersion γ determines the spread of the distribution around its location parameter. When $\alpha = 2$, the $S\alpha S$ probability density function (pdf) becomes the Gaussian pdf. Since it is rare to find $S\alpha S$ noise with $\alpha < 1$ in practical, we restrict our work to the class of $S\alpha S$ distributions where $\alpha \in (1, 2]$. It is assumed that the noise process $w(t)$ is white, which is a common assumption for analytical purposes. In this paper, we define a mixture signal-to-noise ratio (MSNR) as the ratio of the average received signal power to the average noise power

$$\text{MSNR (dB)} = 10\log_{10}\frac{E\left[\|\mathbf{r}(n) - \mathbf{w}(n)\|_{\mathrm{F}}^2\right]}{K\gamma}, \tag{4}$$

where $\mathbf{r}(n) = [r_1(n), \cdots, r_K(n)]^T$ and $\mathbf{w}(n) = [w_1(n), \cdots, w_K(n)]^T$. $\|\cdot\|_{\mathrm{F}}$ is the Frobenius norm, $E\{\cdot\}$ denotes the statistical expectation. The variance of noise is set to dispersion parameter γ in the symmetric alpha-stable noise.

III. FRACTIONAL LOWER-ORDER STATISTICS BASED CORRELATION MATRIX

In this section, we introduce the fractional lower order statistics (FLOS)-based correlation matrix $\mathbf{G_r}$ and show a particular structure of $\mathbf{G_r}$, which includes the information about number of transmit antennas.

The population covariance matrix has been widely used in signal processing as a statistic property of signal correlation. Unfortunately, there are no second-order or higher order moments for non-Gaussian noise and interference. As a result, the performance of the algorithm in the identification of transmit-antenna number using the population covariance matrix will severely degrade in the presence of the alpha-stable noise. For alpha-stable noise, the p-th order statistical moment $E[|X|^p]$ is limited only if $p < \alpha$. Therefore, fractional lower order statistics are introduced to deal with alpha stable noise, including fractional lower order moments, negative-order moment, zero order moment, covariation, etc. The typical fractional lower order correlation (FLOC) based on FLOS was introduced in [27], [28]. The p-th FLOC of random variables X_1 and X_2 obeying $S\alpha S$ distribution can be denoted as

$$R_{X_1 X_2}^p = E\left\{\frac{X_1 X_2^*}{|X_2|^{2-p}}\right\}, 1 < p < \alpha, \tag{5}$$

where $E\{\cdot\}$ denotes the statistical expectation.

According to [29], we introduce fractional lower order statistics (FLOS)-based correlation matrix. The FLOS-based correlation matrix of the received signals can be expressed as

$$\mathbf{G_r} = E\left\{[\mathbf{r}(n)]^{\langle p\rangle}[\mathbf{r}^{\dagger}(n)]^{\langle p\rangle}\right\}$$
$$= \begin{bmatrix} G_{11} & \cdots & G_{1K} \\ \vdots & \ddots & \vdots \\ G_{K1} & \cdots & G_{KK} \end{bmatrix}, \tag{6}$$

where $[r(n)]^{\langle p\rangle} = r(n)\big/|r(n)|^{(2-p)/2}$, and the (i, m)-th element G_{im} of $\mathbf{G_r}$ can be given by

$$G_{im} = E\left\{\frac{r_i(n)r_m^*(n)}{(|r_i(n)||r_m(n)|)^{(2-p)/2}}\right\}, \tag{7}$$

where $r_i(t)$ and $r_m(t)$, $\forall i$ and m are the received signal defined in Section II.

First, we show that $\mathbf{G_r}$ is bounded. Based on Theorem 1 in [29], the elements of the FLOS-based correlation matrix is bounded as

$$-\infty < G_{im} = E\left\{\frac{r_i(n)r_m^*(n)}{(|r_i(n)||r_m(n)|)^{(2-p)/2}}\right\} < \infty, \ 1 < p < \alpha \le 2. \tag{8}$$

Next, we provide a particular structure of the FLOS-based correlation matrix $\mathbf{G_r}$ in Proposition 1.

Proposition 1: The elements of the FLOS-based correlation matrix, G_{im}, can be expressed as

$$G_{im} \simeq \sum_{q=1}^{Q} h_{iq}\Sigma_{qq}h_{mq}^* + \varpi_w^2\delta_{im}, \tag{9}$$

where

$$\Sigma_{qm} \simeq \delta_{qm}E\left\{\frac{s_q(n)\left(\sum_{q=1}^{Q}s_q(n) + w_m(n)\right)^*}{\left(|r_i(n)|\left|\sum_{q=1}^{Q}h_{mq}s_q(n) + w_m(n)\right|\right)^{\frac{(2-p)}{2}}}\right\}, \tag{10}$$

$$\varpi_w^2 = E\left\{\frac{w_i(n)\left(\sum_{q=1}^{Q}h_{mq}s_q(n) + w_m(n)\right)^*}{\left(|r_i(n)|\left|\sum_{q=1}^{Q}h_{mq}s_q(n) + w_m(n)\right|\right)^{\frac{(2-p)}{2}}}\right\}, \tag{11}$$

in which δ_{qm} is the Kronecker delta. From (11), the FLOS-based correlation matrix $\mathbf{G_r}$ can be rewritten as

$$\mathbf{G_r} = \mathbf{H}\boldsymbol{\Sigma}\mathbf{H}^{\dagger} + \varpi_w^2\mathbf{I}. \tag{12}$$

Proof: See Appendix A. ∎

Because $\mathbf{G_r}$ is a nonnegative definite Hermitian matrix, it can be decomposed to a diagonal form as

$$\mathbf{G_r} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^{\dagger}, \tag{13}$$

where $\mathbf{U}$ is a $K \times Q$ unitary matrix and $\mathbf{\Lambda}$ stands for a diagonal matrix consisting of the eigenvalues as

$$\mathbf{\Lambda} = \mathrm{diag}\left(\lambda_1, \lambda_2, \cdots, \lambda_Q, \lambda, \cdots, \lambda\right), \quad (14)$$

and

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_Q \geq \lambda_{Q+1} = \cdots \lambda_K. \quad (15)$$

From (15), the number of transmit antennas can be detected by employing the multiplicity of the smallest eigenvalues of $\mathbf{G_r}$. However, the FLOS-based correlation matrix $\mathbf{G_r}$ is not available in practice, which is can be estimated from a finite number of the received signals. The estimator of FLOS-based correlation matrix can be estimated as

$$\hat{\mathbf{G}}_{\mathbf{r}} = \begin{bmatrix} \hat{G}_{11} & \cdots & \hat{G}_{1K} \\ \vdots & \ddots & \vdots \\ \hat{G}_{K1} & \cdots & \hat{G}_{KK} \end{bmatrix}. \quad (16)$$

$$\hat{G}_{im} = \frac{1}{N}\sum_{n=1}^{N}\frac{r_i\left(n\right)r_m^*\left(n\right)}{\left|r_i\left(n\right)\right|^{\frac{2-p}{2}}\left|r_m\left(n\right)\right|^{\frac{2-p}{2}}}. \quad (17)$$

Relying on eigenvalue decomposition, the eigenvalues of $\hat{\mathbf{G}}_{\mathbf{r}}$ can be expressed as

$$\ell_1 \geq \ell_2 \geq \cdots \ell_Q \geq \ell_{Q+1} \geq \cdots \geq \ell_K. \quad (18)$$

In (18), it is obvious that the noise eigenvalues are not all equal to the smallest eigenvalues, whose multiplicity is not equal to $K - Q$. Hence, it is infeasible to determine transmit-antenna number by observing the smallest eigenvalues. To remedy this difficult, we utilize statistical properties of eigenvalues to develop a hypothesis testing based scheme for detecting the number of transmit antennas, which will be presented in Section IV.

IV. TRANSMIT ANTENNA NUMBER IDENTIFICATION BASED ON FLOS-BASED CORRELATION MATRIX

A. Pertinent Statistical Properties of Eigenvalues

Assume that matrix $\mathbf{A}$ is a sample covariance matrix of observations $\mathbf{Z}$. Also, $\mathbf{Z}$ has a zero-mean Gaussian distribution. According to the random matrix theory, $\mathbf{A} = \mathbf{Z}\mathbf{Z}^{\dagger}$ is nearly a Wishart random matrix. According to [21], for large random matrix, the empirical distribution function of the eigenvalues of $\mathbf{A}$ with m real eigenvalues can be given by

$$F^A\left(x\right) = \frac{\Theta\left(\mathbf{A}\right) \leq x}{m}, \quad (19)$$

where $\Theta\left(\mathbf{A}\right)$ represents the number of eigenvalues of A.

In the noise only condition, when $\alpha, p \to 2$, we further assume that $\mathbf{w}\left(n\right)/\langle \mathbf{w}\left(n\right)\rangle^{p-2}$ approximately follows Gaussian distributions with zero mean, and the matrix $\hat{\mathbf{G}}_{\mathbf{w}}$ has Wishart distribution. Furthermore, the empirical distribution function of the eigenvalues of $\hat{\mathbf{G}}_{\mathbf{w}}$ converges to a Marcenko-Pastur distribution in the asymptotic regime, i.e., $K, N \to \infty, c = K/N$. The Marcenko-Pastur density function can be described as [20]

$$dF^{\mathbf{G_w}}\left(x\right) = \max\left\{0, \left(1 - \frac{1}{c}\right)\right\}\delta\left(x\right)$$
$$+ \frac{\sqrt{\left(x - a_-\right)\left(a_+ - x\right)}}{2\pi\varpi^2 x c}\prod\left[a_-, a_+\right]\left(x\right)dx. \quad (20)$$

Using (20), one can show that the noise eigenvalues become closer to each other when the number of samples N is significantly larger than the number of receive antennas Q, i.e., $c = K/N \to \infty$. As mentioned before, all the eigenvalues of $\hat{\mathbf{G}}_{\mathbf{w}}$ only depend on ϖ^2 in the noise only condition. However, when the signal is present, the signal eigenvalues are greater than noise eigenvalues in the population covariance matrix $\hat{\mathbf{G}}_{\mathbf{r}}$. In [30], the features of eigenvalues are described as the spiked population model, where all eigenvalues are the same except for a finite number of eigenvalues. In addition, Li *et al.* [21] reported that the distribution of the $K - Q$ noise eigenvalues is closely approximated by the Marcenko-Pastur distribution when $K \geq Q$. Considering that λ_i is the i-th eigenvalue of $\mathbf{G_r}$ and ℓ_j is the j-th largest eigenvalues of $\hat{\mathbf{G}}_{\mathbf{r}}$. According to [30], ℓ_j tends to $\lambda_j\left(1 + \frac{\varpi^2 c}{\lambda_j - \varpi^2}\right)$ as K and N both tend to infinity for some c. In addition, for a well-behaved function $\varphi\left(y\right)$, the random variable $\vartheta = \frac{1}{K-Q}\sum_{i=Q+1}^{K}\varphi\left(\ell_i\right)$ converges with probability one to

$$\lim_{\substack{K, N \to \infty \\ c = K/N}} \vartheta = \int \varphi\left(y\right)dF^{\mathbf{G_w}}\left(y\right), \quad (21)$$

It can be readily seen from the above equation that the average of $\varphi\left(y\right)$ over the noise eigenvalues of the sample covariance matrix converges to $E\left\{\varphi\left(y\right)\right\}$ where y is a random variable with Marcenko-Pastur distribution. Thus, the r-th order moment of the eigenvalues of $\mathbf{G_w}$ can be given by [30]

$$E\{\vartheta^r\} = \int x^r dF^{\mathbf{G_w}}(x) = \varpi^{2r}\sum_{j=0}^{r-1}\frac{c^j}{j+1}\binom{r}{j}\binom{r-1}{j}. \quad (22)$$

B. Transmit Antenna Number Identification Scheme

Based on the above discussion, we achieve the pertinent statistical properties of r-th order moments of eigenvalues using the random matrix theory. Subsequently, the test statistic of the eigenvalue weighting is conceived by using the distribution function of the r-th order moments of the eigenvalues. Finally, a hypothesis testing procedure is employed to determine the number of transmit antennas, which can test the multiplicity of the noise eigenvalues by comparing the test statistic with the decision threshold.

We first propose a weighted r-th order moment of eigenvalue-based detector. The test statistic is given by

$$T_\ell\left(k\right) = \frac{1}{K-k}\sum_{j=k+1}^{K}\frac{\left(\ell_j - \varpi_w^2(k)\right)^+}{\varpi_w^{2(r+1)}(k)}\ell_j^r, \quad (23)$$

where $\left(x\right)^+ = \max\left(0, x\right)$, $\varpi_w^2(k) = \frac{1}{K-k}\sum_{j=k+1}^{K}\ell_j$. According to (15) and (18), when $N \to \infty$ and $k \geq Q$, we have $\ell_j \approx \ell_Q \approx \varpi_w^2 = \varpi_w^2(Q)$. Hence $T_\ell\left(k\right) = 0$. For $k < Q$, we have $\ell_j > \ell_Q \approx \varpi_w^2$ and $T_\ell\left(k\right) > 0$. Hence, we can detect the number of transmit antennas by checking the test statistic $T_\ell\left(k\right)$.

Let us then consider a serial binary hypothesis test by using the theoretical distribution of the test statistic. We can estimate the number of transmit antennas using a serial binary hypothesis test. The decision criterion can be set as

$$\begin{cases} T_\ell\left(k\right) > \psi_\ell, & \text{under } \mathcal{H}_1 \\ T_\ell\left(k\right) \leq \psi_\ell, & \text{under } \mathcal{H}_0, \end{cases} \quad (24)$$

where $T_\ell(k)$ represents the test statistic with $k = 1, 2, ..., K - 1$. ψ_ℓ denotes the decision threshold. The hypothesis $\mathcal{H}_1$ represents that the eigenvalue ℓ_j is a signal eigenvalue for the test statistic $T_\ell(k)$. The null hypothesis $\mathcal{H}_0$ represents that the eigenvalue ℓ_j is a noise eigenvalue.

In practice, it is difficult to obtain enough information about the transmitted signals at the receive side, especially at the cognitive receiver. Hence, we cannot obtain the distribution function of $T_\ell(k)$ under the hypothesis $\mathcal{H}_1$. As a result, the decision threshold ψ_ℓ cannot be set based on the detection probability $P_d = P\{T_\ell(k) > \psi_\ell | \mathcal{H}_1\}$ [21]. To set the decision threshold ψ_ℓ, we analyze r-th order moments of the random variables of test statistics $T_\ell(k)$ in the null hypothesis $\mathcal{H}_0$. Details on the decision threshold setting will be given in the next section. In conclusion, the main procedures of the proposed weighted r-th order moments of the eigenvalue-based algorithm are summarized in Algorithm 1.

Algorithm 1 Transmit antenna number identification based on FLOS-based correlation matrix

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1: Compute the FLOS-based correlation matrix of the received signal according to (16) and (17);
2: Obtain the absolute value eigenvalues ℓ_j of the matrix $\hat{\mathbf{G}}_{\mathbf{r}}$, and sort eigenvalues ℓ_j from large to small;
3: Compute the decision statistic $T_\ell(k)$ according to (23);
4: Initialize $k = 1$
5: Set the decision threshold ψ_ℓ based on (34) and (40);
6: **If** $T_\ell(k) \leq \psi_\ell$;
7: $\hat{Q} = k$.
8: **else**
9: Increment $k = k + 1$ and go to Step 5.
10: **end**.

V. Decision Threshold Analysis

A. Largest Eigenvalues Approximation Approach

The test statistic $T_\ell(k)$ can be regarded as the sum of $K - k$ independent random variables. We propose the largest eigenvalue to approximate the weight eigenvalues. We first derive the first-order and second-order moments of the test statistic $T_\ell(k)$ under the null hypothesis $\mathcal{H}_0$. Let

$$
\begin{aligned}
T_\ell(k)|_{k=Q} &= \frac{1}{K-k} \sum_{j=k+1}^{K} \frac{(\ell_j - \varpi_w^2)^+}{\varpi_w^{2(r+1)}} \ell_j^r \\
&\simeq \frac{1}{K-k} \sum_{j=k+1}^{K} \frac{(\ell_j - \varpi_w^2)}{\varpi_w^{2(r+1)}} \ell_j^r \\
&\simeq \frac{1}{K-k} \sum_{j=k+1}^{K} \frac{(\ell_{\max}(\mathbf{G_w}) - \varpi_w^2)}{\varpi_w^{2(r+1)}} \ell_j^r \quad (25)\\
&= \frac{(\ell_{\max}(\mathbf{G_w}) - \varpi_w^2)}{\varpi_w^2} \frac{1}{K-k} \sum_{j=k+1}^{K} \frac{\ell_j^r}{\varpi_w^{2r}} \\
&= \frac{(\ell_{\max}(\mathbf{G_w}) - \varpi_w^2)}{\varpi_w^2} \Delta_r,
\end{aligned}
$$

where $\Delta_r = \frac{1}{K-k} \sum_{j=k+1}^{K} \frac{\ell_j^r}{\varpi_w^{2r}}$.

The first-order and second-order moment of the test statistic $T_\ell(k)$ under the hypothesis $\mathcal{H}_0$ can be obtained as

$$
E\{T_\ell(k)\} \simeq \frac{\ell_{\max}(\mathbf{G_w}) - \varpi_w^2}{\varpi_w^2} E\{\Delta_r\}, \quad (26)
$$

and

$$
E\{T_\ell^2(k)\} \simeq \left(\frac{\ell_{\max}(\mathbf{G_w}) - \varpi_w^2}{\varpi_w^2}\right)^2 E\{\Delta_r^2\}. \quad (27)
$$

According to the distribution of the largest eigenvalue theorem, $\ell_{\max}(\mathbf{G_w}) \simeq \frac{\varpi_w^2}{N}(\sqrt{N} + \sqrt{K})^2$ when $N \to \infty$ and $\alpha, p \to 2$. Hence, the mean and the variance of the test statistic $T_\ell(k)$ under the hypothesis $\mathcal{H}_0$ can be expressed as

$$
\mu_{T_\ell} = \frac{K + 2\sqrt{NK}}{N} E\{\Delta_r\}, \quad (28)
$$

and

$$
\begin{aligned}
\sigma_{T_\ell}^2 &= E\{T_\ell^2(k)\} - E^2\{T_\ell(k)\} \\
&= \left(\frac{K+2\sqrt{NK}}{N}\right)^2 (E\{\Delta_r^2\} - E^2\{\Delta_r\}) \\
&= \frac{(K+2\sqrt{NK})^2}{N^2} Var\{\Delta_r\},
\end{aligned} \quad (29)
$$

where $E\{\Delta_r\}$ and $Var\{\Delta_r\}$ are the mean and the variance of Δ_r, respectively. Based on [30], when $\alpha, p \to 2$, K and N tend to infinity (for $c = K/N$), the distribution of Δ_r converges to a normal distribution as

$$
\Delta_r \sim \mathcal{N}\left(\mu_{\Delta_r}, \frac{\sigma_{\Delta_r}^2}{K^2}\right). \quad (30)
$$

Table I gives μ_{Δ_r} and $\sigma_{\Delta_r}^2$ for $r = 2, 3, 4$ [30].

According to the central limit theorem (CLT) for sufficiently large N, $T_\ell(k)$ approximately follows Gaussian distribution as

$$
T_\ell(k)|_{k=Q} \sim \left(\mu_{T_\ell(k)}, \sigma_{T_\ell(k)}^2\right), \quad (31)
$$

and

$$
\frac{T_\ell(k)|_{k=Q} - \mu_{T_\ell(k)}}{\sigma_{T_\ell(k)}} \sim \mathcal{N}(0, 1). \quad (32)
$$

According to [20] and [21], for the normal distribution, we can set a threshold t, that is

$$
-t \leq \frac{T_\ell(k)|_{k=Q} - \mu_{T_\ell(k)}}{\sigma_{T_\ell(k)}} \leq t. \quad (33)
$$

Then, the decision threshold ψ_l^a can be obtained as

$$
\psi_\ell^a = \frac{K + 2\sqrt{NK}}{N} \mu_{\Delta_r} + t_a \frac{(K+2\sqrt{NK})}{N} \frac{\sigma_{\Delta_r}}{K}. \quad (34)
$$

From (34), it is obvious that the decision threshold ψ_ℓ^a is related to K, N, t_a and the parameters μ_{Δ_r} and σ_{Δ_r}. For given r, μ_{Δ_r} and σ_{Δ_r} are dependent on a Polynomial of variable $c = K/N$. When $\alpha, p < 2$, ϖ_w^2 contains information on the MSNR, and t_a will be set according to different MSNR.

TABLE I

	$r=2$	$r=3$	$r=4$	$r=5$
μ_{Δ_r}	$c+1$	c^2+3c+1	c^3+6c^2+6c+1	$c^4+10c^3+20c^2+10c+1$
$\sigma^2_{\Delta_r}$	$2c^2$	$3c^2\left(6+13c+6c^2\right)$	$36c^2\left(1+4c+2c^2\right)\left(c^2+4c+2\right)$	$20c^2\left(5+20c+15c^2+2c^3\right)\left(5c^3+20c^2+15c+2\right)$

B. Gaussian Approximation Approach

The distribution of $T_\ell(k)$ under the $\mathcal{H}_0$ hypothesis can be approximated by a Gaussian distribution. Let us first rewrite the test statistic $T_\ell(k)$ as

$$
\begin{aligned}
T_\ell(k)|_{k=Q} &= \frac{1}{K-k}\sum_{j=k+1}^{K}\frac{\left(\ell_j-\varpi_w^2\right)^+}{\varpi_w^{2(r+1)}}\ell_j^r \\
&\simeq \frac{1}{K-k}\sum_{j=k+1}^{K}\frac{\left(\ell_j-\varpi_w^2\right)}{\varpi_w^{2(r+1)}}\ell_j^r \\
&= \frac{1}{K-k}\sum_{j=k+1}^{K}\left(\frac{\ell_k^{r+1}}{\varpi_w^{2(r+1)}}-\frac{\ell_k^r}{\varpi_w^{2r}}\right) \\
&= \frac{1}{K-k}\sum_{j=k+1}^{K}\frac{\ell_j^{r+1}}{\varpi_w^{2(r+1)}}-\frac{1}{K-k}\sum_{j=k+1}^{K}\frac{\ell_j^r}{\varpi_w^{2r}} \\
&= \Delta_{r+1}-\Delta_r,
\end{aligned} \tag{35}
$$

where $\Delta_{r+1}=\frac{1}{K-k}\sum_{j=k+1}^{K}\frac{\ell_j^{r+1}}{\varpi_w^{2(r+1)}}$.

Similarly, when $\alpha,p\to 2$, the distribution of Δ_{r+1} converges to a normal distribution as

$$
\Delta_{r+1}\sim\mathcal{N}\left(\mu_{\Delta_{r+1}},\frac{\sigma^2_{\Delta_{r+1}}}{K^2}\right). \tag{36}
$$

According to the properties of Gaussian distribution, the mean and variance of $T_\ell(k)$ can be given by

$$
\mu_{T_\ell(k)}=\mu_{\Delta_{r+1}}-\mu_{\Delta_r}, \tag{37}
$$
$$
\sigma^2_{T_\ell(k)}=\left(\sigma^2_{\Delta_{r+1}}+\sigma^2_{\Delta_r}\right)/K^2. \tag{38}
$$

We approximate the distribution of $T_\ell(k)$ to be Gaussian with sufficiently large K. The normalized $T_\ell(k)$ asymptotically follows a standard Gaussian distribution as

$$
\frac{T_\ell(k)|_{k=Q}-\mu_{T_\ell(k)}}{\sigma_{T_\ell(k)}}\sim\mathcal{N}(0,1). \tag{39}
$$

Similar to Section V-A, we define a threshold t_b for a standard Gaussian distribution. The decision threshold can be expressed as

$$
\psi_\ell^b=\mu_{\Delta_{r+1}}-\mu_{\Delta_r}+t_b\left(\frac{\sigma^2_{\Delta_{r+1}}+\sigma^2_{\Delta_r}}{K^2}\right)^{\frac{1}{2}}. \tag{40}
$$

In (40), the decision threshold ψ_ℓ^b is simply a function of the parameters $\mu_{\Delta_{r+1}}, \mu_{\Delta_{r+1}}, \sigma^2_{\Delta_{r+1}}$ and $\sigma^2_{\Delta_r}$. The parameters rely on a Polynomial of variable $c=K/N$. For $\alpha,p<2$, t_b is a value related to the MSNR.

Accordingly, the decision threshold ψ_ℓ^a and ψ_ℓ^b depend on the number of samples N, the number of receive antennas K, and the threshold t_a and t_b. We note that ψ_ℓ^a and ψ_ℓ^b will be set according to the MSNR because ϖ_w^2 contains the signal amplitude. This means that the proposed scheme require MSNR, while it avoids the need for a priori information about the pilot patterns and channel knowledge.

VI. ASYMPTOTIC CONSISTENCY ANALYSIS

In this section, the asymptotic consistency of the proposed method is investigated. As mentioned in Section III, the FLOS-based matrix $\mathbf{G_r}$ can be closely approximated by a sample matrix $\hat{\mathbf{G}}_\mathbf{r}$. When $N\to\infty$, the sample matrix tends to the FLOS-based matrix, and the eigenvalues of $\hat{\mathbf{G}}_\mathbf{r}$ converge to the eigenvalues of $\mathbf{G_r}$ with probability 1.

As mentioned in Section V, the decision threshold ψ_ℓ^a and ψ_ℓ^b are dependent on μ_{Δ_r} and $\sigma^2_{\Delta_r}$. For $N\to\infty$ and $\alpha,p\to 2$, we have $\mu_{\Delta_r}\to 1$ and $\sigma^2_{\Delta_r}\to 0$. Based on (34) and (40), we obtain

$$
\lim_{N\to\infty}\psi_\ell^a=0^+, \tag{41}
$$

and

$$
\lim_{N\to\infty}\psi_\ell^b=0^+. \tag{42}
$$

Under the $\mathcal{H}_1$ hypothesis, when $1\le k<Q$, the limit of test statistic $T_\ell(k)$ can be expressed as

$$
\lim_{N\to\infty}T_\ell(k)=\lim_{N\to\infty}\frac{1}{K-k}\sum_{j=k+1}^{K}\frac{\left(\ell_j-\varpi_w^2(k)\right)^+}{\varpi_w^{2(r+1)}(k)}\ell_j^r>0. \tag{43}
$$

According to the above inequality, $T_\ell(k)>\psi_\ell^a$ and $T_\ell(k)>\psi_\ell^b$ are always satisfied for $N\to\infty$. Based on the decision criterion in Section II, for the decision threshold ψ_ℓ^a and ψ_ℓ^b, we have

$$
\lim_{N\to\infty}\left\{\hat{Q}\ge Q\right\}=1. \tag{44}
$$

In the null hypothesis $\mathcal{H}_0$, when $Q\le k\le K-1$, the limit of test statistic can be expressed as

$$
\lim_{N\to\infty}T_\ell(k)=\lim_{N\to\infty}\frac{1}{K-k}\sum_{j=k+1}^{K}\frac{\left(\ell_j-\varpi_w^2(k)\right)^+}{\varpi_w^{2(r+1)}(k)}\ell_j^r=0. \tag{45}
$$

As a result, $T_\ell(k)<\psi_\ell^a$ and $T_\ell(k)<\psi_\ell^b$ are always true for $N\to\infty$. Similar to the analysis process under the $\mathcal{H}_1$ hypothesis, for the decision threshold ψ_ℓ^a and ψ_ℓ^b, we obtain

$$
\lim_{N\to\infty}\left\{\hat{Q}>Q\right\}=0. \tag{46}
$$

Therefore, according to (44) and (46), we have

$$
\lim_{N\to\infty}\left\{\hat{Q}=Q\right\}=1. \tag{47}
$$

Hence, the proposed estimator is asymptotically consistent when the sample size N goes to infinity with Q and K fixed.
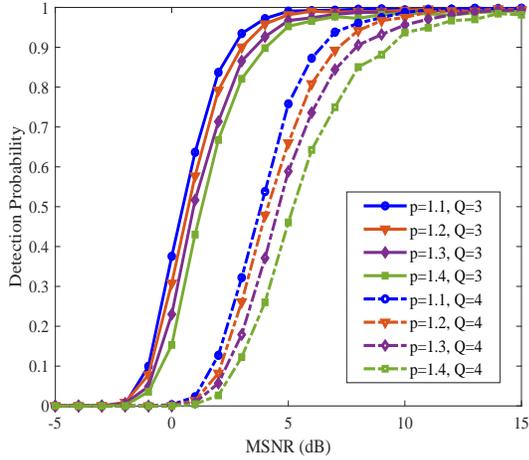
Fig. 2.   Probability of identification versus MSNR for different parameters p when considering decision threshold ψ_ℓ^a
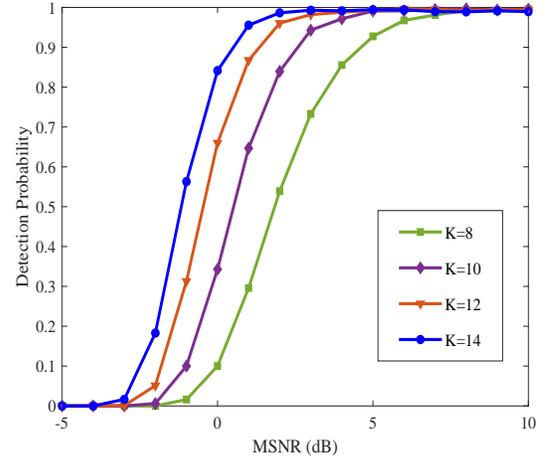

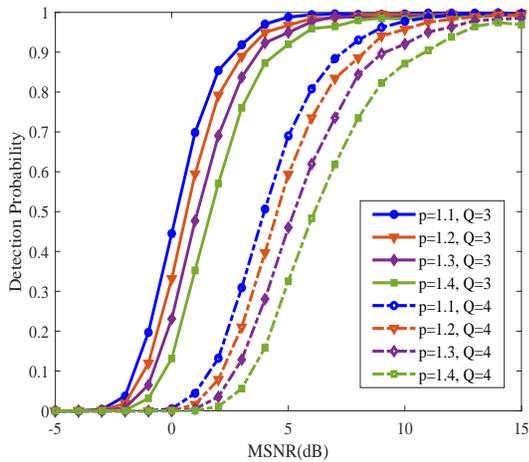Fig. 4.   Probability of identification versus MSNR for different receive antennas K when considering decision threshold ψ_ℓ^a

Fig. 3.   Probability of identification versus MSNR for different parameters p when considering decision threshold ψ_ℓ^b
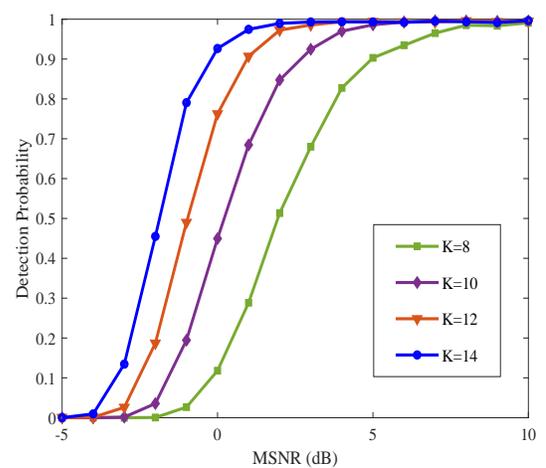

Fig. 5.   Probability of identification versus MSNR for different receive antennas K when considering decision threshold ψ_ℓ^b

VII. SIMULATION RESULTS AND DISCUSSION

In this section, numerical results are presented to validate the performance of the proposed method for the identification of transmit-antenna number in MIMO Systems with alpha-stable noise. In our simulation, we consider a MIMO system that employs linear space-time block coding with multiple transmit antennas. Flat fading channel is considered in the simulation. The parameters r and α are $r = 4$ and $\alpha = 1.9$. Moreover, the threshold t_a and t_b for the decision threshold ψ_ℓ^a and ψ_ℓ^b are set by Monte Carlo method in different MSNR ranges, respectively. All results are calculated using 2000 Monte Carlo trials. The probability of correct identification $P_d = \Pr\left[\hat{Q} = Q\right]$ is employed as the performance measure.

In Figs. 2-3, the probability of identification P_d is plotted versus MSNR for different parameter p in symmetric alpha-stable noise. In this simulation, the number of receiver antennas and the number of received samples are equal to $K = 10$ and $N = 800$, respectively. Fig. 2 shows the effect of parameter p on identification probability P_d when the test

threshold is ψ_ℓ^a. As observed from Fig. 2, the identification probability P_d is improved with the decrease of parameter p for transmit-antenna number $Q = 3$ and $Q = 4$. When MSNR is close to 4dB and transmit-antenna number equals $Q = 3$, the probability of identification is close to 96% for $p = 1.1$ and close to 90% for $p = 1.4$. In Fig. 3, the effect of parameter p on the identification probability P_d is presented for the test threshold ψ_ℓ^b. As can be seen, a smaller parameter p corresponds to better identification performance for transmit-antenna number $Q = 3$ and $Q = 4$. With $p = 1.1$, the probability of correct identification is close to 96% for an MSNR equals to 4dB and $Q = 3$. According to the results shown in Figs. 2 and 3, we conclude that the proposed method achieves superior identification performance for parameter $p = 1.1$ compared with other p. Unless otherwise mentioned, the parameter p is set to 1.1 in the following simulation experiments.

Figs. 4-5 show the effect of the number of receive antennas K on the probability of identification P_d for the proposed
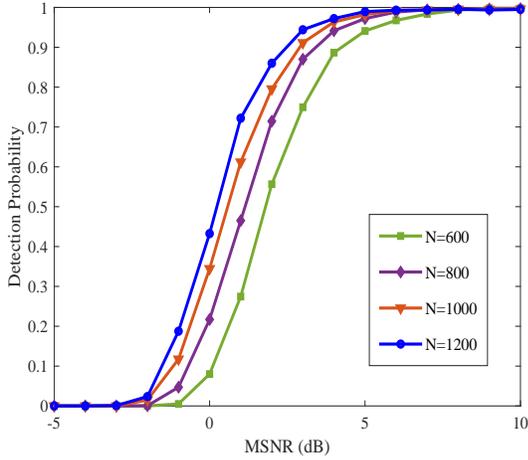
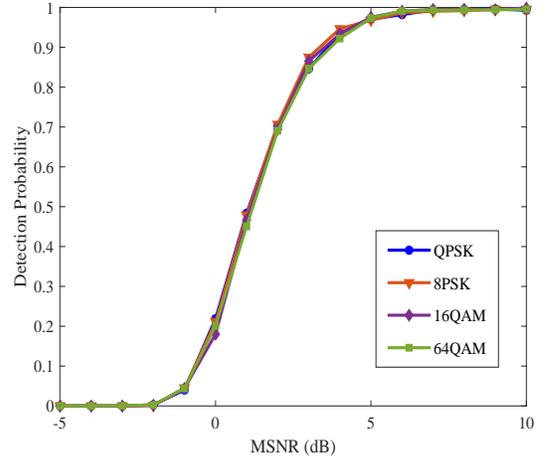Fig. 6.   Probability of identification versus MSNR for different received samples N when considering decision threshold ψ_ℓ^a

Fig. 8.   Probability of identification versus MSNR for different modulation formats when considering decision threshold ψ_ℓ^a
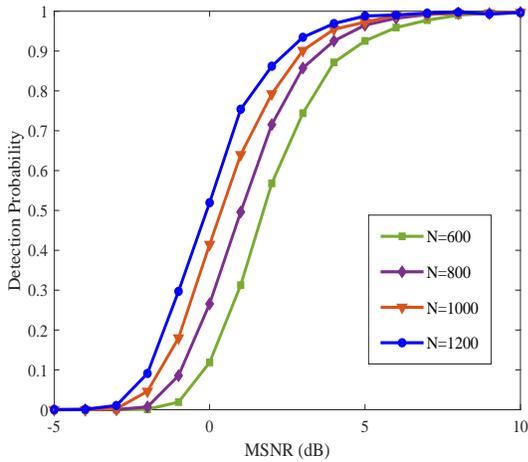

Fig. 7.   Probability of identification versus MSNR for different received samples N when considering decision threshold ψ_ℓ^b
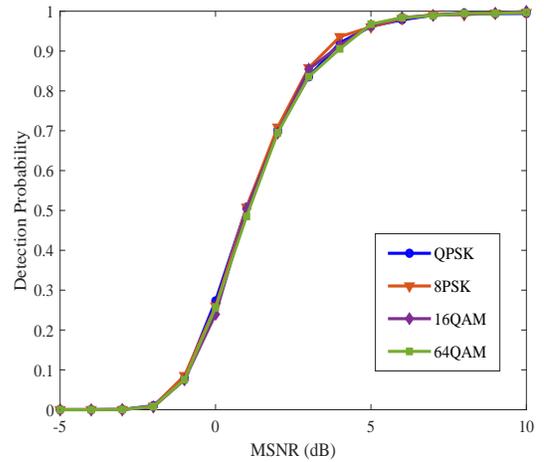

Fig. 9.   Probability of identification versus MSNR for different modulation formats when considering decision threshold ψ_ℓ^b

method when test threshold ψ_ℓ^a and ψ_ℓ^b. With $N = 800$, we evaluate the identification probability of proposed method in alpha-stable noise when receive antennas K is set to 8, 10, 12 and 14. In Fig. 4, for $K = 14$ receiver antennas and an MSNR close to 1 dB, the probability of correct identification is over 90% when transmit-antenna number $Q = 3$. The probability of correct identification P_d is close to 90% at MSNR equals to 0dB in Fig. 5 when $Q = 3$ and $K = 14$. These results show that the identification probability P_d of the proposed method improves with the number of receive antennas K increases. This is because that increasing the number of receive antennas K will increase the effective post-processing MSNR and obtain better performance.

Figs. 6-7 illustrate the effect of the number of received samples N, on the probability of identification P_d for the proposed identification method when decision threshold ψ_ℓ^a and ψ_ℓ^b. In this simulation, we have evaluated the behavior of our identification method for different received samples, where $N = 600$, $N = 800$, $N = 1000$ and $N = 1200$. As can be

seen, the probability of correct identification improves when increasing the number of received samples N. For instance, the probability of correct identification P_d is over 90% when the number of received samples $N = 1200$ at MSNR=3dB, where the number of transmit antennas and receiver antennas are equal to $Q = 3$ and $K = 9$. This can be easily explained, as increasing the number of receive samples N will reduce noise level and achieves better identification performance.

Figs. 8-9 depict the effect of the modulation scheme on the probability of identification P_d of the transmit-antenna number Q for the proposed method. The probability of correct identification P_d, is shown versus MSNR for test threshold ψ_ℓ^a and ψ_ℓ^b. It is observed in Figs. 8-9 that, the performance of the proposed algorithm is not dependent on these modulation formats, which are mandatory for most of the wireless standards. For example, the probability of correct identification P_d is over 90% for different modulation formats at MSNR=5dB. This is mainly because that the proposed scheme is developed rely on the eigenvalues of the FLOS-based matrix, which is
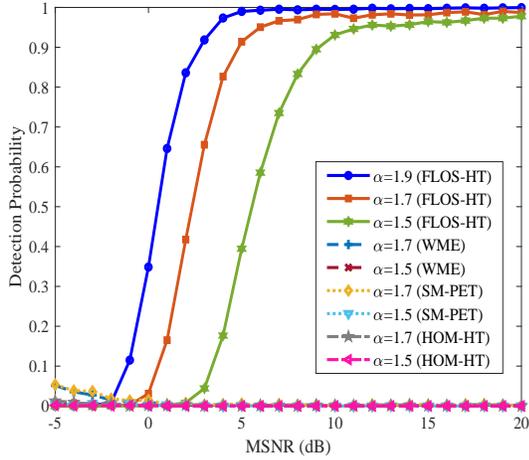
Fig. 10. Probability of identification versus MSNR for different noise characteristic exponent α when considering decision threshold ψ_ℓ^a
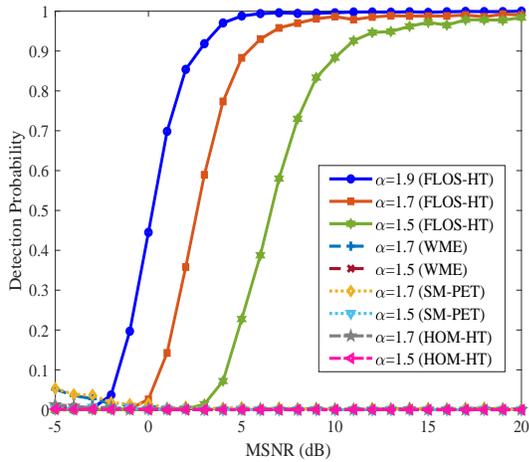

Fig. 11. Probability of identification versus MSNR for different noise characteristic exponent α when considering decision threshold ψ_ℓ^b

independent on the modulation format.

In Figs. 10-11, the probability of identification of the transmit-antenna number is plotted versus MSNR. For comparison, the performance of the HOM-HT algorithm [21], WME and SM-PET algorithms [20] with symmetric alpha-stable noise are also plotted in these figures. From Figs. 10-11, it can be shown that the proposed algorithm (FLOS-HT) is robust against symmetric alpha-stable noise and significantly outperforms the existing algorithms when MSNR is large. In addition, it can be observed that the performance of the proposed algorithm is dependent on the value of α in Figs. 10-11. The correct identification probability increases with an increase in α values. For example, we observed that the probability of correct identification P_d degraded from close to 99% to 20%, when noise characteristic exponent α values decreased from 1.9 to 1.5 for MSNR= 5dB. Specifically, for the same α values ($\alpha = 1.5$), the proposed method achieves good identification performance in relatively very heavy-tailed impulsive noise ($\alpha = 1.5$).

VIII. CONCLUSION

We developed a novel algorithm to detect the number of transmit antennas for MIMO cognitive radio systems in alpha-stable noise. We introduced the FLOS-based correlation matrix of the received signals and provided pertinent statistical properties of eigenvalues. Then, we construct a test statistic rely on the eigenvalues of the FLOS-based correlation matrix and utilized a serial binary hypothesis test to determine the number of transmit antennas. Furthermore, the decision threshold was derived based on the largest eigenvalues and Gaussian approximation, respectively. The proposed algorithm has the advantage of avoiding the need for a priori information about the pilot patterns and channel knowledge. Simulation experiments demonstrated that the algorithm can achieve a good performance in the presence of symmetric alpha-stable noise, and has good robustness to the change of characteristic exponent. Future works include investigating robust detection of transmit-antenna number for multi-cell or cell-free MIMO systems.

APPENDIX A
PROOF OF PROPOSITION 1

Substituting the received signal $r_k(n)$ into the elements of the FLOS-based matrix G_{im}, we have

$$G_{im} = E\left\{ \frac{r_i(n) r_m^*(n)}{(|r_i(n)| |r_m(n)|)^{(2-p)/2}} \right\}$$

$$= E\left\{ \frac{\left(\sum_{q=1}^{Q} h_{iq} s_q(n) + w_i(n)\right)\left(\sum_{q=1}^{Q} h_{mq} s_q(n) + w_m(n)\right)^*}{\left(|r_i(n)| \left|\sum_{q=1}^{Q} h_{mq} s_q(n) + w_m(n)\right|\right)^{\frac{(2-p)}{2}}} \right\} \quad (48)$$

Since the transmitted signal $s_q(n)$ and the noise $w_m(n)$ are independent, the elements of the FLOS-based matrix G_{im} in (49) can be further expressed as (50) at the top of the next page.

According to the process in [29], the elements of the FLOS-based matrix G_{im} can be approximately written as

$$G_{im} \simeq \sum_{q=1}^{Q} h_{iq} E\left\{ \frac{s_q(n)\left(\sum_{q=1}^{Q} s_q(n) + w_m(n)\right)^*}{\left(|r_i(n)| \left|\sum_{q=1}^{Q} h_{mq} s_q(n) + w_m(n)\right|\right)^{\frac{2-p}{2}}} \right\} h_{mq}^*$$

$$+ E\left\{ \frac{w_i(n)\left(\sum_{q=1}^{Q} h_{mq} s_q(n) + w_m(n)\right)^*}{\left(|r_i(n)| \left|\sum_{q=1}^{Q} h_{mq} s_q(n) + w_m(n)\right|\right)^{\frac{2-p}{2}}} \right\} \quad (50)$$

Applying variable changes, the elements of the FLOS-based matrix, G_{im}, can be expressed as

$$G_{im} \simeq \sum_{q=1}^{Q} h_{iq} \Sigma_{qq} h_{mq}^* + \varpi_w^2 \delta_{im}, \quad (51)$$

$$G_{im} \simeq E\left\{ \frac{\sum_{q=1}^{Q} h_{iq}s_q(n)\left(\sum_{q=1}^{Q} h_{mq}s_q(n)+w_m(n)\right)^*}{\left(|r_i(n)|\left|\sum_{q=1}^{Q} h_{mq}s_q(n)+w_m(n)\right|\right)^{\frac{2-p}{2}}} \right\} + E\left\{ \frac{w_i(n)\left(\sum_{q=1}^{Q} h_{mq}s_q(n)+w_m(n)\right)^*}{\left(|r_i(n)|\left|\sum_{q=1}^{Q} h_{mq}s_q(n)+w_m(n)\right|\right)^{\frac{2-p}{2}}} \right\}. \tag{49}$$

where

$$\Sigma_{qm} \simeq \delta_{qm}E\left\{ \frac{s_q(n)\left(\sum_{q=1}^{Q} s_q(n)+w_m(n)\right)^*}{\left(|r_i(n)|\left|\sum_{q=1}^{Q} h_{mq}s_q(n)+w_m(n)\right|\right)^{\frac{(2-p)}{2}}} \right\}, \tag{52}$$

$$\varpi_w^2 = E\left\{ \frac{w_i(n)\left(\sum_{q=1}^{Q} h_{mq}s_q(n)+w_m(n)\right)^*}{\left(|r_i(n)|\left|\sum_{q=1}^{Q} h_{mq}s_q(n)+w_m(n)\right|\right)^{\frac{(2-p)}{2}}} \right\}, \tag{53}$$

in which δ_{qm} is the Kronecker delta. The FLOS-based matrix $\mathbf{G_r}$ is rewritten as

$$\mathbf{G_r} = \mathbf{H\Sigma H}^\dagger + \varpi_w^2 \mathbf{I}. \tag{54}$$

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