# Automated Analysis of the Origin of Movement: An Approach Based on Cooperative Games on Graphs

Ksenia Kolykhalova, Giorgio Gnecco, Marcello Sanguineti, Gualtiero Volpe, and Antonio Camurri

***Abstract*—In this work, a computational method is proposed to automatically investigate the perception of the origin of full-body human movement and its propagation. The method is based on a mathematical game built over a suitably defined graph structure representing the human body. The players of this game are the graph vertices, which form a subset of body joints. Since each vertex contributes to a shared goal (i.e., to the way in which a specific movement-related feature is transferred among the joints), a cooperative game-theoretical model (specifically a transferable-utility game) is adopted, which is able (via the Shapley value) to measure the relevance of the various joints in human movement when performing full-body movement analysis. The method is theoretically investigated and applied to a motion capture dataset obtained from subjects who performed expressive movements. Finally, the method is validated through an online survey, in which several dancers/nondancers participated. The results show the capability of the proposed approach to represent the evolution of the most important joint responsible for originating each dancer's movement.**

***Index Terms*—Automated analysis of the perception of the origin of movement, cooperative game theory, full-body movement analysis, graph theory, transferable-utility (TU) games.**

## I. Introduction

THIS research aims at contributing to the design of systems for the automated analysis of expressive full-body human movement by means of an approach grounded on the integration of techniques from both cooperative game theory and graph theory. The importance of full-body movements in conveying affective expressions and social signals is widely recognized by the scientific community [1]–[3], and a growing number of applications exploiting full-body expressive movement and nonverbal social signals are available. Research on computational models of expressive human movement benefits from trans-disciplinary approaches, integrating biomechanics and neuroscience [4], experimental psychology, and theories from the arts and humanities [5]. The possibility to automatically measure movement qualities demonstrated to be very important in many different interactive applications, including therapy and rehabilitation in autism, and in cognitive and motor disabilities [6].

The analysis of the origin of movement is an important component in the understanding and modeling of expressive movement. For example, the leaning forward of an arm can have very different expressive meanings depending on the origin of movement: a "punch" originates from the foot, a "push away" may originate from the shoulder, and a "caress," from the hand. All these movements are basically a leaning forward of an arm, the very different dynamics of which are explained also in terms of the origin of movement. Measuring this feature can be useful for the applications reported earlier: for example, in rehabilitation, the detection of the origin of movement can help in enabling a patient to learn how to perform a movement (e.g., how to stand up from a chair) correctly to avoid injuries.

According to the leading joint hypothesis [7], the central nervous system organizes multijoint movements according to a hierarchical control process, where the muscle torque at one leading joint (which may be termed the *physical origin of movement*) is responsible for powerful interaction torques at the other subordinate joints. In dance and sports, the awareness and discovery of the physical origin of movement may contribute to enhanced performance and effectiveness of movement expressivity.

From the perspective of an observer of (full-body) movement, however, the *perceived origin of movement* is also an important means to convey and communicate expressivity. The perceived origin of movement is the point at which a movement appears to originate from the point of view of an observer. It refers to a specific body part, which can be identified as a distal or proximal part of the body. The perception of expressive body movements may be influenced by a perceived leading joint. In dance education, an important aspect concerns learning how to induce the perception of the origin of movement in an audience as one of the components to convey the qualities of movement. Nevertheless, despite a consolidated tradition in dance training and sport, according to the authors' knowledge, the perceived

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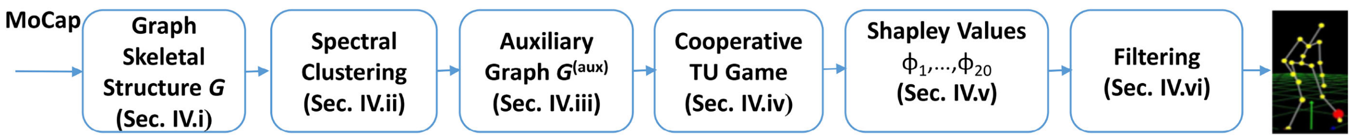


Fig. 1. Conceptual architecture of the proposed method.

origin of movement is still poorly addressed in the scientific literature.

In this article, we propose an approach based on the integration of graph and game theory to contribute toward the analysis of the perceived origin of full-body human movement and its propagation. Games restricted to graphs, first investigated in [8], are able to represent naturally occurring situations in which any two players from among a coalition of players can coordinate with each other only if they are joined by a path (i.e., by a channel of communication). Examples of studies in which graph-theoretical and game-theoretical tools are combined can be found in several recent works [9]–[11], with the aim to investigate vertex relevance in graphs. In our approach, we model the human body as a graph and apply game theory to measure in real time the most relevant joint of the body from which movement originates.

This article is organized as follows. Section II presents the problem and the conceptual framework developed. Section III reports the graph-theoretical and game-theoretical definitions and notations. Section IV details the proposed method of analysis of the perceived origin of human movement. Section V describes some theoretical properties. Section VI discusses the implementation of the proposed approach. Section VII provides details on its evaluation against a dataset of expressive movements and presents the results of an online survey designed to validate the proposed method. Finally, Section VIII provides conclusions and future work. The proofs of the theoretical results are reported in the Appendix. The research described in this article extends thoroughly its preliminary short conference version [13] (Sections V, VII, and VIII and the Appendix are completely novel, whereas Sections II–IV, and VI are significant extensions of some of the material contained in [13]).

## II. Conceptual Architecture

The main aim of this work is to contribute to the automated analysis of the perceived origin of full-body human movement. In particular, we present an approach grounded on both cooperative game theory and graph theory.

The method proposed in this work (the steps of which are reported in Fig. 1; see Section IV for a detailed description of each step) represents the human body by means of an undirected graph, in which the vertices are the joints and the edges are both physical and nonphysical connections between these body joints. Moreover, the edges are associated with weights, the values of which depend on a feature extracted from motion capture data. On the one hand, *physical links* represent connections between consecutive physical body joints, such as the forearm. On the other hand, *nonphysical links* model the dependencies between joints that are not physically connected, solely derived from correlations observed in the chosen movement feature between these joints: for example, a hand moving toward the head, followed by a movement of the head in response in the same direction, reveals a nonphysical link between the hand and the head. Nonphysical links, therefore, play the role of potential bridges joining body parts that are not directly connected within the skeletal structure but exhibit correlated dynamics during the movement performed (see paragraph i) in Section IV for more details about how this issue is modeled in this work).

Starting from the graph representing the skeletal structure augmented by nonphysical links, we define a suitable mathematical game [14] in which the vertices (i.e., the body joints) are the players and the edges model the communication channels (through which movement can propagate) between these players. Body movement is, therefore, represented by a game constructed on the graph. A cooperative game model is proposed, since both the vertices and the edges contribute to the overall movement. Then, the Shapley value [14]—which is a classical solution concept from cooperative game theory able to provide a ranking of the players that represents their relevance in the game—is computed for all the players of the game and adopted as a measure of vertex relevance in the graph to estimate how much each vertex contributes to a shared goal (i.e., to the way in which a specific movement-related feature is transferred among the joints). The possibility to know, moment by moment, which joint(s) is the most representative in the ongoing full-body movement (i.e., those with the highest Shapley value) constitutes precious information for the automated analysis of expressiveness in movement. For example, the joints with the highest Shapley value are candidates to be the *perceived origin* of movement propagating in the body, and among other issues, they can provide useful cues to detect which parts of the body are most relevant for the analysis of expressive movement and worth a detailed observation by means of further analysis techniques (possibly at a finer scale), as well as to inform automated techniques of movement prediction.

The proposed method of movement analysis is evaluated against a dataset of approximately one hundred fragments of motion capture recordings, which constitute a repository of stimuli of expressive movements useful also for further research studies on movement analysis. Validation of our approach includes an online survey (based on the data repository) in which participants with different levels of expertise in dance took part.

Exploiting this approach to automatically investigate movement features associated with expressive gesture communication can allow the development of multimodal interfaces based on full-body human interaction, which are also capable of supporting nonverbal expressive, affective, and social communication.

## III. Concepts From Graph Theory and Game Theory

In this section, some basic graph-theoretical and game-theoretical definitions and notations are reported.

A *graph* $G := (V, E)$ is made of a set $V$ of *vertices* (which are also called *nodes*) and of a set $E \subseteq V \times V$ of *edges*. An edge connecting two vertices $v, \hat{v} \in V$ is denoted by $e_{v,\hat{v}}$. In this work, *undirected graphs* are considered, for which $e_{v,\hat{v}} \in E$ implies $e_{\hat{v},v} \in E$ (hence, they can be identified as the same edge), and there is no edge connecting any vertex $v \in V$ with itself. The *neighbors* of any vertex $v \in V$ are defined as the vertices to which it is linked by an edge. Their set is denoted by $N(v)$. A graph is *connected* if, given any two vertices $v, \hat{v} \in V$, there exists a *path* (i.e., a sequence of consecutive edges in the graph that join distinct vertices) from $v$ to $\hat{v}$. A graph is weighted if a real number $w(e)$ is associated with any edge $e$ (because in the undirected case one identifies the edges $e_{v,\hat{v}}$ and $e_{\hat{v},v}$, the weights $w(e_{v,\hat{v}})$, and $w(e_{\hat{v},v})$ are equal by definition). In this work, the case of nonnegative weights $w(e)$ is considered. The notation $G := (V, E, w)$ is used to denote a weighted (undirected) graph. Finally, let $V' \subseteq V$ be a subset of vertices of the graph $G$, and let $E(V') \subseteq E$ denote the set of all the edges in $G$ directly connecting the vertices in $V'$. The *subgraph induced by* $V'$ is defined as the ordered pair $(V,' E(V'))$. Similarly, one defines the *weighted subgraph induced by* $V'$.

The notation $(V, c)$ is used to denote a *cooperative transferable-utility (TU) game in characteristic function form*, in which $V$ is the set of all players, whereas $c : 2^V \to \mathbb{R}$ is called the *characteristic function of the game*. It associates with each subset $V' \subseteq V$ of players a real number, which represents its *payoff* (or *utility* or *value*). For the case of the empty set, one sets $c(\emptyset) := 0$. A peculiar feature of TU games is that their payoffs can be transferred from any player to any other one without incurring any loss (for instance, by means of a common currency, the value of which is the same for each player). In the approach proposed in this work, the vertices of a connected graph $G$ are the players of a suitable cooperative TU game. A subset $V' \subseteq V$ is also called a *coalition* of players, whereas $V$ is the *grand coalition* (i.e., the coalition comprising all the players). Since they are in a one-to-one correspondence, the two terms "coalition" and "induced subgraph" are herein often used interchangeably. Still, both terms are exploited, since in the proposed cooperative TU game, the value assumed by its characteristic function on each coalition is constructed based on the properties of the associated induced subgraph.

Under the assumption that the grand coalition $V$ has been established, a *solution concept* of a cooperative TU game is defined as a rule to divide $c(V)$, i.e., the utility derived from cooperation in the grand coalition, among all the players of the game. The *Shapley value* [14] is one of the most well-known solution concepts. It can be interpreted as an appropriately weighted average marginal payoff of each player at the time this specific player joins a random coalition it does not belong to. The Shapley value $\phi_i(c)$ of each player $i$ $(i = 1, \ldots, |V|)$ has the following expression:

$$\phi_i(c) := \sum_{V' \subseteq V \setminus \{i\}} \frac{|V'|!(|V| - |V'| - 1)!}{|V|!} (c(V' \cup \{i\}) - c(V')). \tag{1}$$

In the case of our automated investigation of expressive gestures, the Shapley values on a suitably defined graph game, given an appropriate choice for its characteristic function, represent a way to measure the *relevance* (or *centrality*) of its vertices (see, e.g., [9] and more specifically Table 1 and Figure in [10, Sec. 4.1.1] for an illustrative application of (1) to a graph comprising four vertices). As explained in more detail in the following section, we conjecture that the Shapley value of a joint at a given time might be a cue related to the relevance of that joint in the current evolution of movement. The joint with the highest Shapley value at a given moment might be the candidate joint from which movement originates or, in any case, a sort of pivot with respect to the other body parts.

## IV. Details of the Proposed Method

As already mentioned, we hypothesize that the Shapley value associated with a joint at a given instant is related to the relevance of that joint in the ongoing movement and in particular to its role as a joint from which movement originates, from a perceptual point of view. Therefore, we search for joints that separate clusters, where each cluster is characterized by similar values of a movement feature. To clarify this issue, let us consider the situation in which one moves an arm while all other body parts remain at rest. In this case, the shoulder corresponding to that arm may be interpreted as a very relevant joint because, although being at rest (in one cluster), it plays, in a sense, a relevant role in the control of the arm movement (being the arm in another cluster). Hence, our method tends to attribute the largest relevance to a vertex that connects two clusters of joints or even a larger number of these clusters. Here, each cluster represents a subset of connected joints associated with similar values of a movement feature and is identified by applying a suitable clustering technique.

Our proposed method is expected to be relevant to automatically investigate the (perceived) origin of movement and its propagation due to the presence of the following two main ingredients: i) its game-theoretical component, which provides an ensemble view and emerges quite naturally as a possible way to model joint interaction; and ii) its use of graph models, which is well suited to represent an enlarged skeletal structure (in the sense that the pure skeletal graph is enlarged by the presence of bridge edges).

In summary, our proposed method works as follows.

1) First, a subset of relevant vertices (joints) is constructed: these are the vertices of the original body graph that separate the clusters found by a specific clustering technique (spectral clustering), which is suitable to address graph-based data representations. To define the edge weights to which spectral clustering is applied, values of the chosen movement feature are used. Then, edges connecting

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different clusters are exploited to define the topology of an auxiliary graph.

2) Second, a suitable cooperative TU game is built, starting from the weighted edges of the auxiliary graph (this construction requires specifying only a functional form for the characteristic function of the game). For the specific choice of the characteristic function reported later in (2), it is shown (in Proposition V.2) that the Shapley value can be interpreted as a measure of vertex centrality for the auxiliary graph (and, consequently, for the original body graph, since they have the same vertices). Since both the clusters in the original body graph and the edge weights in the auxiliary graph come from the comparison of values of a movement feature in pairs of joints, the characteristic function of the game is clearly derived from movement-related data.

The process of construction of the body graph and of the cooperative game and the computation of the Shapley values are detailed in the following steps.

i) First, a graph $G$ modeling the body structure is defined. Its vertices are the body joints. Among its edges, the ones that belong to the skeletal structure are named *physical edges*. For every frame of a recording session, positive weights are attributed to these edges, exploiting the current similarity in the values of a given movement-related feature. More specifically, the weight associated with a physical edge is chosen to be inversely proportional to the sum of a negligible positive constant (which is used to avoid dividing by 0) and the modulus of the difference between the values assumed by the movement-related feature on the two vertices linked by that physical edge. *Nonphysical edges* (also called *bridge edges*) are also inserted in the graph structure to join vertices that otherwise would be not directly connected by a physical edge. In this case, positive weights are also associated with these nonphysical edges. These weights are proportional to the current similarity in the feature values assumed on the two respective vertices, with a much smaller constant of proportionality (5 times smaller, according to the implementation of the proposed method) than the one adopted to define the weights associated with the physical edges. We make this choice because, different from physical edges, nonphysical ones should be associated with a large weight only when a much larger similarity of feature values occurs between the two respective vertices.

ii) The weighted graph defined earlier is clustered using a specific clustering technique suitable for a graph-based data representation, namely, spectral clustering. We recall here that the main idea of clustering consists of making more similar data points belong (whenever possible) to the same subset (called a cluster) while assigning less similar data points (whenever possible) to different clusters [15]. These requirements are typically translated into suitable optimization problems. The following are distinguishing features of spectral clustering:
   a) the data points are modeled as vertices of a (possibly weighted) graph;
   b) vertex similarity is expressed via the (possibly weighted) adjacency matrix of the graph;
   c) spectral properties (i.e., properties related to eigenvalues/eigenvectors) of the (possibly weighted) Laplacian matrix of the graph are used to cluster the data points.

   For more details about spectral clustering, and for its specific implementation in this work, interested readers are referred, respectively, to [14, Ch. 8] and [16]. As a consequence of the application of spectral clustering to the proposed weighted graph, vertices belonging to each cluster are expected to be associated with similar values of the feature. In contrast, edges connecting different clusters should correspond to vertices whose feature values are significantly different.

iii) Then, for each frame, a suitable auxiliary graph is defined as follows. Its vertices coincide with those of the original body graph, whereas its edges constitute the subset of physical edges belonging to the original body graph that link joints associated with different clusters. Then, weights are attributed to these physical edges. These weights are proportional to how much the feature values of the two associated joints are dissimilar (in the specific implementation reported in this work, they are set equal to the modulus of the difference in these feature values). In the following, the notation $G^{(\mathrm{aux})} := (V, E^{(\mathrm{aux})}, w^{(\mathrm{aux})})$ is used to denote the weighted auxiliary graph, whereas the notation $N^{(\mathrm{aux})}(v)$ is used to denote the set of neighbors of a vertex $v$ in the weighted auxiliary graph.

iv) A cooperative TU game on the weighted auxiliary graph is constructed. The value $c(V')$ associated with a generic coalition $V'$ is chosen as the summation of all the weights (in the weighted auxiliary graph $G^{(\mathrm{aux})}$) associated with the physical edges belonging to the subgraph of the auxiliary graph that is induced by $V'$. More precisely, the characteristic function is chosen as follows:

$$c(V') = \sum_{v,\hat{v}\in V,'\hat{v}\in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \,. \qquad (2)$$

   Note also that [17] considers cooperative TU games whose characteristic function has the form (2). However, these games are not proposed therein as a way to analyze human movement nor is the construction of an auxiliary graph considered.

v) For the cooperative TU game constructed earlier, the Shapley values of all its players (joints) are computed. Then, these Shapley values are used to rank the players. For each specific frame, its most relevant joint is defined as the one having the largest rank, in case there exists only a single joint having that property. Otherwise, if several joints have the maximal rank, the one associated with the smallest index (where the label is assigned according to any *a priori* given indexing of the joints) is produced as the output (in this phase, a random index is not adopted to prevent adding noise to the model). In a similar way, as a by-product of the proposed procedure, one can also find the joint with the second-largest Shapley value (again, the

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vertex with the smallest index is considered in the case of multiple choices). Moreover, it could occur that the two vertices having the two largest Shapley values are connected in the original body graph by a physical edge. In that case, that physical edge could be considered as the most relevant one in the body graph. This scenario would occur, for instance, if the spectral clustering step mentioned above produced only two clusters, separated by that specific edge. A generalization of this situation is reported at the end of Section V-B.

vi) A final filtering step is performed on the Shapley values determined earlier. More precisely, one keeps only the joints that are automatically evaluated by the method as the most relevant ones for a prespecified number of adjacent frames (51, according to the method implementation).

It is worth observing that the analysis of movement performed in this work (which investigates the origin of movement and how it propagates) is complementary to that of other works, such as [18], which involves the application of pose estimation to action recognition. The proposed approach is also different: the method developed in [18] is also based on clustering but at a different level (it involves clustered trajectories) and has different goals. Moreover, it also involves a supervised component.

## V. Theoretical Analysis

The cooperative TU game on the weighted auxiliary graph introduced in Section IV has some interesting properties, which we investigate in this section. In addition to being meaningful from the mathematical point of view, these properties exhibit the nice feature of making the model match the requirements of the problem under investigation. Hence, after stating these properties, we discuss their interpretations in terms of the analysis of movement. Finally, we mention possible extensions of the mathematical framework.

### A. Some Properties of the Mathematical Model

We recall that a characteristic function $c$ of a cooperative TU game is *monotonic* if, for every two subsets $V_1 \subseteq V_2 \subseteq V$, one has $c(V_2) \geq c(V_1)$. It is *superadditive* if, for every two disjoint subsets $V_1, V_2 \subseteq V$, one has $c(V_1 \cup V_2) \geq c(V_1) + c(V_2)$. A cooperative TU game is *convex* if, for every two subsets $V_1 \subseteq V_2 \subseteq V$ and every $v \in V \setminus V_2$, one has $c(V_1 \cup \{v\}) - c(V_1) \leq c(V_2 \cup \{v\}) - c(V_2)$.

The following proposition states some properties of the characteristic function $c$ of the cooperative TU game on the weighted auxiliary graph, provided by (2), and of the cooperative TU game itself.

*Proposition V.1:* For a cooperative TU game whose characteristic function $c$ is defined by (2), the following properties hold:

1) $c$ is monotonic;
2) $c$ is superadditive; and
3) the cooperative TU game is convex.

*Proof:* See the Appendix. ■

The following proposition provides a particularly simple expression for the Shapley values in the proposed game.

*Proposition V.2:* For the cooperative TU game whose characteristic function $c$ is defined by (2), one has the following expression for the Shapley value of any player $i$ ($i = 1, \ldots, |V|$) associated with the vertex $v_i \in V$ in the weighted auxiliary graph

$$\phi_i(c) := \sum_{\hat{v} \in N^{(\text{aux})}(v_i)} w^{(\text{aux})}(e^{(\text{aux})}_{v_i,\hat{v}}) \,. \tag{3}$$

*Proof:* See the Appendix. ■

### B. Interpretation of the Properties in the Context of Human Movement Analysis

According to the choice (2) of the characteristic function $c$, the utility of each subset of vertices measures the overall movement dissimilarity of adjacent vertices in the auxiliary subgraph induced by that subset. Hence, the properties stated in Proposition V.1 can be interpreted as follows.

Monotonicity implies that among all possible coalitions, the grand coalition is the one with the maximum utility. Within the framework of movement analysis, monotonicity refers to the fact that by enlarging the subset, the sum of the movement dissimilarities cannot decrease. Hence, all the marginal payoffs (and thus, all the Shapley values of the joints) are nonnegative.

Superadditivity guarantees that the Shapley value is *individually rational*, i.e., that the Shapley value of each player is larger than or equal to the value of the coalition made only of that player (hence, the player has interest in joining the grand coalition). Referring to movements, superadditivity means that by merging two disconnected components, the overall movement dissimilarity may be larger than the sum of the movement dissimilarity in each component. This occurs because one also has to take into account the additional movement dissimilarity between vertices that became adjacent after merging the two components.

The convexity of a cooperative TU game implies nonemptiness of the *core* (which is another solution concept for cooperative TU games; see, e.g., [14] for its definition and theoretical properties) and, in particular, the belonging of the Shapley value to the core. As an element of the core, the Shapley value satisfies also the *coalitional rationality*: for each subset of players, the sum of their Shapley values is larger than or equal to the value of the coalition comprising only those players (hence, this subset is interested in joining the grand coalition). At first glance, convexity does not appear to have a straightforward interpretation in our context. However, note that it leaves open the possibility of exploiting other solution concepts from cooperative game theory in the analysis of human movement analysis, such as other elements of the nonempty core.

We now consider Proposition V.2. It implies that ranking the joints based on the Shapley value of the cooperative TU game with the characteristic function (2) is equivalent to ranking them based on their *weighted degree centrality* on the weighted auxiliary graph. This implication also shows that, as desired, the proposed method tends to identify, as the most relevant vertex, one that links directly two or more different clusters. Indeed, this vertex tends to have a large weighted degree in the weighted auxiliary graph. Note also that, since for the specific game proposed in this work, the Shapley value is equivalent to a weighted form of degree centrality defined on the auxiliary graph, its evaluation is feasible, from a computational point of view, not only offline but also online. Indeed, for this evaluation, one does not need to take into account all possible coalitions occurring in (1). Finally, Proposition V.2 also implies the following:

1) vertices that do not separate clusters in the original body graph always have zero Shapley values;
2) it may occur that the maximum Shapley value is achieved on two vertices separating (via a physical edge) the same two clusters: in this case, the physical edge may be interpreted as the most relevant one in the body graph.

Both cases were observed when testing the proposed method on the motion capture dataset described in Section VI.

### C. Extensions of the Theoretical Analysis

The mathematical framework can be extended to the use of characteristic functions able to capture the propagation of different movement features other than (2). More precisely, the chosen characteristic function should be tailored to the kind of movement quality one is analyzing. For instance, it can be designed to measure the connectivity of the subgraph associated with any subset of vertices. This can be quantified in terms of spectral properties of the normalized Laplacian matrix of the subgraph (see, e.g., [19]). A motivation of this choice is the following: the second smallest eigenvalue of the normalized Laplacian matrix of a subgraph is related to the properties of diffusion processes on this subgraph, which can be exploited to model movement propagation.

A further comment has to do with the rational nature of players, which is one hypothesis at the basis of classic cooperative game theory [14]. Since in our context the players are joints, a nonrational model for them might provide other insights into the analysis of movement, from a different perspective. Indeed, vertex relevance measures from cooperative games were applied in several contexts in which the players were not necessarily modeled as rational/intelligent entities. For instance, the Shapley value was applied to investigate vertex relevance in gene regulatory networks [20] and transportation networks [10], [11], [12]. It was also proposed as a way to provide feature selection in machine-learning problems in which the players are features and the characteristic function measures the predictive power of each subset of features [21].

## VI. System Implementation

The proposed method was tested on a motion capture dataset originating from a larger set obtained by capturing the expressive movements performed by two professional dancers. This multimodal dataset was recorded in the context of the H2020-ICT-2015 EU Project WhoLoDance, Project no. 688865, and is composed of 127 trials (or recordings), acquired with the goal of analyzing movement, determining the features associated with it, and designing computational techniques for their evaluation. The recordings were acquired using a Qualisys motion capture system with 13 infra-red cameras synchronized with two video cameras in the frontal and lateral views. The two professional dancers were equipped with 1 microphone, 5 accelerometers, and 64 infra-red reflective markers. After their acquisition, the data were cleaned and postprocessed via the Qualisys Track Manager native software using a cubic polynomial interpolation for trajectories with gaps in the data. Finally, annotations of the origin, path, and destination of each movement were produced by experts. The expressive movements performed were not related to a specific dance style, being normal full-body movements, e.g., leaning an arm toward a target or turning toward a direction, characterized by a clear origin of movement, enabling the detection of the origin even by a nonexpert observer (though its *automatic* detection is still not a trivial task). The choice of dancers as movement executors was motivated by their full awareness and control of movement details and their higher motor skills with respect to nontrained people, which allowed reducing the amount of noise with respect to alternative performances by nonexperts.

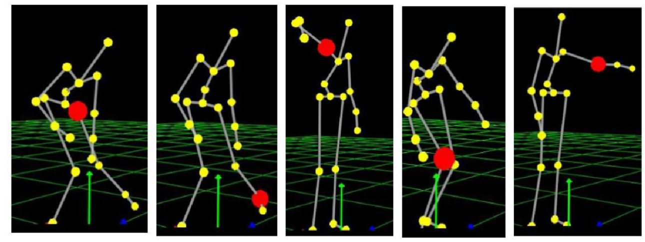

Fig. 2. Example visualization of the results obtained by the method proposed in this work, based on a 20-joint skeletal structure.

The following steps were performed to implement the proposed method.

i) The original dataset (constituted, for each frame of the recordings, by the $x, y, z$ coordinates of the 64 reflective markers) was transformed into a smaller-dimensional motion capture dataset corresponding to a reduced number (20) of joints (see Fig. 2 for some sample frames). The adoption of this simplified skeletal structure is justified by the following:

1) as demonstrated in a previous study [22], a simplified skeletal structure is already able to convey relevant information on expressive movements;
2) a very similar skeletal structure is currently used by Microsoft Kinect SDK, OpenPose, and Orbbec Body Tracking SDK;
3) through the combination of several markers into a single joint in the simplified skeletal structure, the risk of missing markers is reduced;
4) finally, in the implementation, we focus on a holistic analysis of movement limited to gross motion, for which we hypothesize that a simplified skeletal structure provides sufficient information.

ii) Subsequently, based on the location of each of the 20 joints above, each joint's speed (i.e., magnitude of the associated instantaneous velocity vector) was computed automatically for each frame and then chosen as the movement-related feature to be exploited by the proposed approach.

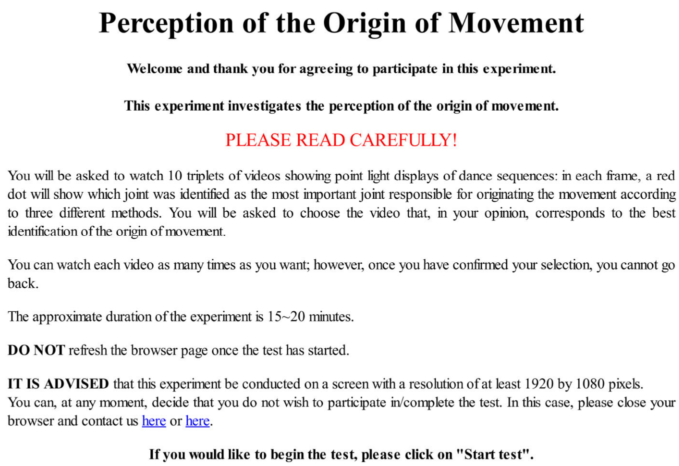


Fig. 3. Web tool for the evaluation of the method: introductory page.

iii) For each frame of the recordings, the Shapley values associated with all 20 joints were computed using the proposed computational method, which was coded in MATLAB. Then, the most relevant joint (i.e., the one evaluated as the most relevant one by the proposed method) was determined automatically. To avoid a too-small average cluster size (e.g., less than five vertices), the maximal number of clusters to be identified by spectral clustering was set to 4, taking into account the small number (20) of vertices in the simplified graph structure (the actual number of clusters being determined automatically by the specific clustering technique). For a more complex skeletal structure, the maximal number of clusters to be identified could be increased without changing the lower bound on the average cluster size.

iv) To make the evaluation of the results easier, they were visualized, with the most relevant joint extracted by the proposed method highlighted in red frame by frame (see Fig. 2 for an example). This last step was implemented by exploiting The EyesWeb XMI open platform.[1]

## VII. Validation

To validate the proposed approach, an online survey was designed to demonstrate the effectiveness of the method as a way to identify the most relevant joints during a movement sequence. More precisely, we tried to evaluate the relevance of the approach as a way to measure *movement propagation* and, particularly, to automatically analyze the *perceived origin of movement*.

### A. Survey Design

A survey website[2] was developed to collect user ratings on the developed method. The protocol implemented to collect survey data is described as follows. As a user visits the webpage, a description of the task to be performed is given (see Fig. 3). Then, the user can choose to proceed with the task or quit the survey at any moment. No sensitive data are collected during the procedure.

Once the user agrees to participate and perform the task, a series of triplets of videos is presented (see Fig. 4). Each of the three videos in a triplet displays a skeletal representation of a dancer performing the same full-body expressive movement. Each video has one highlighted joint (in red). This joint corresponds to the most relevant joint according to one of the following criteria:

1) joint with the maximum Shapley value;
2) joint with the maximum speed;
3) random choice.

The identity of the joint highlighted in red is possibly updated by each criterion every second (making it difficult for the user to guess when the criterion applied in a specific video is, e.g., a random choice). The order of the three criteria is randomized among the three videos so that the specific criterion applied to each video is not predictable by the user. For a fair evaluation, the criteria themselves are also completely unknown to the user (i.e., the user has no idea how they are named and how they work). We constructed the survey in this way because the use of three different types of stimuli allowed us to investigate whether the proposed method for the automated investigation of the origin of full-body human movement and its propagation was perceived better than naive visualization of the joint with maximum speed (with this being more likely to be selected by nonexpert dancers than by expert dancers) and, of course, was better than a random choice.

During the survey, the participant is asked to choose the video that better represents the evolution of the most relevant joint responsible for originating the dancer's movement. Once a user has selected one video, he/she is asked to declare how confident he/she is in his/her choice by selecting a value from 1 to 5 on a 5-point Likert scale (levels: not confident, not so confident, neutral, confident, very confident). The participant can see all the videos as many times as desired and has to answer both questions (video choice and confidence level) before proceeding to the next triplet of videos. Each participant has to rate ten triplets of videos proposed from a selection of one hundred triplets using a Latin square selection method.

The results of the questionnaire are saved on a csv file, including the unique ID of each participant. The file is generated when the participant agrees to perform the task. For each of the ten proposed triplets of videos, the following information is collected:

1) ID of the full-body expressive movement proposed;
2) order used to propose the three alternative videos, and the corresponding extraction method used (largest Shapley value, largest speed, random choice);
3) participant's choice from among the three videos;
4) participant's confidence in that choice.

### B. Participants

The website was submitted to people with three different levels of expertise in dance: professionals, semiprofessionals, and novices/nondancers. A total of 22 people took part in the

[1][Online]. Available: https://www.infomus.org/eyesweb.it.php
[2][Online]. Available: https://www.infomus.org/Tools/OriginOfMovementThreeVideos/index.php

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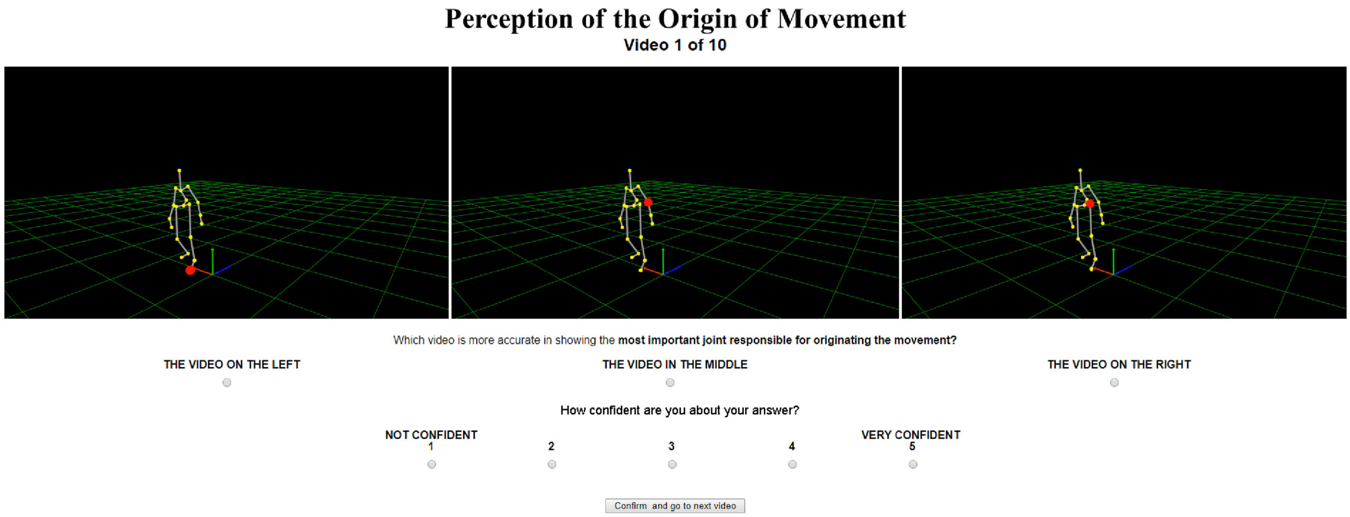


Fig. 4. Website for the evaluation of the proposed method: best method selection.

TABLE I
PARTICIPANTS' CHOICES (IN PERCENTAGES) AVERAGED OVER THE PARTICIPANTS OF EACH GROUP PLUS/MINUS THE ASSOCIATED EMPIRICAL STANDARD DEVIATIONS (IN PARENTHESIS)

| **Participants\Method** | **Shapley value** | **Speed** | **Random** |
|---|---|---|---|
| **Professionals** | 90 ($\pm$5.35) | 8.75 ($\pm$3.54) | 1.25 ($\pm$3.54) |
| **Semi-professionals** | 83.33 ($\pm$8.16) | 10 ($\pm$8.94) | 6.67 ($\pm$5.16) |
| **Novices/non-dancers** | 67.5 ($\pm$8.86) | 25 ($\pm$11.95) | 7.5 ($\pm$11.65) |
| **All Participants** | 80 ($\pm$12.34) | 15 ($\pm$11.44) | 5 ($\pm$8.02) |

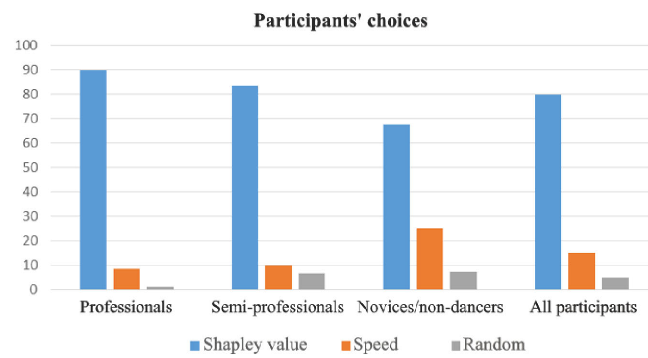


Fig. 5. Participants' choices (in percentages).

evaluation. Each participant self-evaluated his/her own level of expertise. The general information about the participants is as follows.

1) Professionals: 8 participants (3 male, 5 female), with a mean age of 42.75 years (std 9.56 years).
2) Semiprofessionals: 6 participants (3 male, 3 female), with a mean age of 30 years (std 4.47 years).
3) Novices/nondancers: 8 participants (6 male, 2 female), with a mean age of 35.5 years (std 7.4 years).

In the first two cases, the dancers were, respectively, experts and amateurs in contemporary dance.

### C. Results of the Survey

The detailed results of the chosen type from among the three types of different stimuli are presented in Table I and in the diagram shown in Fig. 5. Both demonstrate that the results of the validation of the proposed method are promising. Indeed, the Shapley value method was selected in the large majority of cases. A chi-square test with two degrees of freedom shows that the preference for the three categories of videos was not equally distributed in the subpopulations of professional dancers (80 observations, $\chi^2 = 157.90$, $p \leq 0.05$), semiprofessional dancers (60 observations, $\chi^2 = 74.71$, $p \leq 0.05$), and novices/nondancers (80 observations, $\chi^2 = 71.35$, $p \leq 0.05$) and in the whole population (220 observations, $\chi^2 = 218.90$, $p \leq 0.05$). For each application of the test, the number of observations is equal to ten times the number of participants in the specific category.

Based on the opinions of dancers with different levels of expertise, we can conclude that the joints obtained from the computation of the maximum Shapley value seemed to be very related to the concept of perceived origin of movement in dance, more so than the ones obtained from either the computation of the maximum speed or a random choice.

Despite their different levels of professionalism in dance, the participants selected, in 80% of the cases, the video highlighting the joint with the maximum Shapley value as the most effective video in showing the joint that was the most likely for the origin of movement.

The results also show that our proposed method was selected more often by more experienced dancers. In other words, the higher the level of expertise was, the more frequent the selection of the video representing the largest Shapley value joint. Interestingly, the selection frequency of the joint with the maximum speed increased as the level of expertise of the participant in the survey decreased, as expected.

Moreover, the participants were typically quite confident in the choices they made. In 41.48% of the cases, they were confident, and in 45.45% , they were neutral. A diagram of the confidence level of the participants is shown in Fig. 6.

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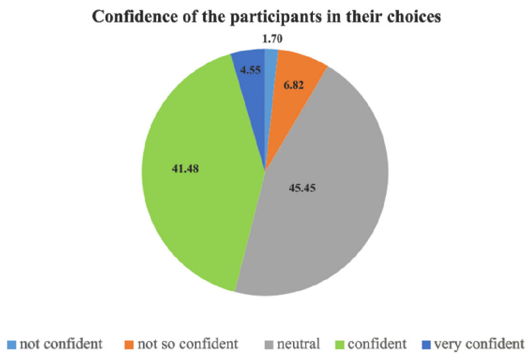


Fig. 6. Diagram of respondents' confidence levels.

TABLE II
FIRST TEN JOINTS ORDERED NONINCREASINGLY WITH RESPECT TO THEIR NORMALIZED SHAPLEY VALUES (IN PARENTHESIS) FOR TWO SELECTED FRAMES

| $1^{st}$ **frame** | $2^{nd}$ **frame** |
|---|---|
| shoulder centre (1.00) | right elbow (1.00) |
| head (0.46) | right shoulder (1.00) |
| right ankle (0.40) | right hip (0.53) |
| right knee (0.40) | right knee (0.53) |
| left elbow (0.38) | left ankle (0.27) |
| left shoulder (0.38) | left knee (0.27) |
| spine (0.31) | shoulder centre (0.19) |
| left ankle (0.31) | left shoulder (0.10) |
| left knee (0.30) | head (0.08) |
| right shoulder (0.22) | all the other joints (0) |

We conclude by reporting in Table II, for two selected frames,[3] the first ten joints, ordered nonincreasingly with respect to their Shapley values, normalized with respect to the maximum Shapley value in each frame. The associated normalized Shapley values are also reported in the table. The movements associated with the two frames are, respectively, a sudden leaning down to the left with the trunk and the head, followed by a rotation to the right, with a final rising of the trunk and head, where the shoulder centre is clearly the origin of movement; and a rotation and extension toward the right of the performer, where the right elbow and right shoulder are clearly leading the movement of the whole body. In both cases, the origin of movement is correctly identified by the proposed approach. The two frames also illustrate, respectively, the following situations, which were quite often observed during the analysis of the specific motion capture dataset: a) a case in which the first and second largest Shapley values are very well separated and (at least) the first largest one is uniquely achieved; and b) a case in which there are actually two joints with the largest Shapley value, but these joints are connected by a physical edge in the body graph, and the second largest Shapley value (which, in the second column, corresponds to the third joint) is very well separated from the first one.

## VIII. CONCLUSION

In this article, we proposed a computational method to analyze the perceived origin of full-body human movement and its propagation, which is based on game-theoretical and graph-theoretical techniques. In the proposed approach, the body graph is first transformed into a suitable auxiliary graph, which has the same vertices as those of the original body graph but different weighted edges, able to represent the dissimilarity between clusters of joints associated with a similar movement behavior. Then, due to the specific choice of the characteristic function of the cooperative TU game constructed on the auxiliary graph, the Shapley value provides a relevance index for its vertices. This measure is related to movement of the body, as the weights of the edges of the auxiliary graph are defined in terms of the movement properties.

The results of the application of the proposed approach to a dataset of movement segments obtained in the framework of this study are promising: indeed, the method was able to extract meaningful joints from the dataset, according to the results of a survey analysis, in which people with different levels of expertise in dance participated.

Visualization of the results showed the relevance of the joints extracted via the proposed approach with respect to the concept of the perceived origin of full-body expressive movement, as it was defined by dancers in the context of the EU Project WhoLoDance. We also observed differences between joints with the maximum Shapley value and joints with the maximum speed. In the opinion of dance experts, who participated in discussions during the project, the joint from which movement originates can be different from the joint with the maximum speed. Typically, the body parts with maximum speed are in the extremities of the body: left/right hand or left/right foot. Instead, the body joints with the maximum Shapley value are typically related to the same body parts but typically not in their ends, meaning the left/right shoulder, left/right knee, etc. (as Fig. 2 illustrates, they have some variability depending on the specific movement performed[4]). This may suggest the possibility of cases of convergence between the perceived and physical origins of movements. Indeed, according to the proximal-to-distal movement organization model reported in [7], [23], and [24], proximal joints tend to be leading joints more often than distal joints, since proximal segments are characterized by higher inertia and more massive musculature with respect to distal segments. The presence of different leading joints at different times is also in accordance with [24].

The approach proposed in this article can enable substantial advancements in the analysis of movement and in understanding the mechanisms involved in social affective communication. It also provides an effective computational model for doing so. The proposed method is promising in several aspects as follows.

[3]The file `2016-03-21_t011.tsv` with both movements is available at https://entimement.dibris.unige.it/repository (seconds 42 and 21, respectively).

[4]For example, in one of the recordings (for which the movement performed was this: the dancer reached the lower and upper parts of the space with either the left or right hand), the vertices with the maximum Shapley value occurred the following numbers of times (over a total of 3165 frames): head: 11 times; hip centre: 35 times; left ankle: 112 times; left elbow: 305 times; left foot: 0 times; left hand: 2 times; left hip: 233 times; left knee: 290 times; left shoulder: 62 times; left wrist: 21 times; right ankle: 151 times; right elbow: 466 times; right foot: 0 times; right hand: 16 times; right hip: 174 times; right knee: 357 times; right shoulder: 109 times; right wrist: 65 times; shoulder centre=neck: 717 times; spine: 39 times.

1) Novelty: this is the first time one explores the possibility of using a computational method based on techniques from both graph theory and game theory for the analysis of the perception of the origin of full-body human movement and its propagation.
2) Good theoretical properties: the characteristic function of the cooperative TU game, which was adopted in the formulation of the proposed method, satisfies desirable properties, i.e., monotonicity, superadditivity, and convexity.
3) Validation outcomes: according to the results of a survey, in which several participants with different levels of expertise in dance took part, the joints with the maximum Shapley value were relevant with respect to the concept of the perceived origin of full-body expressive movement.

A software library for the EyesWeb system implementing the approach presented in this article is under development for real-time interactive multimodal applications in therapy and rehabilitation and in sports, entertainment, and the performing arts. Furthermore, this library will be experimented on for applications requiring the automated measurement and prediction of full-body social signals in small groups in the framework of the H2020-FET-Proactive EU Project EnTimeMent, Project no. 824160.

Possible future developments, able to overcome the current limitations of this method, involve the following.

1) The use of movement-related features different from speed (or of a higher dimensional feature vector) to compute the Shapley value for a comparison with the results obtained using speed as a feature.
2) A comparison with other methods, such as placing the alternatives on joints close to the one with the highest Shapley value.
3) The design of additional validation methods, such as asking the participants to freely identify one joint as the origin of movement (a task which is expected to be more difficult for nonexpert dancers). In this case, the information provided by the experts' annotations could be used to assess the generalization capability of a classifier trained to predict the origin of movement based on the time series of Shapley values produced by the method itself and on a subset of training supervised examples.
4) The application of the proposed method to a more complex skeletal structure (for which each cluster of joints is associated with a specific joint in the simpler 20-joint skeletal structure), making it possible to analyze movement in parallel at a finer interacting spatio-temporal scale in a multiple-scale approach (in line with the objectives of the EU Project EnTimeMent). In this way, one could compare the Shapley value of a joint in the simpler structure with the sum of the Shapley values of the associated joints in the more complex structure (a smaller Shapley value would be expected for each of the latter joints). It is worth mentioning that in the case of a more complex skeletal structure, validation through an online survey could be still performed, but it may be more difficult to find suitable participants, because a more complex skeletal structure could make it more difficult for participants to detect the origin of movement.
5) The investigation of possible constraints on the intervals of variation in the Shapley values associated with different joints due to physical constraints on the movement of specific joints. In future developments, the results of this analysis could be useful for Shapley value prediction.

## Appendix

*Proof of Proposition V.1:*

1) This follows from (2) and the fact that for every two subsets $V_1 \subseteq V_2 \subseteq V$, one has $E^{(\mathrm{aux})}(V_1) \subseteq E^{(\mathrm{aux})}(V_2)$, and the weights of the edges of the weighted auxiliary graph $G^{(\mathrm{aux})}$ are nonnegative.
2) For every two disjoint subsets $V_1 \subseteq V_2 \subseteq V$, one has

$$\begin{aligned}
&c(V_1 \cup V_2) \\
&= \sum_{v,\hat{v} \in V_1 \cup V_2, \hat{v} \in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \\
&= \sum_{v,\hat{v} \in V_1, \hat{v} \in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \\
&\quad + \sum_{v,\hat{v} \in V_2, \hat{v} \in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \\
&\quad + \sum_{v \in V_1, \hat{v} \in V_2, \hat{v} \in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \\
&\quad + \sum_{v \in V_2, \hat{v} \in V_1, \hat{v} \in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \\
&= c(V_1) + c(V_2) \\
&\quad + \sum_{v \in V_1, \hat{v} \in V_2, \hat{v} \in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \\
&\quad + \sum_{v \in V_2, \hat{v} \in V_1, \hat{v} \in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \\
&\geq c(V_1) + c(V_2)
\end{aligned} \tag{4}$$

due to the nonnegativeness of the weights of the edges of the weighted auxiliary graph $G^{(\mathrm{aux})}$.
3) Let $V_1 \subseteq V_2 \subseteq V$ and $v \in V \setminus V_2$. One has

$$\begin{aligned}
&c(V_1 \cup \{v\}) - c(V_1) \\
&\quad = 2 \sum_{\hat{v} \in V_1, \hat{v} \in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \\
&\quad \leq 2 \sum_{\hat{v} \in V_2, \hat{v} \in N^{(\mathrm{aux})}(v)} w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}}) \\
&\quad = c(V_2 \cup \{v\}) - c(V_2)
\end{aligned} \tag{5}$$

since $V_1 \subseteq V_2$ and the insertion of the vertex $v$ in $V_1$ (respectively, in $V_2$) causes an increase in the value of the characteristic function equal to two times the sum of the (nonnegative) weights of the edges in the weighted auxiliary graph linking $v$ with the vertices of $V_1$ (respectively, of $V_2$). ■

*Proof of Proposition V.2:* The proof, which follows from [16, Th. 1], is based on the form of the characteristic function (2) and the linearity of the Shapley value with respect to the characteristic function. It also exploits the interpretation of the Shapley value as the average marginal utility of a player when it enters a random coalition $V'$, considering all possible coalitional scenarios, and assuming that all the possible orders of the players are equally likely [14].

We first focus on the case in which the auxiliary graph contains a single edge that joins the vertices $v$ and $\hat{v}$. One can note that the vertex $v$ enters the random coalition $V'$ before the vertex $\hat{v}$ half of the time, whereas it enters the random coalition $V'$ after the vertex $\hat{v}$ the other half of the time. In the first case, the marginal utility of $v$ is 0, whereas in the second case, it equals $2w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}})$ (because the edge is considered two times, i.e., as $e^{(\mathrm{aux})}_{v,\hat{v}}$ and $e^{(\mathrm{aux})}_{\hat{v},v}$). Hence, its average marginal utility is $w^{(\mathrm{aux})}(e^{(\mathrm{aux})}_{v,\hat{v}})$. Then, the proof is completed by combining this result with the fact that the Shapley value depends linearly on the characteristic function, which in the case of (2) is linear with respect to the edge weights. ■

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