Effect of Railway Track Segmentation Method on the Optimal Solution of Tamping Planning Problem

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Abstract

The safety and continuality of the railway network are guaranteed by carrying out a lot of maintenance interventions on the railway track. One of the most important of these actions is tamping, where railway infrastructure managers focus on optimizing tamping activities in ballasted tracks to reduce the maintenance cost. To this end, this article presents a mixed integer linear programming model of the Tamping Planning Problem (TPP) and investigates the effect of track segmentation method on the optimal solution by three scenarios. It uses an opportunistic maintenance technique to plan tamping actions. This technique clusters many tamping works through a time period to reduce the track possession cost as much as possible. CPLEX 12.6.3 is used in order to solve the TPP instances exactly. The results show that the total number of machine preparations increases by increasing the number of track segments. It is also found that the total costs increase by 6.1% and 9.4% during scenarios 2 and 3, respectively. Moreover, it is better to consider the whole railway track as a single segment (as in scenarios 1) that consists of a set of sections during the tamping planning in order to obtain the optimal maintenance cost.

Keywords: Maintenance; Tamping Planning Problem; Railway Track; Model; CPLEX.

1. Introduction

Tamping planning is an important issue in railway infrastructure maintenance planning and can profoundly influence in the funding aspect and operational flexibility of a railway track. The tamping planning problem (TPP) seeks to decide on when to tamp which section to minimize the total cost across a finite planning horizon. In addition, the tamping process is expensive [1] and has major economic impact on maintenance projects. Thus, the management and optimization of tamping works over time become a vital challenge to the infrastructure managers in order to reduce maintenance costs (Andrade & Teixeira 2011 [2]; Daddow et al. 2020 [3]; Heinicke et al. 2015 [4]; Letot et al. 2016 [5]; Macke & Higuchi 2007 [6]; Miwa 2002 [7]).

In this problem, the studied railway track is assumed to divide into a number of equal sections with their private conditions (i.e. standard deviation of longitudinal level). Each condition increases over time and it is required to

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reduce by performing a tamping operation before it transcends a particular limit. This reduction is called "condition recovery" and calculated based on specific standards. In the mathematical model of this problem, opportunistic maintenance concept is used when planning the tamping jobs. One component of the cost structure in the objective function of the tamping operations is that whenever any tamping operation is being carried out on the track a track possession cost has to be paid. This cost is considered because the tamping works prevent the railway traffic. This cost leads rise to opportunities for more than the required tamping to be planned in a time period to reduce the track possession time as much as possible. Moreover, the problem here considers many budgetary and operational aspects during the mathematical formulation such as resource limitations and tamping machine preparation. The optimal solution represents the optimum tamping schedule that the infrastructure manager aims to obtain.

In the past, many papers investigated the TPP in railway infrastructure. Oyama and Miwa (2006) [8] proposed a mathematical model for this problem to maximize the improvement of railway track condition under limited maintenance costs. After that they evaluated the applicability of the model by three different scenarios from the railway system in Japan. Oh et al. (2006) [9] developed an optimization model for this problem in the Korean railway system regarding many operational restrictions such as the constraints on blocking times. Another memorable article in this area was found in Vale et al. (2012) [10]. They minimized the sum of tamping activities over a planning horizon of two years. They investigated two scenarios of calculations based on the length of planning horizon through each one of them. Finally, they performed two calculations in the first scenario; each one consists of one year with four time periods. While in the second scenario, they carried out a single calculation considering two years with eight periods. Gustavsson (2015) [11] extended the previous model by minimizing the sum of maintenance costs instead of the sum of tamping jobs over the planning horizon. In addition, he utilized disaggregated constraints in the model for the track horizontal layout to increase the effectiveness of the branch-and-bound algorithm.

Wen et al. (2016) [1] also improved Vale's model taking into account the impact of previous tamping interventions on the condition recovery value, minimizing the maintenance cost instead of the total number of tamping activities, and utilizing the discounted total cost in the objective function to improve the track condition at the end of planning horizon as much as possible. Daddow et al. (2017) [12] extended Gustavsson's model by considering the effect of unused life for track sections due to early tamping and including the budget constraints in the model. Moreover, they investigated the impact of maximum and minimum limits for the available resources on the optimal tamping schedules. Another excellent article to optimize the tamping operations in ballasted tracks has been recently done by Khajehei et al. (2020) [13]. They adopted the concept of opportunistic maintenance planning to reduce the total tamping cost. Then, the authors used a genetic algorithm technique to obtain the optimal solution of the problem. At last, some others presented tamping planning models such as Zhang et al. (2013) [14], Famurewa et al. (2015) [15], Daddow et al. (2020) [3], Vale and Ribeiro (2014) [16], Quiroga and Schnieder (2010) [17], Bakhtiary et al. (2020) [18], Peralta et al. (2018) [19] and Lee et al. (2018) [20].

The previous researches are interesting and valuable but to the best of our knowledge no study is yet available on the TPP that investigates the impact of railway track segmentation on the optimal solution and the total maintenance cost. Therefore, we present in this paper an optimization model of this problem and evaluate it under three different scenarios of segmentation. The first scenario includes a single calculation, considering the whole track as a single segment with equal successive sections. The second scenario encompasses two calculations, each one with one segment that equals to the half-length of the track. The third scenario involves three calculations, each one with one segment that equals to the one-third-length of the track. Then, sub-calculations of the second and the third scenarios are calculated to obtain the real total cost of the final optimal solution for each one of them. Final results of these scenarios are then compared and investigated.

The paper is structured as follows. Section 2 provides the TPP description and Section 3 presents its mathematical formulation. In Section 4, the computational results in terms of comparison between the above-mentioned scenarios are discussed. Conclusion is finally presented in Section 5.

2. Problem Description

In this paper, we tackle the TPP in ballasted tracks with economic and operational constraints in which the aim is to minimize the total maintenance cost and satisfy the service requirements. The considered railway track consists of a set of consecutive sections with the same length (i.e. 200 m) and the proposed model will be evaluated under three segmentation scenarios as presented in detail in Section 4. Additionally, the standard deviation of longitudinal level demonstrates the track condition for any section. This parameter is assumed to increase linearly over time and it is required to perform a tamping action to decrease before surpasses the track condition limit. This limit depends on the maximum admissible train speed over the regarded track [10]. In this article, the condition recovery after tamping is calculated according to the Office for Research and Experiments (ORE, 1988) [21]. It equals to $a.(sd_i^{j-1} + dr_i) + b$, where: sd_i^{j-1} is the standard deviation of longitudinal level for the track section i at the end of time period j-1, dr_i represents the degradation rate of the section i, $(sd_i^{j-1} + dr_i)$ is the standard deviation of longitudinal level for the

section i at the time of tamping process, while a and b are real parameters to calculate the recovery value. As mentioned above the concept of opportunistic maintenance is utilized when scheduling the tamping actions in the TPP model. In this concept, combining tamping interventions as much as possible results in track possession time savings (i.e. it reduces the disturbance of railway traffic).

Moreover, to overcome the dilemma of the limitation of laptop's storage space under a big number of variables and use a big number for the track sections and utilize a long planning horizon (10 time periods in this paper), it is only adopted the following considerations in the model: (1) the track layout impact, (2) the resource constraints and (3) the tamping machine preparation issue.

According to the Union Internationale des Chemin de Fer (UIC, 2008) [22], the tamping activity must begin and end on a straight section. This recommendation considers the impact of the track horizontal layout on the tamping plan, where the track layout consists of two categories of sections as follows: straight sections (S) and curve sections (C). In addition, the impact of resource constraints on the TPP is included to make it more realistic. The resource involves, for example, limited maintenance staff, machines availability, funding, etc. Hence, it is assumed that the total number of planned tamping interventions during one time period is not allowed to exceed a certain limit. Furthermore, the influence of tamping machine preparation for every tamping work on a single section or successive sections is considered in this model. In other words, the machine needs warm-up before tamping and ramped down after tamping. Therefore, the solver will try to cluster the tamping jobs as much as possible to reduce the preparation cost and, thus, the total cost. The preparation cost here encompasses the cost of warming up and ramping down together.

Lastly, it is required to get an optimal tamping schedule minimizing the total cost across the planning horizon. This cost involves the following components: cost of tamping, cost of machine preparation and cost of railway track possession that is calculated whenever any tamping intervention performed in that time period.

3. Mathematical Formulation

This section introduces the mathematical formulation of the TPP in this article. Let I represents the set of railway track sections, let J denotes the set of time periods and let $K_i \subseteq I$ be the smallest set of sequent sections where $i \in K_i$ and the first and last sections through K_i are **straight sections**. In addition, let parameters Tc, MPc and TPc be the tamping cost of each section, the unit tamping machine preparation cost and the track possession cost, respectively. Each section $i \in I$ has an initial standard deviation of longitudinal level sd_i^{ini}, degradation rate dr_i and maximum allowable limit of the standard deviation of longitudinal level sd_i^{max}. Let M be a very large number utilized in the big-M restrictions. Let a and b denote real parameters to determine the value of recovery. The infrastructure manager has a maximum number of tamping actions W^j that can be scheduled through each period $j \in J$. In order to present the mathematical formulation of the studied model, the following decision variables will be utilized:

x_i^j: Binary variable that indicates whether a tamping action is assigned to the section $i \in I$ during the period $j \in J$ ($x_i^j = 1$) or not ($x_i^j = 0$).

p_i^j: Binary variable that denotes whether the tamping machine needs to be prepared at the section $i \in I$ during the time period $j \in J$ ($p_i^j = 1$) or not ($p_i^j = 0$).

z^j: Binary variable that indicates whether the considered railway track is utilized for tamping works at the period $j \in J$ ($z^j = 1$) or not ($z^j = 0$). A time period $j \in J$ such that $z^j = 1$ is called a maintenance opportunity.

sd_i^j: Variable that represents the standard deviation of longitudinal level for the section $i \in I$ during the time period $j \in J$.

re_i^j: Variable that denotes the condition recovery after tamping for the section $i \in I$ during the period $j \in J$.

R_i^j: Auxiliary variable that equals zero if $x_i^j = 0$ and re_i^j if $x_i^j = 1$.

Based on the above notations, the proposed mathematical formulation of the model is as follows:

$$min \sum_{j \in J} \left(\sum_{i \in I} \left(Tc.x_i^j + MPc.p_i^j \right) + TPc.z^j \right) \tag{1}$$

Subject to:

$$sd_i^0 = sd_i^{ini} \qquad \qquad \forall i \in I \tag{2}$$

$$sd_i^{j-1} + dr_i \leq sd_i^{max} \qquad \qquad \forall i \in I, j \in J, j > 0 \tag{3}$$

$$sd_i^j = sd_i^{j-1} + dr_i - R_i^j \qquad \qquad \forall i \in I, j \in J, j > 0 \tag{4}$$

$$R_i^j \leq M.x_i^j \qquad \forall i \in I, j \in J \tag{5}$$

$$0 \leq re_i^j - R_i^j \leq M.(1 - x_i^j) \qquad \forall i \in I, j \in J \tag{6}$$

$$re_i^j = a.(sd_i^{j-1} + dr_i) + b \qquad \forall i \in I, j \in J, j > 0 \tag{7}$$

$$x_i^j \leq z^j \qquad \forall i \in I, j \in J \tag{8}$$

$$x_m^j \geq x_i^j \qquad \forall m \in K_i, i \in I, j \in J \tag{9}$$

$$\sum_{i \in I} x_i^j \leq W^j \qquad \forall j \in J \tag{10}$$

$$p_1^j \geq x_1^j \qquad \forall j \in J \tag{11}$$

$$p_i^j \geq x_i^j - x_{i-1}^j \qquad \forall i \in I, j \in J, i > 1 \tag{12}$$

$$x_i^j, p_i^j, z^j \in \{0,1\} \qquad \forall i \in I, j \in J \tag{13}$$

$$sd_i^j, re_i^j, R_i^j \geq 0 \qquad \forall i \in I, j \in J \tag{14}$$

Equation 1 represents the objective function that aims to minimize the sum of tamping costs (first part), the machine preparation costs (second part) and the track possession costs (third part). Equation 2 initializes the standard deviations of longitudinal level at the time period $j = 0$ and Equation 3 establishes the threshold values of these deviations. Then, sd_i^j is calculated by Equation 4, while R_i^j is determined by Equations 5 to 7. If the track section $i \in I$ has no tamping in the time period $j \in J$ ($x_i^j = 0$), then R_i^j is set to zero by Equation 5. Otherwise, R_i^j is set to re_i^j by Equation 6, where $re_i^j = a.(sd_i^j{}^{-1} + dr_i) + b$ as guaranteed in Equation 7. Equation 8 ensures that the possession cost will be paid each period $j \in J$ in which any tamping operation is being carried out. In Equation 9, it is guaranteed that the tamping intervention will start and end on a straight section. Equation 10 ensures that the maximum number of tamping activities scheduled in any period $j \in J$ cannot exceed the value of W^j. The effect of tamping machine preparation is included through Equations 11 and 12. If the first section in the track requires tamping then the machine requires warming up at that section by Equation 11. Moreover, if any section $i - 1$ does not need tamping work but the section i does, thus, the machine should be warmed up at the section i through Equation 12. Finally, Equations 13 and 14 define all the decision variables involved in the model.

4. Computational Study

4.1. Generation of Instances and Input Data

Ten instances - with different initial condition values for each section - are randomly generated for a computational study of the mathematical model presented in Section 3. For this study, it is established three different segmentation scenarios (1, 2 and 3) for the concern railway track as shown in Figure 1, each with its own number of segments, number of sections in each segment and maximum number of tamping interventions that can be planned during each period (W^j) as presented in Table 1. In this article, it is assumed that W^j is around 85% of the total number of sections for each segment through every scenario in order to achieve a fair occasion for all the scenarios (Table 1). This study aims to perceive the differences of tamping schedules and maintenance costs over the given track between these scenarios. Additionally, the whole length of the track is 30 km and each section has a length of 200 m. The planning horizon consists of ten periods of 90 days. The parameters *a* and *b* are set to 0.4257 and - 0.153, respectively as mentioned in [10]. Figure 2 depicts the related track horizontal layout, while Figure 3 presents the degradation rate for each section. Moreover, the initial condition values of the track sections for instance 3 are shown in Figure 4. The maximum speed of trains on this track varies between 160 and 220 km/h, therefore, the track condition limit for all the sections equals to 1.9 mm based on EN13848-5 (CEN, 2008) [23] (Figure 4). The parameters Tc, MPc and TPc are set to 10,400, 18,000 and 200,000 SEK (Currency of Sweden), respectively. Lastly, Figure 5 shows a flowchart of the developed model.

Table 1. Characteristics of scenarios

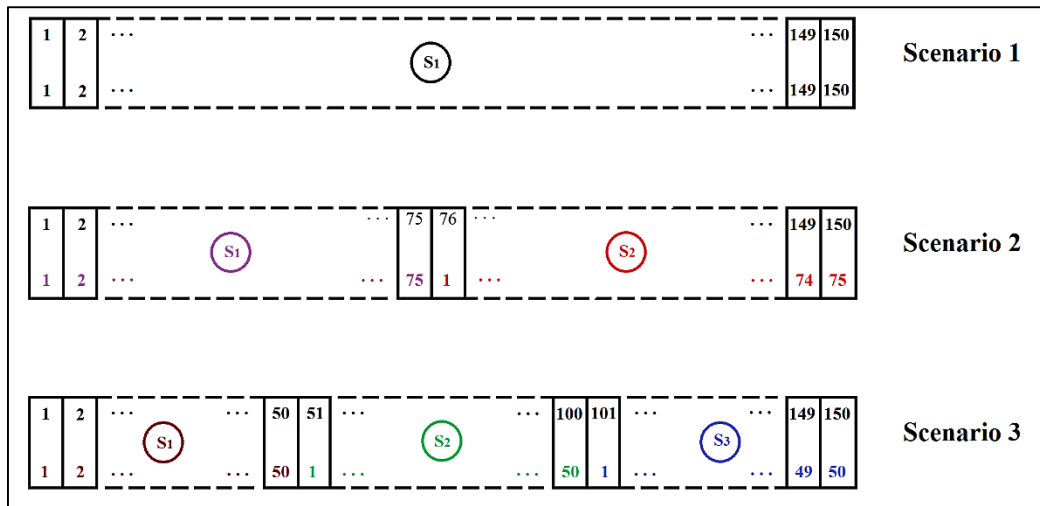| Scenario	Number of segments	Number of sections in each segment	W^j
1	1	150	128
2	2	75	64
3	3	50	43

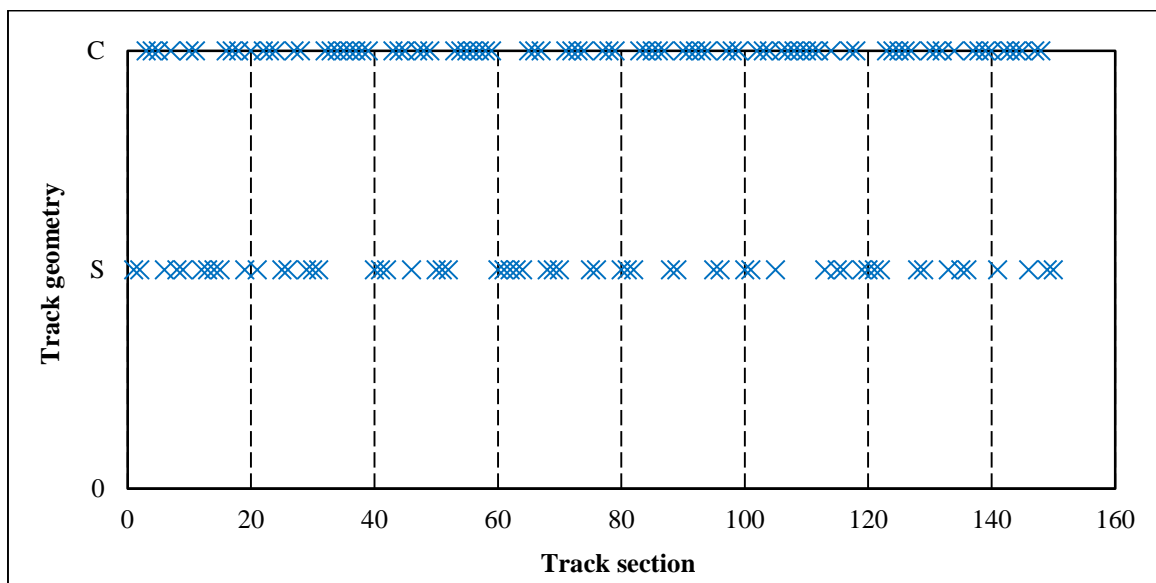**Figure 1. Segmentation scenarios of the concern track**

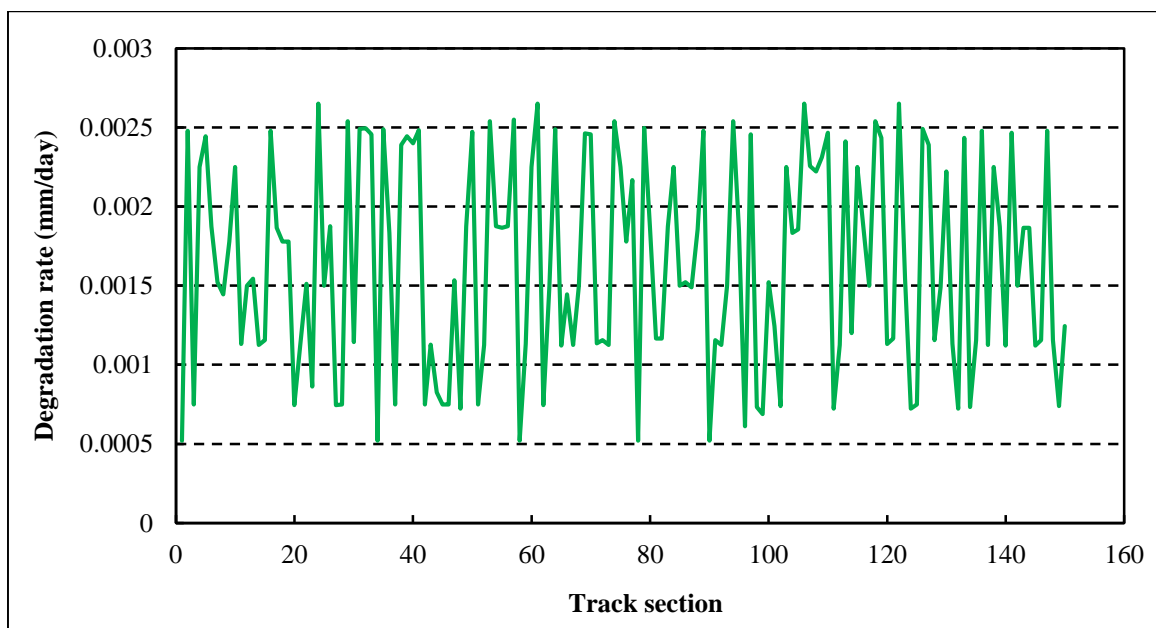**Figure 2. The track horizontal layout**

Figure 3. Degradation rate of the standard deviation of the longitudinal level for track sections
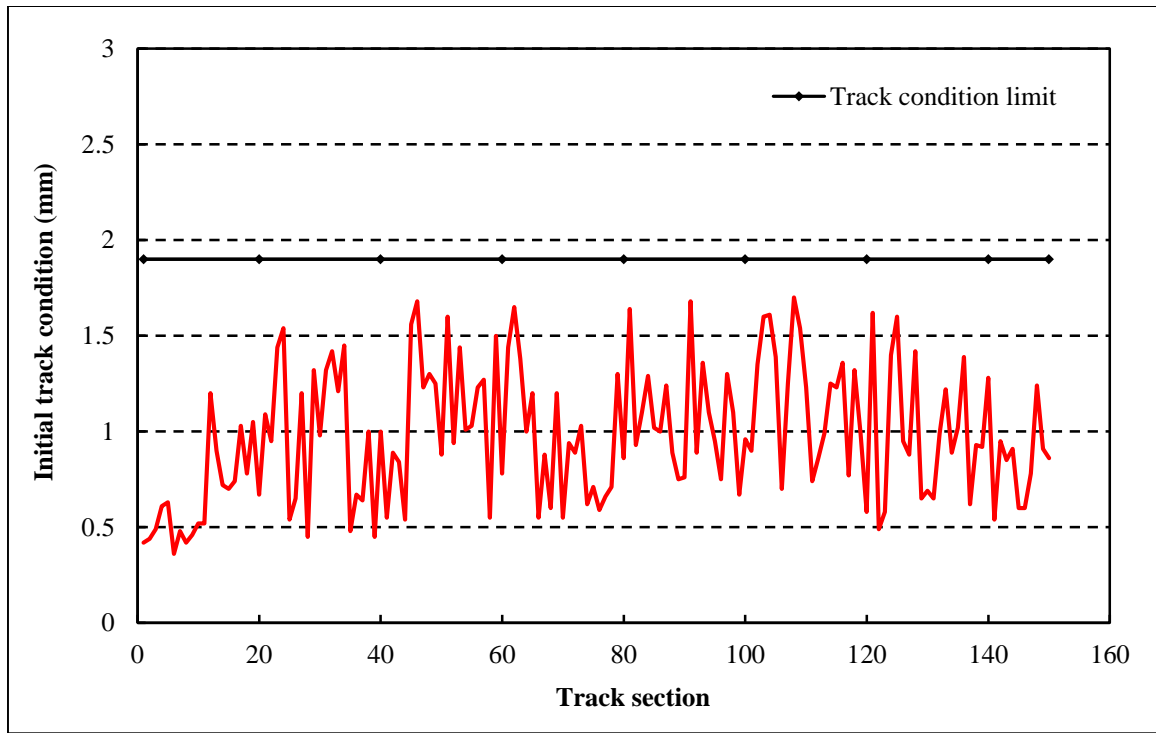
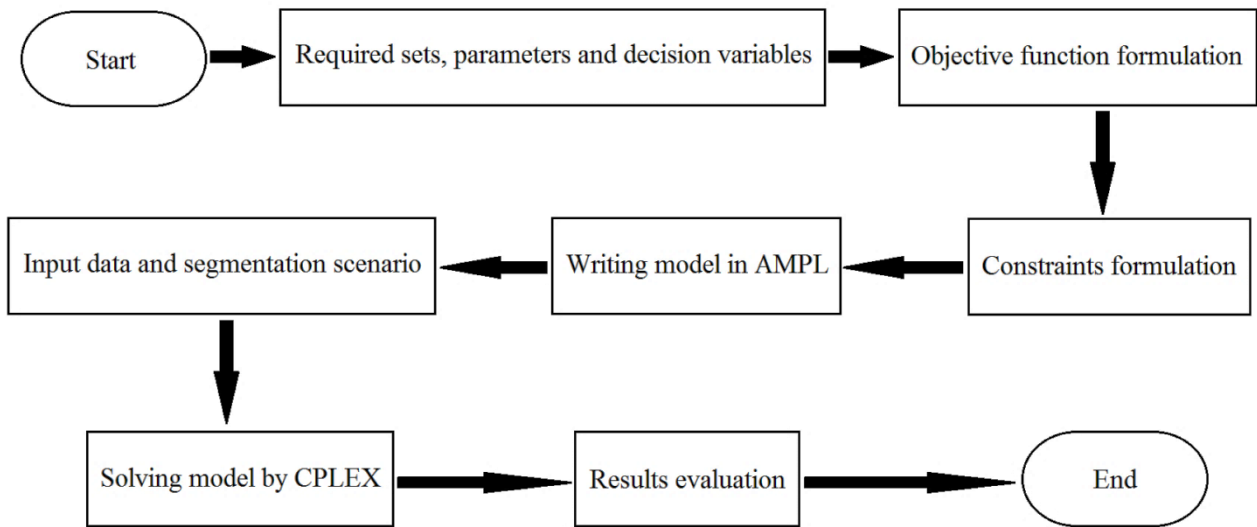**Figure 4. Initial track condition values for instance 3**

Figure 5. Flowchart of the developed model

The mixed-integer linear formulation of the model is implemented in AMPL [24] and the regarding instances are solved using CPLEX 12.6.3 [25]. All experiments are conducted in a laptop Intel(R) Core i5 with 2.40 GHz and 4GB of RAM under Windows 7 ultimate system.

5. Results and Discussion

At first, we want to explain the method of the sub-calculations for both second as well as third scenarios to get the real total cost of the final optimal solution for each one of them. To this end, an example would help to clarify the process better. Table 2 epitomizes the results of instance 3. We outline values of the total number of tamping operations (t.n.x), the total number of machine preparations (t.n.p), the total number of possession periods (t.n.z) and the total cost (t.c) (in SEK) for each scenario. In addition, the possession time periods through each scenario are given in the last column. In scenario 2, it is found that the sections 75 and 76 (segmentation area in scenario 2 as shown in Figure 1) are planned to be tamped together in the fourth and the seventh periods, respectively. Thus, cost of two preparations (2*18,000) must be subtracted from the sum of the costs of both segments solutions because tamping machine does not need to prepare at the section 76 during the actual tamping process (i.e. they will successively be

tamped) in both periods. As well, it can be noted in Table 2 that the fixed costs of the first, the fourth and the seventh periods are paid two times in this scenario (one time during each segment); consequently, fixed cost of three time periods (3*200,000) must also be subtracted from the above-mentioned sum. Therefore, the real total cost herein = 2,856,000 + 3,046,800 − (2*18,000 + 3*200,000) = 5,266,800 SEK. Similarly, for scenario 3, it is found that the number of repeated preparations through the segmentation areas (two areas in scenario 3 as shown in Figure 1) equals to four and the number of multi-paid possession periods during the three segments of this scenario equals to five; thence, the real total cost here = 2,118,400 + 2,342,400 + 2,082,800 − (4*18,000 + 5*200,000) = 5,471,600 SEK. Hence, the bold values in Table 2 for t.n.p, t.n.z and t.c represent their final values after excluding the repeated or multi-paid values through the final solution for each scenario. Finally, Tables 3 and 4 report the number of repeated preparations and multi-paid possession periods in both scenarios for all the instances, respectively. These values are utilized to calculate the real total costs for all final solutions in both scenarios.

Table 2. Results of instance 3

Scenario	Segment	t.n.x	t.n.p	t.n.z	t.c (SEK)	Possession periods
1	1	370	21	4	5,026,000	1.2.4.7
2	1	170	16	4	2,856,000	1.2.4.7
	2	197	11	4	3,046,800	1.4.6.7
	Total	367	**25**	**5**	**5,266,800**	1.2.4.6.7
3	1	106	12	4	2,118,400	1.2.5.6
	2	131	10	4	2,342,400	1.2.4.7
	3	127	9	3	2,082,800	1.4.7
	Total	364	**27**	**6**	**5,471,600**	1.2.4.5.6.7

Table 3. Number of repeated preparations for segmentation areas in scenarios 2 and 3 for all the instances

Instance	Scenario 2	Scenario 3	
	75-76	50-51	100-101
1	2	0	2
2	3	0	2
3	2	1	3
4	3	0	1
5	2	0	1
6	2	0	1
7	2	1	2
8	2	1	1
9	2	0	2
10	2	1	1

Table 4. Number of multi-paid possession periods during the segments solutions in scenarios 2 and 3 for all the instances

Instance	Scenario 2	Scenario 3
1	3	6
2	4	7
3	3	5
4	3	5
5	3	5
6	2	5
7	2	5
8	3	6
9	4	7
10	3	7

Effect of Segmentation Method on the Total Number of Tamping Actions (t.n.x)

The effect of segmentation method on this parameter is investigated by comparing its values obtained through the three scenarios. Figure 6 displays these values given by these scenarios for all the instances. As can be seen, the results herein do not give a specific indicator about their variability between scenarios. However, the comparison between scenario 1 and scenario 3 shows that t.n.x values are decreased in most instances by increasing the number of segments. Further, the largest decrease in the t.n.x values is related to instance 7 with the amount of eleven actions where the t.n.x decreases from 387 to 376. Probably, the reasons for these results are that the initial condition values of the track sections for all the instances in addition to the construction of the studied track horizontal layout (Figure 2).
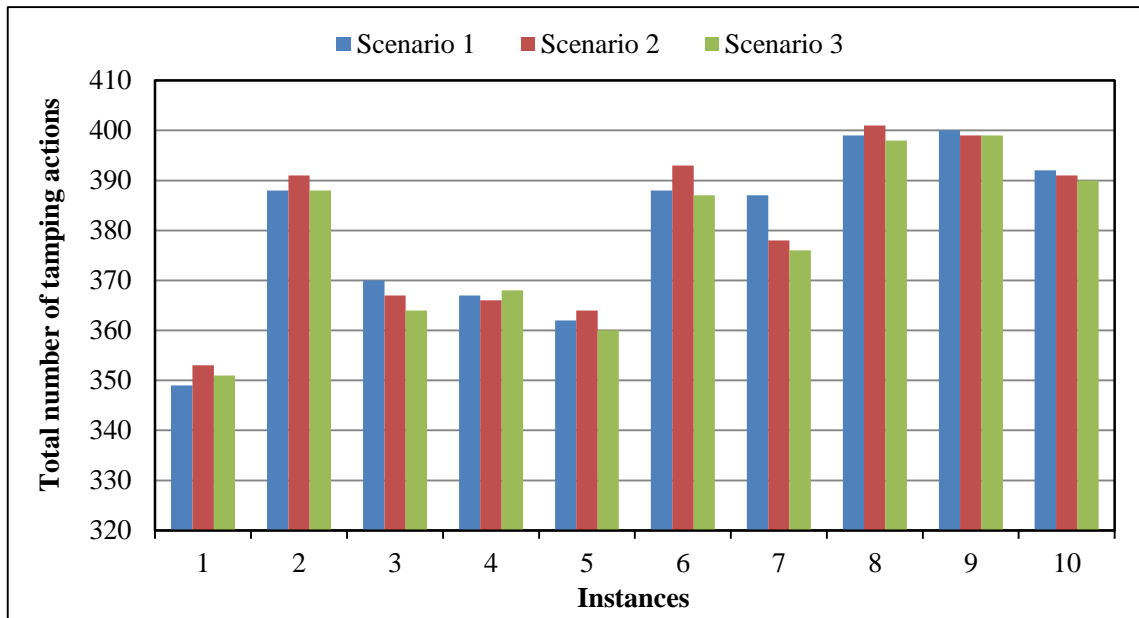
Figure 6. The total number of tamping actions in the three scenarios for all the instances

Effect of Segmentation Method on the Total Number of Preparations (t.n.p)

The t.n.p values obtained by the three scenarios for all the instances are presented in Figure 7. It can be observed that these values increase by increasing the number of segments. For example, by transition from scenario 1 to scenario 2 and from scenario 2 to scenario 3 in instance 4, they increase by two and three preparations, respectively. Also, it is found that the largest increase in the t.n.p values is related to instances 3 and 9 with the amount of six preparations. One of the reasons for these results is the individual scheduling for segments through scenarios 2 and 3.
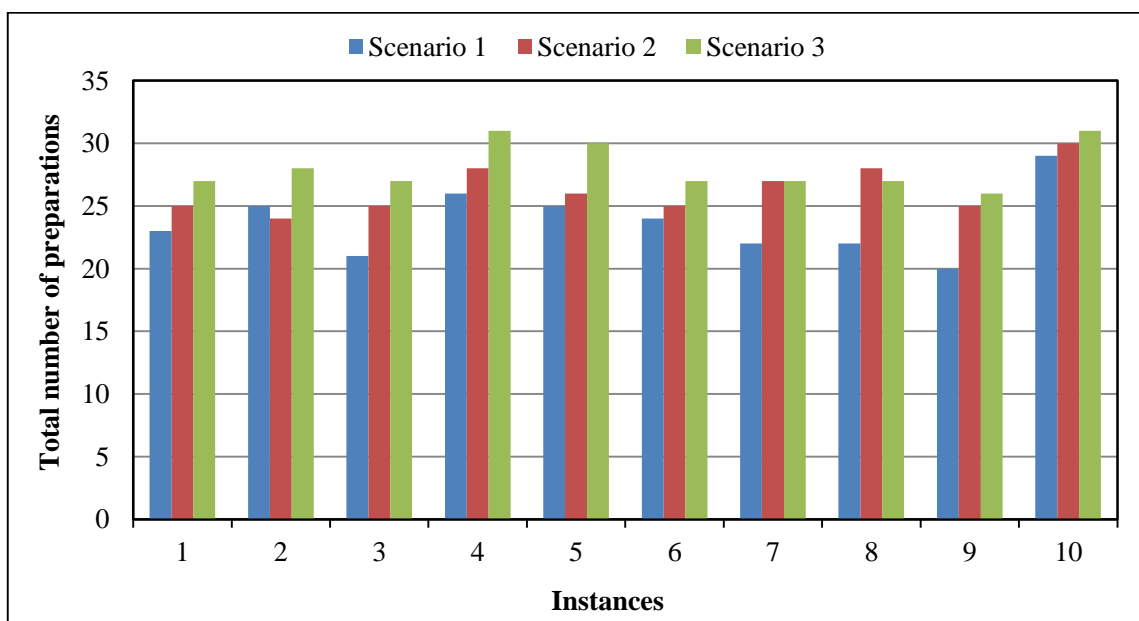
Figure 7. The total number of preparations in the three scenarios for all the instances

Effect of Segmentation Method on the Total Number of Possession Periods (t.n.z)

To achieve a more comprehensive understanding of this effect, Figure 8 reports the t.n.z values given by the three scenarios for all the instances. According to Figure 8, it can be seen that these values for scenario 2 and scenario 3 are equal to or greater than those of scenario 1 because of the independent planning for segments through scenarios 2 and 3. In light of this, it would be interesting for future research to explore and investigate the optimal solution of an improved model of the TPP in which dependent planning constraints for the segments can be included.
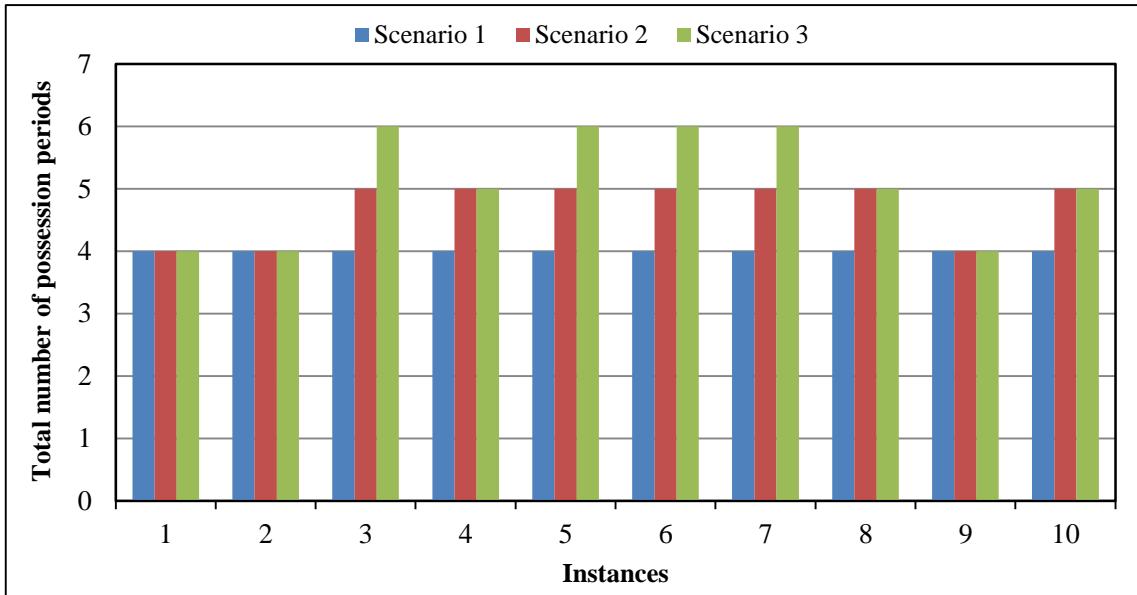

Figure 8. The total number of possession periods in the three scenarios for all the instances

Effect of Segmentation Method on the Total Costs (t.c)

To investigate this effect, Figure 9 shows the t.c values obtained in the three scenarios for all the instances. It can be seen that these values increase by increasing the number of track segments. Furthermore, the maximum increase in the t.c by transition from scenario 1 to scenario 2 is found for instance 8, where it increases by 6.1% (from 5,345,600 SEK to 5,674,400 SEK). Also, the maximum increase from scenario 1 to scenario 3 is found for instance 5, where it increases by 9.4% (from 5,014,800 SEK to 5,484,000 SEK). Hence, it can be concluded that it is important to consider the whole railway track as a single segment (as in scenario 1) that consists of a big number of sections during the tamping planning to obtain the minimum total cost. However, a big size problem would be a vital challenge to solve by a commercial optimization solver (more CPU time for solving); thus, it would be interesting to use heuristic techniques for future research.
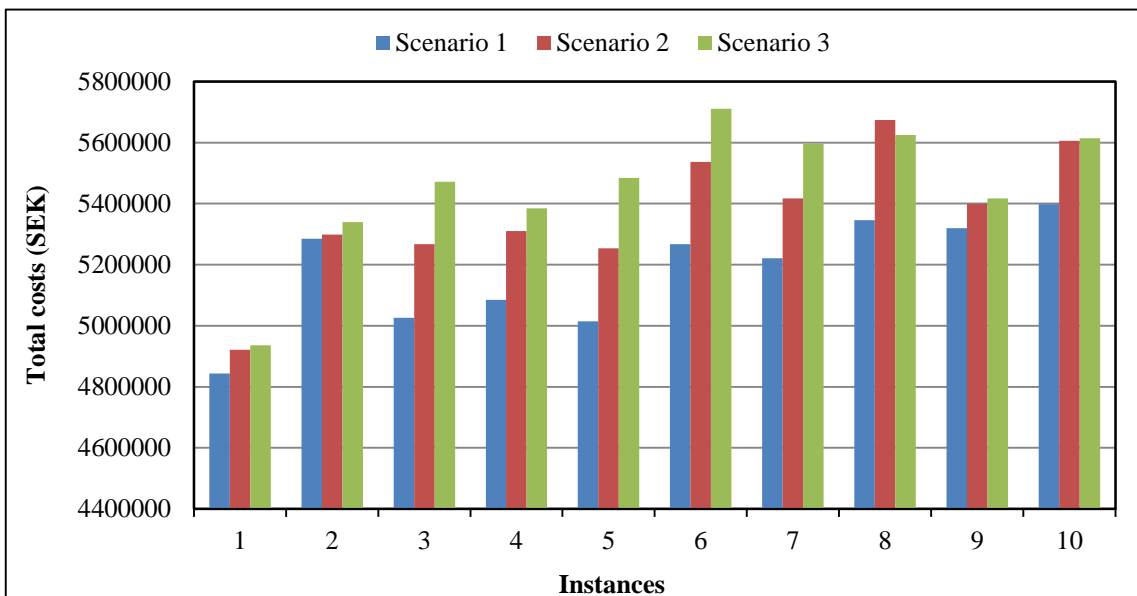

Figure 9. The total costs in the three scenarios for all the instances

Comparison between the Optimal Schedules of the Three Scenarios

Figures 10 to 12 display the optimal schedules provided by these scenarios for instance 3. It can be observed that the tamping jobs are planned during four, five and six time periods by scenario 1, scenario 2 and scenario 3, respectively. Moreover, the segmentation process leads to an increase in the t.n.p by transition from scenario 1 to scenario 2 and from scenario 2 to scenario 3, although the t.n.x decreases for this instance from 370 in scenario 1 to 367 in scenario 2 to 364 in scenario 3. Consequently, scenario 1 provides the best optimal tamping schedule with the lowest maintenance cost.
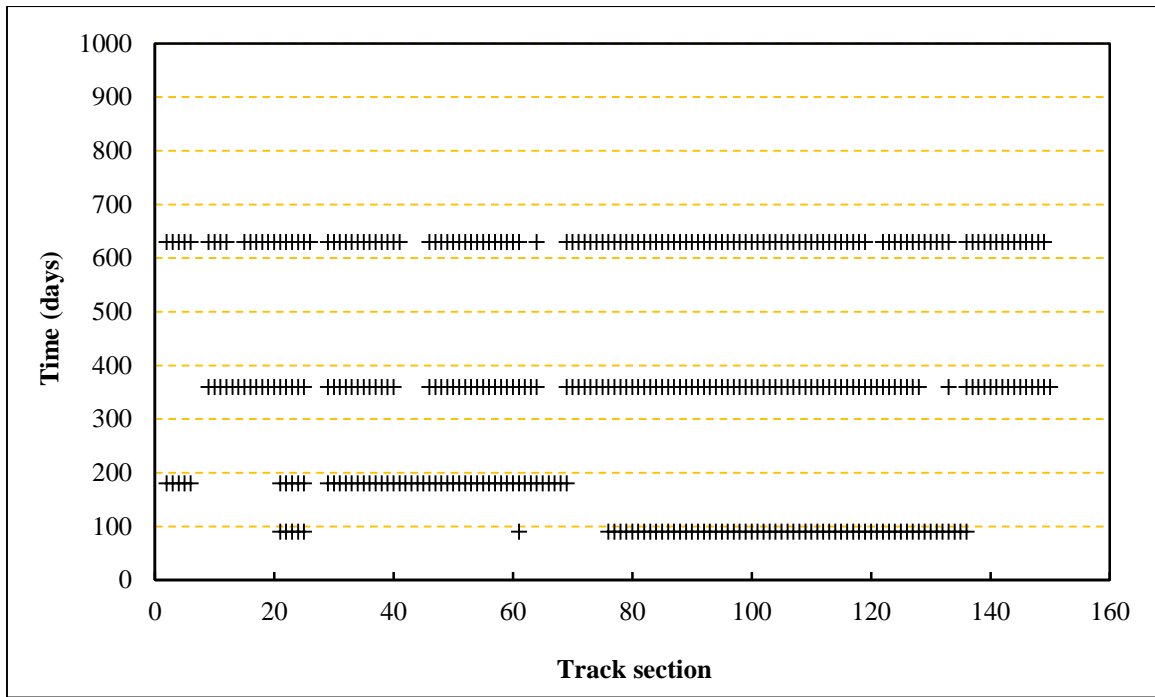
Figure 10. The optimal schedule for instance 3 obtained by scenario 1
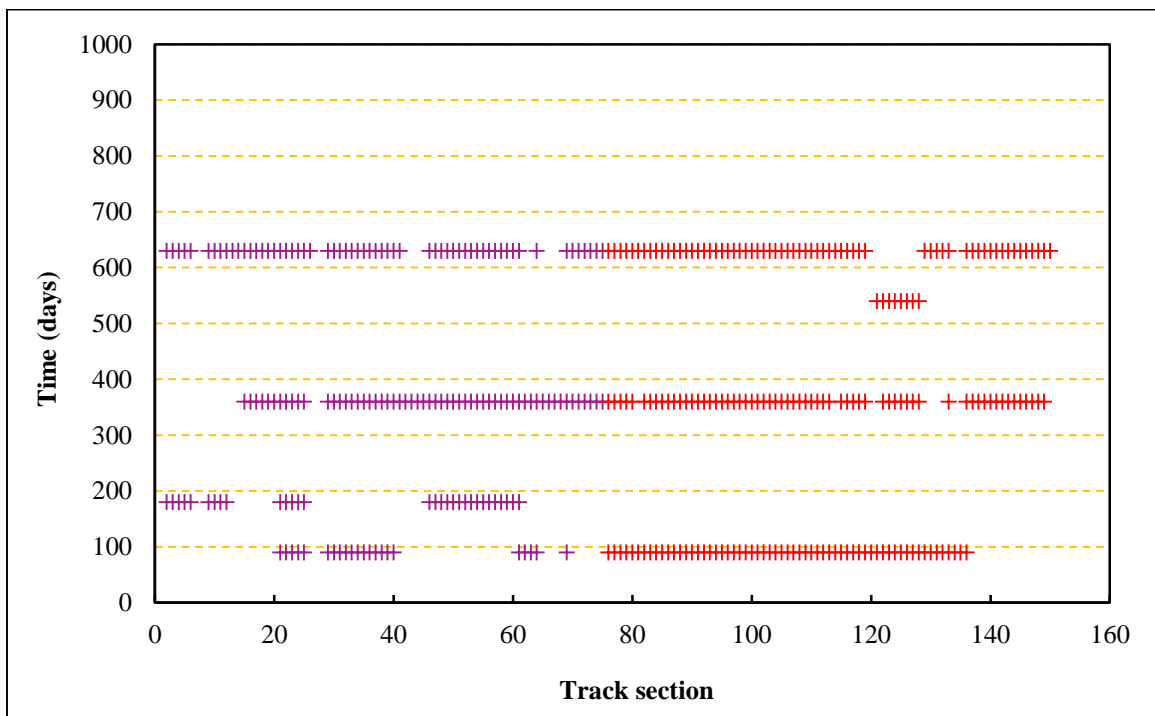
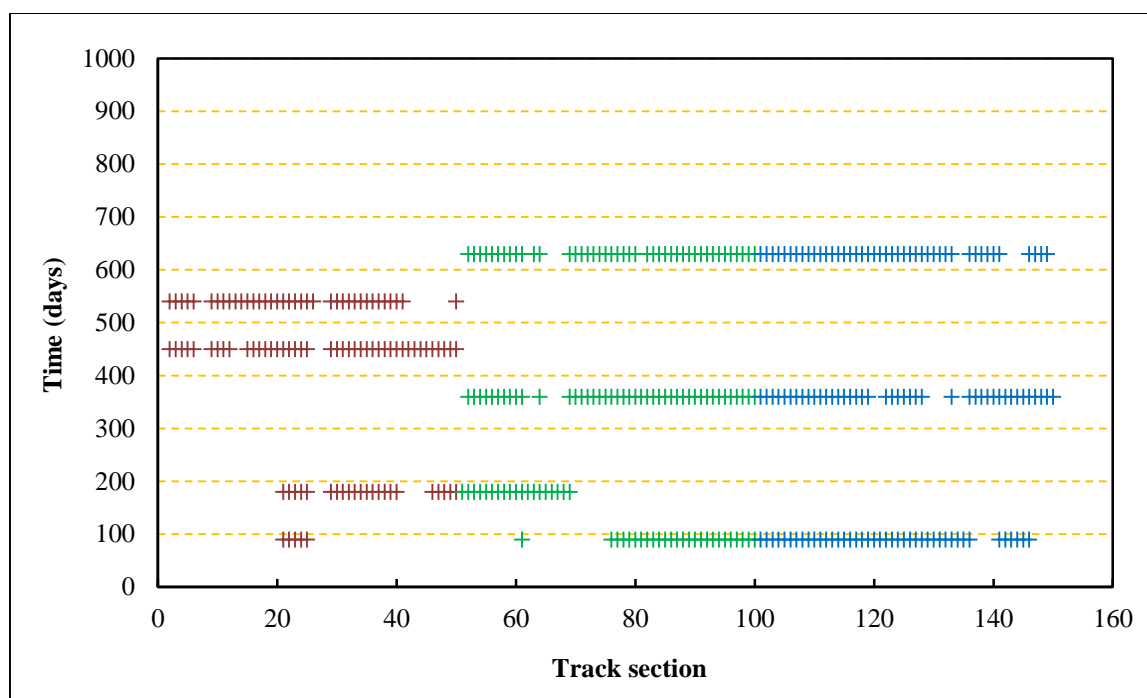**Figure 11. The optimal schedule for instance 3 obtained by scenario 2**

Figure 12. The optimal schedule for instance 3 obtained by scenario 3

6. Conclusion

Railway infrastructure managers seek to optimize the tamping interventions in ballasted tracks to reduce the tamping cost. In this article, an optimization model of the TPP is proposed and the effect of track segmentation method on the optimal solution is investigated by three scenarios. Moreover, the opportunistic maintenance technique is utilized to schedule tamping interventions. Through this technique, it can be clustered many tamping works in a time period to reduce the track possession time for maintenance as much as possible. The analysis of the results shows that by increasing the number of track segments, the total number of machine preparations increases while the total number of possession periods remains equal or increase. Furthermore, the total costs increase by 6.1% and 9.4% during scenarios 2 and 3, respectively. Finally, it is better to consider the whole railway track as a single segment (as in scenario 1) that consists of a big number of sections during the tamping planning to get the minimum total cost.

The model herein plans unnecessary early tamping actions; therefore, it will increase the resultant track condition at the end of planning horizon. Based on this weakness, another research opportunity can be raised by suggesting a suitable technique to avoid an early tamping as much as possible without neglecting the best performance of the model. The suggestion of discounted total cost in the objective function as in Wen et al. (2016) [1] and Daddow et al. (2020) [3] would be an interesting development in future researches. However, this development would make it more challenging the computation of real total cost of final solution across the planning horizon for two or three segments' scenarios. Finally, it would be interesting for future research to explore the optimal solution of the TPP in which dependent planning restrictions for the considered segments can be involved.

7. Declarations

7.1. Author Contributions

Conceptualization, M.D. and X.Z.; methodology, M.D.; software, M.D. and H.A.H.N.; validation, M.D. and H.A.H.N.; formal analysis, M.D.; investigation, M.D.; resources, M.D. and X.Z.; data curation, M.D. and M.A.; writing—original draft preparation, M.D.; writing—review and editing, M.D.; visualization, M.D. and M.A.; supervision, X.Z.; project administration, X.Z.; funding acquisition, X.Z. All authors have read and agreed to the published version of the manuscript.

7.2. Data Availability Statement

The data presented in this study are available in article.

7.3. Funding

7.4. Conflicts of Interest

The authors declare no conflict of interest.

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