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Violations at the Reference Point of Discontinuity: Limitations of Prospect Theory and an Alternative Model of Risk Choices

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Abstract

The tilted S-shaped utility function proposed in Prospect Theory (PT) relied fundamentally on the geometrical notion that there is a discontinuity between gains and losses, and that individual preferences change relative to a reference point. This results in PT having three distinct parameters; concavity, convexity and the reference point represented as a disjoint between the concavity and convexity sections of the curve. The objective of this paper is to examine the geometrical violations of PT at the zero point of reference. This qualitative study adopted a theoretical review of PT and Markowitz's triply inflected value function concept to unravel methodological assumptions which were not fully addressed by either PT or cumulative PT. Our findings suggest a need to account for continuity and to resolve this violation of PT at the reference point. In so doing, an alternative preference transition theory, was proposed as a solution that includes a phase change space to cojoin these three separate parameters into one continuous nonlinear model. This novel conceptual model adds new knowledge of risk and uncertainty in decision making. Through a better understanding of an individual's reference point in decision making behaviour, we add to contemporary debate by complementing empirical studies and harmonizing research in this field.

Keywords:

Preferences;
Prospect Theory;
Value Function;
Discontinuity;
Transitional Phase;
Risky Choice.

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1- Introduction

Without providing a plausible explanation for how the reference point is derived from first principle, Prospect Theory (PT) introduced a discontinuous point between the concave segment (gains) and the convex segment (loses) in the PT curve. This point, referred to as the "status quo", constitute a unique feature and a key assumption of PT [1]. The turn of events that ushered in PT saw the drift away from the once neoclassical expected utility theory (EUT) [2]. Although PT retained certain principles from Markowitz's theory that was structured on EUT [1], it disagreed on other fundamental assumptions, one of which is the formulation of the reference point at the origin [3]. The consensus that both theories are structured on a reference dependent platform is accepted [4, 5], but the configuration of the reference point relative to which outcomes are seen as gains and losses, is regarded as a distinct and fundamental feature in PT [6]. The implications that the reference point is a status quo added a measure of elegance and for the ensuing four decades this assumption was retained from the outset. Few scholars questioned the validity of what exactly determines the reference point and the plausibility of its derivation from first principles. These are major limitations of PT [7, 8].

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The upsurge in interest in project risk management research led scholars to investigate the ontological paradigm associated with its main framework of project management [9-12]. Since its inception in the 1950's many felt that the limitations of the original positivist engineering model of PM needed further expansion into a more universal setting to characterize a wider and diverse spectrum of events [13-16]. Innovation and institutional growth have greatly influenced and contributed to extending the PM field beyond its traditional engineering school orientation [17]. The field has become multifaceted drawing from both related and diverse fields, thus encouraging cross fertilization, debates and lively exchanges. In an ongoing debate since the beginning of the new millennium, 2000, between a group of scholars supporting the theory of the planning fallacy [18, 19] and another group defending the theory of the hiding hand [20, 21], several fundamental limitations on the convergence of cost risks in project management were presented [22]. Proponents of planning fallacy grounded their arguments on PT. A noteworthy observation was PT and its derivatives dominated contemporary behavioural economics discourse.

Surprisingly, these scholarly exchanges were conspicuously silent on the 1952 paper of Markowitz, a Nobel Prize winning economist, who exerted a profound influence in shaping the root causes in economic decision-making theory. Not even a passing reference was made [23-25]. This gap persists due to the fragmented nature of decision-making theories, which manifest in practice as billions of dollars in cost overruns on projects globally. In the pursuit to identify solutions to problems that arose in the executions of projects that were by nature complex, risky and uncertain, and inspired by the relevance of the on-going cost overruns debate, we undertook our own line of investigation and unearthed some salient root limitations that are inherent in prospect theory (PT), the original conceptual frame of the planning fallacy.

This paper examines the root causes from a geometric standpoint, starting from the nonlinearity theory that gave rise to the concept of concavity, which formed a definition for money and economic development. Central to this discussion is the fundamental paper on Expected Utility Theory (EUT) presented some three centuries ago by Bernoulli in 1738 [reprinted in [26]]. We then explore the geometric region of the reference point, which is typically seen as the zero point on the Utility-Wealth (U-W) curve in PT. A critical examination of the (0,0) inflexion region was undertaken. Consequently, based on our findings, we propose a conceptual model, the preference transitionary theory to cohere the curve at the point of discontinuity in PT. The phase change mechanism in the proposed preference transition theory brings continuity to the value-gains system, which is a modification to Markowitz's hypothesis and by extension, a new geometry. This paper is sectioned as follows: Section 2 outlines the theoretical foundations of EUT and PT. Section 3 discusses the theoretical implications of PT. Section 4 reviews the geometric violations of PT at the point of discontinuity. Section 5 provides a general discussion and introduces the preference transition theory.

2- Theoretical Overview

Bernoulli (1954) [26] proposed that a nonlinear function of the utility of an outcome should be used instead of the expected value of an outcome, accounting for risk aversion. This formalized a universal mathematical concept into risk and uncertainty studies possessing a wide range of applications across a multiplicity of domains including economics, social sciences, engineering, and provided an impetus for the development of EUT. The theory sets the standard of rationality in the context of decision-making under risk and uncertainty. EUT has since spawned a multiplicity of studies leading to several modifications and adaptations. By the beginning of 2000 it was estimated that twenty generalisations of EUT existed [9, 27]. This indicates that the field is wide, drawing from many phenomena and perspectives, sometimes in an incoherent way [28, 29].

A significant progress with the EUT paradigm was the von Neumann – Morgenstern (1944) utility axioms [30] which laid the necessary and sufficient conditions for expected utility to operate. These axioms of rational choice were complimented by the works of Savage (1948) on personal probabilities [31]. The fundamental tenet of EUT has developed to the stage that the rational decision– maker can clearly distinguish between two (or more) alternative courses of action by combining mathematically the probability of an event associated with that course of action and the magnitude of the outcome (either gain or loss) emanating from that event [12].

Contradictions made in EUT [32] led Markowitz to advance his arguments by stating explicitly (i.e. "*To tell geometrically* …" [33] p.151) the problems of the geometry. Markowitz hypothesized the triply inflection value mechanism, identifying an upper, mid and lower inflexion points. After the publication of prospect theory, this inflexion point with prospect theory was given the name "Reference Dependent Risk Theory of Choice" [23]. We examine the contrasting format of both methods.

Prospect theory and its geometric violations led to an independent point of reference called the status quo. The importance placed on this reference point, and the overall curve, was further reinforced by Kahneman: *"If prospect theory had a flag, this image would be drawn on it. The graph…". [34] (p.282).*

However, speculative inquiries continued as to why the framers of prospect theory did not follow up on their immediate predecessor, Markowitz, but chose a different perspective [23]. The dormancy of the period is expressed as follows:

> *"He remembered that the economist Harry Markowitz, who would later earn the Nobel Prize for his work on finance, had proposed a theory in which utilities were attached to changes of wealth rather than to states of wealth. Markowitz's idea had been around for a quarter of a century and had not attracted much attention, but we quickly concluded that this was the way to go…" [34],(p.278-279).*

During the last four decades prospect theory has blossomed into one of the most influential behavioural theories of choice in the wider social sciences, particularly in economic psychology [1, 35]. Through planning fallacy suggestions, it has made its way to account for cost overruns in mega projects, and as a result, enhanced project management studies [18, 19, 36]. It has also been applied to issues in international political science and economy [37-41] and has emerged in the areas of international relations [42-45], renewable energy [46] and transportation research [47, 48].

Prospect theory developed as an alternative to the more normative models of rational choice, such as subjective expected utility theory [49], even challenging the extent of rationalism adopted in decision making. A feature of this theory is relying on ideas taken from a branch of studies, psychophysics, which introduce scientific principles into psychological analysis [50]. This broadened the research field of behavioural science to compete with the dominant deterministic technical approach.

3- Theoretical Implications of Prospect Theory

Although considerable attention has been given to the psychological applications of prospect theory and in particular, to the inclusion of a reference point and loss aversion, minimal attention was given to the geometric problems created at the point of discontinuity and its subsequent violations or implications to theory. The hypothesis of prospect theory read as follows:

> *"Thus, we hypothesize that the value function for changes of wealth is normally concave above the reference point (v"(x) < 0, for x>0 and often convex below it (v"(x) >0, for x<0" [1] (p.278).*

We explored the following questions: Where is x = 0? Is x = 0 continuous to the right to concavity? Is x = 0 continuous to the left to convexity? Is the derivatives v" simply mimicking the standard calculus used by Friedman-Savage?

The PT's value function curve presented by [1, 34] is shown in Figure 1 below.

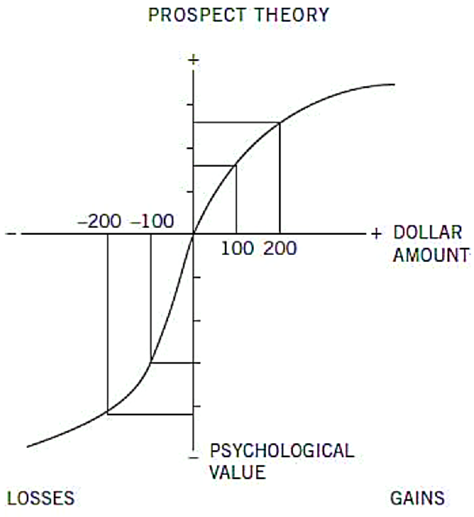

Figure 1. Value function curve [34]

PT as originally proposed was intended to address positive failures of the EU model, and to introduce an alternative to the neoclassical utility function. The theory presumes concave utility over gains and convex utility over losses, a pattern widely seen in lottery and gambling. The value function curve shows that a person is risk-averse for gains, represented by a concave curve, and risk-seeking for losses, as represented by the convex function of the curve. Kahneman [34] identifies the geometry of Figure 1 as having two distinct sections brought together by introducing a "floating" reference point. This value function curve is described as follows:

> *"The graph has two distinct parts, to the right and to the left of a neutral reference point….The slope of the function changes abruptly at the reference point: …"[34] (p.282).*

The reference point is referred to as "the status quo" and normally located at the zero point of the curve. It is a point of discontinuity and challenges the concept of nonlinearity, on which predecessors of expected utility theory [26, 32, 33, 51] developed their original format. The hypothetical value function curve shown in Figure 2 [52] joins, or conflates, both concavity and convexity portions of the value functions. This is described as follows:

"When the value functions for gains and for losses are pieced together, we obtain an s-shaped function of the type displayed…" [52] (p.342).

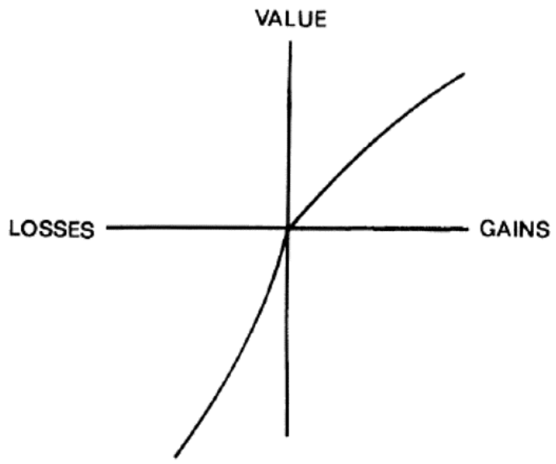

Figure 2. **Hypothetical Value Function S curve [52]**

The original EU model relies on the notion that a single continuous utility function can be used and since there is no distinction between losses and gains, there is no discontinuity between the domains of losses and gains (changes in wealth, x-axis). In contrast, the tilted and kinked, S-shaped utility function that Kahneman & Tversky [1] proposed rests fundamentally on the notion that there is a discontinuity between gains and losses; preferences change relative to a reference point. However, the appeal that their geometry in Figure 2 resembles an S-shaped function is an attempt to mask the discontinuity and replace it with an arbitrary disjoint reference point. This disjoint at the reference point provides a space, a "floating gap", where little attention is given and seldom discussed. But it is clearly a violation of the principle of continuity in S-shaped curves and betrays the concept of nonlinearity as originally suggested by the fundamental axiom of concavity. This violation leads to a number of consequences limiting prospect theory.

3-1- Nonlinearity prior to Prospect Theory

In 1738, Daniel Bernoulli introduced to economics what is known as the fundamental theory: utility represented as a concave function of money. The concavity of the curve is based on a priori deductions that any increase in wealth yields an increase in utility, thus utility is a function of wealth. Provided that other factors remain constant, the corresponding increase in resultant utility is inversely proportionate to "the quantity of goods" previously acquired. "Quantity of goods" was defined as *"any commodity an individual can possess contributing to the adequate satisfaction of any sort of want"* [26] (p.25). Bernoulli proposed that this nonlinear function of the utility of an outcome should be used instead of the expected value of an outcome, accounting for risk aversion. The degree of risk aversion can be measured by the curvature of the utility function (Figure 3).

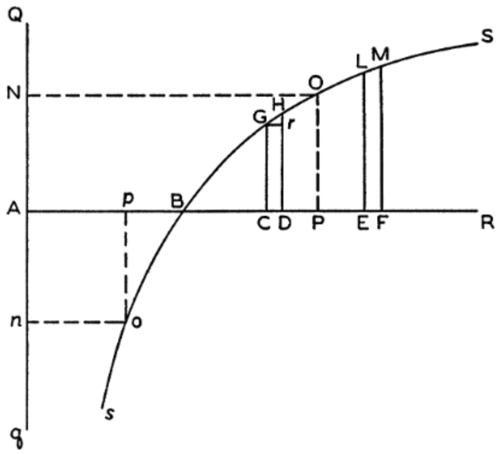

Figure 3. **Bernoulli's (1738) utility vs. wealth curve [26]**

Following this study nonlinearity geometry has taken roots in economic and social science theories. By 1944, some two hundred years later, four axioms were introduced [51] to define a rational decision maker, and in 1948, Friedman and Savage proposed a nonlinear framework, which by general consensus became the acceptable de facto model. The model progressed from concavity to a full nonlinear S-shaped curve. In 1952, Markowitz added to the Friedman – Savage model by introducing the inflexion points in a turning curve [33]. This was the prevailing growth of expected utility theory before the advent of Prospect Theory in 1979. Friedman and Savage set about to explain the existence of insurance and lotteries by the following joint hypothesis [33]:

"*Each individual (or consumer unit) acts as if he (a) ascribed (real) numbers (called utility) to every level of wealth, and (b) acts in the face of known odds so as to maximise expected utility" [33](p.151).*

The utility function is as illustrated in Figure 4.

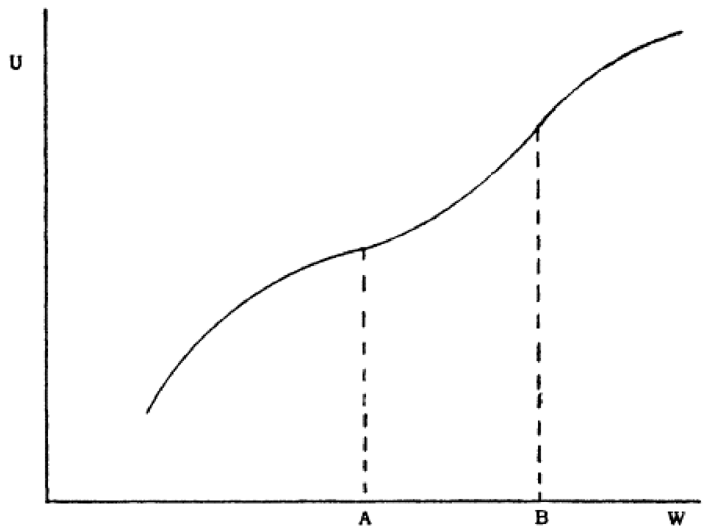

Figure 4. Friedman and Savage Illustration of the S-shaped utility curve [33]

Friedman and Savage's geometric continuous nonlinear utility – wealth S-curve is defined in the positive first quadrant. Based on this nonlinear theory, further properties were added. It is necessary to preserve the continuity to avoid any violation from the fundamental axiom. An essential feature to the hypothesis is that the curve is convex below A and above B and concave between A and B. It is assumed that it is a continuous curve with at least first and second derivatives. Let U be the utility and W be wealth:

Below some point A, $\partial^2 U / \partial W^2 < 0$,

Between A and B, $\partial^2 U / \partial W^2 > 0$,

And above B, $\partial^2 U / \partial W^2 < 0$.

Figure 4 defines the salient features of a simple nonlinear curve. The convex and concave parts are continuously held together through an inflexion point. But, assuming its present role as a point of continuity, a model for monetary outcome can be established. Markowitz's [33] utility model defined the middle inflection point as the individual's "customary wealth", as shown in Figure 5. Customary wealth equals to the present state of wealth with the exception of any windfall gains or losses to present wealth (all things being equal). This utility model shows using the present state of wealth as the reference point that an individual is loss averse. For incremental gains an individual is initially risk seeking then risk averse as represented by the positive quadrant of the curve. To the left of the reference point, in the third quadrant of the graph, an individual is initially risk averse over losses, then risk seeking. The fundamental assumption of the model is that an individual is inherently risk averse, and generally avoids symmetric bets (i.e., $|U(-X)| > U(X), X > 0$) [33].

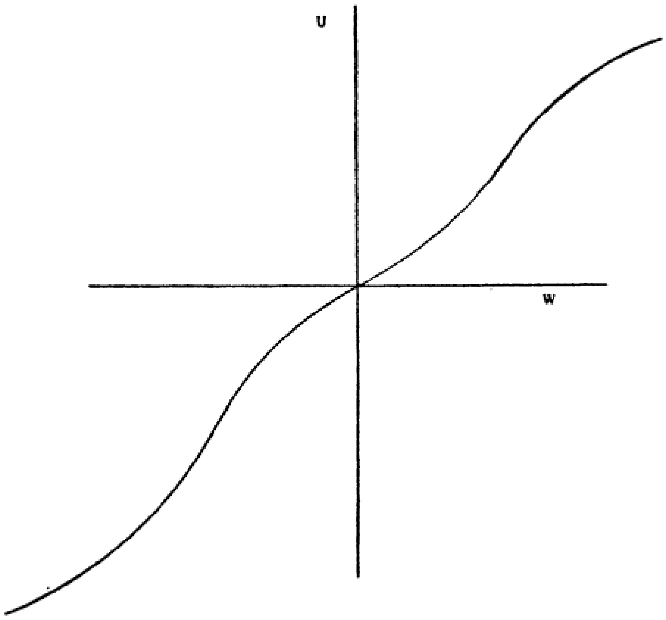

Figure 5. **Markowitz's triply inflection curve [33]**

4- Research Methodology

This theoretical review paper adopts a qualitative case study methodology to examine the assumptions made in PT that promotes a discontinuous region of the reference point. Case studies are suitable for understanding a phenomenon where the boundary between phenomenon and context is ambiguous [53], such as the reference point being endogenous and random in PT. This study also rationalises the "particularity and complexity" of this discontinuous reference point into an integrated system with bounded parts [54]. As such, we employed inductive longitudinal case study approach to synthesize the current evaluation of the geometric limitations of PT, as shown in Figure 1. To promote theorizing and generalization, we harmonise our findings into a parsimonious conceptual model to set the foundation to explain this geometrical violation with the intent of focusing on this phenomenon and promoting further debates and empirical inquiries. The specific case, PT and its violations at the reference point of discontinuity, serve to challenge the current trajectory and popularity of the cross fertilization of PT concepts among various disciplines. Concentrating on the case of the S-curve in PT, we challenge the assumption made by Kahneman and Tversky "piecing together" gains and losses at the reference point that is typically ascribed as the zero point in the cardinal scale. We introduce the concept of encapsulation [55] hindering learning and the continued cross fertilization of ideas between EUT and PT. By focusing on these two parallel theories that share important concepts in the sub-field of decision making with risks, we use a longitudinal theoretical approach of the evolution of utility theory branching into PT.

The main sources of data were gathered by conducting a detailed literature review in this narrow field and adopting the root sources of these works to form the theoretical foundation of this paper. We screened previously published theoretical, systematic and critique articles to identify substantial changes to the theories over time, together with accompanying assumptions and salient root limitations inherent to PT, to develop an agreement around conceptualising the reference point. A heuristic to determine the need for theory articles in a required field is the theory/data ratio [56]. A high ratio suggests the area of study is "theory-heavy" or skewed compared to empirical articles while a low ratio or "theory-light" suggests the area of study is empirical dependant and fewer theory articles. Employing this heuristic in the study of PT yielded a low theory/data ratio, indicating this area of study is empirically driven and additional synthesis is likely to be appreciated to broaden, and/or deepened, current scholarly inquiry. This makes them an ideal case for inquiry and informing theory about the limitations of axiomizing the reference point or the status quo. From our theoretical review we identified three areas of discussion in the limitation of PT: (1) Non-linearity prior to PT; (2) revisiting the S-shaped curve; and (3) encapsulation of and ideological distancing. We now describe these findings in greater detail.

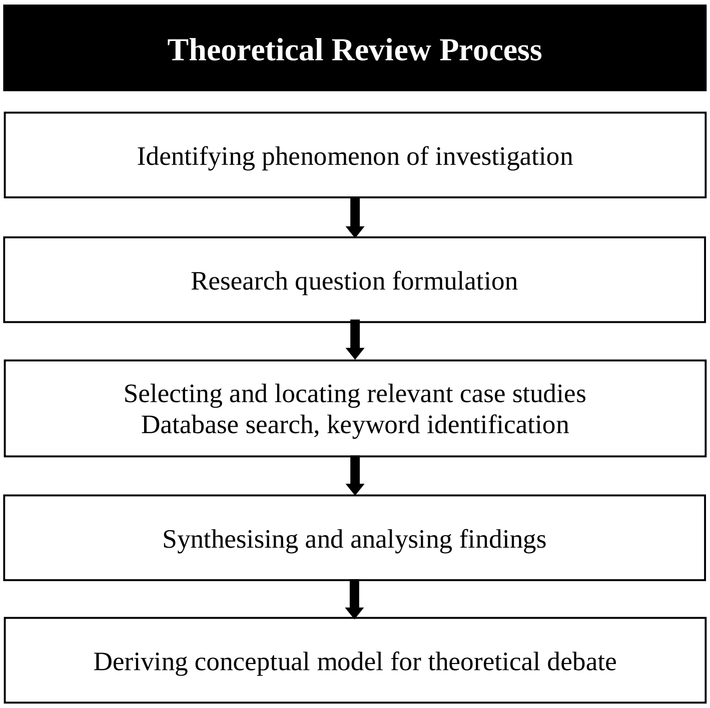

Figure 6. Methodology of the theoretical review

5- Findings

In this section we examine a salient feature of the geometry of what makes one curve continuous as is exhibited in expected utility theory (EUT), and why PT is considered discontinuous. Specifically, the movement from concavity to convexity and vice-versa is defined differently in both theories, where in prospect theory two distinct concave and convex sections are "pieced together" through a reference point [52], while in EUT the continuity is maintained through an inflexion point as in Markowitz [33]. The resultant effect is that the two theories are fundamentally different and rely on values that have contrasting geometric views. Little consideration is given to the geometry of the system at the reference point, exhibiting a free parameter, which is a discontinuous gap.

5-1- Revisiting the S-shaped Curve in Prospect Theory

EUT is structured on the idea of a single continuous nonlinear utility function [32, 33, 51]. Modelled along the EUT format the S-shaped curve is a nonlinear continuous curve. In contrast, the tilted and kinked S-shaped value function curve that Kahneman and Tversky [1] proposed had a completely different geometric outlook and relied fundamentally on the concept that there is a discontinuity between gains and losses implying that preference change relative to a reference point, referred to as the status quo. PT geometric S-shaped value function curve claimed to satisfy the conditions of nonlinearity and the axioms of von Neuman and Morgenstern and Markowitz's geometry. This is best captured by Kahneman and Tversky [52], who stated the following:

> "*Note that the proposed S-shaped value function is steepest at the reference point, in marked contrast to the utility function postulated by Markowitz…" [52] (p. 342).*

PT as originally proposed was intended to address positive failures of the EUT model [1] and it specified that it was essentially an alternative to the neoclassical utility function. PT presumes concave utility over gains and convex utility over losses. It is known as the discontinuous gain – loss reference dependent utility function [57]. The claim made by prospect theory that its curve is S-shaped needs further investigation. We challenge the above statement about "*the proposed S-shaped value function*" and argue that this S-shaped proposition is not conforming to the continuity concept and is shielded by a similar design created by Markowitz's geometry. This shielding technique is patterned after the format known as encapsulation [55].

5-2- Ideological Distancing and Encapsulation of PT

The concept of encapsulation creates the illusion of theory building, like the S-shaped curve, but in fact the graphical pattern of the "S" is being used as a mask to narrowly assimilate the concept, while promoting further self-containment [55]. Markowitz S-shaped curve originated from the S-curve of Friedman-Savage and the traditional concept of continuity as proposed in EUT. PT was searching for a comparative advantage to promote its two distinct and separated curves that were "pieced together" [52] or loosely joined through a discontinuous arrangement of a reference point. PT attempted to exploit the mid-inflexion region in Markowitz's triply inflexion curve. The piecing together of the concave and convex sections of the curves suggests PT violated continuity and appealed to the S-shaped curve as a disguise to mask its infusion.

The advent of PT propelled an ideological distance between the parallel yet contrasting theories of PT and EUT. Ideological distancing is a concept used to show how these dominant theories may, intentionally or unintentionally, fail to acknowledge other salient perspectives and contributions [55]. Both PT and Markowitz triply inflection curve initially assume an individual being risk adverse, avoiding symmetrical bets. However, PT had little interest in Markowitz's findings, and this departure was attributed to the rational-agent model which forms the foundation of EUT and its axioms. Kahneman acknowledged that PT was descriptive in nature with the goal of capturing and explaining *"systematic violations of the axioms of rationality in choices between gambles"* [34]. Kahneman [1] asserted the following about Markowitz:

> *"His treatment, however, retains the expectation principle; hence it cannot account for the many violations of this principle; …" (p. 276).*

For the purposes of our analyses, we present Markowitz's [33] hypothesis:

> *"To summarize my hypothesis: the utility function has three inflection points. The middle inflection point is defined to be at the 'customary' level of wealth. Except in cases of recent windfall gains and losses, customary wealth equals present wealth. The first inflection point is below, the third inflection point is above, customary wealth. The distance between the inflection points is a nondecreasing function of wealth. The curve is monotonically increasing but bounded; it is first concave, then convex, then concave, and finally convex. We may also assume that | U (- X) | > U(X), X > 0 (where X = 0 is customary wealth).* (p. 155).

Markowitz's hypothesis is an expansion of the Friedman-Savage geometric S-curve, which sought to explain the existence of insurance and lotteries, by displaying a nonlinear S-curve in the first positive quadrant of a utility-wealth frame (Figure 4). Although the domain of the utility function can accommodate other applications, most of its present applications have been concerned with monetary outcomes. Indeed, this led early decision framers of the eighteenth century to propose that utility is a concave function of money, and this concept has been retained as fundamental [58-61]. Markowitz's hypothesis broadened the geometry of Friedman- Savage and was the first to propose that utility be defined on gains and losses rather than on final asset positions. This assumption proved to be beneficial and has been implicitly retained in most experimental measurements of utility [25, 62]. His hypothesis also explained the presence of risk seeking in preferences among positive as well as negative prospects. By the use of inflexion points he was able to propose a utility function which has convex and concave regions in both the positive and negative domains. His geometry maintained the continuity that is exhibited in an S-curve. These findings were acknowledged in PT which, from this stage, delinked from the neoclassical EUT to promote a new doctrine. Hence the arrival of a new geometry: two distinct curves 'pieced together' through a floating reference point. The novelty of the ideas spawned a new field in behavioural science and relegated EUT and Markowitz's geometry as neoclassical, as belonging to a bygone era. Hence, little interest is shown on furthering the geometry of Markowitz.

Several suggestions of the value function model have emerged to bring about modifications to both PT and its EU counterpart [63-65]. The process have reached the stage where evolutionary models are suggested [43, 44]. The more recent models tried to unite risky and riskless choice with either EU or PT [4, 66-71]. To generalise the domain, an implicit assumption is made of a continuous value function around the reference point that separates gains and losses. Also, the effort was focused theoretically on loss aversion as the primary mechanism to determine gain-loss problems. This was readily seen when the endowment effect was introduced in riskless choice [66, 72] as a function of the phenomenon of loss aversion, a method that was initially designed for risky choice. The work of [73] lead to substantial evidence that loss aversion may not accurately provide empirical evidence to satisfy the endowment idea. While some effort was made to harness the concept of another major assumption of prospect theory, i.e. the changing curvature of the value function across the loss and gain domain [57], the study was limited to riskless choice.

The general consensus among leading academics is that the geometry of PT at the reference point clearly violates nonlinear continuity, and that substantial modifications are necessary to set the system into a continuous value function. Little attention is paid to this problem, as is evident from a perusal of the extant literature. This is further exacerbated by the fact that the two parallel theories of PT and EU express divergent views, which created an ideological distancing that tend to keep them as separate institutions within a specialised framework.

5-3- The Reference Point Flaws in EUT

A problem originating from the beginning was Bernoulli's introduction of EUT through hypothesising that the logarithmic of a person's wealth is a measure of the satisfaction or utility (u) derivable and he expressed it as [1, 23, 35, 74, 75];

$$u(w) = \ln w \tag{1}$$

But at the zero-point, u (0) = ln 0, which is undefined. This means that the utility function does not pass through the origin [6]. Markowitz's mid-inflexion point attempted to address this flaw through a quadratic utility function, hypothesising:

> *"There is an inflection point at W=0; therefore, the utility function is* ***almost linear*** *in the neighbourhood of W = 0…" (p.157)*

However, the term "almost linear" is in itself ambiguous, and Markowitz did not elaborate nor provide further explanation to this well-known expression attached to the inflexion region. Baillon et al. [3] also recognised the persistent nature of this fundamental flaw on the approach of forming reference points in PT and other reference -dependent theories. For over four decades the extant literature is noticeably silent on this phenomenon. The word 'almost' in the same expression of "almost linear" needs further clarification, to avoid the pitfall of leading to dubious interpretations. One representation of "almost" can be a state of mind, transitioning from perceived alternative choices to actual decision making [76]. It will be clearer now that 'almost' has a meaningful part to play and is not an arbitrary attachment to embellish 'linear'. We presume that 'almost' is a 'transitionary' operation. Thus, the inflexion point has a lot more geometry than what was originally anticipated. Markowitz did suggest this by stating:

> *"We can resolve this dilemma by assuming that in the case of recent windfall gains or losses the second inflection point may, temporarily, deviate from present wealth"[33] (p.155).*

There was room around the inflection point, both to the right, to accommodate gains, and to the left to allow for losses. Markowitz further contended:

> *"To have an exact hypothesis – the sort one finds in physics – we should have to specify two things: (a) the conditions under which customary wealth is not equal to present wealth (i.e., the conditions referred to as recent windfall gains or losses) and (b) the value of customary wealth (i.e., the position of the second inflection point) when customary wealth is not equal to present wealth…. But I do not have such a rule and formula … I leave it to further research…" [33] (p.157).*

This statement is more complicated and demonstrates how 'almost' is part of the actual operation. For example, moving away from the gambles and exploring a wider context in construction management, a project manager faces constant challenges of de-risking cost overrun issues on a construction project based on available data, heuristics in practice, contingencies etc. Each decision has a cost and value implication. There are either/ or choices to confirm a decision, and there is also the choice to reconsider and change a decision after it was "almost" accepted. In this example, 'almost' allows for elasticity in decision making. The elasticity in decision making is part of the process and continues throughout the operation.

5-4- The Reference Dependent of Risky Choices

Markowitz [33] proposed a reference dependent system in a series of choices having the same expected value between a sure thing and a gamble. With the introduction of PT, a non-expected value model, a new model of reference dependent of risky choice emerged. These contrasting theories were termed "reference dependent theory of risky choice (RDRC)" [74]. These prominent theories attempted to address similar issues at the "zero" reference point; Markowitz's model provided for a continuous function at mid-inflexion zero point, while the PT model was framed on a discontinuous point that separates concave gains from convex loss. Markowitz's model relied on the mathematical notion of inflexion points, which carried the concept of "almost linear" at the zero point. In contrast, the zero point in PT was a free and floating discontinuous parameter.

The configuration of sensitivity, or elasticity, and assumptions on elasticity (stake-dependent) of the value function and its variability with outcome magnitude has been investigated from several angles in PT. These are symmetric elasticity [35, 75], decreasing elasticity [23, 74] and constant elasticity on either side of the discontinuity point [1]. Scholten and Read [78] conducted a systematic and large-scale investigation of the choice series designed by Markowitz and PT. They associated gains with decreasing elasticity and uncovered, for losses, decreasing elasticity is not a universal property. They eventually concluded that predictions of both models were inconclusive. This confirmed limitations in the widely accepted constant elasticity of the power value function commonly applied to PT.

However, all contemporary elasticity works made the tacit assumption of continuity at the zero point in PT. This gap and theoretical violation at the point of discontinuity remains unresolved. Our theoretical modification to both PT and Markowitz's utility function proposes a transition phase. This transition phase, a "phase" or state change at and around the zero point, incorporates elasticity with boundary conditions, is not previously associated with PT nor Markowitz's utility function. Our theory demonstrates a continuous geometry by graphical representations, and further research on how the value function domain can be expanded to solve more complicated risky phenomena which PT cannot accommodate is a challenge for future works.

5-5- Theoretical Framework to Address the Reference Point Gap

The concept of nonlinearity presumes a wider definition than a singularity point introduced in PT. The transition from convex to concave contains a region or space for decision making and choice determination between gains- losses, based on an individual's contingencies of the perceived value function of preferences. The existing discontinuity in the PT curve is enlarged and represented in Figure 7.

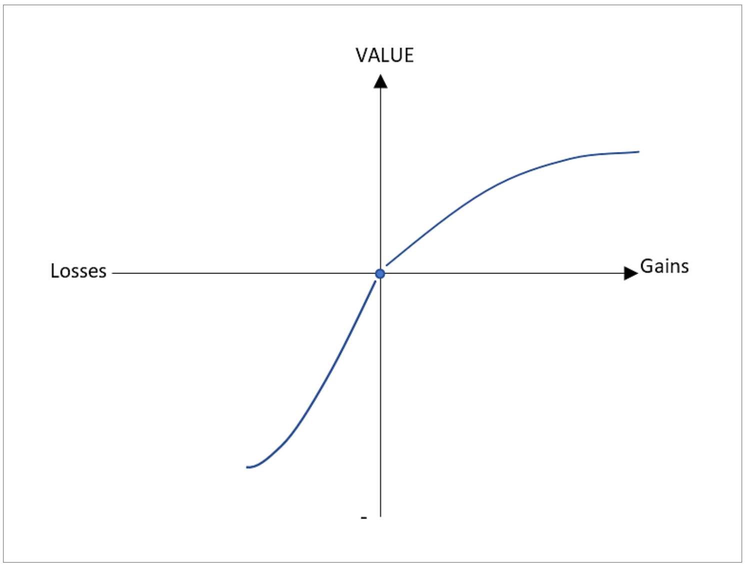

Figure 7. Graph illustrating singularity point of discontinuity in PT

As a result, we establish an initial configuration to our model. We hypothesise that the word "linear" in the expression "almost linear" assumes a change, or a transition, from nonlinearity to linearity. It is a clear distinction of a temporary readjustment from one form to another, in effect, exhibiting continuity through a phase change. At W = 0, we introduce hatch lines as transitionary boundaries (defined with boundary limits) parallel to the vertical utility axis to conceptualise this space. Figure 8 demonstrates an enlargement of this space around the mid-inflexion reference point to show the phase change effect.

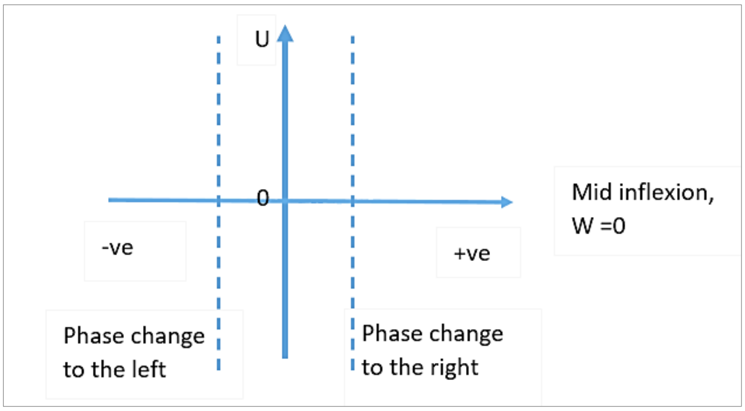

Figure 8. The limits of phase transition boundaries from linearity to non-linearity at the mid-inflexion zero point

To accommodate the "almost linearity" movement from gains to losses and vice versa around the mid-inflexion point of Markowitz's curve, a transition phase change is proposed. A transitionary phase change, based on an individual's choice variables and decision-making preferences allow for the co-joining of these two diverse interpretations .Co-joining permits for causal linkages and removes ad-hoc conflation of value-gain/loss states. We further modify the constant elasticity of substitution utility (CES) function introduced by Solow [77] to include a broader spectrum of risky events outside the range of monetary economics. This is demonstrated below graphically:

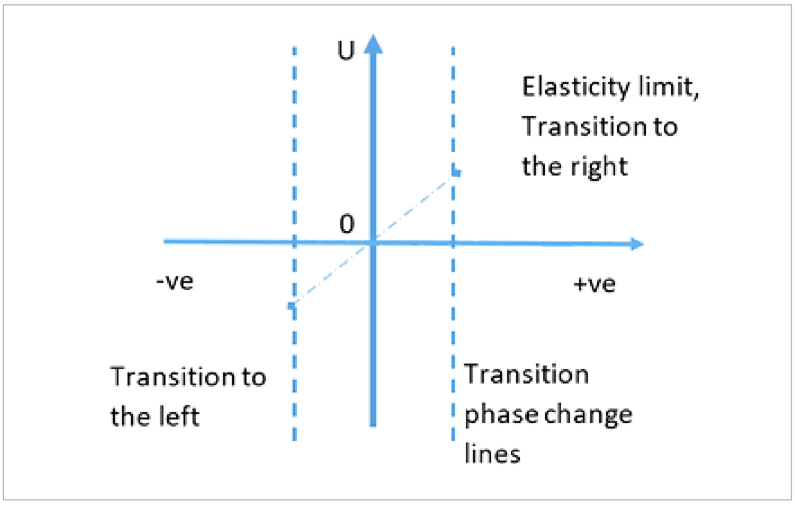

Figure 9. Elasticity of utility based on phase transition boundaries

Having modified the mid-inflexion region, we can apply this geometry to remodel the discontinuity in prospect theory as shown in Figure 10. The geometry of the utility function as being nonlinear and continuous is determined at the points of inflexion. The general characteristic of any inflexion point conveys a mechanism of transitionary linearity (almost linear) to nonlinearity. There exists a middle inflection point located at x = 0 that transitions to the right (first quadrant) as concave, and to the left (third quadrant) as convex.

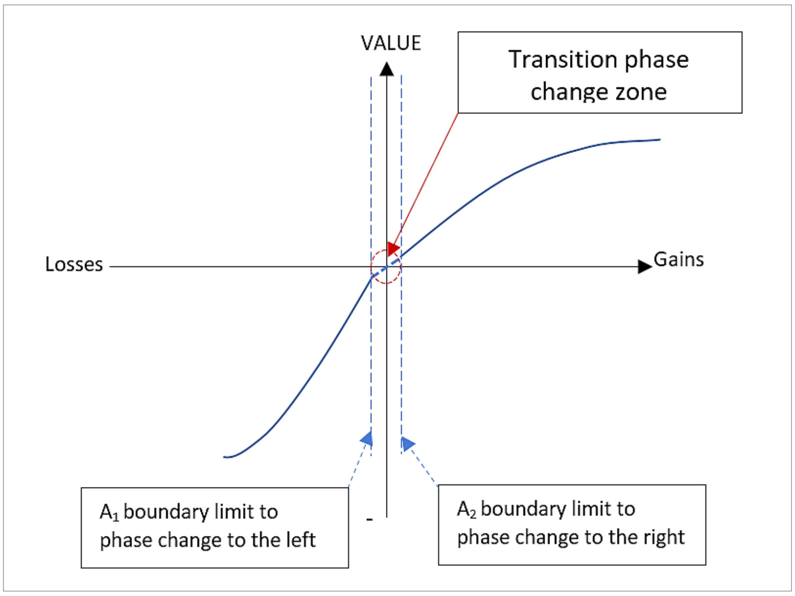

Figure 10. The transition phase change model with constant elasticity at zero point/ reference point

6- Discussion

In the above sections we isolate the region around the reference point in both Markowitz's model and PT, the two leading theories in risk and uncertainty in decision making. We present positive arguments about the flaws and gaps around this point. Among the shortcomings emerging is the arbitrary assembly of preferences which include reference dependence functions, and by extension, a random series of reference points. Despite four decades of research from its initial publication in 1979, PT scholars remain conspicuously silent on a major fundamental violation at the reference point of discontinuity, a region normally referred to as the status quo. This has led to the acceptance of the many variations infused into this theory [8], among which are mixture models addressing heterogeneity of preferences [25, 78]. These models expanded the scope of PT to a full range of algebraic functions (logarithm, quadratic, power and exponential) and their many sub-derivatives. However, the current trend in PT derivations assumes either the reference point is exogenously given at the outset, or endogenous choice-dependent models [8]. Such multiple reference points, created to achieve different risk averseness and preferences, shows the need to stabilise this point by a continuous transitionary format. In this paper we focussed on the geometric configuration at the reference point to synchronise a multiplicity of randomised points emerging from the variety of utility functions.

The geometry of PT is framed on two distinct sections – the concave arm representing gains, and the convex arm denoting losses. These segments are loosely joined, or in Kahneman's words "pieced together", by means of a floating reference point. It should be noted that prospect theory, as originally conceived, chose a reflection principle in mark contrast to the inflexion mechanism by Markowitz. This framework limited continuity at the point of reference. The reference point of PT has similar characteristics to "customary wealth" (or present wealth), as proposed by Markowitz, and both are definable at the zero point (W = 0). A feature of both "customary wealth" and the reference point (x = 0) is that they are not defined as rigid and fix entities but possess some mobility. It behaves as an elastic device between concavity and convexity [23].

The act of "piece together" to derive a nonlinear, continuous, and S-shaped curve in the tradition of calculus theory creates a discontinuous point at x = 0. Thus, what exactly formulates a discontinuous reference independent risk point, commonly referred to as the status quo, remains unspecified [8]. Geometrically, these three parameters – concavity, the reference point and convexity are separable and can be repositioned into our modified transition geometry. The findings demonstrated several issues encapsulated within these questions. They are:

1. Markowitz utility function model followed the neoclassical expected utility framework developed by Friedman – Savage [32]. The Friedman-Savage geometry was a continuous S–shaped curve drawn in the first quadrant. Markowitz produced a mid-inflexion point that provided continuity, as is the case in nonlinear models, and attempted to define the mid-inflexion point flaw through quadratic utility function, hypothesising "almost linear", which in itself, is ambiguous. In an effort to determine the best suitable way to satisfy investors' utility, Levy and Markowitz [79] used twelve different log utility functions and twelve differing exponential functions in their empirical analysis [6], building their models from the standard EUT.

2. In 1979, Kahneman & Tversky [1] introduced PT as an alternative model to EUT. While not proposing a parametric specification of the value function for PT, Kahneman & Tversky proposed the following power value function for illustrative purposes:

$$v\,(x) = \begin{cases} x^{\alpha} & if\ x \geq 0 \\ -\lambda(-x)^{\beta} & if\ \ x < 0 \end{cases} \tag{2}$$

where $0 < \alpha, \beta < 1$ is diminishing absolute sensitivity, and $\lambda > 1$ is constant loss aversion [1]. This value function has constant elasticity, α, for gains, and β for losses, and therefore does not predict the Markowitz's model [23].

3. S – shaped geometry in prospect theory: This contrivance of a continuous nonlinear curve, when it was not, presumes continuity in the S-curve – one concave and one convex – loosely held together at the reference point.

4. The mappings of the inflexion system in Markowitz's framing and the reflection mechanism of prospect theory convey different meanings. However, Kahneman and Tversky developed a reflection mechanism in prospect theory. The reflection is discontinuous at the reference point.

The proposed preference transition theory is an alternative reference dependent-based model that considers the two main reference dependent theories (EUT and PT) as inclusive cases. Although Markowitz's [33] value function model has received limited interest and currently not integrated into PT, it was suitable for theory building. Using Markowitz's assumptions, we reformulated the utility function domain at the mid-inflexion point (x = 0) to accommodate the transitionary change between concavity and convexity and to provide an alternative concept. The analysis proceeded along the lines of geometry, and this model treats the reference point as a parameter, estimated like other model parameters such as utility curvature and loss aversion [3]. The modified S–shaped geometric model proposed in Figure 10 facilitated continuity at the reference point in PT. Additional unknown features, like elasticity, were mentioned with elastic boundary conditions to cojoin the two independent concave and convex sections. The focus of the phase transition was not limited from linearity to nonlinearity as a unidirectional process in the concave region, but consideration was given to the convex region. A complete cycle was envisaged from concave to convex and vice versa. All this is possible if the presumption of continuity through the mid-inflexion region is retained as elastic and harmonic.

7- Conclusion

The aim of this paper was to unravel the characteristics (unknown and unspecified) of the mid inflexion point at x = 0 in prospect theory, and to propose a transitionary concave-convex universal system. We critiqued the origin of the reference point in PT (i.e., x = 0), and investigated the continuity of the curve to the right of the reference point to concavity and to the left of the reference point to convexity. To date, what exactly determines the reference point remains unspecified. Our findings showed that Kahneman & Tversky's statement of "pieced together" in PT, created a discontinuity and a geometric violation at the reference point. This discontinuity in PT leads to limitations in deriving, and accounting for the zero-outcome effect and the absence of behavioural conditionalities in risky decisions. As a result, prospect theory is a limited theory and violates the concept of nonlinearity as originally suggested by the fundamental axiom of concavity in EUT. Inspired by the concept of mixed model suggestions in reference dependent theories, we developed a heterogenous phase change transitionary model, i.e., preference transition theory (shown geometrically in sections 4-5) as an alternative model. To our knowledge, this is the first model to achieve continuity at the reference point. In this respect, we introduce new knowledge applicable to a wide spectrum of disciplines – from economics to behavioural psychology, to science and engineering. There is room to expand the utility function domain from a localised setting into a universal model and to include risky phenomena. We recommend for future works to build on preference transition theory through mixed model and heterogeneity of preferences, thereby generating a fundamental approach to decision making in project studies development.

8- Declarations

8-1- Author Contributions

Conceptualization, A.C. and C.C.; methodology, A.C. and C.C.; software, X.C.; validation, A.C. and C.C.; formal analysis, A.C.; investigation, A.C. and C.C.; resources, A.C. and C.C.; data curation, A.C. and C.C.; writing—original draft preparation, A.C. and C.C.; writing—review and editing, A.C., C.C. and X.C.; visualization, A.C.; supervision, C.C.; project administration, A.C and F.O.; funding acquisition, F.O. All authors have read and agreed to the published version of the manuscript.

8-2- Data Availability Statement

Data sharing not applicable: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

8-3- Funding

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8-4- Acknowledgements

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8-5- Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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