

# Application of Solutions of Uncorrected Tasks' Theory for Managing of Production Processes' Robustness

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## Abstract

The industrial revolution of “Industry 4.0” is currently underway at an active pace. Individualization of provided products and services, transition to single production, the issues of acceptability of production processes at the stage of development and design have become urgent. The aim of this work was to develop a strategy for a fundamental solution to the task of guaranteed acceptability of production processes as an integral quality characteristic.

We have proposed a systematic approach to strategy development. The basis of the approach was the theory of incorrect tasks solving. We adapted signs of correctness of mathematical tasks by J. Adamar to the tasks of ensuring acceptability of production processes (technological and measurement). They were used in the part of identification of properties of display of incorrectness and ways of incorrectness' management.

We have proposed to consider the property of robustness as a generalized index of acceptability for production processes (technological and measurement). We substantiate the equivalence of the concepts of incorrectness of tasks according to J. Adamar and losses of robustness of production processes. We conclude that the developed approaches and techniques of the theory of incorrect tasks can be put in a basis of the system approach to an estimation and management of losses of robustness of production processes. We have proposed a classification of situations of robustness losses in production processes in accordance with the classification of the signs of incorrect tasks by J. Adamar.

We have developed a two-step algorithm for ensuring the robustness of production processes at the stage of their development. It included identification of the sources of robustness losses and management of input factors that cause significant variation in process output. This has given a practical implementation of a strategy to guarantee the acceptability of production processes. We have systematized the sources of potential losses in the robustness of production processes and proposed a two-stage mechanism for managing them. We have justified rational methods of ensuring the robustness of production processes for each stage based on, the practice of uncorrected tasks solving. We have proposed a method for ensuring high efficiency of robustness loss management in certain situations. The principles of G. Taguchi's robust redesigning of production processes formed the basis of our method.

**Keywords:** task incorrectness, production processes, robustness, management.

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# Применение положений теории решения некорректных задач для управления робастностью производственных процессов

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В условиях наступления промышленной революции «Индустрия 4.0», индивидуализации предоставляемой продукции и услуг, перехода к единичному производству вопросы обеспечения приемлемости производственных процессов на стадии разработки и проектирования становятся актуальными. Целью данной работы являлась разработка стратегии принципиального решения проблемы гарантированного обеспечения приемлемости производственных процессов, как интегральной характеристики качества.

Предложен системный подход разработки стратегии, в основу которого положены подходы теории решения некорректных задач. Признаки корректности математических задач по Ж. Адамару адаптированы к задачам обеспечения приемлемости производственных процессов (технологических и измерительных) в части идентификации свойств проявления некорректности и способов управления некорректностью.

Установлено, что для производственных процессов (технологических и измерительных) свойство робастности может рассматриваться как обобщенный показатель приемлемости. Обоснована эквивалентность понятий некорректности задач по Ж. Адамару и потерями робастности производственных процессов. Сделан вывод о том, что в основу системного подхода к оцениванию и управлению потерями робастности производственных процессов могут быть положены наработанные подходы и техники теории некорректных задач. Предложена классификация ситуаций потерь робастности производственных процессов в соответствии с классификацией признаков некорректности задач по Ж. Адамару.

Для практической реализации стратегии гарантированного обеспечения приемлемости производственных процессов разработан двухшаговый алгоритм обеспечения робастности производственных процессов на стадии их разработки, включающий идентификацию источников потерь робастности и управление факторами входа, вызывающими существенную вариацию выхода процесса. Систематизированы источники потенциальных факторов потерь робастности производственных процессов, предложен двухэтапный механизм управления ими. Для каждого этапа обоснованы рациональные методы обеспечения робастности производственных процессов, наработанные практикой решения некорректных задач. Предложен метод обеспечения высокой эффективности управления потерями робастности в определенных ситуациях, в основе которого положены принципы Г. Тагучи по робастному перепроектированию производственных процессов.

**Ключевые слова:** некорректность задач, производственные процессы, робастность, управление.

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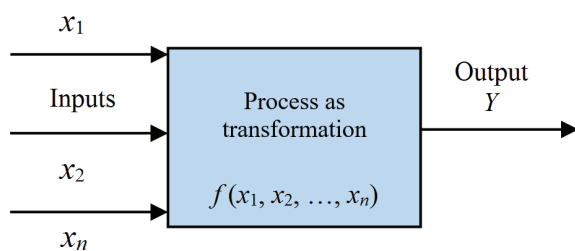
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## Introduction

At its core, any production process (technological or measuring) consists in converting inputs to outputs [1]. This classical definition of the process can be mathematically interpreted as follows: the process converts the inputs “ $x_1, x_2, \dots, x_n$ ” into the output “ $Y$ ” in accordance with the transformation rule  $f$  (Figure 1).



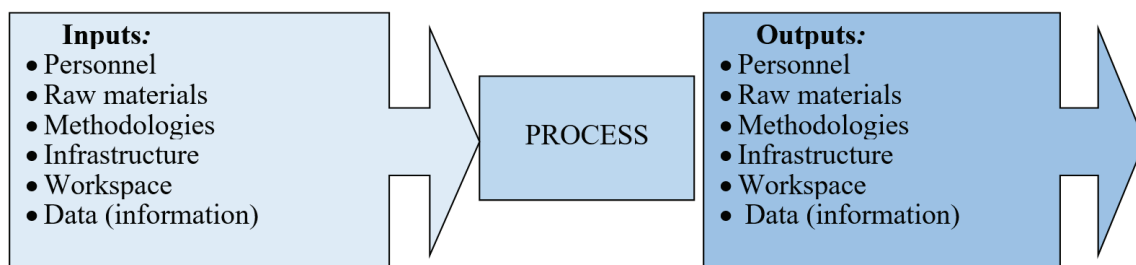
**Figure 1** – Interpretation of process through concepts of function and objects

As follows from Figure 1, the general structure of the process can formally be represented by two components:

- objects – inputs ( $x_1, x_2, \dots, x_n$ ), outputs ( $Y$ ), which answer the question “what?”;
- function – transformation rules  $f(x_1, x_2, \dots, x_n)$ , which answer the question “how?”.

Typical categories of inputs and outputs of the production process are presented in Figure 2.

The quality of products (process output) is determined by one or a set of standardized characteristics: functional (purpose, manufacturability, reliability, etc.), safety, ergonomic, patent, legal, economic, etc.



**Figure 2** – Typical categories of process inputs and outputs

The factors of uncertainty in ensuring the acceptability of a particular process in terms of the concept of qualitatively new industrial production “Industry 4.0” become critical for its successful im-

The modern concept of engineering quality management suggests that product quality (process output) is best managed through the quality of the process itself. Process quality is traditionally defined by a set of characteristics that relate primarily to the ability of a process to produce results that meet predetermined acceptance criteria within a certain period of time [2]. Acceptability implies that the process has a set of properties (suitability, reproducibility, stability, etc.), indicating that over time, under conditions of acceptable changes in the input parameters of the production process or control (subcontractors, personnel, components, materials, conditions, etc.), the quality of the result at the output (standardized characteristics of products or services) will not deteriorate [3].

Obviously, acceptability is an integral characteristic of the quality of the process and, as follows from Figures 1 and 2, is formally determined by the quality of the inputs and the quality of their transformation (function  $f(x_1, x_2, \dots, x_n)$ ). The expression:

$$Y = f(x_1, x_2, \dots, x_n) \quad (1)$$

can be seen as a baseline model of the production process for the purposes of acceptability assessment and management.

According to expression (1), sources of non-compliance with the acceptability criteria in both the short and long term implementation of the process can be:

- input parameters  $x_i$ , e. g. not all influencing parameters are identified or their values are incorrectly defined;
- the coupling function  $f$ , e. g., the coupling function is incorrectly defined or the coefficients of influence of input parameters  $x_i$  on the value of output  $Y$  are incorrectly defined.

plementation [4]. The inevitability of the industrial revolution “Industry 4.0” in terms of individualization of provided products and services, transition to unit production while increasing its productivity and

minimizing the cost, makes it relevant to develop a strategy for a fundamental solution to the task of guaranteeing the acceptability of production processes.

The aim of this work was to develop a strategy for a fundamental solution to the task of guaranteed acceptability of production processes as an integral quality characteristic.

### The concept of incorrect tasks

To develop the strategy of ensuring the acceptability of production processes of different categories, it is rational to consider close, in our opinion, mathematical approaches of the theory of solution of incorrect tasks [5, 6]. “The point of contact” serves as a typical property of the incorrect task in mathematics – instability and uncertainty of the right part of the equation at small changes in the left part, which is associatively close to the concept of loss of process acceptability.

For the first time, the notion of a “correctly posed task” was introduced by J. Adamar in 1923 and referred only to boundary value tasks of mathematical physics. J. Adamar believed that the vast majority of investigations lead to the formation of a mathematical model. Let the model be represented by an abstract equation of the first kind:

$$A \cdot x = y, y \in Y, x \in X, \quad (2)$$

with suitable spaces (finite-dimensional, functional)  $X, Y$ , and with some operator  $A: X \rightarrow Y$ .

According to J. Adamar, the correctness of the task statement is ensured by fulfilling three conditions (signs of correctness):

- 1) existence of the solution of the task – equation (2) is solvable for all “ $y \in Y$ ”, i. e. it exists;
- 2) the solution of the task is singular;
- 3) the solution of the task is stable, i. e. small perturbations of the right part  $x$  correspond to small perturbations of the solution  $y$  in the metric of space  $Y$  [5].

Absence of any of the features points to the incorrectness of the task. For a long time according to J. Adamar it was considered that incorrect tasks have no practical sense and, therefore, cannot be solved. Academician A.N. Tikhonov introduced the notion of conditionally correct task and for the first time applied theoretical developments in the field of correctness – incorrectness for solving actual tasks in the field of exploration geophysics [7, 8]. This gave an impetus to the deve-

lopment and wide application of the theory of solving uncorrected tasks, which is explained by the established pattern that the solved practical tasks are most often uncorrected [8].

However, it should be noted that the theory has not yet been applied to process acceptability tasks. Process acceptability inherently manifests itself through the degree of process output  $Y$  (Figures 1, 2). Accordingly, in the presence of variation  $Y$ , ensuring process acceptability as a conversion of inputs to output  $Y = f(x_1, x_2, \dots, x_n)$  automatically becomes an incorrect task. Note that ensuring acceptability of the process as its stability is particularly important at the design stage. It is obvious that the approaches, methods and techniques of the theory of solving incorrectness tasks, adapted for production processes, should become the basis of a systematic approach to solving the actual task of determining ways to ensure the acceptability of technological and measurement processes already at the stage of their design.

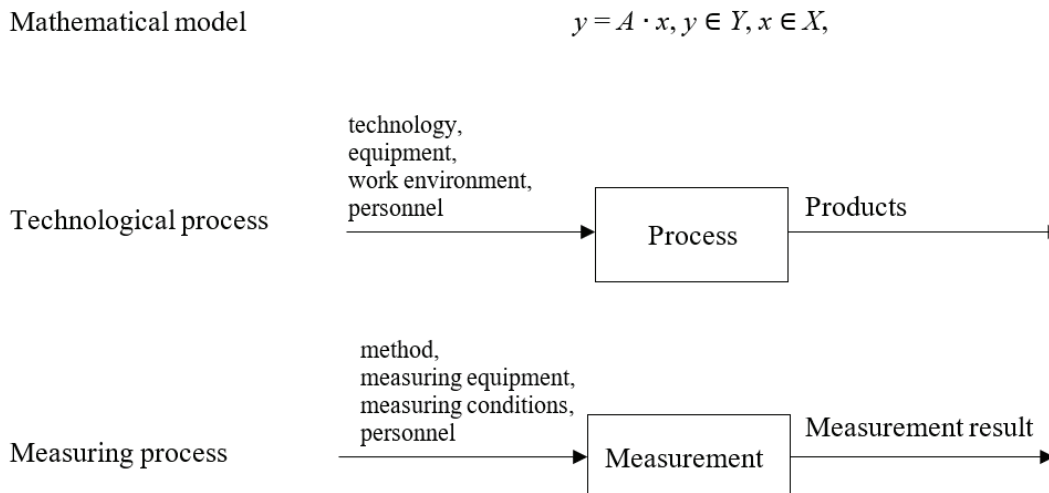
The theory of solution of uncorrected tasks in mathematics, in fact, reflects the kind of relation of two parts of equation (2), which can be associated with input and output of process model. The application of J. Adamar’s provisions to the task of acceptability of production processes (technological and measurement) can be formulated as follows:

- for technological processes the analysis of task incorrectness is associated with the study of the relationship between the process output (products) and inputs (technology, equipment, production environment, personnel, etc.);
- for measurement processes the analysis of task incorrectness is associated with the study of the relation of the process output (measurement result) with inputs (method, measurement equipment, measurement conditions, personnel, etc.).

The applicability of the provisions of the theory of solution of uncorrected tasks in mathematics to the tasks of acceptability of production processes is illustrated in Figure 3.

As can be seen from Figure 3, the analogy of the models leads us to the following conclusion: the provisions of the theory of solution of uncorrected tasks can serve as the basis for the development of a systematic approach to the assessment and management of the acceptability of production processes.

To form it by analogy with the incorrect tasks in mathematics and mechanics, let us consider the situ-



**Figure 3** – Analogy of incorrect task solving models in mathematics and of acceptability of production processes

ations related to each of the incorrectness attributes in the tasks of acceptability of production processes. Let us formulate the methods of incorrectness identification, methods of incorrectness degree estimation and control mechanisms.

### The first sign of incorrectness is the absence of task solution

A classical example of presence of the given sign of incorrectness from the field of mechanics is a task of definition of force of interaction of a brake shoe and a flywheel of the mechanical drive [5]. The mathematical model of interaction force ( $Y$  by analogy with (1)) is proposed as the equilibrium equation of the block, including force and geometrical parameters, and also the parameters characterizing frictional properties of contact pair ( $x_i$  by analogy with (1)) is offered. The model incorrectness is shown in the fact, that the solution of the equation reasonably simulates the force of interaction between brake shoe and flywheel not in all range of possible values of input parameters  $x_i$ . At their certain correlation the model loses its sense – the reaction of interaction becomes negative, which implies the separation of the shoe from the flywheel.

I. e. this sign of incorrectness manifests itself in modeling tasks in the fact that in certain situations (correlations of input parameters  $x_i$ ) the task has no solution.

In the objectives of acceptability of production processes, examples of manifestations of this attribute are:

– for measuring and control processes: the available linear dimensional measurement system with

the help of the caliper does not allow to provide requirements for dimensional control over 200 mm and accuracy grades 8 and less<sup>1</sup>;

– for technological production processes:

- the existing technology of steel castings in a given mould does not allow to meet the requirements to the accuracy of dimensions of class 11 or more and roughness  $Rz = 5$  or more<sup>2</sup>;

- for the safe storage and transport of high concentrations of nitric acid, the use of available stainless steel tanks is functionally unsuitable; a special aluminium alloy is appropriate for the functional purpose.

The given examples show the absence of solving production tasks with the help of existing tools and allow classifying the cases of manifestation of the first sign of incorrectness in relation to production processes:

– unacceptability due to impossibility to provide qualitative indicators (functional purpose);

– unacceptability due to inability to provide quantitative indicators (stability, accuracy, etc.).

Of particular practical interest are the ways to eliminate incorrectness on this feature, developed in model tasks in the field of mathematics and mechanics:

<sup>1</sup> RD 50-98-86 Methodical instructions. Selection of universal measuring instruments of linear dimensions up to 500 mm (by application of GOST 8.051-81); GOST 166-89 Vernier callipers. Specifications

<sup>2</sup> GOST 26645-85 Metal and alloy castings. Dimensions and mass tolerances and machining allowances

- formation of an alternative model (1) [5], which changes the solution of the task cardinally;
- input of new refinement parameters  $x_{n+1}$  of model (1) [5];
- imposing restrictions on the parameters  $x_i$  of the model (1) [5].

Obviously, the above paths constitute a complete formalized set of possible solutions to incorrectness and can serve as a methodological basis for the development of options to solve this kind of tasks in relation to production processes.

For example, in the field of measurements the incorrectness as an unacceptable model of accuracy evaluation of measurement and control processes in chemistry, pharmaceuticals, biology etc. through errors has been eliminated by applying an alternative model of accuracy evaluation through uncertainties of measurement results<sup>3</sup>.

In production technology, it is common practice to ensure the correctness of the existing process as its acceptability by imposing restrictions (tolerances) on the process parameters (cutting modes, production environment conditions (temperature, humidity, noise level, cleanliness of the working area air, etc.)).

### **The second sign of incorrectness – the solution of the task is not unique**

In mathematics and mechanics, analysis tasks are typical. This feature is characteristic in solving the so-called synthesis tasks [5].

For example, in the tasks of the theory of vibrations the typical task of the analysis consists in finding the spectrum of vibrations at given parameters of the vibrating system. The synthesis task in this case would look like this: for a given spectrum it is required to find such parameters of the vibrational system, which acceptably provide this spectrum.

The second feature is exemplified in the acceptability of production processes:

- for measurement and control processes:
  - of the analysis task is to estimate the measurement uncertainty for a particular measurement system (method) that ultimately has a single solution;
  - the synthesis task for this case is the development of measurement system providing the given uncertainty of measurement results;

<sup>3</sup> GOST 34100.3-2017/ISO/IEC Guide 98-3:2008 Uncertainty of measurement. Part 3. Guide to the expression of uncertainty in measurement

- for technological production processes:
  - the task of analysis is the evaluation of accuracy and stability of the particular technological process of gear hobbing by the rolling method, which has a single solution;
  - the task of synthesis for this case is to develop the technological process of gear teeth cutting, providing the specified accuracy and stability.

It is obvious that the synthesis tasks by definition have a sign of multiplicity of solutions. Ways to eliminate incorrectness by this sign, developed in model tasks in the field of mathematics and mechanics:

- input of new refinement parameters  $x_{n+1}$  of model (1) [9];
- imposing constraints on the parameters  $x_i$  of the model (1) [9].

In engineering, the elimination of the second sign of incorrectness as the determination of the best variant of solving the set task from the set of alternatives is realized by empirical methods, among which the most famous and generally accepted is the method of experiment planning. Experiment planning is a procedure for selecting the number and conditions of experiments (physical or computational), necessary and sufficient to solve the task with the required accuracy [10, 11, 12]. The search for the optimal variant of the implementation of the production process under study is one of the most common tasks in engineering, solved by the method of experiment planning. Experiment planning theory offers a fairly wide range of effective techniques for investigating processes and products in order to find the best option, both for technological processes and for measurement and control processes [11, 12].

In the technical sphere it is quite common to use simpler ways to choose a solution, if not optimal, then at least rational. In the field of industrial technology it is comparative testing of alternatives [13]. In the field of measurements, control and tests – these are comparisons of measurement results obtained by different methods or by the same method, but in different conditions<sup>4</sup>.

<sup>4</sup> On ensuring the uniformity of measurements: Law of the Republic of Belarus of 5 September. 1995, No. 3848-XII: in edition of November 11, 2019, No. 254-3; On implementation of metrological assessment in the form of comparisons of measurement results: Decision of the State Committee for Standardization of the Republic of Belarus, November 27, 2020, No. 89

Also, in order to eliminate the second sign of incorrectness, we should not ignore such a method of determining the best solution variant from a set of alternatives methods of expert evaluation, which can be applied in any necessary sphere of activity. Until recently, this approach has not been popular for evaluating the acceptability of production processes, since the existing methods of expert evaluation, due to their subjectivity, depend on the qualification and experience of experts in the matter in question and, therefore, have less credibility than experimental research methods. However, in recent years, expert estimation as a scientific direction has been developing quite dynamically due to an important advantage – cost – effectiveness. New methods, approaches aimed at increasing the reliability of the evaluation results are emerging. This makes the approach quite attractive for solving incorrect evaluation tasks and managing the acceptability of production processes [14].

### The third sign of incorrectness – small changes of model input parameters correspond to large variations in output parameters

A classic example of the presence of the third sign of incorrectness according to J. Adamar from the field of mathematics can be the following two-dimensional system:

$$\begin{cases} z_1 + 7z_2 = 5; \\ \sqrt{2}z_1 + \sqrt{98}z_2 = \sqrt{50}. \end{cases} \quad (3)$$

The system was solved on the computer for different degrees of rounding of irrational numbers  $\sqrt{2}$ ,  $\sqrt{98}$ ,  $\sqrt{50}$  and the determinant  $\Delta$  was simultaneously calculated. The graph (Figure 4) shows the dynamics of changes in the solutions of the system of equations in cases where 50, 200, 400, 600 decimal places were held, respectively, when writing irrational numbers.

It is well seen that solutions of the system of equations (variation of output parameter  $Y$  (1)) at different degrees of parameter rounding (variation of input parameters  $x_i$  (1)) behave very unstable and with increasing number of decimal places do not tend to exact fixed solution of the system.

In acceptability tasks, examples of manifestations of the third feature of incorrectness are:

– for measurement and control processes: “natural” or intentional variation of input factors to a

measurement process by S.W.I.P.E. classification (standard, part, measuring instrument, operator, procedure and production environment) or P.I.S.M.O.E.A. (part, measuring instrument, standard, method, operator, production environment, assumptions) causes inevitable variation of the output – uncertainty of measurement result, which value is normalized as an upper admissible limit [15].

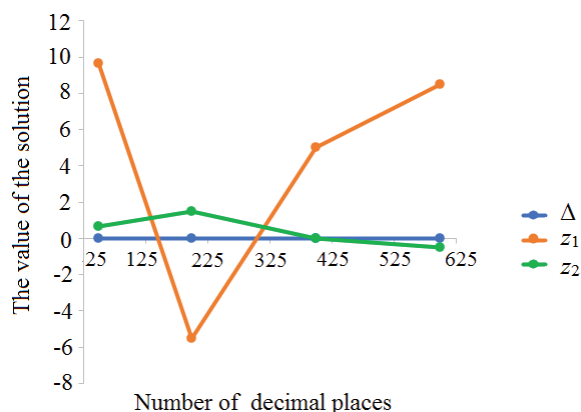


Figure 4 – Solving a system of equations when writing irrational numbers with different numbers of decimal places

Note. A more detailed systematization and analysis of sources of measurement process incorrectness is given in [16].

– for manufacturing processes: “natural” or intentional variation in factors related to the process infrastructure (e. g., variation in the functional characteristics of process equipment), the operating environment (variation in temperature, noise level, air purity in the work area), personnel (variation in qualifications, skills, attention and responsibility), materials and supplies (variation in quality characteristics from one supplier to another) causes variation in outputs to a certain extent.

Note. To assess the quality of technological processes at the stage of validation (accuracy, stability) are widely used indices  $C_p$  and  $P_p$ <sup>5</sup>. The reproducibility index  $C_p$  is defined as the ratio of the tolerance of the process output parameter to the estimate of the intrinsic variability of the process. The suitability index  $P_p$  is defined as the ratio of the tolerance of the controlled parameter to the estimate of the total variability of the process. Both indices obviously allow us to identify the incorrectness of the process in its particular implementation on the third attribute.

<sup>5</sup> GOST R 50779.46-2012 Statistical methods. Process management. Part 4. Process capability and performance estimation

Of practical interest are the ways to eliminate incorrectness on the third sign, developed in modeling tasks in the field of mathematics as model robustness loss tasks:

– fixation (reduction and/or limitation) of the value of parameters  $x_i$  of the model (1) having a large variation;

– reduction of the influence coefficients  $A$  of parameters  $x_i$  of the model (1);

– introduction of new parameters  $x_{n+1}$  into model (1), compensating the influence of parameters  $x_i$ ,  $i = 1, \dots, n$ , on variation of output  $Y$  of model (1) [6].

It can be assumed that the above paths, as well as for the previous signs of incorrectness, constitute a complete formalized set of possible solutions to incorrectness and can serve as a methodological basis for developing options for solving such tasks with respect to the acceptability of production processes.

### Relation of task incorrectness to the notion of robustness

The concept of robustness is quite well established and is widely used not only in mathematics, but also in a number of research areas. Mathematical support of the data processing methods robustness estimation and analysis is an independent direction in mathematics [17], separate aspects of which are also applied in metrology. In particular, STB ISO 5725-5 proposes the robustness methods of data analysis for correctness and precision of measurement (test) methods as an alternative<sup>6</sup>. Robustness of measurement methods is a validation characteristic, necessarily confirmed at attestation (validation) or verification of some methods in the field of analytical chemistry<sup>7</sup>. In this case the robustness is considered as the ability of the method to give the analytical results with acceptable precision and correctness under small deliberate changes of the test method parameters.

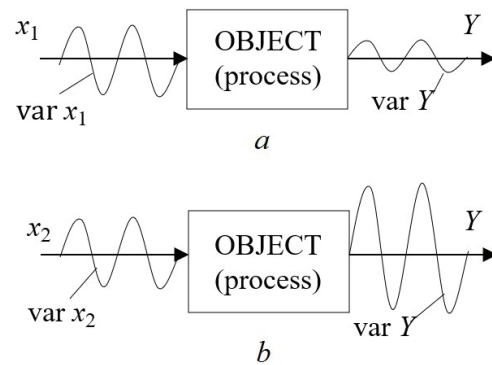
In engineering, the concept of robustness is associated primarily with the methods of G. Taguchi – methods of robust design of industrial products and technological processes, which aim to ensure the stability of their final characteristics (values lie

<sup>6</sup> STB ISO 5725-5-2002 Accuracy (correctness and precision) of methods and results of measurements. Part 5. Alternative methods for determining the precision of a standardized method of measurement

<sup>7</sup> STB 1436-2004 Manufacture of medicinal products. Validation of testing procedures

within the tolerance field) to the variability of input factors [18, 19].

The generalized concept of robustness of the output parameter  $Y$  of the object (model, process, products, etc.) in relation to the input parameters  $x_i$  can be illustrated as follows (Figure 5).



**Figure 5** – Classical understanding of object (model, process, product, etc.) robustness: *a* – the process is robust with respect to factor  $x_1$ ; *b* – the process is not robust with respect to factor  $x_2$

Here  $Y$ ,  $\text{var } Y$  is the output of the production process and its variation.  $x_1$ ,  $x_2$ ,  $\text{var } x_1$ ,  $\text{var } x_2$  are input parameters characterizing the process realization conditions and their variation. It can be argued that the process is robust with respect to parameter  $x_1$  and nonrobust with respect to parameter  $x_2$ . I. e.,  $x_2$  is the factor of the loss of robustness of the process, indicating the presence of the third sign of incorrectness of the model of production process [5].

The equivalence of the notions of loss of robustness and the third sign of incorrectness is obvious. On the other hand it is obvious, that the robustness can be considered as the generalized indicator of acceptability of production processes. I. e. in the wide sense of the word the losses of robustness are close in meaning with all three signs of incorrectness of the tasks considered above. It is possible to assert that all three signs of incorrectness in relation to production processes can be considered as the manifestation of the robustness property.

This allows us to formulate a hypothesis that a systematic approach to the evaluation and management of robustness losses of production processes in the sense of their unacceptability can be based on the developed approaches and techniques of the theory of uncorrected tasks.



## A strategy for assessing and managing the robustness of production processes

Identification in the production process of any of the three signs of incorrectness according to J. Adamar leads to uncertainty of the result:

- in the presence of the first sign, there is no acceptable result of the process;
- in the presence of the second feature, the acceptability of the result is uncertain due to the variety of alternative solutions;
- in the presence of the third feature, the acceptability of the process result is unpredictable due to the large variation in the output.

The first two lead to an uncompromising need to perform any action to eliminate them, since the process must be devoid of these signs of incorrectness, i. e. robust to their manifestation.

The peculiarity of the third sign of incorrectness is the fact that the model of acceptability of the production process is always incorrect. Input factors are, by definition, random quantities, i. e. they have natural variation, which obviously leads to inevitable variation of the output – the result of the process. According to the standard<sup>8</sup> “...variability can be observed in the behaviour and outcome of virtually all processes, even in conditions of apparent stability...”.

It is proposed to divide the robustness losses of the production process into two types, depending on the acceptable degree of output variation:

- loss of robustness of type I: process output variation can be considered acceptable (it corresponds to a given technological accuracy, admissible uncertainty (error));
- loss of robustness of type II: the variation of the process output must be reduced because it exceeds the admissible values.

As a result it is possible to offer the following classification of situations of losses of robustness of production processes as their unacceptability according to classification of signs of incorrectness of tasks on J. Adamar (Figure 6).

In its essence, we obtain that the degree of incorrectness of the task is an analogue of the degree of robustness loss. Accordingly, the significance of the robustness loss factor can be determined by setting a threshold value, for example, through the coefficient  $K$ :

$$K = \Delta_{out} / \Delta_{in}, \quad (4)$$

where  $\Delta_{out}$  is the variation of the “output” of the process;  $\Delta_{in}$  is the variation of the “input” of the process.

If the coefficient  $K$  exceeds some predetermined threshold value, the process acceptability task becomes incorrect and the analyzed process “input” factor can be considered as a robustness loss factor and be treated as a control object.

A systematic approach to ensuring the robustness of production processes involves a two-step algorithm, including:

- identification of sources of robustness losses and corresponding input factors, the variation of which causes variation of the process output;
- management of input factors that cause significant variation in the process output factor.

*Step 1: Identification of sources of robustness loss and corresponding input factors.* The issues of systematic approach to identification of robustness loss factors of measurement processes are discussed in detail in [16]. Measurement processes are characterized by two complex sources of potential robustness loss factors:

- 1) measurement object;
- 2) the actual measurement process.

The measurement object as a source of robustness losses can “give” the following factors of robustness losses:

- parameters characterizing various states of the measurement object (in the nominal scale);
- parameters characterizing various quantitative values of the input characteristics of the measurement object (in the scale of ranks, relations, absolute scale).

The actual measurement process can “give” robustness loss factors related to:

- parameters of the measurement information conversion process;
- parameters of the statistical data processing process.

A similar approach can be proposed for identifying the factors of loss of robustness of technological processes. Obviously, they are also characterized by two complex sources of potential factors of robustness losses:

- 1) processing object (blanks, materials, components);
- 2) the actual technological process.

The processing object as a source of robustness losses in the image and likeness can “give” the following factors of robustness losses:

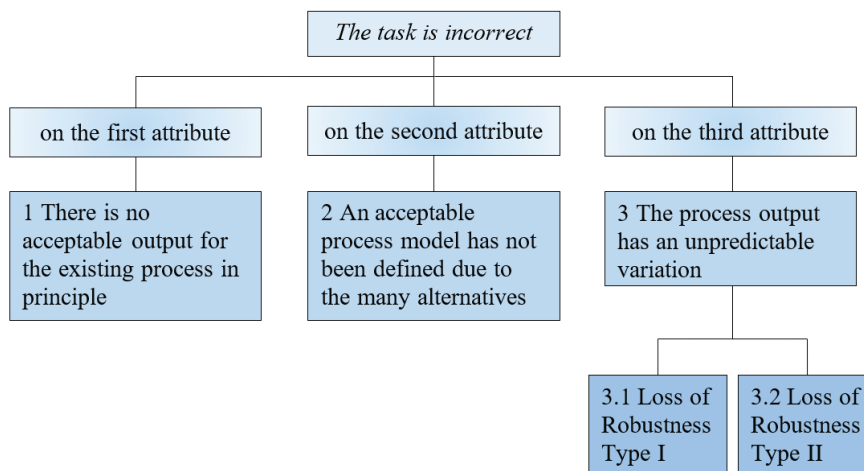
<sup>8</sup> STB ISO/TR 10017-2011 Guidance on statistic methods with respect to STB ISO 9001-2009

- parameters characterizing various states of the processing object (in the nominal scale);
- parameters characterizing various quantitative values of the input characteristics of the processing object (in the scale of ranks, relations, absolute scale).

The actual technological process in the image and likeness can “give” factors of loss of robustness, related to:

- technological process parameters;
- parameters of the process of collecting, processing and analyzing data.

The peculiarity of robustness loss for measuring and technological processes is its latent character due to incomplete information about the process model, since it is impossible (or economically unreasonable) to identify absolutely all influencing factors and the degree of their influence.



**Figure 6** – Classification of situations of loss of robustness of production processes as their unacceptability according to the classification of task incorrectness attributes

*Step 2. Management of input factors that cause significant variation in the process output factor.* Taking into account the classification of robustness loss situations (Figure 6), the following mechanism for managing the robustness of production processes at the stage of their development can be proposed:

Stage 1: Neutralize the appearance of robustness losses of types 1, 2 and 3.2 according to Figure 6.

Stage 2: Verify the process according to the criterion of total output uncertainty assessment compliance with the specified requirements.

To implement stage 1, it is proposed to use the methods developed in the practice of solving incorrect tasks for each of the three signs of incorrectness.

To neutralize the type 1 robustness loss (attribute 1, see Figure 6) the methods formulated above may be used: 1) forming an alternative process model as a cardinal solution of the task; 2) introducing new clarifying input parameters into the process model, decreasing the output variation; 3) imposing restrictions on model parameters, stabilizing the output variation.

To neutralize the type 2 robustness loss (attribute 2, see Figure 6) we can use the methods stated above: 2) introducing new clarifying input para-

eters into the process model, decreasing the output variation; 3) imposing restrictions on model parameters, stabilizing the output variation; as well as experimental researches of process by experiment planning methods (comparison of alternatives) or methods of expert evaluation to define the best variant of process realization in accordance with given requirements.

To neutralize the robustness loss of type 3.2 (attribute 3.2, see Figure 6) methods of influence on process input factors which have significant influence on process output variation can be used: 1) decreasing influence of factor by fixing its value or reducing its weighting coefficient; 2) decreasing influence of factor by controlled change of other factors using their correlation relationship with opposite sign.

Methods for neutralization of robustness losses of types 1 and 2, as well as type 3.2 (method 1) are intuitively understandable (Figure 6). In each case they are individual, but all of them refer to engineering (organizational and technical) measures of providing and adjusting production processes.

The greatest scientific interest is the neutralization of robustness losses of type 3.2 (method 2).

The method implies correction of the production process to such a state, at which the coefficient  $K$  in expression (4) for each identified factor of robustness loss, potentially, will not exceed its threshold value. An interesting solution to the task are the so-called methods of G. Taguchi – methods of robust redesign, which are designed mainly for technological processes [19]. However, they have not become widespread due to a number of disadvantages:

- methods for identifying the factors affecting the robustness of the process output are not defined, which causes the risk of not identifying them;
- it is necessary to investigate a priori the function of influence of each factor on variation of the process output;
- there is a need to implement special active experimentation plans, which are not always economically feasible.

We have substantiated a method that involves an essential modification of G. Taguchi's techniques, which has two main distinctive points [20]:

- the use of passive experiment in the form of collecting and accumulating data on the implementation of the process over a period of time, which ensures high efficiency of the method;
- special processing of the results using the mathematical apparatus of nonparametric regression based on wavelet transforms, which ensures the correctness of the obtained results.

Taken together, the application of the modernized method of G. Taguchi will make it possible to ensure high efficiency of the management of the robustness loss of production processes in situation 3.2 (Figure 6).

Stage 2 is implemented on the assumption that stage 1 has been successfully completed, i. e. all significant influencing factors of the input of the production process (types 1, 2, 3.1 in Figure 6) have been identified and neutralised. In this case, the verification of the process is carried out according to the criterion of conformity of the estimate of the total uncertainty of the output with the specified requirements. Stage 2 is reduced to evaluation of the resulting variation of the output of the production process as a result of the combined effect of input factors recognized at stage 1 as having little effect, and stating the fact that it does not exceed the values specified in the technical requirements. In this case, the process is considered acceptable. For measurement processes these are, for example, such indicators of measurement result (process output) as drift,

precision, uncertainty<sup>9</sup>. For technological processes, these are, for example, such indicators of output as  $C_p$ ,  $P_p$ , stability, etc.<sup>10</sup>. The resulting estimates are used as passport characteristics of the process to which the values obtained in periodic monitoring will be compared in order to certify the process to maintain its acceptability.

## Conclusion

The relevance of developing a strategy for a guaranteed solution to the task of acceptability of production processes under the concept of industrial production "Industry 4.0" is substantiated. The article proposes a systematic approach to the development of the strategy based on the approaches of the theory of solving incorrect tasks. The signs of correctness of mathematical tasks according to J. Adamar are adapted to the tasks of ensuring the acceptability of production processes (technological and measurement) in terms of identifying the properties of the manifestation of incorrectness and methods of incorrectness management.

It has been established that signs of incorrectness in relation to production processes can be considered as a manifestation of the robustness property, therefore robustness can be considered as a generalized indicator of the acceptability of production processes. It is concluded that the basis of a systematic approach to the assessment and management of robustness losses of production processes in the sense of their unacceptability can be based on the established approaches and techniques of the theory of incorrect tasks. The classification of situations of losses of robustness of production processes as their unacceptability in accordance with the classification of signs of incorrectness of tasks on J. Adamar is offered. In order to divide the factors of the loss of robustness into groups depending on the method of control the notion of the degree of task incorrectness as an analogue of the degree of loss of robustness is introduced.

We have proposed a two-step algorithm for ensuring the robustness of production processes, which includes the identification of sources of robustness losses and the management of input factors that cause

<sup>9</sup> GOST 34100.3-2017/ISO/IEC Guide 98-3:2008 Uncertainty of measurement. Part 3. Guide to the expression of uncertainty in measurement

<sup>10</sup> GOST R 50779.46-2012 Statistical methods. Process management. Part 4. Process capability and performance estimation

a significant variation in the output of the process. The sources of potential factors of loss of robustness of production processes are determined by the criterion of completeness and non-redundancy using a systematic approach. We have formulated a two-stage mechanism for managing the robustness of production processes, taking into account the classification of robustness loss situations. Rational methods are proposed to ensure the robustness of production processes for each stage, which have been developed by the practice of solving incorrect tasks. We have substantiated the method, which is a modification of the methods of robust redesign of G. Taguchi, providing high efficiency of managing losses of robustness of production processes in certain situations.

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