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題　名

Intrafamily transfer and human capital investment in China:
An OLG Model Analysis

中国における家族内移転と人的資本投資：OLGモデル分析

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1 Introduction

Public pension plays an important role as the protection of elderly. However, with the rapid acceleration in the aging population, it has a great impact on public pension system. Due to decades of one-child policy and prolonged human life expectancy, the proportion of elderly is increasing every year in China. As of the end of 2019, China's population of elderly aged over 65 years old was 12.6%, and it will reach 27.5% by 2050 predicted by United Nations.

Zhao and Mi(2019) argue that if the current pension system remain unchanged, the pension gap will continue to exist and does not seem to be sustainable. Although the Chinese government has introduced a series of social security reforms, changes in demographics and one-child policy make challenges to the government's ability to deliver sufficient old-age support. At present, 57% of elderly rely on family support, while only 25% of elderly rely on pension, both in the urban and the rural areas according to the Chinese census data. Furthermore, Herd et al.(2010) point out that the effective replacement rate is about 40% and will continue to decline further. These problems are more serious in rural areas and even in some poverty areas the concept of raising children to prevent the elderly has been very serious. Therefore, the traditional habit of transferring family income from children to parents remains strong to substitute for the lack of social security system.

In 2016, China's government introduced the universal two-child policy to relieve pressure on pension fund. However, China's fertility rate continues to decrease and it's far from expected according to the data released by the National Bureau of Statistics. Coeurdacier et al.(2014) did a research on the relationship between relaxing fertility controls and expanding social security using a three-period overlapping generation model in China. They came to a conclusion that if children are an important support for the elderly, an expansion of social security benefits reduces fertility, in other words, abandoning fertility restrictions may not be as effective in helping to finance pension reform.

On the other hand, human capital is the key factor to the development of a country. Hundreds of scholars have studied this issue and confirmed it. Now not only the government but also families are paying more attention to children's education. But in some rural areas, because of poverty and low level of education, parents would ignore children's education, thus forming a vicious circle. Becker(2009) notes that investment in human capital is a radical solution that breaks the poverty cycle. When we consider China's economic growth, it's necessary to consider the two-way altruism of such human capital investment from parents to children and intrafamily transfer from children to parents. Zhu et al.(2014) asserts that under China's poor functioning formal social security system and the one-child policy, parents depend on their children for old age support, so that the decreasing in the number of children prompted by the one child policy resulted in parents investing more in their children's education to ensure retirement consumption.

In addition, Sinn(2004) discusses pension and human capital from the view of insurance. The result is that individuals will have moral hazard after joining the pension insurance, thereby reducing human capital investment compared to when not joining the insurance. And in a one-way altruistic situation where parent-to-child human capital investment exists but child-to-parent intrafamily transfer cannot be enforced, human capital investment is raised in a generous PAYGO system. This analysis focuses on the equilibrium in a game situation.

In this paper, we expand Sinn(2004)'s research to Overlapping Generations Model from a macro perspective. We built a model combined China's current situation of the importance of intrafamily support, considering the two-way altruism of such human capital investment from parents to children and transfer from children to parents, and explore the relationship between intrafamily transfer and human capital investment. The conclusion is that there are two balanced growth path. The one is a low growth path with child-to-parent transfer. The another is a high growth path with parent-to-child transfer.

The paper is structured as follows. Section 2 describes the model and computes equilibrium. Section 3 exports the steady state. Section 4 analyzes the steady state combined with Chinese situation. Section5 concludes.

2 The model

We use an Overlapping Generations Model of closed economy introduced intrafamily transfer and human capital. The economy consists of three types of agents, households, firms and the government.

2.1 Households

Consider a household that lives for three periods of childhood, youth, and old. The agent in childhood receives education and does not consume. The education becomes human capital and affects his wage for the next period. Then he works in youth age, retires in old age.

The preference of household born in period $t-1$ is described by the following utility

$$U(c_{y.t}, c_{o,t+1}, h_{t+1}, f_{t+1}; t) = \ln c_{y,t} + \beta \ln c_{o,t+1} + \gamma n V(h_{t+1}, f_{t+1}; t+1), \tag{1}$$

where $c_{y,t}$ is consumption in youth age in period t, $c_{o,t+1}$ is consumption in old age in period $t+1$, $0 < \beta < 1$, denotes the subjective discount factor, function V is indirect utility of next generation, $n > 0$ means population growth, and $\gamma > 0$ represents the strength of parent-to-child altruism. It means the agents care not only for their own consumption but for the utility of their children.

When working, the agent earns effective labor income and pay a fraction of gross wages to the public pension system. When retired, the agent receives interest payments on saving, public pension and he receives ($f_t > 0$) or pays ($f_t < 0$) family income transfer. The budget constraints are

$$c_{y,t} = (1-\tau_t) w_t h_t - s_t - e_t \cdot n - f_t, \tag{2}$$

$$c_{o,t+1} = r_{t+1} s_t + \sigma_{t+1} + f_{t+1} \cdot n, \tag{3}$$

where $0 < \tau < 1$ is the contribution rate to the PAYGO pension system; w_t is the wage rate in the period t; h_t is human capital, s_t is saving, e_t is education investment, f_t is a transfer to his parents if $f_t > 0$ or from his parents if $f_t < 0$, r_{t+1} is interest rate in the period $t+1$, σ_{t+1} is pension benefits.

The human capital production technology is linear with respect to human capital of the previous generation and education investment. Human capital in the period $t+1$ is

$$h_{t+1} = (1-\theta)h_t + De_t, \tag{4}$$

where $\theta(\in [0,1])$ is depreciation rate, and $D > 0$.

household utility maximization

In order to solve the problem of maximizing household utility, the calculation is divided into three steps.

First step, given e_t, h_{t+1}, f_{t+1}, we solve

$$\max_{(c_{y,t}, c_{o,t+1}, s_t)} \ln c_{y,t} + \beta \ln c_{o,t+1}$$

s.t.

$$c_{y,t} = (1-\tau_t)w_t h_t - s_t - e_t \cdot n - f_t$$

$$c_{o,t+1} = r_{t+1}s_t + \sigma_{t+1} + f_{t+1} \cdot n.$$

Maximization of utility subject to the budget constraints gives the following Euler equation of consumption:

$$\frac{1}{c_{y,t}} = \frac{\beta r_{t+1}}{c_{o,t+1}}. \tag{5}$$

By substituting Eq.(5) into Eq.(2) and (3), we can get consumption function of youth age and old age are

$$c_{y,t} = c_y(h_t, f_t, h_{t+1}, f_{t+1}; t) \equiv \frac{1}{1+\beta}\left[(1-\tau_t)w_t h_t - e(h_t, h_{t+1})n - f_t + \frac{\sigma_{t+1} + f_{t+1} \cdot n}{r_{t+1}}\right], \tag{6}$$

$$c_{o,t+1} = c_o(h_t, f_t, h_{t+1}, f_{t+1}; t) \equiv \frac{\beta r_{t+1}}{1+\beta}[(1-\tau_t)w_t h_t - e(h_t, h_{t+1})n - f_t] + \frac{\beta}{1+\beta}(\sigma_{t+1} + f_{t+1} \cdot n), \tag{7}$$

respectively. Saving function is

$$s_t = s_t(h_t, f_t, h_{t+1}, f_{t+1}; t) \equiv \frac{\beta}{1+\beta}\left[(1-\tau_t)\, w_t h_t - e_t(h_t, h_{t+1})n - f_t\right] - \frac{\sigma_{t+1} + f_{t+1}\cdot n}{(1+\beta)r_{t+1}}. \tag{8}$$

Then, from Eq.(4)

$$e_t = e(h_t, h_{t+1}) \equiv \frac{1}{D}[h_{t+1} - (1-\theta)h_t]. \tag{9}$$

By substituting (6) and (7) into (1), the Bellman equation is

$$V(h_t, f_t; t) = \max_{(h_{t+1}, f_{t+1})} \ln c_y(h_t, f_t, h_{t+1}, f_{t+1}; t) + \beta \ln c_o(h_t, f_t, h_{t+1}, f_{t+1}; t) + \gamma n V(h_{t+1}, f_{t+1}; t+1). \tag{10}$$

Second step, derive the conditions of the solution of the Bellman equation. The first-order condition of the Bellman Eq.(10) is

$$\frac{1}{c_{y,t}}\frac{\partial c_{y,t}}{\partial h_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \frac{\beta}{c_{o,t+1}}\frac{\partial c_o}{\partial h_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \gamma n \frac{\partial V}{\partial h_{t+1}}(h_{t+1}, f_{t+1}; t+1) = 0, \tag{11}$$

$$\frac{1}{c_{y,t}}\frac{\partial c_{y,t}}{\partial f_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \frac{\beta}{c_{o,t+1}}\frac{\partial c_o}{\partial f_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \gamma n \frac{\partial V}{\partial f_{t+1}}(h_{t+1}, f_{t+1}; t+1) = 0. \tag{12}$$

Let $h_{t+1} = g^h(h_t, f_t; t)$, $f_{t+1} = g^f(h_t, f_t; t)$ be the state transition function in the optimal planning, then rewrite Eq.(10)

$$\begin{aligned} V(h_t, f_t; t) &= \ln c_y(h_t, f_t, g^h(h_t, f_t), g^f(h_t, f_t); t) + \beta \ln c_o(h_t, f_t, g^h(h_t, f_t), g^f(h_t, f_t); t) \\ &+ \gamma n V(g^h(h_t, f_t), g^f(h_t, f_t); t+1). \end{aligned} \tag{13}$$

Differentiate respect to h_t and f_t,

$$\begin{aligned}
\frac{\partial V}{\partial h_t} =& \frac{1}{c_{y,t}} \cdot \frac{\partial c_y}{\partial h_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) \\
&+ \frac{1}{c_{y,t}} \cdot \frac{\partial c_y}{\partial h_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) \cdot \frac{\partial g^h}{\partial h_t}(h_t, f_t) \\
&+ \frac{1}{c_{y,t}} \cdot \frac{\partial c_y}{\partial f_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) \cdot \frac{\partial g^f}{\partial h_t}(h_t, f_t) \\
&+ \frac{\beta}{c_{o,t+1}} \cdot \frac{\partial c_o}{\partial h_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) \\
&+ \frac{\beta}{c_{o,t+1}} \cdot \frac{\partial c_o}{\partial h_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) \cdot \frac{\partial g^h}{\partial h_t}(h_t, f_t) \\
&+ \frac{\beta}{c_{o,t+1}} \cdot \frac{\partial c_o}{\partial f_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) \cdot \frac{\partial g^f}{\partial h_t}(h_t, f_t) \\
&+ \gamma n \frac{\partial V}{\partial h_{t+1}}(h_{t+1}, f_{t+1}; t+1) \cdot \frac{\partial g^h}{\partial h_t}(h_t, f_t) \\
&+ \gamma n \frac{\partial V}{\partial f_{t+1}}(h_{t+1}, f_{t+1}; t+1) \cdot \frac{\partial g^f}{\partial h_t}(h_t, f_t) \\
=& \frac{1}{c_{y,t}} \cdot \frac{\partial c_y}{\partial h_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \frac{\beta}{c_{0,t+1}} \cdot \frac{\partial c_o}{\partial h_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) \\
&+ \underbrace{\left[\frac{1}{c_{y,t}} \cdot \frac{\partial c_y}{\partial h_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \frac{\beta}{c_{o,t+1}} \cdot \frac{\partial c_o}{\partial h_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \gamma n \frac{\partial V}{\partial h_{t+1}}(h_{t+1}, f_{t+1}; t+1)\right] \cdot \frac{\partial g^h}{\partial h_t}(h_t, f_t; t)}_{=0(11)} \\
&+ \underbrace{\left[\frac{1}{c_{y,t}} \cdot \frac{\partial c_y}{\partial f_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \frac{\beta}{c_{c,t+1}} \cdot \frac{\partial c_o}{\partial f_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \gamma n \frac{\partial V}{\partial f_{t+1}}(h_{t+1}, f_{t+1}; t+1)\right] \cdot \frac{\partial g^f}{\partial h_t}(h_t, f_t; t)}_{=0(12)} \\
=& \frac{1}{c_{y,t}} \cdot \frac{\partial c_y}{\partial h_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \frac{\beta}{c_{o,t+1}} \cdot \frac{\partial c_o}{\partial h_t}(h_t, f_t, h_{t+1}, f_{t+1}; t).
\end{aligned} \tag{14}$$

Similarly, we can get

$$\frac{\partial V}{\partial f_t}(h_t, f_t; t) = \frac{1}{c_{y,t}} \cdot \frac{\partial c_y}{\partial f_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) + \frac{\beta}{c_{o,t+1}} \cdot \frac{\partial c_o}{\partial f_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) \tag{15}$$

Third step, calculate the differential part concretely. First, calculate Eq.(14) and Eq.(15). From Eq.(6), (7), (8), we can get

$$\begin{aligned}
\frac{\partial c_y}{\partial h_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) &= \frac{1}{1+\beta}\left[(1-\tau_t) w_t + \frac{n(1-\theta)}{D}\right], \\
\frac{\partial c_o}{\partial h_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) &= \frac{\beta r_{t+1}}{1+\beta}\left[(1-\tau_t) w_t + \frac{n(1-\theta)}{D}\right], \\
\frac{\partial c_y}{\partial f_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) &= -\frac{1}{1+\beta}, \\
\frac{\partial c_o}{\partial f_t}(h_t, f_t, h_{t+1}, f_{t+1}; t) &= -\frac{\beta r_{t+1}}{1+\beta}.
\end{aligned}$$

By substituting them into Eq.(13) and (14), and rewrite them with Eq.(5)

$$\frac{\partial V}{\partial h_t}(h_t, f_t; t) = \frac{1}{c_{y,t}} \cdot \left[(1-\tau_t) w_t + \frac{n(1-\theta)}{D}\right], \tag{16}$$

$$\frac{\partial V}{\partial f_t}(h_t, f_t; t) = -\frac{1}{c_{y,t}}. \tag{17}$$

respectively. Then, use them to calculate Eq.(10) and (11). From Eq.(6), (7), (8), we can get

$$\frac{\partial c_y}{\partial h_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) = -\frac{1}{1+\beta}\frac{n}{D},$$
$$\frac{\partial c_o}{\partial h_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) = -\frac{\beta r_{t+1}}{1+\beta}\frac{n}{D},$$
$$\frac{\partial c_y}{\partial f_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) = \frac{1}{1+\beta}\cdot\frac{n}{r_{t+1}},$$
$$\frac{\partial c_o}{\partial f_{t+1}}(h_t, f_t, h_{t+1}, f_{t+1}; t) = \frac{\beta}{1+\beta}\cdot n.$$

Let Eq.(15) and (16) advanced by one period and substitute them into Eq.(10) and (11) respectively, and organize them using Eq.(5), then these become as follows

$$-\frac{1}{c_{y,t}}\cdot\frac{n}{D} + \gamma n\cdot\frac{1}{c_{y,t+1}}\cdot\left[(1-\tau_{t+1})\,w_{t+1} + \frac{n(1-\theta)}{D}\right] = 0, \tag{18}$$

$$\frac{1}{c_{y,t}}\cdot\frac{n}{r_{t+1}} - \gamma n\cdot\frac{1}{c_{y,t+1}} = 0. \tag{19}$$

From the above, the conditions for the optimal choice of generation t are obtained. The individual optimal choice $(c_{y,t}, c_{o,t+1}, e_t, h_{t+1}, f_{t+1})$ satisfies the following.

$$c_{y,t} = \frac{1}{1+\beta}\left[(1-\tau_t)w_t h_t - e_t\cdot n - f_t + \frac{\sigma_{t+1} + f_{t+1}\cdot n}{r_{t+1}}\right], \tag{6'}$$

$$c_{o,t+1} = \frac{\beta r_{t+1}}{1+\beta}[(1-\tau_t)w_t h_t - e_t\cdot n - f_t] + \frac{\beta}{1+\beta}(\sigma_{t+1} + f_{t+1}\cdot n), \tag{7'}$$

$$h_{t+1} = (1-\theta)h_t + De_t, \tag{4}$$

$$-\frac{1}{c_{y,t}}\cdot\frac{n}{D} + \gamma n\cdot\frac{1}{c_{y,t+1}}\cdot\left[(1-\tau_{t+1})\,w_{t+1} + \frac{n(1-\theta)}{D}\right] = 0, \tag{18}$$

$$\frac{1}{c_{y,t}}\cdot\frac{n}{r_{t+1}} - \gamma n\cdot\frac{1}{c_{y,t+1}} = 0, \tag{19}$$

$$s_t = \frac{\beta}{1+\beta}[(1-\tau_t)\,w_t h_t - e_t\cdot n - f_t] - \frac{\sigma_{t+1} + f_{t+1}\cdot n}{(1+\beta)r_{t+1}}. \tag{8'}$$

Combined Eq.(18) and (19), we can get

$$\frac{1}{D} = \frac{1}{r_{t+1}}[(1-\tau_{t+1})w_{t+1} + \frac{n(1-\theta)}{D}]. \tag{20}$$

2.2 Firms

A representative competitive firm in period t produces final goods using two factors of production, physical capital stock, K_t, and human capital stock, H_t. The production technology is Cobb-Douglas.

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha}, \tag{21}$$

where A is a scale parameter, and $0 < \alpha < 1$, is capital share.

The profit in the period t is

$$\Pi_t = AK_t^{\alpha} H_t^{1-\alpha} - w_t H_t - R_t K_t. \tag{22}$$

Maximize (21) subject to (22) and solve the profit maximization problem, the first-order condition is obtained as follows.

$$w_t = (1-\alpha)AK_t^{\alpha} H_t^{-\alpha} = (1-\alpha)A\hat{k}_t^{\alpha}, \tag{23}$$

$$r_t = \alpha AK_t^{\alpha-1} + 1 - \delta = \alpha A\hat{k}_t^{\alpha-1} + 1 - \delta, \tag{24}$$

where $r_t = R_t + 1 - \delta$, and δ is capital depreciation rate, and $\hat{k}_t \equiv \frac{K_t}{H_t}$.

2.3 Government

Assume that the pension system is a pay-as-you-go system. Budget constraint is

$$\tau_{t+1} w_{t+1} H_{t+1} = \sigma_{t+1} N_t \Rightarrow n\tau_{t+1} w_{t+1} h_{t+1} = \sigma_{t+1}. \tag{25}$$

2.4 Equilibrium

Labor market clearing condition in period t is

$$H_t = N_t h_t. \tag{26}$$

Capital market clearing condition in period t is

$$K_{t+1} = N_t s_t \Rightarrow \hat{k}_{t+1} \cdot nh_{t+1} = s_t. \tag{27}$$

Then substitute (8')(9)into (27), one obtains

$$\hat{k}_{t+1} \cdot nh_{t+1} = \frac{\beta}{1+\beta}\left[(1-\tau_t)\, w_t h_t - \frac{1}{D}[h_{t+1} - (1-\theta)h_t]\cdot n - f_t\right] - \frac{\sigma_{t+1} + f_{t+1}\cdot n}{(1+\beta)r_{t+1}}. \tag{28}$$

Goods market clearing condition in period t is

$$Y_t = C_t + I_{K,t} + I_{H,t} = N_t c_{y,t} + N_{t-1}c_{o,t} + K_{t+1} - (1-\delta)K_t + e_t \cdot nN_t$$

$$\Rightarrow A\hat{k}_t^{\alpha} N_t h_t = N_t c_{y,t} + N_{t-1}c_{o,t} + \hat{k}_{t+1}N_{t+1}h_{t+1} - (1-\delta)\hat{k}_t N_t h_t + e_t \cdot nN_t. \tag{29}$$

Divide by N_t

$$A\hat{k}_t^{\alpha} h_t = c_{y,t} + \frac{1}{n}c_{o,t} + \hat{k}_{t+1}nh_{t+1} - (1-\delta)\hat{k}_t h_t + e_t \cdot n. \tag{30}$$

Shift the period to $t+1$

$$A\hat{k}_{t+1}^{\alpha} h_{t+1} = c_{y,t+1} + \frac{1}{n}c_{o,t+1} + \hat{k}_{t+2}nh_{t+2} - (1-\delta)\hat{k}_{t+1} h_{t+1} + e_{t+1} \cdot n. \tag{31}$$

From Eq.(5)(19), we can get $c_{y,t+1} = \frac{\gamma}{\beta}c_{o,t+1}$, and substitute it into Eq.(31)

$$A\hat{k}_{t+1}^{\alpha} h_{t+1} = \frac{n\cdot\gamma+\beta}{n\cdot\beta}c_{o,t+1} + \hat{k}_{t+2}nh_{t+2} - (1-\delta)\hat{k}_{t+1} h_{t+1} + e_{t+1} \cdot n. \tag{32}$$

Substitute Eq.(3) into Eq.(32)

$$A\hat{k}_{t+1}^{\alpha} h_{t+1} = \frac{n\cdot\gamma+\beta}{n\cdot\beta}[r_{t+1}\hat{k}_{t+1}\cdot nh_{t+1} + \sigma_{t+1} + f_{t+1}\cdot n] + \hat{k}_{t+2}nh_{t+2} - (1-\delta)\hat{k}_{t+1} h_{t+1} + e_{t+1} \cdot n. \tag{33}$$

Hence, optimal household selection condition, capital market equilibrium condition and goods market equilibrium condition are as follows.

$$\frac{1}{D} = \frac{1}{r_{t+1}}[(1-\tau_{t+1})w_{t+1} + \frac{n(1-\theta)}{D}], \tag{20}$$

$$\hat{k}_{t+1} \cdot nh_{t+1} = \frac{\beta}{1+\beta}[(1-\tau_t)w_t h_t - \frac{1}{D}[h_{t+1} - (1-\theta)h_t]\cdot n - f_t] - \frac{\sigma_{t+1} + f_{t+1}\cdot n}{(1+\beta)r_{t+1}}, \tag{28}$$

$$A\hat{k}_{t+1}^{\alpha} h_{t+1} = \frac{n\cdot\gamma+\beta}{n\cdot\beta}[r_{t+1}\hat{k}_{t+1}\cdot nh_{t+1} + \sigma_{t+1} + f_{t+1}\cdot n] + \hat{k}_{t+2}nh_{t+2} - (1-\delta)\hat{k}_{t+1} h_{t+1} + e_{t+1} \cdot n. \tag{33}$$

Then substitute Eq.(9)(23)(24)(25) into (20)(28)(33)

$$\frac{1}{D} = \frac{1}{\alpha A\hat{k}_{t+1}^{\alpha-1} + 1 - \delta}[(1-\tau_{t+1})(1-\alpha)A\hat{k}_{t+1}^{\alpha} + \frac{n(1-\theta)}{D}], \tag{34}$$

$$\begin{aligned}\hat{k}_{t+1}\cdot nh_{t+1} = {} & \frac{\beta}{1+\beta}\left[(1-\tau_t)(1-\alpha)A\hat{k}_t^{\alpha}h_t - \frac{1}{D}[h_{t+1}-(1-\theta)h_t]\cdot n - f_t\right] \\ & - \frac{n\tau_{t+1}(1-\alpha)A\hat{k}_{t+1}^{\alpha}h_{t+1} + f_{t+1}\cdot n}{(1+\beta)(\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta)},\end{aligned} \tag{35}$$

$$\begin{aligned}A\hat{k}_{t+1}^{\alpha}h_{t+1} = {} & \frac{n\cdot\gamma+\beta}{n\cdot\beta}[(\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta)\hat{k}_{t+1}\cdot nh_{t+1} + n\tau_{t+1}(1-\alpha)A\hat{k}_{t+1}^{\alpha}h_{t+1} + f_{t+1}\cdot n] \\ & + \hat{k}_{t+2}nh_{t+2} - (1-\delta)\hat{k}_{t+1}h_{t+1} + \frac{1}{D}[h_{t+2}-(1-\theta)h_{t+1}]\cdot n.\end{aligned} \tag{36}$$

Assume that human capital grows at an endogenous rate, so we define the growth rate g_t by $h_{t+1} = g_t h_t$. Dividing Eq.(35) by h_t and Eq.(36) by h_{t+1}, we rewrite the three equilibrium conditions as follows.

$$\frac{1}{D} = \frac{1}{\alpha A\hat{k}_{t+1}^{\alpha-1} + 1 - \delta}[(1-\tau_{t+1})(1-\alpha)A\hat{k}_{t+1}^{\alpha} + \frac{n(1-\theta)}{D}], \tag{37}$$

$$\hat{k}_{t+1}\cdot ng_t = \frac{\beta}{1+\beta}\left[(1-\tau_t)(1-\alpha)A\hat{k}_t^{\alpha} - \frac{n}{D}[g_t-(1-\theta)] - \hat{f}_t\right] - \frac{n\tau_{t+1}(1-\alpha)A\hat{k}_{t+1}^{\alpha}g_t + \hat{f}_{t+1}g_t\cdot n}{(1+\beta)(\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta)}, \tag{38}$$

$$\begin{aligned}A\hat{k}_{t+1}^{\alpha} = {} & \frac{n\cdot\gamma+\beta}{n\cdot\beta}[(\alpha A\hat{k}_{t+1}^{\alpha-1}+1-\delta)\hat{k}_{t+1}\cdot n + n\tau_{t+1}(1-\alpha)A\hat{k}_{t+1}^{\alpha} + \hat{f}_{t+1}\cdot n] + \hat{k}_{t+2}ng_{t+1} \\ & - (1-\delta)\hat{k}_{t+1} + \frac{n}{D}[g_{t+1}-(1-\theta)],\end{aligned} \tag{39}$$

where $\hat{f}_t \equiv \frac{f_t}{H_t}$.

3 Steady state

The steady state (balanced growth path) is characterized by constant state variables: $\hat{k}_t = \hat{k}, \tau_t = \tau, g_t = g, \hat{f}_t = f$. To facilitate calculation and analysis, we make the following assumptions.

Assumption 1 Full capital depreciation: $\delta = 1$.

Assumption 2 $\beta > \frac{n\gamma\alpha(1-\alpha)^{-1}}{1-n\gamma-n\gamma\alpha(1-\alpha)^{-1}}$, guarantees that there is an intersection.

Thus, the three equilibrium conditions become

$$\frac{1}{D} = \frac{1}{\alpha A\hat{k}^{\alpha-1}}[(1-\tau)(1-\alpha)A\hat{k}^{\alpha} + \frac{n(1-\theta)}{D}], \tag{40}$$

$$\hat{k}\cdot ng = \frac{\beta}{1+\beta}\left[(1-\tau)\,(1-\alpha)A\hat{k}^{\alpha} - \frac{n}{D}[g-(1-\theta)] - \hat{f}\right] - \frac{n\tau(1-\alpha)A\hat{k}^{\alpha}g + \hat{f}g\cdot n}{(1+\beta)\alpha A\hat{k}^{\alpha-1}}, \tag{41}$$

$$A\hat{k}^{\alpha} = \frac{n\cdot\gamma+\beta}{n\cdot\beta}[\alpha A\hat{k}^{\alpha-1}\hat{k}\cdot n + n\tau(1-\alpha)A\hat{k}^{\alpha} + \hat{f}\cdot n] + \hat{k}ng + \frac{n}{D}[g-(1-\theta)]. \tag{42}$$

About Eq.(41)

The first relationship between g and $\hat{f}$ from capital market clearing condition. Differentiating with respect to $\hat{f}$, and we get

$$g' = -\frac{\beta\alpha A\hat{k}^{\alpha-1} + gn}{n(1+\beta)\alpha A\hat{k}^{\alpha} + \beta nD^{-1}\alpha A\hat{k}^{\alpha-1} + n\tau(1-\alpha)A\hat{k}^{\alpha} + \hat{f}n} < 0. \tag{43}$$

Thus Eq.(41) is downward sloping with respect to $\hat{f}$.

About Eq.(42)

The second relationship between g and $\hat{f}$ from goods market clearing condition. Differentiating with respect to $\hat{f}$, and we get

$$g' = -\frac{n\cdot\gamma+\beta}{n\cdot\beta(\hat{k}+D^{-1})} < 0. \tag{44}$$

Thus Eq.(42) is a downward sloping straight line with respect to $\hat{f}$.

Check the intersection of the Eq.(41) and (42)

From Eqs.(41) and (42)

$$g = \frac{\beta\alpha A\hat{k}^{\alpha-1}[(1-\tau)(1-\alpha)A\hat{k}^{\alpha} + nD^{-1}(1-\theta) - \hat{f}]}{A\hat{k}^{\alpha}n[\alpha(1+\beta)+\tau(1-\alpha)] + nD^{-1}\beta\alpha A\hat{k}^{\alpha-1} + \hat{f}n}, \tag{41'}$$

$$g = -\frac{(n\gamma+\beta)\beta^{-1}}{\hat{k}n + D^{-1}n}\hat{f} + \frac{A\hat{k}^{\alpha} - (n\gamma+\beta)\beta^{-1}[\alpha A\hat{k}^{\alpha} + \tau(1-\alpha)A\hat{k}^{\alpha}] + D^{-1}n(1-\theta)}{\hat{k}n + D^{-1}n}. \tag{42'}$$

When $\tau = 0$ and $\hat{f} = 0$, horizontal intercept of Eq.(41) and Eq.(42) are

$$\hat{f}_1 = (1-\alpha)A\hat{k}^{\alpha} + D^{-1}n(1-\theta) = \frac{(1-\alpha)A\hat{k}^{\alpha}(n\gamma+\beta)\beta^{-1} + D^{-1}n(1-\theta)(n\gamma+\beta)\beta^{-1}}{(n\gamma+\beta)\beta^{-1}}, \tag{45}$$

$$\hat{f}_2 = \frac{A\hat{k}^{\alpha} - (n\gamma+\beta)\beta^{-1}\alpha A\hat{k}^{\alpha} + D^{-1}n(1-\theta)}{(n\gamma+\beta)\beta^{-1}}. \tag{46}$$

Compare magnitude relationship

$$\hat{f}_1 - \hat{f}_2 = \frac{(n\gamma+\beta)\beta^{-1}[A\hat{k}^{\alpha} + D^{-1}n(1-\theta)] - [A\hat{k}^{\alpha} + D^{-1}n(1-\theta)]}{(n\gamma+\beta)\beta^{-1}} > 0. \tag{47}$$

Hence, $\hat{f}_1 > \hat{f}_2$

When $\tau = 0$ and $\hat{f} = 0$, vertical intercept of Eq.(41) and Eq.(42) are

$$g_1 = \frac{\alpha\beta[(1-\alpha)A\hat{k}^{\alpha} + D^{-1}n(1-\theta)]}{(1+\beta)\alpha\hat{k}n + \alpha\beta nD^{-1}}, \tag{48}$$

$$g_2 = \frac{\alpha\beta[(1-\alpha)A\hat{k}^{\alpha} + D^{-1}n(1-\theta)] - n\gamma\alpha^2 A\hat{k}^{\alpha}}{\alpha\beta\hat{k}n + \alpha\beta D^{-1}n}. \tag{49}$$

Compare magnitude relationship

$$g_2 - g_1 = \frac{\alpha^2 A\hat{k}^{\alpha+1}n[\beta(1-\alpha) - n\gamma\alpha(1+\beta)] + \alpha^2\beta n^2 D^{-1}[\hat{k}(1-\theta) - \gamma\alpha A\hat{k}^{\alpha}]}{(\alpha\beta\hat{k}n + \alpha\beta D^{-1}n)[(1+\beta)\alpha\hat{k}n + \alpha\beta nD^{-1}]}. \tag{50}$$

From Eq.(40)

$$\hat{k}^{\alpha} = \frac{D^{-1}\alpha A\hat{k}^{\alpha-1} - n(1-\theta)D^{-1}}{(1-\tau)(1-\alpha)A}. \tag{40'}$$

When $\tau = 0$

$$\hat{k}^{\alpha} = \frac{D^{-1}\alpha A\hat{k}^{\alpha-1} - n(1-\theta)D^{-1}}{(1-\alpha)A}. \tag{51}$$

Substitute it to Eq.(50)

$$g_2 - g_1 = \frac{\alpha^3 nD^{-1}\alpha A\hat{k}^{\alpha}[\beta - n\gamma\alpha(1+\beta)(1-\alpha)^{-1} - \beta n\gamma] + n^2\alpha^2(1-\theta)D^{-1}\hat{k}n\gamma\alpha(1+\beta)(1-\alpha)^{-1}}{(\alpha\beta\hat{k}n + \alpha\beta D^{-1}n)[(1+\beta)\alpha\hat{k}n + \alpha\beta nD^{-1}]}. \tag{52}$$

If $\beta > \frac{n\gamma\alpha(1-\alpha)^{-1}}{1-n\gamma-n\gamma\alpha(1-\alpha)^{-1}}$, $g_2 > g_1$, so Eqs.(41) and (42) have an intersection. This is the assumption 2 that guarantees the intersection.

Therefore, the figure is as follows.

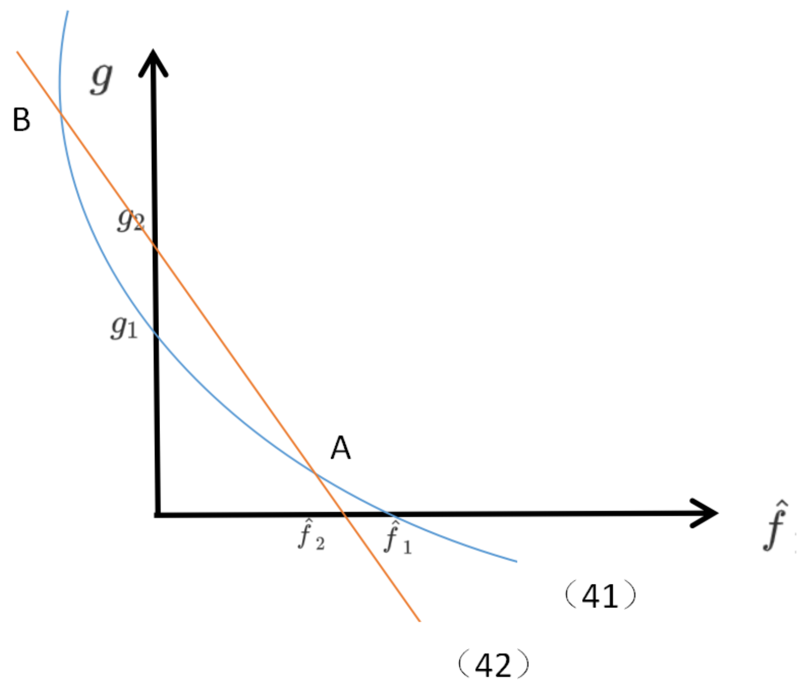

Figure 1: Steady State

Hence, the model has two steady states. One has a higher growth rate with parent-to-child transfer. Another has a lower growth rate with child-to-parent transfer.

4 Discussion

In this section, we first describe some intuition of the result we obtain in the previous section. After that, we discuss the result that the economy without public pension has two balanced growth path, corresponding it to the situation in China[1].

Multiple steady states and self-fulfilling prophecy The intuition of the result that our model economy have two steady states is as follows. First, goods market clearing condition (42) requires that an increasing of child-to-parent transfer $\hat{f}$ brings down human capital growth g. This is because the altruism from parents to children makes their consumption complements[2]. It means that when transfer from children to parents, $\hat{f}$, increases, aggregate consumption increases. The increase of aggregate consumption crowds out physical capital investment and education expenditure in goods market[3], so that growth rate of human capital, g, falls. Second, capital market clearing condition (equivalently, saving function) (41) requires, as well as (42), that an increasing of child-to-parent transfer $\hat{f}$ brings down human capital growth g. The increasing of $\hat{f}$ causes a less disposable income of young individuals, and it lowers saving and education expenditure. Furthermore, The increasing of $\hat{f}$ causes a more disposable income of old, it implies that young individuals decreases saving and education expenditure. These two negative relationships between $\hat{f}$ and g allows multiple steady state. Suppose that the economy is on the low-growth steady state with child-to-parent transfer ($\hat{f} > 0$, and now each individual expects that other families reverse their strategy of intrafamily transfer, that is, they expect $\hat{f} < 0$. This reverse of expectation about intrafamily transfer increases their saving and education expenditure (Eq. (41)), so that they expect that growth rate rises. The expectation about higher growth rate forms their expectation that $\hat{f}$ decreases (eq. (42)). Hence, the expectation that $\hat{f}$ may be confirmed by equilibrium conditions (41) and (42). It implies that the reverse of expectation about $\hat{f}$ may be self-fulfilling.

One of our original results is that intrafamily transfer causes the multiple steady

[1]As written in Introduction, public pension benefit is at a very low level and many individuals do not participate in public pension, so that our model economy with $\tau = 0$ can be thought of as China economy.

[2]From Eqs. (5) and (19), $c_{y,t+1} = \frac{\gamma}{\beta} c_{o,t+1}$.

[3]Physical-human capital ratio $\hat{k}$ is constant, given τ. See Eq. (34). Hence, physical capital investment and education expenditure are positively correlated.

states. Previous macroeconomic research on intrafamily transfer do not the possibility of multiple steady state. Coeurdacier et al. (2014) consider an OLG economy with intrafamily transfer to analyze China's social security reform. However, in their model the amount of intrafamily transfer is determined by an ad hoc function, so that it does not depend on their consumption level. As a result of that, in their model consumption of young and that of old do not comove, hence, there is not multiple steady states. Sinn (2004) uses the model with human capital investment and intrafamily transfer, which is very close to ours. However, Sinn (2004) considers a two-period game between a parent and children, not an infinite-horizon dynamic general equilibrium model, hence he do not focus on any steady states.

Implications to macroeconomy in China Next, we discuss the result that the economy without public pension has two balanced growth path, corresponding it to the situation in China.

Point A in Figure 1 represents a steady state at which parents provide education to their children ($e > 0$) and children transfer goods to their parents ($f > 0$). This long-run equilibrium corresponds to equilibrium under *the private enforcement rules of the traditional family*[4]. It represents the current situation in China well. Particularly, we can see this situation in a lot of rural and poor areas in China. On the other hand, Point B in Figure 1 is also a long-run equilibrium. In this steady state, parents not only provide education ($e > 0$) but also transfer goods to their children ($f < 0$). We refer to this long-run equilibrium as *the 'rotten-kids' equilibrium*[5].

In the literature of intrafamily transfer (private pension) and public pension, the 'rotten kids' problem, i.e., the neglect or mistreatment of old people by their children, has been thought of as a result of the loosing of family ties after modern industrialization. Sinn (2004) represents the 'rotten kids' problem by assuming that parents (or the legal system) cannot enforce the transfers from their children. This assumption means that adding the constraint that $f_t \leq 0$ to individual's utility maximization problem. In our model, however, the 'rotten kids' problem arises in the form of a self-fulfilling steady state, without the lack of enforcement. Therefore, in our model, differ from in

[4]This terminology is named after the description at Proposition 5 in Sinn (2004)

[5]This terminology is named after Becker (1974, 1976, 1981). The rotten kids problem is a problem whether ungrateful children cause an ineffeciency to their family or whole economy. See, for example, Bergstrom (1989), Cornes and Silva (1999), Kolpin (2006).

Sinn (2004), the 'rotten kids' are a result of parents' voluntary choice.

Sinn (2004) shows that in the lack-of-enforcement model, PAYGO public pension alleviates underinvest problem by 'rotten kids'. However, welfare effects of pension would depend on whether the 'rotten kids' arises by the lack of enforcement or by parents' voluntary choice. Hence, in order to consider the China's social security system, it is needed that research on welfare effects of pension in OLG models with intrafamily transfer and compare the one in the lack-of-endorcement model. Even from a positive perspective, some results in our model differ from in Sinn (2004). In Sinn (2004)'s model, equilibrium under the lack of enforcement is underinvested for human capital. On the other hand, in our model human capital growth rate in the 'rotten-kids' equilibrium is higher than that in the equilibrium under the private enforcement rules of the traditional family. It implies that it is uncertain whether the Sinn (2004)'s normative result – PAYGO system alleviates the 'rotten kids' problem – holds in dynamic general equilibrium models.

5 Conclusion

In this paper, we consider an overlapping generations model with intrafamily transfer and human capital. We show that this economy has two steady states (balanced growth path). One is the equilibrium under the private enforcement rules of the family, in which goods are transferred from children to parents and human capital investment is relatively low. Another is the 'rotten-kids' equilibrium, in which goods are transferred from parents to children and human capital investment is relatively high. Accordingly, (i) 'rotten-kids' equilibrium may realize by self-fulfilling expectation, (ii) The realization of 'rotten-kids' equilibrium causes higher growth, (iii) 'rotten-kids' may not detriment economic welfare, and (iv) The introduction of public pension may not be a solution to the 'rotten-kids' problem.

With regard to China, in the near future intrafamily transfers from children to parents might decrease by loosing of family ties. Public pension system is thought of as a solution of this problem in this literature. However, in this paper we present a model against that conventional wisdom. Whether public pension system can alleviate the 'rotten-kids' problem (underinvest problem caused by the lack of enforcement for children) depends on welfare effect of public pension at the 'rotten-kids' equilibrium in general equilibrium models. This task is left to future research.

References

[1] Coeurdacier, N., Guibaud, S. and Jin, K. (2014) " Fertility Policies and Social Security Reforms in China", *IMF Economic Review*, 62, 371–408.

[2] Herd, R., Hu, H. and Koen, V. (2010) " Providing greater old-age security in China", *OECD Economics Department Working Papers*, No. 750, OECD Publishing.

[3] Becker, G.S. (2009) " Human Capital: a Theoretical and Empirical Analysis, With Special Reference to Education", *University of Chicago press.*

[4] Becker, G. S. (1974) "The Theory of Social Interactions", *Journal of Political Economy*, 82(6), 1063–93.

[5] Becker, G. S. (1976) "Altruism, Egoism, and Genetic Fitness: Economics and Sociobiology", *Journal of Economic Literature*, 14(3), 817–26.

[6] Becker, G. S. (1981) "A Treatise on the Family", *Harvard University Press.*

[7] Bergstrom, T. C. (1989) "A Fresh Look at the Rotten Kid Theorem—and Other Household Mysteries", *Journal of Political Economy*, 97, 1138–59.

[8] Cornes, R. C., and E. C. D. Silva (1989) "Rotten Kids, Purity, and Perfection", *Journal of Political Economy*, 97, 1138–59.

[9] Kolpin, V. (2006) "The modeling and analysis of rotten kids", *Social Choice and Welfare,* 26, 23–30.

[10] Sinn. HW. (2004) " The Pay-As-You-Go Pension System as a Fertility Insurance and Enforcement Device", *Journal of Public Economics*, v88(7-8,Jul), 1335-1357.

[11] Zhao, Q., Mi, H. (2019) " Evaluation on the Sustainability of Urban Public Pension System in China", *Sustainability 11*, no. 5: 1418.

[12] Zhu, X., Whalley, J. and Zhao, X. (2014) " Intergenerational transfer, human capital and long-term growth in China under the one child policy", *Economic Modelling*, Volume 40, 275-283.