Formation control of non-identical multi-agent systems

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Article Info

ABSTRACT

The problem considered in this work is formation control for non-identical linear multi-agent systems (MASs) under a time-varying communication network. The size of the formation is scalable via a scaling factor determined by a leader agent. Past works on scalable formation are limited to identical agents under a fixed communication network. In addition, the formation scaling variable is updated under a leader-follower network. Differently, this work considers a leaderless undirected network in addition to a leader-follower network to update the formation scaling variable. The control law to achieve scalable formation is based on the internal model principle and consensus algorithm. A biased reference output, updated in a distributed manner, is introduced such that each agent tracks a different reference output. Numerical examples show the effectiveness of the proposed method.

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1. INTRODUCTION

Cooperative control of multi-agent systems (MASs) has received much attention from the research community in recent years. One active area in MASs is achieving state or output consensus among agents [1]–[4], and others, wireless sensor networks [5]–[7] and others, and task scheduling [8], [9], and others. One extension of consensus is multi-agent formation. In the literature of multi-agent formation, the formation is produced by introducing a bias to the consensus control law of each agent, see for example [10]–[15]. The bias is introduced so that the states or outputs of agents differ by the bias amount. Another formation control problem is by considering obstacle or collision avoidance [16], [17] and multi-robot navigation [18]–[22]. However, in the approaches mentioned above, the formation size cannot be controlled or changed during the operation of MAS. The environment where MAS operates may change over time, for example, MAS may encounter a path narrower than its current formation size. In this situation, the ability to adjust the formation size is essential for MAS in continuing its operation.

Past works on formation control with formation size adjustment (scalable formation problem) are sparse. The works Coogan and Arcak [23] and Coogan *et al.* [24] solve the scalable formation problem of double integrator agents by introducing an auxiliary state that acts as a multiplier to the bias in the standard formation control law. In one of the methods discussed by [24], the auxiliary state is updated by using a consensus algorithm. Whereas in [23], the auxiliary state is updated by estimating the desired formation scaling factor (known only by leader agent (s)) by monitoring the relative position of agents' neighbor. In [23], both the single link and multi-link method in monitoring the neighbors' position is discussed, while in

[24], only the single link method is discussed. When the auxiliary state reaches consensus, all agents have similar multiplier to their formation parameter. Thus, the whole group reach a formation where its size is a multiplication to the consensus value of the auxiliary state. In [24], the leader-follower network is used so that the consensus value of the auxiliary state is dictated by the initial value of the leader agent's auxiliary state. The work [25] discusses scalable formation for single integrator and double integrator agents by using a self-loop communication weight. The formation parameter is embedded in the self-loop communication weight. However, the communication network is considered to be fixed.

Scalable formation control considering identical linear system agents is discussed in [26]. In [26], a consensus-based scalable formation algorithm is used as a dynamic compensator. Similar to [24], the work [26] also uses auxiliary state as the multiplier to the formation parameter. In addition to the scalable formation, the work [26] allow for orientation adjustment. This additional adjustment is achieved by adding a variable multiplied to the desired position in the dynamic compensator. The works [23]–[26] consider fixed communication networks and identical agents in their formulation. Moreover, the network used to update the auxiliary state (formation scaling factor variable) is a leader-follower network. Scalable formation for heterogeneous agents under time-varying network can be found in [27]. The work considers continuous time agent model and leader-follower network.

Unlike the works [23]–[26] that consider scalable formation for identical agents under fixed network, this work considers scalable formation for non-identical agents under a time-varying communication network. Moreover, unlike the work [27], this work considers leaderless undirected network in addition to leader-follower network in updating the formation scaling variable. Non-identical agents' consideration is important in the formulation of MAS since not all agents are made similar; and also, non-identical agents formulation allows for different agents to cooperatively work under similar network (for example, drone and ground robot). Time-varying communication consideration is important since the communication network among agents may be changing during the operation due to weather, packet drop, and signal loss. The time-varying network formulation is important to guarantee the MAS algorithm to work under these situations (under some common network assumption, for example jointly connected network).

Leaderless network formulation is needed on a situation where leader-follower network is not suitable for MAS operation. An example is when the formation leader (the agent that determine the formation size) must be changed during the operation. In scalable formation under leader-follower network, the leader agent is the formation leader, and the leader agent does not receive information from other agents. Suppose agent q is the leader agent in the network. When the formation leader is changed to another agent; since there is no information coming to agent q, then the whole agents cannot reach consensus (spanning tree condition is violated). Formation leader change during the operation is needed when the formation leader experiences faulty which makes it unable to continue its purpose.

The control input in this work is based on the internal model principal approach described by [3]. The internal model principle approach is to devise a reference output which will be tracked by the output of each agent. A bias is then introduced in the reference output so that agents track different output and hence reach a formation. Moreover, a virtual state is multiplied to the reference output for the purpose of resizing the formation. Two network cases in updating the virtual state, leader-follower and leaderless undirected (both are time-varying networks), are discussed in this work. In achieving scalable formation for leaderless undirected network, the result from [28] is used to estimate the consensus time duration. This time duration is needed by the formation leader to know when it can change the formation size into the new one. It is shown that the consensus time duration is a function of the desired error bounds between the desired and actual formation scaling factor. The proposed work is expected to contribute in the application of search and rescue [29]–[32], area surveillance [33], [34], and others.

Non-negative and positive integer sets are indicated by $\mathbb{Z}_0^+$, and $\mathbb{Z}^+$ respectively. Let $M, L \in \mathbb{Z}^+$ with $M > L$. Then $\mathbb{Z}_0^M := \{0,1,2,\cdots,M\}$ and $\mathbb{Z}_L^M := \{L, L + 1, \cdots, M\}$. Meanwhile, $\mathbb{R}, \mathbb{R}^n, \mathbb{R}^{n \times m}$ refer respectively to the sets of real numbers, n-dimensional real vectors and n by m real matrices. I_n is the $n \times n$ identity matrix with 1_n being the n-column vector of all ones (subscript omitted when the dimension is clear). Given a set C, $|C|$ denotes its cardinality. The transpose of matrix M and vector v are indicated by M' and v', respectively. Additional notations are introduced when required in the text.

2. RESEARCH METHOD

In this section, the problem will be formally presented, and the proposed distributed algorithm will be discussed. The proposed distributed algorithm is based on the internal model principle which has the advantage to be used when the communication network is time-varying, and the agent's model are non-identical. Since the proposed algorithm involves the use of graph, a brief discussion on the graph notation is

discussed in the beginning of this section. The problem definition and the proposed algorithm are presented subsequently.

2.1. Graph notation

The network of N nodes described by a time-varying graph is $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ with vertex set $\mathcal{V} = \{1, 2, \cdots, N\}$ and edge set $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$. When the graph sequence $\mathcal{G}(t)$ is a directed graph, the pair $(j, i) \in \mathcal{E}(t)$ if node j points towards node i at time t. On the other hand, when the graph sequence $\mathcal{G}(t)$ is undirected, $(j, i) \in \mathcal{E}(t)$ if node j is adjacent to node i at time t and vice versa. The set of neighbors of node i at time t is $\mathcal{N}_i(t) := \{j \in \mathcal{V} : (j, i) \in \mathcal{E}(t), i \neq j\}$. We define $d_i(t) = |\mathcal{N}_i(t)|$ to be the number of neighbors of agent i at time t.

Remark 1. The largest possible $d_i(t)$ for all $i \in \mathbb{Z}^N$, $t \geq 0$, is $N - 1$.

The union of the graph sequence $\mathcal{G}(t)$ at the time interval $[t_a, t_b]$ is defined as $\mathcal{G}_{t_a}^{t_b} = (\mathcal{V}, \cup_{t_a}^{t_b} \mathcal{E}(t))$.

The adjacency matrix $\mathcal{A}(t) = [a_{ij}(t)]$ of $\mathcal{G}(t)$ is the $N \times N$ matrix whose (i, j) entry is 1 if $(j, i) \in \mathcal{E}(t)$, and 0 otherwise. When the graph $\mathcal{G}(t)$ is undirected, then $a_{ij}(t) = a_{ji}(t)$ for all $i, j \in \mathcal{V}$. The standard Perron matrix $P(t) = [p_{ij}(t)]$ associated with to graph $\mathcal{G}(t)$ is defined by

$$p_{ij}(t) = \begin{cases} 1 - \gamma \cdot d_i(t) & i = j, \\ \gamma \cdot a_{ij}(t) & i \neq j, \end{cases}$$

where $\gamma = \frac{1}{N}$, and this imply that the off-diagonal term of $P(t)$ is either $\frac{1}{N}$ or 0. By the above construction, it is easy to see that $P(t)$ is a nonnegative matrix and its row sum equals one, implying that $P(t)$ is a stochastic matrix, and also its diagonal entries are positive.

2.2. Problem formulation

Consider N non-identical agents where each agent i is given by the discrete-time model (1) and (2):

$$x_i(t + 1) = A_i x_i(t) + B_i u_i(t), \ i \in \mathbb{Z}^N \tag{1}$$

$$y_i(t) = C_i x_i(t), \ i \in \mathbb{Z}^N \tag{2}$$

where $x_i(\cdot) \in \mathbb{R}^{n_i}$, $y_i(\cdot) \in \mathbb{R}^p$ and $u_i(\cdot) \in \mathbb{R}^{m_i}$ are the state, output, and control signal of agent i. Meanwhile, A_i, B_i, C_i are the state matrix, input matrix, and output matrix of agent i, respectively. The variables $n_i \in \mathbb{Z}^+, p \in \mathbb{Z}^+, m_i \in \mathbb{Z}^+$ are the vector's dimension of the state x_i, output y_i, and control input u_i. By non-identical agents, it means that A_i, B_i, C_i are not necessarily similar to A_j, B_j, C_j for $i, j \in \mathbb{Z}^N$. Also note that the output dimension for all agents is the same and this is necessary for agents to reach output-formation. The assumptions on the systems' matrices are:

Assumption (A1). The pair (A_i, B_i) and (A_i, C_i) are stabilizable and detectable for all $i \in \mathbb{Z}^N$.

This work aims to coordinate the N agents in a distributed manner to form a scalable formation via a scaling factor (scalable formation). Let $\delta_i \in \mathbb{R}^p$ and $\delta_j \in \mathbb{R}^p$ be the desired outputs of agents i and j defined on a local coordinate system, then the scalable formation objective is such that:

$$\lim_{t \to \infty}(y_i(t) - y_j(t)) = \alpha(\delta_i - \delta_j), \qquad \forall \, i, j \in \mathbb{Z}^N,$$

where $\alpha \in \mathbb{R}^+$ is the desired scaling factor of the formation that is determined by a leader agent (formation leader).

The communication network among agents is represented by a time-varying graph $\mathcal{G}(t)$ with two cases being considered:
- Case leader-follower (LF). $\mathcal{G}(t)$ is a directed graph with $\kappa \in \mathbb{Z}^N$ as the leader node. This implies that $a_{\kappa j}(t) = 0$ for all $t \geq 0$ and $j \in \mathbb{Z}^N$.
- Case leaderless undirected (LLU). $\mathcal{G}(t)$ is an undirected graph.

Remark 2. Let $e_\kappa \in \mathbb{R}^N$ be a vector of zeros except its κth entry being 1. The consequences of Case LF is that the κth row of $P(t)$ is e'_κ for all $t \geq 0$. Thus, $e'_\kappa P(t) = e'_\kappa$ holds for all $t \geq 0$. This implies that e'_κ is the left eigenvector of $P(t)$ corresponding to its eigenvalue of 1 for all $t \geq 0$.

Remark 3. In case LLU, $P(t)$ is a doubly stochastic matrix. Therefore, $1'_N$ is the left eigenvector of $P(t)$ corresponding to its eigenvalue of 1 for all $t \geq 0$, i.e., $1'_N P(t) = 1'_N$ for all $t \geq 0$.

2.3. Proposed control input for case LF

The proposed control input to the ith agent (1)-(2) is based on the internal model principal approach discussed in [3]:

$$u_i(t) = K_i \hat{x}_i(t) + L_{\lambda_i} \lambda_i(t) + L_{w_i} w_i(t), \quad i \in \mathbb{Z}^N \tag{3}$$

where $K_i \in \mathbb{R}^{m_i \times n_i}$ is a feedback matrix designed such that $(A_i + B_i K_i)$ is Schur for all $i \in \mathbb{Z}^N$, while $L_{\lambda_i} \in \mathbb{R}^{m_i}$ and $L_{w_i} \in \mathbb{R}^{m_i \times \bar{n}}$ are the feedback vector and feedback matrix from the internal model principle. The state $\hat{x}_i \in \mathbb{R}^{n_i}$ in (3) is the estimate of x_i obtained from

$$\hat{x}_i(t + 1) = A_i \hat{x}_i(t) + B_i u_i(t) + H_i\big(\hat{y}_i(t) - y_i(t)\big), \quad i \in \mathbb{Z}^N \tag{4}$$

$$\hat{y}_i(t) = C_i \hat{x}_i(t), i \in \mathbb{Z}^N \tag{5}$$

where $\hat{y}_i$ is the estimated output, and $H_i \in \mathbb{R}^{n_i \times p}$ is feedback matrix designed such that $(A_i + H_i C_i)$ is Schur for all $i \in \mathbb{Z}^N$. The variables $\lambda_i \in \mathbb{R}$ and $w_i \in \mathbb{R}^{\bar{n}}$ in (3) are the states of reference generator, with λ_i as the formation scaling factor variable of agent i and w_i as the common trajectory among all agents. The dynamics of λ_i and w_i are

$$\lambda_i(t + 1) = \sum_{j=1}^{N} p_{ij}(t)\lambda_j(t), \quad i \in \mathbb{Z}^N \tag{6}$$

$$w_i(t + 1) = S \sum_{j=1}^{N} p_{ij}(t)w_j(t), \quad i \in \mathbb{Z}^N \tag{7}$$

In (6)-(7), $p_{ij}(t)$ is the entry of the Perron matrix $P(t)$. In (7), S is the state matrix of the reference generator state w_i with the following assumption made on S:
Assumption (A2). Eigenvalues of S lie on the unit circle.
Meanwhile, the assumption on graph $\mathcal{G}(t)$ for case LF is.
Assumption (A3). The graph $\mathcal{G}(t)$ is uniformly connected.
The graph sequence $\mathcal{G}(t)$ is said to be uniformly connected if there exist a finite time horizon $T > 0$ such that for all t, the graph $\mathcal{G}_t^{t+T}$ contains a spanning tree, or that there exist a directed path from at least one node to every other node. (A3) is a standard assumption on consensus problem under directed time-varying graph (see for example [35] and [1]).

To achieve scalable formation, the output of agent i will be made to track its reference-output defined in (8):

$$y_{i,ref}(t) = \delta_i \lambda_i(t) + Q w_i(t) \tag{8}$$

where $\delta_i \in \mathbb{R}^p$ ($\delta_i \neq \delta_j$; $i, j \in \mathbb{Z}^N$) is the desired output of agent i defined on the local coordinate system and Q is the output matrix of w_i. Refer to [35] and [1], under (A3), the N agents (6)-(7) reach consensus exponentially, or that $\lambda_i(t) \to \lambda_\infty$ and $w_i(t) \to \bar{w}(t)$ exponentially for all $i \in \mathbb{Z}^N$, where $\bar{w}(t)$ is a solution to $w_0(t + 1) = S w_0(t)$ for some $w_0 \in \mathbb{R}^{\bar{n}}$. Since $\lambda_i(t) \to \lambda_\infty$ and $w_i(t) \to \bar{w}(t)$ for all $i \in \mathbb{Z}^N$, then $y_{i,,ref}(t) \to \delta_i \lambda_\infty + Q\bar{w}(t)$ for all $i \in \mathbb{Z}^N$. Thus, we can see that asymptotically $y_{i,ref}(t)$ contains two components. The first component, $\delta_i \lambda_\infty$, is the formation component scalable by λ_∞ and the second component, $Q\bar{w}(t)$, is the common (consensus) trajectory component.

If each y_i tracks $y_{i,ref}$ asymptotically, which will be shown shortly, then asymptotically the outputs of agents i and j, $i, j \in \mathbb{Z}^N$ will differ by $\lambda_\infty(\delta_i - \delta_j)$, i.e. $\lim_{t \to \infty}(y_i(t) - y_j(t)) = \lambda_\infty(\delta_i - \delta_j)$ for all $i, j \in \mathbb{Z}^N$. This implies that λ_∞ is the formation scaling factor. The following lemma shows that for Case LF, suppose node κ is the leader node, then agent κ is also the leader agent that can determine λ_∞ (formation leader):
Lemma 1. Let node $\kappa \in \mathbb{Z}^N$ be the leader node in the leader-follower graph $\mathcal{G}(t)$. Given system of N agents (6). Suppose (A3) holds. Then $\lambda_i(t) \to \lambda_\kappa(t_0)$ for all $i \in \mathbb{Z}^N$, where $\lambda_\kappa(t_0)$ is the initial condition of λ_κ and t_0 is a moving initial time (t_0 is not necessarily equals zero). Lemma 1 is a special case of the discrete-time case in [2]. The proof of Lemma 1 can be shown by taking results from [2] and utilizing Remark 2. Thus, the proof is omitted.

By noting that asymptotically $y_{i,ref}(t) = \delta_i \lambda_\infty + Q\bar{w}(t)$, where λ_∞ is a constant and $\bar{w}(t)$ is a solution to the dynamics $w_0(t + 1) = S w_0(t)$, to ensure each agent tracks their own reference-output, the feedback vector L_{λ_i} and feedback matrix L_{w_i} are designed via the well-known internal model principle [36].

L_{λ_i} in (3) is feedback vector computed by $L_{\lambda_i} = \Gamma_{\lambda_i} - K_i \Pi_{\lambda_i}$, where Γ_{λ_i} and Π_{λ_i} are the solutions to the following Francis equations:

$$A_i \Pi_{\lambda_i} + B_i \Gamma_{\lambda_i} = \Pi_{\lambda_i} \tag{9}$$

$$C_i \Pi_{\lambda_i} = \delta_i \tag{10}$$

On the other hand, L_{w_i} is feedback matrix computed by $L_{w_i} = \Gamma_{w_i} - K_i \Pi_{w_i}$, where Γ_{w_i} and Π_{w_i} are the solutions to the following Francis equations:

$$A_i \Pi_{w_i} + B_i \Gamma_{w_i} = \Pi_{w_i} S \tag{11}$$

$$C_i \Pi_{w_i} = Q \tag{12}$$

To guarantee solvability of (9)-(10) and (11)-(12) for any δ_i and Q, the following assumption is needed [36]:
Assumption (A4). The matrices:

$$\begin{bmatrix} A_i - I_{n_i} & B_i \\ C_i & 0 \end{bmatrix} \text{ and } \begin{bmatrix} A_i - \xi I_{n_i} & B_i \\ C_i & 0 \end{bmatrix}$$

are full-row rank for all $i \in \mathbb{Z}^N$ and for every eigenvalue ξ of S.

2.4. Proposed control input for case LLU

The case LLU is similar to Case LF. The controller, observer, dynamics of λ_i and w_i are similarly given by (3), (4)-(5), (6)-(7). All assumptions are also needed except (A3). Instead of (A3), the assumption on the communication graph is
Assumption (A5). The graph $\mathcal{G}(t)$ is uniformly strongly connected.
The graph sequence $\mathcal{G}(t)$ is said to be uniformly strongly connected if there exist a finite time horizon $T > 0$ such that for all t, the graph $\mathcal{G}_t^{t+T}$ is strongly connected, or that there exist a directed path between any two nodes. Since an undirected graph is bidirectional, an undirected graph that contains a spanning tree has the properties that any two nodes in the graph is connected by a path, which means that the graph is strongly connected. Therefore, (A5) is used here instead of (A3).

We will now discuss the method so that a designated leader agent is able to determine the formation scaling factor. Since case LLU is similar to case LF, except of the network topology, it can be shown that the consensus value of λ is also the formation scaling factor. We first state the following result which is the undirected network version of Lemma 1:

Lemma 2. Given system of N agents (6) where the communication graph $\mathcal{G}(t)$ is undirected. Suppose *(A5)* holds, then $\lambda_i(t) \to \frac{1}{N}\sum_{j=1}^{N} \lambda_j(t_0)$ for all $i \in \mathbb{Z}^N$. The proof of Lemma 2 can be shown by taking results from [2] and noting Remark 3. Thus, the proof is omitted.

By Lemma 2, for case LLU, the formation scaling factor is the average of the initial condition of λ. To make the consensus value of λ equal to the desired formation scaling factor, then the leader agent must know the initial states of λ_i for all i which is too demanding. Our approach is to have the leader agent to wait until λ reach consensus with some reasonable tolerance, and then the leader agent uses its own λ as the estimate of the λ of other agents. In other words, the problem is to compute the waiting time, $\bar{t}$, before the leader agent can change its λ to a new value such that the error bound between the desired formation scaling factor, λ_d, and the actual formation scaling factor, λ_∞, is known. We shall assume here that although the leader agent updates its λ using (6), the leader agent is authorized to reset its λ to any real number at any time.

Let agent $\kappa \in \mathbb{Z}^N$ be the designated leader agent. We start the analysis by first defining the choice of the reset value of λ_κ or $\lambda_\kappa(t_{0_{new}})$, followed by the analysis of the error bound between λ_d and λ_∞, and the computation of $\bar{t}$. Suppose $\nu > 0$ is chosen where it is desired that $|\lambda_\infty - \lambda_d| \leq \nu$. Suppose further at t^*, $|\lambda_i(t^*) - \lambda_j(t^*)| \leq \nu$ for all $i, j \in \mathbb{Z}^N$, where $t^* = \bar{t} + t_{0_{old}}$, where $t_{0_{old}}$ is the previous value of t_0. Let agent κ reset its λ at $t = t^*$ to

$$\lambda_\kappa^* = N\lambda_d - (N-1)\lambda_\kappa(t^*) \tag{13}$$

where $\lambda_\kappa(t^*)$ is computed from (6). The following lemma shows that $|\lambda_\infty - \lambda_d| \leq \nu$ holds:
Lemma 3. Consider system of N agents (6). Let agent $\kappa \in \mathbb{Z}^N$ reset its λ at $t = t^*$ by (13) and suppose $|\lambda_i(t^*) - \lambda_j(t^*)| \leq \nu$ holds for all $i, j \in \mathbb{Z}^N$, then $|\lambda_\infty - \lambda_d| \leq \nu$ holds.

Proof from Lemma 2, the steady state valu*e of* λ is the average of its the initial states. Let the new t_0, $t_{0_{new}} = t^*$, we can write the following for λ_i, $i \in \mathbb{Z}^N$:

$$\text{Lim}_{t \to \infty} \lambda_i(t) = \frac{1}{N} \sum_{j=1}^{N} \lambda_j(t_{0_{new}}) = \frac{1}{N} \left(\sum_{j=1; j \neq \kappa}^{N} \lambda_j(t_{0_{new}} + \lambda_\kappa^*) \right)$$

by replacing λ_κ^* using (13), we obtain

$$\begin{aligned}
\text{Lim}_{t \to \infty} \lambda_i(t) &= \frac{1}{N} \left(\sum_{j=1; j \neq \kappa}^{N} \lambda_j(t^*) + N\lambda_d - (N-1)\lambda_\kappa(t^*) \right) \\
&= \frac{1}{N} \left(N\lambda_d + \sum_{j=1; j \neq \kappa}^{N} \left(\lambda_j(t^*) - \lambda_\kappa(t^*) \right) \right) \\
&= \lambda_d + \frac{1}{N} \sum_{j=1; j \neq \kappa}^{N} \rho_j = \lambda_\infty,
\end{aligned} \quad (14)$$

where $\rho_j = \lambda_j(t^*) - \lambda_\kappa(t^*)$. Since $|\lambda_\kappa(t^*) - \lambda_j(t^*)| \leq v$ for all $j, j \neq \kappa$, then $-v \leq \rho_j \leq v$. Thus, (14) can be written as:

$$\begin{aligned}
& \lambda_d - \frac{N-1}{N} v \leq \lambda_\infty \leq \lambda_d + \frac{N-1}{N} v \\
& \Rightarrow \lambda_d - v \leq \lambda_\infty \leq \lambda_d + v \\
& \Rightarrow |\lambda_\infty - \lambda_d| \leq v.
\end{aligned}$$

Then, the next problem is to compute the value of $\bar{t}$ such that at $t^* = \bar{t} + t_{0_{old}}$, $|\lambda_i(t^*) - \lambda_j(t^*)| \leq v$ holds. There are two steps that will be involved to compute $\bar{t}$. For the first step, we first write the solution of (6) for all agents as :

$$\lambda(t+1) = P(t) \cdots P(t_0)\lambda(t_0) = P(t, t_0)\lambda(t_0)$$

Let $T \in \mathbb{Z}$ be the bounded communication interval where the union of graph sequence $\mathcal{G}(t)$ is strongly connected at least every T time steps. The following lemma [28] shows that, under (A5), the difference between $P(t, t_0)$ and $1_N q(t_0)$ decays geometrically where $q(t_0) = [q_1(t_0), \cdots, q_N(t_0)]' \in \mathbb{R}^N$ is a stochastic vector.

Lemma 4. [28] Consider the product of the stochastic matrices $P(t, t_0)$. Let (A5) be satisfied, then for each $t_0 \geq 0$ there is a stochastic vector $q(t_0)$ such that for all $i, j \in \mathbb{Z}^N$ and $t \geq t_0$.

$$\left| [P(t, t_0)]_{ij} - q_j(t_0) \right| \leq 2 \left(\left(1 - \frac{1}{N^{NT}} \right)^{\frac{1}{T}} \right)^{t - t_0}$$

The reader is referred to [28] for the proof of Lemma 4. In view of Lemma 2, we know that $\lim_{t \to \infty} P(t, t_0) = \frac{1}{N} 1_N 1_N'$. Applying this to Lemma 4 and using $t_{0_{old}}$ as the initial time, under (A5), we get $\left| [P(t^*, t_{0_{old}})]_{ij} - \frac{1}{N} \right| \leq \mu$, with

$$\mu = 2 \left(\left(1 - \frac{1}{N^{NT}} \right)^{\frac{1}{T}} \right)^{\bar{t}} \quad (15)$$

where $\bar{t} = t^* - t_{0_{old}}$. If μ is known, then $\bar{t}$ can be computed using (15). The second step is to derive $\bar{t}$ as a function of v where $|\lambda_i(t^*) - \lambda_j(t^*)| \leq v$ for all $i, j \in \mathbb{Z}^N$, which will be given next.

Let $\left| [P(t^*, t_{0_{old}})]_{ij} - \frac{1}{N} \right| \leq \mu$, for all $i, j \in \mathbb{Z}^N$ then $\left| [P(t^*, t_{0_{old}})]_{ij} - [P(t^*, t_{0_{old}})]_{\ell j} \right| \leq 2\mu$ for all $i, j, \ell \in \mathbb{Z}^N$. Since each $\lambda_i(t^*) = \sum_{j=1}^{N} [P(t^*, t_{0_{old}})]_{ij} \lambda_j(t_{0_{old}})$ and $\lambda_\ell(t^*) = \sum_{j=1}^{N} [P(t^*, t_{0_{old}})]_{\ell j} \lambda_j(t_{0_{old}})$, $i, \ell \in \mathbb{Z}^N$, we can write $\lambda_i(t^*) - \lambda_k(t^*) = \sum_{j=1}^{N} \left([P(t^*, t_{0_{old}})]_{ij} - [P(t^*, t_{0_{old}})]_{kj} \right) \lambda_j(t_{0_{old}})$, thus

$$\left|\lambda_i(t^*) - \lambda_\ell(t^*)\right| \le \sum_j^N \left|\left(\left[P(t^*, t_{0_\text{old}})\right]_{ij} - \left[P(t^*, t_{0_\text{old}})\right]_{\ell j}\right)\lambda_j(t_{0_\text{old}})\right|$$

$$= \sum_{j=1}^N \left|\left(\left[P(t^*, t_{0_\text{old}})\right]_{ij} - \left[P(t^*, t_{0_\text{old}})\right]_{\ell j}\right)\right|\left|\lambda_j(t_{0_\text{old}})\right|$$

$$\le \sum_{j=1}^N 2\mu\left|\lambda_j(t_{0_\text{old}})\right| = 2\mu\left\|\lambda(t_{0_\text{old}})\right\|_1 = \nu$$

for all $i, \ell \in \mathbb{Z}^N$. Substituting (15) to the above result, we have

$$\bar{t} = \frac{\ln\left(\frac{\nu}{4\left\|\lambda(t_{0_\text{old}})\right\|_1}\right)}{\ln\left(\left(1 - \frac{1}{N^{NT}}\right)^{\frac{1}{T}}\right)} \tag{16}$$

Remark 5. The value $\left\|\lambda(t_{0_\text{old}})\right\|_1$ for $t_{0_{old}} = 0$ can be estimated by defining the ball that contains the possible initial value of λ_i for all i. Whereas for the subsequent $t_{0_{old}} > 0$, $\left\|\lambda(t_{0_\text{old}})\right\|_1$ can be estimated by utilizing the previous value of ν and λ_d.

3. RESULTS AND DISCUSSION
3.1. Main theorem and discussion for case LF
By having all necessary results established in section 2.3, we can present the main result of case LF:
Theorem 1: Given system of N agents (1)-(2) with control input $u_i(t)$ given by (3), observer dynamics given by (4)-(5), reference generator dynamics given by (6)-(7) where the communication graph $\mathcal{G}(t)$ is directed leader-follower with $\kappa \in \mathbb{Z}^N$ as the leader node, and reference-output $y_{i,ref}(t)$ given by (8). Let λ_d be the desired scaling factor of the formation and setting $\lambda_\kappa(t_0) = \lambda_d$. Suppose (A1)-(A4) are satisfied, then:
a. $y_i(t) \to y_{i,ref}(t)$ exponentially for all $i \in \mathbb{Z}^N$
b. $y_{i,ref}(t) \to \delta_i\lambda_d + Q\bar{w}(t)$ exponentially for all $i \in \mathbb{Z}^N$, and
c. $\lim_{t \to \infty}(y_i(t) - y_j(t)) = \lambda_d(\delta_i - \delta_j)$ for all $i, j \in \mathbb{Z}^N$
Proof:
a. We first define $\tilde{x}_i(t) = x_i(t) - \Pi_{\lambda_i}\lambda_i(t) - \Pi_{w_i}w_i(t)$, and $\zeta_i(t) = \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(\lambda_i(t) - \lambda_j(t))$, $\phi_i(t) = \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(w_i(t) - w_j(t))$ for notational simplification. By noting the definition of $p_{ij}(t)$, we can write the time evolution of $\tilde{x}_i$ as:

$$\tilde{x}_i(t+1) = A_i x_i(t) + B_i\left(K_i\hat{x}_i(t) + L_{\lambda_i}\lambda_i(t) + L_{w_i}w_i(t)\right)$$
$$-\Pi_{\lambda_i}\left(\lambda_i(t) - \gamma\,\zeta_i(t)\right) - \Pi_{w_i}S\left(w_i(t) - \gamma\,\phi_i(t)\right)$$

By replacing $L_{\lambda_i} = \Gamma_{\lambda_i} - K_i\Pi_{\lambda_i}$, $L_{w_i} = \Gamma_{w_i} - K_i\Pi_{w_i}$, and $\hat{x}_i = x_i - \epsilon_i$, we can write

$$\tilde{x}_i(t+1) = (A_i + B_iK_i)x_i(t) - B_iK_i\epsilon_i(t) + B_i\left(\left(\Gamma_{\lambda_i} - K_i\Pi_{\lambda_i}\right)\lambda_i(t) + \left(\Gamma_{w_i} - K_i\Pi_{w_i}\right)w_i(t)\right)$$
$$-\left(A_i\Pi_{\lambda_i} + B_i\Gamma_{\lambda_i}\right)\lambda_i(t) + \Pi_{\lambda_i}\gamma\zeta_i(t) - \left(A_i\Pi_{w_i} + B_i\Gamma_{w_i}\right)w_i(t) + \Pi_{w_i}\gamma\phi_i(t).$$

After some algebra we have

$$\tilde{x}_i(t+1) = (A_i + B_iK_i)\tilde{x}_i(t) - B_iK_i\epsilon_i(t) + \Pi_{\lambda_i}\gamma\zeta_i(t) + \Pi_{w_i}\gamma\phi_i(t)$$

From [1] and [35], under (A3), we know that $\zeta_i(t)$, and $\phi_i(t)$ approach zero exponentially for all $i \in \mathbb{Z}^N$. As for ϵ_i, we can see that:

$$\epsilon_i(t+1) = x_i(t+1) - \hat{x}_i(t+1)$$
$$= A_i(x_i(t) - \hat{x}_i(t)) + H_iC_i(x_i(t) - \hat{x}_i(t))$$
$$= (A_i + H_iC_i)\epsilon_i(t)$$

since $(A_i + H_i C_i)$ is Schur for all $i \in \mathbb{Z}^N$, $\epsilon_i(t)$ approach zero exponentially. Lastly, since $A_i + B_i K_i$ is also Schur, from (17), $\tilde{x}_i(t) \to 0$ exponentially for all $i \in \mathbb{Z}^N$. From the above analysis, $x_i(t) \to \Pi_{\lambda_i} \lambda_i(t) + \Pi_{w_i} w_i(t)$ exponentially. Whereas, from (10) and (12) we have $C_i \Pi_{\lambda_i} = \delta_i$ and $C_i \Pi_{w_i} = Q$. Thus, $y_i(t) \to \delta_i \lambda_i(t) + Q w_i(t)$ exponentially. And so, $y_i(t) \to y_{i,ref}(t)$ exponentially for all $i \in \mathbb{Z}^N$ is shown.

b. From [1] and [35], under (A3), λ and w reach consensus exponentially. From [35], w converge to some trajectories $\bar{w}(t)$, which is a solution to the dynamics $w_0(t+1) = S w_0(t)$. Whereas for λ, from Lemma 1, $\lim_{t \to \infty} \lambda_i(t) = \lambda_\kappa(t_0)$ and since $\lambda_\kappa(t_0) = \lambda_d$, therefore $\lim_{t \to \infty} \lambda_i(t) = \lambda_d$. Thus, $y_{i,ref}(t) = \delta_i \lambda_i(t) + Q w_i(t) \to \delta_i \lambda_d + Q \bar{w}(t)$ exponentially for all $i \in \mathbb{Z}^N$ is shown.

c. From proof a, we have that $\lim_{t \to \infty} y_i(t) = y_{i,ref}(t)$ holds. Whereas from proof b, we have that $\lim_{t \to \infty} y_{i,ref}(t) = \delta_i \lambda_d + Q \bar{w}(t)$. Therefore, $\lim_{t \to \infty} y_i(t) = \delta_i \lambda_d + Q \bar{w}(t)$. Evaluating $\lim_{t \to \infty} \left(y_i(t) - y_j(t) \right) = \lambda_d (\delta_i - \delta_j)$, the result is shown. (q.e.d.)

Remark 4. In Case LF, the leader node is also the leader agent that can change the formation scaling factor. The leader agent can change its λ at any time and it is guaranteed that the scaling factor of the formation will converge to the λ of the leader agent.

By using algorithm proposed in this section, agents are not just achieving scalable formation, but the formation also moves along trajectory of $\bar{w}(t)$. By noting that eigenvalues of S lie on imaginary axis, the trajectory of $\bar{w}(t)$ could be in a form of step (constant) function, ramp function, and sinusoid function. When $\bar{w}(t)$ is a step function, it can then be used to shift the formation to a different location. Meanwhile, when $\bar{w}(t)$ is a ramp function, it can be used to make the formation moves continuously like a group of birds flying in a formation. Lastly, when $\bar{w}(t)$ is a sinusoid function, the formation can be made to move in a circle. This is useful when we want the MAS to perform surveillance while in formation. These three functions can be combined to create a composite trajectory that suit the purpose of the operation.

We note that assumption (A4) is needed to guarantee the existence of feedback matrices L_{λ_i} and L_{w_i}. These matrices are needed to track the reference output, $y_{i,ref}$. Therefore, if assumption (A4) is violated, the scalable formation cannot be achieved. It is easy to see that to satisfy (A4), the rank of the matrices stated in assumption (A4) must be $n_i + p$. From here, we also know that $m_i \geq p$ is necessary for assumption (A4) to be satisfied, where m_i is the number of columns of B_i (it is also the dimension of u_i and p is the number of rows of C_i. This means that the number of control input u_i must be at least equal to the number of outputs it will regulate. For example, a 2-dimensional problem where $y_i \in \mathbb{R}^2$, needs at least 2 inputs that deal with the dynamics in both dimensions.

3.2. Main theorem and discussion for case LLU

By having all necessary results established in section 2.4, we can state the main result of case LLU:
Theorem 2: Given system of N agents (1)-(2) with control input $u_i(t)$ given by (3), observer dynamics given by (4)-(5), reference generator dynamics given by (6)-(7) where the communication graph $\mathcal{G}(t)$ is undirected with T as the bounded communication interval, and reference-output $y_{i,ref}(t)$ given by (8}). Let λ_d be the desired scaling factor of the formation, the desired error bound between λ_∞ and λ_d be ν, the value $\bar{t}$ be computed using (16), and also let agent κ be designated as the leader agent and setting λ_κ at $t^* = \bar{t} + t_{0_{old}}$, for all $t_{0_{old}} \geq 0$ following (13). Suppose (A1)-(A2) and (A4)-(A5} are satisfied, then:

a. $y_i(t) \to y_{i,ref}(t)$ exponentially for all $i \in \mathbb{Z}^N$,
b. $y_{i,ref}(t) \to \delta_i \lambda_\infty + Q \bar{w}(t)$ exponentially for all $i \in \mathbb{Z}^N$, and
c. $\lim_{t \to \infty} (y_i(t) - y_j(t)) = \lambda_\infty (\delta_i - \delta_j)$ for all $i, j \in \mathbb{Z}^N$,

where $|\lambda_\infty - \lambda_d| \leq \nu$.
Proof:

a. Case LLU uses the same set of control input, observer dynamics, reference generator dynamics and reference-output as Case LF. They only differ in the communication graph type. Therefore, the first point of Theorem 2 can be proven in the same way as in the first point of Theorem 1.

b. From [1] and [35], under (A5), λ and w reach consensus exponentially to some consensus value λ_∞ and some consensus trajectories $\bar{w}(t)$. These show that $y_{i,ref}(t) \to \delta_i \lambda_\infty + Q \bar{w}(t)$ exponentially holds for all $i \in \mathbb{Z}^N$.

c. From proof a, $\lim_{t \to \infty} y\_i(t) = y_{i,ref}(t)$ holds. Whereas from proof b, we have that $\lim_{t \to \infty} y_{i,ref}(t) = \delta_i \lambda_\infty + Q \bar{w}(t)$. Therefore, $\lim_{t \to \infty} y_i(t) = \delta_i \lambda_\infty + Q \bar{w}(t)$. Evaluating $\lim_{t \to \infty} \left(y_i(t) - y_j(t) \right) = \lambda_\infty (\delta_i - \delta_j)$, the premise holds.

Having computed $\bar{t}$ using (16), by choosing the reset value λ_κ at $t^* = \bar{t} + t_{0_{\text{old}}}$ following (13), from Lemma 3 we have that $|\lambda_\infty - \lambda_{\text{d}}| \leq \nu$ for the chosen value of $\nu > 0$. (q.e.d)

Remark 6. For case LLU, to change the formation scaling factor, the leader agent cannot change its λ to a new value at any time. Furthermore, the leader agent cannot determine the formation is scaling factor for the first consensus process. However, in case LLU, any agent can be designated as the leader agent during the operation as long that agent is given the authority to reset its λ. In other words, the leader agent can be changed during the operation by just giving the reset authority to the new leader agent.

As we will see in the numerical examples, due to the waiting time $\bar{t}$, scalable formation under leaderless undirected network has longer time interval between different formation size compared to the case under leader-follower network. The most important step in computing $\bar{t}$ is in defining ν (the error bound) and computing $\left\|\lambda\left(t_{0_{\text{old}}}\right)\right\|_1$. The (16) is applicable to any other systems utilizing consensus algorithm under time-varying undirected network. Thus, the result presented in this work can be used in general situation. One example is on consensus network; any agent that can compute $\bar{t}$ using (16) can estimate the consensus value of the network by just computing $\bar{t}$ and monitoring its own state.

3.3. Numerical examples for cases LF and LLU

Two numerical examples of the proposed method will be given in this subsection, one for each case. For both examples, $N = 3$, and agent 3 is chosen as the leader agent. The communication network consists of two graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ with $\mathcal{A}_1 = [a_{ij}(1)]$ and $\mathcal{A}_2 = [a_{ij}(2)]$ being the associated adjacency matrices. $\mathcal{G}(t)$ switches between $\mathcal{G}_1$ and $\mathcal{G}_2$ at every time step starting from $\mathcal{G}_1$, and the graphs are chosen such that $\mathcal{G}_1 \cup \mathcal{G}_2$ satisfies (A3) and (A5) for each example. The system considered in both examples are the discrete-time version of the ones used in [3]:

$$A_1 = \begin{bmatrix} 1 & 0.0998 & 0.0047 \\ 0 & 0.9953 & 0.0905 \\ 0 & 0.0905 & 0.8144 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0.0978 & 0.0123 \\ 0 & 0.9387 & 0.2200 \\ 0 & 0.3667 & 0.4987 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0.0995 & 0.0296 \\ 0 & 0.9852 & 0.5881 \\ 0 & 0.0490 & 0.9558 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}', \quad S = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

Where $B_1 = B_2 = B_3 = B$, $C_1 = C_2 = C_3 = C$, and $Q = [1 \ 0]$. Additionally, $\delta_1 = 1, \delta_2 = 2$ and $\delta_3 = 5$. The first state in the system above is the position, while the second and third states are the velocity and the actuator state. With the above choice of systems, (A1), (A2) and (A4) are satisfied by all agents. The formation in the examples is a one-dimensional formation since the output of agents has dimension of 1. Two-dimensional formation is possible when the output of agents has dimension of 2. Simulation result for each case will be given in the following.

3.4. Numerical example for case LF

For Case LF, $a_{12}(1) = a_{23}(2) = 1$, while other entries of $\mathcal{A}_1$ and $\mathcal{A}_2$ are zeros. Note that $\mathcal{G}_1 \cup \mathcal{G}_2$ is uniformly connected which satisfies (A3), and node 3 is the leader node. To show how the proposed method perform to achieve the scalable formation determined by agent 3, the initial condition for $x, \hat{x}, w$ and λ are set to be arbitrary except for initial state of λ_3 which are $\lambda_3(0) = 1, \lambda_3(150) = 2$ and $\lambda_3(300) = 0.5$. Figure 1 shows the time evolution of y_i and we can see that agents track a ramp trajectory with the same gradient. Agents are also separated by $\alpha(\delta_i - \delta_j)$, $i, j \in \mathbb{Z}^3$, where $\alpha \in \mathbb{R}^+$ is determined by agent 3.

3.5. Numerical example for case LLU

For Case LLU, $a_{12}(1) = a_{21}(1) = a_{23}(2) = a_{32}(2) = 1$, while other entries of $\mathcal{A}_1$ and $\mathcal{A}_2$ are zeros. Note that $\mathcal{G}_1 \cup \mathcal{G}_2$ is uniformly strongly connected which satisfies (A3). Meanwhile, the bounded communication interval $T = 2$. To show how the proposed method perform to achieve the scalable formation determined by agent 3, the initial condition for $x, \hat{x},$ and w are set to be arbitrary. The initial condition for $\lambda_i(0)$, $i \in \mathbb{Z}^3$ is specified to be within a ball of $\mathcal{B}(0,0.1)$. Let λ_{d} be 5 and 10 and let the actual formation scaling factor to be $0.9 \lambda_{\text{d}} \leq \lambda_\infty \leq 1.1 \lambda_{\text{d}}$. These imply that the first $\nu = 0.5$ and the second $\nu = 1$. To compute the first and the second $\bar{t}$, we use $|\lambda(t_0)|_1 = 0.3$ and $|\lambda(t_0)|_1 = 16$ respectively.

From the data above, the first $\bar{t} = 1280$ and the second $\bar{t} = 6060$. Therefore, we set $\lambda_3(t_0)$ for $t_0 = 1280$ and $t_0 = 7340$ following (13). Figure 2 shows the time evolution of y_i and we can see that agents track a ramp trajectory with the same gradient. Agents are also separated by $\alpha(\delta_i - \delta_j)$, $i, j \in \mathbb{Z}^3$, where $\alpha \in \mathbb{R}^+$ is determined by agent 3.
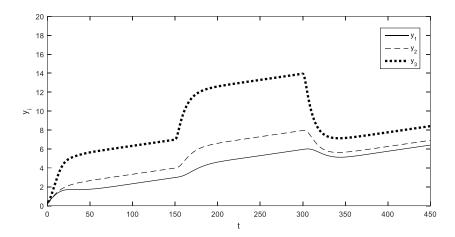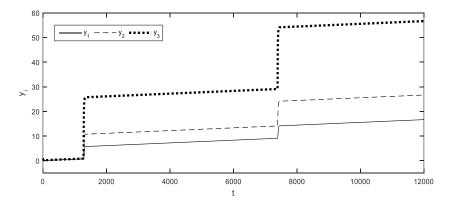
Figure 1. Plots of $y_i(t)$ againts t for case LF

Figure 2. Plots of $y_i(t)$ againts t for case LLU

4. CONCLUSION

Scalable formation for non-identical linear system agents under a time-varying network has been presented in this paper for two graph cases. The first graph case is LF or leader- follower network. This is the common network setting used in any other scalable formation problem. Leader-follower network makes the formation scaling adjustment easy during the operation since the formation size will always follow the initial state of the leader's virtual state (λ). The leader agent can change the formation size at any time. However, changing leader agent during operation is not possible due to the leader- follower network and spanning tree restriction. The second graph case is LLU or leaderless undirected network. This is the first work on scalable formation that uses undirected network, and it allows for a leader agent to determine the formation size. Although the leader agent cannot change the formation size any time (it needs to wait for $\bar{t}$), the leader agent can be changed during the operation, and it is useful when the leader agent is damaged or having technical issues. The presented method can also be used to estimate the consensus value of an undirected network for more general problem.

The scalable formation is achieved, via internal model principle, by having agents track a reference-output with bias. The bias has two components which are δ_i and λ_i. The former is a static component where the desired output of agents is specified, whereas the latter is a dynamic component updated in a distributed manner. To make the reference- tracking possible, feedback matrices need to be computed and the system's matrices must satisfy certain requirements (assumptions (A1) and (A4)). The contribution from this work, scalable formation formulation for non- identical agents, makes formation with size adjustment possible to be developed using different type of agents (with different mathematical model). Several types of agents can work cooperatively to be used in surveillance, searching mission, and defense formation. Thus, the proposed method opens more application for MASs. We take note that the output-feedback internal model principle used in this work is not robust to model mismatch. A robust scalable formation for non-identical agents is a possible future research direction.

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