Infinite Singletons and the Logic of Freudian Theory

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Abstract

The aim of this paper is to advance a formal description of the implicit logic grounding of the psychoanalytic theory. We therefore propose a new interpretation of the logical features of the Freudian unconscious process, starting from the Bi-logic formulation put forward by the Chilean psychoanalyst Matte Blanco. We conceive the universal undifferentiated state of the deep psychoanalytic Unconscious in terms of particular sets named infinite singletons, and we show how they can represent the logical foundations for a formal description of the Primary process. We first disclose some implicit assumptions underlying the common logical language. In doing so, we discover an unexpected presence of symmetry even in the most basic of logical and verbal structures. In the approach derived, we show that infiniteness, not finiteness, is the primary mode of sets, and therefore, of thinking. The pivotal consequence of this model is that the unconscious elements cannot be characterised in the absence of external reality, which produces the collapse of infinite sets and allows for the emergence of linguistic representations. Finally, we discuss how the model could represent a platform to formalise further developments of psychoanalytic theory, in particular with respect to the shift from the First to the Second Topics in Freudian theory.

"Freud's fundamental discovery is not that of the unconscious…
but that of a world—which he unfortunately called the unconscious—
ruled by entirely different laws from those governing conscious thinking".
(Matte Blanco, 1975, p. 93)

Introduction

In this paper, we offer a detailed account of the development of a logical formalism that sheds light on some aspects of unconscious processes. It is based on an idea by the Chilean psychoanalyst Ignacio Matte Blanco (1908–1995), who understood the intrinsic advantages and values of Freud's original theoretical proposal and put forward the Bi-logic theory (Matte Blanco, 1975, 1988), as a hypothesis to describe the logic systems underlying the unconscious mental functioning.

In our view of Bi-logic, the pivotal element of the unconscious mode is the absence of characterisation, namely the impossibility of distinguishing elements of a set in the absence of a link to an external reality. We show that "without characterisation, every set is infinite, namely: *infiniteness, not finiteness, is the primary mode of sets*". The indefiniteness in our

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model is therefore linked to infiniteness. Technically, our view is achieved analysing some implicit assumptions and shortcuts underlying the usual logical thinking. To start, we consider logical duality, that is the algebraic counterpart of the bivalent (true/false) setting of logic, to create a negationless environment that is symmetric and can contain elements with an infinite character, i.e., infinite singletons (see below). In a recently published paper, Saad (2020) puts forward the hypothesis that unconscious thinking is carried out based on intuitionistic negationless logic and relates it to different defence mechanisms such as projection and reactive formation (for a wider discussion of this issue see Matte Blanco's (1988) last work on Bi-logic called "Thinking, Feeling and Being". In our model infinite singletons can grasp the positive elements describing the primary process, namely condensation and displacement.

The psychoanalytic theory proposed by Freud in his work on dreaming (Freud, 1953), is an apparatus based on the idea of psychic determinism and a fundamental insight about the nucleus of human experience not being phenomenological. In the process of theory-genesis Freud introduced constructs such as dreamwork, primary and secondary process, condensation and displacement – all examples of the building of the objects of scientific study from within the discipline itself rather than from the everyday, commonsensical experience, exactly as happens in natural and some social sciences.[2] However, the development of the psychoanalytic movement and the preeminent role assumed by the clinical practice within it led to a progressive marginalisation of such a scientific attitude. This in turn determined a major decrease in the inherent capacity of psychoanalysis, insofar as scientific discipline, to generate new, durable insights. Our work goes, therefore, in the direction of abstraction and formalization of the psychoanalytic theory. We are convinced that psychoanalysis as a discipline can benefit from the formal clarification of the theory, which implies a validation in terms of internal coherence, from which both theoretical and clinical research can take advantage. Our proposal, therefore, is not pointing to new phenomena that could enrich the basis of empirical data which have inspired Matte Blanco, but directly to his theory.

In psychology, this type of modelling efforts follows Salvatore's (2016) quest for a return to the Theory-driven psychology – "a psychology which gives up those concepts defined in terms of phenomenical experience and models from within itself objects and categories it assumes as target and means of investigation". (2016, p. 7) In our view, the notion of *stylized facts* can be useful in the quest to restore the explanatory power of a theory. Introduced by Kaldor (Kaldor, 1961, quoted in Arroyo Abad & Khalifa, 2016) in his discussion on economic growth, stylized facts are to be considered an essential building block in the process of theory-construction. Facts *determined by* and not *inferred from* data are stylized facts[3]. By ignoring individual

[2] For example, in economics there are concepts of market forces, elasticity and so on which are not objects of direct experience rather conceptual tools for a better understanding of the phenomena themselves.

[3] To further define the stylized facts, we can contrast them to so-called bare facts, which are the facts as we usually intend them, "well-confirmed statements about empirical regularities that are typical of economic explanation" (Arroyo Abad and Khalifa, 2016, p. 145). Thus, we can say that there is a direct inferential link between bare facts and the data. However, bare facts are considered inadequate for the process of theory building – they are too limiting, too qualifying and "unable to be properly summarized" (Arroyo Abad and Khalifa, 2016, p. 145). Therefore, in working with bare facts theorists find themselves in such a situation where for further progress to be achieved it is necessary to relax some of the aforementioned qualifications.

details and historical noise, stylized facts address the underlying mechanisms and processes thus setting the foundation for the theory-genesis. In order to connect the dots (Ellemers, 2013) proper models must be developed.

This interdisciplinary paper is based on constructs that require an effort from the part of the reader. Such effort will hopefully be justified since it can allow for a clearer comprehension of the functioning of the unconscious processes. The paper is structured as follows, section 1 introduces the theoretical background, including elements of both Freudian and Matte Blanco's theories, section 2 offers the necessary logical tools for the comprehension of our argument, section 3 introduces the concept of infinite singleton, section 4 describes a model of the symmetric unconscious based on the infinite singleton and proposes some considerations on different types of the infinite (namely the asymmetric and symmetric infinite) and section 5 offers a formal representation of the model. Finally, we discuss some of the possible implications and developments for the formalisation of the psychoanalytic theory.

Theoretical Background

Freud and thinking. The psychoanalytic work – ever since Freud's days – has been focused on associative chains and thought sequences, which analysis has enabled him to grasp the idea of existence of the unconscious mind. Accessing unconscious processes through conscious elaboration has opened the path to a radically different vision of the human being. Freud's description of the characteristics of the unconscious mind was based on a meticulous comparison of the manifestations of unconscious processes and classical logic. The analysis revealed the inherent illogicality of the unconscious – "the fundamental rules of logic have no weight in the unconscious, it could be called the Realm of the Illogical" (Freud, 1964b, p. 168). He considered the unconscious to be a structure, a system, essentially independent from the ego and uniqueness of individual mental content: "to sum up: *exemption from mutual contradiction*, *primary process* (mobility of cathexes), *timelessness*, and *replacement of external by psychical reality* – these are the characteristics which we may expect to find in processes belonging to the system Ucs[4]" (Freud, 1964, p. 187).

Freud's description of this new world was nevertheless characterised by a certain degree of intrinsic ambiguity. He tackled the problem from various points of view – dynamic, energetic and topological – and the theoretical prism he built was influenced by such an approach. These considerations represent the nucleus of the early psychoanalytic contribution to the study of thinking and remain crucial for the understanding of the mental elaboration present in instances such as dreams, psychotic and some neurotic symptoms.

Matte Blanco: Origins of Bi-logic. Difficulties in the development of this model subsequently led to the gradual abandonment of the structural unconscious and its replacement with two new constructs – the Id and the repressed unconscious, which nevertheless did not possess the elegance and the explanatory potential of the old *Ucs*. It wasn't until the 1950s and the works of the Chilean psychoanalyst Ignacio Matte Blanco that any significant development was made in this vital area of psychoanalytic knowledge. Matte Blanco aspired to sew a new "set of clothes for psychoanalysis, which finds itself in a situation comparable to that of an adolescent who has outgrown his clothes and feels restricted, hampered in his movements and

[4] Following Freud, we will use the abbreviations *Ucs* for the system unconscious and *Cs* for the consciousness.

uncomfortable" (1975, p. 3). His ideas are systematically represented for the first time in his magnum opus "The Unconscious as Infinite Sets" of 1975.

Starting from the Freudian works and the ideas of M. Klein, he laid out a new theoretical framework, based on the awareness that none of the cases of the aforementioned illogicality of the mind resulted from a chaotic or randomly operating process. On the contrary, it was clear that they were subject to rules which, although distant from ordinary, classical logic, still produced an order of some kind. His research led to a conception of the human mind essentially as an interplay of two distinct, polarised modalities. He reformulated the Freudian conscious-unconscious duality in terms of the underlying logics of these processes and this is why his theory is known as Bi-logic. Matte Blanco's conception of mental activity is represented by a dichotomous model of "sameness registration and difference discrimination" (Iurato, 2018, p. 124), as two main aspects of thinking. According to him, any psychic act is essentially a combination of these two modalities, at various degrees. The intertwining of these two fundamental modalities gives rise to all mental structures, however complex they may be.

The two principles of Bi-logic. For Matte Blanco, thinking is essentially a process of establishing relations between elements of both external and internal reality: "thinking is actually propositional activity which may lead to simple propositions, to rules, relations, etc. (uni-, bi-, tri-, etc. -positional propositional functions). As relations usually constitute the most frequent aspect of propositional activity, for the sake of brevity I shall, in future, designate thinking (propositional activity) simply as establishment of relations" (Matte Blanco, 1975, p. 225).

Conscious thinking predominately entails relations of analysis and discrimination. This is vital as the first task of consciousness is to locate the self in the world of other objects. From a logical standpoint, these relations are to be considered asymmetric, since they establish differences between elements. The process of ordinary thinking can be conceptualised as following mainly the rules of classical, Aristotelian logic. Consciousness is segmenting and categorizing reality into its constitutive elements to know it. This classificatory activity is present on all levels of thought. However, while the *Cs* is engaged with individual elements, the *Ucs* generalises, treating every individual thing as a member of a class, which in turn belongs to a wider class and so on. Matte Blanco formalised this observation as the generalization principle, the first law of the logic of *Ucs*. The choice of the term *principle* reflects its deep, fundamental character and its importance in mental life.

The next Matte Blanco's discovery is a crucial one, because it represents the core mechanism of the functioning of *Ucs*. As soon as we move towards less conscious processes, the aspect of sameness and likeness starts to become more and more prominent. Matte Blanco states that the *Ucs* processes work by seeking similarities between elements. According to him, the aforementioned sameness registration can be usefully represented by the principle of symmetry, that is, *Ucs* "treats the asymmetrical relations as if they were symmetrical" (Matte Blanco, 2975, p. 38). The word symmetry refers to the sameness, identity between two things and their fundamental indistinguishability. Thus, since the relation of contradiction is nevertheless a relation, the *Ucs* treats opposites as identical.

The *Ucs* activity is hence synthetic, or *condensative* as Freud (1900) would say: it homogenises the experience. Each element is indistinguishable from any other member of the same class. The *Ucs* does not deal with individual elements, only with classes to which they belong. To

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provide an example, for the child the mother is not a single, individual person, it is rather a summum of all of the attributes of all members of its defining class – the class of mothers. Therefore, the individual thing is identified by the class it belongs to. Based on Dedekind's observation that if a set[5] is equivalent to its part, the set itself is necessarily infinite (Dedekind, 1901, p. 64), Matte Blanco's formal explanation accounts for the infinite, all-or-nothing character of *Ucs* processes.

Building on the explanatory capabilities of both principles of generalization and symmetry, the functioning of *Ucs* is seen as a creation of a hierarchy of ever-growing classes, structured as "bags of symmetry": "as the mind and the unconscious deal with various classes, we can say that there are as many 'bags' of symmetry surrounded by films of asymmetry as there are classes in our unconscious" (Matte Blanco, 1975, p. 302). Thus, we can conceptually encompass the totality of thinking processes on the basis of their underlying logic – the *Cs* activity is essentially asymmetric, it heterogenises and separates, whereas the *Ucs* activity is symmetric, homogenizing and relies on sameness and identity. As already mentioned, these two modalities are inseparable and ubiquitous, as the pure form of either is more of a theoretical extreme than an observable clinical fact. Complete asymmetry would implicate the paralysis of the thought process and total symmetry would mean a collapse of psychic coherence and the loss of self (Matte Blanco, 1988). This inextricable interplay of asymmetric and symmetric logic in all mental activity has lead Matte Blanco to conceive the human mind as a stratified structure where the consciousness, most sensitive to differences and individualities, represents the highest level, and the lower levels are characterised by increasing levels of symmetry and preference for similarities (for a discussion of some very relevant genetic and clinical implications of the theory see Rayner, 1981; Grotstein, 1995; Fink, 1989; Matte Blanco, 1989; Charles, 2003; Bria & Lombardi, 2008; Saad, 2020).

The Bi-logic dilemma. Speaking of the *Ucs*, we are confronted with processes whose knowable aspects can be described by something that, although described in logical terms, is alien to any type of ordinary logic, i.e., the principle of symmetry. However, for an accurate description of the *Ucs* phenomena, both the principle of symmetry and the classical logic are needed: neither would suffice if taken separately. Matte Blanco stressed this point, along with his desire to abandon this hybrid, twofold approach and return to a singular, "unitary super-logic", which must be able to contain symmetry (Matte Blanco, 1988, p. 66). His desire remained unaccomplished, and the problem lives on as the "Bi-logic dilemma" (Matte Blanco, 1988, p. 66).

Revisiting Some Logical Tools in View of Bi-Logic

In the following, we shall see the re-interpretation of Matte Blanco's proposal in a suited logical language, that can enable us to better analyse some of the key points of the logic of the unconscious and its relations with conscious thinking.

Formal logic, dating back to Aristotle, abstracts the form from the content and treats the concrete only through abstraction. To determine if something is true or false, formal logic begins by ascertaining its logical form and Aristotle distinguishes three orders of forms—the inference, the proposition, and the term. At the first level, that of inferences, he is almost exclusively concerned with syllogisms, which are schemes of reasoning, consisting of exactly three propositions, two premises (minor and major) and the conclusion. The proposition, which

[5] Terms set, class, category, group are used as synonyms in this regard.

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can be defined as any declarative statement, which in the given context has a meaning and can be true or false, represents second order in formal logic. The third and final order concerns the internal structure of propositions and deals with subjects and predicates. The formal language introduced in the XIX century and still in use nowadays, comprises propositional logic (dealing with propositions) and predicate or first-order logic (dealing with predicates).

Observations on the use of logical connectives. The propositions can be atomic (their truth value does not depend on the truth or falsity of any other proposition) or compound. The compound propositions are built using atomic propositions and a set of propositional connectives: conjunction, disjunction, negation, and implication. In addition to the propositional connectives, the first-order language includes quantifiers, which represent the formal equivalent of terms like "all of" and "some of". Quantifiers regard sets of elements and aim to establish a truth about some or every element of the set. Before exploring some of these notions in greater detail, we must remark that this classification is based on our conscious experience, since it depends on the idea of verification of truth, which in turn implies a contact with the external reality.

Let us start with a very simple example: George, who is making a sandwich, adds some ham first and then he adds some cheese. Based on the two previous items of information and without any further verification, he can assert the following: "some ham is in my sandwich AND some cheese is in my sandwich", or simply "ham AND cheese are in my sandwich". This is the truth he expects to verify when eating the sandwich. Our example describes the logical conjunction "AND", which forms a true proposition when both conjuncts (there is ham in the sandwich *and* there is cheese in the sandwich) are true.

With respect to finding a correspondence with the elements of reality, the example of conjunction is the simplest one. As we will see, the correspondence issue, in other cases, may be considered controversial. For instance, the logical connective "NOT" subsumes linguistic particles and expressions that describe the negation of a proposition. Since classical logic is bivalent (something can be either true or false and not both at the same time) the negation of a proposition is true if and only if the proposition is false, and the double negation of a proposition is equivalent to the proposition itself. Going on with the previous example, if George makes a sandwich just with ham, then he can assert "there is NOT cheese in my sandwich" without tasting it. However, we notice that tasting the sandwich carries a piece of positive information about the presence of ham and no positive information about cheese, that is, negation seems to be a pure logical artifact.

Furthermore, logical disjunction, described by the connective "OR", is true when either of the disjuncts is true. As an example of how a disjunction is formed, we could imagine George making two distinct sandwiches—one with ham and the other with cheese—and putting them in a bag. Then, if he were to pick up one of them randomly from the bag, he could assert "there is ham OR cheese in my sandwich". However, after tasting the sandwich, George could be more precise, thus making the disjunction irrelevant. As a matter of fact, the truth contained in the disjunction is more about the bag than about the sandwiches!

Finally, we consider the implication connective, describing logical consequence "IF … THEN …". The implication is true when the conclusions are true, given that the assumptions are true. We notice that the implication requires a certain level of abstraction. It is located at the level of rules, and it concerns the truth about the rule itself, regardless of the subject involved. Let us consider, as an example, the following rule: “if you earn money, then you have to pay taxes”. The meaning of the rule is general, and it applies to everyone.

Logical connectives can link and modify logical propositions and allow us to infer what the true elements in the external reality are. However, we must add that the semantics of logical connectives also carries an idea of truth that goes beyond the mere verification, and hence hides components of thinking independent of the reality itself. As we have noticed in the disjunction and implication examples, even at this level, there is an impelling need for some amount of abstraction. Namely, it seems to us that a certain degree of symmetry is implied in the construction of the logical connectives as well.

The language of predicate logic. Predicates, in the first-order language, are propositions which express properties or relations about certain objects. For example, in the statement “red is a beautiful colour”, the “[…] is a beautiful colour” part is a unary predicate. There are also binary, ternary and n-ary predicates – in the proposition “3 is greater than 6”, the “[…] is greater than […]” part is a binary predicate. In the cases just considered the object is defined, that is, the predicate is applied to a *closed* term (a constant), and the corresponding predicate is closed as well. On the other side, the language also needs *open* terms to describe properties of objects that are not univocally defined. Open terms consist of variables or contain variables. Subsequently, open predicates are predicates that contain open terms. They do not have a definite truth value: for example, the sentences “x is a positive number”, or “a person has got a degree” cannot be judged as true or false. This is possible only when the variables (“x” or “a person”) are substituted by a closed term, for example “3 is a positive number”, or “George has got a degree”.

As already mentioned, first-order language adopts quantifiers, which are usually classified in two kinds: the universal one “FOR ALL” and the existential one “THERE EXISTS”. If we re-use once again the gastronomic example, we can imagine George with a bag full of sandwiches, making some general assertions about them: “all sandwiches in the bag have cheese”, “no sandwich in the bag has cheese”, “at least one sandwich in the bag has ham”, and so on. Therefore, quantifiers are necessary to *close* open predicates and hence assign a truth value to them, by specifying the range of the variable (in George’s case, his bag).

First-order language revealed its weaknesses very soon after its introduction. In the first decades of the XX century, the results proved by Gödel and by Löwenheim–Skolem discovered a gap between the formal, *object* level and the informal, *meta-level*. That is, in logic, one needs to distinguish between the object level, which is the study of logic in a specific logic system (i.a., intuitionist logic, fuzzy logic) and the meta-level, which is the study of logic systems themselves. The limitative results were proved in model theory (Löwenheim–Skolem theorems) and in proof theory (i.e., Gödel theorems, for an in-depth review see Mangione & Bozzi, 1993). It was shown that, in the formal logical systems, the notion of truth is not in accordance with the notion of derivability, namely there are true and underivable sentences. A gap is created between semantics and syntax, between the informal, metalinguistic level, and the formal one. In particular, it became clear that the formal notion of term, the key element of first-order language, is insufficient and misleading. This is easily understandable considering the difference between variable and parameter, that everyone that solved equations in high school is familiar with. Solving an equation means finding the values of x for which the equality is true. This means that the letter x must be conceived as a variable that gives different values to the expressions involved in the equation. On the other side, when solving the equation, one applies some uniform rules, independent of the constants contained in the equation itself. This can be performed informally, or it can be formalised by describing the rules themselves, and adopting other letters (i.e., the parameters) to represent the constant part of the equation. Then the status of a parameter is that of … a variable constant! Therefore, we can say that the parameter is a variable at the metalevel. The point is that this difference is lost in the first-order language.

Nevertheless, since human beings can…flee to the metalevel when they need, and hence abandon the cage given by the formalism to understand what they are doing, the formal apparatus has been maintained essentially for its convenience. However, this solution is not always convenient nor reasonable. It is not convenient, for example, in developing artificial intelligence, where one viable solution was proposed by the introduction of different kinds of probabilistic reasoning or different formalisms derived from mathematics (Wang, 2012). On the other hand, it is certainly not reasonable when we need to treat the roots of our thinking. In this case, it is necessary to perform the analysis of the role of the formalism with respect to the implicit assumptions and shortcuts underlying our usual thinking.[6]

Duality, Symmetry, and Infinite Singletons

Duality in logic. De Morgan laws state that conjunction and disjunction are *dual* connectives, that is: the negation of the conjunction of two propositions is the disjunction of their negation, and the negation of the disjunction of two propositions is the conjunction of their negation (Mangione and Bozzi, 1993). Going back to our informal example, the negation of "a sandwich with ham and cheese" is "a sandwich without ham or without cheese", the negation of "a sandwich with ham or cheese" is "a sandwich without ham and without cheese". Consequently, in propositional classical logic, it is possible to avoid the introduction of a primitive connective for negation and to define negation by adopting duality: we just need pairs of opposite elementary propositions. Then, by De Morgan laws, the negation of a conjunction of two elementary propositions is given by the disjunction of its opposites and the negation of a disjunction by the conjunction of its opposites, and so on. This means that there is a formalisation of classical logic that only includes the negation implicitly. Namely, bivalence is given by duality. So, the analysis of duality can be considered a natural formal platform that allows to avoid negation and approach symmetry.[7]

In classical logic, De Morgan's laws are extended to quantifiers as well, namely the negation of a universal proposition is equivalent to the existential quantification on the negation of its predicate and the negation of an existential proposition is equivalent to the universal quantification on the negation of its predicate (idem). For example, to express the negation of the sentence "every day of last week was sunny" one says, "there was a day of last week which

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[6] We must recognise that these considerations, especially the concept of symmetry, refer to aspects of thinking so deeply embedded in the very structure of it, that sometimes they seem either nonsensical or trivial. As the young fish who asked, "What is water?" in the famous speech by David Foster Wallace, we believe that the apparently obvious and trivial aspects of reality are seldom such, exceptionally less so when they concern our inner realities.

[7] A more careful analysis of duality is possible by adopting more specific logical tools such as *sequent calculus* (Gentzen, 1969; Sambin, Battilotti & Faggian, 2000). We are confident this could allow to develop technical results for a deeper comprehension of the border of the symmetric mode.

was not sunny". However, a different perspective is proposed by intuitionistic logic, where the truth of an existential statement is established only based on the actual construction of the object witnessing the truth of the predicate. Then the truth cannot be established simply by proving the negation of a universal quantifier, as duality would allow.

In addition, in intuitionistic logic the negation of a proposition is true when assuming that proposition as true would yield false conclusions[8]. As an example, let us consider the sentence "every day of last week was sunny". John, an intuitionistic logician, was away on holiday last week and wants to discover if the sentence is true or not. John adopts the intuitionistic approach to negation: he starts from the assumption that, if the whole week was indeed sunny, the ground in his garden would be dry. However, when he came back, he found the garden was wet and therefore he can conclude that the week was not completely sunny, even without knowing when exactly it had rained. Then he can assert the negation of the sentence. It is important to notice that, to conclude the negation of "every day of last week was sunny", John is exploiting the ability to see the week *as a whole thing*, not partitioned into different moments of time, even if the week is described adopting a quantification on the days of the week. This possibility is crucial for symmetry.

At the beginning of the XX century, the intuitionistic logic was developed based on the constructive notion of truth. Intuitionism did not criticise the formalisation of the language based on couples of dual connectives. However, the above example hints that there can be a way to overcome duality even inside first-order language, that is based on a *shift* in the approach to the domain of quantifiers.

Symmetry and infinite singletons. When defining the unconscious mode, Matte Blanco adopted two logical tools: the analysis of binary relations and the definition of infinite set. Actually, there is a third basic element in logic: connectives. Here we aim to explore the conditions under which the logical connectives are reduced to a unique, symmetric connective. To start, we do have a symmetric connective in first-order language: any quantifier applied to a domain of one element only (singleton). In such a case, the existential and universal quantifier coincide. For example, the father of an only child, who is good in maths, could say "all my children are good in maths" or "there exists a child of mine who is good in maths", equivalently. However, quantifying on a singleton is usually avoided. From our usual, conscious point of view, it is more informative to use the predicate attributing the considered property to the element, as in "my daughter/son is good in maths", where the element itself is univocally characterised.

Let us see this point in general. We assume that u is the unique element of the set U, and A is any property. Then the universal proposition

"FOR ALL x belonging to U x has the property A"

is equivalent to the existential proposition

"THERE EXISTS x belonging to U such that x has the property A;

since they are both equivalent to the property A applied to the element u, represented by a closed term of the language. To recognise the equivalence between the universal and existential quantifier by reducing of the quantification to a predicate with a closed term, hence, depends on the characterisation of the element u. That is, the domain U coincides with the singleton

[8] This approach to negation is possible in classical logic as well, even if in this case it coincides with the result of duality.

$\{u\}$[9] since, if x is any element of U, then x is identified with u. As a matter of fact, *the characterisation* of the element eliminates the action of the quantifier. By *characterising* we mean isolating well-defined individual entities. This happens when we admit and recognise something as an *external reality*, that is, when we refer to objects of the external reality that do not depend on ourselves and our descriptions. However, to develop the idea of an effective symmetric connective, let us exclude the possibility of referring to an external reality. In other words, let us suppose that we cannot characterise. This would mean that there are conditions where it is not possible to describe an element of a set by simply assigning it an identity (as if a description of the set was to be produced in the absence of an *external reality)*. In that case, to describe a set, we can consider a domain, where for every property A, there is equivalence between existential quantification ("THERE EXISTS x belonging to U such that x has the property A") and its corresponding universal quantification ("FOR ALL x belonging to U x has the property A"). Every nonempty set for which such an equivalence is assumed is provably a singleton.[10] This means that U has a unique element, even if it is not specified.

We are accustomed to the extensional notion of set: the set is characterised by its elements and two sets with the same elements are indistinguishable. Therefore, it seems impossible to characterise something as a set without specifying its elements. However, as already mentioned, in logic one needs to distinguish between the meta-level and the object level. The point is that the notion of finite/infinite is level-sensitive: one can consider a set which is finite at the meta-level and cannot be proved as such at the formal, object level. This happens because the characterisation of the elements of a set may occur at the meta-level and not at the object level. Indeed, to judge a given set $V = \{u_1, \dots, u_n\}$ as finite, we need to count its elements, and hence to separate two parts: the part that has already been counted from the part that has not been counted yet. If characterising the elements is not possible, the counting process is not possible as well, and hence the very notion of finiteness is not available[11]. Even in the case of a set U with a unique element u, if we cannot characterise, we cannot count it as 1, as strange as it is! The invisible man described by Wells (1897) was *there.* Yet, he did not become visible until he was covered in some opaque fabric: so, even when we *see him,* we are actually only seeing the clothes that are covering him.

At this point, we have all the elements to introduce the notion of *infinite singleton.* It is a set positively described by the equivalence between the universal and existential quantification for every predicate, regardless of the possibility to denote the element of its domain by a closed term. In this way, *normal* singletons are a particular case of the infinite ones. In other terms, without characterisation, every set is infinite, namely: infiniteness, not finiteness, is the primary mode of sets. We recall that a set is infinite when there exists a proper part of its which is idempotent to the set itself. Therefore, this mathematical definition of infinite set can emerge once we can separate the existential from the universal quantifier, on one side, and distinguish the part from the whole, on the other. In singletons, every non-empty part is idempotent to the whole set. As mentioned earlier, for a set to represent the symmetric mode, it *must have only symmetric relations*—therefore such a set is necessarily a singleton (since, as soon as one can

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9 The standard mathematical notation for a set is the so-called listing one, where the elements are placed between a pair of curly braces and separated by commas. In the case of singleton, there is a single element between braces.

10 That is, any two of its elements are provably equal. See section 5 for a proof.

11 A similar situation is found in quantum mechanics: indistinguishable particles form Bose condensates (Ancona, 1992, p. 132).

distinguish two different elements in a set, one can put order between them, and hence define a non-symmetric relation). The infinite singleton satisfies this condition and, paradoxically, we find that it represents a stronger notion of infinite. A *symmetric infinite*, indeed.

Infinite singletons and the logic of the symmetric mode. Infinite singletons are defined as those sets U which make the universal and existential quantification coincide. As we have seen, this yields two possibilities:

1) when one cannot characterise (in absence of the correspondence with an *external reality*), a generic infinite singleton emerges;
2) when one can characterise (provide an identity u for the element of U) the result is a finite singleton $\{u\}$.

We note that, in the first case, the set is described only by the membership relation while, in the second case, it is described also by an identification: "x belongs to U" becomes "$x = u$". As we have seen, any singleton, by definition, satisfies the symmetry principle. Then, to describe the symmetric mode, we need to exclude the finite singletons. To do so, it is enough to add the generalisation principle. Indeed, the generalisation process cannot be applied to a finite singleton $\{u\}$, because its identification with its unique element u yields that all elements different from u are excluded.

By exploiting the gap between the object level–meta level in logic, on one side, and gap between the asymmetry–symmetry in psychology on the other, we can now show that infinite singletons, together with the generalisation principle, produce the logical features of the primary process. Given that the symmetry cannot distinguish between elements (otherwise it would create a non-symmetric relation), we first notice that any two infinite singletons U and V cannot be distinguished in the symmetric mode. To distinguish two sets means to find an element that belongs to one and not to the other. This is impossible in the symmetric mode since the generalisation principle can establish a membership, but it cannot exclude one. Then, as observed by Matte Blanco (1988), when symmetry finds no obstacle at all, one gets to one object only. Therefore, any two infinite singletons are forced to *condense together*.

More so, infinite singletons do not have a complement, since considering "the set of the elements excluded from U" does not have any meaning in a system in which exclusion is not conceived. In addition, let us see that conceiving exclusion is equivalent to conceiving negation. For, let us consider the set U of elements satisfying the property A. Then, the set of elements excluded from U, that is the complement of U, is the set of elements that do not satisfy A, namely they satisfy its negation. Then, if a system is capable of excluding membership, the complement of U is available, and hence the negation of A is available. Conversely, if a system is capable of negation, then the set of elements satisfying the negation of A, that is the complement of U, is available. We conclude that infinite singletons, in presence of generalisation, do not allow negation.

Furthermore, as a corollary of the definition of infinite singleton, we see that the displacement of a property from an element to "another", inside the infinite singleton, is necessary. For, if an element of U has a certain property A, then "THERE EXISTS x in U which has the property A", but then "FOR ALL x in U x has the property A", by the definition of infinite singleton itself.

Matte Blanco (1975) considered the symmetric mode as the primary mode of experience. Rayner (1981) referred it to pre-object, symbiotic states of mind and, more recently, it was recognised—quite paradoxically—as "the source of any possible determination or predication" (Bria & Lombardi, 2008, p. 712).

Following our model, we claim that this primary mode corresponds to an absence of characterisation, which, by the way, could be reminiscent of the fetal condition (Grotstein, 1978; Fink, 1989). Let us explore what happens when external reality comes in. The system, besides structuring the world in terms of classes of indistinguishable elements, becomes capable of characterizing and identifying individual entities. Therefore, when characterization becomes possible, the system becomes capable of recognising that the membership relationship "x belongs to U" is equal to the identity relationship "$x = u$". Then the complement of U becomes conceivable as a separate entity.

This resembles the Kleinian conceptualisation in which the first representation of experience is a bivalent one (good breast vs. bad breast) (Klein, 1946). A similar idea is found for the spin observable in the quantum model (Battilotti, 2014a,b), where the set of elements not belonging to U is characterised as having the *opposite* value with respect to U. In this view, bivalence is interpreted as a sort of *natural collapse* of symmetry, hence it is close to symmetry, but it represents *the end of the infinite mode*.

Finally, we mention another view of the collapse of symmetry, which can preserve some aspects of the infinite mode. As we have mentioned, Matte Blanco conceives mental activity as an inextricable interplay of asymmetric and symmetric logic; in particular, he considers the mathematical infinite as an asymmetric infinite, namely a way for the infinite to survive out of the symmetric mode. This clearly hints at the possibility that, in our rational reasoning, we can partially preserve the symmetric mode under the form of infinite mode. While Matte Blanco does not refer to different specific logic systems, we suggest that intuitionistic logic contains a way to preserve an infinite mode out of symmetry and without bivalence. As Kurt Gödel (Gödel, 1986a) proved, intuitionistic logic is infinite valued, since its semantics cannot be characterised by a finite number of truth values. In particular, as mentioned in section 3, intuitionistic logic is not bivalent and, therefore, its negation is not definable in terms of duality. Reconsidering the example "every day of last week was sunny", its negation in the realm of intuitionistic logic requires the ability to see the week as a whole thing and not partitioned into different items of time. This is true even if the week is described by adopting the universal quantification on the domain of the days of the week. The example reveals traces of the symmetric infinite in our conscious thinking, *where negation is formulated.*

Our approach leaves open a very relevant issue, that must be included in the modeling effort, and that is related to the emergence of bivalence from the fundamental symmetric mode (a solution to a structurally similar issue was explored within a quantum formalism in Battilotti, 2014). There is an ongoing research effort to envisage a pathway leading from the logic of infinite singletons to the intuitionistic definition of negation, and therefore to bi-logic: in brief, the proposal assumes that reconciliation between infinite and finite is possible considering a further abstraction of the definition of infinite singleton within a modal logic, in which a predicate is introduced to be interpreted as "it is necessary that", "it must be", namely *a prescription*. We are therefore proposing to develop a modal approach in which prescription should 'contain' the infinite element, inherited from infinite singletons. The modal logic approach under study (Battilotti, *in preparation*) is consistent with the evolution of the

Freudian theory, which in moving from the First to the Second Topics, as expressed in The Ego and the Id (Freud, 1923), introduces a normative instance moderating the encounter of the psychic dimension with the external reality. However, going back to the present goal, in the following section, we will provide the formal presentation of the statements introduced in the paper; this section can be skipped without any loss of comprehension or generality by the reader who is not interested in it.

Formal representation of duality, infinite singletons and their properties. The language of propositional logic consists of elementary propositions, represented by small letters (literals): $p_1, p_2, p_3, \dots, p_n, \dots$ and propositional connectives, negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$)[12]. Compound propositions $\neg A$, $A \wedge B$, $A \vee B$ are formed by applying connectives to simpler propositions, A and B, in turn originated from literals.

Two propositions A and B are equivalent when by assuming A one can derive B and conversely. In the following, we shall denote logical consequence by $\vdash$ and logical equivalence by $\equiv$. De Morgan laws state that

$$\neg(A \wedge B) \equiv \neg A \vee \neg B \text{ and } \neg(A \vee B) \equiv \neg A \wedge \neg B;$$

and we say that conjunction and disjunction are dual connectives. Negation is formally avoided by adopting a wider set of literals, containing, for each literal p, its negation $p^{\perp}$. Indeed, compound propositions are formed by using conjunction and disjunction only: initially $\neg(p_1 \wedge p_2)$ is given by $p_1^{\perp} \vee p_2^{\perp}$ and $\neg(p_1 \vee p_2)$ is given by $p_1^{\perp} \wedge p_2^{\perp}$, and so on. This language confines negation to literals and shows that bivalence can be read as duality.

We now see the formal description of the elements of predicate logic used in the paper. A term denotes an object, that may be characterised or not. In the first case, it is a constant denoted by $c, k, u, \dots$ and is *closed*, otherwise, it is a variable, and is *open*. A predicate P applied to some terms: $P(t_1, \dots, t_n)$ represents a relation applied to some objects. In particular, the equality $=$ is a binary predicate and properties A are unary predicates. In the following, we refer to equality, membership relation and properties only. As we said, open predicates cannot be given a truth value unless the quantification on a given domain is specified. The universal proposition "for all elements of the domain D, the property A holds" is written formally $(\forall x \in D)A(x)$, where $\forall$ is the universal quantifier and $x \in D$ (x belongs to D) denotes the membership relation between x and D. The existential proposition "there exists some element of the domain D, such that the property A holds" is written formally $(\exists x \in D)A(x)$, where $\exists$ is the existential quantifier. Then, the definition of infinite singleton is the following: the domain V is an infinite singleton if and only if, for every property A:

$$(\exists x \in V)A(x) \equiv (\forall x \in V)A(x).$$

We see that any two elements of an infinite singleton V are equal. Let us assume $z \in V$ for some element z even if not specified. One can see that $z \in V$ is equivalent to $(\exists x \in V)x = z$, which in turn is equivalent to $(\forall x \in V)x = z$, by the definition of infinite singleton applied to the property $x = z$ "to be equal to z". The last says that any element x is equal to the generic element z.

Requiring that the singleton V is recognised as finite by a system, namely that there is a constant element u such that the equality $V = \{u\}$ is proved in the system itself, amounts to the

12 Implication, in classical logic, can be obtained as the disjunction of the negation of the antecedent with the consequence.

acceptance of the consequence $x \in V \vdash x = u$. Then we can have two different levels: in one V is finite since the consequence is asserted, in the other it is infinite since the consequence is not asserted. Finally, we would like to note that a formal analysis would show that the equivalence $(\exists x \in V)A(x) \equiv (\forall x \in V)A(x)$ is rewritten as the consequence:
$(z \in V) \wedge A(y) \vdash A(z) \vee (y \notin V)$

As we have seen, in the symmetric mode the disjunct $y \notin V$ is not the case, then one is forced to conclude the disjunct $A(z)$, providing a formal derivation of displacement in the symmetric mode. On the contrary, if the singleton is finite: $V = \{u\}$, the above consequence is rewritten as $z = u \wedge A(y) \vdash A(z) \vee y \neq u$ where $\neq$ is the negation of equality. This means that, when z is equal to u and a property A is true for y, then the same property is not necessarily true for y, unless y itself coincides with z and u. This is reasonably true and excludes any significant form of displacement in the finite case.

Concluding Remarks and Forthcoming Research

In this paper, we have shown that the implicit assumptions concerning the *extensionality of sets* and *the duality of logical language* hide an important distinction, which leads to the discovery of the symmetric mode. In the formal approach just derived, we have shown that infiniteness, not finiteness, is the primary mode of sets, and therefore, of thinking. This statement is reminiscent of some positions about the nature of thinking that have been expressed by different authors at different points in the development of psychoanalytic theory. Lou Andreas-Salomé (1921) proposed a theory of Primary Narcissism as an original state in which identity has not yet emerged from an undifferentiated state, grounded in pre-natal and infantile experience, in which we perceive ourselves as the whole and the whole as ourselves. She depicts the human being as a plant that longs for the Sun (i.e., the differentiated state) while being, in the meantime, grounded in the soil of this universal undifferentiated state. In his essay “The Ego and Thinking” (1929) Imre Hermann, a Ferenczian scholar, proposed the idea that the pure form of thinking would correspond to the deep Unconscious, sheltered from perception, that can only be described in logical terms. Finally, we would like to mention the Jungian concept of the psychoid nature of the Archetype as discussed in the essay “On the nature of the psyche” (Jung, 2001).

Notice that by describing an infinite singleton as *a set for which the language has no term to denote the unique object it contains*, we are actually saying, in formal terms, that no word is available for the thing we have in mind. As a matter of fact, we are finding a way to keep separate what Freud, already in his pre-analytical work “On Aphasia” (2002), had recognised as *different levels of representation,* namely *word presentation* and *thing presentation.* In his own words “that’s why the idea of the object does not appear to us as closed, and indeed hardly as closable, while the word concept appears to us as something that is closed though capable of extension” (Freud, 2002, p. 80). As well known, this was the basis for Freud’s subsequent theoretical and clinical work. We plan to further develop the analysis of the links between Matte Blanco and Freud’s representation theory by referring to the formal approach just introduced (Battilotti, Borozan & Lauro Grotto, *in preparation*).

In general, developing a further analysis of the logical role of infinite singletons could allow reaching the formalisation of other aspects of the Freudian theory. In fact, the overwhelming power of symmetry in the model should be somehow limited (Saad, 2020; Lauro Grotto, 2021) to reproduce the mixture of conscious and unconscious processes that is typical of human thinking, as described by Matte Blanco in the Bi-logic approach. In the genetic development of thinking the normative dimension plays a crucial role in this respect: as postulated by Freud, the development of the Oedipal dynamics assumes a structural status with the introduction of the Super-Ego in the Second Topic (Freud, 1961). In abstract terms, the notion of infinite singleton can be tackled from the point of view of criticism of the notion of term, as we have seen above. In this sense, two possible paths worth considering are Gödel's modal system S4 (Gödel, 1986b) and Girard's linear logic (Girard, 1987) both introduced to overcome the problem of incompleteness determined by first-order language. The modality of S4 could offer the opportunity to include an abstract and subsequently a normative element in the formalization. On the other side, the features of the unconscious thinking deriving from the quantitative aspects of the theory, namely the ones related to the economic point of view and the degree of investment could be described by developing infinite singletons in the modalities of linear logic. In fact, linear logic was the original proposal in a project to realise the aforementioned *unity of logic*. As already noticed, a similar idea was in Matte Blanco (1988): for, more than once he said that the solution of the "Bi-logical dilemma" was in the proposal of a "unitary super-logic" (Matte Blanco, 1988, p. 66). We hope that a formal approach rooted in the analysis of elementary elements allows for new developments in this sense. Finally, topological models of unconscious thinking have been widely explored in the last two decades (Iurato, 2018; Iurato et al. 2016; Khrennikov, 1998, 2002; Lauro Grotto, 2007, 2017; Murtagh, 2012a, 2012b, 2014) and since one of the most productive research methods in logic consists in establishing a correspondence between logical systems and topological structures, we hope such a correspondence can be found for our models.

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