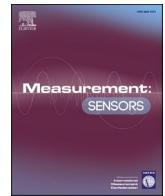


Feasibility study of multi-point two-dimensional profile measurement by 3-2-1 and 3×3 sensor layout

著者	Fujiwara Ryotaro, Shimizu Hiroki
journal or publication title	Measurement: Sensors
volume	18
page range	100358-1-100358-4
year	2021-11-24
URL	http://hdl.handle.net/10228/00008665

doi: <https://doi.org/10.1016/j.measen.2021.100358>



Feasibility study of multi-point two-dimensional profile measurement by 3-2-1 and 3×3 sensor layout

ARTICLE INFO

Keywords

Multi-point method
Profile measurement
Software datum
On-machine measurement
Flatness

ABSTRACT

New multi-point methods, the six-point method of 3-2-1 sensor layout and the nine-point method of 3 × 3 sensor layout, have been proposed as the two-dimensional profilometry for a machined flat surface. Monte Carlo simulations were carried out and properties of proposed methods were examined. As a result, it was shown that the six-point method, the minimum configuration that calculates the planar shape with only six displacement sensors, can obtain the equivalent result to that of the simple nine-point method. In the nine-point method with averaging, it was confirmed that the maximum and average values of the standard deviation of the reconstructed profile were reduced to 68% and 81%, respectively compared with simple nine-point method. The improved nine-point method that averages the pitching error to improve the data connection accuracy also proposed and this method reduced standard deviation, but effectiveness is limited.

1. Introduction

In order to process a machined plane with high straightness and flatness, it is necessary to measure the waviness of the cross-sectional profile and feedback the measurement results for corrective machining. On-machine measurement using the stage of the machine tool is effective for corrective machining, but it is necessary to remove the motion error of the stage from the obtained profile by an error separation technique.

As error separation techniques in one-dimensional (1D) profile measurement, the two-point method [1,2], the three-point method [3], and their variations [4] are known. The motion error is removed mathematically from the outputs of the multiple displacement sensors mounted on the same stage table. However, the two-dimensional (2D) profile of a plane cannot be obtained by simply arranging the 1D profiles because the height and inclination of the baseline are different. In other words, in order to obtain the 2D profile of a plane, it is necessary to add sensors that relate the 1D profiles along the main scanning lines. As multi-point methods for planar profile measurement, four-point method [5] and a method for using 4 × 4 sensors [6] were proposed.

To extend the sequential two-point method to planar profile measurement, the authors have proposed planar measurement methods using displacement sensors in a 2 × 2 layout and angle sensors [7].

In this paper, the authors have proposed two measurement methods for planar form measurement: the one is the six-point method which uses a sensor unit with six displacement sensors arranged 3-2-1 layout and the other is the nine-point method which uses a sensor unit with nine displacement sensors arranged 3 × 3 layout. In addition, the redundancy of using many sensors can be used to reduce accumulation of random errors by averaging and to improve the accuracy of the data connection. The error characteristics of each method and the methods for reducing the accumulation of random error were investigated using

Monte Carlo simulation.

2. Principle of six-point method and nine-point method

The six-point method and the nine-point method are based on the three-point method for straightness measurement. In the six-point method, six displacement sensors are arranged in a 3-2-1 layout as shown in Fig. 1, and the sensor unit is raster-scanned to obtain height data. The number of each sensor is defined as shown in the upper right of the figure and coordinate and measurement points also figured.

$m_n(s, t)$ means the height data measured by the n th sensor at the coordinate (s, t) . After scanning the sensor unit in the X-axis direction (main scanning direction), the table is returned to $X = 1$, each sensor is moved up one line in the Y-axis direction (sub scanning direction), and the scanning in the X-axis direction is repeated again. When scanning a sensor unit with sensors placed at six points, the sensor outputs m_1 to m_6 at each position are calculated considering the translational motion error $E_z(x)$, the pitching error $E_p(x)$, and the rolling error $E_r(x)$.

From these outputs, the recurrence formulas for the sample profile $F(x)$, formulas (1) to (6), are obtained. Note that each component of motion error is eliminated in these equations.

When the sensor unit is scanned in the X-axis direction

$$F(x+2, y) = 2F(x+1, y) - F(x, y) + m_1(x, y) - 2m_2(x+1, y) + m_3(x+2, y) \quad (1)$$

$$F(x+1, y+1) = F(x+1, y) + F(x, y+1) - F(x, y) + m_1(x, y) - m_2(x+1, y) - m_4(x, y+1) + m_5(x+1, y+1) \quad (2)$$

<https://doi.org/10.1016/j.measen.2021.100358>

Available online 24 November 2021

2665-9174/© 2021 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

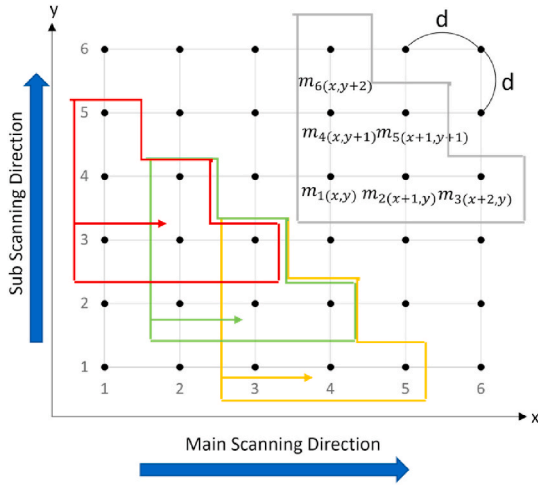


Fig. 1. Six-point method.

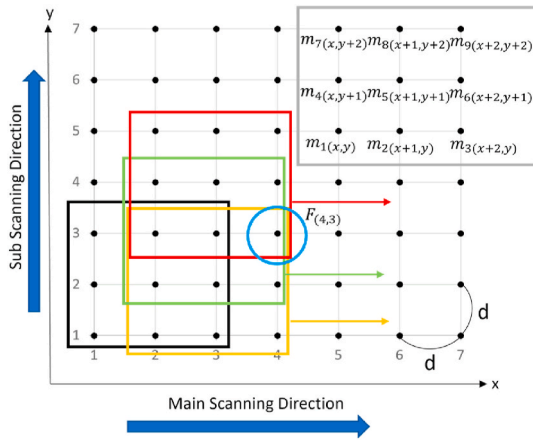


Fig. 2. Nine-point method.

$$F(x, y + 2) = 2F(x, y + 1) - F(x, y) + m_1(x, y) - 2m_4(x, y + 1) + m_6(x, y + 2) \quad (3)$$

When the sensor unit is scanned in the Y-axis direction,

$$F(x + 2, y) = 2F(x + 1, y) - F(x, y) + m_1(x, y) - 2m_2(x + 1, y) + m_3(x + 2, y) \quad (4)$$

$$F(x + 1, y + 1) = F(x + 1, y) + F(x, y + 1) - F(x, y) + m_1(x, y) - m_2(x + 1, y) - m_4(x, y + 1) + m_5(x + 1, y + 1) \quad (5)$$

$$F(x, y + 2) = 2F(x, y + 1) - F(x, y) + m_1(x, y) - 2m_4(x, y + 1) + m_6(x, y + 2) \quad (6)$$

Using these equations, the 2D cross-sectional profile of the entire plane can be obtained sequentially from the heights of the initial points and the sensor outputs during the scan.

In the nine-point method, the sensor unit is composed of nine displacement sensors arranged in a 3×3 layout as shown in Fig. 2, and the height data are acquired by raster scanning. The sensor arrangement and number are defined as shown in the upper right of the figure.

From these sensor outputs, the recurrence formulas for the sample profile $F(x)$, formulas (7) to (12), are obtained.

When the sensor unit is scanned in the X-axis direction

Table 1
Standard deviation of each motion error.

	E_x	E_p	E_r
X-axis direction	1.0 μm	3.0 μm	1.0 μm
Y-axis direction	5.0 μm	9.0 μm	3.0 μm

$$F(x + 2, y) = 2F(x + 1, y) - F(x, y) + m_1(x, y) - m_2(x + 1, y) + m_3(x + 2, y) \quad (7)$$

$$F(x + 2, y + 1) = 2F(x + 1, y + 1) - F(x, y + 1) + m_4(x, y + 1) - m_5(x + 1, y + 1) + m_6(x + 2, y + 1) \quad (8)$$

$$F(x + 2, y + 2) = 2F(x + 1, y + 2) - F(x, y + 2) + m_7(x, y + 2) - m_8(x + 1, y + 2) + m_9(x + 2, y + 2) \quad (9)$$

When the sensor unit is scanned in the Y-axis direction,

$$F(x, y + 2) = 2F(x, y + 1) - F(x, y) + m_1(x, y) - m_4(x, y + 1) + m_7(x, y + 2) \quad (10)$$

$$F(x + 1, y + 2) = 2F(x + 1, y + 1) - F(x + 1, y) + m_2(x + 1, y) - m_5(x + 1, y + 1) + m_8(x + 1, y + 2) \quad (11)$$

$$F(x + 2, y + 2) = 2F(x + 2, y + 1) - F(x + 2, y) + m_3(x + 2, y) - m_6(x + 2, y + 1) + m_9(x + 2, y + 2) \quad (12)$$

Using these equations, the 2D cross-sectional profile of the entire plane can be obtained sequentially from the heights of the initial points and the sensor outputs during the scan.

3. Simulation and results

In order to confirm the validity of reconstructing calculations and the effect of reducing the influence of accumulated errors, measurement simulations using the six-point method and the nine-point method were performed using a numerical analysis software (MATLAB, MathWorks Inc.).

The measurement range is 360 mm square with 13×13 (169) points at 30 mm intervals. The profile model is represented by equation (13): In this profile model, two sinusoidal waves along the X-axis and along the Y-axis, those wavelengths are 720 mm and amplitudes are 25 μm , were added to the plane (maximum height is 50 μm).

$$F(x, y) = 25 \sin(\pi \times x / 360) + 25 \sin(\pi \times y / 360) \quad (13)$$

To each displacement sensor output, a random error represented by a normally distributed random number with an average of 0 μm and a standard deviation of 0.05 μm was added. As translational motion errors, pitching errors, and rolling errors shown in Table 1 were given when the sensor unit scans in the X- and Y-axis directions. In this table,

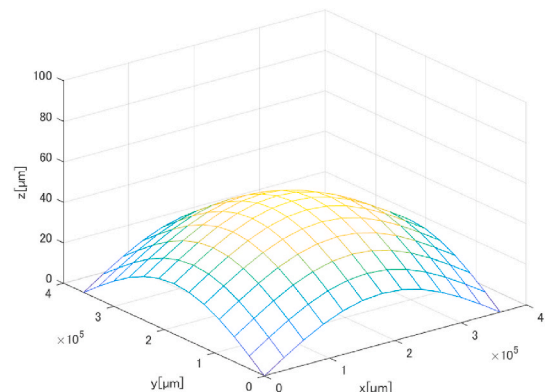


Fig. 3. Reconstructed 2-D profile.

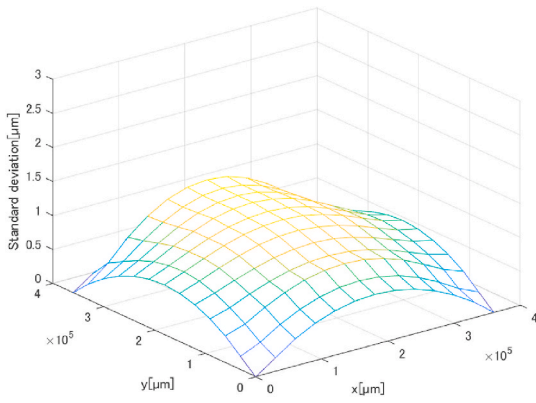


Fig. 4. Standard deviation of residual error by six-point method.

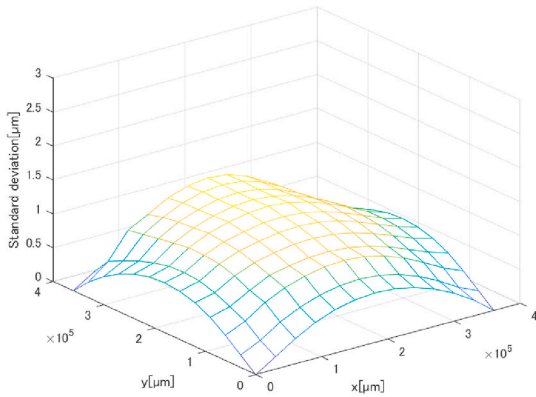


Fig. 5. Standard deviation of residual error by nine-point method without averaging.

the pitching and rolling errors are expressed as the values corresponding to the vertical displacement of the adjacent sensor.

After calculating the sample profile using the multipoint method, the residual error was calculated after correcting a vertical shift and inclinations about the X-axis and the Y-axis based on the height of the three points $(x, y) = (0, 0), (0, 360),$ and $(360, 0)$ for comparison with the original profile. Monte Carlo simulations with 10000 repetitions were performed, and the standard deviation of the residual error was calculated in micrometer for each measurement point.

3.1. Confirmation of principle

First, we checked whether the six-point and nine-point methods for planar profile measurement are feasible. Fig. 3 shows the result of reconstructing the sample profile without adding any random error to the sensor outputs $m_1 \sim m_9$. The original sample profile is reproduced and the residual error is only due to rounding error.

3.2. Results of six-point method

A result of the Monte Carlo simulation of the six-point method is shown in Fig. 4. The vertical axis indicates the standard deviation of the residual error, the difference between the calculated profile and the sample profile.

The maximum value of the standard deviation is $1.269 \mu\text{m}$ at the point $(x, y) = (180, 210)$ and the mean value of the standard deviation is $0.809 \mu\text{m}$.

As the result of data connection of 13×13 (169) points, the random error of $0.05 \mu\text{m}$ added to each sensor output was accumulated, and a cumulative error of approximately 25 times appeared at the maximum

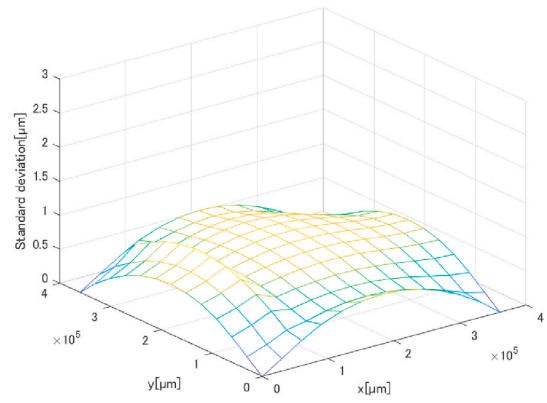


Fig. 6. Standard deviation of residual error by nine-point method with averaging.

point near the center.

In this method, the height data along the main scanning direction were obtained by the three-point method from the outputs of sensors 1 to 3, and connects them using sensors 4 to 6. In other words, this result corresponds to the result of simply connecting the height data calculated by the three-point method with the relation between the lines. Though the six-point method is the minimum configuration without using an external angle monitor or a reference mirror, it can successfully reconstruct the 2D profile.

3.3. Results of simple nine-point method

At first, simulations were carried out simply applying Eqs. (7)–(12). The result is shown in Fig. 5.

The maximum value of the standard deviation is $1.295 \mu\text{m}$ at the point $(x, y) = (180, 240)$, and the mean value of the standard deviation is $0.822 \mu\text{m}$.

The maximum value of the cumulative error was about 26 times the random error given to each sensor.

The result of the nine-point method was approximately the same as those of the six-point method for both the maximum and mean values of the standard deviation. This is because, except for the difference in data connection in the sub scanning direction, the simple nine-point method also connects 1D data, calculated by the three-point method, to each other like the six-point method. Since the connected data are equivalent, it is considered that the results are almost the same. Therefore, the result of the simple nine-point method will be set as the reference for comparison.

3.4. Accuracy improvement with averaging

In the nine-point method, multiple height data can be calculated at most points.

The point (4, 3) in Fig. 2 is taken as an example for explanation. The height of the sample profile $F(4, 3)$ can be calculated from the sensor output data of $m_3(4, 3), m_6(4, 3),$ and $m_9(4, 3)$ respectively. The number of height calculation results obtained for each measurement point is shown below.

- 3 times: $m_3(x, y), m_6(x, y), m_9(x, y)$ at $x \geq 4, y \geq 3$
- 2 times: $m_3(x, y), m_6(x, y)$ at: $x \geq 4, y = 2$

By averaging the results of these multiple height calculations, random error can be expected to be reduced.

The results of the averaged nine-point method is shown in Fig. 6.

The maximum value of the standard deviation is $0.878 \mu\text{m}$ at the point $(x, y) = (210, 210)$, which is 68% of the simple nine-point method. The mean value of the standard deviation is $0.668 \mu\text{m}$, which is 81% of the simple nine-point method.

In the averaged nine-point method, three height data are averaged at

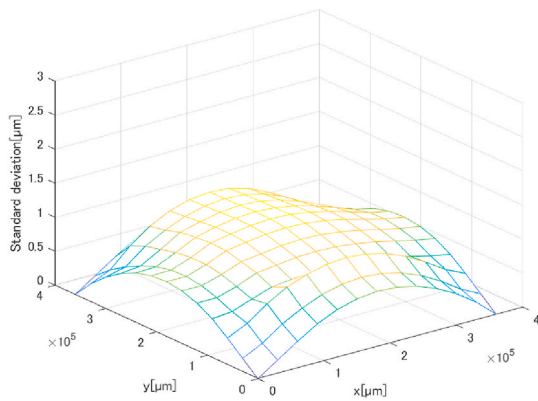


Fig. 7. Standard deviation of residual error by improved nine-point method.

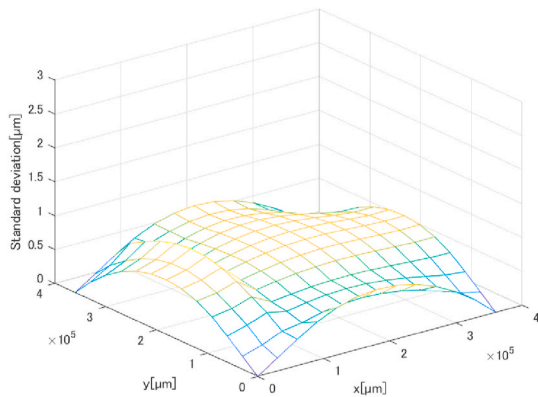


Fig. 8. Standard deviation of residual error by improved nine-point method with averaging.

11×10 points, which is 65% of the total 169 points of 13×13 . In general, averaging three completely independent values would be expected to reduce the random error by $1/\sqrt{3}$ (approximately 58%), but since only 65% of the points can be averaged three times, the rate of error reduction is lower than that.

3.5. Improved nine-point method

In the nine-point method, when moving in the main scanning direction, the three-point method can be applied to three sets of data from sensors 1–3, 4–6, and 7–9, and three independent sets of translational motion error and pitching error can be calculated. As the pitching error is not affected by the rolling error, three measurement results can be averaged. By using the averaged pitching data, it is expected to reduce the influence of random errors in the data connection and allows for more accurate data connection. The simulation result of the improved nine-point method are shown in Fig. 7.

The maximum value of the standard deviation without averaging is $1.161 \mu\text{m}$ at the point $(x, y)=(180, 240)$, which is 90% of the simple nine-point method. The mean value of the standard deviation is $0.790 \mu\text{m}$, which is 96% of the simple nine-point method. In this result, the standard deviation decreases for both the maximum value and the mean value, but the reduction rate is not large.

As the improved nine-point method also provides multiple measurement data at most of the measurement points, the results by applying the averaging method is shown in Fig. 8.

The maximum value of the standard deviation of the is $0.864 \mu\text{m}$ at the point $(x, y)=(210, 210)$, which is 67% of the simple nine-point method. The mean value of the standard deviation is $0.665 \mu\text{m}$, which is 81% of the simple nine-point method. This result is almost the same as that of the averaging method, and the effect of the improved method was not clear. The reason is that only the pitching error was averaged this time. It is expected that a larger reduction rate can be obtained by adopting an algorithm that averages the corrected translational error in consideration of the rolling error.

4. Conclusions

The authors has been proposed the principle of the six-point method using a 3-2-1 layout of displacement sensors and the nine-point method using a 3×3 layout of displacement sensors.

Numerical simulations to confirm the validity of the proposed principle. As a result, it was shown that the six-point method, the minimum configuration that calculates the planar shape with only six displacement sensors, can obtain the equivalent result to that of the simple nine-point method.

In the nine-point method with averaging, it was confirmed that the maximum and average values of the standard deviation of the reconstructed profile were reduced to 68% and 81%, respectively compared with the simple nine-point method. The improved nine-point method, which averages three data of the pitching errors to improve the accuracy of data connections, was also proposed and this method reduced the standard deviation, but effectiveness is limited.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Numbers JP17K06082, JP21K03791.

References

- [1] D.J. Whitehouse, "Some theoretical aspects of error separation techniques in surface metrology"1976, *J. Phys. E Sci. Instrum.* 9 (1976) 531–536.
- [2] H. Tanaka, H. Sato, Basic characteristics of straightness measurement by sequential two-point method, *J. JSME Int. J. Ser. CC* 48 (436) (1982) 1930–1937 (in Japanese).
- [3] W. Gao, S. Kiyono, On-machine profile measurement of machined surface using the combined three-point method, *JSME Int. J. Ser. C* 40 (2) (1997) 253–259.
- [4] C. Elster, I. Weingartner, M. Schulz, Coupled distance sensor systems for high-accuracy topography measurement, *Precis. Eng.* 30 (1) (2006) 32–38.
- [5] I. Weingartner, C. Elster, System of four distance sensors for high-accuracy measurement of topography, *Precis. Eng.* 28 (2004) 164–170.
- [6] L. Lahousse, S. Leleu, J. David, O. Gibaru, S. Ducourtieux, Z calibration of the LNE ultra precision coordinate measuring machine, in: 7th EUSPEN International Conference, 2007.
- [7] H. Shimizu, R. Yamashita, T. Hashiguchi, T. Miyata, Y. Tamaru, Square layout four-point method for two-dimensional profile measurement and self-calibration method of zero-adjustment error, *Int. J. Autom. Technol.* 12 (5) (2018) 707–713.

Ryotaro Fujiwara, Hiroki Shimizu*
Kyushu Institute of Technology, Fukuoka, Japan

* Corresponding author.

E-mail addresses: p104108r@mail.kyutech.jp (R. Fujiwara), shimizu.hiroki885@mail.kyutech.jp (H. Shimizu).