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Construction of Concepts Images from Mathematical Models Obtained with the Tracker Software

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Construção de Imagens de Conceitos a partir de Modelos Matemáticos obtidos com o Programa Tracker

Resumo

Este trabalho tem por objetivo investigar o potencial de aliar a metodologia de Modelagem Matemática com o uso do programa Tracker com estudantes da graduação em Matemática-Licenciatura na reconstrução de modelos físicos, para construção de conceitos de função. Para isso foi realizado um estudo qualitativo de caráter exploratório, envolvendo atividades de campo. Nossos resultados parciais apontam para uma maior compreensão na construção dos modelos, já que os modelos matemáticos podem ser recriados a partir da correlação a modelos físicos já validados, também verificou-se que o programa Tracker possibilita uma excelente percepção visual de diferentes imagens de conceitos, que compreendemos favorecer a construção de definição de novos conceitos ou modificação e ampliação de conceitos já construídos no cognitivo do aluno.

Palavras-chave: Mathematical Modeling; Mathematics Teaching; Tracker Software.

Abstract

This work aims to investigate the potential of combining the Mathematical Modeling methodology with the use of the software Tracker with undergraduate students in Mathematics in the reconstruction of physical models for the construction of function concepts. For this, a qualitative of exploratory character study was carried out, involving field activities. Our results point to a greater understanding in the construction of the models, since the mathematical models can be recreated from the correlation to physical models already validated. We also verified that the Tracker software enables an excellent visual perception of different concept images, that we understand to favor the construction of definition of new concepts or modification and expansion of concepts already built in the student's cognitive.

Keywords: Modelagem Matemática; Ensino de Matemática; Programa Tracker.

MSC: 97B50; 97D50; 97B50.

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1 INTRODUCTION

Currently, there is a lot of discussion about the teaching adopted by the teachers from mathematics courses. The greatest difficulty for students in understanding the mathematics presented is in mentally building what is approached in an expository way, regardless of the education level.

Teaching mathematics in nowadays requires confronting students with problems using methods that provide opportunities to create strategies, build hypotheses and solve problems. Tall and Vinner [1] argue that a given concept should not be developed from its formal definition. For its understanding it is necessary that the student is familiarized with the process of its formalization, before it is presented. Then the authors argue that must be presented different visions of a certain concept, enabling to construct various mental representations, so they can appropriate concepts images what the students are able to produce their own definitions of concepts.

In this work we combine the Mathematical Modeling with the use of the Tracker software to help in the construction of concepts image, as this software allows the exploration of models that represent physical phenomena generated from different situations. This way, we define the problem: What is the potential of combining the software Tracker in the construction of mathematical models?

Thus, our objective is to investigate the potential of combining the Mathematical Modeling methodology with the use of the software Tracker with undergraduate students in Mathematics in the reconstruction of physical models for the construction of functions concepts.

2 CONCEPT IMAGE AND CONCEPT DEFINITION

Tall and Vinner [1] argue that a given concept should not be worked from its formal definition and for that they developed the theory of concept image and concept definition. For such a concept to be understood it is necessary the student has a previous familiarization with its formalization.

When being stimulated to work with an object, the subject is provoked and several mental representations start to appear, such as: visual images, impressions, experiences and properties which can be elaborated through thought processes about these mental representations [2]. The authors refer to this process as concept image, because they are the ones that "(. . .) describe the total cognitive structure that is associated with the concept" [1, p. 152]. This concept is developed over the years from the people experiences and can be changed as new experiences are experienced.

The Concept Definition is formed from the Concept Image and is understood by the authors as any form of representing the concept image through words. "It may also be a personal reconstruction by the student of a definition. It is then the form of words that the student uses for his own explanation of his (evoked) concept image." [1, p. 2].

In this sense the authors defend that for a formal definition to be understood by the student it is essential they had the opportunity to experience different experiences which help in the construction of the image of the concept to be studied. In this way we seek in this experience to offer opportunities for students from the manipulation and graphic visualization of the studied phenomena to reinforce their conceptual images in order to be able to build their own concept definition.

3 MATHEMATICAL MODELING

For Almeida, Silva and Vertuan [3] a mathematical model is a conceptual, descriptive or explanatory system which occurs from a language or mathematical structure with the purpose of describing or explaining the behavior of the other system making predictions about it.

In this perspective the authors understand the mathematical model is the set of symbols and mathematical relationships that seek to translate a phenomenon or situation in real life and the model can be represented by numerical expressions or formulas, diagrams, graphs or geometric representations, algebraic equations, tables, software, etc.

For Bassanezi [4, p. 16] the Mathematical Modeling "consists of the art of transforming problems of reality into mathematical problems and solving them by interpreting their solutions in the language of the real world", and he proposes a sequence of steps to be followed to the construction of the desired model, they are:

Experimentation: information on the experiment will be compiled. For the author in this step the modeler's knowledge and experience are trivial to direct the other steps.

Abstraction: its purpose is to obtain mathematical models for the situation or problem addressed in the experiment. At this time, the selection of variables and the relationships between them describe the evolution of the system. The formulation of the problem must be clear and operational being constituted through a scientific question that must make the relationship between the variables or events involved in the phenomenon. At this point, the hypothesis is built which directs the investigation.

Resolution: in this stage the mathematical model is manipulated which is linked to the degree of complexity contained in its formulation. Furthermore, computational resources provide opportunities for better understanding of the phenomenon.

Validation: it is the stage where checking the acceptance or rejection of the proposed model. Now, the models and the hypotheses will be tested with the experimental data comparing the solutions and predictions with the data obtained in the real system. The acceptance or not of the model will depend on the factors that the modeler conditions the objectives and available resources.

Modification: it is the last step in the modeling process. Here the models can be improved if necessary and their reformulation becomes essential in the process.

4 METHODOLOGY

Our research is characterized as qualitative of exploratory character involving field activities. Initially, a bibliographic survey of the existing literature on the Tracker was made [5, 6], after studies on Mathematical Modeling [4] and about use of Digital Technologies [7].

Inspired by physical studies, the students chose two phenomena based on their community reality. Thus, we chose to investigate the skateboard tricks movement and the child swing movement. For this purpose, the two movements were filmed to obtain data and later discuss and build the models. Finally, different physical phenomena were investigated using the Tracker software and the construction of models of these movements related to the studied phenomena.

These models were constructed about Bassanezi steps [4], where the studies were carried out remotely in periodic meetings between teachers and students where doubts and discussions could be stimulated through virtual contact in a group of mobile app and in discussion rooms.

5 RESULTS AND DISCUSSIONS

5.1 The First Activity

The first activity to be analyzed is related to a very common sports practice in the squares in Itaqui city, located in the Rio Grande do Sul state, Brazil: skateboard tricks. First, the students filmed the trick and later its analysis was performed on the Tracker. During the trick study they concluded more than one function would be needed to represent it, dividing into three movements as can be seen in Figure 1.

The next phase was the individual analysis of each movement observing their data and graphs separately. The first movement made the students relate the phenomenon like an oblique launch [8, 9], and they found it could be described by linear and quadratic functions, presented in the Equations (5.1). Thus, they found the Equations (5.2) in the curves adjustment by the Tracker to represent the movement as can be seen in Figure 2.

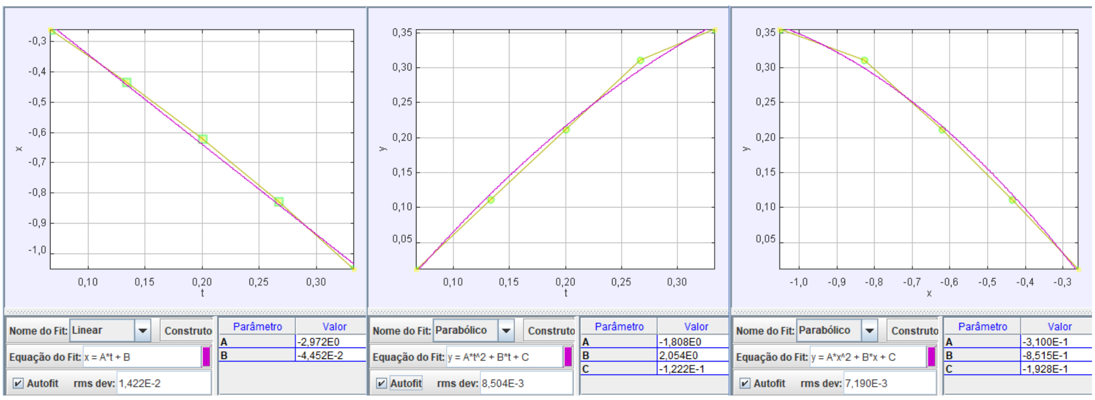
Figure 1: Skate movement analysis. Movement 1 in yellow; Movement 2 in red; Movement 3 in blue.

$$
\begin{aligned}
x(t) &= x_0 + v_{0x}t \\
y(t) &= y_0 + v_{0y}t - \frac{1}{2}gt^2 \\
y(x) &= y_0 + v_{0y}\left(\frac{x - x_0}{v_{0x}}\right) - \frac{1}{2}g\left(\frac{x - x_0}{v_{0x}}\right)^2
\end{aligned} \tag{5.1}
$$

where x and y represent the horizontal and vertical positions (m), t represents the time (s), (x_0, y_0) represents the starting position, v_{0x} and v_{0y} represent the initial velocity components, g represents the gravity acceleration.

$$
\begin{aligned}
x(t) &= -0.04452 - 2.972t \\
y(t) &= -0.1222 + 2.054t - 1.808t^2 \\
y(x) &= -0.1928 - 0.8515x - 0.31x^2
\end{aligned} \tag{5.2}
$$

Figure 2: Curves of the jump movement in the box with the relations $x(t)$, $y(t)$ and $y(x)$.



The curve adjustments in the Tracker software also show the Root Mean Square Error (dev rms in Figure 2)

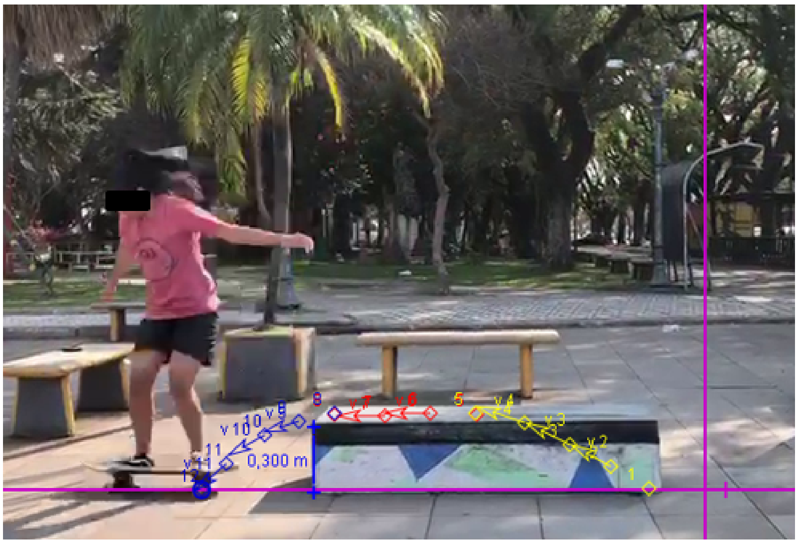
$$E = \sqrt{\frac{\sum_{i=1}^{n}(\widehat{y}_i - y_i)^2}{n}},$$

where $\widehat{y}_i$ represent the calculated values, y_i the experimental values and n the number of data. We can visualize for the adjusted equations (5.2) we have respectively $E = \{0.01422; 0.008504; 0.00719\}$.

Observing the velocity intensity data $v(t)$ and velocity angle $\theta(t)$ in relation to the Ox axis the software presents vectors (as shown in Figure 3) and average data calculated at each three points. As there are only three data at the first movement no one curve was adjusted then we highlight the approximate initial data, $v_0 = 3,105$ m/s and $\theta_0 = 151,2^{\circ}$.

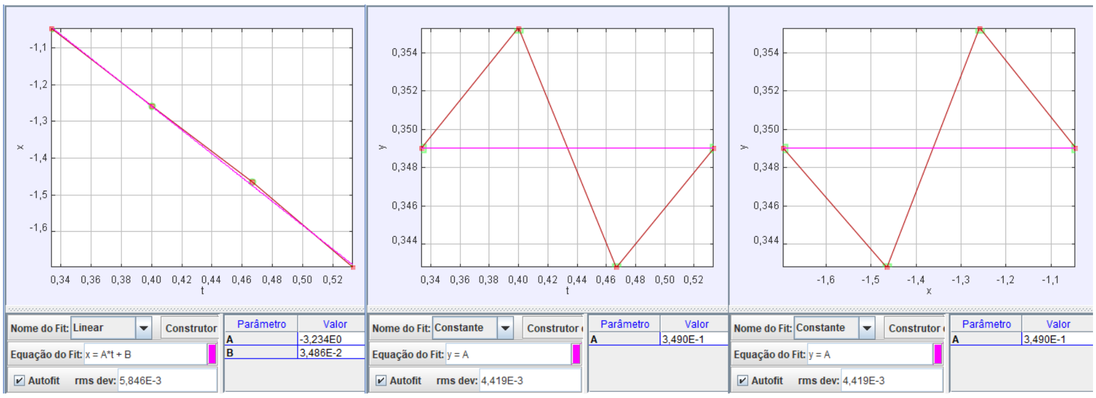
The analysis of the second movement describes a rectilinear trajectory; therefore, it was associated with uniform horizontal movement [10], which is described, by linear function and constants according the Equations (5.3). The adjusted equations can be seen in Figure 4 and presented by the Equations (5.4). We highlight the constant relations are confirmed by the small variation in height y. The errors are given by $E = \{0.005846; 0.004419\}$, $v_0 = 3,141$ m/s and $\theta_0 = 180,9^{\circ}$.

Figure 3: Velocity vectors in skate movement.



$$x(t) = x_0 + v_{0x}t$$
$$y(t) = y(x) = y_0 \tag{5.3}$$

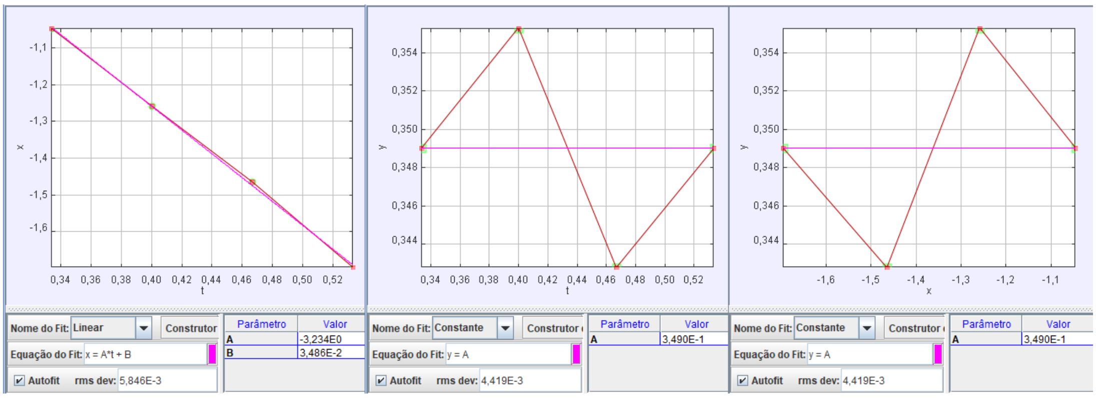
Figure 4: Curves of the movement on the box with the relations $x(t)$, $y(t)$ and $y(x)$.



$$\begin{aligned} x(t) &= 0.03486 - 3.234t \\ y(t) &= y(x) = 0.349 \end{aligned} \tag{5.4}$$

Finally, the descent of the box was interpreted as a horizontal launch [8] with the movement described by linear and quadratic functions which can also be represented analytically through the Equations (5.1). The curves adjustments can be seen in Figure 5 which generated the Equations (5.5) that analytically describe the observed movements with errors given by $E = \{0.01378; 0.00853; 0.003738\}$. We still got $v_0 = 2,506$ m/s and $\theta_0 = 197,4^o$.

Figure 5: Curves of the third movement with the relations $x(t)$, $y(t)$ and $y(x)$.



$$\begin{aligned} x(t) &= -0.4689 - 2.316t \\ y(t) &= -0.5135 + 3.615t - 3.717t^2 \\ y(x) &= -2.133 - 2.964x - 0.8836x^2 \end{aligned} \tag{5.5}$$

5.2 The Second Activity

This activity is part of the student's observation about the existing leisure in their neighborhoods specifically the moment of coming and going of a child swing. Just like the first phenomenon it was also filmed and inserted in the Tracker for further discussion and analysis as can be seen in Figure 6.

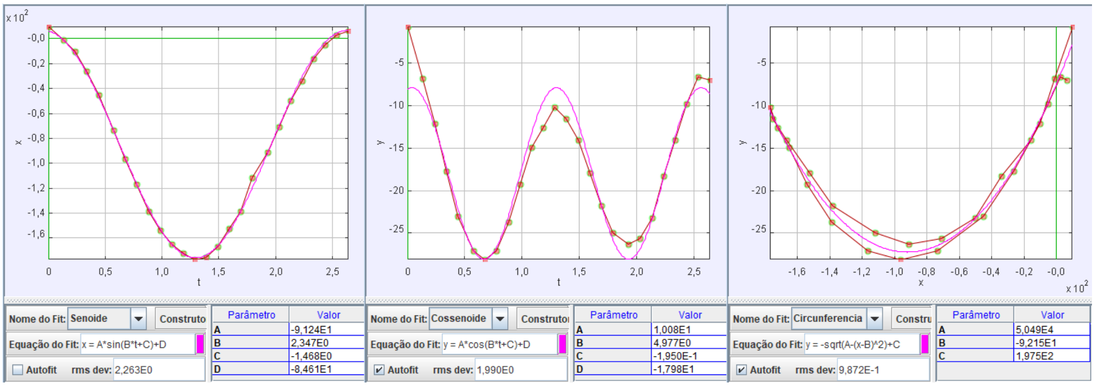
Figure 6: Child swing movement analysis.

When analyzing the movement of a child sitting on a swing it was realized that the movement can be compared to that of a simple pendulum [10]. The movement can be described by sine (or cosine) functions and by a root function (from a semi-circumference) as shown in Equations (5.6). Using the Tracker software the functions that represent the movement in a given period were adjusted which can be viewed in Figure 7 and described in the Equations (5.7).

$$\begin{aligned} x(t) &= A \cdot \sin\left(\sqrt{\frac{g}{L}}\, t\right) \\ y(t) &= B \cdot \cos\left(\sqrt{\frac{g}{L}}\, t\right) \\ y(x) &= -\sqrt{r - (x - x_c)^2} + y_c \end{aligned} \tag{5.6}$$

where x and y represent the horizontal and vertical positions (cm), t represents the time (s), L represents the rope length, A and B represent amplitude, g represents the gravity acceleration,(x_c, y_c) represents the center and r the circumference radius.

Figure 7: Child Curves of the child swing movement with the relations $x(t)$, $y(t)$ and $y(x)$.



The sine and cosine functions of the Equations (5.7) needed to be expanded with a new parameter D for adjustment in an equation $A \cdot \cos(Bt+C)+D$ due to the chosen positions of the coordinate axes. Adjustment errors are given by $E = \{2.263; 1.99; 0.9872\}$.

$$
\begin{aligned}
x(t) &= -91.24 \cdot \mathrm{sen}(2.347t - 1.468) - 84.61 \\
y(t) &= 10.08 \cdot \cos(4.977t - 0.195) - 17.98 \\
y(x) &= -\sqrt{50490 - (x + 92.15)^2} + 197.5
\end{aligned} \tag{5.7}
$$

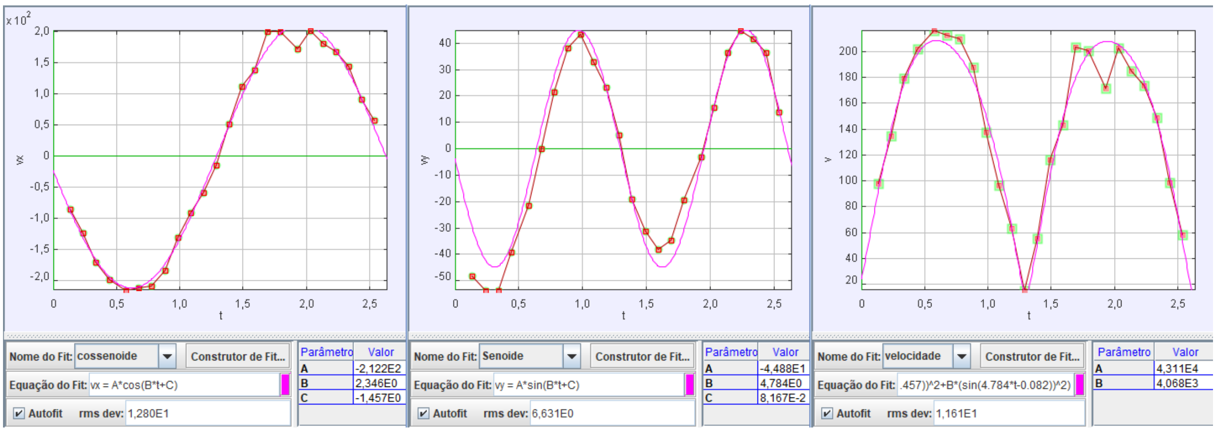
As the child swing movement contains more available speed points although they are averages for every three points, the Equations (5.8) was considered which they were adjusted in the Tracker by Equations (5.9) and can be seen in Figure 8. Such adjustments showed the following errors $E = \{12.8; 6.631; 11.61\}$.

$$
\begin{aligned}
v_x(t) &= A \cdot \sin\left(\sqrt{\frac{g}{L}}\, t\right) \\
v_y(t) &= B \cdot \cos\left(\sqrt{\frac{g}{L}}\, t\right) \\
v(t) &= \sqrt{v_x^2 + v_y^2}
\end{aligned} \tag{5.8}
$$

where v represents the velocity intensity, v_x and v_y represent the velocity components.

$$
\begin{aligned}
v_x(t) &= -212.2 \cdot \cos(2.346t - 1.457) \\
v_y(t) &= -44.88 \cdot \mathrm{sen}(4.784t - 0.082) \\
v(t) &= \sqrt{43110 \cdot \cos^2(2.346t - 1.457) + 4068 \cdot \sin^2(4.784t - 0.082)}
\end{aligned} \tag{5.9}
$$

Figure 8: Velocity curves of the child swing movement.



6 CONCLUSIONS

Returning to the problem described in this work our studies have shown some limitations and advantages in relation to the study of these phenomena analyzed from the use of Modeling combined with the Tracker software. We can point out the students had a greater graphic perception because the software allows a detailed analysis of the movement presenting different images and relations of the same concept.

However, there were some difficulties in its use because they without previous contact in addition to relating the studied phenomena to physical equations and discussing a mathematical language of their interpretation. Thus, we observe that we can explore more observable variables, mainly with regard to the comparison of the adjusted equations and the physical models already validated.

Our studies are still incipient and we are convinced we need to deepen our research in order to explore this method further. However, there is a great potential to unite mathematical modeling and digital tools in the physical phenomena analysis. It allows the student a greater understanding in the models construction since the mathematical models can be recreated from the correlation to physical models already validated and the Tracker software enables an excellent visual perception of different concepts images.

REFERENCES

[1] D. Tall and S. Vinner, “Concept image and concept definition in mathematics with particular reference to limit and continuity”, Educational Studies in Mathematics, vol. 12, no. 2, pp. 151–169, 1981. https://doi.org/10.1007/BF00305619

[2] A. S. G. Leão and V. Bisognin, “Construção do conceito de função no ensino fundamental por meio da metodologia de resolução de problemas”, Educação Matemática em Revista, vol. 1, no. 10, pp. 27–35, 2009.

[3] L. W. Almeida, K. P. Silva, and R. E. Vertuan, Modelagem Matemática na Educação Básica. São Paulo: Contexto, 2012.

[4] R. C. Bassanezi, Ensino-aprendizagem com modelagem matemática. São Paulo: Contexto, 2002.

[5] R. Magarinus, L. Buligon, and M. M. Martins, "Uma proposta para a introdução do ensino de funções através da utilização do programa Tracker", Ciência e Natura, vol. 37, no. 3, pp. 481–498, 2015.

[6] D. Brown, R. Hanson, and W. Christian, "Tracker: Video analysis and modeling tool", 2021. Access date: 20 jun. 2021.

[7] M. C. Borba and A. Chiari, Tecnologias Digitais e Educação Matemática. São Paulo: Livraria da Física, 2013.

[8] M. A. Cavalcante and C. R. C. Tavolaro, "Estudo do lançamento horizontal utilizando o computador para aquisição e análise de dados", Caderno Brasileiro de Ensino de Física, vol. 14, no. 3, pp. 276–287, 1997.

[9] D. Halliday, R. Resnick, and J. Walker, Fundamentos de Física: Mecânica, vol. 1. São Paulo: LTC, 10 ed., 2016.

[10] A. R. Souza, D. F. V. Arévalo, E. R. Viveiros, and O. L. C. Abril, Ensino de Ciências e Matemática IV: temas de investigação, ch. Uso de GeoGebra para analisar o movimento harmônico simples por meio do pêndulo simples. São Paulo: UNESP, 2010.

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