

Analytical and numerical approach on quenching friction-induced oscillations in MDOF-systems by the use of high-frequency excitation

Simon Keller^{1,*} and Wolfgang Seemann¹

¹ Karlsruhe Institute of Technology (KIT), Institute of Engineering Mechanics, Kaiserstraße 10, 76131 Karlsruhe, Germany

Dry friction is well known to lead to undesired, self-excited vibrations which negatively influence a process or cause discomfort. One attempt to quench these oscillations are superposed high-frequency vibrations. A system consisting of two visco-elastically coupled masses on a moving belt subject to high-frequency excitation is analyzed. A multiple scales method is used to find an analytical approach of the system dynamics, which leads to the equation for the slow motion of the system, which is of interest. Linearizing about the quasi-equilibrium and analyzing the eigenvalues, statements about the stability of the new equilibrium point can be given. These semi-analytical results are compared to the numerical solution of the full system and the influence of certain parameters is outlined.

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1 Introduction

Systems subject to dry friction often show self-excited oscillations due to the unsteady friction force and its negative slope at a small relative velocity. This property holds for systems with one degree of freedom, as well as for systems with multiple degrees of freedom. The application of an additional external excitation at a high frequency can influence the system dynamics in such way, that the self-exciting effect is suppressed and a stable equilibrium point can exist. For SDOF-systems, this is analytically shown in [3] and in [2] good accordance between experiment and simulations is shown. In [1], this approach is extended to a MDOF-system and first investigations on the stabilizing effect of high-frequency excitation are outlined.

2 Model and analytical approach

To analyze the capability of high-frequency excitation to quench friction-induced oscillations in a MDOF-system, the model depicted in figure 1 is used. The model is already reduced to dimensionless parameters ζ (mass ratio), γ (stiffness ratio),

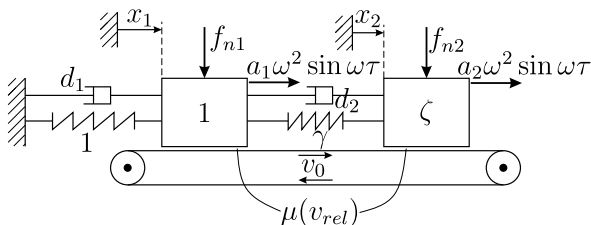


Fig. 1: Model of two coupled masses on a moving belt.

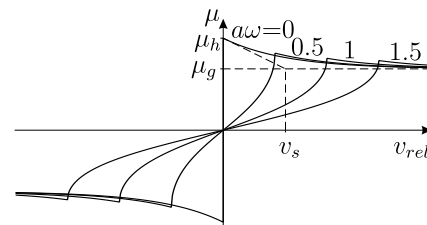


Fig. 2: Effective friction coefficient depending on the macroscopic relative velocity v_{rel} and velocity amplitude $a\omega$.

d_i (damping coefficients), f_{ni} (normal forces), v_0 (belt velocity), a_i (excitation amplitudes) and ω (excitation frequency), according to [1]. For the friction contact, Coulomb friction is used, where the friction coefficient μ depends on the relative velocity. The exponential form

$$\mu(v_{rel,i}) = \text{sign}(v_{rel,i}) (\mu_g + (\mu_h - \mu_g) \exp(-|v_{rel,i}|/v_s)) , \quad v_{rel,i} = \dot{x}_i - v_0$$

is used, which is depicted in figure 2 at $a\omega = 0$. According to [3] and [1], a multiple scales approach is used, where additionally to the slow time τ a fast time scale $\theta = \omega\tau$ is introduced. Furthermore, the motion is divided into a slow part and a fast, small part $\underline{x}(\tau) = \underline{z}(\tau) + \underline{\varphi}(\theta)/\omega$. This leads to the ordinary differential equation of the slow motion given by equation (1). Here, the positive definite matrices M (mass), K (damping) and C (stiffness) coincide with those of the full system. The derivation of this equation is shown in [1]. The influence of the high-frequency excitation and the friction force is included by the term $\langle \mathcal{r}(v_{rel}) \rangle$, which is the mean of the friction force, averaged over one period of the excitation force.

$$M\ddot{\underline{z}}_{,\tau\tau} + K\dot{\underline{z}}_{,\tau} + C\underline{z} = \langle \mathcal{r}(v_{rel}) \rangle , \quad \langle \mathcal{r}(v_{rel}) \rangle = \frac{1}{2\pi} \int_0^{2\pi} \mathcal{r}(v_{rel}) d\theta , \quad r_i(v_{rel,i}) = -f_{n,i} \mu(v_{rel,i}) \quad (1)$$

* Corresponding author: e-mail simon.keller@kit.edu, phone +49 721 608 42660, fax +49 721 608 46070



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The differential equation for the slow motion is linearized about its equilibrium point z_e , which leads to two additional entries in the damping matrix \tilde{K} , which can be interpreted as damping against the environment

$$z_e = C^{-1} \langle r(v_{rel} = -v_0) \rangle, \quad \Delta z = z - z_e, \quad M \Delta z_{,\tau\tau} + \tilde{K} \Delta z_{,\tau} + C \Delta z = \underline{0},$$

$$\tilde{K} = K - \left. \frac{\partial \langle r(v_{rel}) \rangle}{\partial z_{,\tau}} \right|_{z_{,\tau}=0} = K + \begin{pmatrix} f_{n1} \bar{\mu}'_1 & 0 \\ 0 & f_{n2} \bar{\mu}'_2 \end{pmatrix}.$$

The magnitudes $\bar{\mu}'_i$ describe the slope of the effective friction force at the belt velocity (cf. figure 2) for each mass, respectively. If the respective mass is excited and $v_0 < a\omega$, the slope is positive, which has a damping effect. Otherwise, the slope is negative, which may lead to a self-exciting effect. By applying an exponential ansatz, a statement about the stability of z_e can be made by using HURWITZ-criteria. This yields conditions for $\bar{\mu}'_1$ and $\bar{\mu}'_2$, depending on the remaining parameters.

3 Results

In figures 3 and 4, two stability maps for the equilibrium point z_e are depicted. The dots represent the stability behavior of z_e for stable (black) and unstable (gray) solutions resulting from the numerical solving. The thick black line dividing the areas results from linearization and HURWITZ-criteria. For the numerical solution, the initial conditions are chosen as the equilibrium point z_e plus a small perturbation. Numerically and analytically approached results show very good accordance. When both masses are excited and $v_0 < a_i\omega$, both slopes are positive, the related point is in the upper right quadrant, which is always stable. When only one mass is excited, one slope is positive, while the other one is negative, which yields a point in the lower right or upper left quadrant. As seen, there are still stable areas, which means that the system can be stabilized even if only one mass is excited. Figure 3 is for zero damping, whereas figure 4 contains additional external damping, which increases the valid region for a stable equilibrium point. If the masses are strongly coupled, i. e. $\gamma > 1$, the area for stable

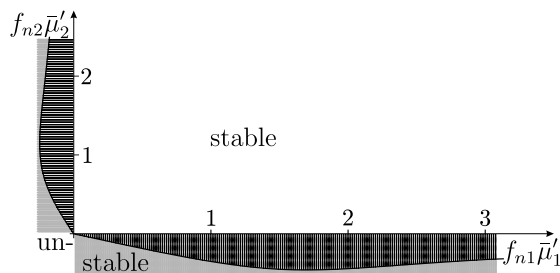


Fig. 3: Stability map, determined analytically and numerically, $\zeta = 1$, $\gamma = 1$, $d_1 = d_2 = 0$, $a = 0.01$, $\omega = 100$, $v_0 = 0.5$, $\mu_h = 0.6$, $\mu_g = 0.4$, $v_s = 0.6$.

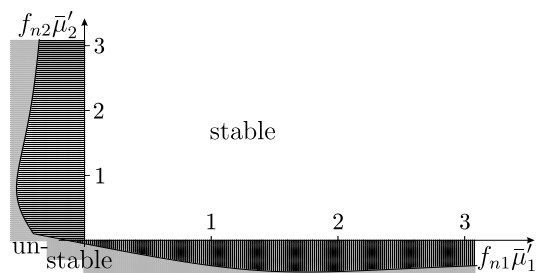


Fig. 4: Stability map with additional external damping, determined analytically and numerically, $d_1 = d_2 = 0.05$.

points increases. For a weak coupling, i. e. $\gamma < 1$, both slopes have to be positive in order to stabilize the equilibrium point. If the mass ratio is not unity, the area of stable points decreases in such way, that at least the lighter mass has to be excited to enable the stabilizing effect. For the shown results, not only the behavior concerning stability shows good accordance between numerical and analytical approach, but also the course of the time function.

4 Conclusions

Based on the procedure in [1], the MDOF-system subject to dry friction was analyzed in a more general way. The importance of the slopes of the effective friction forces and their impact on the stability of the equilibrium point was outlined by a stability map. The self-excitation due to dry friction can be suppressed by external high-frequency excitation, even if only one mass is excited. The comparison between numerical and analytical approach shows that the multiple scales method and linearization yield good results concerning the stability of the equilibrium point. Additional external damping enlarges the parameter range for stable equilibrium points. A stiff coupling between the two masses also increases the stable area, whereas a weak coupling decreases it. A mass ratio differing from unity results in the fact, that at least the lighter mass has to be excited in order to enable a stabilization of the equilibrium point.

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