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Math and the Mouse: Explorations of Mathematics and Science in Walt Disney World

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ABSTRACT: Math and the Mouse is an intensive, collaborative, project-driven, study away course that runs during the three-week May Experience term at Furman University and has many of the attributes of a course-based undergraduate research experience in mathematics. We take twelve students to Orlando, Florida to study the behind-the-scenes mathematics employed to make Walt Disney World operate efficiently. Students learn techniques of mathematical modeling (mostly resource allocation, logistics, and scheduling models), statistical analysis (mostly probability, clustering, data collection, and hypothesis testing), and flow management (queuing theory and some beginning flow dynamics) in an applied setting. Through planned course modules, collaborative activities, conversations with guest speakers, and three group projects, one of which is of the students' choosing, this academic experience provides an engaged learning experience that shows how material from eleven academic courses comes together in connection with real-world applications.

Keywords: Course-based undergraduate research experience, mathematical modeling, statistical analysis, flow management, data analytics, optimization, algorithm design, decision-making, study away experience, Maymester, Walt Disney World

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1 Introduction

Furman University is a private liberal arts college in Greenville, South Carolina. In addition to its commitment to providing students access to an excellent well-rounded education, Furman supports and encourages opportunities for students to engage in high-impact experiences that enhance their education outside the traditional classroom. These often occur as research, internship, study away, and community engagement opportunities, but at times, they do blend together. Often, students are participating in these high-impact experiences alongside Furman peers and faculty. This helps students develop meaningful mentoring relationships and support networks across campus in a deeper way than often develops in a traditional classroom.

One of these experiences is Math and the Mouse: Explorations of Mathematics and Science in Walt Disney World. We designed this to be an intensive, collaborative, project-driven, study away course that runs during the optional three-week May Experience term at Furman University. We (three mathematics professors) take twelve students to Orlando, Florida to study the behind-the-scenes mathematics that is used to make Walt Disney World operate efficiently. The Walt Disney World Resort consists of four theme parks: Magic Kingdom, Epcot, Disney's Hollywood Studios, and Disney's Animal Kingdom. It also includes two water parks, dozens of onsite hotels, hundreds of restaurants and shops, four golf courses, a sports complex, and numerous transportation systems (including monorails, buses, boats, trains, trams, and gondolas). Students learn techniques of mathematical modeling (mostly resource allocation, logistics, and scheduling models), statistical analysis (mostly probability, clustering, data collection, and hypothesis testing), and flow management (queuing theory and some beginning flow dynamics) in an applied setting. Students are asked to complete projects in each of these areas and a final project of their own design. These elements combine to offer a students an impactful course-based undergraduate research experience (CURE) in mathematics.

We have run four iterations of the Math and the Mouse course: May 2014, 2016, 2018, and 2019. The main focus of this article will be the most recent offerings in May 2018 and 2019, but many of the themes carry throughout all course iterations, even if the execution of the courses differed in some ways from year to year. It is worth noting that even though this is a "three-week" course, because this is a study away course, we utilize every day of the term, including weekends and holidays, and also utilize many of the waking hours for both structured and unstructured course activities. Students earn two credits for completing this course (traditional semester-long courses at Furman are four credits). This course does not satisfy any graduation or major requirements beyond accumulating credit hours, so it is an elective course for all students who take it. Finally, while tuition for the course is covered as part of the comprehensive fee that all students pay each academic year, the student pays for the travel, food, and park expenses (usually $3500-$4000), although scholarships are available to help defray this cost.

Students are selected to participate in the course via an application and interview process. Each student who applies gets an interview with the professors where we investigate the student's interest in mathematics and analytics as well as their past interactions with Walt Disney World. We typically ask students about a project that they are curious to explore while on the experience. The acceptance rate for the course is around 50% with preferences towards students whose interests in the course stem from curiosity about the park operations or mathematical modeling involved. The students have come from many different majors: applied mathematics, biology, business, chemistry, computer science, economics, health sciences, mathematics, music, neuroscience, physics, politics and international affairs, and Spanish. While their backgrounds differ, there is a common curiosity and a shared interest in developing quantitative understanding of how math, statistics, and operations research are used to solve real-world problems. In the context of Walt Disney World, the ideas we introduce are often unseen to the casual vacationer because they help aid efficiency and safety upon which the enjoyment of activities of their vacation is built.

We have taught 48 students in Math and the Mouse across the four iterations, 71% of which were female and 29% male. This gender breakdown is not typical in mathematics courses, but study away experiences do tend to skew towards higher female participation. As Auchincloss et al. point out, research experiences have the opportunity to be additionally impactful to women and minorities because stronger professional relationships form with faculty and fellow peers, and this helps enhance the socio-emotional networks upon which they rely as they navigate the challenges of their time in college and careers beyond [ALB]. The depth of relationships created among students and between students and faculty in this course are much richer than the typical advising, mentoring relationships we see with our traditional classroom students. Given that 77% of our students are either finishing their first or second years when they take this course, having an immersive, study away CURE early in their career affords them a few years to continue developing the mentoring relationships with faculty while still an undergraduate.

Given the wide range of interests and an age range that spans from rising sophomores to recently graduated seniors, we serve students with a variety of mathematical experiences and maturity levels. There is a prerequisite of Calculus I for the course, and within multiple cohorts of 12 students, we have had some students who took Calculus I as their first and only math class in college as well as those who have already completed a math major and are pursuing graduate school in mathematics. The immersive, collaborative nature of our course helps to engage students of all levels to work together, to help each other, and explore the world around them.

As educators, it is astounding to watch students rise to the challenges we ask of them, watch them interact with and present their work to Disney professionals, and grow in their confidence over the short span of three weeks. For some students that we continue working with after the class is over, the longer term growth is also incredibly rewarding to observe. As advisors and mentors of these students, we are proud that they are pursuing their interests after college, often by attending graduate school or working in a field related to content they saw in Math and the Mouse. Of the students who have graduated or clarified their post-graduation plans, 78% of the students from all four cohorts have gone on to pursue a graduate degree or find a job in a field related to the topics in the course (33% pursued opportunities in data analytics, data science, or statistics, 17% in operations research or industrial engineering, 14% in mathematics or applied mathematics, and 14% in computer programming).

As we further introduce the course in subsequent sections, let us be clear about the level of the content. By needing to meet the needs of both the first year student straight out of Calculus I while simultaneously meeting the needs of the graduated math major, the use of collaborative assignments and activities coupled with reflection, blog writing, and in-depth group projects allows everyone to grow from the experience, both academically and personally. Students develop an intense sense of self-efficacy, ownership, and pride over their work in this course, but they work tirelessly to earn it. Because our student cohorts are selected through an application process, we do benefit from the fact that all of our students are very interested in learning what we are studying and are completely committed to working hard during waking hours for 21 days straight. This persistence and resilience does not come easily, but it serves the students well through their college years and beyond.

2 Course Topics and Design

The course centers around mathematical topics that are often important in business settings, i.e. using mathematics to make decisions. We break the course into thirds with a project due at the end of each of the three weeks. The first week focuses on data analytics. Data analytics is a growing field of study within applied mathematics and related disciplines, and Disney uses analytics in a variety of settings, including ticket pricing, customer experiences, and insights from MagicBand[1] data. Along with focusing on the gathering, visualization, and analysis of data, we also discuss clustering, rating/ranking data (also known as scoring), and regression analysis.

The second week is spent learning about modeling and algorithm design. Optimization is a field where practitioners seek to mathematically model real-world scenarios, create techniques and tools to analyze these models, and construct optimal solutions. From assigning each of ten thousand employees to a job for each time period of each day to determining schedules of bus routes throughout the park, Disney makes use of optimization models to plan a variety of operations. Some of the models that we introduce include resource allocation/workforce scheduling models (such as assigning employees/characters to tasks/locations), network flow models (such as modeling tours of the park), and transportation/routing models (finding optimal routes through the park depending on congestion). Once a mathematical optimization model is built, we show the students how to generate an optimal solution, if possible, to their models. Through a series of activities and a project, we introduce students to the broad area of algorithm design where students study techniques that generate optimal or near-optimal solutions to the classes of models above.

[1] A MagicBand is an RFID-enabled wristband that facilitates theme park entry, FastPass redemption (a type of attraction reservation), opening hotel room doors, and payment options.

The final week focuses on decision-making under uncertainty as well as some miscellaneous topics. Almost every decision made by managers at Disney is made under uncertainty of conditions. Variation in the form of crowd size and composition makes planning workforce needs difficult, and Disney has made decisions about how to distribute FastPasses (attraction reservations) and Extra-Magic Hours (extra operating hours for those customers who stay in onsite hotels) to try control crowd size variability. We introduce uncertainty through learning some elementary probability distributions (the binomial, the geometric, and the normal distribution). We apply the first two of these distributions to the study of simple, discrete queues. The normal distribution is used to introduce hypothesis testing. Students are given examples of hypothesis tests that companies like Disney might conduct. For example, one hypothesis might be to study the impact on wait times for Rock 'n' Roller Coaster after the introduction of the Slinky Dog Dash roller coaster in Disney's Hollywood Studios. In the next section, we will discuss projects from recent course iterations that represent each of these broad topics.

Each class day is structured so that part of the day is spent in class and part of the day is spent in the parks. Class time is spent introducing new material and activities. Topics covered are presented in Table 1.

Data, Clustering, and Regression	Operations Research Models
Rating and Ranking	Linear Programming
Excel's Optimization Solver	Knapsack Problem
Data Visualization (Tableau)	Finding Optimal Solutions
Network Optimization	Probability and Discrete Distributions
Construction Heuristics	Normal Distribution
Genetic Algorithms	Statistics and Hypothesis Testing
Agent-Based Modeling	Theme Park Physics

Table 1: Course Topics

Park time is spent reinforcing the topic idea from the current material. For example, when introducing algorithm design to the students we have them participate in a group activity we call the Traveling Tourist Problem (TTP) [BHH]. The TTP is a time-dependent version of the Traveling Salesman Problem (TSP) that is faced by theme park guests who try to design a route through the park to minimize the amount of time it takes them to ride every attraction. The problem is more difficult than the traditional TSP because the time one spends waiting in line at an attraction depends on the time of day the guest enters the queue for that attraction. For example, if one wants to ride Space Mountain quickly, showing up within the first hour of park opening will keep the wait time to a minimum, but showing up to Space Mountain three hours after opening might cost a guest an hour or more waiting in the queue line. For the TTP activity, we break the students into groups of three and give them an Excel spreadsheet that lists the average wait time for each attraction for one of the four WDW parks broken down into fifteen-minute time intervals. Figure 1 shows how the wait time fluctuates across time periods for an average day in May 2019[2] [TP1]. Each line represents a different attraction in the Magic Kingdom. This data is provided by Touring Plans, an analytics-based company that offers vacationers data-driven advice for things such as hotels, park navigation, and the like.

In the TTP activity, each group of three students is required to plan a route through the park to ride a predetermined list of attractions. The number of attractions changes by park, but it is usually in the range of 18-22. As they work together to formulate a plan, they are required to record the heuristic approach they are employing to design their route. For instance, if they are choosing the next attraction to visit based on walking distance, projected wait time, or something else, they must try to design an algorithm to solve the specific instance of the TTP. The following day, each team, including the professor team, actually races through the park implementing their route to see who designed the fastest route in practice. Figure 2 shows a winning route through the Magic Kingdom from the 2018 class. As you can see, winning routes result from not making decisions based on distance as the route zig-zags throughout the park, but rather balances distance and anticipated wait times. Afterwards, students are asked questions about what went wrong, what went right, or under what conditions might their designed route be the

[2]Note that the offerings of this course reported in this paper preceded the COVID-19 pandemic, so this data is not representative of current wait times for a number of reasons.

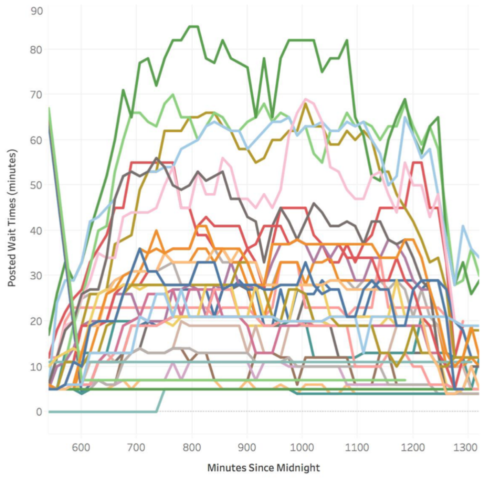

Figure 1: Predicted wait time data for 34 Magic Kingdom attractions for a typical day in May 2019 (data source: [TP1]).

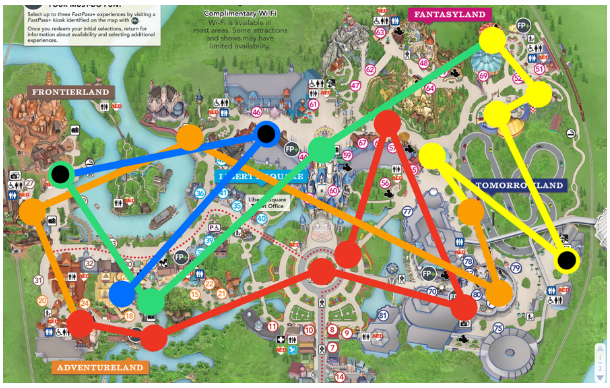

Figure 2: Route taken by the winning group for the Traveling Tourist Problem (TTP) activity. The colors show time progression from park open (red) to end of tour (blue). The black circles represent where the group used a FastPass.

most efficient. To show the relevance of the problem, the Time-Dependent Traveling Salesman Problem is the same problem faced by FedEx, UPS, and Amazon Prime drivers daily.

Another park activity that we use is to have students gather data while standing in line so that at a later point in the course we can visualize posted versus actual wait times or perhaps the relative rates of admittance of the FastPass and regular lines. An additional park activity was inspired by watching a YouTube video made by Touring Plans [TP2]. They were testing what order to visit the attractions Soarin', Frozen Ever After, and Test Track at Epcot right at 'rope drop' (Disney-speak for when the park allows guests to start accessing attractions for the day). Their results, however, were inconclusive due to a breakdown of Test Track during the day they visited. We performed the same experiment on several occasions while visiting Epcot at park opening. Dividing students into pairs to control for pace, we had the student groups ride the three attractions in the order of one of the six permutations of the three rides to determine which permutation was the fastest. After gathering data on several occasions, we used this activity to introduce hypothesis testing in the course.

We have been fortunate to meet and develop relationships with professionals at Disney and companies tangential to Disney through networking at conferences. To further reinforce the relevancy of the material, activities, and projects in the course, the students have several meetings with these professionals, including several alumni of the course who work at Disney. These meetings allow for opportunities for the students to network and hone communication skills within a professional setting. These meetings also reinforce the relevancy of the material as professionals often talk about the larger-scale projects that they have completed in the same area.

In addition to communicating project work to industry professionals, students have the opportunity to communicate with a general public audience by authoring entries on the course blog [Blog]. The students take turns doing this, and the activity serves many purposes. For the student authors it provides a chance to think about how the activities of the day fit into what we are learning and present it to a general audience. It also gives family members and friends of the course participants a way to stay up to date on the happenings of this unique study away course. While the blog writing provides some moments for reflection, because it has a public audience and is written by student pairs, we also ask students to write individual reflection pieces not intended for a public audience. We interlace these prompted writing exercises with the demands of other activities or project work, but in general, they provide a good opportunity for students to reflect and connect the things they are learning about it class with the hands-on activities and projects they are exploring.

3 Course Projects

The group projects are designed in consultation with professionals at Disney that work in the areas of analytics and decision sciences. Since the projects are smaller versions of ones that these professionals would work on themselves, the projects are highly relevant. All projects are completed in groups, and the groups change for each project. This gives students the opportunity to collaborate with students from a variety of backgrounds. Students are able to discuss their analysis and presentation with the professors at any time, including while standing in line or transporting to and from the parks. In fact, these out-of-class conversations are often the most insightful. Professors' evaluations of the students include making sure each student is participating and contributing. Every student must exhibit a significant role in the project execution and analysis as well as in the creation and delivery of each presentation. The expectation is that each student is able to understand the project's goals, analysis, and results, and articulate such information on demand.

3.1 Project #1: Mobile Mickey Bar Ice Cream Stand (Data Analytics)

In this project, we ask the students to map out the best location for where a mobile Mickey Bar ice cream stand should be placed in the Magic Kingdom for each 15 minute time period of the day. We let the student decide on what a "best" location means in this context as there could be competing objectives. Students are separated into three teams and each team concentrates their efforts to locate one stand in separate, non-overlapping areas of the Magic Kingdom. Applying some of what they have learned about clustering, the students are required to have the stand located near the clustering centroid of a k-means clustering for their portion of the park. This helps to simulate the spatial distribution of demand. After each group finds where to place their stand in their part of the park throughout the day, they form one large group to collaboratively find where to best place three concession stands throughout the day, unrestricted by park area.

To estimate crowd densities in the park by time of day, students use wait time data, supplied by Touring Plans, for an average day in May as a proxy for density. Using Excel, students perform a k-means clustering analysis, weighted by crowd densities, in their designated area for each 15-minute interval during the day and see how the centroid moves throughout the day. If the clustering algorithm says that the stand should be located on top of an attraction, restaurant, or shop, they are provided path locators where carts can be placed nearby. Students are required to create a visual of their results using Tableau and create and give a professional presentation of their results. Figure 3 provides a visual representation of each group's solution to the first part of the project (where each group has been assigned a portion of the park). During the presentations, students field questions from the audience and consider things like what real-world conditions might occur that would make their cart's placement dictated by the clustering algorithm less than ideal.

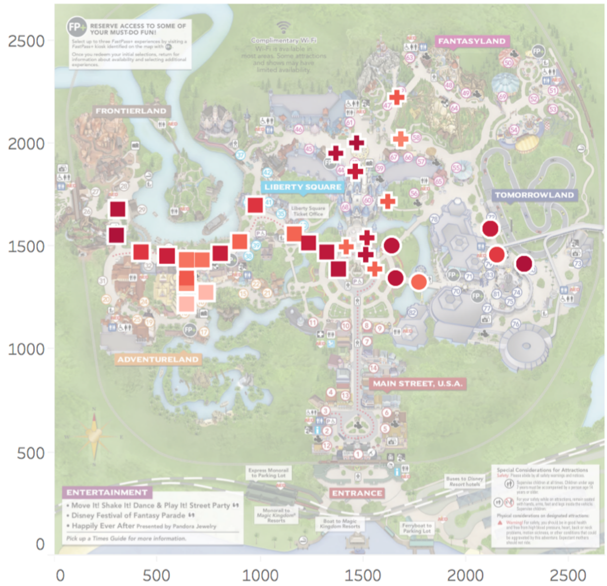

Figure 3: Time progression of each group's solutions for where to place their Mickey Bar ice cream stands. Time progression moves from lighter pink shades at park opening to dark red shades at park closing.

3.2 Project #2: Workforce Scheduling (Optimization Modeling and Algorithms)

Our second project asks students to schedule workers to a restaurant in the Magic Kingdom throughout the day. Students are given a chart of the number of workers needed in each fifteen-minute segment from 8:00am to 2:00am each day. We break the project into two stages to have the students struggle with creating decision variables, objective functions, and constraints in this real-world problem setting. The second stage uses the same basic design as the first but increases the difficulty associated with the model. Students use Excel's Solver Add-In to find a solution to their models. The Solver functionality of Excel allows students to enter linear or nonlinear (including integer) programming models and produce an optimal or near optimal solution quickly.

1. In stage 1 of the project, students are asked to design a workforce schedule that minimizes the number of shifts needed to cover all of the workload at a Disney restaurant during the day. The students must design constraints that are subject to the following restrictions:
   (a) Each cast member gets a continuous shift of eight hours (32 time periods) that can begin in any time period.
   (b) Each cast member gets a 60-minute break that can either be assigned to begin after working three hours or after working five hours.
   (c) Non-break shift times should completely cover the workload in each time period.
2. Stage 2 modifies some constraints to allow for each cast member to get a continuous shift of eight hours (32 time periods) that can begin in any time period, and each cast member gets a 60-minute break that cannot be assigned within 90 minutes of the start or end of the eight-hour shift.

As with the first project, students work in groups of four to set up a model and program it into Excel. They must make and give a professional presentation of their results. Students are also encouraged to think of different objectives where their solution might change.

3.3 The Final Project

The last project in the course is of the students' design and highlights the scientific method. This final project is where the attributes of a CURE are best on display after the prior coursework and experiences.

Throughout the course the students are encouraged to observe areas of the parks that they encounter decision-making on the part of Disney and formulate research questions. On several occasions they have journaling and brainstorming opportunities to come up with questions and hypotheses. The professors have many conversations with the groups to condense the questions to something that can be completed in the time remaining. Once questions have been formed, each group must decide how best to gather data related to their hypothesis question. Although the students are divided into groups, all students and the professors help each group gather data. The groups then perform the analysis, change hypotheses, and gather more data until the question is settled. As with previous projects, they make a presentation about their conclusions. Examples of three recent final projects are presented in Section 4.

4 Examples of Independent Research

In this section we look at three different final projects from the May 2018 and 2019 iterations of this course highlighting the different types of questions that students find interesting. These projects also highlight that students choose different tools of the course to apply to their final projects.

4.1 Optimization: Maximizing Mickeys in the Magic Kingdom

The genesis of the idea for the Maximizing Mickeys project was the observation that wait times for character autographs in the parks are quite long. Families enjoy meeting Mickey and pals, and they are willing to spend time in line to do so. Not surprisingly, lines for popular characters like Mickey can be especially long. Interestingly, in 2014, months after the 2013 release of the movie Frozen, the wait times to meet Anna and Elsa were regularly in the five-hour range! The students in our 2018 cohort considered the idea of spreading the demand for Mickey around the parks so that no single line would be too long. Knowing that Disney magic would be spoiled if it were possible to see two different Mickeys at the same time, the goal of the project was to use concepts from network (graph) theory to find the maximum number of Mickeys that could be spread around the park in such a way that no two could be spotted simultaneously.

The students began their work with a Magic Kingdom map, and they identified places in the park where it would be appropriate for Mickey to stand and greet people. They labeled 103 spots, and these became the nodes in their network (see blue dots in Figure 4). The next thing was to determine the lines of sight between these spots, and so the students took to the field. They visited each of the sites and made note of which other sites could be seen. When sites A and B were in sight of one another, they added an edge between the nodes corresponding to those sites. In total, they added 226 edges.

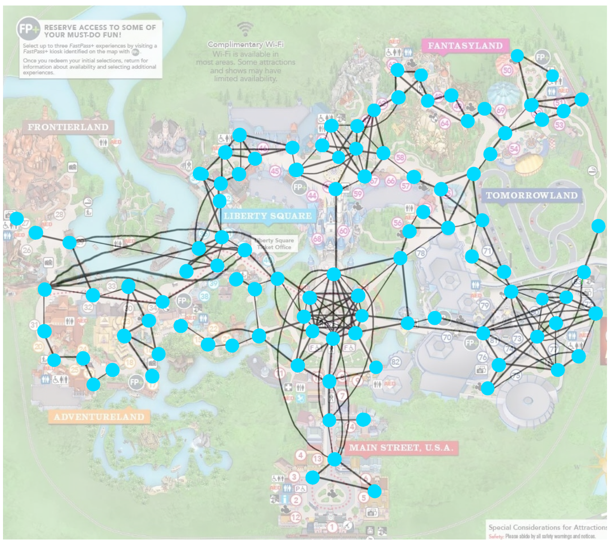

Figure 4: The 103 possible locations in the Magic Kingdom where a character could greet guests. The edges in this network represent sight lines between possible character greet locations.

During our class time, the students learned about independent sets of vertices in a network. Such a set is one for which no vertex in the set is joined by an edge to any other vertex in the set. In our Mickey example, an independent set would represent locations such that no Mickey could see any other Mickey. While this is important, it is also the case that we want to avoid two Mickeys being seen from a single spot. This led to a discussion of 2-independent sets, which are sets for which no two vertices are within two steps of one another. Identifying a maximum 2-independent set in our Mickey network, then, would guarantee that no guest who sees one Mickey could view another Mickey at a different location at the same time, and thus provide the positions for our team of Mickeys.

Finding such sets can be difficult, especially in large networks. Our students learned how to create a digital representation of their network, and they were able to use software (Mathematica) to identify a maximum 2-independent set. Using this set, they determined that they could safely place 25 Mickeys around the park (denoted by red stars in Figure 5). Of course, there would likely be problems in getting

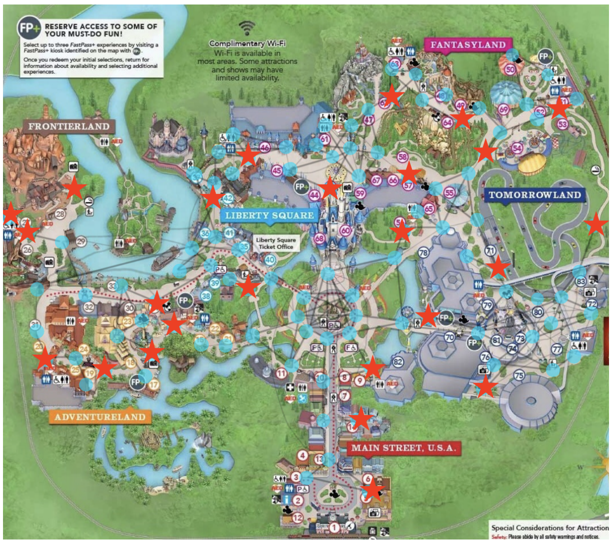

Figure 5: The red stars represent the 25 locations in the maximum 2-independent set where Mickeys can be placed to avoid one guest seeing two Mickeys simultaneously.

all of these Mickeys to their spots without being seen together. And so while it may not be feasible to place this many in the park, the project idea and the strategies and tools used were great for the students to encounter.

4.2 Algorithm Design: Online Bin Packing at Tower of Terror

The Tower of Terror is a popular thrill ride that looms over Sunset Boulevard at Disney's Hollywood Studios. It is set in an old Hollywood hotel and takes guests to the Twilight Zone thanks to a trip on an erratic service elevator. The immersive theming and thrills coming from randomized drop sequences make this a popular attraction. As such, wait times can be long and observant guests may notice that there are empty seats in each elevator. While students in our 2018 cohort were experiencing this attraction over the class's first few visits to Hollywood Studios, they began to question the efficiency of the loading process. Those conversations eventually developed into a research project exploring whether Disney's algorithm for seating guests in the Tower of Terror service elevator vehicles was the most efficient for optimizing guest throughput or could they design a better one.

The 21 seats on a Tower of Terror service elevator are arranged in three rows. The back row has seven consecutive seats in one row whereas the first and second rows have seven seats split into three and four seats on either side of an aisle (see Figure 6). As such, this seating configuration provides a more complicated organization challenge for the cast members than a ride like Rock 'n' Roller Coaster, where each row only has two seats. The queue ends with a cast member who directs each party to their assigned position in the service elevator loading area. The cast member asks the party at the front of the queue how many are in their party and then makes an assignment. If there are not enough available

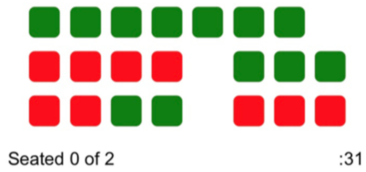

Figure 6: Schematic of seating configuration on Tower of Terror with 21 seats organized in three rows, two of which have a center aisle (image from [Bou]).

seats left in the elevator to seat the party at the front of the line, the cast member moves sequentially back further into the line a bit to find a party that will fit in the remaining empty seats. There is only a limited amount of time to search the queue before the elevator is launched so that the ride can stay on schedule. The first attempt at filling the elevator is equivalent to an online bin packing problem in the mathematical literature, where someone is trying to arrange items of various sizes that arrive sequentially into a bin of finite volume. When the cast member looks beyond the party at the front of the queue, the online nature of the bin packing problem is modified.

The students were interested in determining if the way that Disney cast members were filling the elevators is most efficient or if a different algorithm could employ a better fill rate. To explore this, the students first collected data from the actual operations of the loading process at Tower of Terror. They observed:

1. Party size data

2. Number of open seats before looking deeper into the line

3. Number of open seats after looking deeper into the line

They also asked cast members their approach to seating parties of various sizes in the service elevator configuration. It was reported to our students that they put even parties on the side of the car with four seats (left as shown in Figure 6) and odd parties on the side of the car with three seats (right). The students labeled this approach to filling a service elevator as Algorithm 1. They developed two other approaches to filling the elevator:

1. Algorithm 2: Put parties of size one or three on the right, parties of size two or four on the left, and split larger parties into smaller groups of size two, three, or four.

2. Algorithm 3: Fill left to right regardless of party size.

Notice that Algorithms 1 and 2 are similar, but differ in their treatment of larger parties. Since the students were not able to run the actual placement of guests into service elevators, they had to simulate the seating process. Earlier in the course, we gave students an activity on algorithm design that got them thinking about this idea of online bin packing problems. We commissioned a website [Bou] that simulated party sizes based on sampling from a variety of party size distributions. It allowed users to seat parties and kept track of the statistics of each packing of a service elevator for aggregate analysis. The students working on this Tower of Terror project utilized the simulation component and asked their classmates to help them collect the simulation data upon which they would ultimately use hypothesis testing to determine if they could find a better algorithm than Disney uses in practice.

They customized the simulation website to use the observed distribution of party sizes when filling the simulated queue. Then they asked their peers to run many simulations of an assigned algorithm (1, 2, or 3) and they analyzed the simulated data to compare with the data collected onsite in Hollywood Studios. In practice, the Disney cast members launched elevators that had 9.06% of seats that were empty before they looked back the queue (online bin packing problem) and 3.10% that were empty after utilizing more of the queue (modified online bin packing). Table 2 shows the percentage of empty seats for each algorithm as well as the observed data, delineated whether it was before or after the cast member bypassed the first person in the queue (online versus modified bin packing).

Algorithm	% Empty (Online)	% Empty (Modified)
Disney Observed (Alg. 1)	9.06%	3.10%
Simulated Algorithm 1	7.69%	0.84%
Simulated Algorithm 2	7.47%	1.92%
Simulated Algorithm 3	11.65%	3.76%

Table 2: Percentage of seats left empty when elevator launches for the online scenario (when only the party at the front of the line is considered for seating) and the modified scenario (where a small number of parties are considered for filling the elevator after bypassing the first party in line).

The simulations using Algorithms 1 and 2 performed better than the Disney cast members in practice at a statistically significant level (all p-values< 0.02), while Algorithm 3 performed worse. One explanation for the improved performance of the simulated tests for Algorithms 1 and 2 compared with the observed Disney performance is that there are time constraints on the Disney employees where they can only realistically look a certain distance into the queue before they must launch the elevator. In general, these findings are consistent with the students' hypotheses.

4.3 Design of Experiments and Hypothesis Testing: Dominance on Toy Story Midway Mania

Toy Story Midway Mania is an interactive ride at Disney's Hollywood Studios that combines virtual carnival games with a zippy indoor ride. Guests sit in four-person carts where two pairs of players face in opposite directions and are whisked from one video game screen to the next shooting at targets to accrue points. At the end of the ride, overall scores and accuracy percentages are displayed in the cart. The ride is enjoyed by young and old and our class is no exception.

Riding Toy Story Mania piqued the curiosity of one of our students from the 2019 cohort. She wondered if there would be a noticeable difference in points and accuracy percentages between using her dominant hand to shoot versus using her non-dominant hand. Further, she questioned whether there would be a difference in points accrued from the left-seat of the ride vehicle versus from the right-seat. The students in her group had to grapple with the issues associated with data collection in this environment. Essentially they were confronting the stumbling blocks associated with a matched pairs design of experiments (without knowing what that is exactly). They needed to try to keep external factors the same from ride to ride. To accomplish this, they paired students and faculty based on historical scores from the ride (high scorers with high scorers, etc.). The asked pairs to ride the attraction several times together switching seats and shooting hand and report score and accuracy totals back to the group. Since in each ride, a guest is only really playing against the person seated in the adjacent seat in the cart, the students reasoned that there would be consistency over trials. The students collected 80 total data points, 40 using dominant shooting hands and 40 using non-dominant shooting hands, 40 from the left side of the cart and 40 from the right side.

With the gathered data, students performed a test of hypothesis for a difference in mean scores and a difference in accuracy proportions. These tests showed a significant difference between the mean score with the dominant hand versus non-dominant hand and a significant difference between the mean score from the left side of the cart versus the right side. Each of these tests had a p-value well below 0.01. The mean score for the dominant hand was 179,403 versus a non-dominant mean of 152,938. The mean score for the left side of the cart was 178,180 versus right side mean of 153,310. They also found a significant difference between accuracy percentages for the non-dominant hand (34%) versus dominant (31%) and right side (34%) versus left side (31%). The students conjectured that people would focus harder on the task while using their non-dominant hand, increasing their accuracy but decreasing their firing rate. To try to explain the high-scores but lower accuracy from the left side of the cart, they students conjectured that there are more targets on the left-hand side of screens and that some targets move from the left side of the screen to the right side of the screen. For those left-to-right moving targets, there is a high possibility that the target would not survive to the right side of the screen.

The project emphasizes the students' use of scientific practices on their own in the course. Students proposed a hypothesis for a park activity of interest; they designed an experiment, gathered and analyzed real-world data, developed interpretations and communicated their results. To our knowledge this study had never been done before and might be of particular interest to readers of travel guides or blogs about the park (references). They did all of this with a limited knowledge of statistics and hypothesis testing, but collaboratively developed the intuition together with the instructors. In this way, they are highly motivated to discover a lot of the intricacies of the subject in a hands-on course-based undergraduate research experience.

5 Conclusion

The description of our course structure, activities, and projects already presented in this paper demonstrate that Math and the Mouse has the signature elements of a CURE. Using the guidelines that Auchincloss et al. provide in Table 2 of [ALB], students are exposed to multiple viewpoints of scientific practices. The activities and projects throughout the course are a combination of student- and faculty-driven explorations. Typically, the most student-driven work is done in the final project, both in the design and execution of the work. There are *many* uncertainties involved in the projects and activities in this class in that many elements of things (crowds, weather, etc.) are out of our control, but impact our ability to get results in an efficient manner, which is central to the condensed timeline of our course. Having the flexibility and resilience to adapt to these challenges is of utmost importance for both the instructors and the students. When we do get the data that forms the basis of a project or activity, it is inherently messy. There are often human factors in the collection of the data, but the subject of the data often relies on studying the behavior of people, an inherently unpredictable thing.

Students also face many challenges on the course. Some students have to get used to always working on a team. For some who like to work alone, working on a team brings frustrations associated with differing timelines and work ethics. Students who come to the course with a limited mathematical background can sometimes feel like their contribution is less than others. In each of the above challenges we try to encourage the students to ask questions and become teachers to help all of the participants in the group understand the mathematics involved in the project. We also help the students understand some of the unique gifts that each student brings to the course. Finally, one of the biggest challenges students face on the experience is exhaustion. We have built more “me time” into the course to combat this, and this generally helps, but occasionally there are students who opt-out of certain experiences to recharge.

The scope of work that the students engage with during this course extends beyond the course. They often make these connections through two primary avenues: first, through immersing themselves in the application with enough time and space to be curious; and second, through interacting with guest speakers, including Furman Math and the Mouse alumni. The entire course is collaborative between students and faculty, with a level of access to faculty that is not seen in traditional on-campus classes. The study away aspect of the course provides this extended access, but the fact that the course is offered in the three-week May Experience term helps because students are only permitted to take one May Experience course at a time, so their attention is not divided between classes and extracurricular activities for the duration of the program. The professors have an incredible role in this course and it certainly falls in the realm of guidance and mentorship. The mentoring relationships formed during this program between faculty and students tend to grow and expand after the three-week duration of the course. It is often the case that students will pursue more traditional summer research opportunities with Furman faculty in later summers, often with one or more of the professors that led the Math and the Mouse program. Because of the relationships and interests formed during the course, we often are chosen by students to become their academic advisors if/when they declare their major in the Mathematics Department.

Sometimes the content of the Math and the Mouse course as well as the students carry over into a summer research capacity. In our 2014 cohort, we had two talented Math and Computer Science double majors interested in further pursuing the algorithmic side of the Time-Dependent Traveling Salesman Problem (TDTSP) that the Traveling Tourist Problem introduced them to. They chose to expand their work on the TDTSP into their final project in the course and ultimately into a summer research project that led to a collaboration with Len Testa (President of Touring Plans and a computer scientist by training) and a research publication in a scholarly peer-reviewed journal [BFH]. In another summer research project, a few students in the 2018 cohort used Touring Plans survey data in an effort to develop a hotel recommender system for Touring Plans customers to use when choosing their hotel at the Walt

Disney World Resort.

Throughout the whole Math and the Mouse course experience, students face challenges, setbacks, and struggles, but they are paired with collaboration, support, and encouragement that ultimately build resilience, persistence, and self-efficacy. These long-lasting effects on the students continue to develop through deep connections with their peers and faculty directors of the Math and the Mouse program long beyond the three weeks we spend together in Florida.

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