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Holographic current correlators at finite coupling and scattering off a supersymmetric plasma

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Abstract

By studying the effect of the $\mathcal{O}(\alpha'^3)$ string theory corrections to type IIB supergravity, including those corrections involving the Ramond-Ramond five-form field strength, we obtain the corrected equations of motion of an Abelian perturbation of the AdS$_5$-Schwarzschild black hole. We then use the gauge theory/string theory duality to examine the coupling-constant dependence of vector current correlators associated to a gauged $U(1)$ sub-group of the global $\mathcal{R}$-symmetry group of strongly-coupled $\mathcal{N}=4$ supersymmetric Yang-Mills theory at finite temperature. The corrections induce a set of higher-derivative operators for the $U(1)$ gauge field, but their effect is highly suppressed. We thus find that the $\mathcal{O}(\alpha'^3)$ corrections affect the vector correlators only indirectly, through the corrected metric. We apply our results to investigate scattering off a supersymmetric Yang-Mills plasma at low and high energy. In the latter regime, where Deep Inelastic Scattering is expected to occur, we find an enhancement of the plasma structure functions in comparison with the infinite 't Hooft coupling result.

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1 Introduction

The gauge theory/gravity duality [1, 2, 3, 4] is a powerful tool that may hold the key to unlocking the mysteries of strongly-coupled field theory. Although the applicability of the duality has so far been limited to theories with a high degree of symmetry, it is nonetheless important to understand the strong-coupling regime of these theories fully, as they share many features with QCD. A very important and phenomenologically relevant example of a system which can be studied using the AdS/CFT correspondence is the quark-gluon plasma (QGP). This system, which can be obtained as a result of the collision of two heavy nuclei, such as those collisions achieved at RHIC, has shown tantalizing hints of ideal hydrodynamic behaviour (for some review articles see [5, 6, 7, 8, 9, 10, 11]).

On the other hand, if one applies the gauge theory/string theory duality to calculate the hydrodynamic properties of the large N limit of an $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma with gauge group $SU(N)$, one finds results that may be close to those measured for the QGP [5]. A lot of attention has therefore been devoted to the study of the various transport coefficients of supersymmetric plasma in the hydrodynamic regime, in an effort to more accurately model the behaviour of the quark-gluon plasma studied at RHIC. This programme of approaching the real-world QGP using AdS/CFT techniques requires taking into account many non-trivial aspects, such as the fact that the QGP contains fundamental quarks, whereas $\mathcal{N} = 4$ supersymmetric Yang-Mills theory only has fields in the adjoint representation of the gauge group. Another major ingredient that must be included is going beyond the infinite 't Hooft parameter limit, as the QGP is governed by large yet finite coupling. On the supergravity side, this corresponds to adding string-theoretic higher-curvature corrections to the gravitational background.

The hydrodynamic regime describes the behaviour of plasma at distance scales $\gg 1/T$, where T is the temperature of the plasma. One may choose to study plasma in the opposite regime, namely for distance scales $\ll 1/T$, by using a different physical probe. One can deduce many useful properties of plasma by considering its response to a hard probe, such as a photon or a parton. In this work we are concerned with Deep Inelastic Scattering (DIS) in strongly-coupled $\mathcal{N} = 4$ supersymmetric plasma at finite 't Hooft coupling. As is well-known, DIS is a key process for investigating hadronic structure. A lepton scatters from a hadron through the emission of a virtual photon with four-momentum q^μ. This photon then probes the hadron structure at distances of order $\sqrt{1/q^2}$, thus giving us valuable information about the distribution of momenta amongst the various partons comprising the hadron. The relevant quantity for studying deep inelastic scattering is the matrix element of two electromagnetic currents inside the hadron. In particular, the hadronic tensor is defined in terms of structure functions which can be extracted from the imaginary part of the two-point function of the electromagnetic current.

There are two distinct regimes for this process, separated by the coupling strength of the theory. At weak coupling the appropriate description is given by perturbative QCD. The operator product expansion of the current-current correlator is dominated by twist-two operators in the perturbative regime. On the other hand, at strong coupling, the operator product expansion is dominated by double trace operators, and the process can be studied using the gauge/string theory duality [12].

The situation for plasma is different but entirely analogous to that of scattering off a single hadron. One may now view the scattering as taking place off "quasiparticles" which constitute the plasma. Clearly, a knowledge of the structure functions governing this process yields valuable information on the dynamics of plasma in an important regime which is not probed in the hydrodynamic limit. In particular, deep inelastic scattering in $\mathcal{N} = 4$ SYM plasma at infinite 't Hooft coupling has been reported in several recent articles [13, 14, 15, 16, 17, 18, 19, 20, 21]. In the holographic dual description, finite 't Hooft coupling corrections to the structure functions of an $\mathcal{N} = 4$ SYM plasma can be investigated by considering the effect of higher-curvature terms on the vector fluctuations of the metric. This is the subject of the present article.

So far, the community has focussed a lot of its effort on finite 't Hooft coupling corrections to the transport properties of plasma in the hydrodynamic regime (see for instance [22, 23, 24, 25, 26, 27, 28, 29, 30, 31]). Such transport properties include the shear viscosity and the mass-density diffusion constants, both of which can be obtained by studying tensor fluctuations of the supergravity metric with higher-curvature corrections. One the other hand, the vector fluctuations of the metric yield quantities such as the charge diffusion and conductivity. The finite coupling corrections to these quantities have been considered so far only for the cases where the additional curvature terms have been of mass-dimension four and six. The case of $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma, where the stringy corrections are known, and are found to yield dimension eight operators, has not been considered. Therefore, the formalism we develop below for computing the effect of these dimension eight corrections on the vector fluctuations, can be applied to the computation of the charge conductivity and diffusion constants for $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma with 't Hooft coupling corrections. In this paper, we are not concerned with the hydrodynamic regime, choosing instead to focus on the effect of the finite coupling corrections on the structure functions of the plasma. We will consider the effect of the corrections on the charge transport properties in a future work [32].

Let us define the premise of the paper more carefully. We investigate the full effect of the $\mathcal{O}(\alpha'^3)$ string theory corrections to the retarded correlators of the vector currents associated with a gauged $U(1)$ sub-group of the global $\mathcal{R}$-symmetry group of $\mathcal{N} = 4$ supersymmetric YM theory. This allows us to compute the structure functions F_s with $s = 1, 2$ of a strongly-coupled $\mathcal{N} = 4$ supersymmetric YM plasma with gauge group $SU(N)$ for finite values of the 't Hooft coupling. The structure functions are extracted from the imaginary part of the retarded current-current commutator

$$R_{\mu\nu}(q) = i \int d^4x \, e^{-iq\cdot x} \, \Theta(x_0) \, < [J_\mu(x), J_\nu(0)] > \,, \tag{1}$$

where $\Theta(x_0)$ is the Heaviside function, while $J_\mu(x)$ is the conserved current associated with the gauged $U(1)$ subgroup mentioned above. The expectation value is understood as a thermal average over the statistical ensemble of an $\mathcal{N} = 4$ SYM plasma at temperature T. It is assumed that in this plasma the tensor $R_{\mu\nu}(q)$ plays an analogous role as the hadronic tensor does in deep inelastic scattering off a single hadron. In that case, the imaginary part of the hadronic tensor allows us to extract the structure functions of the hadron, which in perturbative QCD describe the partonic nature of the hadron.

The tensor structure of the retarded current-current commutator can be derived from two properties: $J_\mu(x)$-current conservation and $R_{\mu\nu}(q) = R_{\nu\mu}(-q)$, so that

$$R_{\mu\nu}(q) = \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2}\right) R_1 + \left[n_\mu \, n_\nu - \frac{n\cdot q}{Q^2}(n_\mu q_\nu + n_\nu q_\mu) + \frac{q_\mu q_\nu}{(Q^2)^2}(n\cdot q)^2\right] R_2 \,, \tag{2}$$

where the flat four-metric $\eta_{\mu\nu}$ is chosen with mostly plus signature $(-1, 1, 1, 1)$, while n^μ is the plasma four-velocity and Q^2 is the virtuality, defined as $Q^2 = q^2 - \omega^2$. In the plasma rest frame $n^\mu = (1, 0, 0, 0)$. We have also defined $q^\mu = (\omega, 0, 0, q)$ as the momentum transfer. Thus, $q \cdot n = -\omega$ and this is a negative quantity.

We define the DIS plasma structure functions as follows

$$F_1(x_B, Q^2) \equiv \frac{1}{2\pi} \, \mathrm{Im} R_1(x_B, Q^2) \,, \tag{3}$$

and

$$F_2(x_B, Q^2) \equiv \frac{-(n\cdot q)}{2\pi T} \, \mathrm{Im} R_2(x_B, Q^2) \,. \tag{4}$$

Notice that we have defined a new Bjorken variable which involves the temperature

$$x_B = -\frac{Q^2}{2(q\cdot n)T} = \frac{Q^2}{2\omega T} \,. \tag{5}$$

This paper has two main results: the first is that the 't Hooft coupling corrections to the retarded current-current correlators for the electromagnetic $U(1)$ group enhance the plasma structure functions. This is the physical result of the formalism developed below, as far as DIS is concerned.

On a more formal level, at distance scales $\ll 1/T$, the computations reported here imply that the higher-curvature $\mathcal{O}(\alpha'^3)$ operators affect the on-shell action of the vector perturbations only through the correction to the metric. This applies to the full set of corrections involving the metric and the Ramond-Ramond five-form field strength appearing in type IIB supergravity. We show below that the corrections to the Maxwell equations due to these terms only appear at very high powers of the radial coordinate u, so that their effect vanishes at the boundary $u \to 0$.

The paper is organized as follows. In section 2 we briefly describe the basic action and metric setup for the computation of the retarded current-current correlators at infinite 't Hooft coupling. In section 3 we study the string theory corrections to the metric, introducing also the *ansatz* for the vector fluctuations of the ten-dimensional metric and the Ramond-Ramond five-form field strength. In section 4 we perform a detailed study of the contributions of the type IIB string theory action at $\mathcal{O}(\alpha'^3)$ which contain the Ramond-Ramond five-form field strength. In section 5 we derive the Maxwell equations for vector fluctuations with the $\mathcal{O}(\alpha'^3)$ corrected metric. We then consider deep inelastic scattering and explicitly show the enhancement of the longitudinal and transverse plasma structure functions due to the mentioned string theory corrections. In the conclusions presented in section 6 we discuss our results. The full expression of the C^4 term when vector fluctuations are included is presented in Appendix A. In Appendix B we provide a detailed analysis of the equations of motion, along with the subtleties arising from the higher derivative terms introduced by the higher-curvature corrections to the gauge field action. In Appendix C we give details of how to solve the equations of motion with finite 't Hooft coupling corrections for the low energy regime, the physical interpretation of which is a multiple scattering series.

2 Infinite 't Hooft coupling

At infinite 't Hooft coupling, the string theory holographic dual to finite-temperature $\mathcal{N} = 4$ SYM theory is the AdS-Schwarzschild black hole solution with a five-sphere as the internal space. This is a solution of type IIB supergravity with only the leading curvature terms, namely the Einstein-Hilbert action coupled to the dilaton and the five-form field strength:

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[R_{10} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4.5!} (F_5)^2 \right] . \tag{6}$$

It is easy to check that the solution of this system for a constant dilaton and N units of five-form flux through the five-sphere is given by the metric

$$ds^2 = \frac{(\pi T R)^2}{u} (-f(u)dt^2 + d\vec{x}^2) + \frac{R^2}{4u^2 f(u)} du^2 + R^2 d\Omega_5^2 \, , \tag{7}$$

where $f(u) = 1 - u^2$ and R is the radius of the AdS_5 and the five-sphere. In these coordinates the AdS-boundary is at $u = 0$ while the black hole horizon is at $u = 1$. We denote the AdS_5 coordinates by the indices m, where $m = \{(\mu = 0, 1, 2, 3), 5\}$.

The AdS/CFT correspondence stipulates that supergravity fields are dual to certain field-theory operators. In this work we are concerned with the conserved current $J_\mu(x)$ associated with a gauged $U(1)$ subgroup of the $SU(4)$ $\mathcal{R}$-symmetry group possessed by the $\mathcal{N} = 4$ SYM theory [33]. The supergravity field corresponding to $J_\mu(x)$ is the s-wave (massless) mode of the vector fluctuation about the background of Eq.(7). More precisely, we introduce off-diagonal fluctuations $G_{\mu a}$ of the metric, where a is an index on the five-sphere, and plug this fluctuating metric into the ten-dimensional action Eq.(6), making sure that we are picking a specific Abelian subgroup [34, 35]. We employ the following *ansatz* for the perturbed metric [35, 36, 37], for a general case which includes the black brane metric with obvious substitutions, where we have imposed that the internal metric is the five-sphere

$$\begin{aligned} ds^2 &= \left[g_{mn} + \frac{4}{3} R^2 \, A_m A_n\right] \, dx^m dx^n + R^2 \, d\Omega_5^2 \\ &\quad + \frac{4}{\sqrt{3}} R^2 \left(\sin^2 y_1 \, dy_3 + \cos^2 y_1 \, \sin^2 y_2 \, dy_4 + \cos^2 y_1 \, \cos^2 y_2 \, dy_5\right) \, A_m \, dx^m \, . \end{aligned} \tag{8}$$

We write the metric of the unit five-sphere as $d\Omega_5^2$ where

$$d\Omega_5^2 = dy_1^2 + \cos^2 y_1 \, dy_2^2 + \sin^2 y_1 \, dy_3^2 + \cos^2 y_1 \, \sin^2 y_2 \, dy_4^2 + \cos^2 y_1 \, \cos^2 y_2 \, dy_5^2 \, . \tag{9}$$

Also, we use the reduction *ansatz* for the five-form field strength $F_5 = G_5 + *G_5$, where:

$$G_5 = -\frac{4}{R}\epsilon_5 + \frac{R^3}{\sqrt{3}} \left(\sum_{i=1}^{3} d\mu_i^2 \wedge d\phi_i\right) \wedge \bar{*}F_2 \, , \tag{10}$$

while $F_2 = dA$ is the Abelian field strength and ϵ_5 is the volume form of the five-dimensional metric of the AdS-Schwarzschild black hole. The Hodge dual $*$ is taken with respect to the ten-dimensional metric, while $\bar{*}$ denotes the Hodge dual with respect to the 5D metric piece of the black hole. In addition

$$\mu_1 = \sin y_1 \, , \qquad \mu_2 = \cos y_1 \, \sin y_2 \, , \qquad \mu_3 = \cos y_1 \, \cos y_2 \, , \tag{11}$$

$$\phi_1 = y_3 \, , \qquad \phi_2 = y_4 \, , \qquad \phi_3 = y_5 \, . \tag{12}$$

Inserting the above *ansätze* for the type IIB supergravity fields into the zeroth-order supergravity action of Eq.(6) and discarding all the higher (massive) Kaluza-Klein harmonics of the five-sphere, we are then left with the following action for the zero-mode Abelian gauge field A_m:

$$S = -\frac{N^2}{64\pi^2 R} \int d^4x \, du \, \sqrt{-g} \, g^{mp} \, g^{nq} \, F_{mn} \, F_{pq} \, , \tag{13}$$

where the Abelian field strength is $F_{mn} = \partial_m A_n - \partial_n A_m$, the partial derivatives are $\partial_m = \partial/\partial x^m$, while $x^m = (t, \vec{x}, u)$, with t and $\vec{x} = (x_1, x_2, x_3)$ refer to the Minkowski coordinates, and $g \equiv \det(g_{mn})$, where the latter is the metric of AdS-Schwarzschild black hole.

The equations of motion derived from the above action are just the Maxwell equations for the bulk five-dimensional Abelian gauge fields A_m on the AdS-Schwarzschild spacetime Eq.(7). By studying the bulk solutions of these equations subject to certain boundary conditions that we will specify shortly, we can obtain the retarded correlation functions [38, 39] of the operator $J_\mu(x)$. At the level of this section, these correlators would pertain to the infinite 't Hooft coupling limit. Our aim is to obtain the leading coupling-constant dependence of these correlators. We now describe how this is achieved.

3 The $\mathcal{O}(\alpha'^3)$ string theory corrections

We would like to derive the leading order α'-corrected action for the vector fluctuations of the metric. The higher-curvature corrections on the supergravity side correspond to finite-coupling corrections in the field theory. In other words, for any given field-theoretic observable $\mathcal{O}$, we can write a series $\mathcal{O}_0+\mathcal{O}_1/\lambda^{n_1}+\cdots$, where λ is the 't Hooft coupling, and n_1 is a positive number which indicates that the lowest order correction to the result at infinite coupling $\mathcal{O}_0$ need not begin at order one. The inclusion of higher-derivative corrections to the supergravity must take place at the level of the ten-dimensional action, through the evaluation of stringy corrections to Eq.(6). The leading corrections were found to begin at $\mathcal{O}(\alpha'^3)$. There is a large volume of literature on these corrections, and the initial application to holography was at zero temperature [40], where the metric was found to remain $\mathrm{AdS}_5\times S^5$, verifying certain non-renormalization theorems of CFT correlators. At finite temperature [41, 42], much of the work focussed on the corrections to the thermodynamics of the black hole. The corrections were then revisited in references [43, 44, 45], where the computation of the α'-corrected metric was improved and attempts were made to address the issue of the completeness of the corrections at leading order in α'. More recently, the conjectured lower bound for the ratio of the shear viscosity to the entropy of any material [46, 47][3], has prompted interest in the higher curvature corrections to the supergravity duals of gauge theories, primarily in the spin-2 sector of the fluctuations [22, 26]. In [50, 27] the higher curvature corrections to the dual of $\mathcal{N}=4$ SYM were parsed thoroughly to determine how they affect the metric. Our case is slightly more complicated, because we must use the corrected metric as our background and must also evaluate the action for the vector fluctuations of the metric, thereby obtaining the corrected Lagrangian for the field A_μ. There are therefore two distinct parts to the calculation: the first part consists of obtaining the minimal gauge-field kinetic term using new perturbed *and* corrected metric and five-form *ansätze*. The second part of the computation consists of obtaining the corrections to the gauge field Lagrangian coming directly from the higher-derivative operators. The reason why these two steps are distinct is that the first step will require insertion of the corrected perturbation *ansätze* into the minimal 10D supergravity two-derivative part Eq.(6). The second step requires insertion of the *uncorrected* perturbation *ansätze* into the higher-curvature terms in ten dimensions.

[3]More recent studies have shown that the conjectured universal lower bound does not hold when certain higher-derivative corrections are included. For a discussion see [48, 49] and references therein.

The corrections to the 10D action are given by [27]

$$S_{10}^{\alpha'} = \frac{R^6}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[\gamma e^{-\frac{3}{2}\phi} W_4 + \cdots \right] , \tag{14}$$

where γ encodes the dependence on the 't Hooft coupling λ through the definition $\gamma \equiv \frac{1}{8}\,\xi(3)\,(\alpha'/R^2)^3$, with $R^4 = 4\pi g_s N \alpha'^2$. Setting $\lambda = g_{YM}^2 N \equiv 4\pi g_s N$, γ becomes

$$\gamma \equiv \frac{1}{8}\,\xi(3)\,\frac{1}{\lambda^{3/2}} \,. \tag{15}$$

The W_4 term is a dimension-eight operator, and is given by

$$W_4 = C^{hmnk}\, C_{pmnq}\, C_h{}^{rsp}\, C^q{}_{rsk} + \frac{1}{2}\, C^{hkmn}\, C_{pqmn}\, C_h{}^{rsp}\, C^q{}_{rsk} \,, \tag{16}$$

where $C^q{}_{rsk}$ is the Weyl tensor. The dots in Eq.(14) denote extra corrections containing contractions of the five-form field strength F_5, which we can schematically write as $\gamma(C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4)$, where C is the Weyl tensor and $\mathcal{T}$ is a tensor found in [27] and composed of certain combinations of F_5. The authors of [27] showed that the metric itself is only corrected by W_4, essentially due to the vanishing of the tensor $\mathcal{T}$ on the uncorrected supergravity solution. After taking into account the contribution of this term to the Einstein equations, one finds the corrected metric [41, 42, 44]

$$ds^2 = \left(\frac{r_0}{R}\right)^2 \frac{1}{u} \left(-f(u)\, K^2(u)\, dt^2 + d\vec{x}^2\right) + \frac{R^2}{4u^2 f(u)}\, P^2(u)\, du^2 + R^2 L^2(u)\, d\Omega_5^2 \,, \tag{17}$$

where

$$K(u) = \exp\left[\gamma\,(a(u) + 4b(u))\right] , \tag{18}$$

$$P(u) = \exp\left[\gamma\, b(u)\right] , \tag{19}$$

$$L(u) = \exp\left[\gamma\, c(u)\right] . \tag{20}$$

and

$$a(u) = -\frac{1625}{8}\, u^2 - 175\, u^4 + \frac{10005}{16}\, u^6 \,, \tag{21}$$

$$b(u) = \frac{325}{8}\, u^2 + \frac{1075}{32}\, u^4 - \frac{4835}{32}\, u^6 \,, \tag{22}$$

$$c(u) = \frac{15}{32}\,(1 + u^2)\, u^4 \,. \tag{23}$$

Notice that r_0 is related to the temperature by

$$r_0 = \frac{\pi T R^2}{(1 + \frac{265}{16}\gamma)} \,, \tag{24}$$

so that there is a hidden but important γ-dependence inside r_0. The reader should be aware that there is some confusion in the literature regarding the α'-corrected metric, and we refer the reader to [42] for a discussion. We use the metric of [42], and our conventions follow theirs' closely, with the obvious change of coordinate $u = r_0^2/r^2$.

Now that we know the corrected metric, we are able to obtain the minimal kinetic term of the $U(1)$ gauge field. To this end, we must construct the corrected versions of Eq.(8) and Eq.(10) and insert them into the two-derivative supergravity action Eq.(6). The metric *ansatz* we use is that of Eq.(8) with the appropriate corrected substitutions, and the imposition $R \to R\,L(u)$ to take account of the non-factorisability of the corrected metric. As for the *ansatz* for F_5 we use the fact that we are only interested in the terms which are quadratic in the gauge-field perturbations in order to define the following *ansatz*, which is a direct extension of the unperturbed *ansatz* of Eq.(10)

$$G_5 = -\frac{4}{R}\overline{\epsilon} + \frac{R^3 L(u)^3}{\sqrt{3}} \left(\sum_{i=1}^{3} d\mu_i^2 \wedge d\phi_i \right) \wedge \overline{*} F_2\,. \tag{25}$$

Note that we are not interested in the part of G_5 which does not contain the vector perturbations. This part is denoted by $\overline{\epsilon}$, and only contributes to the potential of the metric, and is thus accounted for by the use of the corrected metric in the computation. Therefore, the only difference between this *ansatz* and the uncorrected one as far as the gauge field is concerned is the warp factor $L(u)$, which starts at $\mathcal{O}(u^4)$ and will be seen to drop out of all of our results. Inserting the metric *ansatz* and the F_5 *ansatz* into the action Eq.(6), we obtain the kinetic term for the gauge field A_m, as expected:

$$S = -\frac{N^2}{64\pi^2 R} \int d^4x\, du\, \sqrt{-g}\, L^7(u)\, g^{mp}\, g^{nq}\, F_{mn}\, F_{pq}\,, \tag{26}$$

where the dependence on the dimensionless factor $L(u)$ is acquired by the proper reduction from ten dimensions [52], and ultimately arises as a consequence of the non-factorisability of the corrected metric [42]. The determinant factor $\sqrt{-g}$ refers to the five-dimensional part of the 10D metric of Eq.(8), and all 5D indices are raised and lowered by that metric.

We have thus completed the first step in our programme, that of obtaining the minimal gauge kinetic term from the two-derivative supergravity action. The next step is to obtain the effect of the eight-derivative corrections of Eq.(14). Concretely, we must determine the five-dimensional operators that arise once the perturbed metric and five-form field strength *ansätze* are inserted into Eq.(14). Crucially, we are able to use the uncorrected *ansätze* Eq.(8) and Eq.(10) in this step, because using the corrected ones results in terms of even higher order in γ. The salient point to take from the discussion in the next section is that a simple operator analysis together with an analysis of the equations of motion reveals that the contributions arising directly from the ten-dimensional higher-curvature operators *will not* contribute to the on-shell action in this work. We explain this important statement in the next section, where we also introduce the explicit expressions for the ten-dimensional eight-derivative corrections.

4 The higher-curvature operators

We first introduce the explicit expressions of the eight-derivative corrections. In addition to $W_4 = C^4$, we have the terms that we denoted above by $\gamma(C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4)$. The tensor C is the Weyl tensor, and it only depends on the metric. The six-tensor $\mathcal{T}$ is defined in terms of the self-dual field $F^+ = (1 + *)F_5/2$ via

$$\mathcal{T}_{ABCDEF} = i\nabla_A F^+_{BCDEF} + \frac{1}{16}\left[F^+_{ABCMN}F^{+\ \ MN}_{DEF} - 3F^+_{ABFMN}F^{+\ \ MN}_{DEC}\right], \tag{27}$$

where there is implicit antisymmetry in $[A, B, C]$ and $[D, E, F]$ in addition to symmetry under the interchange $[A, B, C] \leftrightarrow [D, E, F]$. Note that for the purposes of this section we write the ten-dimensional indices in capital letters, reserving small caps for the AdS-Schwarzschild coordinates, and denoting the coordinates of the five-sphere by indices with a tilde $\tilde{a}$. The six-tensor $\mathcal{T}$ is a complicated object in terms of its index structure, but it is a rather simple object when viewed from the point-of-view of the 5D effective field theory obtained upon integrating out the sphere. The sheer size of the six-tensor $\mathcal{T}$ means that it is very difficult (impossible) to compute its contribution to the 5D gauge field Lagrangian directly. We will therefore adopt a different approach below. We are interested in evaluating C_{ABCD} and $\mathcal{T}_{ABCDEF}$ on the perturbed *ansätze* Eq.(10) and Eq.(8). We write the two tensors C_{ABCD} and $\mathcal{T}_{ABCDEF}$ as

$$\begin{aligned} C_{ABCD} &= C^{(0)}_{ABCD} + C^{(1)}_{ABCD} + C^{(2)}_{ABCD}, \\ \mathcal{T}_{ABCDEF} &= \mathcal{T}^{(0)}_{ABCDEF} + \mathcal{T}^{(1)}_{ABCDEF} + \mathcal{T}^{(2)}_{ABCDEF}, \end{aligned} \tag{28}$$

where the superscript (i) on each term in the right hand side of this equation denotes the power of F_{ab} contained within that term, where F_{ab} is the field strength of the $U(1)$ gauge field. A crucial property of the six-tensor $\mathcal{T}_{ABCDEF}$ is that it vanishes when evaluated on the uncorrected F_5 and G_{MN} with no vectorial perturbations. In other words, $\mathcal{T}^{(0)}$ is zero. Given that we are only interested in operators which are quadratic in F_{ab}, we can then completely discard the operators $C\mathcal{T}^3 + \mathcal{T}^4$, which is a massive simplification. We then focus on the terms given by $\gamma(C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2)$. We first note that C^4 only contains the metric and no factors of the F_5 *ansatz*. Our strategy will be to discuss the terms $C^3\mathcal{T} + C^2\mathcal{T}^2$ and draw general conclusions about the 5D operators that can arise from them. We will then compute C^4 explicitly using the metric *ansatz*, and show that it confirms our conclusions about the expected class of operators. This is not a surprising outcome in a certain sense: the terms containing $\mathcal{T}$ are nothing but the supersymmetric completion of the C^4 term. Our final conclusion will be that these operators all come in with a very high power in u, the radial coordinate. Given that the holographic partition function is evaluated in the ultraviolet, *i.e.* in the limit $u \to 0$, we can prove that the only contributing operator is the minimal kinetic term of Eq.(26).

The important point to remember in the following analysis is that we use the *uncorrected* metric and five-form *ansätze*. This has crucial consequences, the first being that the Weyl tensor factorizes on the unperturbed metric because the latter is a direct product. Moreover, the five-sphere has vanishing Weyl tensor. In addition, the tensor $\mathcal{T}_{MNPQRS}$ is such that terms with one AdS index and five internal S^5 indices are zero, and terms with one internal index and five AdS indices are also zero. Finally, the fact that we are only interested in terms that have two factors of F_{ij}, where the latter is the $U(1)$ gauge field strength, simplifies the analysis considerably. The final deduction that we require below is that the terms coming from $\mathcal{T}^{(1,2)}$ will give rise to five-dimensional operators composed solely of g_{ab}, F_{ij}, $\nabla_k F_{ij}$ and the five-dimensional Levi-Civita tensor ϵ_{abcde}, where all the indices are AdS-Schwarzschild indices. This is a direct consequence of the *ansatz* for $\mathcal{T}^{(1,2)}$: at most it has two-derivatives, and at least one of them must reside in the field strength F_{ij}. Therefore, we cannot obtain terms that go like R_{ij}, the Ricci tensor of the AdS-Schwarschild space, or like R_{ijkl}, because both of these require two derivatives acting on the metric components. This will become clear when we discuss the details of the analysis.

4.1 $C^2\mathcal{T}^2$ terms

In this section we will examine the ten-dimensional eight-derivative terms and determine the five-dimensional gauge-field and gravity operators that will result upon dimensional reduction. The approach is based on a counting of the derivative terms and symmetries of each particular term. Let us begin with the $C^2\mathcal{T}^2$ term. According to [50] there are eight of these terms, with different Lorentz contractions. They are given by:

$$
\begin{aligned}
C^2\mathcal{T}^2 \quad = \quad & C_{ABCD}\, C_{ABCE}\, \mathcal{T}_{DGFHIJ}\, \mathcal{T}_{EFGHIJ} \\
& + (C_{ABCD}\, C_{ABEF}\, \mathcal{T}_{CDGHIJ}\, \mathcal{T}_{EFGHIJ} + C_{ABCD}\, C_{AECF}\, \mathcal{T}_{BEGHIJ}\, \mathcal{T}_{DFGHIJ} \\
& + C_{ABCD}\, C_{AECF}\, \mathcal{T}_{BGHDIJ}\, \mathcal{T}_{EGHFIJ}) \\
& + [C_{ABCD}\, C_{AEFG}\, \mathcal{T}_{BCEHIJ}\, \mathcal{T}_{DFHGIJ} + C_{ABCD}\, C_{AEFG}\, \mathcal{T}_{BCEHIJ}\, \mathcal{T}_{DHIFGJ} \\
& + C_{ABCD}\, C_{AEFG}\, \mathcal{T}_{BCFHIJ}\, \mathcal{T}_{DEHGIJ} + C_{ABCD}\, C_{AEFG}\, \mathcal{T}_{BCHEIJ}\, \mathcal{T}_{DFHGIJ}] + h.c.
\end{aligned}
\tag{29}
$$

where we have neglected the numerical coefficients of the terms and left the metric tensors implicit. We are only interested in operators quadratic in the gauge-field A_a. Immediately it follows that the only contribution can arise via $[C^{(0)}]^2[\mathcal{T}^{(1)}]^2$, so that all of the metric factors and the Weyl factors are non-fluctuating. There are two types of contributions: those with only F_{ij} and those with F_{ij} and $\nabla_k F_{ij}$.

4.1.1 $C^2\mathcal{T}^2$ terms with no ∇F^+

The first term (see [50]) is given by

$$C^2\mathcal{T}^2 = \frac{30240}{86016} G_{(0)}^{KA} G_{(0)}^{BL} G_{(0)}^{MC} G_{(0)}^{NF} G_{(0)}^{PH} G_{(0)}^{GQ} G_{(0)}^{IR} G_{(0)}^{JS} \, C^{(0)}_{ABC}{}^{D} \, C^{(0)}_{KLM}{}^{E} \, \mathcal{T}^{(1)}_{DGFHIJ} \, \mathcal{T}^{(1)}_{ENPQRS} \, . \tag{30}$$

The two-tensor $G_{(0)}^{ka} G_{(0)}^{bl} G_{(0)}^{mc} C^{(0)\,d}_{abc} \, C^{(0)}_{klm}{}^{e} = C^{(0)\,d}_{abc} \, C^{abce}_{(0)}$ is diagonal and has no S^5 indices. The entries are of the form $(u^5, u^5, u^5, u^5, u^6)$. Therefore, if we want to consider the five-dimensional gauge-invariant operators coming from the above equation, then it is clear that the $C^{(0)\,d}_{abc} \, C^{abce}_{(0)}$ term gives us simply $\hat{C}_{lmn}{}^{i} \, \hat{C}^{lmnj}$, where $\hat{C}$ is the Weyl tensor evaluated on the AdS-Schwarzschild space only. Now, the remaining piece is

$$G_{(0)}^{NF} G_{(0)}^{PH} G_{(0)}^{GQ} G_{(0)}^{IR} G_{(0)}^{JS} \mathcal{T}^{(1)}_{DGFHIJ} \, \mathcal{T}^{(1)}_{ENPQRS} \, .$$

The factors of $\mathcal{T}^{(1)}$ can only give F_{ij} and G_{kl}, where everything now is in the AdS-Schwarzschild space. Therefore the two operators that can be obtained after integrating out the S^5 from this piece are simply

$$\hat{C}_{lmn}{}^{i} \, \hat{C}^{lmnj} \; g^{pr} \, F_{ir} F_{jp} \, , \tag{31}$$

and

$$\hat{C}_{lmn}{}^{i} \, \hat{C}^{lmnj} g_{ij} F^2 \, . \tag{32}$$

Schematically, the contribution of such operators is $u^4(u^2 F_{xz}^2 + u^3 F_{xu}^2)$.

This manner of computation can then be extended to all of the ten-dimensional operators of the form $C^2\mathcal{T}^2$. For example, the next term in the list is given by is

$$C^2\mathcal{T}^2 = G_{(0)}^{GK} G_{(0)}^{HL} G_{(0)}^{IM} G_{(0)}^{JN} \, C^{(0)\,CD}_{AB} \, C^{ABEF}_{(0)} \, \mathcal{T}^{(1)}_{CDGHIJ} \, \mathcal{T}^{(1)}_{EFKLMN} \, . \tag{33}$$

To get the contribution of this operator to the five-dimensional effective theory we must enumerate the various types of operators coming from two factors of F_{ij} and as many factors of the metric as we need, as well as the factor $C^{(0)\,ij}_{st} \, C^{stkl}_{(0)}$, evaluated in AdS-Schwarzschild. These will have the form

$$\hat{C}_{st}{}^{ij} \, \hat{C}^{stkl} \, F_{ij} F_{kl} \, , \quad \hat{C}_{st}{}^{ij} \, \hat{C}^{stkl} \, g_{ik} \, g_{jl} \, F^2 \, , \quad \hat{C}_{st}{}^{ij} \, \hat{C}^{stkl} g_{ik} \, g^{mn} \, F_{jn} F_{lm} \, , \tag{34}$$

where we have used the fact that $C_2^{ijkl} = C^{(0)\,ij}_{st} \, C^{stkl}_{(0)}$ obeys $C_2^{ijkl} = C_2^{klij} = -C_2^{jikl} = -C_2^{ijlk}$. The contribution of these operators is again of the schematic form $u^4(u^2 F_{xz}^2 + u^3 F_{xu}^2)$.

The final type of term is that given by

$$C^2\mathcal{T}^2 = G_{(0)}^{LH} G_{(0)}^{IM} G_{(0)}^{JN} \, C^{(0)\,BCD}_{A} \, C^{AEFG}_{(0)} \, \mathcal{T}^{(1)}_{BCEHIJ} \, \mathcal{T}^{(1)}_{DLMFGN} \, . \tag{35}$$

We must now enumerate all the operators that will contain the six-tensor $C^{(0)\,bcd}_{a} \, C^{aefg}_{(0)}$ and two factors of F_{ij} and all the necessary metric factors. For example, we have

$$\hat{C}_{a}{}^{bcd} \, \hat{C}^{aefg} \, g_{cf} \, g_{dg} \, F_{bm} \, g^{mn} \, F_{ne} \, . \tag{36}$$

Again, the explicit u-dependence of this operator is of the same form as that of the previous two.

4.1.2 $C^2\mathcal{T}^2$ terms with ∇F^+

Let us extend this kind of analysis to the term in $\mathcal{T}^{(1)}$ which goes like ∇F^+. In this case, we can now build operators from F_{ij} and $\nabla_k F_{ij}$. The analysis then follows exactly as before. For example, from the Weyl tensor with two up indices, we obtain

$$[C^2_{(0)}]^{AB}(\mathcal{T}^{(1)}\mathcal{T}^{(1)})_{ab} \longrightarrow \hat{C}_{lmn}{}^{i}\,\hat{C}^{lmnj} g^{bf}\, g^{cg}\,\nabla_i\, F_{bc}\,\nabla_j F_{fg}\,. \tag{37}$$

One can check that this operator gives us terms like $u^7(\partial_z^2 A_x)^2$. We also get $u^8(\partial_z\partial_u A_x)^2$, and $u^9(\partial_u^2 A_x)^2$. In principle, we may also have terms that involve the Levi-Civita tensor, such as

$$\hat{C}_{lmn}{}^{i}\,\hat{C}^{lmnj}\, g_{ij}\,\epsilon_{abcde}\,\nabla^a F^{bc}\, F^{de}\,. \tag{38}$$

The point here is that these operators enter with a very high power dependence in u. The reason is that we need at least five factors of g^{ab} to contract the indices, which then means that this operator will enter with at least u^9 in its coefficient, rendering it harmless.

Also, there are now terms with odd numbers of derivatives acting on the gauge field. These come from the connection piece in the covariant derivative ∇. For example, from the above operator we have

$$\hat{C}_{lmn}{}^{i}\,\hat{C}^{lmnj}\, g^{bf}\, g^{cg}\,\Gamma^s_{ab}\, F_{sc}\,\nabla_j F_{fg}\,. \tag{39}$$

This term then gives us contributions of the form $u^7\partial_z A_x\,\partial_u\partial_z A_x$. Of course, we may have other contractions amongst the terms, but the overall effect is the same. The crucial point is the high power of u which enters into these higher derivative operators. Another operator is given by

$$\hat{C}_{lmn}{}^{i}\,\hat{C}^{lmnj}\, g_{ij}\, g^{ab}\, g^{cf}\, g^{dg}\,\nabla_a F_{cd}\,\nabla_b F_{fg}\,. \tag{40}$$

A simple counting of powers of u reveals that these two operators contribute at the same order.

Consider now the contribution of the $C^2_{(0)}$ term with four indices up. One may be tempted to think that the u-dependence drops, but that is not the case. Consider for example a situation where the contractions are such that we have the following operator

$$\hat{C}_{st}{}^{ij}\,\hat{C}^{stkl}\, g^{ab}\,\nabla_a F_{ij}\,\nabla_b F_{kl}\,. \tag{41}$$

By direct computation of $\hat{C}_{st}{}^{ij}\,\hat{C}^{stkl}$, we may show that the contributions of this operator enter at the same power in u. Again, operators with the Levi-Civita tensor are not ruled out, so we may obtain

$$\hat{C}_{st}{}^{ij}\,\hat{C}^{stkl}\,\epsilon_{abcij}\,\nabla^a F^{bc}\, F_{kl}\,. \tag{42}$$

The index contractions imply that the least power with which this operator contributes is then u^9, as before. The same argument follows for the contribution of $C^2_{(0)}$ with six indices up.

4.2 $C^3\mathcal{T}$ terms

The $C^3\mathcal{T}$ term is uniquely given by

$$C^3\mathcal{T} = C^{JKMN}\, C_{KL}{}^{RS}\, C_J{}^{PLQ}\, \mathcal{T}_{MNPQRS} + h.c.\,. \tag{43}$$

The compactification of this term will receive two types of contributions. In the first, all the gauge-field dependence will reside in $\mathcal{T}$. This term is then written as $C^3_{(0)}\mathcal{T}^{(2)}$, where $\mathcal{T}^{(2)}$ here denotes the part of $\mathcal{T}$ containing two powers of the gauge field. The other type of contribution will be that where the quadratic dependence on the gauge field is shared between the C^3 factor and the $\mathcal{T}$ tensor, and we denote this by $C^{(1)}C^2_{(0)}\mathcal{T}^{(1)}$. We begin with the former.

4.2.1 $C^3_{(0)}\mathcal{T}^{(2)}$

This term is given by

$$C^3\mathcal{T} = C^{JKMN}\, C_{KL}{}^{RS}\, C_J{}^{PLQ}\, \mathcal{T}^{(2)}_{MNPQRS}\,. \tag{44}$$

Again, the vanishing of the Weyl tensor on the S^5 means that the compactification of this term is straightforward. The Weyl tensors simply go to the AdS ones, so that all the indices in the above expression become AdS indices. Moreover, $\mathcal{T}^{(2)}$ cannot contain pieces of the form ∇F, as is clear from the definition of the tensor $\mathcal{T}$. Thus, the $\mathcal{T}^{(2)}$ piece is restricted to providing two factors of F_{ij}, as well as factors of the metric tensor. A simple five-dimensional operator resulting from the compactification would then be

$$\hat{C}^{jkmn}\, \hat{C}_j{}^{plq}\hat{C}_{kl}{}^{rs}\, g_{mp}\, g_{rq}\, g_{ns}\, F^2\,. \tag{45}$$

The resulting contribution is then $u^6(u^2F^2_{xz} + u^3F^2_{xu})$. Another operator is given by

$$\hat{C}^{jkm}{}_s\, \hat{C}_j{}^{plq}\hat{C}_{kl}{}^{rs}\, F_{mr}F_{pq}\,. \tag{46}$$

We find this to be given by $u^8(F^2_{xz} + uF^2_{xu})$, exactly as for the previous operator. There are many other contractions for the indices inside this operator, but they all contribute at the same order in u.

4.2.2 $C_{(1)}C^2_{(0)}T^{(1)}$

First, write the following shorthand notation for the contraction of three Weyl tensors

$$C^{JK}{}_C{}^N\, C_{JE}{}^{LQ}C_{KLH}{}^S = [C^3_{(0)}]_C{}^N{}_E{}^Q{}_H{}^S\,. \tag{47}$$

In these terms the gauge-field dependence enters directly into the Weyl tensor itself. These are in principle very complicated terms. This can be schematically written as

$$G^{CM}_{(0)}G^{EP}_{(0)}G^{HR}_{(0)}\Big[[C_{(1)}C_{(0)}C_{(0)}]_{C}{}^{N}{}_{E}{}^{Q}{}_{H}{}^{S} + [C_{(0)}C_{(1)}C_{(0)}]_{C}{}^{N}{}_{E}{}^{Q}{}_{H}{}^{S} + [C_{(0)}C_{(0)}C_{(1)}]_{C}{}^{N}{}_{E}{}^{Q}{}_{H}{}^{S}\Big]$$
$$\mathcal{T}^{(1)}_{MNPQRS}\,. \tag{48}$$

where for example we have for the second term:

$$G^{CM}_{(0)}G^{EP}_{(0)}G^{HR}_{(0)}\Big[C_{(0)}C_{(1)}C_{(0)}\Big]_{C}{}^{N}{}_{E}{}^{Q}{}_{H}{}^{S} = G^{CM}_{(0)}G^{EP}_{(0)}G^{HR}_{(0)}C^{JK}_{(0)}{}_{C}{}^{N}\,C^{(1)\,LQ}_{JE}C^{(0)}_{KLH}{}^{S}\,. \tag{49}$$

We raise the indices and consider the tensor $C^{JKM}_{(0)}{}_{S}\,C^{(1)\,PLQ}_{J}C^{(0)\,RS}_{KL}$. The indices J, M, N, L, R, S are all AdS indices. To get a non-zero result, we then require that both P and Q are internal or AdS indices. But the tensor $C^{(1)\,PLQ}_{J}$ is off-diagonal, and so P and Q cannot be AdS indices. Therefore, we can write the contribution of this term as

$$C^{jkm}_{(0)}{}_{s}\,C^{(1)\tilde{p}l\tilde{q}}_{j}C^{(0)\,rs}_{kl}\,T^{(1)}_{mn\tilde{p}\tilde{q}rs}\,. \tag{50}$$

where $\tilde{p}$ and $\tilde{q}$ are internal S^5 indices. Examining the tensor $C^{(1)\tilde{p}l\tilde{q}}_{j}$, we find that it only has first derivatives of the gauge field A_a. We must now determine what manner of AdS-Schwarzschild tensors can come from $C^{(1)\tilde{p}l\tilde{q}}_{j}$ upon integrating out the five-sphere. The fact that $C^{(1)\tilde{p}l\tilde{q}}_{j}$ contains two internal indices means that the only available tensor is again simply of the form $g^{la}F_{ja}$. For example, we cannot obtain $\hat{R}^{la}F_{ja}$ because this one contains three derivatives, but the maximum number of derivatives contained in the Weyl tensor is two.

From the $\mathcal{T}^{(1)}_{mn\hat{p}\hat{q}rs}$ term we get the usual suspects, namely terms like F_{ij} and G_{ab}. In principle, we may obtain terms of the form $\epsilon_{abcde}\nabla^a F^{bc}$ as well. For example, we have the operator:

$$\hat{C}^{jkmn}\,\hat{C}_{kl}{}^{rs}\,g^{la}\,g_{nr}\,F_{aj}\,F_{ms}\,. \tag{51}$$

We may also have an operator like

$$\hat{C}^{jkmn}\,\hat{C}_{kl}{}^{rs}\,g^{la}\,g_{nr}F_{aj}\,\epsilon_{msdef}\,\nabla^d F^{ef}\,. \tag{52}$$

The latter operator actually contributes at a very high order in u, because of the factors of the metric which contract the Levi-Civita tensor with the ∇F tensor.

4.3 The contribution of the operators as a series in u

Having determined the form of the five-dimensional operators descending from the eight-derivative corrections to type IIB supergravity, we are able to compute their contribution to the gauge-field Lagrangian explicitly. Given that the relevant quantity as far as holography is concerned is the on-shell action evaluated on the boundary of the space, it is sufficient to exhibit the low-u dependence of the contributions. We find the following terms:

$$\begin{aligned}&u^6(\partial_\alpha A_\beta)^2+\cdots+u^7(\partial_u A_\beta)^2\\&+u^7(\partial_\alpha A_\beta)(\partial_u\partial_\gamma A_\beta)+u^7(\partial_\alpha\partial_\gamma A_\beta)^2\\&+u^8(\partial_u A_\beta)(\partial_u\partial_u A_\beta)+\cdots+u^8(\partial_\alpha\partial_u A_\beta)^2+\cdots+u^9(\partial_u^2 A_\beta)^2+\cdots\end{aligned}\tag{53}$$

where the $\cdots$ denote terms which have coefficients that contain a higher u-dependence. The notation here is such that Greek indices α, β, γ denote the four-dimensional Minkowski slices of the AdS-Schwarzschild space, *i.e.* the directions t, x, y, z. The crucial point is that the inclusion of these terms does not affect the on-shell action of perturbations whose typical length scale is much smaller than $1/T$, as we discuss in the next section.

5 The corrected equations of motion

We have argued above that inserting the perturbed metric and five-form field strength into the eight-derivative corrections results in a slew of operators for the gauge field, all of which contain at least two factors of the gauge field strength F_{ab} and two factors of the AdS-Schwarzschild Weyl tensor $\hat{C}_a{}^{bcd}$. In a certain sense the higher dimensionality of the corrections is then replaced by high dependence on the radial coordinate u. In principle, we should be able to organize the result of the dimensional reduction of the $\mathcal{O}(\alpha'^3)$ corrections into a series of eight-derivative gauge invariant operators quadratic in F_{mn} with fixed coefficients, in the manner of [29, 30]. However, whereas the latter references consider operators that are at most carrying six derivatives, our case goes up to eight derivatives, and the large number of such distinct operators, coupled with the sheer size of the expression produced even just by the C^4 correction (see Appendix A), means that such a programme is unfeasible. We therefore rely on the results of the previous section, considering the terms with the lowest u-dependence. Let us illustrate the behaviour of these terms schematically: the action in the presence of the corrections at small u is given by

$$\begin{aligned}\text{action}\ \propto\ &\int d^4x\,du\,\frac{1}{2u^3}\left[-u^2\left(\partial_t A_x\right)^2+u^2\left(\partial_i A_x\right)^2+\frac{4u^3r_0^2}{R^4}\left(\partial_u A_x\right)^2\right.\\&\left.+\gamma B_1 u^9\left(\partial_u^2 A_x\right)^2+\gamma B_2 u^8\left(\partial_u^2 A_x\right)\left(\partial_u A_x\right)+\cdots\right],\end{aligned}\tag{54}$$

where the first line is the contribution of the minimal F^2 kinetic term, and the terms with coefficients B_i arise directly from the eight-derivative term itself, with the dots denoting extra terms arising from the eight-derivative corrections but containing less u-derivatives (the terms considered here are the most problematic). Due to the high positive power of u in the terms coming from $S_{C,\mathcal{T}}$, we find that none of the terms produced affect the solutions of the equations of motion in the ultraviolet. They all enter with at least u^6 in their coefficient, rendering them irrelevant at small u. To see this, recall that the relevant quantity for holography is the on-shell action evaluated at the boundary of the space $u = 0$. Now, it is easy to show that the gauge field must behave like $A_a = a + b\,u + c\,u\,\log(u) + \cdots$ near the boundary, because the ultraviolet boundary is a regular singular point of the equations of motion, with indices $\sigma = 0, 1$. If we take this form and plug it into Eq.(54), and then take the limit $u \to 0$, the only contributing terms will be those coming from the minimal kinetic term. We refer the reader to Appendix B for more details of this argument, and a thorough examination of the equations of motion. Further, this conclusion means that we may eliminate all terms of $\mathcal{O}(u^4)$ or higher from $L(u)$, $P(u)$ and $K(u)$ as defined in Eq.(17), as they will not contribute to the *on-shell* action. The effect of this is dramatic: it means that for the purposes of this computation we can assume that the corrected metric is factorisable, because $L(u) = 1 + \mathcal{O}(u^4)$, and therefore drops out of the entire computation. A final important observation is that the overarching $SU(4)$ gauge symmetry of the vector fluctuations ensures that, at least at quadratic order in the Lagrangian, no mixing with other fluctuations can occur. For fields which are sourced by the $\mathcal{O}(\alpha'^3)$ corrections themselves, any effect on our calculations will contribute to even higher power of α' and therefore does not enter into what follows [27].

The upshot of the preceding arguments is that we may compute the equations of motion solely using the minimal F^2 term and the corrected metric, retaining only $\mathcal{O}(u^2)$ corrections in the functions $P(u)$, $K(u)$ and setting the warp factor $L(u)$ to one, as it is given by $1 + \mathcal{O}(u^4) + \cdots$. Given these enormous simplifications, we may now present the corrected equations of motion for the gauge fields. We first fix the gauge $A_u = 0$, and choose the perturbation as a plane-wave propagating in the x_3-direction [14]. Thus, an appropriate *ansatz* for the gauge field is

$$A_\mu(t, \vec{x}, u) = e^{-i\omega t + iqx_3}\, A_\mu(u)\,. \tag{55}$$

The equations derived from Eq.(13) are then given by

$$\varpi\, A_0' + \kappa\, f\, K^2(u)\, A_3' \;=\; 0\,, \tag{56}$$

$$A_i'' + \frac{f'}{f}\, A_i' + \partial_u \left(\log \left[\frac{K(u) L^7(u)}{P(u)} \right] \right) A_i' + \left[\frac{\varpi^2 - \kappa^2\, f\, K^2(u)}{u\, f^2 K^2(u)} \right] P^2(u) A_i \;=\; 0\,, \tag{57}$$

$$A_0'' + \partial_u \left(\log \left[\frac{L^7(u)}{P(u)\, K(u)} \right] \right) A_0' - \frac{\kappa}{u\, f}\, P^2(u)\, (\kappa\, A_0 + \varpi\, A_3) \;=\; 0\,. \tag{58}$$

where we have defined $\varpi = \omega R^2/(2r_0)$ and $\kappa = qR^2/(2r_0)$. Defining $\tilde{a}(u) \equiv A_0'(u)$ we may recast Eq.(58) into

$$\tilde{a}'' + \frac{(u\, f)'}{u\, f}\, \tilde{a}' + \partial_u \left(\log \frac{L^7(u)}{P^3(u) K(u)} \right) \tilde{a}' \;+\; \frac{P^2(u)}{u\, f^2} \left(\frac{\varpi^2 - \kappa^2\, f K^2(u)}{K^2(u)} \right) \tilde{a}$$

$$+\frac{P^2(u)}{uf}\,\partial_u\left(\frac{uf}{P^2(u)}\partial_u \log\left[\frac{L^7(u)}{P(u)K(u)}\right]\right)\tilde{a} \;=\; 0\,. \quad (59)$$

In order to solve the above equations we have to impose certain boundary conditions. For the non-vanishing $U(1)$ gauge fields we have generic boundary conditions at $u = 0$. Specifically, from Eq.(58) we obtain

$$\lim_{u\to 0}\left[u\,\tilde{a}'(u)\right] = \kappa\,(\kappa\,A_0 + \varpi\,A_3)|_{u=0} = \kappa^2\,A_L(0)\,. \quad (60)$$

On the other hand, at $u = 1$ the appropriate boundary condition that must be imposed is equivalent to selecting only solutions that describe waves going into the black hole, such that there is no reflection off the horizon [38, 39]. At zero temperature, this condition is consistent with the requirement of regularity of the solutions at the AdS horizon $u \to \infty$.

Once we know the solutions of the equations of motion, the next step is to evaluate the on-shell action by inserting the solutions into Eq.(13), which, after integration on u and using the boundary conditions above, gives [4]

$$S_{on-shell} = -\frac{N^2\,r_0^2}{16\pi^2 R^4}\int d^4x\left[\tilde{a}\left(A_0 + \frac{\varpi}{\kappa}\,A_3\right)|_{u=0} - A_i\,\partial_u A_i(u)|_{u=0}\right]\,. \quad (61)$$

Defining the on-shell action density

$$S_{on-shell} = \int d^4x\,\hat{S}_{on-shell}\,, \quad (62)$$

one may now obtain the desired current-current correlator by differentiating with respect to the boundary value of the gauge field $A_\mu \equiv A_\mu(u = 0)$, so that

$$R_{\mu\nu} = \frac{\partial^2 \hat{S}_{on-shell}}{\partial A_\mu\,\partial A_\nu}\,. \quad (63)$$

The results of this section can therefore be used to compute the fully-corrected solution of the equations of motion for the gauge fields A_μ at order $\mathcal{O}(\alpha'^3)$, in any desired regime of the parameters of the system, provided that the length scale of the perturbation is much smaller than $1/T$. By holography, this enables us to obtain the behaviour of the electromagnetic current-correlator at finite 't Hooft coupling. We now proceed to solve the equations of motion of the gauge fields in the regime appropriate for deep inelastic scattering.

5.1 Solving the bulk equations

At this point we need to solve the Maxwell equations for the bulk $U(1)$ gauge field. To avoid confusion, we define $\kappa_0 = q/2\pi T$ and $\varpi_0 = \omega/2\pi T$. The quantities denoted with a subscript $_0$

[4] We remind the reader that, following the prescription of [38, 51, 14], we have dropped the contribution to the on-shell action coming from the horizon at $u = 1$.

are those corresponding to the case where $\mathcal{O}(\alpha'^3)$ corrections are not considered (*i.e.* infinite 't Hooft coupling limit). It is also convenient to define $\mathcal{K}^2 = \kappa^2 - \varpi^2$ (recall that the virtuality Q is given by $Q^2 = q^2 - \omega^2$, so $\mathcal{K} = QR^2/(2r_0)$). We also define $\mathcal{K}_0^2 = \kappa_0^2 - \varpi_0^2 = Q^2/(2\pi T)^2$. It is then convenient to recast the EOMs as a time-independent Schrödinger-like equation [5]

$$\psi'' - V(u)\,\psi = 0\,. \tag{64}$$

For this purpose we define the function $\psi(u) = \Omega(u)\,\tilde{a}(u)$ and by choosing

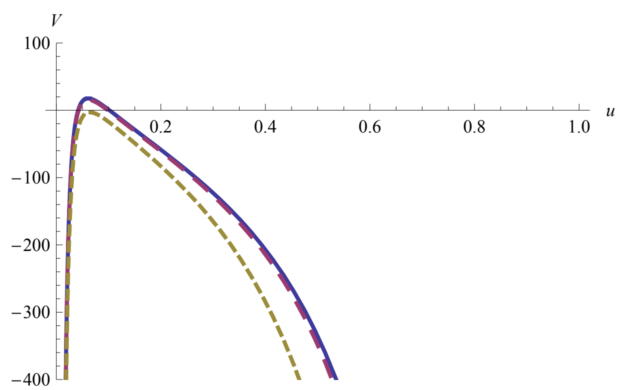
$$\Omega(u) = \left[\frac{u\,f(u)\,L^7(u)}{P^3(u)\,K(u)}\right]^{1/2}\,, \tag{65}$$

we obtain the Schrödinger-like equation (64) with the potential given by

$$V(u) = \frac{\Omega''(u)}{\Omega(u)} - \left\{\frac{P^2(u)}{uf}\,\partial_u\left(\frac{uf}{P^2(u)}\partial_u \log\left[\frac{L^7(u)}{P(u)\,K(u)}\right]\right) + \frac{P^2(u)}{u\,f^2}\,\left(\frac{\varpi^2 - \kappa^2\,fK^2(u)}{K^2(u)}\right)\right\}. \tag{66}$$

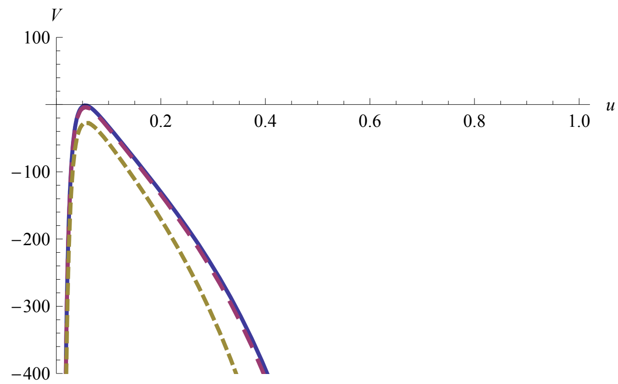
Let us firstly try to intuitively understand the relation between the gravity and field theory descriptions by analyzing the parametric dependence of the potential barrier given by the above potential.

1. *a*



[5] We here neglect the terms in the Lagrangian coming from the higher-derivative corrections, as they do not influence the final result.

1. b



1. c

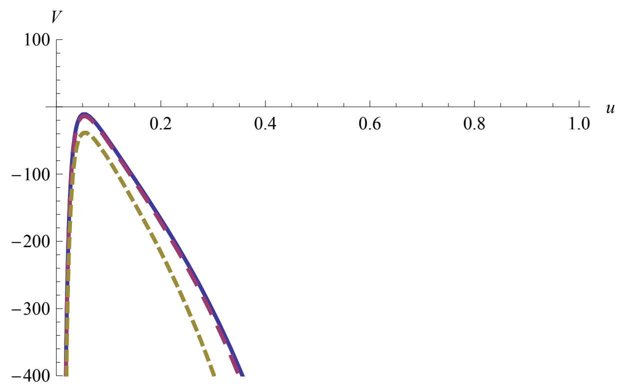


Figure 1. The potential barrier for the A_0 gauge field.

There are special regions of parameter space for which the physical behaviour is rather distinct, and they are distinguished by the following ratio, which we denote by r_q, and which is defined in terms of the physical dimensionfull quantities of the plasma, namely ω, q and T:

$$r_q = \frac{\kappa_0}{\mathcal{K}_0^3} = \frac{q/2\pi T}{(Q/2\pi T)^3} = \frac{\kappa}{\mathcal{K}^3}\left(1+\gamma\frac{265}{16}\right)^2 . \tag{67}$$

In Figure 1 we plot the $\mathcal{O}(\alpha'^3)$-corrected potential barrier $V(u)$ as a function of the variable u, for different parametric values of the ratio r_q and different values of the 't Hooft coupling, as explained below. We also plot the potential barrier for the same values of this ratio without string theory corrections, thus allowing us to see the effect of finite coupling explicitly. In the limit of infinite coupling, the potential becomes

$$\lim_{\lambda\to\infty} V(u) = \frac{1}{u(1-u^2)^2}\left[-\frac{1}{4u}\left(1+6u^2-3u^4\right)+\mathcal{K}_0^2-\kappa_0^2\,u^2\right] . \tag{68}$$

The potential barrier for the longitudinal mode A_0 without α' corrections is plotted with a solid line. We can distinguish among three possible parametric situations in terms of the ratio r_q. In the first case (figure 1.a) the ratio r_q is 1.14, which gives a non-vanishing potential barrier. This case corresponds to intermediate energies where the structure functions of deep inelastic scattering are expected to be very small. For extremely large values of λ the "tunneling effect" through the potential barrier is very small. For finite values of λ the height of the potential barrier decreases, depending on the actual value of λ, thus enhancing the tunneling effect as the value of the 't Hooft coupling decreases. In fact, we have used the values $\lambda = 50$ (dashed line) and $\lambda = 10$ (dotted line) to show explicitly this effect in each figure. In the figure 1.b $r_q = 1.539$ and we see that for this limiting case the height of the barrier vanishes for $\lambda \to \infty$. Figure 1.c shows the potential for $r_q = 1.71$. This case corresponds to the high energy scattering process, where the potential barrier disappears and the wave can propagate all the way towards the black hole horizon and can thus be absorbed. This implies that the retarded current-current correlation function acquires an imaginary part thereby giving non-vanishing structure functions. We confirm that the $\mathcal{O}(\alpha'^3)$-corrections to the potential decrease the height of the barrier independently of the $\kappa/\mathcal{K}^3$ values. This enhances the probability of complete propagation of the wave to the black hole horizon, therefore increasing the imaginary part of the tensor $R_{\mu\nu}$ as the value of λ decreases. Notice that for the above figures we have used the values $\lambda = 50$ which gives $\gamma \simeq 0.00042$, while for $\lambda = 10$ it gives $\gamma \simeq 0.00474$. In addition, the height of the barrier is very sensitive to the value of the 't Hooft coupling as can be seen from the figures.

We now focus on the transvers modes $A_i(u)$. From equation (57), we can define

$$\phi(u) = \Sigma(u)\, A_i(u)\,, \tag{69}$$

where

$$\Sigma(u) = \left(\frac{K(u)\, L^7(u)\, f}{P(u)}\right)^{1/2} . \tag{70}$$

Proceeding as in the previous case, we obtain a time-independent Schrödinger-like equation for A_i

$$\phi''(u) - V(u)\,\phi(u) = 0\,, \tag{71}$$

where the potential is

$$V(u) = \frac{\Sigma''}{\Sigma} - \frac{P^2(u)}{u\, f^2}\left(\frac{\varpi^2 - \kappa^2\, f K^2(u)}{K^2(u)}\right), \tag{72}$$

which for $\lambda \to \infty$ reduces to

$$\lim_{\lambda\to\infty} V(u) = \frac{\mathcal{K}_0^2 - u\,(u\kappa_0^2 + 1)}{u\,(u^2-1)^2} . \tag{73}$$

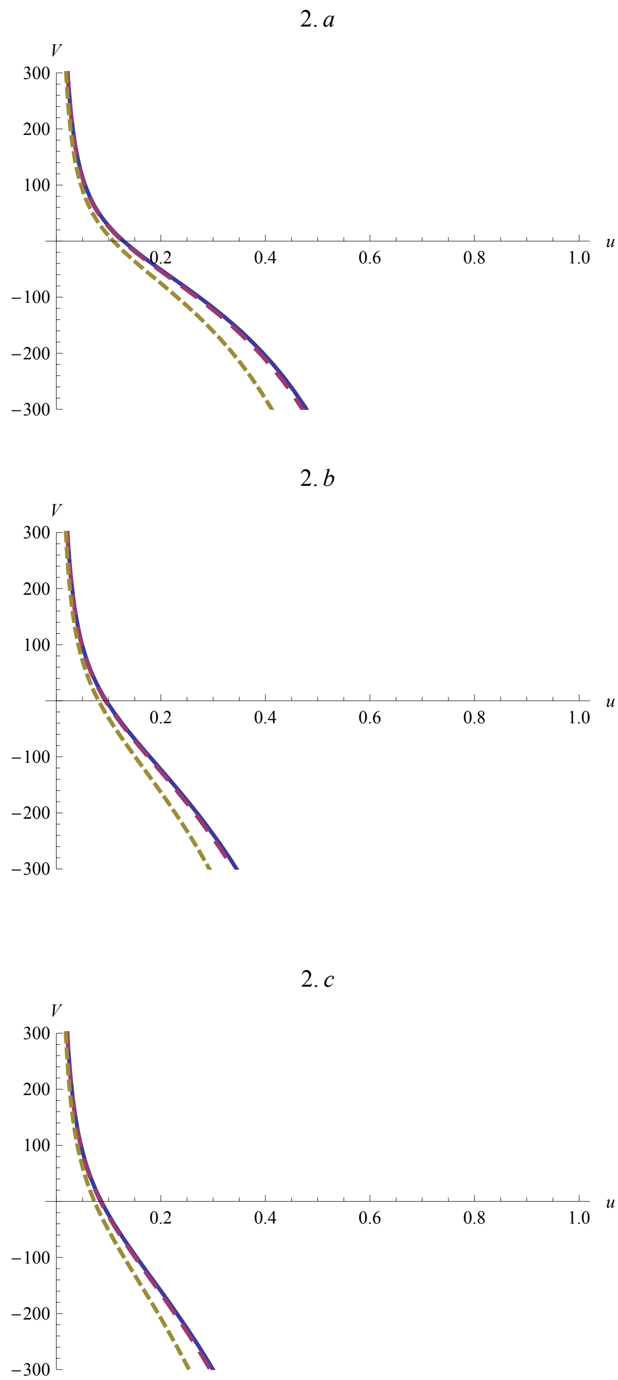


Figure 2. The potential barrier for the transverse modes.

In figure 2 we display the potential barrier for the transverse modes A_i. From top to bottom, these figures correspond to the ratios $r = 1.14,\ 1,539$ and 1.71, respectively. In each figure we have three curves corresponding to $\lambda \to \infty$ (solid line), $\lambda = 50$ (dashed line) and $\lambda = 10$ (dotted line) as in figure 1. It is clear that we again have a reduction in the barrier height for decreasing λ, which implies an enhancement of the structure functions.

Having physically motivated the expected enhancement in our results, we now compute the current-current correlation functions, reading off the structure functions from the imaginary parts. Interestingly, one can consider two different parametric regimes.

Let us briefly study the low energy regime, leaving details of the computation to the Appendix C. We focus on the parametric region where $\kappa \ll \mathcal{K}^3$, which is equivalent to the low temperature regime $qT^2 \ll Q^3$. In addition, by restricting the radial coordinate u to a small region $0 \leq u \leq 1/\mathcal{K}^2 \ll 1$, equations (57) and (59) can be solved in perturbation theory. The physical interpretation is given by a multiple scattering series at low energy [14].

The on-shell 5D action of Eq.(62) together with the on-shell action density (61) can be split into two terms:

$$S_{on-shell} = S^{(0)}_{on-shell} + S^{(1)}_{on-shell} \,, \tag{74}$$

where $S^{(0)}_{on-shell}$ is the zero temperature contribution to the on-shell action, while the other term is proportional to T^4. Thus, within the parametric region mentioned above and with the regularization scheme used in [14] the on-shell action becomes

$$S^{(0)}_{on-shell} = -\frac{N^2}{64\pi^2} \log\left(\frac{Q^2}{\Lambda^2}\right) \left[(q\,A_0 + \omega\,A_3)^2 - Q^2\,\mathcal{A}_T \cdot \mathcal{A}_T\right]_{u=0} \,, \tag{75}$$

where Λ is a regulator in the gauge theory. As expected, this expression is not corrected by the effect of the $\mathcal{O}(\alpha'^3)$ term in the 10D action. Therefore, the corresponding expression for the retarded current-current correlator at zero temperature gets no α'-corrections. For $R^{(0)}_{\mu\nu}$ we obtain

$$R^{(0)}_{\mu\nu} = \frac{N^2 Q^2}{32\pi^2} \log\left(\frac{Q^2}{\Lambda^2}\right) \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2}\right) \,, \tag{76}$$

which is real. These are indeed the expected results since at zero temperature the $\mathrm{AdS}_5 \times S^5$ metric is left uncorrected by the higher derivative corrections to the classical supergravity action [40].

The second term in the on-shell action above is

$$S^{(1)}_{on-shell} \simeq \frac{N^2\pi^2 T^4}{30}\,\frac{q^2}{Q^6}\,(1+15\gamma)\,\left[(q\,A_0 + \omega\,A_3)^2 + \frac{3}{2} Q^2\,\mathcal{A}_T \cdot \mathcal{A}_T\right]_{u=0} \,. \tag{77}$$

We thus see that $R^{(1)}_{\mu\nu}$ is corrected at finite 't Hooft coupling. The α'-corrections do not introduce an imaginary part into the retarded current-current correlator for low energies and, therefore, the plasma structure functions in the present regime vanish[6].

[6]There is, however, a very small contribution to the structure functions in this regime which is due to the "barrier tunneling" effect, which is similar to the contribution reported in [14].

In the next section we explore the role of the α'-corrections in the high energy regime, where deep inelastic scattering is expected to occur.

5.2 High-energy scattering

In this section we consider the high energy regime where $\kappa \gg \mathcal{K}^3$. We examine the equations of motion of the gauge fields, and we keep only the leading terms in an expansion in powers of u. The idea is to compute the corrections to the equations of motion in a double expansion in u and γ, simultaneously applying the condition $\kappa \gg \mathcal{K}^3$. This is easy to do in practice, and a quick calculation reveals that the corrected equations of motion are given by

$$a(u)'' + \left[\frac{1}{u} + \mathcal{O}(u)\right] a'(u) + \left(\left[1 + \frac{325}{4}\gamma\right] \kappa^2 u + \mathcal{O}(u^2)\right) a(u) = 0 \;, \tag{78}$$

and

$$\partial_u{}^2 A_i(u) + \left(\left[1 + \frac{325}{4}\gamma\right] \kappa^2 u + \mathcal{O}(u^2)\right) A_i(u) = 0 \,. \tag{79}$$

The analysis is then simplified considerably if one defines the variable $k = (1 + \gamma\, 325/8)\kappa$, where it is crucial to keep in mind the relations $\kappa = qR^2/(2r_0)$ and $r_0 = \pi T R^2/(1+\gamma\, 265/16)$.

Let us first consider the solution of Eq.(78). Defining $\xi = (2/3)ku^{3/2}$, we find that the general solution is given by $a(\xi) = c_1 J_0(\xi) + c_2 N_0(\xi)$. To fix the constants $c_{1,2}$, we must impose the appropriate boundary conditions. At the U.V. we have the generic Dirichlet condition demanded by AdS/CFT. At the I.R. we must impose the incoming wave boundary condition. One notices that an important simplification occurs [14]: it is the fact that this infrared b.c. can be imposed at relatively small values of $u \ll 1$. The argument is that, at high energies, the absence of a potential barrier means that there is no mechanism to generate reflected waves at intermediate values of $u < 1$, since it would necessarily describe reflection off the black hole. We proceed as follows: notice firstly that although $u \ll 1$ the argument of the Bessel functions is large since $\xi = (2/3)ku^{3/2}$ and $k \gg K^3$, for all the values far beyond the peak of the potential $u \gg u_0 \approx 1/k^{2/3}$. In that region one can asymptotically expand the Bessel functions and get: $J_0(\xi) \approx \sqrt{2/(\pi\xi)} \cos(\xi - \pi/4)$ and $N_0(\xi) \approx \sqrt{2/(\pi\xi)} \sin(\xi - \pi/4)$. Remembering the time-dependence $e^{-i\omega t}$, we see that the solution becomes an outgoing wave if $c_1 = -ic_2$. We have thus fixed the ratio of c_1 to c_2, and the magnitude of the latter is then fixed by the U.V. boundary condition. The same arguments can be applied to solve the equation of A_i, as explained in [14].

We finally have that the solutions for equations (78) and (79) which obey the incident-wave condition at the black hole horizon $u = 1$ and the conditions demanded by the AdS/CFT correspondence at the boundary $u = 0$ are given by [14]

$$a(u) = -i\frac{\pi}{3}\, k^2\, H_0^{(1)}\left(\frac{2}{3}ku^{3/2}\right) \mathcal{A}_L(0) \,, \tag{80}$$

and

$$A_i(u) = \frac{i\pi}{\Gamma(1/3)} \left(\frac{k}{3}\right)^{1/3} \sqrt{u}\, H^{(1)}_{1/3}\left(\frac{2}{3}ku^{3/2}\right) \mathcal{A}_T(0)\,, \tag{81}$$

where $H^{(1)}_\nu(x)$ is the first Hankel function defined by $H^{(1)}_\nu(x) = J_\nu(x) + iY_\nu(x)$, where J_ν, Y_ν are the usual Bessel functions of order ν. The next step is to evaluate the on-shell action for these field configurations. Using the following form for the action density

$$\hat{S}_{on-shell} = -\frac{N^2\, r_0^2}{16\pi^2 R^4} \left[\tilde{a}\left(A_0 + \frac{\varpi}{\kappa} A_3\right)|_{u=0} - A_i\, \partial_u A_i(u)|_{u=0}\right]\,, \tag{82}$$

we obtain

$$\begin{aligned}\hat{S}_{on-shell} \;=\; & -\frac{N^2\, r_0^2}{48\pi^2 R^4} \\ & \left\{k^2\left[2\left(\xi_E + \ln\left(\frac{k}{3}\right)\right) - i\pi\right]\mathcal{A}_L^2(0) + \frac{9\pi}{\Gamma^2(1/3)}\left(\frac{k}{3}\right)^{2/3}\left[\frac{1}{\sqrt{3}} - i\right]\mathcal{A}_T^2(0)\right\}\,,\end{aligned} \tag{83}$$

where ξ_E is the Euler-Gamma function $\xi_E = 0.57\cdots$. Remembering that

$$F_1 = \frac{1}{2\pi}\,\mathrm{Im}R_1\,, \tag{84}$$

we can express the structure function F_1 in terms of the physical momentum q and the temperature T, obtaining

$$F_1 \simeq \left(1 + \frac{5}{8}\xi(3)\lambda^{-3/2}\right) \frac{3N^2T^2}{16\Gamma^2(1/3)} \left(\frac{q}{6\pi T}\right)^{2/3}\,, \tag{85}$$

where λ is the 't Hooft couppling and $\xi(3) \approx 1.20$. Observe that our result is enhanced in comparison with the zero-order result of [14]. From this we can define the transverse structure function as $F_T \equiv 2x_B F_1$, where $x_B = Q^2/(2\omega T)$.

Now, we can similarly obtain the longitudinal structure function F_L

$$F_L \simeq \left(1 + \frac{325}{32}\xi(3)\lambda^{-3/2}\right) \frac{N^2 Q^2 x_B}{96\pi^2}\,, \tag{86}$$

which is related to F_2 through $F_L \equiv F_2 - 2x_B F_1$, where

$$F_2 = \frac{(-n\cdot q)}{2\pi T}\,\mathrm{Im}R_2 = \frac{\omega}{2\pi T}\,\mathrm{Im}R_2\,. \tag{87}$$

The parametric estimates of the above equations are similar to those of [14] where they do not include finite 't Hooft coupling corrections. Thus, in our case we have

$$F_T \propto \left(1 + \frac{5}{8}\xi(3)\lambda^{-3/2}\right) \frac{N^2T^2}{x_B} \left(\frac{x_B^2 Q^2}{T^2}\right)^{2/3}\,, \tag{88}$$

$$F_L \propto \left(1 + \frac{325}{32}\xi(3)\lambda^{-3/2}\right) \frac{N^2T^2}{x_B} \left(\frac{x_B^2 Q^2}{T^2}\right) . \tag{89}$$

We see that in the small-x_B regime at $x_B \ll T/Q$, $F_L \ll F_T$. As in [14], this result looks quite different from the results of DIS from a single hadron at weak and strong coupling, where in the high energy limit the transverse and longitudinal structure functions are parametrically of the same order.

6 Conclusions

In this work we have investigated the behaviour of holographic vector current-current correlators when the leading higher derivative corrections to the low energy type IIB supergravity are included. These corrections enter at order $\gamma \propto \alpha'^3$, and are built out of both the metric and the five-form field strength. The metric at leading order in γ is not corrected by the higher-derivative terms containing F_5, but only by the $\gamma\, C^4$ term.

By considering vector perturbations of the metric and the five-form field strength around the corrected AdS-Schwarzschild background, we derive the modified Maxwell equations for the Abelian gauge field which is dual to the vector current of a gauged $U(1)$ subgroup of the $\mathcal{R}$-symmetry group of $\mathcal{N} = 4$ SYM at finite temperature[7]. Although the higher curvature corrections induce a large number of additional operators to the gauge field Lagrangian (*i.e.* additional to the minimal kinetic term), a careful examination reveals that these operators enter with very high powers of the radial coordinate u (u^6 or higher). Given that the on-shell action for the gauge field reduces to a boundary term to be evaluated in the ultraviolet $u \to 0$, we thus find that the *direct* effect of the extra operators vanishes on-shell. The net result is that the influence of the higher-curvature string theory corrections is indirect, and is effected by the modification to the metric only. We would like to emphasize this result: the higher derivative operators induced in five dimensions *do not* affect the on-shell action in the high energy (scattering) regime. The latter is only affected by the modifications of the metric through the minimal gauge-field kinetic term.

It is interesting to emphasize the distinct effects of the two sets of $\mathcal{O}(\alpha'^3)$ corrections. On the one hand, there is the $\gamma W_4 = \gamma C^4$ term which only involves the Weyl tensor and thus the metric. This term couples to the Einstein equations so that its presence modifies the metric, yielding the corrections computed in [41, 42]. The supersymmetric completion of the C^4 term brings a full set of $\mathcal{O}(\alpha'^3)$ corrections which contain the Ramond-Ramond five-form field strength, as discussed above. These do not modify the background metric, as discussed in [50, 27]. At the quadratic level for the vector fluctuations, we have argued that the full set of ten-dimensional higher-curvature terms only produce an indirect effect on the on-shell

[7] For the zero temperature case one recovers the results of the low energy type IIB supergravity, *i.e.* when no α' corrections are included. This is in full agreement with expectations, since at zero temperature the system reduces to the simpler $\mathrm{AdS}_5 \times S^5$ metric solution.

action for the gauge fields. One may speculate that this simplification is a consequence of the maximal symmetry of the problem at hand. Clearly, the case of $\mathcal{N} = 4$ SYM is very special, in that the S^5 has vanishing Weyl tensor, so that all of the higher-derivative operators we found contained at least two powers of the AdS-Schwarzschild Weyl tensor. Moreover, the tensor $\mathcal{T}$ vanishes for the AdS-Schwarzschild $\times S^5$ background, which also simplified our analysis considerably and eliminated a large class of five-dimensional operators. This is not necessarily the case for other internal manifolds, implying that for complicated cases one may actually find five-dimensional operators that directly influence the holographic correlation functions.

We have applied our results to compute the structure functions governing deep inelastic scattering off an $\mathcal{N} = 4$ SYM plasma. We have found an enhancement of all of the relevant structure functions. The same trend is found in deep inelastic scattering off a single hadron [12].

There are other interesting applications that can be addressed with the results obtained here. Among them would be a computation of the leading 't Hooft coupling corrections to the electric conductivity and the charge diffusion constant of strongly-coupled $\mathcal{N} = 4$ SYM. We will report on these issues in a forthcoming paper [32].

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A Appendix: Vector fluctuations and the W_4-term

The fluctuation *ansatz* of Eq.(8) and that of Eq.(10) ensure that we pick a specific $U(1)$ subgroup of the $SU(4)$ $\mathcal{R}$-symmetry group of the dual field theory. Plugging the *ansätze* into the two-derivative Lagrangian of type IIB supergravity gives the gauge kinetic term for the $U(1)$ field, as explained in section 5. The substitution of the *ansatz* into the eight-derivative operators of Eq.(14) results in a large expression, which, in principle, can be placed into a series of higher-derivative operators quadratic in the $U(1)$ field strength F_{mn} (we have ignored operators with higher factors of the field strength, as they do not contribute to the linearized equations of motion), as we discussed in section 5. Unfortunately, such a scheme is made prohibitively difficult by the sheer size of the expression. We here include the expression obtained when the metric *ansatz* is inserted into the C^4 term, to confirm the arguments of section 5. Setting $A_u = 0$ using the gauge symmetry, we first decompose the W_4 term as follows,

$$W_4 \;=\; \frac{u^6}{6\pi^6 f(u)^2 R^8 T^6}\left[W_{xx} + W_{yy} + W_{tt} + W_{zz} + W_{zt}\right] , \tag{90}$$

where $W_{\alpha\beta}$ contains only the quadratic combination $A_\alpha A_\beta$. Observe that, at least for the C^4 term computed here, the A_x and A_y perturbations do not mix with any others, so that symmetry entails that W_{yy} can be obtained from W_{xx} simply by replacing $A_x \to A_y$ in the latter. In what follows we denote the various derivatives acting on gauge fields using the notation $A_\mu^{(l,m,n)}(u) = \partial_t^l \partial_z^m \partial_u^n A_\mu(t,z,u)$. In this appendix we exhibit the results for W_{xx} and W_{tt} by way of illustrating the type of operators that are induced. For W_{xx} we obtain:

$$\begin{aligned}
W_{xx} \;=\; & \Big(64\pi^4 T^4 u f(u)^2 \left(78u^4 - 67u^2 + 14\right) A_x^{(0,0,1)}(u)^2 + 528\pi^4 T^4 u^3 f(u)^4 A_x^{(0,0,2)}(u)^2 \\
& +16\pi^2 T^2 f(u)^2 \left(7u^2 - 12\right) A_x^{(0,1,0)}(u)^2 - 704\pi^2 T^2 u f(u)^3 A_x^{(0,1,0)}(u) A_x^{(0,1,1)}(u) \\
& -384\pi^2 T^2 u^2 f(u)^3 A_x^{(0,1,1)}(u)^2 - 120\pi^2 T^2 u^2 f(u)^3 A_x^{(0,0,2)}(u) A_x^{(0,2,0)}(u) \\
& +16\pi^2 T^2 \left(21u^4 - 45u^2 + 16\right) A_x^{(1,0,0)}(u)^2 + 33u f(u)^2 A_x^{(0,2,0)}(u)^2 \\
& +96u f(u) A_x^{(1,1,0)}(u)^2 - 128\pi^2 T^2 u^2 f(u)^2 A_x^{(1,0,1)}(u)^2 \\
& -64\pi^2 T^2 u f(u) \left(u^2 + 3\right) A_x^{(1,0,0)}(u) A_x^{(1,0,1)}(u) + 33u A_x^{(2,0,0)}(u)^2 \\
& +2u f(u) \left[68\pi^2 T^2 u f(u) A_x^{(0,0,2)}(u) + 15 A_x^{(0,2,0)}(u)\right] A_x^{(2,0,0)}(u) \\
& +16\pi^2 T^2 u f(u) \Big\{ f(u) \left[4\pi^2 T^2 u f(u) \left(28 - 53u^2\right) A_x^{(0,0,2)}(u) + 5\left(7u^2 - 4\right) A_x^{(0,2,0)}(u)\right] \\
& - \left(12 - 37u^2\right) A_x^{(2,0,0)}(u) \Big\} A_x^{(0,0,1)}(u) \Big) .
\end{aligned} \tag{91}$$

For the contribution quadratic in A_t, we get

$$W_{tt} \;=\; 384\pi^2 T^2 \, u^2 \, f^2(u) \, A_t^{(0,1,1)}(u)^2 \;+\; 7u f(u) A_t^{(0,2,0)}(u)^2 \;+\; 33u A_t^{(1,1,0)}(u)^2$$

$$
\begin{aligned}
&+228\pi^2T^2u^2f(u)A_t^{(1,0,1)}(u)^2 - 912\pi^4T^4u^3f(u)^3A_t^{(0,0,2)}(u)^2\\
&+960\pi^4T^4uf(u)^2\left(7u^2-2\right)A_t^{(0,0,1)}(u)^2 + 184\pi^2T^2u^2f^2(u)A_t^{(0,2,0)}(u)A_t^{(0,0,2)}(u)\\
&+64\pi^2T^2u\Big[f(u)(5-7u^2)A_t^{(0,1,0)}(u)A_t^{(0,1,1)}(u)\\
&+5\left(-12\pi^2T^2uf(u)A_t^{(0,0,2)}(u)+A_t^{(0,2,0)}(u)\right)A_t^{(0,0,1)}(u)\Big]\,. \qquad (92)
\end{aligned}
$$

The other contributions are similar in structure, so we shall not present them here.

B Appendix: The equations of motion

As we saw in the last section, the eight-derivative $\mathcal{O}(\alpha'^3)$ corrections introduce a multitude of higher derivative operators, and we must take account of them properly to solve the equation of motion within perturbation theory. The situation is entirely analogous to that studied by Buchel, Liu and Starinets in [22]. In that paper, the authors were concerned with the tensor perturbations of the metric, but the logic is the same. When we derive the equations of motion from an action which contains higher derivative terms like $A_x''A_x''$, dangerous boundary terms like $\delta A_x'$ and $\delta A_x''$ will be introduced. These threaten to ruin the consistency of the variational principle, necessitating the addition of boundary localized terms that ensure that all variations are simply proportional to δA_x, so that the Dirichlet problem is well-posed. Such an idea is familiar from Einstein gravity, where the problem is made consistent by adding the exterior curvature term. In our case, as in [22], we must add *boundary* terms by hand to make the variational procedure consistent. At the end of the day, these terms do not contribute to the physical answers we seek in this work, as will become clear shortly.

To see how this works in detail, it is convenient to introduce the Fourier transform of the field A_x

$$
A_x(t,\vec{x},u) = \int \frac{d^4k}{(2\pi)^4}\, e^{-i\omega t+iqz}\, A_k(u)\,. \qquad (93)
$$

Upon inserting this into the total action for the gauge field A_x (*i.e.* the action containing both the two derivative term F^2 and the higher-curvature terms in the appendix above), we obtain the expression:

$$
\begin{aligned}
S_{\text{total}} \;=\; &-\frac{N^2r_0^2}{16\pi^2R^4}\int\frac{d^4k}{(2\pi)^4}\int_0^1 du\,\Big[\gamma A_W A_k''A_{-k} + (B_1+\gamma B_W)A_k'A_{-k}'\\
&+\gamma C_W A_k'A_{-k} + (D_1+\gamma D_W)A_kA_{-k} + \gamma E_W A_k''A_{-k}'' + \gamma F_W A_k''A_{-k}'\Big]\,. \qquad (94)
\end{aligned}
$$

The coefficients B_1 and D_1 arise directly from the minimal kinetic term F^2. The subscript W indicates that the particular coefficient comes directly from the eight-derivative corrections. Moreover, B_1 and D_1 contain some γ-dependence, but they are non-vanishing in the $\gamma \to 0$

limit, while every other coefficient vanishes in that limit. We will discuss the explicit form of the various coefficients shortly. First, we remark that upon varying the action of Eq.(94), we obtain the equations of motion:

$$A\,A_k'' + CA_k' + 2DA_k - \partial_u\left(2BA_k' + CA_k + FA_k''\right) + \partial_u^2\left(AA_k + 2EA_k'' + FA_k'\right) = 0\,, \quad (95)$$

where $B = B_1 + \gamma B_W$ and so on. We may write this action in the form

$$A_k'' + p_1 A_k' + p_0 A_k = \mathcal{O}(\gamma)\,, \quad (96)$$

where we have moved all γ-dependence to the right. A careful examination of the variation of Eq.(94) then convinces us that in order to remove all dangerous variations to order γ^2 we must add the boundary term

$$S_b = -\frac{N^2 r_0^2}{16\pi^2 R^4}\int \frac{d^4k}{(2\pi)^4}\int_0^1 du\,\partial_u\left[-\gamma A_W\,A_k A_{-k}' + \gamma E_W\left(p_1 A_k' + 2p_0 A_k\right)A_{-k}' - \gamma\frac{F_W}{2}A_k' A_{-k}'\right]. \quad (97)$$

After appending this necessary term to the action Eq.(94), we may then write the action as

$$S_{\text{total}} = -\frac{N^2 r_0^2}{16\pi^2 R^4}\int \frac{d^4k}{(2\pi)^4}\int_0^1 du\,\left[\frac{1}{2}A_{-k}\mathcal{L}\,A_k + \partial_u\Phi\right]. \quad (98)$$

where $\mathcal{L}A_k = 0$ is simply the equation of motion of Eq.(95), and Φ is a boundary term. Upon evaluating the on-shell action, the only surviving term is the boundary term, as we expect from holography. This is given by

$$\begin{aligned}\Phi \;=\;& (B-A)A_k' A_{-k} + \frac{1}{2}(C - A')A_k A_{-k} \\ & - E' A_k'' A_{-k} + EA_k'' A_{-k}' - EA_k''' A_{-k} + E\left(p_1 A_k' + 2p_0 A_k\right)A_{-k}' - \frac{F'}{2}A_k' A_{-k}\,. \end{aligned} \quad (99)$$

Therefore, the strategy is to solve the equation of motion Eq.(95), then insert the solution into the action, which leaves us only with the boundary term Φ. This is what was done in the main text of the paper, with massive simplifications arising from the fact that the only term which contributes to the boundary action is B_1 from the first term of Eq.(99). This is because the coefficients A_W, B_W, C_W, E_W and F_W all have high positive powers of u, which, coupled with the regularity of the solutions of $A_k(u)$ at $u = 0$, means that their contribution to the boundary term vanishes. Let us now list the various coefficients used in this section, for the contribution of C^4 calculated in the previous appendix. First we have the coefficients with no γ-dependence p_0 and p_1, given by

$$p_0 = \frac{\varpi_0^2 - f(u)\kappa_0^2}{uf^2(u)} \quad \text{and} \quad p_1 = \frac{f'(u)}{f(u)}\,, \quad (100)$$

where $\varpi_0 = \omega/(2\pi T)$ and $\kappa_0 = q/(2\pi T)$. For the coefficients originating from the F^2 term in the action of the gauge field, we obtain

$$
\begin{aligned}
B_1 &= \frac{K(u)f(u)L^7(u)}{P(u)}\,, \\
D_1 &= -K(u)P(u)L^7(u)\left[\frac{\varpi^2 - f(u)K^2(u)\kappa^2}{uf(u)K^2(u)}\right]\,,
\end{aligned}
\tag{101}
$$

where $\varpi = \omega R^2/(2r_0)$ and $\kappa = qR^2/(2r_0)$. For the terms originating from the higher curvature term in the action, we have the following expressions, here evaluated only for the C^4 operator, retaining only the A_x fluctuation:

$$
\begin{aligned}
A_W &= -2u^5\left[15f(u)\kappa_0^2 - 17\varpi_0^2\right]\,, \\
B_W &= -4u^4\left[(14 - 67u^2 + 78u^4) - 8u\varpi_0^2 - 24uf(u)\kappa_0^2\right]\,, \\
C_W &= -4\frac{u^4}{f(u)}\left[3f(u)(3u^2-8)\kappa_0^2 - 41u^2\varpi_0^2\right]\,, \\
D_W &= -\frac{u^3}{f^2(u)}\Big[33uf^2(u)\kappa_0^4 + 33u\varpi_0^4 + 126uf(u)\kappa_0^2\varpi_0^2 \\
&\quad +4f^2(u)(7u^2-20)\kappa_0^2 + 4(16 - 45u^2 + 2u^4)\varpi_0^2\Big]\,, \\
E_W &= -33u^6 f^2(u)\,, \\
F_W &= 4u^5 f(u)(53u^2 - 28)\,.
\end{aligned}
\tag{102}
$$

We remind the reader that although we have computed these coefficients explicitly only for the C^4 operator (see appendix A), the full set of 10D eight-derivative terms are expected to *have the same leading power dependence in* u, as explained in section 4. To demonstrate that none of these coefficients contribute to the on-shell action, we write $A_k = A_0 + \gamma A_1$, in order to solve the equations of motion perturbatively in γ. Now, the equation of motion for A_0 is then simply given by

$$
A_0'' + p_1 A_0' + p_0 A_0 = 0\,. \tag{103}
$$

This equation has a regular singular point at $u = 0$, with indices $\sigma = 0, 1$. Expanding around the point $u = 0$, we can therefore write a general solution of the equation in the form $A_0 = a + bu + cu\log(u) + \cdots$. Now, because we are only interested in the on-shell action to $\mathcal{O}(\gamma)$, and given that $A_W, B_W, C_W, D_W, E_W, F_W$ in Eq.(99) all start at $\mathcal{O}(\gamma)$, it is clear that the only term contributing to the on-shell action is $B_1\, A_k'\, A_{-k}$. We must however also show that the solution of the equation of motion of A_1 in the ultraviolet region is unaffected by the terms $A_W, B_W, C_W, D_W, E_W, F_W$. Plugging the ansatz $A_k = A_0 + \gamma A_1$ into the EOM of Eq.(95), we get the equation of motion of A_1 as

$$
\partial_u\left[2B_1|_{\gamma\to 0}A_1'\right] - 2D_1|_{\gamma\to 0}A_1 = V(A_0)\,. \tag{104}
$$

where

$$
\begin{aligned}
& A_W\, A_0'' + C_W A_0' + 2\left((D_1 - D_1|_{\gamma\to 0}) + D_W\right) A_0 \\
& -\partial_u \left(2(B_1 - B_1|_{\gamma\to 0}) A_0' + 2B_W A_0' + C_W A_0 + F_W A_0''\right) \\
& +\partial_u^2 \left(A_W A_0 + 2E_W A_0'' + F_W A_0'\right) \,=\, V(A_0)\,.
\end{aligned} \tag{105}
$$

We are only interested in the solution of A_1 in the region of small u. Taking into account the power dependence of the coefficients $A_W, B_W, C_W, D_W, E_W, F_W$ and the behaviour of A_0 near the boundary, we find that the only contributing factors in the potential $V(A_0)$ are those given by $2\left((D_1 - D_1|_{\gamma\to 0})\right) A_0 - \partial_u \left(2(B_1 - B_1|_{\gamma\to 0}\right) A_0'$. We have therefore showed that the only relevant terms are B_1 and D_1, both of which arise from the minimal kinetic operator F^2. This completes the analysis for A_x. The exercise is entirely analogous for the other fluctuations.

C Appendix: Scattering at low energy and finite 't Hooft coupling

In this appendix we perform the analysis of the EOMs for low energies, $\kappa \ll \mathcal{K}^3$, which is equivalent to considering the low temperature regime $qT^2 \ll Q^3$ when the virtuality Q^2 is fixed. For the small u region $0 \le u \le 1/\mathcal{K}_0^2 \ll 1$, together with the assumption that $A_i\, \kappa_0^2 \gg 2\, A_i'$ and similarly for a, in the limit $\lambda \to \infty$, equations (57) and (59) become

$$
A_i'' - \frac{\mathcal{K}_0^2}{u}\, A_i = -\, \kappa_0^2\, u\, A_i\,, \tag{106}
$$

and

$$
a'' + \frac{1}{u} a' - \frac{\mathcal{K}_0^2}{u}\, a = -\kappa_0^2\, u\, a\,. \tag{107}
$$

Now, consider the case when the 't Hooft coupling is finite. Within the same level of approximation as before, and within the parametric region $0 \le u \le 1/\mathcal{K}^2 \ll 1$, equations (57) and (59) become

$$
A_i'' - \frac{\mathcal{K}^2}{u} A_i = -\left(1 + \frac{325}{4}\,\gamma\right)\, \kappa^2\, u\, A_i\,, \tag{108}
$$

and

$$
a'' + \frac{1}{u}\, a' - \frac{\mathcal{K}^2}{u}\, a = -\left(1 + \frac{325}{4}\,\gamma\right)\, \kappa^2\, u\, a\,. \tag{109}
$$

The next step is to solve these equations in perturbation theory. For that purpose it is convenient to define $\xi = 2\,\mathcal{K}\, u^{1/2}$, so that the equations (108) and (109) become

$$
\left(\frac{d^2}{d\xi^2} + \frac{1}{\xi}\,\frac{d}{d\xi} - 1 - \frac{1}{\xi^2}\right)\, h(\xi) = -\left(1 + \frac{325}{4}\,\gamma\right)\, \left(\frac{\kappa^2\,\xi^4}{16\,\mathcal{K}^6}\right)\, h(\xi)\,, \tag{110}
$$

and

$$\left(\frac{d^2}{d\xi^2}+\frac{1}{\xi}\frac{d}{d\xi}-1\right)a(\xi)=-\left(1+\frac{325}{4}\gamma\right)\left(\frac{\kappa^2\,\xi^4}{16\,\mathcal{K}^6}\right)a(\xi)\,, \tag{111}$$

respectively, where in equation (110) we have redefined $A_i(\xi)=A_i(0)\,\xi\,h(\xi)$.

In order to study these equations perturbatively, let us begin with equation (110). The zeroth order solution is $h^{(0)}(\xi)=K_1(\xi)$, where K_1 is the modified Bessel function of order one. The function $\xi\,h^{(0)}(\xi)$ approaches one at the AdS boundary. The general solution is given as a sum of the general solution of the homogeneous ordinary differential equation and the convolution of the Green's function with the source of equation (110)

$$h(\xi)=h^{(0)}(\xi)-\int_0^\infty d\xi'\,G(\xi,\,\xi')\left(1+\frac{325}{4}\gamma\right)\left(\frac{\kappa^2\,\xi'^4}{16\,\mathcal{K}^6}\right)h(\xi')\,, \tag{112}$$

where $G(\xi,\,\xi')$ is the Green's function corresponding to the differential operator of Eq.(110) with appropriate boundary conditions. This is the same Green's function as given in [14]. The $\mathcal{O}(u)$-perturbation theory solution for the transverse modes can be expressed as $A_i(u)=A_i^{(0)}(u)+A_i^{(1)}(u)$, where

$$\begin{aligned} A_i^{(0)}(u) &= 2\,\mathcal{K}\,A_i(0)\,u^{1/2}\,K_1[2\,\mathcal{K}\,u^{1/2}]\,, &(113)\\ A_i^{(1)}(u) &= A_i(0)\,\frac{\kappa^2}{5\,\mathcal{K}^4}\left(1+\frac{325}{4}\gamma\right)u\,. &(114)\end{aligned}$$

For small values of u, we can expand the modified Bessel function. Thus we obtain

$$A_i^{(0)}(u)\simeq A_i(0)\,(1+u\,\mathcal{K}^2\,[\log\mathcal{K}^2-1+\log u+2\,\xi_E])\,, \tag{115}$$

where ξ_E is the Euler-Gamma function which is $0.57\cdot\cdots$.

The solution for the longitudinal modes in perturbation theory can be expressed as $a(u)=a^{(0)}(u)+a^{(1)}(u)$, where

$$\begin{aligned} a^{(0)}(u) &= -2\,\kappa^2\,A_L(0)\,K_0[2\,\mathcal{K}\,u^{1/2}]\,, &(116)\\ a^{(1)}(u) &= -2\,A_L(0)\,\frac{\kappa^4}{15\,\mathcal{K}^6}\left(1+\frac{325}{4}\gamma\right). &(117)\end{aligned}$$

For small values of u we can expand the modified Bessel function as follows

$$a^{(0)}(u)\simeq\kappa^2\,A_L(0)\,[\log\mathcal{K}^2+\log u+2\,\xi_E]\,. \tag{118}$$

Inserting these approximations for the transverse and longitudinal components of the gauge fields into the on-shell action, we obtain the results for the low energy scattering presented in section 5.1.

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