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SPATIAL VARIATION OF FUNDAMENTAL CONSTANTS: TESTING MODELS WITH THERMONUCLEAR SUPERNOVAE

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Since Dirac stated his *Large Number Hypothesis* the space-time variation of fundamental constants has been an active subject of research. Here we analyze the possible spatial variation of two fundamental constants: the fine structure constant α and the speed of light c. We study the effects of such variations on the luminosity distance and on the peak luminosity of Type Ia supernovae (SNe Ia). For this, we consider the change of each fundamental constant separately and discuss a dipole model for its variation. Elaborating upon our previous work, we take into account the variation of the peak luminosity of Type Ia supernovae resulting from the variation of each of these fundamental constants. Furthermore, we also include the change of the energy release during the explosion, which was not studied before in the literature. We perform a statistical analysis to compare the predictions of the dipole model for α and c variation with the Union2.1 and JLA compilations of SNe Ia. For this, we also allow the nuisance parameters of the distance estimator μ_0 and the cosmological density matter $\Omega_{\rm m}$ to vary. As a result of

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our analysis we obtain a first estimate of the possible spatial variation of the speed of light c. On the other hand, we find that there is no significant difference between the several phenomenological models studied here and the standard cosmological model, in which fundamental constants do not vary at all. Thus, we conclude that the actual set of data of Type Ia supernovae does not allow to verify the hypothetical spatial variation of fundamental constants.

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1. Introduction

The Standard Model of Cosmology describes, in good agreement with the observations, the evolution of the Universe. The Standard Model is based on the assumption that fundamental constants – the fine structure constant α, the proton-to-electron mass ratio μ and the gravitational constant G among others – remain truly constant throughout space and time. However, there are other competing theories that do not rely in this, otherwise reasonable, hypothesis. Indeed, theories that predict a variation of fundamental constants have been developed over the years. They can be divided into two categories. The first one of these families are grand-unification theories. Within this theoretical framework fundamental constants are slowly varying functions of low-mass dynamical scalar fields – see, for instance, Refs. 1, 2, 3, and references therein. The second family of formulations corresponds to low-energy effective theories. These are phenomenological models specifically proposed to study a potential variation of fundamental constants. In these theories the "parameter" whose variation is going to be studied is replaced by a scalar field. Furthermore, each phenomenological model assumes that only a fundamental coupling varies at a time, either the fine structure constant α,[4–6] the speed of light c[7–9] or the electron mass.[10]

Different versions of the theories mentioned above predict different variations for the fundamental constants. Thus, if these theories turn out to be correct, the fundamental constants are expected to depend weakly on time, or vary on different space lengths. In essence, this is equivalent to say that searching for possible variations in the fundamental constants of nature would eventually allow us to test whether the laws of physics are the same everywhere at any time. Therefore, it is important to analyze the observational effects that may arise from changes of the fundamental constants, and to develop new methods to constrain these hypothetical variations in order to check the validity of such theories.

Numerous experiments and observations have attempted to establish whether the fundamental couplings are indeed constant. The experimental studies can be grouped in two broad classes. The first of them comes from purely local methods, whereas the second one is based in the observation of astronomical phenomena. The former category includes geophysical methods such as the natural nuclear reactor that operated about 1.8×10^9 yr ago in Oklo, Gabon,[11–13] the analysis of natural long-lived β decays in old minerals and meteorites,[14–16] and laboratory measure-

ments, that include for instance detailed comparisons of several atomic clocks with different isotopes.[17–19] The latter family of methods is based mainly, but not only, on the analysis of spectra from high-redshift quasar absorption systems.[20–25] Besides, further constraints on a hypothetical variation of α can be obtained by comparing X-ray results and Sunyaev-Zeldovich measurements in galaxy clusters.[26–30] Moreover, the variation of α in the early universe can be constrained as well from primordial nucleosynthesis[31,32] and from the Cosmic Microwave Background (CMB) fluctuation spectrum.[33–36] We recommend the interested reader a careful reading of the reviews of Refs. 2 and 3 for extensive discussions of the many observational techniques.

Evidence for a dipole spatial variation of α was obtained using the combined observations of distant quasars using the KECK/HIRES[37] and VLT/UVES[38,39] telescopes. However, subsequent analyses[40] showed that long-range wavelength distortions can mimic the effect of the reported variation of α. On the other hand, Ref. 41 performed an independent analysis using the observed data set, together with other observational results and found that the observations were consistent with a dipole variation of α. Finally, recent work on two ZnII and three CrIII transitions has provided us with the first method to test a variation of α that is not influenced by long-range distortions.[42,43] Even though these latest results showed no convincing evidence for a variation of α, Ref. 43 concluded that their quasar sample is too small to rule out the dipole model. Otherwise, the spatial variation of α has been analyzed using different methods.[26,29,30,36,44,45] In particular, in addition to the analyses mentioned above, Ref. 36 obtained constraints from the CMB radiation, while Ref. 44 studied the effect of the CMB modulation on the orbital motion of the major bodies of the Solar System. Also, Ref. 45 studied a Finslerian Universe where both α and the luminosity distance of Type Ia supernovae require a dipole variation.

On the other hand, there is some evidence of a dipole anisotropy in other cosmological observables. For instance, Refs. 46 and 47 proposed a dipole model for the deviation of distance modulus of SNe Ia with respect to the standard value of the Λ_{CDM} model, and performed a statistical analysis using the Union 2 and Union 2.1 samples. Their results show that the dipole that best fits the SNe Ia sample has a similar direction than the dipole that results from the quasar sample. Nonetheless, none of these authors considered the effects on the luminosity distance of SNe Ia of a variation of α. On the other hand, Ref. 48 have identified a direction of the maximum temperature asymmetry (MTA) of the WMAP7 reduced map and found that the direction of the asymmetry is consistent with that found by the previously mentioned analyses.[46,47] However, Ref. 49 concluded that using two different methods, i.e., dipole-fitting and hemisphere comparison, the preferred directions coming from the Union 2 data are approximately opposite. We note, however, that later the two methods have been critically reviewed, and it was found that the dipole-fitting method is statistically significant while the hemisphere comparison method is strongly biased by the distribution of data points in the sky.[50] Furthermore,

Ref. 51 preformed a statistical analysis to constrain the amplitude and direction of anisotropy of the SNe Ia using a new data set, the JLA compilation.[52] They applied a Markov Chain Monte Carlo method and they derived a dipole direction that is not consistent with the results of Ref. 47. Nevertheless, they have obtained consistent results when applying the same method to the Union 2.1 compilation. In summary, the anisotropy derived from SNe Ia strongly depends on both the employed data sets, and on the methods used to analyze the data.

Additionally, there is some observational evidence that could be interpreted as a hint for deviations from large-scale statistical isotropy such as the large-scale alignment in the QSO optical polarization data[53] and the alignment of low multipoles in the CMB angular power spectrum.[54] These observations have intensified the interest in the spatial variation of fundamental constants.

In a previous work,[55] we studied the effects of a possible spatial variation of α on the luminosity distance of SNe Ia. In that work we included the variation of the peak luminosity resulting from the variation of α, which was not analyzed before. In particular, we used the previous analysis performed by Ref. 56, who considered the dependence of the mean opacity of the expanding photosphere of SNe Ia on the value of α, and in addition we evaluated the effects of a varying α on the precise value of the Chandrasekhar limit. Both physical effects change the luminosity distance of SNe Ia. In this work we go one step further and consider the variation of two different fundamental constants separately.

The authors of Ref. 57 have discussed the differences between theories where e is the varying fundamental constant with respect to those where c varies. One of the most important differences is that in the former case, the Weak Equivalence Principle is violated while in the latter it is not. On the other hand, models where the variation in c is spatial were analyzed in Ref. 58. Therefore, we consider first a variation of the fine structure constant α, and then a varying speed of light c. It is important to stress that the variation of α considered in this paper is different from that considered in our previous work.[55] In Ref. 55 we considered the spatial variation of α, while keeping all other fundamental constants fixed. In contrast, in this paper we analyze the variation of α through the variation of the electron charge e. As a consequence the dependence of the relevant quantities for the peak luminosity with α, namely the opacity of the expanding photosphere, and the Chandrasekhar mass, differ from those obtained in our previous analysis. Furthermore, we investigate the effect of a possible variation of the fundamental constants on the energy release during the supernova explosion. In addition, we propose a dipole model for the spatial variation of each fundamental constant considered in this paper. Finally, we perform a statistical analysis using the distance modulus of SNe Ia obtained from the Union 2.1[59] and the JLA[52] compilations to check if the models are compatible with observations taking into account the emerging estimates of the nuisance parameters and of the cosmological density matter. The reason for this is that the standardization of the SNe Ia depends on the theoretical model used for the distance modulus, in the sense that it influences the nuisance parameters

determination which accompany the stretch, color and host-mass corrections.

Our paper is organized as follows. In Sect. 2 we analyze the dependence of the peak luminosity of SNe Ia peak with the variation of fundamental constants and how a possible change modifies the the distance modulus. It follows Sect. 3, where we present the dipole model. Then, in Sect. 4 we present our results. Lastly, in Sect. 5 we summarize our main findings and we draw our conclusions.

2. The Distance Modulus of Thermonuclear Supernovae

SNe Ia are among the most energetic and interesting phenomena in our universe. Owing to their large luminosities, they can be observed up to very high redshifts. Moreover, a sizeable number of them are nowadays routinely detected by dedicated surveys. All this makes them suitable astronomical objects to test the possible spatial variation of fundamental constants. But not only that, the spectra and light curves of normal SNe Ia are very homogeneous. This arises because the light curve of a SNe Ia can be understood in terms of the capture and thermalization of the products of radioactive disintegration of ^{56}Ni and ^{56}Co. SNe Ia reach their peak luminosity in approximately 20 days after explosion.[60] Moreover, it is observationally found that there is a tight correlation between their peak bolometric magnitudes and the decline rates of their light curves. All these features make SNe Ia one of the best standard candles known today.

An other important property of SNe Ia is that they are detected in all types of galaxies. It follows from this, and from the homogeneity of the observed characteristics of SNe Ia, that normal SNe Ia share the same explosion mechanism. The details of this mechanism are still the subject of active research. However, it is well established that the homogeneity of the light curve is essentially due to the narrow spread of nickel masses ($M_{Ni} \sim 0.6 \, M_\odot$) produced in the explosion of a carbon-oxygen white dwarf with a mass close to the Chandrasekhar limit. Consequently, the observational properties of SNe Ia are primarily determined by the precise value of the Chandrasekhar limiting mass.

2.1. *The dependence of the intrinsic properties of SNe Ia on α and c*

The maximum mass of a stable white dwarf star is given by the Chandrasekhar limit. Above this mass, the pressure of degenerate electrons cannot balance the gravitational force. The value of the Chandrasekhar mass is $\simeq 1.44 \, M_\odot$ and can be expressed as:

$$M_{Ch} = \frac{w_3^0 \sqrt{3\pi}}{2} \left(\frac{\hbar c}{G} \right)^{3/2} \frac{1}{(\mu_e m_H)^2},$$ (1)

where μ_e is the average molecular weight per electron, m_H is the mass of the hydrogen atom, w_3^0 is a constant and G is the gravitational constant. In our analysis we will assume that the variation of α follows from a variation in e. Note that

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the Chandrasekhar limiting mass depends only on c. Consequently, if this constant varies, the mass limit will change accordingly, and so will do the mass of nickel synthesized in the explosion. This, in turn, will eventually lead to a different peak luminosity of SNe Ia, and will ultimately affect the determination of distances to distant supernovae. In particular, a small variation of c results in a variation of the Chandrasekhar mass:

$$\frac{\delta M_{\text{Ch}}}{M_{\text{Ch}}} = \frac{3}{2} \frac{\delta c}{c}. \tag{2}$$

At this point we would like to emphasize that the treatment adopted here differs from that used in our previous work in which we assumed a varying α. When this is the case, there is a dependence of the Chandrasekhar mass with α. However, in the present work the variation of α arises from a hypothetical variation of the electron mass. Therefore, there is no dependence of the Chandrasekhar mass with α.

One of the main improvements that incorporates this work is the study the energy released during the explosion. The energy released in a SNe Ia outburst comes from the difference of nuclear binding energies of nickel and cobalt. The the leading contribution is the Coulomb term:

$$E = \frac{3}{5} \frac{e^2}{r_0} \frac{Z^2}{A^{1/3}}, \tag{3}$$

where Z is the atomic number, A is the number of nucleons, and r_0 an empirical constant. Therefore:

$$\frac{\delta E}{E} = \frac{\delta \alpha}{\alpha}. \tag{4}$$

Thus, the energy released depends only on the value of α, and not on c.

The peak luminosity of SNe Ia not only depends on the energy released during the explosion, but also on the opacity of the expanding photosphere. Actually, the emitted photons do not escape immediately because the material ejected in the outburst is optically thick. At early times, the opacity is dominated by the line of opacity κ_i rather than the electron scattering contribution κ_e. It should be noted that $\kappa_i \sim \alpha$ while $\kappa_e \sim \alpha^2$. However, since our analysis focuses on the long-term variations of the observed luminosities due to a hypothetical variation of α, we adopt the treatment of Ref. 56, which is a simplified model of the explosion. We note, nevertheless, that a more detailed analysis can be found in Ref. 61. Within this approximation:

$$\kappa \sim \kappa_e = \frac{n_e}{\rho} \sigma_{\text{Th}}, \tag{5}$$

where n_e is the number of electrons, ρ is the density and σ is the Thomson cross section:

$$\sigma_{\text{Th}} = \frac{8\pi}{3} \left(\frac{e^2}{mc^2} \right)^2, \tag{6}$$

being m the electron mass. This expression can be rewritten in terms of α:

$$\sigma_{\text{Th}} = \frac{8\pi}{3}\left(\frac{\alpha\hbar}{mc}\right)^2,\qquad(7)$$

and therefore:

$$\frac{\delta\kappa}{\kappa} = 2\frac{\delta\alpha}{\alpha}.\qquad(8)$$

Also, from Eq. (6) we find that:

$$\frac{\delta\kappa}{\kappa} = -4\frac{\delta c}{c}.\qquad(9)$$

Thus, the opacity depends on α and c.

In Table 1, we show a summary of the previous analysis. There we list the different dependencies of the peak bolometric magnitude on fundamental constants. In the following we calculate how the peak bolometric magnitude scales on α and c, taking into account all the dependencies just described.

Table 1. Variation of the physical quantities involved in the supernovae explosion due to the variation of fundamental constants.

Varying constant	$\frac{\delta M_{\text{Ch}}}{M_{\text{Ch}}}$	$\frac{\delta E}{E}$	$\frac{\delta\kappa}{\kappa}$
α	0	$\frac{\delta\alpha}{\alpha}$	$2\frac{\delta\alpha}{\alpha}$
c	$\frac{3}{2}\frac{\delta c}{c}$	0	$-4\frac{\delta c}{c}$

2.2. *The peak luminosity and its dependence on α and c*

The relation between the fundamental constants and the peak bolometric magnitude of SNe Ia can be obtained using simple analytical arguments. We follow closely the procedure of Ref. 55 which relies on the analysis of Ref. 56, but this time taking into account all the dependencies on α and c. The peak luminosity of the optical light curve is essentially proportional to the energy deposition rate of the $^{56}\text{Ni}\rightarrow^{56}\text{Co}\rightarrow^{56}\text{Fe}$ decay chain inside the photosphere of the exploding star at the time t_{peak}, which is the time where the diffusion and expansion timescales are similar ($t_{\text{peak}} \sim t_{\text{exp}} \sim t_{\text{dif}}$). It is important to note that the energy deposition rate depends mainly on t_{peak}. Moreover, the γ-ray deposition function can be developed in a power series:

$$\frac{\delta G}{G} = \frac{1.6}{1.6 + 3.6}\frac{\delta\tau}{\tau},\qquad(10)$$

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where τ is the optical depth, for which we adopt $\tau_{\mathrm{peak}} \sim 3.6$. Then, the change of the peak luminosity as a function of t_{peak} and τ_{peak} is given by:

$$\frac{\delta L_{\mathrm{peak}}}{L_{\mathrm{peak}}} = -\frac{\delta t_{\mathrm{peak}}}{t_{\mathrm{peak}}} + \eta \frac{1.6}{1.6 + 3.6} \frac{\delta \tau_{\mathrm{peak}}}{\tau_{\mathrm{peak}}}, \tag{11}$$

being

$$\eta = 1 + 4G(t_{\mathrm{peak}}) - 10.5G(t_{\mathrm{peak}})^2 + 6G(t_{\mathrm{peak}})^3. \tag{12}$$

In what follows, we will analyze the dependence of t_{peak} and τ_{peak} with each fundamental constant considered in this paper.

2.2.1. *Model A: varying α*

A variation of the fine structure constant would result in a change of the opacity and of the energy of the explosion. Accordingly, t_{peak} will be also modified. This happens because

$$t_{\mathrm{peak}} = \left(\frac{3\kappa}{4\sqrt{2}\pi c}\right)^{1/2} \left(\frac{M_{\mathrm{Ch}}^3}{E}\right)^{1/4}, \tag{13}$$

and

$$\tau_{\mathrm{peak}} = \frac{\sqrt{2}c}{2} \left(\frac{M_{\mathrm{Ch}}}{E}\right)^{1/2}, \tag{14}$$

For additional details we refer the interested reader to Ref. 55, but we note that in Eq. (14) of this paper a factor $1/2$ is missing. Then,

$$\frac{\delta t_{\mathrm{peak}}}{t_{\mathrm{peak}}} = \frac{1}{2}\frac{\delta\kappa}{\kappa} - \frac{1}{4}\frac{\delta E}{E}, \tag{15}$$

and

$$\frac{\delta \tau_{\mathrm{peak}}}{\tau_{\mathrm{peak}}} = -\frac{1}{2}\frac{\delta E}{E}. \tag{16}$$

Combining Eqs. (4), (8), (11), (15) and (16), the variation of the peak luminosity in terms of the variation of the fine structure constant α can be written as:

$$\frac{\delta L_{\mathrm{peak}}}{L_{\mathrm{peak}}} \simeq -0.8269\frac{\delta\alpha}{\alpha}. \tag{17}$$

In Fig. 1 we compare our results with those of Ref. 56 and Ref. 55. As can be seen, the dependence of $\delta L_{\mathrm{peak}}/L_{\mathrm{peak}}$ on $\delta\alpha/\alpha$ obtained by Ref. 55 has a smaller slope than the one calculated by Ref. 56. Conversely, the relation between the peak luminosity and the variation of α computed in this paper has a slope stepper than previous estimates. This, clearly, is due to the fact that in the present work we do not only take into account the dependence of the opacity on α. Also, here we consider the dependence of the energy released in the supernova outburst, which was not taken into account by Ref. 56. However, we stress that our results and those of Ref. 56 show that a decrease of the value of α translates into an increase of the luminosity of thermonuclear supernovae. Thus a smaller (larger) value of α makes SNe Ia brighter (fainter).

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2.2.2. *Model B: varying c*

According to the previous discussion – see Table 1 – within this scenario both the opacity and the Chandrasekhar mass vary. Using Eqs. (13) and (14) we get:

$$\frac{\delta t_{\text{peak}}}{t_{\text{peak}}} = -\frac{1}{2}\frac{\delta c}{c} + \frac{1}{2}\frac{\delta \kappa}{\kappa} + \frac{3}{4}\frac{\delta M_{\text{Ch}}}{M_{\text{Ch}}}, \tag{18}$$

and

$$\frac{\delta \tau_{\text{peak}}}{\tau_{\text{peak}}} = \frac{\delta c}{c} + \frac{1}{2}\frac{\delta M_{\text{Ch}}}{M_{\text{Ch}}} = \frac{7}{4}\frac{\delta c}{c}. \tag{19}$$

Combining Eqs. (2), (9), (11), (18) and (19) we can obtain the variation of the peak luminosity due to a variation on the speed of light c:

$$\frac{\delta L_{\text{peak}}}{L_{\text{peak}}} \simeq 1.6442\frac{\delta c}{c}. \tag{20}$$

Note that an increase of the value of c translates into an increase of the luminosity of thermonuclear supernovae. Thus a larger (smaller) value of c would make SNe Ia brighter (fainter).
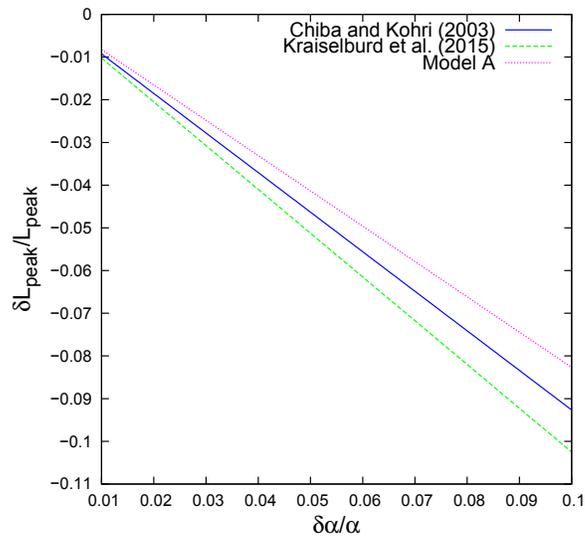
Fig. 1.   Peak luminosity of distant SNe Ia as a function of $\delta\alpha/\alpha$. Here Model A corresponds to the case in which the changes of the energy released during the explosion and the opacity of the expanding photosphere are considered.

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2.3. *The distance modulus*

The distance modulus of a supernova in the case of a varying fundamental constant can be expressed as:

$$\mu = 25 + 5 \log \left(\frac{d_L}{\text{Mpc}} \right) + \delta M, \tag{21}$$

where d_L is the standard distance modulus that depends on the cosmological parameters and redshift z, and δM is a correction taking into account such variation:

$$\delta M = -2.5 \frac{\delta L_{\text{peak}}}{L_{\text{peak}}}. \tag{22}$$

For our calculations we adopt a flat Λ_{CDM} model with $\Omega_R = 0.0$ and $H_0 = 67.31 \, \text{Mpc}^{-1} \, \text{km} \, \text{s}^{-1}$. The latter is the best fit value obtained by the Planck collaboration.[62] It was derived using the Cosmic Microwave Background (CMB) temperature ($30 < l < 2508$), the low-l polarization data ($2 < l < 29$), together with the position of the peak of the Baryon Acoustic Oscillations.[63–65] Besides, the matter density Ω_m is a free parameter in our analysis.

3. Dipole Models

In subsequent sections we will compare the observed distance moduli of SNe Ia with the theoretical predictions. For such comparison we use the previously explained cosmological model, but including a possible variation of fundamental constants – see Sect. 2. To account for the variation of fundamental constants we adopt a dipole model. At this point we would like to emphasize that even though dipole models are controversial, the recent observational evidence is not conclusive, and does not allow to safely discard them.

In particular, we adopt the following expression for the spatial variation of α:

$$\frac{\delta \alpha}{\alpha} = A_\alpha + B_\alpha \cos \theta_\alpha, \tag{23}$$

where $\cos \theta_\alpha = \vec{r} \cdot \vec{D}$, $\vec{D}$ is the direction of the dipole, $\vec{r}$ is the position on the sky, A_α is a constant (a monopole term) and B_α is the amplitude of the dipole term. Likewise, for the variation of c we adopt a similar expression:

$$\frac{\delta c}{c} = A_c + B_c \cos \theta_c \tag{24}$$

4. Results

We now proceed to compare the predictions of the phenomenological dipole models discussed earlier with the data of the Union 2.1[59] and JLA[52] compilations of SNe Ia. These datasets do not incorporate either luminous supernovae nor fast and bright transients, Ca-rich transients or .Ia supernovae, but only "normal" or classical thermonuclear supernovae that follow closely the canonical relationship between

its intrinsic brightness and the decline rate of the light curve.[66] Hence the intrinsic dispersion of the corresponding Hubble diagram is much smaller. Specifically, using these high-quality datasets we consider the values of A, B and $\vec{D}$ introduced in Sect. 3 as free parameters, and we obtain the best-fitting values using the observational data provided by these compilations.

To quantify the agreement between the theoretical results and the observed data we use a χ^2 test. Acording to Ref. 59, the distance modulus estimator for the Union 2.1 compilation can be expressed as:

$$\mu_{\mathrm{o}} = m_b^* - (M_b - \widetilde{\alpha}X_1 + \widetilde{\beta}C + \widetilde{\delta}P), \tag{25}$$

where m_b^* corresponds to the observed peak magnitude in the B band, X_1 and C refer to the deviation from the average light-curve shape and the mean SN Ia BV color respectively [a], and $P = P(m_*^{\mathrm{host}} < m_*^{\mathrm{Threshold}})$ is the probability that the true mass of the host galaxy is less than a certain mass threshold. Besides, M_b, $\widetilde{\alpha}$, $\widetilde{\beta}$ and $\widetilde{\delta}$ are the nuisance parameters.

In this case the χ^2 estimator is:

$$\chi^2_{\mathrm{Union2.1}} = \sum_i \frac{\left(\mu_{\mathrm{t_i}} - \mu_{\mathrm{o_i}}\right)^2}{\sigma^2_{\mathrm{o_i}}}, \tag{26}$$

where the subscript i refers to each observational data point, whereas $\sigma_{\mathrm{o_i}}$ the total errors including systematics and sample dependent effects are taken from the covariance matrix of the Union 2.1 compilation.[59]

On the other hand, the distance modulus for the JLA compliation reads:[52]

$$\mu_{\mathrm{o}} = m_b^* - (M_b + \Delta_{\mathrm{M}} - \widetilde{\alpha}X_1 + \widetilde{\beta}C), \tag{27}$$

where m_b^*, X_1 and C where described above and Δ_{M} is a nuisance parameter which is set to 0 if the star host mass is lower than $10^{10} M_\odot$. Again, M_b, $\widetilde{\alpha}$, $\widetilde{\beta}$ are nuisance parameters. Then, the χ^2 estimator computed using the expression of Ref. 52 is:

$$\chi^2_{\mathrm{JLA}} = (\mu_{\mathrm{o}} - \mu_{\mathrm{t}})^{\mathrm{T}} S^{-1} (\mu_{\mathrm{o}} - \mu_{\mathrm{t}}), \tag{28}$$

with S the covariance matrix of μ_{o}. The data of these magnitudes used for the calculation as well as the covariance matrix are taken from the JLA compilation.

We calculate the reduced χ^2 – that is the value of χ^2 divided by the number of degrees of freedom, ν – for the standard cosmological model for the case in which we do not consider any hypothetical variation of the fundamental constants. For the complete data set of the Union 2.1 compilation (580 data points) we obtain $\chi^2/\nu = 1.11$ while for the JLA compilation (740 data points) we obtain $\chi^2/\nu = 0.96$. As already noted in previous works[62, 67] the agreement between the cosmological

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[a]The light-curve parameters X_1 and C result from the fit of a model of the SNe Ia spectral sequence to the photometric data (for details see Ref. 59).

parameters obtained by the Planck collaboration and those obtained using the JLA compilation is considerably better than those obtained when the Union 2.1 dataset is employed. However, this discrepancy is within the 1σ error bar.

Next, we perform the statistical analyzes to compare the theoretical prediction for the distance modulus μ_{t} including the possible variation of each fundamental constant (α or c) according to the phenomenological dipole model described in Sect.3 with observational data from SNIa. For each data set we consider the following free parameters:

(1) For the analyzes performed with data from the JLA compilation: the matter density in units of the critical density Ω_m, the nuisance parameters $\widetilde{\alpha}$, $\widetilde{\beta}$, M_b and ΔM and the dipole model parameters A, B, the right ascension $R.A$ and declination δ of the dipole model

(2) For the analyzes performed with data from the Union 2.1 compilation: the matter density in units of the critical density Ω_m, the nuisance parameters $\widetilde{\alpha}$, $\widetilde{\beta}$, M_b and $\widetilde{\delta}$ and the dipole model parameters A, B, the right ascension $R.A$ and declination δ of the dipole model

Table 2.   Results for the parameters of the dipole model for the spatial variation of α, the SNe Ia nuisance parameters and Ω_{m} obtained from the statistical analysis with the 1σ error. Δ refers to Δ_{M} for the analysis performed with the JLA data and to $\widetilde{\delta}$ for the Union2.1 data. The size of the JLA dataset is 740 SNe Ia. The size of the Union2.1 dataset is 580 SNe Ia. For the standard model the value of the goodness of fit is χ^2_ν=0.96 for JLA and χ^2_ν=1.11 for Union2.1.

Dataset	A_α $(\times 10^{-2})$	B_α $(\times 10^{-2})$	R.A. (h)	δ (°)	$\widetilde{\alpha}$ $(\times 10^{-1})$	$\widetilde{\beta}$	Δ $(\times 10^{-2})$	M_{b}	Ω_{m} $(\times 10^{-1})$	χ^2_ν
JLA	-1.42 ± 0.84	0.22 ± 0.47	—	—	1.25 ± 0.06	2.63 ± 0.07	-5.32 ± 0.45	-19.11 ± 0.02	3.26 ± 0.27	0.86
Union2.1	5.77 ± 0.92	-3.11 ± 1.16	14 ± 4	-70 ± 15	1.04 ± 0.07	2.30 ± 0.06	-3.20 ± 1.66	-19.51 ± 0.01	3.03 ± 0.24	0.84

In Table 2 we show the results of the statistical analyses described before for the case where the luminosity distance is modified through α variation. A quick look at this table reveals that the Union 2.1 compilation seems to favor a dipole model, since the value of χ^2 is smaller than the one obtained with a standard Λ_{CDM} cosmological model. For the JLA compilation, even though the phenomenological dipole model results in a lower value of the reduced χ^2, the differences between the values of χ^2 are not statistically significant. Thus, we judge that this dataset does not yield significant evidence favoring the phenomenological dipole model.

It should be noted as well that the JLA dataset does not allow to derive the direction of the dipole. The reason for this turns out to be that the magnitude of the dipole B_α is small compared with the uncertainties. In fact, the results for the confidence limits on the dipole term B_α is consistent with 0 for the JLA analysis. Consequently, there is no difference between the theoretical distance moduli calculated for different dipole directions. This is not the case for the Union 2.1 analysis and therefore, the statistical analysis performed with this dataset yields limits on the direction of the dipole, although with sizeable uncertainties (75% in right as-

cension and 12% in declination). We also note that there is no degeneracy between the free parameters A_α and B_α, for the analysis performed with the JLA data set while there is a little degeneracy for the Union2.1 case (see Fig. 2).

Table 2 also shows the values of the SNe Ia nuissance parameters and the cosmological density matter from the statistical analysis considering α as a varying constant for both compilations. For Union 2.1, the estimates of $\widehat{\delta}$ and Ω_m are consistent at 1σ level with those obtained by Ref. 59, while $\widehat{\alpha}$ and $\widehat{\beta}$ show a 2σ level consistency. Besides, the M_b estimates are only compatible at 5σ level with those of Ref. 59. On the other hand, for JLA compilation, the estimates of two parameters Ω_m and Δ_M are in agreement with the ones from Ref. 52 at 1σ level, $\widehat{\alpha}$ and M_b values are consistent at 2σ level, and only the $\widehat{\beta}$ estimates are consistent at 4σ level. It should be mentioned that only the Ω_m estimates of Refs. 59 and 52 are consistent with each other at 1σ level.

Finally, in Fig. 3 we compare the individual distance moduli obtained using the phenomenological model for α variation with the observed data using the SNe Ia nuissance parameters and Ω_m estimates from the statistical analysis explained above. The upper panels correspond to the Union 2.1 dataset, whereas the bottom plots correspond to the JLA dataset. Also, the left panels display the distance moduli as a function of right ascension, the central panels show the individual distance moduli as a function of declination and, finally, in the right panels we plot the relative differences between the predictions and the observed data as a function
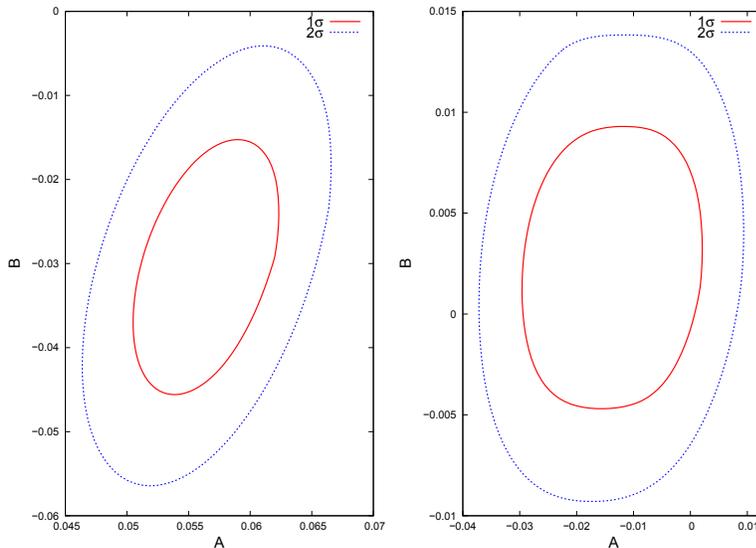
Fig. 2.   Results of the statistical analysis for a hypothetical variation of α. We show the 1σ (68%) and 2σ (95%) confidence contours for the free parameters A_α and B_α of the phenomenological dipole model obtained using the Union 2.1 data set (left panel) and the JLA data set (right panel).

of the redshift. The grey points are the observed data, while the red ones are the results of our theoretical calculations.

We continue our analysis examining a possible variation of c. In Table 3 we present the results of the statistical analysis considering c as a free parameter and using the Union 2.1 and the JLA datasets respectively. As in the case for the spatial variation of α, the value of χ^2/ν does not differ significantly from the value obtained using the standard model when the JLA dataset is used. However, when the Union 2.1 dataset is used, the differences between the dipole model and the standard cosmological model is sizeable. As it occurs for the case of a varying α, the JLA sample does not allow to establish a preferred direction for the dipole. The reason for this is the same previously discussed for the case of a varying α. It is important to emphasize that these results set an independent upper bound to a possible spatial variation of c. To the best of our knowledge this is the first constraint on such hypothetical variation.

In addition, we add the SNe Ia nuissance parameters and the cosmological density matter estimates from the statistical analysis considering varying c for both compilations in Table 3. For Union 2.1, the values of $\widehat{\delta}$ and $\Omega_{\rm m}$ are compatible with those from Ref. 59 at 1σ level, while $\widehat{\alpha}$ and $\widehat{\beta}$ show a 2σ level consistency. Finally,
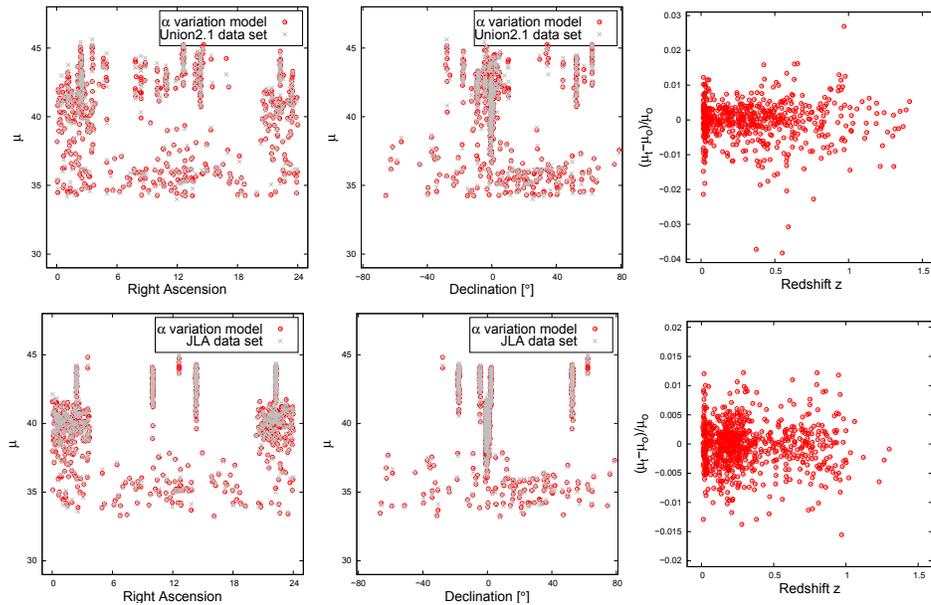
Fig. 3.   Distance modulus as a function of the right ascension (left panels) and declination (middle panels) for the Union 2.1 (upper panels) and JLA (lower panels) datasets. We compare the predictions of the phenomenological dipole model for a variation of α (red circles) with the observed data (gray symbols). In the right panels we show the relative differences between the predictions and the observed data as a function of the redshift.

Table 3.   Same as Table 2, but for a model in which a variation of c is adopted.

Dataset	$A_{\rm c}$ $(\times 10^{-1})$	$B_{\rm c}$ $(\times 10^{-2})$	R.A. (h)	δ $(°)$	$\widetilde{\alpha}$ $(\times 10^{-1})$	$\widetilde{\beta}$	Δ $(\times 10^{-2})$	$M_{\rm b}$	$\Omega_{\rm m}$ $(\times 10^{-1})$	χ^2_ν
JLA	-2.49 ± 0.09	0 ± 0.4	—	—	1.26 ± 0.06	2.63 ± 0.07	-5.29 ± 2.56	-20.22 ± 0.03	3.26 ± 0.47	0.88
Union2.1	1.9 ± 0.05	1.47 ± 0.6	14 ± 5	-70 ± 14	1.04 ± 0.07	2.30 ± 0.06	-3.25 ± 2.10	-18.61 ± 0.02	3.01 ± 0.25	0.84

the estimates of $M_{\rm b}$ are not consistent at all. Besides, for JLA compilation, the estimates of $\Delta_{\rm M}$ and $\Omega_{\rm m}$ are in agreement with the ones from Ref. 52 at 1σ level while the constraints on $\widehat{\alpha}$ are consistent at 2σ level, and only the $\widehat{\beta}$ estimates are consistent at 4σ level. Finally, there is no agreement between our estimates on M_b and those of Ref. 52 within $5\,\sigma$. However, it is importan to stress, that the Union2.1 and JLA results were obtained assuming a standard cosmological model which is not the case for the analyses performed in this paper. Most important, the estimates on M_b provided by Cepheid based calibrations of the SN Ia peak luminosity (and therefore independent of the assumed cosmological model)[68] are consistent within $5\,\sigma$ with the estimates of Table 3.

5. Summary and Conclusions

In this paper we have analyzed the dependence of the distance modulus of Type Ia supernovae on the fundamental constants α and c. In our analysis we have included the dependence on a possible variation of these constants of the energy released in the explosion. This study had not been performed before. Besides the dependence of the SNe I standardization with the distance modulus predicted by the theoretical model here exposed, we have also had the possibility of estimating $\Omega_{\rm m}$ and the nuisance parameters of SNe Ia. Using the scaling laws resulting from this study we have examined the possibility of obtaining upper bounds to a hypothetical variation of these fundamental constants. To do this we have used a phenomenological model that accounts for a possible anisotropy of cosmological observables. Specifically, we have assumed that these fundamental constants, as suggested by some recent observations, have a dipolar dependence. We then compared the predictions of our theoretical analysis with the most up-to-date compilations of observational data for thermonuclear supernovae, namely the Union 2.1 and JLA datasets. The reason to choose Type Ia supernovae for our analysis is threefold. On one hand, the peak luminosities of SNe Ia depend on α and c. Therefore, a variation of these constants directly translates into a different peak bolometric magnitude. This, in turn, means that the distance modulus is modified. On the other hand, normal SNe Ia are a very homogeneous class of objects that can be observed up to very large distances. Consequently, any possible variation of the fundamental constants analyzed in this work would become prominent. Finally, the last reason to adopt Type Ia supernovae for our analysis is that we have databases of observational measurements for a large number of them.

Our results show that the JLA compilation of SNe Ia data favors a standard

cosmological model, in which none of these fundamental constants varies. On the contrary, when the Union 2.1 dataset is employed there is marginal evidence for such variations. Specifically, the JLA dataset does not allow to obtain a preferred dipole direction. This is not the case when the Union 2.1 compilation is used. However evidence for such variation is weak at the 3σ level, even in the case in which the best data of the Union 2.1 database is employed. Thus, we conclude that at 3σ, the parameters of the supernova data are consistent with a null variation of the fundamental constants. This can be viewed from a different perspective. The analysis of the observational data can be used to set upper limits to the spatial variation of the fundamental constants. We obtained upper bounds to the spatial variation of α and c. These upper limits are $\delta\alpha/\alpha \sim 10^{-2}$ and $\delta c/c \sim 10^{-1}$. To the best of our knowledge, here we have reported the first upper limit to a hypothetical spatial variation of c. Hence, we judge that this is perhaps the main result of our calculations. On the other hand, the best current upper limit on the spatial variation of α are $\delta\alpha/\alpha \sim 10^{-5}$, and was obtained using data from quasar absorption systems for a range of redshifts $0.3 < z < 2.8$.[39,43] Clearly, this limit is considerably more stringent that that obtained here. However, we emphasize that the upper bound reported here has been obtained using a totally independent method and a different dataset (the range of redshifts are $0.623 < z < 1.415$ for the Union2.1 and $0.05 < z < 1$ for JLA), and therefore complements (and is compatible with) the previous upper limit.

Acknowledgments

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