# Plasma photoemission from string theory

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**Abstract**

Leading 't Hooft coupling corrections to the photoemission rate of the planar limit of a strongly-coupled $\mathcal{N} = 4$ SYM plasma are investigated using the gauge/string duality. We consider the full $\mathcal{O}(\alpha'^3)$ type IIB string theory corrections to the supergravity action, including higher order terms with the Ramond-Ramond five-form field strength. We extend our previous results presented in [1]. Photoemission rates depend on the 't Hooft coupling, and their curves suggest an interpolating behaviour from strong towards weak coupling regimes. Their slopes at zero light-like momentum give the electrical conductivity as a function of the 't Hooft coupling, in full agreement with our previous results of [2]. Furthermore, we also study the effect of corrections beyond the large $N$ limit.

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# Contents

# 1 Introduction

Electrically charged particles in a quark gluon plasma (QGP) emit photons. An analysis of these photons can lead to very valuable information about the medium in which they are produced. On the one hand, transport coefficients related to the electric charge such as electrical conductivity and charge diffusion constant characterize the dynamics of long wavelength, low frequency fluctuations in a plasma. They are effectively related to ultra-soft photons, *i.e.* those with momentum much smaller than the equilibrium temperature of the medium, $T$. Ultra-soft photons eventually probe the hydrodynamical regime of the plasma, with momentum $k \leq \lambda^2 T$, where $\lambda$ is the 't Hooft coupling defined as $\lambda \equiv g_{YM}^2 N$, where $g_{YM}$ is the SYM theory coupling and $N$ the rank of its gauge group, $SU(N)$ in the present case. On the other hand, it is possible to scrutinize a thermally equilibrated plasma for a long range of emitted photon wavelengths. This precisely gives shape to the photoemission rate from a plasma as a function of the energy of the photons. It includes ultra-soft, soft and hard photons, thus providing extremely useful information about the dynamical structure of the medium.

For a weakly coupled QCD plasma, transport coefficients and photoemission rates have been calculated using perturbative quantum field theory in [3, 4, 5, 6, 7, 8] and references therein. These references are particularly important since Arnold, Moore and Yaffe have obtained the first complete leading order results for the photoemission rates in QCD [5]. They conclude that, in addition to well known $2 \longleftrightarrow 2$ particle processes, near-collinear *Bremsstrahlung* and inelastic pair annihilation also make leading order contributions. The Landau-Pomeranchuk-Migdal (LPM) suppression, which is the effect produced by multiple soft scatterings, may occur during the emission of the photon and has important implications on the consistent treatment of the above mentioned processes. The LPM effect leads to an $\mathcal{O}(1)$ suppression of these near-collinear processes.

There are indications, however, that the QGPs produced at the Relativistic Heavy Ion Collider (RHIC) and at the Large Hadron Collider (LHC) are in the strongly-coupled regime of QCD [9, 10, 11, 12, 13, 14, 15, 16, 17]. This is where the gauge/string duality enters. This duality allows us to compute properties of a strongly coupled gauge theory in terms of a weakly coupled holographic dual string theory description [18, 19, 20]. We ought to admit that at present there is no complete or exact holographic string theory dual model which accounts for all the relevant properties of real QCD, not even in the planar limit of the gauge theory. For reasons which shall be explained below, the holographic string theory dual model which has been considered so far for these investigations is in fact dual to the large $N$ limit of the strongly-coupled $SU(N)$ $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) plasma.

The holographic dual model of the planar limit of the strongly-coupled $SU(N)$ $\mathcal{N} = 4$ SYM plasma is defined in terms of a type IIB supergravity background given by a direct product of an Anti-de Sitter-Schwarzschild black hole in five dimensions ($AdS_5 BH$) times a five sphere $S^5$. There is a number of considerations to take into account at the moment of extrapolating this dual description of the large $N$ limit of $\mathcal{N}=4$ SYM theory in order to make contact with QCD. Firstly, as it is well known at zero temperature these theories are very different. Indeed, the field content is different: while QCD has three colour degrees of freedom and three flavours, matter is in the fundamental representation of the gauge group $SU(3)$, it shows colour confinement, has explicit and spontaneous chiral symmetry breaking, and displays a discrete spectrum; on the other hand, in the $\mathcal{N}=4$ SYM theory all their fields transform in the adjoint representation of $SU(N)$, it is not a confining theory, conformal symmetry is preserved at quantum level, and it is a supersymmetric theory with the maximal number of supersymmetries in four dimensions. At finite equilibrium temperature, $T$, above the critical temperature of QCD, $T_c$, where hadrons become a deconfined QGP, there are two regimes. For $T >> T_c$, again the two types of plasmas related to these two theories behave very differently too: while in QCD the coupling runs to weak coupling, leading to a free gas of quarks and gluons; in the case of the $\mathcal{N}=4$ SYM plasma, the coupling, which remains constant, is strong. Thus, it leads to a strongly coupled plasma. However, in the intermediate region where $T$ is just above $T_c$, both plasmas behave somewhat similarly. In this case QCD behaves as a strongly coupled plasma of gluons and fundamental matter. These degrees of freedom are deconfined, there is screening and the correlation lengths are finite. Interestingly, the $\mathcal{N}=4$ SYM plasma shares those properties because it is a strongly coupled plasma of gluons and adjoint matter fields, it is also deconfined, shows screening, and has finite correlation lengths. Moreover, quantum field theories lattice calculations indicate that for certain properties the similarities can be made even quantitatively (see for instance [21] and references therein). Therefore, one may assume that for $T > T_c$ but not $T >> T_c$, there is a parametric region where one can focus on in order to describe the rates for the emission of photons from a thermally equilibrated SYM plasma using the gauge/string duality at finite yet strong 't Hooft coupling.

A very important step towards the understanding of the photoemission process and electric charge transport coefficients of QGP in terms of the $\mathcal{N}=4$ SYM plasma has been done in a very nice paper by Caron-Huot, Kovtun, Moore, Starinets and Yaffe [22]. They consider the two limiting situations: for very large and very small 't Hooft coupling. In the strongly coupled case they consider the pure type IIB supergravity description of the large $N$ limit of the $\mathcal{N}=4$ SYM plasma, which we summarize in section 3. In the opposite limit they consider a perturbative quantum field theory description of the $\mathcal{N}=4$ SYM plasma, using similar ideas as in [5]. In section 5 we shall briefly review some perturbative results of [22].

In the light of the new experimental findings suggesting that the QGP plasma at RHIC and LHC is in the strongly coupled regime of QCD, a more realistic outlook requires a consideration of the 't Hooft coupling expansion around the infinitely strongly-coupled regime of the plasma. On the string theory side, we must therefore consider the full $\mathcal{O}(\alpha'^3)$ type IIB string theory corrections to the supergravity action. It includes a number of terms which arise from the supersymmetric completion to the standard power-four ten-dimensional Weyl-tensor. These new terms are constructed from a rank-6 tensor which contains the Ramond-Ramond five-form field strength. This is indeed a very complicated task from the technical point of view. However, it is worth carrying out since it yields the precise structure of the 't Hooft coupling corrections to the strong coupling regime. Using this procedure we have obtained very interesting results, which we briefly describe here and present in full detail in section 4.

Our results show the following features. Firstly, the slopes of the photoemission rates, which at zero light-like momentum give the electrical conductivity as a function of the 't Hooft coupling, are in full agreement with our previous results of [2]: the electrical conductivity increases as the 't Hooft coupling decreases. This concerns the hydrodynamic regime of the plasma. Secondly, for higher momentum, the height of the peaks decrease as the 't Hooft coupling increases (*i.e.* as we approach the limit of infinite coupling), their maxima are shifted towards the ultraviolet and the photoemission rate curves cross downwards the limiting strongly coupled curve for momentum around 3 times the equilibrium temperature. Another important feature which comes from our results is that the number of emitted photons increases as the 't Hooft coupling weakens. These features show an interpolating trend from the supergravity calculation of the strongly coupled gauge theory towards the perturbative quantum field theory calculation in the weakly coupled $\mathcal{N}=4$ supersymmetric Yang-Mills plasma. In addition to describing these effects in more detail in the general discussion and conclusions section below, we will also consider the effect of including non-planar perturbative $1/N$ corrections from higher derivative terms in the type IIB action as well as non-perturbative contributions due to D-instanton effects.

In section 2 we briefly describe generalities about the formalism behind the calculation of plasma photoemission rates based on the computation of two-point correlators of electromagnetic currents. A review of strongly coupled $\mathcal{N}=4$ SYM plasma results entirely obtained within pure type IIB supergravity, *i.e.* with no string theory corrections, is presented in section 3. In section 4, which is the longest section of the paper, we introduce details of the formalism and results from our calculations of the leading 't Hooft coupling corrections to the photoemission rate of a strongly-coupled $\mathcal{N}=4$ SYM plasma using the gauge/string duality. Section 5 is devoted to a very brief review of results in the weakly coupled regime. The material of this section is used in the last section of the article in order to carry out a general discussion of our results.

# 2 Derivation of photoemission rates in SYM plasmas

In this section we very briefly review the formalism needed in order to derive the photoemission rate in plasmas from thermal field theory. Since we expect to be able to compare our results with those of reference [22], when possible we mainly follow its notation through this paper. Also, the same assumptions as in [22] are considered here: the plasma is in thermal equilibrium; we do not include prompt photons produced by the initial scattering of partons from the colliding nuclei; and, the electromagnetic coupling constant, $e$, is considered small enough in order to ensure that photons are not to be re-scattered.

Consider the minimal coupling of a photon to the electromagnetic current $J_\mu^{\rm em}$ of the SYM plasma. Recall that the $SU(N)$ $\mathcal{N}=4$ gauge supermultiplet is $\{\mathcal{A}_\mu^a, \psi_p, \phi_{pq}\}$, where $a$ is the $SU(N)$ colour index, $p$, $q = 1, \cdots, 4$, and all the fields transform in the adjoint representation of the gauge group. They are $SU(N)$ gauge bosons, 4 Weyl fermions and 6 real scalars, respectively. Furthermore, since there is an anomaly free global $SU(4)$ $R$-symmetry, there is an associated global $R$-symmetry current, $J_\mu$. The way to consider the electromagnetic coupling is by adding a $U(1)$ gauge field $A^\mu$ which couples to the conserved current of a $U(1)$ subgroup of the full $SU(4)$ $R$-symmetry group [22], under the assumption that, to leading order in $e$, $J_\mu^{\rm em} \equiv J_\mu$. Thus, the Lagrangian can be written as

$$L = L_{SYM} + e\, J_\mu^{\rm em}\, A^\mu - \frac{1}{4} F_{U(1)}^2 \,, \tag{1}$$

where $L_{SYM}$ is the Lagrangian of the $\mathcal{N}=4$ SYM theory and $F_{U(1)}^2$ is the kinetic term of the photon field.

We denote the photon four-momentum as $K \equiv (k^0, \vec{k})$, which is a null four-vector having its time component fixed by the on-shell condition $k^0 = |\vec{k}|$. We use the mostly plus signature for the Minkowski metric in four dimensions, denoted by $\eta_{\mu\nu} = (-,+,+,+)$. First, let us consider the Wightman function of electromagnetic currents defined as

$$C_{\mu\nu}^{<}(K) = \int d^4X\, e^{-iK\cdot X} \; < J_\mu^{\rm em}(0) J_\nu^{\rm em}(X) > , \tag{2}$$

which in thermal equilibrium is related to the spectral density $\chi_{\mu\nu}(K)$ by

$$C_{\mu\nu}^{<}(K) = n_b(k^0)\, \chi_{\mu\nu}(K) \,, \tag{3}$$

with the Bose-Einstein distribution function $n_b(k^0) = 1/(e^{\beta k^0} - 1)$. In addition, the spectral density is given by the imaginary part of the retarded current-current correlation function

$$\chi_{\mu\nu}(K) = -2\, {\rm Im}\, C_{\mu\nu}^{\rm ret}(K) \,. \tag{4}$$

The number of photons which are produced per unit time per unit volume is denoted by $\Gamma_\gamma$. At leading order in $e$ the photoemission rate is given by

$$d\Gamma_\gamma = \frac{e^2}{2|\vec{k}|}\, \eta^{\mu\nu}\, C_{\mu\nu}^{<}(K)\, |_{k^0=|\vec{k}|} \; \frac{d^3k}{(2\pi)^3} \,. \tag{5}$$

Notice that this formula for the photoemission rate holds to leading order in the electromagnetic coupling $e$. On the other hand, and very importantly for our purposes, it is valid non-perturbatively in all other interactions, *i.e.* the strong interaction [22].

It is worth mentioning that the slope of the photoemission rate in the zero-frequency limit is proportional to the electrical conductivity of the plasma, $\sigma$, which can also be determined by the current-current correlator using the Kubo formula:

$$\sigma = \lim_{k^0 \to 0} \frac{e^2}{6T} \, \eta^{\mu\nu} \, C^{<}_{\mu\nu}(k^0, \vec{k} = 0) \, . \tag{6}$$

In the next section we describe the computation of the plasma photoemission rate in infinitely strongly-coupled plasma.

# 3 Review of photoemission rates at strong 't Hooft coupling

The $AdS_5 BH \times S^5$ background, which is an exact solution of type IIB supergravity, is given by

$$ds^2 = \left(\frac{r_0}{R}\right)^2 \frac{1}{u} \left(-f(u)\, dt^2 + d\vec{x}^2\right) + \frac{R^2}{4u^2 f(u)} \, du^2 + R^2 \, d\Omega_5^2 \, , \tag{7}$$

where $f(u) = 1 - u^2$, and $R$ is the radius of the $AdS_5$ and the five-sphere. The $AdS$-boundary is at $u = 0$ and the black hole horizon is at $u = 1$. For the $AdS_5$ coordinates we use indices $m = \{(\mu = 0, 1, 2, 3), 5\}$. It is well known that this is the holographic dual background to the large $N$ limit of the $SU(N)$ $\mathcal{N} = 4$ SYM theory at finite temperature $T$.

As mentioned, the purpose of the present work is to investigate the $\mathcal{O}(\lambda^{-3/2})$ 't Hooft coupling corrections to the photoemission rate of a $SU(N)$ $\mathcal{N} = 4$ SYM plasma produced by the leading order $\alpha'^3$ corrections to the pure type IIB supergravity calculation. In this section we briefly review some of the calculations of [22], which are applicable for the $\lambda \to \infty$ limit. The idea is to obtain the correlation functions of two $R$-symmetry currents using the methods developed in references [23, 24].

The general form of the correlator at finite temperature is obtained by taking into account rotation and gauge invariance:

$$C^{\rm ret}_{\mu\nu}(K) = \Pi^{\rm T}(k^0, k) \, P^{\rm T}_{\mu\nu}(K) + \Pi^{\rm L}(k^0, k) \, P^{\rm L}_{\mu\nu}(K) \, , \tag{8}$$

where the transverse and longitudinal projectors are defined such that $P^{\rm T}_{0\mu}(K) = 0$, $P^{\rm T}_{ij}(K) = \delta_{ij} - k_i k_j / k^2$, and $P^{\rm L}_{\mu\nu}(K) = P_{\mu\nu}(K) - P^{\rm T}_{\mu\nu}(K)$, with $P_{\mu\nu} = \eta_{\mu\nu} - K_\mu K_\nu / K^2$. We use the notation for the photon light-like momentum defined in the previous section and $k = |\vec{k}|$. The trace of the spectral function is

$$\chi^\mu_\mu(k^0, k) = -4 \, {\rm Im}\Pi^{\rm T}(k^0, k) - 2 \, {\rm Im}\Pi^{\rm L}(k^0, k) \, . \tag{9}$$

For light-like momentum only $\Pi^{\rm T}$ contributes. Therefore, it is the only relevant part for the computation of the photoemission rate.

The gauge/string duality establishes a precise prescription to compute a two-point correlator of conserved currents in a strongly coupled SYM theory. The idea is the following: the insertion of an operator of the SYM theory at the $AdS$-boundary induces a fluctuation of a certain ten-dimensional background field. Specifically, using the gauge/string duality prescription, a global $U(1)$ symmetry current in the SYM theory couples to a $U(1)$ gauge field in the bulk, $A_m$. From the SYM theory point of view the $U(1)$ group is a subgroup of the $SU(4)$ $R$-symmetry group of the $\mathcal{N}=4$ SYM theory. Recall that the $SU(4)$ group is isomorphic to the $SO(6)$ group, which obviously is the global symmetry which generates rotations among the 6 real scalars of the vector supermultiplet of the gauge theory. On the other hand, from the supergravity side, the isometry group of the five sphere is $SO(6)$. Thus, there is a $U(1)$ subgroup, which is related to vector fluctuations of the metric, whose gauge field is precisely the $A_\mu$ Abelian gauge field. Therefore, the point is to solve the linearised equations of motion for the vector perturbations of the metric. The definition of the two-form field strength is $F_{mn}=\partial_m A_n-\partial_n A_m$. With the identification $E_i\equiv F_{0i}$ one can write down the EOMs for the vector fluctuation by splitting them into the transverse $(x,y)$, and longitudinal $(z)$ components as follows:

$$E''_{x,y}-\frac{2u}{f(u)}\,E'_{x,y}+\frac{\varpi_0^2-\kappa_0^2 f(u)}{uf^2(u)}E_{x,y}=0\,, \tag{10}$$

$$E''_z-\frac{2\varpi_0^2 u}{f(u)(\varpi_0^2-\kappa_0^2 f(u))}\,E'_z+\frac{\varpi_0^2-\kappa_0^2 f(u)}{uf^2(u)}E_z=0\,, \tag{11}$$

where primes denote derivatives with respect to the radial coordinate $u$, and one defines $\varpi_0\equiv k^0/(2\pi T)$ and $\kappa_0\equiv k/(2\pi T)$. The solution of these EOMs have been discussed in [22], so here we just quote their results in the following equations. First, notice that the correlators are determined by the boundary term of the five-dimensional on-shell Maxwell action

$$S_B=\frac{N^2T^2}{16}\lim_{u\to 0}\int\frac{d^4K}{(2\pi)^4}\left[\frac{f(u)}{\kappa_0^2 f(u)-\varpi_0^2}E'_z(u,K)E_z(u,-K)-\frac{f(u)}{\varpi_0^2}E'_{x,y}(u,K)\cdot E_{x,y}(u,-K)\right] \tag{12}$$

and by applying the Lorentzian AdS/CFT prescription [23] it turns out that the transverse component which is the only one actually needed for the computation of the photoemission rate is given by [22]

$$\Pi^T(k^0,k)=-\frac{N^2T^2}{8}\lim_{u\to 0}\frac{E'_x(u,K)}{E_x(u,K)}\,. \tag{13}$$

For light-like momenta there is an analytical solution to the EOM above which can be written in terms of a hypergeometric function

$$E_x(u)=(1-u)^{-i\varpi_0/2}\,(1+u)^{-\varpi_0/2}\,{}_2F_1\left(1-\frac{1}{2}(1+i)\varpi_0,\,-\frac{1}{2}(1+i)\varpi_0;\,1-i\varpi_0;\,\frac{1}{2}(1-u)\right)\,. \tag{14}$$

Thus, the trace of the spectral function for light-like momenta is

$$\chi^{\mu}{}_{\mu}(k^0 = k) = \frac{N^2 T^2 \varpi_0}{8} \; | \, {}_2F_1 \left( 1 - \frac{1}{2}(1+i)\varpi_0, \, 1 + \frac{1}{2}(1-i)\varpi_0; \, 1 - i\varpi_0; \, -1 \right) |^{-2} \; . \quad (15)$$

In addition, the electrical conductivity is given by

$$\sigma = e^2 \, \frac{N^2 T}{16\pi} \, , \quad (16)$$

which has been obtained from the Kubo formula quoted in the previous section.

Finally, the photoemission rate is given by

$$\frac{d\Gamma_\gamma}{dk} = \frac{\alpha_{\rm em} N^2 T^3}{16\pi^2} \, \frac{(k/T)^2}{e^{k/T} - 1} \; | \, {}_2F_1 \left( 1 - \frac{(1+i)k}{4\pi T}, \, 1 + \frac{(1-i)k}{4\pi T}; \, 1 - \frac{ik}{2\pi T}; \, -1 \right) |^{-2} \, , \quad (17)$$

which holds in the large $N$ limit and for large $\lambda$ (where the supergravity approximation is valid, $1 << \lambda << N$), and is valid for the whole range of photon energies.

Now, we proceed to investigate the leading 't Hooft coupling corrections to these expressions and analyse their physical implications.

# 4 't Hooft coupling corrections to photoemission rates

In this section we present the general corrections to type IIB supergravity action at leading order in $\alpha'$. Firstly, in subsection 4.1 we describe the formalism needed to account for higher derivative corrections to the effective IIB action. Then, we focus on $\mathcal{O}(\alpha'^3)$ string theory corrections and develop the vector perturbations we need for the computation of the current-current correlators. In subsection 4.2 we carry out the computation of 't Hooft coupling corrections to photoemission rates, whose results we show in subsection 4.3. Our results concerning the effects of leading $1/N$ corrections and non-perturbative instanton contributions are restricted to the electrical conductivity of the plasma, and are presented in the discussion and conclusions, in the last section of the paper.

## 4.1 Higher derivative corrections to the effective IIB action and vector perturbations

To begin with, we consider the leading type IIB string theory corrections to the supergravity action $S_{IIB}^{SUGRA}$ which are given in the term $S^3_{\mathcal{R}^4}$. The total action that we shall consider is

$$S_{IIB} = S_{IIB}^{SUGRA} + S^3_{\mathcal{R}^4} \, . \quad (18)$$

At the strong 't Hooft coupling limit the holographic dual model is derived from type IIB supergravity, *i.e.* for $\alpha' \to 0$. This contains the Einstein-Hilbert action coupled to the dilaton and the Ramond-Ramond five-form field strength

$$S_{IIB}^{SUGRA} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[ R_{10} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4.5!} (F_5)^2 \right] . \tag{19}$$

Effects of higher curvature terms which includes $\mathcal{O}(\alpha'^3)$, perturbative $1/N$ corrections as well as instanton corrections were considered in the presence of D3-branes in type IIB string theory by Green and Stahn in [25]. This reference proposes a supersymmetric completion of the $C^4$ term, where $C$ is the ten-dimensional Weyl tensor, leading to the following correction:

$$S_{\mathcal{R}^4}^3 = \frac{\alpha'^3 g_s^{3/2}}{32\pi G} \int d^{10}x \int d^{16}\theta \sqrt{-g}\, f^{(0,0)}(\tau,\bar{\tau}) [(\theta\Gamma^{mnp}\theta)(\theta\Gamma^{qrs}\theta)\mathcal{R}_{mnpqrs}]^4 + c.c. \,, \tag{20}$$

where $\tau$ is the complex scalar field given by $\tau_1 + i\tau_2 \equiv a + ie^{-\phi}$, with $a$ being the axion and $e^\phi = g_s$ the string coupling. The function $f^{(0,0)}(\tau,\bar{\tau})$ is the so-called modular form. The tensor $\mathcal{R}$ tensor is defined in terms of the Weyl tensor and

$$F^+ = (1+*)F_5/2 \,, \tag{21}$$

as given in [25, 26, 27, 28]

$$\mathcal{R}_{mnpqrs} = \frac{1}{8} g_{ps} C_{mnqr} + \frac{i}{48} D_m F^+_{npqrs} + \frac{1}{384} F^+_{mnpkl} F^{+\ kl}_{qrs} \,. \tag{22}$$

The action (20) was arrived at using the fact that the physical content of type IIB supergravity can be arranged in a scalar superfield $\Phi(x,\theta)$, where $\theta_a$, with $a = 1, \cdots, 16$, is a complex Weyl spinor of $SO(1,9)$. The matrices $\Gamma$ have the usual definitions [27].

The modular form is presented in [29] and is given by the following expression

$$f^{(0,0)}(\tau,\bar{\tau}) = 2\zeta(3)\tau_2^{3/2} + \frac{2\pi^2}{3}\tau_2^{-1/2} + 8\pi\tau_2^{1/2} \sum_{m\neq 0, n\geq 0} \frac{|m|}{|n|} e^{2\pi i |mn| \tau_1} K_1(2\pi|mn|\tau_2) \,, \tag{23}$$

where $K_1$ is the modified Bessel function of second kind which comes from the non-perturbative D-instantons contributions. Recall that the zeta function $\zeta(3)$ is the coefficient of the first perturbative correction in the Eisenstein series of the modular form. Note that in the background we consider with $N$ coincident parallel D3 branes, the axion vanishes, thus $\tau_1 = 0$, while $\tau_2 = g_s^{-1}$. Therefore, for small values of $g_s$ the modular form becomes

$$f^{(0,0)}(\tau,\bar{\tau}) = 2(4\pi N)^{3/2} \left( \frac{\zeta(3)}{\lambda^{3/2}} + \frac{\lambda^{1/2}}{48N^2} + \frac{e^{-8\pi^2 N/\lambda}}{2\pi^{1/2} N^{3/2}} \right) . \tag{24}$$

It is interesting to mention that Green and Stahn also have shown that the D3-brane solution in supergravity does not get renormalised by higher derivative terms [25]. Previously Banks and Green had shown that $AdS_5 \times S^5$ is a solution to all orders in $\alpha'$ [30].

Now, we focus on the large $N$ limit of the dual $SU(N)$ $\mathcal{N}=4$ SYM theory. Later on, in the conclusions, we shall return to the consequences of the general corrections to the electrical conductivity.

The finite leading 't Hooft coupling corrections are accounted for by the following action [28]

$$S_{IIB}^{\alpha'} = \frac{R^6}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[ \gamma e^{-\frac{3}{2}\phi} \left( C^4 + C^3 \mathcal{T} + C^2 \mathcal{T}^2 + C \mathcal{T}^3 + \mathcal{T}^4 \right) \right] , \tag{25}$$

obtained from the action (20) in the large $N$ limit, where $\gamma \equiv \frac{1}{8}\,\zeta(3)\,(\alpha'/R^2)^3$, where $R^4 = 4\pi g_s N \alpha'^2$. Since $\lambda = g_{YM}^2 N \equiv 4\pi g_s N$, we get $\gamma = \frac{1}{8}\,\zeta(3)\,\frac{1}{\lambda^{3/2}}$. This action was computed in [27], using the methods of [31].

The $C^4$ term is a dimension-eight operator, defined as follows:

$$C^4 = C^{hmnk}\, C_{pmnq}\, C_h{}^{rsp}\, C^q{}_{rsk} + \frac{1}{2}\, C^{hkmn}\, C_{pqmn}\, C_h{}^{rsp}\, C^q{}_{rsk} \,, \tag{26}$$

where $C^q{}_{rsk}$ is the Weyl tensor. The tensor $\mathcal{T}$ is defined by

$$\mathcal{T}_{abcdef} = i \nabla_a F^+_{bcdef} + \frac{1}{16} \left( F^+_{abcmn} F^+_{def}{}^{mn} - 3 F^+_{abfmn} F^+_{dec}{}^{mn} \right) , \tag{27}$$

where the indices $[a, b, c]$ and $[d, e, f]$ are antisymmetrized in each squared brackets, and symmetrized with respect to interchange of $abc \leftrightarrow def$ [27].

At finite temperature the metric only gets corrections from the $C^4$ term. This is so because the tensor $\mathcal{T}$ vanishes on the uncorrected supergravity solution [28]. The solution to the Einstein equations derived from the pure supergravity action (19) is an $AdS_5BH \times S^5$ background. There are $N$ units of flux of $F_5$ through the sphere, and the volume form of $S^5$ is denoted by $\epsilon$. On the field theory side, $N$ is the rank of the gauge group, and it corresponds to the number of parallel D3-branes whose back-reaction deforms the space-time leading to the above metric in the near horizon limit. The current operator $J_\mu(x)$ is dual to the $s$-wave mode of the vectorial fluctuation on this background.

Next, we have to obtain the Lagrangian for the vectorial perturbation in this background. Thus, we must construct a consistent perturbed *Ansatz* for both the metric and the Ramond-Ramond five-form field strength, such that a $U(1)$ subgroup of the $SU(4)$ $R$-symmetry group is obtained [32, 33, 34]. Then, by plugging this consistent perturbation *Ansatz* into the full action (up to $\mathcal{O}(\alpha'^3)$) and integrating out the five-sphere, one obtains the desired action for the $U(1)$ gauge field in the $AdS_5BH$. Therefore, by studying the bulk solutions of the Maxwell equations in the $AdS_5BH$ with certain boundary conditions we can obtain the retarded correlation functions [23, 24, 22] of the operator $J_\mu(x)$.

Higher-curvature corrections to the type IIB supergravity action correspond to finite 't Hooft coupling corrections in the field theory. Suppose that we are interested in a certain observable of the gauge theory, $\mathcal{O}$. If one carries out a series expansion of it in inverse powers of the 't Hooft coupling one schematically can write it as: $\mathcal{O}_0 + \mathcal{O}_1/\lambda^{n_1} + \cdots$. The power $n_1$ is a positive number corresponding to the leading $\alpha'^3$ correction to the type IIB supergravity action. In the present case, we consider that $\mathcal{O}$ is the product of two electromagnetic currents. Thus, we obtain the leading correction in $\lambda$ using the gauge/string duality. The leading order corrections come from terms $\mathcal{O}(\alpha'^3)$ in the ten-dimensional action. It is important to recall that these corrections dot no modify the metric at zero temperature [30]. At finite temperature things are different as shown in [35, 36] where corrections to the metric were obtained, and then further improved in [26, 37, 38].

Higher curvature corrections on the spin-2 sector of the fluctuations have been investigated in [39, 40, 41, 42], among other references. They are relevant to the computation of the viscosity and mass-diffusion constants of the plasma.

In our case, we investigate vector fluctuations of the background. The method to carry out the calculation consists of two steps. Firstly, we have to obtain the minimal gauge-field kinetic term using the vector-perturbed metric including the $\alpha'^3$ corrections to it, and the same for the five-form field strength. Then, the corrections to the gauge field Lagrangian coming directly from the higher-derivative operators have to be computed. The reason why these two steps are different is that the first one will require insertion of the corrected perturbation *Ansätze* into the minimal ten-dimensional type IIB supergravity two-derivative part Eq.(19). The second step requires insertion of the *uncorrected* perturbation *Ansätze* into the higher-curvature terms in ten dimensions.

Our plan here is to start from the corrected metric and $F_5$ solutions, then proposing an *Ansätze* for the perturbations that may be inserted into Eq.(19).

As mentioned before, the only piece of the $\mathcal{O}(\alpha'^3)$-action which affects the metric is the $C^4$ term. This induces the following corrected metric [35, 36, 37]

$$ds^2 = \left(\frac{r_0}{R}\right)^2 \frac{1}{u}\left(-f(u)\, K^2(u)\, dt^2 + d\vec{x}^2\right) + \frac{R^2}{4u^2 f(u)}\, P^2(u)\, du^2 + R^2 L^2(u)\, d\Omega_5^2\,, \qquad (28)$$

where we have used similar notation as for Eq.(7). The functions of $u$ in the above metric are

$$K(u) = \exp\left[\gamma\left(a(u) + 4b(u)\right)\right], \quad P(u) = \exp\left[\gamma\, b(u)\right], \quad L(u) = \exp\left[\gamma\, c(u)\right], \qquad (29)$$

where there are the following exponents, which are functions of the radial coordinate

$$\begin{aligned} a(u) &= -\frac{1625}{8}\, u^2 - 175\, u^4 + \frac{10005}{16}\, u^6\,, \\ b(u) &= \frac{325}{8}\, u^2 + \frac{1075}{32}\, u^4 - \frac{4835}{32}\, u^6\,, \\ c(u) &= \frac{15}{32}\,(1+u^2)\, u^4\,. \end{aligned} \qquad (30)$$

In addition, the radius of the black hole horizon gets corrections given by

$$r_0 = \frac{\pi T R^2}{(1+\frac{265}{16}\gamma)}\,. \tag{31}$$

$T$ has been already identified as the physical equilibrium temperature of the plasma. Thus, having obtained the corrected metric Eq.(28), we have to focus upon the appropriate perturbation *Ansätze*. The vectorial perturbation we are interested in enters the metric and the $F_5$ solution, in contrast to the metric tensor perturbations - needed for mass-transport phenomena in the hydrodynamical regime of the plasma - the latter only enter the metric *Ansatz*, but not the $F_5$ *Ansatz*. This observation obviously makes the computation of the corrections to the mass-transport coefficients much more straightforward compared with the electric charge-transport coefficients as well as other plasma properties beyond the hydrodynamical domain.

We first obtain the kinetic term for the gauge fields. For this purpose we plug the corrected *Ansatz* into the two-derivative supergravity action Eq.(19). The metric *Ansatz* reads

$$\begin{aligned} ds^2 &= \left[g_{mn} + \frac{4}{3}R^2 L(u)^2 A_m A_n\right] dx^m dx^n + R^2 L(u)^2\, d\Omega_5^2 + \frac{4}{\sqrt{3}} R^2 L(u)^2 \\ &\quad \times \left(\sin^2 y_1\, dy_3 + \cos^2 y_1\, \sin^2 y_2\, dy_4 + \cos^2 y_1\, \cos^2 y_2\, dy_5\right) A_m\, dx^m\,, \end{aligned} \tag{32}$$

where the metric of the unit five-sphere is given by

$$d\Omega_5^2 = dy_1^2 + \cos^2 y_1\, dy_2^2 + \sin^2 y_1\, dy_3^2 + \cos^2 y_1\, \sin^2 y_2\, dy_4^2 + \cos^2 y_1\, \cos^2 y_2\, dy_5^2\,. \tag{33}$$

Notice that since we are only interested in the terms which are quadratic in the gauge-field perturbations we can write the $F_5$ *Ansatz* as follows

$$G_5 = -\frac{4}{R}\bar{\epsilon} + \frac{R^3 L(u)^3}{\sqrt{3}} \left(\sum_{i=1}^{3} d\mu_i^2 \wedge d\phi_i\right) \wedge \bar{*} F_2\,, \tag{34}$$

where $F_2 = dA$ is the Abelian field strength and $\bar{\epsilon}$ is a deformation of the volume form of the metric of the $AdS_5$-Schwarzschild black hole. We should mention that we are not interested in the part of $G_5$ which does not contain the vector perturbations because it only contributes to the potential of the metric, and is thus accounted for by the use of the corrected metric in the computation. The Hodge dual $*$ is taken with respect to the ten-dimensional metric, while $\bar{*}$ denotes the Hodge dual with respect to the five-dimensional metric piece of the black hole. In addition, we have the usual definitions for the coordinates on the $S^5$

$$\begin{aligned} &\mu_1 = \sin y_1\,, && \mu_2 = \cos y_1\, \sin y_2\,, && \mu_3 = \cos y_1\, \cos y_2\,, \\ &\phi_1 = y_3\,, && \phi_2 = y_4\,, && \phi_3 = y_5\,. \end{aligned} \tag{35}$$

By inserting these *Ansätze* into Eq.(19), and discarding all the higher (massive) Kaluza-Klein harmonics of the five-sphere, we get the following action for the zero-mode Abelian gauge field $A_m$

$$S_{IIB}^{SUGRA} = -\frac{\tilde{N}^2}{64\pi^2 R} \int d^4x\, du\, \sqrt{-g}\, L^7(u)\, g^{mp}\, g^{nq}\, F_{mn}\, F_{pq}\,. \qquad (36)$$

Above we have written the Abelian field strength, defined as $F_{mn} = \partial_m A_n - \partial_n A_m$, the partial derivatives are $\partial_m = \partial/\partial x^m$, while $x^m = (t, \vec{x}, u)$, with $t$ and $\vec{x} = (x_1, x_2, x_3)$, are the Minkowski coordinates, and $g \equiv \det(g_{mn})$, which only involves the metric of $AdS_5$-Schwarzschild black hole. Also notice that $L(u)$ straightforwardly comes from the dimensional reduction [43]. The volume of the five-sphere has been included in $\tilde{N}$.

Now, we should get the effect of the eight-derivative corrections of Eq.(25). In order to achieve this we must determine the five-dimensional operators that arise once the perturbed metric and five-form field strength *Ansätze* are inserted into Eq.(25). As in [44], we use the uncorrected *Ansätze* at this point. Indeed, we can do it because using the corrected ones generates terms of even higher order in $\gamma$. Clearly, the uncorrected *Ansätze* are derived from the ones displayed here by taking $L(u), K(u), P(u) \to 1$ and $\bar{\epsilon} \to \epsilon$. Next, we explain how to calculate the explicit contributions from the ten-dimensional operators, leading to the photoemission rates.

### 4.2 't Hooft coupling corrections to photoemission rates

In order to calculate the 't Hooft coupling corrections to photoemission rates we now perform the explicit dimensional reduction on $S^5$, including the leading type IIB string theory corrections discussed in the previous subsection. This is done along the lines of our previous work [2][3]. For this purpose it is necessary to write explicitly all the terms of the full set of higher derivative ten-dimensional operators which come from the supersymmetric completion obtained in [25]. We use the definitions introduced in [27]

$$C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4 \equiv \frac{1}{86016}\sum_i n_i\, M_i\,. \qquad (37)$$

Thus, we can write the two contributions to the $C^4$ term as follows

$$C^4 = -\frac{43008}{86016} C_{abcd}C_{abef}C_{cegh}C_{dgfh} + C_{abcd}C_{aecf}C_{bgeh}C_{dgfh}\,. \qquad (38)$$

Repeated indices means usual Lorentz contractions. In order to extract the quadratic terms in the vectorial fluctuations of the metric we should notice that they can straightforwardly

[3] The main difference with respect to our previous calculation of the electrical conductivity of plasma in [2] is that while for the conductivity it is only needed to consider the dependence $A_m(u)$, for the photoemission rate it is necessary to consider the dependence $A_m(t, z, u)$ which is not a trivial extension of our former calculations in [2]. Thus, having the $A_m(t, z, u)$ dependence implies actually a much more complicated calculation.

be computed by expanding the ten-dimensional Weyl tensor as $C = C_0 + C_1 + C_2$, where the sub-indices label the number of times that the Abelian gauge field occurs. Obviously, a similar expansion can be made for the $\mathcal{T}$ tensor: $\mathcal{T} = \mathcal{T}_0 + \mathcal{T}_1 + \mathcal{T}_2$. In addition, from a straightforward explicit calculation on the present background it can be shown that all the components of $\mathcal{T}_2$ are zero. This fact is responsible of an important simplification of the actual computations. Also, $\mathcal{T}_0$ is zero for any compactification which contains a five-dimensional Einstein manifold [28], and therefore it vanishes in the case we consider here.

Now, let us look at terms of the form $C^3\mathcal{T}$;

$$C^3\mathcal{T} = \frac{3}{2}\, C_{abcd}C_{aefg}C_{bfhi}\mathcal{T}_{cdeghi}\,. \tag{39}$$

Their only possible contributions comes in fact from terms like $C_1C_0^2\mathcal{T}_1$, $C_0C_1C_0\mathcal{T}_1$ and $C_0^2C_1\mathcal{T}_1$.

Then, let us study operators like $C^2\mathcal{T}^2$. We find a few contractions which can be collected in the following terms

$$\begin{aligned} C^2\mathcal{T}^2 = \frac{1}{86016} \quad & \big(30240\, C_{abcd}C_{abce}\mathcal{T}_{dfghij}\mathcal{T}_{efhgij} + 7392\, C_{abcd}C_{abef}\mathcal{T}_{cdghij}\mathcal{T}_{efghij} \\ & -4032\, C_{abcd}C_{aecf}\mathcal{T}_{beghij}\mathcal{T}_{dfghij} - 4032\, C_{abcd}C_{aecf}\mathcal{T}_{bghdij}\mathcal{T}_{eghfij} \\ & -118272\, C_{abcd}C_{aefg}\mathcal{T}_{bcehij}\mathcal{T}_{dfhgij} - 26880\, C_{abcd}C_{aefg}\mathcal{T}_{bcehij}\mathcal{T}_{dhifgj} \\ & +112896\, C_{abcd}C_{aefg}\mathcal{T}_{bcfhij}\mathcal{T}_{dehgij} - 96768\, C_{abcd}C_{aefg}\mathcal{T}_{bcheij}\mathcal{T}_{dfhgij}\big)\ . \end{aligned} \tag{40}$$

The vanishing result of $\mathcal{T}_0$ implies that terms like $C_0C_1\mathcal{T}_0\mathcal{T}_1$ also vanish. Then, the only possible type of contribution from these terms is of the form $C_0^2\mathcal{T}_1^2$. Making use of the same arguments all the terms like $C\mathcal{T}^3$ and $\mathcal{T}^4$ include a factor $\mathcal{T}_0$ and, therefore, are not present in a reduction upon a generic five-dimensional Einstein manifold [44].

Now, we proceed to explicitly calculate the operators above. Firstly, we must calculate the ten-dimensional Weyl tensor with and without vector fluctuations. Secondly, we need to obtain $\mathcal{T}_1$, and by its definition it can be separated into one piece which contains the covariant derivative, defined by

$$(\nabla F_5)_{abcdef} = i\nabla_a F^+_{bcdef}\,, \tag{41}$$

and a second piece which does not contain covariant derivatives which reads

$$\bar{\mathcal{T}}_{abcdef} = \frac{1}{16}\left(F^+_{abcmn}F^{+\ \ mn}_{def} - 3F^+_{abfmn}F^{+\ \ mn}_{dec}\right)\ . \tag{42}$$

So, we can write this tensor as $\mathcal{T}_1 = \nabla F_5 + \bar{\mathcal{T}}$.

Let us define $F^+ = F_{(e)} + F_{(m)}$. Thus, with the obvious meaning of the electric and magnetic contribution, for the electric part we have

$$F_{(e)} = -\frac{4}{R}\epsilon + \frac{R^3}{\sqrt{3}}\left(\sum_{i=1}^{3} d\mu_i^2 \wedge d\phi_i\right) \wedge \bar{*}F_2 \,, \tag{43}$$

where $\bar{*}$ indicates the Hodge dual operation with respect to the $AdS_5$-Schwarzschild black hole metric. It is convenient to split the electric part into the background plus a fluctuation,

$$F_{(e)} = F_{(e)}^{(0)} + F_{(e)}^{(f)} \,, \tag{44}$$

and similarly for the magnetic terms. Therefore, in components we have

$$(F_{(e)}^{(0)})_{\mu\nu\rho\sigma\delta} = -\frac{4}{R}\sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma\delta} \,, \tag{45}$$

where $g$ is the determinant of the $AdS$ piece of the metric, in fact $g = \det g_{AdS} = -r_0^4/(2u^3R^3)$. The Hodge dual gives

$$(F_{(m)}^{(0)})_{abcde} = -\frac{4}{R}R^5\sqrt{\det S^5}\epsilon_{abcde} \,. \tag{46}$$

Let us focus on the fluctuation. Actually, for this calculation we only need the $U(1)$ gauge component $A_x(t,z,u)$, where in this notation $t = x_1$ and $z = x_4$. Notice that if we were interested in the electrical conductivity it is enough to consider the $A_x(u)$ dependence, which largely simplifies the calculation [2] in comparison with the actual calculation of the photoemission rates that we make in this work. Therefore, we have to deal with the following non-vanishing components of the two-form field strength: $F_{tx}$, $F_{zx}$ and $F_{ux}$, all of them with the full dependence on $t$, $z$ and $u$ $AdS$-coordinates. We use the following definition: $F = dA = \frac{1}{2!}F_{\mu\nu}/\sqrt{3}\,dx^\mu \wedge dx^\nu$.

So, the fluctuations of the electric part induce fluctuations in the $F_5$ Ramond-Ramond field strength which are given by

$$(F_{(e)}^{(f)})_{y_iy_jtyz} = Fe_{ux}(t,z,u)\,b_{ij}\,\epsilon_{y_iy_jtyz} \,, \tag{47}$$

$$(F_{(e)}^{(f)})_{y_iy_jyzu} = Fe_{tx}(t,z,u)\,b_{ij}\,\epsilon_{y_iy_jyzu} \,, \tag{48}$$

$$(F_{(e)}^{(f)})_{y_iy_jtyu} = Fe_{zx}(t,z,u)\,b_{ij}\,\epsilon_{y_iy_jtyu} \,, \tag{49}$$

where

$$Fe_{ux}(t,z,u) = -\frac{R^3}{\sqrt{3}}\frac{1}{2}\sqrt{-g}\,(2F_{ux}G^{xx}G^{uu}) \,, \tag{50}$$

$$Fe_{tx}(t,z,u) = \frac{R^3}{\sqrt{3}}\frac{1}{2}\sqrt{-g}\,(2F_{tx}G^{tt}G^{xx}) \,, \tag{51}$$

$$Fe_{zx}(t,z,u) = \frac{R^3}{\sqrt{3}}\frac{1}{2}\sqrt{-g}\,(2F_{zx}G^{zz}G^{xx}) \,, \tag{52}$$

where the pairs $(ij)$ are (13), (14), (15), (24) and (25). The indices $i, j$ run over the coordinates of $S^5$, and correspond to the coordinates $x_6, x_7, x_8, x_9$ and $x_{10}$. The $b_{ij}$ functions are:

$$\begin{aligned} &b_{13} = 2\sin y_1 \cos y_1\,, \quad b_{14} = -2\sin^2 y_2 \sin y_1 \cos y_1\,, \quad b_{15} = -2\cos^2 y_2 \sin y_1 \cos y_1\,, \\ &b_{24} = 2\cos^2 y_1 \sin y_2 \cos y_2\,, \quad b_{25} = -2\cos^2 y_1 \sin y_2 \cos y_2\,. \end{aligned} \tag{53}$$

The fluctuations on the magnetic part are obtained after performing the ten-dimensional Hodge dual operation on the corresponding electrical fluctuations above. We present the full expression in the appendix.

The kinetic term of the gauge field coming from the Ramond-Ramond five-form field strength becomes

$$-\frac{1}{4\cdot 5!}\,F_5^2 = -\frac{2}{3}\,R^2\,F^2 - \frac{8}{R^2}\,, \tag{54}$$

which is exactly what is expected. Recall that the scalar curvature piece $R_{10}$ of the action gives $-1/3\,R^2\,F^2$, where $F^2$ denotes $F_{\mu\nu}F^{\mu\nu}$.

As we have seen in our previous paper [44], the eight-derivative $\mathcal{O}(\alpha'^3)$ corrections introduce a large number of higher-derivative operators after the compactification on a general five-dimensional Einstein manifold is done. We must take account of them properly to solve the equation of motion within perturbation theory. The situation is entirely analogous to that studied in [39], where the authors were concerned with the tensor perturbations of the metric, but the rationale is the same. We have discussed this for vectorial perturbations of the metric in [2, 44]. Lagrangian for the transverse mode $A_x$ reads

$$\begin{aligned} S_{\text{total}} \;=\; & -\frac{\tilde{N}^2 r_0^2}{16\pi^2 R^4}\int \frac{d^4k}{(2\pi)^4}\int_0^1 du\,\Big[\gamma A_W A_k'' A_{-k} + (B_1 + \gamma B_W) A_k' A_{-k}' \\ & + \gamma C_W A_k' A_{-k} + (D_1 + \gamma D_W) A_k A_{-k} + \gamma E_W A_k'' A_{-k}'' + \gamma F_W A_k'' A_{-k}'\Big]\,, \end{aligned} \tag{55}$$

where we have introduced the following Fourier transform of the field $A_x$,

$$A_x(t, \vec{x}, u) = \int \frac{d^4k}{(2\pi)^4}\, e^{-i\omega t + iqz}\, A_k(u)\,. \tag{56}$$

There are also a number of boundary terms that must be included for this higher-derivative Lagrangian to make sense, and this is discussed in detail in [39, 45]. The coefficients $B_1$ and $D_1$ arise directly from the minimal kinetic term $F^2$. The subscript $W$ indicates that the particular coefficient comes directly from the eight-derivative corrections, and the functions $A_W \to F_W$ are written below. Moreover, $B_1$ and $D_1$ contain some $\gamma$-dependence, but they are non-vanishing in the $\gamma \to 0$ limit, while every other coefficient vanishes in that limit. The equation of motion is given by

$$A_x'' + p_1 A_x' + p_0 A_x = \gamma \frac{1}{2f(u)} G(A_x)\,, \tag{57}$$

where

$$G(A_x) = A_W A_x'' + C_W A_x' + 2(\delta D_1 + D_W) A_x - \partial_u (2\delta B_1 A_x' + 2B_W A_x' + C_W A_x + F_W A_x'') + \partial_u^2 (A_W A_x + 2E_W A_x'' + F_W A_x'), \tag{58}$$

where $B_1 - B_1|_{\gamma\to 0} = \delta B_1$ and $D_1 - D_1|_{\gamma\to 0} = \delta D_1$. First we have the coefficients with no $\gamma$-dependence $p_0$ and $p_1$, given by

$$p_0 = \frac{\varpi_0^2 - f(u)\kappa_0^2}{u f^2(u)} \quad \text{and} \quad p_1 = \frac{f'(u)}{f(u)}, \tag{59}$$

where $\varpi_0 = k_0/(2\pi T)$ and $\kappa_0 = k/(2\pi T)$. For the coefficients originating from the $F^2$ term in the action of the gauge field, we obtain

$$\begin{aligned} B_1 &= \frac{K(u) f(u) L^7(u)}{P(u)}, \\ D_1 &= -K(u) P(u) L^7(u) \left[\frac{\varpi^2 - f(u) K^2(u)\kappa^2}{u f(u) K^2(u)}\right], \end{aligned} \tag{60}$$

where $\varpi = k_0 R^2/(2r_0)$ and $\kappa = k R^2/(2r_0)$. At this stage it is convenient to reduce the equation to a second-order differential equation using a simple trick [48]. The idea is that $\gamma A_x'' = -\gamma (p_1 A_x' + p_0 A_x) + \mathcal{O}(\gamma^2)$. Thus, we may reduce the entire RHS of the equation of motion to terms which are first or zeroth order in derivatives. The resulting equation is given by

$$A_x'' + \left[p_1 - \frac{\gamma}{2f(u)}[\theta_1(u) - p_1\theta_2(u)]\right] A_x' + \left[p_0 - \frac{\gamma}{2f(u)}[\theta_0(u) - p_0\theta_2(u)]\right] A_x = \mathcal{O}(\gamma^2), \tag{61}$$

where

$$\begin{aligned} \theta_0(u) &= 2(\delta D_1 + D_W) - C_W' + A_W'' - 4E_W' p_0' + 2E_W (p_1 p_0' - p_0''), \\ \theta_1(u) &= 2A_W' - 2(\delta B_1 + B_W)' + F_W'' - 4E_W' (p_1' + p_0) + 2E_W [p_1(p_1' + p_0) - p_1'' - 2p_0'], \\ \theta_2(u) &= 2A_W - 2(\delta B_1 + B_W) + F_W' + 2E_W'' - 4E_W' p_1 + 2E_W [p_1^2 - 2p_1' - p_0]. \end{aligned} \tag{62}$$

In order to solve Eq.(61), the first step is to examine the singularity structure of the equation at the horizon $u = 1$. As usual, we change variables to $x = 1 - u$, so that the singularity is at $x = 0$, then insert the functional form $A_x = x^\beta$. We obtain the indicial equation:

$$\beta^2 + \left(\frac{\omega}{4\pi T}\right)^2 = 0. \tag{63}$$

This is of course the same indicial equation that would have been obtained in the infinite 't Hooft coupling limit. Thus, as long as the Lagrangian originates from a gauge-invariant series of operators, then the indicial equation is unchanged. The fact that the indicial equation is unchanged is a consequence of the gauge-invariance in five dimensions, which is in turn a consequence of the $U(1)$ isometry of the internal manifold $S^5$, but it is not a consequence of supersymmetry. This behaviour is expected [40, 46, 47, 48] for scalar and tensor fluctuations of the metric.

At this point, we have to solve the equation of motion for $A_x$. First, we have to specify the functions $A_W, B_W, C_W, D_W, E_W$ and $F_W$ in the action Eq.(55). We have computed them explicitly and obtained the following expressions:

$$A_W(t,z,u) = \frac{4\gamma}{9}u^5\left[157\left(u^2-1\right)\kappa_0^2+275\varpi_0^2\right], \tag{64}$$

$$B_W(t,z,u) = \frac{\gamma}{9}u^4\left[u\left(-26214u^3+29423u+8844\varpi_0^2+6012\kappa_0^2\left(u^2-1\right)\right)-9853\right], \tag{65}$$

$$C_W(t,z,u) = \frac{4\gamma}{9}u^4\left[\kappa_0^2\left(3360u^2-3046\right)-\frac{\left(1543u^2+4540\right)\varpi_0^2}{u^2-1}\right], \tag{66}$$

$$\begin{aligned} D_W(t,z,u) = \frac{\gamma u^3}{9\left(u^2-1\right)^2}&\left[-3872u\varpi_0^4+\left(1191u^4+3857u^2-5384\kappa_0^2\left(u^2-1\right)u+3796\right)\varpi_0^2\right. \\ &\left.+\kappa_0^2\left(u^2-1\right)^2\left(-1512u\kappa_0^2+5241u^2-2332\right)\right], \end{aligned} \tag{67}$$

$$E_W(t,z,u) = -\frac{3872\gamma}{9}u^6\left[u^2-1\right]^2, \tag{68}$$

$$F_W(t,z,u) = -\frac{2\gamma}{9}u^5\left(u^2-1\right)\left[9719u^2-6397\right]. \tag{69}$$

Finally, to enable us to carry out analytic computations of the photoemission spectrum in the high-momentum limit, and as an aide to the understanding of the physics behind the corrected equation of motion, we can rewrite the EOM in the Schrödinger basis. We transform the field variable as follows:

$$\begin{aligned} A_x(u) &= \Sigma(u)\Psi(u), \\ \Sigma(u) &= \frac{1}{288}\sqrt{f(u)}\left(u^2\gamma\left(u^2\left(37760\kappa_0^2u-87539u^2+343897\right)-11700\right)+288\right). \end{aligned} \tag{70}$$

giving us the Schrödinger-like equation:

$$\begin{aligned} \Psi''(u) &= V(u)\Psi(u) \\ V(u) &= -\frac{1}{144\left(u^2-1\right)^2}\left[144(u\kappa_0^2+1)+\gamma(u^2-1)\left(1838319u^6-4752055u^4\right.\right. \\ &\left.\left.+2098482u^2+\kappa_0^2\left(1011173u^4+245442u^2-16470\right)u-11700\right)\right]. \end{aligned} \tag{71}$$

We can study the structure of the relative difference between this $\gamma$-corrected potential and the uncorrected one (for $\gamma = 0$ which corresponds to 't Hooft coupling going to infinity).

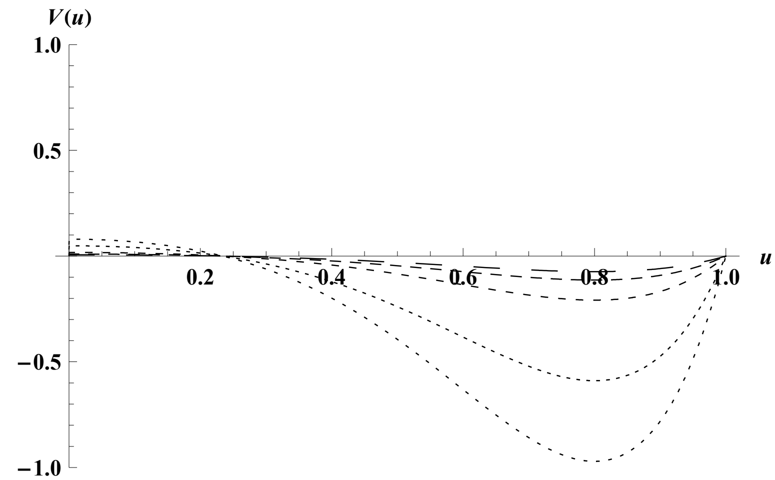


Figure 1: Difference between the numerical potential minus the analytical one for $\lambda \to \infty$, divided by the analytical one for $\lambda \to \infty$, as a function of the radial coordinate of the black hole $u$. Long-dashed, dashed, small-dashed, tiny-dashed, and dotted lines correspond to decreasing values of $\lambda = 200, 150, 100, 50$ and 35, respectively.

Figure 1 shows the difference between the numerical potential minus the analytical one for $\lambda \to \infty$, divided by the analytical one for large $\lambda \to \infty$, as a function of the radial coordinate of the black hole, $u$. Long-dashed, dashed, small-dashed, tiny-dashed, and dotted lines correspond to decreasing values of $\lambda = 200, 150, 100, 50$ and 35, respectively. Notice the smooth behaviour of the corrected potential as a function of $\lambda$ and also the fact that they coincide exactly at $u = 1$. This is behind the fact that the indicial equation is unchanged from the uncorrected case. Notice also that there is an exact cancelation of all $\kappa^4$ as well as $\kappa_0^4$ terms in the potential, which is consequence of the supersymmetric structure of the higher derivative corrections in the ten-dimensional action[4]. As a consequence, the plasma structure functions show a slight enhancement (at ultra-high momenta) from their values at $\lambda \to \infty$ as the coupling decreases, as has been shown in our previous work [45]. The next step is to actually solve the EOM for $A_x$. We do this by a numerical solution of Eq.(71). With this numerical solution we compute the trace of the spectral function and the photoemission rate for any value of the 't Hooft coupling at finite yet strong coupling. We show our results in the next subsection.

[4]Notice that although in Eq.(71) we have set the light-like momenta condition, the validity of the statement about the power four-momentum and frequency terms cancelation is quite general, and we have explicitly checked that.

### 4.3 Results of photoemission with 't Hooft coupling corrections

First, let us describe our results for the trace of the spectral function. Its asymptotic behaviour can be evaluated analytically for low- and high-momentum, and numerically for the remaining momentum domain. This gives [1, 22]

$$\frac{\chi^{\mu}_{\mu}(\kappa_0)}{\frac{1}{2}N^2T^2} = \begin{cases} \left(1+\frac{14993}{9}\gamma\right)\kappa_0 + \mathcal{O}(k^3) & \kappa_0 \ll 1 \\ \frac{3^{5/6}}{2}\frac{\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{3}\right)}\left(1+5\gamma\right)\kappa_0^{2/3} + \mathcal{O}(1) & \kappa_0 \gg 1 \end{cases} . \tag{72}$$

The coefficient of $\kappa_0$ in the low-momentum regime of Eq.(72) means that the electrical conductivity of the strongly-coupled plasma is enhanced by a factor $\left(1+\frac{14993}{9}\gamma\right)$ due to the finite $\lambda$ corrections [44]. This is as expected from the perturbative computations in [22] since the weakly-coupled plasma has a larger mean-free-path per collision, allowing more efficient diffusion of electric charge, and hence a higher electrical conductivity. On other hand, for the higher momentum, the results of [22] imply that the spectral function at weak coupling should go like $\kappa_0^{1/2}$ in the ultraviolet. Given the fact that the spectral function at $\lambda \to \infty$ goes like $\kappa_0^{2/3}$, in that regime one would have expected our result in Eq.(72) to display some smooth interpolation between $\kappa_0^{1/2}$ and $\kappa_0^{2/3}$. We do not obtain such an interpolation, finding instead that the finite coupling corrections do not change the momentum-dependence for large momentum. Moreover, we find an enhancement by a factor $(1+5\gamma)$ in that regime (see also [45]). The fact that the leading $\kappa_0$ behaviour is unchanged by the corrections could have been seen from the Schrödinger-like potential above: the only $\kappa_0$-dependence is $\kappa_0^2$, identically to the $\lambda \to \infty$ case. Terms like $\kappa_0^4$, which could have changed the high-momentum functional dependence of $\chi^{\mu}_{\mu}(\kappa_0)$, vanish. Figure 2 shows the trace of the spectral function ${\chi^{\mu}}_{\mu}$ divided by $\kappa_0$ as a function of the light-like momentum $k$ for the cases when $\lambda$ is large. Solid, long-dashed, dashed and small-dashed lines correspond to decreasing values of $\lambda = \infty, 200, 150,$ and 100, respectively.

In figure 3 we show the photoemission rates of a strongly-coupled $\mathcal{N}=4$ supersymmetric Yang-Mills plasma as a function of photon momentum divided by the equilibrium temperature, $k/T$. In fact the curves show $d\Gamma_\gamma/dk$ divided by $\alpha_{\rm em}(N^2-1)T^3$. Different curves correspond to different large values of the 't Hooft coupling: solid, long-dashed, dashed, small-dashed, tiny-dashed, and dotted lines correspond to decreasing values of $\lambda$ from $\lambda=\infty$ (in fact it is the analytical expression from supergravity with no string theory corrections), and then $\lambda = 200, 150, 100, 50$ and 35, respectively. These curves have been obtained using the gauge/strings duality, considering the full $\mathcal{O}(\alpha'^3)$ type IIB string theory corrections to the supergravity action. It is evident that the behaviour of the photoemission rates depend upon the values of the 't Hooft coupling. Their slopes at zero momentum give the electrical conductivity as a function of the 't Hooft coupling in full agreement with our previous results of [2]. The height of the peaks decrease as the 't Hooft coupling increases, their maxima are shifted towards the ultraviolet and the photoemission rate curves cross downwards

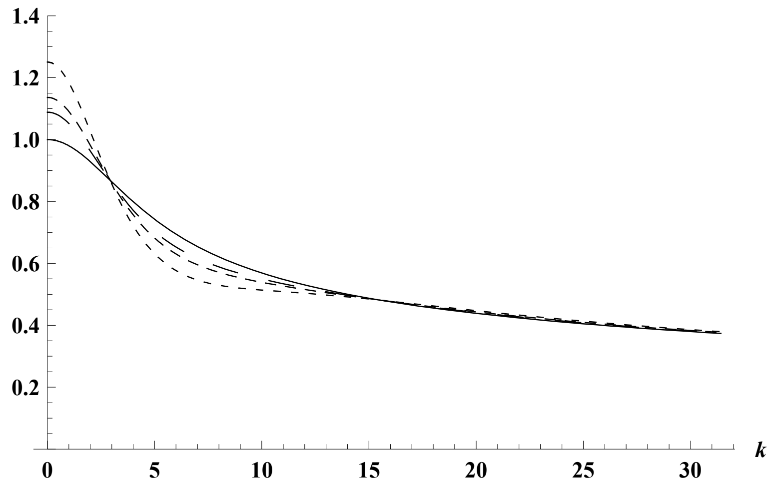


Figure 2: Trace of the spectral function $\chi^\mu_\mu$ divided by $\kappa_0$ as a function of $k$ for the cases when $\lambda$ is large. Solid, long-dashed, dashed and small-dashed lines correspond to decreasing values of $\lambda = \infty, 200, 150,$ and 100, respectively.

the limiting strongly coupled (pure supergravity) curve for momentum around three times the equilibrium temperature. These features are expected from perturbative quantum field theory calculations in the weakly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma and from the supergravity calculation of the large $N$ strongly coupled theory. However, at much higher momentum, all these curves cross upwards the extreme strongly coupled one. This behaviour is exemplified in figure 4 for the large 't Hooft coupling case (solid line) compared with $\lambda = 50$ (tiny-dashed line), for a relatively large photon frequency in comparison with the equilibrium temperature, actually the crossing occurs around $k \approx 17.4T$. For larger values of the momentum all curves approach the solid one. This result is in agreement with our former results on deep inelastic scattering structure functions from $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma with string theory corrections [45]. The reason for such a behaviour comes from the fact that at $\mathcal{O}(\alpha'^3)$ in string theory, the Schrödinger-like potential describing the dynamics of the photo-production gets no corrections like the fourth power of momentum and frequency. This is caused by an exact cancelation of this power of momentum contributions from the $C^4$-term and its supersymmetric completion at $\mathcal{O}(\alpha'^3)$ in the string theory type IIB action.

# 5 Weakly coupled SYM plasma photoemission rates

In this section we very briefly describe the results for the weakly coupled regime of SYM obtained by [22]. We include this in order to be able to compare our results in the previous

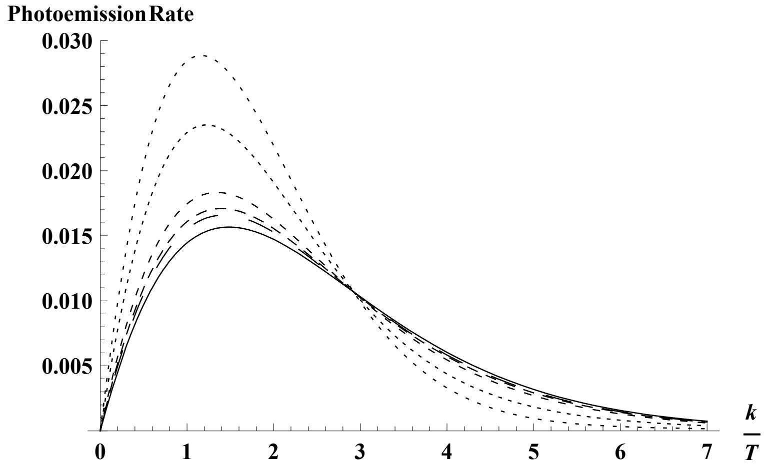


Figure 3: Photoemission spectrum for different values of $\lambda$, as a function of the light-like momentum of the emitted photon divided by the equilibrium temperature, $k/T$. Notice that in fact we show the curves for $d\Gamma_\gamma/dk$ divided by $\alpha_{\rm em}(N^2-1)T^3$. Solid, long-dashed, dashed, small-dashed, tiny-dashed, and dotted lines correspond to decreasing values of $\lambda$, from $\lambda = \infty$ (in fact it is the analytical expression from supergravity with no string theory corrections), and $200, 150, 100, 50$ and 35, respectively.

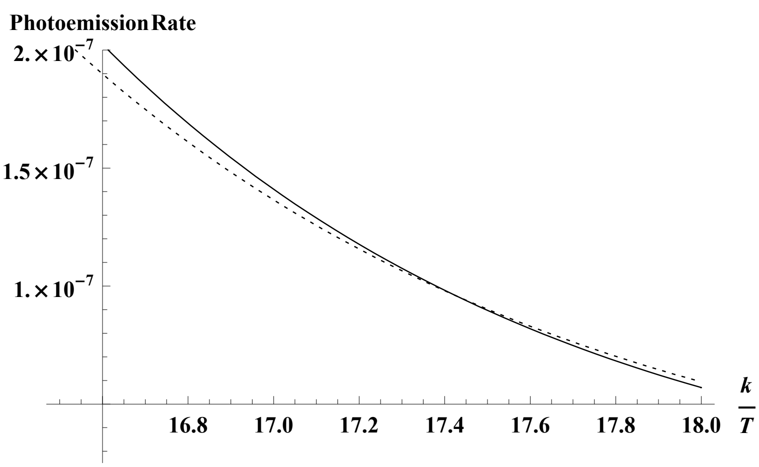


Figure 4: Photoemission spectrum as a function of the light-like momentum of the emitted photon divided by the equilibrium temperature, $k/T$. As before we show the curves for $d\Gamma_\gamma/dk$ divided by $\alpha_{\rm em}(N^2-1)T^3$. Solid and tiny-dashed lines correspond to values of $\lambda = \infty$ and 50, respectively. This figure shows the typical large photon momentum behaviour having a crossing around $k/T \approx 17.4$, for larger momentum the tiny-dashed curve approaches the solid line.

section for large values of $\lambda$ and see how they help to understand a broader picture of the plasma structure in terms of the 't Hooft coupling.

The computations of the spectral function in the weakly coupled regime has been done using perturbative SYM theory [22], also previous results for perturbative QCD were obtained in [5, 4]. For light-like momenta the contribution to the trace of the spectral function appears at two-loop level. The key point is that in a thermal system the expansion of physical quantities in powers of the 't Hooft coupling is not the same as the diagrammatic expansion in loops. Basically, what happens is that there is sensitivity to energy and momentum scales which are parametrically small in comparison with the equilibrium temperature. In the situation where one considers light-like momentum this complication already arises at the first non-trivial order. To deal with this, it is necessary to carry out an infinite resummation of diagrams in order to find the leading order weak-coupling photon production rate. This rate can be split into a contribution from a Compton-like $2 \leftrightarrow 2$ scattering process and near-collinear *Bremsstrahlung* and pair-annihilation processes, which are further corrected due to the Lipatov-Pomeranchuk-Migdal suppression effect. The complete presentation of all these processes and effects for QCD is given in [5, 4] and references therein. Also in [22] is presented a detailed comparison between QCD and SYM in the pertubative domain of both theories. Here we just quote some of their results relevant for our discussion.

Firstly, notice that the differential photoemission rate can be recast into the emission rate per unit volume as a function of the photon momentum

$$\frac{d\Gamma_\gamma}{dk} = \frac{\alpha_{\rm em}}{\pi} k\, \eta^{\mu\nu}\, C^<_{\mu\nu}(K)\,. \tag{73}$$

At low frequencies the trace of the spectral function approaches a constant which is proportional to the electrical conductivity of the plasma:

$$\frac{d\Gamma_\gamma}{dk} = \frac{\sigma T}{\pi^2} k\,. \tag{74}$$

In figure 5 we show the photoemission rates per unit volume per unit time, divided by $\alpha_{\rm em}(N^2-1)T^3$, both in the non-perturbative and perturbative regimes. The non-pertubative case is the same as before in figure 3 in the previous section, and we repeat it here for comparison with the weakly coupled regime of the SYM theory.

It is very interesting the fact that for weakly coupled $\mathcal{N}=4$ supersymmetric Yang-Mills plasma the hydrodynamical regime in which Eq.(74) holds is as narrow as $k/T \leq \lambda^2$, with $\lambda$ small. Its slope $\sigma T/\pi^2$ is parametrically large and the photoemission rate has a maximum. For larger photon momentum we take the expression from [22]

$$\frac{d\Gamma_\gamma}{dk} = \frac{(N^2-1)\alpha_{\rm em}}{4\pi^2} k\, n_f(k)\, m_\infty^2 \left[\ln(T/m_\infty) + C_{\rm tot}(k/T)\right]. \tag{75}$$

being $n_f(k)$ the fermion statistical factor (with a minus sign), while the thermal correction to the hard fermion propagation in the medium is given by $m_\infty^2 = \lambda T^2$. The integral called

$C_{\rm tot}(k/T)$ was numerically solved in [22] and also can be written in the following form:

$$C_{\rm tot}(k/T) = \frac{1}{2}\ln(2k/T) + C_{2\longleftrightarrow 2}(k/T) + C_{\rm brem}(k/T) + C_{\rm pair}(k/T)\,. \tag{76}$$

The numerical results from [22] are reproduced quite accurately by the expressions:

$$\begin{aligned} C_{2\longleftrightarrow 2}(k/T) &\simeq 2.01\,T/k - 0.158 - 0.615\,e^{-0.187k/T}\,, \\ C_{\rm brem}(k/T) + C_{\rm pair}(k/T) &\simeq 0.954\,(T/k)^{3/2}\,\ln(2.36 + T/k) + 0.069 + 0.0289\,k/T\,, \end{aligned} \tag{77}$$

which hold in the range $0.2 < k/T < 20$. In figure 5 we have also shown a small-dashed stiff curve which corresponds to the value of $\lambda = 0.2$ and a long-dashed curve for $\lambda = 0.5$, both obtained using the combined expressions Eq.(76) and Eq.(77). For weak coupling, it can be seen that the photoemission rates decrease as the coupling decreases, except for very low frequencies, where they are very much enhanced compared to the strongly coupled regime. On the other hand, for large photon momentum the strongly coupled plasma displays larger photo-production rate. This effect is due to the suppression which is produced by the $m_\infty$ factor, proportional to the 't Hooft coupling, in the weakly coupled expression.

# 6 General discussion and conclusions

In this work we have investigated 't Hooft coupling corrections to the photoemission rate of a strongly-coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma by the means of the gauge/string duality. We consider the full $\mathcal{O}(\alpha'^3)$ type IIB string theory corrections to the supergravity action. The behaviour of the photoemission rates depend upon the values of the 't Hooft coupling. Their slopes at zero momentum give the electrical conductivity as a function of the 't Hooft coupling. Beyond the hydrodynamical regime of the plasma, as discussed before, the peak of the photoemission is enhanced by the corrections, and the momentum of maximal emission shifts towards the infrared, taking the corrected curves closer to the weakly coupled result. Simple numerical analysis on the light-like spectral function yields that the maximal rate is given by

$$\left.\frac{d\Gamma_\gamma}{dk}\right|_{max} \simeq 0.0156695\left(1 + \left[1115.3 - \frac{265}{8}\right]\gamma\right) + \mathcal{O}(\gamma^2)\,, \tag{78}$$

in units of $\alpha_{\rm em}N^2T^3$, where we have made explicit the factor $-265/8\gamma$ coming from the overall normalization of the action. For the peak displacement we estimate the position of the peak $k_{max}$ as

$$k_{max} \simeq 1.48469\,(1 - 842.425\gamma)\,T + \mathcal{O}(\gamma^2)\,, \tag{79}$$

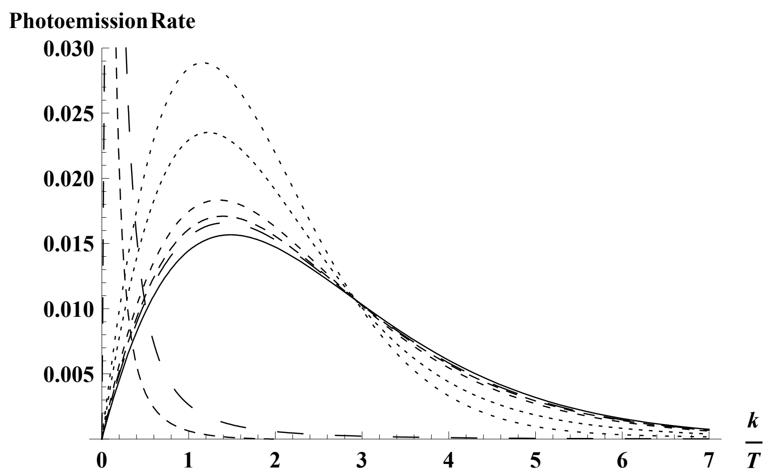


Figure 5: Photoemission spectrum for different values of $\lambda$, as a function of the light-like momentum of the emitted photon divided by the equilibrium temperature, $k/T$. Notice that in fact we show the curves for $d\Gamma_\gamma/dk$ divided by $\alpha_{\rm em}(N^2-1)T^3$. Solid, long-dashed, dashed, small-dashed, tiny-dashed, and dotted lines correspond to decreasing values of $\lambda =$ very large (in fact it is the analytical expression from supergravity with no string theory corrections), to $200, 150, 100, 50$ and 35, respectively. On the other hand, we show two additional curves corresponding to the weakly coupled SYM obtained in [22]: a small-dashed line and a long-dashed line, which represent the perturbative SYM plasma for $\lambda = 0.2$ and $0.5$, respectively.

which turns out to be independent of the overall normalization of the action, making it an excellent candidate for comparing disparate gauge theories. Furthermore, we obtain the total number of photons emitted, given by the area under the curves in figure 3. This is enhanced by a factor

$$\frac{N_{\rm total}(\gamma)}{N_{\rm total}(0)} \simeq 1 + \left[461.941 - \frac{265}{8}\right]\gamma + \mathcal{O}(\gamma^2)\,, \tag{80}$$

due to the fact that the weakly-coupled theory dominates in the infrared, where Bose-suppression (due to $n_b(k)$) is small.

These features are expected from perturbative quantum field theory calculations in the weakly coupled $\mathcal{N}=4$ supersymmetric Yang-Mills plasma and from the supergravity calculation of the large $N$ strongly coupled theory [22]. There is a ($\lambda$-independent) crossover point around $k/T \sim 2.92$, where the corrected curves dip below the $\lambda \to \infty$ result. However, at much higher momentum, all these curves cross upwards the extreme strongly coupled one, leading to that the asymptotic values of the $\lambda$-corrected curves for large $k/T$ are given by $(1+5\gamma)$ times the infinite coupling result, as in Eq.(72). Although the range of momentum of figure 3 does not extend to cover this asymptotic behaviour, we show this behaviour in figure 4. This means that the finite-$\lambda$ corrections enhance the photoemission rate in the deep ultraviolet domain, contrary to the expectations of [22]. Obviously, we are not guaranteed that the weakly-coupled result should be approached by strongly-coupled corrections computed in perturbation theory, especially not for a situation where the functional dependence on momenta is expected to be different, so we are not unduly concerned by this apparent discrepancy [1]. It would be very revealing to understand these cross-over points, as well as their scaling with $\lambda$.

The overall picture which emerges from our calculations is the following: the peaks of the photoemission rates are displaced towards low frequencies as $\lambda$ decreases. It implies that soft photons are produced more efficiently at weaker values of the 't Hooft coupling, in the strong coupling regime. On the other hand, for harder photons the emission is dominated by stronger values of the coupling. But then, due to the effect described in the previous paragraph, the t' Hooft coupling corrected curves dominate for much larger values of the ratio $k/T$. Still there is a notorious separation of the behaviour at the weak and strong coupling regimes: for the asymptotic behaviour of the weak coupling regime the fall off goes proportional to $k^{3/2}e^{-k/T}$, while for all the strongly coupled curves it falls down as $k^{5/2}e^{-k/T}$.

Let us consider very briefly what happens if we consider finite $N$ corrections. Using the modular form introduced in subsection 4.1, since it factorises out from the Green-Stahn action (20) we obtain the corrections to the plasma conductivity (recall that this is proportional to the slope of the photoemission rate at zero frequency)

$$\sigma = \sigma_0 + \frac{14993}{72}\,\sigma_0\left(\frac{\zeta(3)}{\lambda^{3/2}} + \frac{\lambda^{1/2}}{48N^2} + \frac{e^{-8\pi^2 N/\lambda}}{2\pi^{1/2}N^{3/2}}\right)\,. \tag{81}$$

where as we have seen $\sigma_0 = e^2 T N^2/16\pi$. At this point one may wonder whether this result can be compared with those obtained from lattice QCD. Obviously, any statement in the context of the present work has to be considered with several caveats, coming from differences between QCD and the $\mathcal{N}=4$ SYM theory. Said that, it is possible to make contact with lattice QCD at some extent. We must take into account that in lattice QCD calculation $N=3$ typically, and there are other differences with respect to the planar limit of the SYM plasma. In a recent estimation of the electrical conductivity it was found $\sigma \simeq 0.4 e^2 T$, above $T_c$ of quenched lattice QCD [49]. A more recent calculation [50] shows that $1/3 e^2 T \leq \sigma \leq e^2 T$ from the vector current correlation function from lattice computations at temperatures about 1.5 to 2 $T_c$, the values of $\alpha_s = g_{YM}^2/4\pi$ are between 0.3 and 0.4, where these values were obtained by matching the Debye mass screening in QCD and in $\mathcal{N}=4$ SYM at finite $T$. If we use the parametrization $\sigma = \rho e^2 T$ and extract $\rho$ from our equation (81), using naively $N=3$, and evaluating the electrical conductivity for $\lambda = 11.3$, 15.08 and $6\pi$, which lead to $\alpha_s$ =0.3, 0.4 and 0.5, respectively, we obtain $\rho$ =1.64, 1.28 and 1.101, respectively. So, we can see how close is the lowest value 1.101 to the upper value of the conductivity obtained from lattice QCD in [50].

The studies and results of photoemission and electrical conductivity presented in this work exclusively concern the $\mathcal{N}=4$ SYM plasma in thermal equilibrium. Beyond thermal equilibrium it is possible to carry out very interesting computations such as the one presented in [51], where the authors consider production of prompt photons from an out-of-equilibrium $\mathcal{N}=4$ SYM plasma, including $\mathcal{O}(\alpha'^3)$ string theory corrections. The work of [51] merges the formalism introduced in [52] about the photo-production from a out-of-equilibrium strongly coupled plasma with the one corresponding to $\mathcal{O}(\alpha'^3)$ string theory corrections to photo-production [1]. The AdS/CFT description of dilepton production from an out-of-equilibrium plasma has also been considered [53]. Papers [51, 52, 53] are based on a model of holographic thermalisation which involves the gravitational collapse of a thin shell in $AdS_5$ in a quasi-static approximation as in [54, 55]. On the other hand, one can also consider other approach to holographic thermalisation with a dynamical shell as in [56, 57]. Based on [56, 57], thermal and electromagnetic quenches for the specific case of $AdS_4$ have been studied [58]. More recently, a systematic study of holographic thermalisation from a collapsing shell of charged pressureless dust has been considered in [59], which allows one to consider a chemical potential effect on the thermalisation time scale of the plasma.

There are very interesting directions for further extensions of the work presented here. One is to look for similar strong coupling corrections to the electrical diffusion constant, with the idea of discussing about the possibility of a universal bound for electrical charge transport coefficients [43, 46]. Also, effects of leading 't Hooft coupling corrections to dilepton production from a strongly coupled plasma would be interesting to study in this context. Another interesting situation is the boost invariant plasma. In that case corrections coming from $F_5$ could have modified the ratio of shear viscosity over entropy density, however in [28] it was concluded that corrections from $F_5$ do not modify the results in this case. However, likely the situation would be different if one refers to electric charge transport properties. In addition, the inclusion of an $R$-charged black hole allows to study the plasma at finite chemical potential. In this case, as Paulos has shown [27], the EOM are modified by the $F_5$ corrections. However, while the analysis using only $C^4$-term is prohibitively complicated, Paulos has shown that the inclusion of its supersymmetric completion gives a simple result. It would be interesting to see if something similar happens for properties related to electric charge in the same background.

It would also be very interesting to carry out similar calculation as for the corrections to electrical conductivity and photoemission rates for other backgrounds with deformations from the conformal ones [60, 61] and with no conformal symmetry [62, 63]. It would also be very interesting to be able to compute higher-order corrections to the photoemission rate for plasmas with fundamental quarks. In that case there are at least two different and very interesting possibilities. One is the case for a D3D7 plasma which basically consists of the embedding of D7 branes in the background of a large number of D3 branes [64] at finite temperature. There are two possible embeddings for this: the Minkowski and the black hole ones, and there is a Hawking-Page transition which is associated with glueball to deconfined gluons transition and at higher temperature there is a second transition temperature at which mesons melt down giving a QGP [65]. Another very different system which actually is closer to QCD than the D3D7 brane system is the D4D8-anti D8-brane system proposed by Sakai and Sugimoto [66], which can also be heated up in order to obtain a QGP [67]. $\alpha'$ corrections to D-brane solutions have been investigated in [37].

## Acknowledgments

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## Appendix: The magnetic part of $F_5$ for vector fluctuations

### The vector fluctuation $F_{ux}$

Let us first consider the vector fluctuation $F_{ux}$. The magnetic part is a ten-dimensional Hodge dual of the electric one. In components we can write

$$F^f_{(m)uxy_{\nu_1}y_{\nu_2}y_{\nu_3}} = \sqrt{|\det G_{10}|}\, F^f_{(e)y_iy_jtyz}\, G^{y_iy_i}\, G^{y_jy_j}\, G^{tt}\, G^{yy}\, G^{zz}\, \epsilon_{y_iy_jtyzxu\nu_1\nu_2\nu_3}\,. \tag{82}$$

Note that this component as a form reads:

$$F^f_{(m)} = \frac{\sqrt{|\det G_{10}|}}{5!5!}\, F^f_{(e)y_iy_jtyz}\, G^{y_iy_i}\, G^{y_jy_j}\, G^{tt}\, G^{yy}\, G^{zz}\, \epsilon_{y_iy_jtyzxu\nu_1\nu_2\nu_3}\, du\wedge dx\wedge dy^{\nu_1}\wedge dy^{\nu_2}\wedge dy^{\nu_3}\,, \tag{83}$$

$\epsilon_{y_iy_jtyzxu\nu_1\nu_2\nu_3}$ determines the sign of each piece of the magnetic components as follows.

So, let us label the magnetic components by the indices of the vector fluctuation in the metric, *i.e.* $ux$ in this case we have

$$F^f_{(m)ux} \;=\; \sqrt{|\det g_{AdS}|}\, F^f_{(e)ux}\, G^{tt}\, G^{yy}\, G^{zz}\, (m^{ux}_{13} + m^{ux}_{14} + m^{ux}_{15} + m^{ux}_{24} + m^{ux}_{25})\,. \tag{84}$$

Then

$$\begin{aligned}
m^{ux}_{13} &= \epsilon_{13tyzxu245}\sqrt{\det S^5}\, b_{13}\, G^{y_1y_1}\, G^{y_3y_3} = -\sqrt{\det S^5}\, b_{13}\, G^{y_1y_1}\, G^{y_3y_3}\,,\\
m^{ux}_{14} &= \epsilon_{14tyzxu235}\sqrt{\det S^5}\, b_{14}\, G^{y_1y_1}\, G^{y_4y_4} = +\sqrt{\det S^5}\, b_{14} G^{y_1y_1}\, G^{y_4y_4}\,,\\
m^{ux}_{15} &= \epsilon_{15tyzxu234}\sqrt{\det S^5}\, b_{15}\, G^{y_1y_1}\, G^{y_5y_5} = -\sqrt{\det S^5}\, b_{15}\, G^{y_1y_1}\, G^{y_5y_5}\,,\\
m^{ux}_{24} &= \epsilon_{24tyzxu135}\sqrt{\det S^5}\, b_{24}\, G^{y_2y_2}\, G^{y_4y_4} = -\sqrt{\det S^5}\, b_{24}\, G^{y_2y_2}\, G^{y_4y_4}\,,\\
m^{ux}_{25} &= \epsilon_{25tyzxu134}\sqrt{\det S^5}\, b_{25}\, G^{y_2y_2}\, G^{y_5y_5} = +\sqrt{\det S^5}\, b_{25}\, G^{y_2y_2}\, G^{y_5y_5}\,.
\end{aligned} \tag{85}$$

### The vector fluctuation $F_{tx}$

As before, the magnetic part is a 10d Hodge dual of the electric one. In components we have

$$F^f_{(m)txy_{\nu_1}y_{\nu_2}y_{\nu_3}} = \sqrt{|\det G_{10}|}\, F^f_{(e)y_iy_jyzu}\, G^{y_iy_i}\, G^{y_jy_j}\, G^{uu}\, G^{yy}\, G^{zz}\, \epsilon_{y_iy_jyzutx\nu_1\nu_2\nu_3}\,, \tag{86}$$

As a form it reads:

$$F^f_{(m)} = \frac{\sqrt{|\det G_{10}|}}{5!5!}\, F^f_{(e)y_iy_jyzu}\, G^{y_iy_i}\, G^{y_jy_j}\, G^{uu}\, G^{yy}\, G^{zz}\, \epsilon_{y_iy_jyzutx\nu_1\nu_2\nu_3}\, dt\wedge dx\wedge dy^{\nu_1}\wedge dy^{\nu_2}\wedge dy^{\nu_3}\,. \tag{87}$$

Again, let us label the magnetic components by the indices of the vector fluctuation in the metric, *i.e.* $tx$

$$F^f_{(m)tx} \quad = \quad \sqrt{|\det g_{AdS}|}\, F^f_{(e)tx}\, G^{uu}\, G^{yy}\, G^{zz}\, (m^{tx}_{13} + m^{tx}_{14} + m^{tx}_{15} + m^{tx}_{24} + m^{tx}_{25})\,. \tag{88}$$

Then we have,

$$\begin{aligned}
m^{tx}_{13} &= \epsilon_{13yzutx245}\sqrt{\det S^5}\, b_{13}\, G^{y_1y_1}\, G^{y_3y_3} = -\sqrt{\det S^5}\, b_{13}\, G^{y_1y_1}\, G^{y_3y_3}\,,\\
m^{tx}_{14} &= \epsilon_{14yzutx235}\sqrt{\det S^5}\, b_{14}\, G^{y_1y_1}\, G^{y_4y_4} = +\sqrt{\det S^5}\, b_{14}\, G^{y_1y_1}\, G^{y_4y_4}\,,\\
m^{tx}_{15} &= \epsilon_{15yzutx234}\sqrt{\det S^5}\, b_{15}\, G^{y_1y_1}\, G^{y_5y_5} = -\sqrt{\det S^5}\, b_{15}\, G^{y_1y_1}\, G^{y_5y_5}\,,\\
m^{tx}_{24} &= \epsilon_{24yzutx135}\sqrt{\det S^5}\, b_{24}\, G^{y_2y_2}\, G^{y_4y_4} = -\sqrt{\det S^5}\, b_{24}\, G^{y_2y_2}\, G^{y_4y_4}\,,\\
m^{tx}_{25} &= \epsilon_{25yzutx134}\sqrt{\det S^5}\, b_{25}\, G^{y_2y_2}\, G^{y_5y_5} = +\sqrt{\det S^5}\, b_{25}\, G^{y_2y_2}\, G^{y_5y_5}\,.
\end{aligned} \tag{89}$$

**The vector fluctuation $F_{zx}$**

Similarly with the two previous cases the ten-dimensional Hodge dual of electrical part of $F_5$ induced by the vector fluctuation $F_{zx}$ in components is given by

$$F^f_{(m)zxy_{\nu_1}y_{\nu_2}y_{\nu_3}} = \sqrt{|\det G_{10}|}\, F^f_{(e)y_iy_jtyu}\, G^{y_iy_i}\, G^{y_jy_j}\, G^{tt}\, G^{yy}\, G^{uu}\, \epsilon_{y_iy_jtyuxz\nu_1\nu_2\nu_3}\,. \tag{90}$$

This component as a form reads:

$$F^f_{(m)} = \frac{\sqrt{|\det G_{10}|}}{5!5!}\, F^f_{(e)y_iy_jtyu}\, G^{y_iy_i}\, G^{y_jy_j}\, G^{tt}\, G^{yy}\, G^{uu}\, \epsilon_{y_iy_jtyuxz\nu_1\nu_2\nu_3}\, dx\wedge dz\wedge dy^{\nu_1}\wedge dy^{\nu_2}\wedge dy^{\nu_3}\,. \tag{91}$$

Now, let us label the magnetic components by the indices of the vector fluctuation in the metric, *i.e.* $zx$

$$F^f_{(m)zx} \quad = \quad \sqrt{|\det g_{AdS}|}\, F^f_{(e)ux}\, G^{tt}\, G^{yy}\, G^{uu}\, (m^{zx}_{13} + m^{zx}_{14} + m^{zx}_{15} + m^{zx}_{24} + m^{zx}_{25})\,. \tag{92}$$

Then

$$\begin{aligned}
m^{zx}_{13} &= \epsilon_{13tyuxz245}\sqrt{\det S^5}\, b_{13}\, G^{y_1y_1}\, G^{y_3y_3} = -\sqrt{\det S^5}\, b_{13}\, G^{y_1y_1}\, G^{y_3y_3}\,(-1)\,,\\
m^{zx}_{14} &= \epsilon_{14tyuxz235}\sqrt{\det S^5}\, b_{14}\, G^{y_1y_1}\, G^{y_4y_4} = +\sqrt{\det S^5}\, b_{14}\, G^{y_1y_1}\, G^{y_4y_4}\,(-1)\,,\\
m^{zx}_{15} &= \epsilon_{15tyuxz234}\sqrt{\det S^5}\, b_{15}\, G^{y_1y_1}\, G^{y_5y_5} = -\sqrt{\det S^5}\, b_{15}\, G^{y_1y_1}\, G^{y_5y_5}\,(-1)\,,\\
m^{zx}_{24} &= \epsilon_{24tyuxz135}\sqrt{\det S^5}\, b_{24}\, G^{y_2y_2}\, G^{y_4y_4} = -\sqrt{\det S^5}\, b_{24}\, G^{y_2y_2}\, G^{y_4y_4}\,(-1)\,,\\
m^{zx}_{25} &= \epsilon_{25tyuxz134}\sqrt{\det S^5}\, b_{25}\, G^{y_2y_2}\, G^{y_5y_5} = +\sqrt{\det S^5}\, b_{25}\, G^{y_2y_2}\, G^{y_5y_5}\,(-1)\,.
\end{aligned} \tag{93}$$

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