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Kinematical and nonlocality effects on the nonmesonic weak hypernuclear decay

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Abstract

We make a careful study about the nonrelativistic reduction of one-meson-exchange models for the nonmesonic weak hypernuclear decay. Starting from a widely accepted effective coupling Hamiltonian involving the exchange of the complete pseudoscalar and vector meson octets (π, η, K, ρ, ω, K^*), the strangeness-changing weak $\Lambda N \to NN$ transition potential is derived, including two effects that have been systematically omitted in the literature, or, at best, only partly considered. These are the kinematical effects due to the difference between the lambda and nucleon masses, and the first-order nonlocality corrections, i.e., those involving up to first-order differential operators. Our analysis clearly shows that the main kinematical effect on the local contributions is the reduction of the effective pion mass. The kinematical effect on the nonlocal contributions is more complicated, since it activates several new terms that would otherwise remain dormant. Numerical results for ${}^{12}_{\Lambda}\mathrm{C}$ and ${}^{5}_{\Lambda}\mathrm{He}$ are presented and they show that the combined kinematical plus nonlocal corrections have an appreciable influence on the partial decay rates. However, this is somewhat diminished in the main decay observables: the total nonmesonic rate, Γ_{nm}, the neutron-to-proton branching ratio, Γ_n/Γ_p, and the asymmetry parameter, a_Λ. The latter two still cannot be reconciled with the available experimental data. The existing theoretical predictions for the sign of a_Λ in ${}^{5}_{\Lambda}\mathrm{He}$ are confirmed.

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1. Introduction

The free decay of a Λ hyperon occurs almost exclusively through the mesonic mode, $\Lambda \to \pi N$, with the nucleon emerging with a momentum of about 100 MeV/c. Inside nuclear matter ($p_{\mathrm{F}} \approx 270$ MeV/c) this mode is Pauli blocked, and, for all but the lightest Λ hypernuclei ($A \geqslant 5$), the weak decay is dominated by the nonmesonic channel, $\Lambda N \to NN$, which liberates enough kinetic energy to put the two emitted nucleons above the Fermi surface. In the absence of stable hyperon beams, these nonmesonic decays offer the only way available to investigate the strangeness-changing weak interaction between hadrons. (For reviews on hypernuclear decay, see Refs. [1–3].)

The simplest model for this process is the exchange of a virtual pion [4], and in fact this can reproduce reasonably well the total (nonmesonic) decay rate, $\Gamma_{\mathrm{nm}} = \Gamma_n + \Gamma_p$, but fails badly for other observables like the ratio of neutron-induced ($\Lambda n \to nn$) to proton-induced ($\Lambda p \to np$) transitions, Γ_n/Γ_p, and the asymmetry parameter a_Λ. The deficiency of this model is attributed to effects of short range physics, which should be quite important in view of the large momentum transfers involved (~ 400 MeV/c). Although there have been some attempts to account for this fact by making use of quark models to compute the shortest range part of the transition potential [5–9], most of the theoretical work opted for the addition of other, heavier mesons in the exchange process [10–23]. None of these models gives fully satisfactory results. Inclusion of correlated two-pion exchange has not been completely successful either [24,25]. Nor have the addition of uncorrelated two-pion exchange, two-nucleon induced transitions or medium effects, treated within the nonrelativistic [26–31] or relativistic [32] propagator approaches, been of much help.

Here, we concentrate on the line of one-meson-exchange (OME) models [4–25]. We do not explicitly discuss hybrid models, i.e., those involving quark degrees of freedom [5–9]. However, much of the theoretical developments we present could be generalized to include them. Also two-pion exchange [24,25] could be brought into our general framework. The main ingredients of OME models are the effective baryon–baryon–meson weak and strong Hamiltonians. These are constructed in the language of relativistic field theory, but in almost all calculations (exceptions are Refs. [14–16]) one has proceeded to make a nonrelativistic reduction for the extraction of the transition potential. This often involves some further approximations, like neglecting the nonlocality in this potential, and balancing by hand the distorted kinematics in the OME Feynman amplitudes, resulting from the difference between initial and final baryon masses. This not only alters the different terms in the transition potential, but also eliminates several of them. Our main purpose here is to assess the relative importance of these effects.

The paper is organized as follows. Most of the formalism is developed in Section 2. In Subsection 2.1, we explain the construction of the nonrelativistic transition potential, taking due care of the kinematics and including nonlocal terms, and, in Subsection 2.2, we give a motivation for not neglecting a priori the lambda–nucleon mass difference. In Subsections 2.3 and 2.4, the explicit expressions for the local and first-order nonlocal

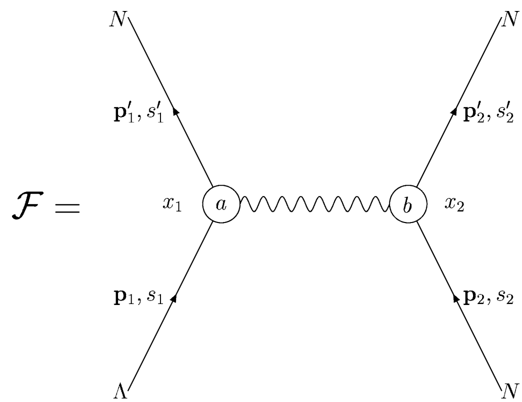

Fig. 1. OME Feynman amplitude in coordinate space.

contributions to the transition potential due to the exchange of a π, ρ, K or K^* meson are derived and commented. The ones corresponding to the η and ω exchanges can be easily obtained by analogy with those of the π and ρ, respectively, thus allowing the inclusion of the full pseudoscalar and vector meson octets, as deemed necessary by the present day consensus. In Subsection 2.5, we describe how to take the finite size effects into account by means of form factors. Our numerical results are reported in detail and discussed in Section 3. The phenomenological way to include short range correlations is presented in Subsection 3.1, together with the main expressions for the calculation of the transition rates in the extreme particle–hole model of Ref. [21], and all this is applied to the decay of ${}^{12}_{\Lambda}$C. In Subsection 3.2, we do the same for ${}^{5}_{\Lambda}$He and compute also the asymmetry parameter. Finally, Section 4 summarizes our main conclusions. Some useful formulas are collected in Appendix A and a specific point relating to our phase conventions is discussed in Appendix B.

2. OME transition potential

2.1. General discussion

The transition rate for the nonmesonic weak decay of a hypernucleus in its ground state $|I\rangle$, having energy E_I, to a residual nucleus in any of the allowed final states $\langle F|$, having energies E_F, and two outgoing nucleons is given by Fermi's golden rule,

$$\Gamma = 2\pi \sum_{s'_1 s'_2 F} \int\int \frac{d^3\boldsymbol{p}'_1}{(2\pi)^3} \frac{d^3\boldsymbol{p}'_2}{(2\pi)^3} \delta(E'_1 + E'_2 + E_F - E_I) \left|\langle \boldsymbol{p}'_1 s'_1 \boldsymbol{p}'_2 s'_2, F|\hat{V}|I\rangle\right|^2. \quad (1)$$

To construct the transition potential $\hat{V}$ in one-meson-exchange models, one starts from the free space Feynman amplitude depicted in Fig. 1, where $x = (t, \boldsymbol{x})$ denotes space–time coordinates, $\boldsymbol{p}$, momentum, and s, spin and, eventually, other intrinsic quantum numbers (such as isospin). In the remainder of this subsection we will consider a general situation, i.e., without specifying which baryons are propagating in each of the four legs, or which meson is being exchanged, or yet the exact nature of the couplings at the two vertices. This will be particularized to Λ-hypernuclear decay in the subsections that follow.

Vertices a and b correspond to coupling Hamiltonians of the general form ($c = a$ or b)

$$\mathcal{H}^c(x) = g_c \bar{\psi}(x)\big[\Gamma^c(\partial)\phi(x)\big]\psi(x), \tag{2}$$

where ψ and ϕ stand for the baryon and meson fields, respectively, g_c is a coupling constant and Γ^c may contain differential operators, in which case they are understood to be acting on the boson field only. The Feynman rules give

$$\mathcal{F} = (2\pi)^4\delta(E_1' + E_2' - E_1 - E_2)\delta(\boldsymbol{p}_1' + \boldsymbol{p}_2' - \boldsymbol{p}_1 - \boldsymbol{p}_2)F, \tag{3}$$

where $E_i = \sqrt{M_i^2 + \boldsymbol{p}_i^2}$ ($i = 1, 2$) for the incoming baryons, having masses M_i, and similarly (primed quantities) for the outgoing ones, and F is the Feynman amplitude in momentum space. Choosing the CM frame,

$$-\boldsymbol{p}_1 = \boldsymbol{p}_2 = \boldsymbol{p}, \qquad -\boldsymbol{p}_1' = \boldsymbol{p}_2' = \boldsymbol{p}', \tag{4}$$

this can be put in the form

$$iF(\boldsymbol{p}', \boldsymbol{p}; s_1's_2's_1s_2) = \chi_{s_1'}^\dagger \chi_{s_2'}^\dagger V(\boldsymbol{p}', \boldsymbol{p})\chi_{s_1}\chi_{s_2} \tag{5}$$

with

$$V(\boldsymbol{p}', \boldsymbol{p}) = \bar{u}_1'(-\boldsymbol{p}')\bar{u}_2'(\boldsymbol{p}')\mathcal{V}(q)u_1(-\boldsymbol{p})u_2(\boldsymbol{p}), \tag{6}$$

where $u_i(\boldsymbol{p}_i)\chi_{s_i}$ and $\chi_{s_i'}^\dagger \bar{u}_i'(\boldsymbol{p}_i')$ ($i = 1, 2$) are the momentum eigenspinors and their conjugates for the incoming and outgoing baryons, respectively, and, denoting the meson propagator by $\mathcal{D}$,

$$-i\mathcal{V}(q) = \big[-ig_a\Gamma^a(iq)\big]i\mathcal{D}(q)\big[-ig_b\Gamma^b(-iq)\big]. \tag{7}$$

We have introduced the 4-momentum transfer in the CM frame, $q = (\omega, \boldsymbol{q})$, with

$$\omega = \frac{1}{2}(E_1 - E_1' + E_2' - E_2), \qquad \boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}. \tag{8}$$

Notice that we have directed q from vertex a to vertex b.

The nonrelativistic transition potential $\hat{V}$ is given by the identification

$$\langle \boldsymbol{p}'|\hat{V}|\boldsymbol{p}\rangle = V(\boldsymbol{p}', \boldsymbol{p}), \tag{9}$$

where an expansion up to quadratic terms in momentum/mass is implied. To this end it is convenient to change the momentum variables to $\boldsymbol{q}$, defined in Eq. (8), and

$$\boldsymbol{Q} = \frac{m\boldsymbol{p}' + m'\boldsymbol{p}}{m + m'}, \tag{10}$$

where m and m' are the initial and final reduced masses,

$$\frac{1}{m} = \frac{1}{M_1} + \frac{1}{M_2}, \qquad \frac{1}{m'} = \frac{1}{M_1'} + \frac{1}{M_2'}. \tag{11}$$

In this transformation, the following relations hold:

$$\frac{\boldsymbol{p}'^2}{2m'} + \frac{\boldsymbol{p}^2}{2m} = \frac{1}{2}\left(\frac{1}{m'} + \frac{1}{m}\right)\boldsymbol{Q}^2 + \frac{\boldsymbol{q}^2}{2(m+m')}, \tag{12}$$

$$\boldsymbol{p}' \cdot \boldsymbol{r}' - \boldsymbol{p} \cdot \boldsymbol{r} = \boldsymbol{Q} \cdot (\boldsymbol{r}' - \boldsymbol{r}) + \boldsymbol{q} \cdot \left(\frac{m'\boldsymbol{r}' + m\boldsymbol{r}}{m+m'}\right), \tag{13}$$

where

$$\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1 \quad \text{and} \quad \boldsymbol{r}' = \boldsymbol{r}'_2 - \boldsymbol{r}'_1 \tag{14}$$

are, respectively, the initial and final relative coordinates.

The contribution of any given meson, i, to Eq. (9) has the general form

$$V_i(\boldsymbol{p}', \boldsymbol{p}) = \frac{v_i(\boldsymbol{p}', \boldsymbol{p})}{\boldsymbol{q}^2 + \mu_i^2 - \omega^2}, \tag{15}$$

where μ_i is the meson mass. The nonrelativistic expansion of the numerator in Eq. (15) poses no problem, but the denominator does not truly have such an expansion.[2] Therefore, it needs a special treatment. Recall that, strictly speaking, the Feynman amplitude F in Eq. (5) is defined only for energy-conserving transitions, i.e., for $E_1 + E_2 = E'_1 + E'_2$, and in this case one has, in the nonrelativistic approximation,

$$\frac{\boldsymbol{p}^2}{2m} - \frac{\boldsymbol{p}'^2}{2m'} \cong M'_1 + M'_2 - M_1 - M_2. \tag{16}$$

This relation together with Eq. (12) allow us to write, again in the nonrelativistic approximation,

$$\omega \cong M_0 - \frac{\boldsymbol{q}^2}{2M_q} - \frac{\boldsymbol{Q}^2}{2M_Q}, \tag{17}$$

where we have introduced the kinematical masses M_0, M_q and M_Q, given by

$$\begin{aligned}
M_0 &= \frac{1}{2}\left[1 + \frac{1}{2}\left(\frac{M_1 - M_2}{M_1 + M_2} + \frac{M'_1 - M'_2}{M'_1 + M'_2}\right)\right](M_1 - M'_1) \\
&\quad + \frac{1}{2}\left[1 - \frac{1}{2}\left(\frac{M_1 - M_2}{M_1 + M_2} + \frac{M'_1 - M'_2}{M'_1 + M'_2}\right)\right](M'_2 - M_2), \\
\frac{1}{M_q} &= \frac{1}{4}\left(\frac{M_1 - M_2}{M_1 + M_2} - \frac{M'_1 - M'_2}{M'_1 + M'_2}\right)\frac{1}{m+m'}, \\
\frac{1}{M_Q} &= \frac{1}{4}\left(\frac{M_1 - M_2}{M_1 + M_2} - \frac{M'_1 - M'_2}{M'_1 + M'_2}\right)\left(\frac{1}{m} + \frac{1}{m'}\right).
\end{aligned} \tag{18}$$

It is clear that, whenever the absolute values of the differences of baryon masses are much smaller than the corresponding sums, as is the case, e.g., for hypernuclear decay, one can take advantage of the inequalities $|M_0/M_q| \ll 1$ and $|M_0/M_Q| \ll 1$ to write the following approximation:

$$\frac{1}{\boldsymbol{q}^2 + \mu_i^2 - \omega^2} \cong \frac{1}{1 + M_0/M_q}\frac{1}{\boldsymbol{q}^2 + \tilde{\mu}_i^2} - \frac{M_0/M_Q}{(1 + M_0/M_q)^2}\frac{\boldsymbol{Q}^2}{(\boldsymbol{q}^2 + \tilde{\mu}_i^2)^2}, \tag{19}$$

[2] The nonrelativistic approximation is for the *baryon* dynamics.

where we have introduced the *effective meson mass*

$$\tilde{\mu}_i = \sqrt{\frac{\mu_i^2 - M_0^2}{1 + M_0/M_q}}. \tag{20}$$

The net result is that the nonrelativistic approximation, as defined here, will reduce $V(\boldsymbol{p}', \boldsymbol{p})$ to a quadratic polynomial in $\boldsymbol{Q}$,

$$V(\boldsymbol{p}', \boldsymbol{p}) \cong V^{(0)}(\boldsymbol{q}) + \boldsymbol{V}^{(1)}(\boldsymbol{q}) \cdot \boldsymbol{Q} + \boldsymbol{Q} \cdot \mathsf{V}^{(2)}(\boldsymbol{q}) \cdot \boldsymbol{Q}, \tag{21}$$

whose coefficients $V^{(0)}$, $\boldsymbol{V}^{(1)}$ and $\mathsf{V}^{(2)}$ are themselves, excluding the denominators that come from the meson propagators, polynomials of degree 2, 1 and 0, respectively, in $\boldsymbol{q}$. This $\boldsymbol{Q}$ dependence translates, in the coordinate representation, into an expansion in the nonlocality of the transition potential. To see this, we make use of Eq. (9) to write[3]

$$\begin{aligned} \langle \boldsymbol{r}' | \hat{V} | \boldsymbol{r} \rangle &= \int \frac{d^3\boldsymbol{p}'}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \langle \boldsymbol{r}' | \boldsymbol{p}' \rangle \langle \boldsymbol{p}' | \hat{V} | \boldsymbol{p} \rangle \langle \boldsymbol{p} | \boldsymbol{r} \rangle \\ &\equiv \int \frac{d^3\boldsymbol{p}'}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} e^{i\boldsymbol{p}'\cdot\boldsymbol{r}'} e^{-i\boldsymbol{p}\cdot\boldsymbol{r}} V(\boldsymbol{p}', \boldsymbol{p}). \end{aligned} \tag{22}$$

Changing the integration variables to $(\boldsymbol{q}, \boldsymbol{Q})$, making use of Eq. (13) and truncating, for simplicity, the quadratic polynomial in Eq. (21) at the linear term, this gives

$$\langle \boldsymbol{r}' | \hat{V} | \boldsymbol{r} \rangle \cong \left[V^{(0)}(\boldsymbol{r}') - i \boldsymbol{V}^{(1)}\left(\tfrac{m'\boldsymbol{r}' + m\boldsymbol{r}}{m+m'}\right) \cdot \nabla' \right] \delta(\boldsymbol{r}' - \boldsymbol{r}), \tag{23}$$

where

$$V^{(0)}(\boldsymbol{r}) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} V^{(0)}(\boldsymbol{q}), \tag{24}$$

$$\boldsymbol{V}^{(1)}(\boldsymbol{r}) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \boldsymbol{V}^{(1)}(\boldsymbol{q}). \tag{25}$$

This means that for any state of relative motion, Ψ,

$$\langle \boldsymbol{r} | \hat{V} | \Psi \rangle = \hat{V}(\boldsymbol{r}) \Psi(\boldsymbol{r}), \tag{26}$$

with the *transition potential in coordinate space*, $\hat{V}(\boldsymbol{r})$, given, as an operator in wave-function space, by

$$\hat{V}(\boldsymbol{r}) = V^{(0)}(\boldsymbol{r}) + \hat{V}^{(1)}(\boldsymbol{r}) \tag{27}$$

where $V^{(0)}(\boldsymbol{r})$ is the *local potential*, and the differential operator

$$\hat{V}^{(1)}(\boldsymbol{r}) = -i \frac{m}{m+m'} \left(\nabla \cdot \boldsymbol{V}^{(1)}(\boldsymbol{r}) \right) - i \boldsymbol{V}^{(1)}(\boldsymbol{r}) \cdot \nabla, \tag{28}$$

[3] To do this we have to assume that Eqs. (9) and (21) hold for $\boldsymbol{p}$ and $\boldsymbol{p}'$ unrestricted, although $V(\boldsymbol{p}', \boldsymbol{p})$ was extracted from the Feynman amplitude F in Eqs. (3) and (5), which, being related to a T-matrix, is unambiguous only for energy-conserving transitions.

its *first-order nonlocality correction*.[4] For our purposes here it will be sufficient to stop at this order, and we will not consider the second-order corrections, that would come from the last term in Eq. (21).

Before closing this subsection, let us make a brief comment on our treatment of the ω^2 term in the meson propagator in Eq. (15), more specifically, on the expression we use for the time component, ω, of the 4-momentum transfer in Eq. (8). We are extracting the transition potential from the Feynman diagram in Fig. 1, in which the baryons are on their mass shells. However, as already alluded to in footnote 3, we need to extend these OME amplitudes to the off-shell region, for which there is no unique procedure. In the meson-exchange theory for the strong NN force, this is done by treating the two interacting particles through the Bethe–Salpeter equation [33], and this ambiguity appears in the choice of which one of its various tridimensional reductions to use. This issue, which is related to meson retardation effects, has been much discussed in the past [34–41], but remains unsettled. We have followed the general philosophy of Machleidt and collaborators [38–40], to the effect that, for the case of similar masses, the best choice is to use tridimensional reductions that treat the two particles symmetrically, like the Blankenbecler–Sugar [42] or the Thompson [43] equations. One, then, puts the two interacting particles equally off-shell in the CM frame, and fixes the time-components of the relative 4-momenta in the initial and final two-particle propagators at the values $p_0 = \frac{1}{2}(E_2 - E_1)$ and $p_0' = \frac{1}{2}(E_2' - E_1')$, respectively. (See, for instance, Eq. (2.28) in Ref. [37], remembering our convention in Eq. (4).) This leads directly to our expression for ω in Eq. (8). Notice that, for strictly equal masses, this gives $\omega = 0$, being, therefore, equivalent, in this case, to the instantaneous approximation. Our choice for ω in Eq. (8) leads, in the nonrelativistic approximation, to Eq. (17) and, consequently, to the expansion of the meson propagators in Eq. (19). We are confident that this is appropriate for processes not too far off the energy-shell. In other situations that might occur, for instance, in a fully microscopic treatment of short-range correlations, this point should be reexamined.

2.2. *Kinematical effects*

In computing the OME Feynman amplitudes contributing to the strong NN force it is standard practice [39] to avoid the kinematical complications due to the difference between the neutron and proton masses by setting

$$M_n, M_p \to M \equiv (M_n + M_p)/2, \tag{29}$$

which can be justified by the small value of the ratio

$$\frac{M_n - M_p}{M} = 0.0014. \tag{30}$$

[4] Notice that, despite its name, this has a local piece, namely, the first term in Eq. (28), where $(\nabla \cdot \boldsymbol{V}^{(1)}(\boldsymbol{r})) \equiv \mathrm{div}\, \boldsymbol{V}^{(1)}(\boldsymbol{r})$.

The analogous practice is followed in the calculation of the transition potentials for the weak decay of Λ-hypernuclei [18]. In this case, however, one equally sidesteps the lambda-nucleon mass difference, by setting, at the vertex where the Λ decays,

$$M_\Lambda, M \to \bar{M} \equiv (M_\Lambda + M)/2, \tag{31}$$

despite the fact that the corresponding ratio,

$$\frac{M_\Lambda - M}{\bar{M}} = 0.17, \tag{32}$$

is nowhere as small.

Undoubtedly, this approximation considerably simplifies the calculations. However, in view of the nonnegligible value of the ratio (32), it seems appropriate to investigate the effects of the latter approximation. To this end, we examine below, for each meson in the pseudoscalar and vector octets, the expression for the nonrelativistic OME transition potential obtained by accepting the approximation in Eq. (29), but not that in Eq. (31). This gives for the kinematical masses (18)

$$\begin{aligned} M_0 &= \frac{1}{4}\left(\frac{M_\Lambda - M}{M_\Lambda + M}\right)(3M_\Lambda + M) = 92.18\ \text{MeV}, \\ \frac{1}{M_q} &= \frac{1}{2}\left(\frac{M_\Lambda - M}{3M_\Lambda + M}\right)\frac{1}{M} = 2.196 \times 10^{-5}\ \text{MeV}^{-1}, \\ \frac{1}{M_Q} &= \frac{1}{4}\left(\frac{M_\Lambda - M}{M_\Lambda + M}\right)\left(\frac{3M_\Lambda + M}{M_\Lambda M}\right) = 8.800 \times 10^{-5}\ \text{MeV}^{-1}, \end{aligned} \tag{33}$$

where we have used [44] $M = 938.92$ MeV and $M_\Lambda = 1115.68$ MeV. The approximation (31) would have set M_0, $1/M_q$ and $1/M_Q$ to zero.

2.3. Contributions of nonstrange mesons

For the nonstrange mesons, we have, acting respectively at the vertices a and b in Fig. 1, weak (W) and strong (S) coupling Hamiltonians that we take to be the same as those in Ref. [18]. For the pion they are

$$\mathcal{H}^W_{\Lambda N\pi} = iG_F\mu_\pi^2 \bar{\psi}_N (A_\pi + B_\pi \gamma_5)\boldsymbol{\phi}^{(\pi)} \cdot \boldsymbol{\tau} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_\Lambda, \tag{34}$$

$$\mathcal{H}^S_{NN\pi} = i g_{NN\pi} \bar{\psi}_N \gamma_5 \boldsymbol{\phi}^{(\pi)} \cdot \boldsymbol{\tau} \psi_N, \tag{35}$$

where $G_F\mu_\pi^2 = 2.21 \times 10^{-7}$ is the Fermi weak coupling-constant, A_π and B_π are fitted to the free Λ decay, and $g_{NN\pi}$ is taken from OME models for the strong NN force. The isospurion $\binom{0}{1}$ is used to enforce the $\Delta T = 1/2$ rule for isospin violation, observed in the free Λ decay [18]. For the rho meson, we have

$$\mathcal{H}^W_{\Lambda N\rho} = G_F\mu_\pi^2 \bar{\psi}_N \left[\left(A_\rho \gamma^\nu \gamma_5 + B^V_\rho \gamma^\nu + B^T_\rho \frac{\sigma^{\mu\nu}\partial_\mu}{2\bar{M}}\right)\boldsymbol{\phi}^{(\rho)}_\nu \cdot \boldsymbol{\tau}\right]\begin{pmatrix} 0 \\ 1 \end{pmatrix}\psi_\Lambda, \tag{36}$$

$$\mathcal{H}^S_{NN\rho} = \bar{\psi}_N \left[\left(g^V_{NN\rho}\gamma^\nu + g^T_{NN\rho}\frac{\sigma^{\mu\nu}\partial_\mu}{2M}\right)\boldsymbol{\phi}^{(\rho)}_\nu \cdot \boldsymbol{\tau}\right]\psi_N. \tag{37}$$

Table 1
Coupling constants, masses (μ_i) and cutoff parameters (Λ_i) for the nonstrange mesons. The weak couplings are in units of $G_F\mu_\pi^2$. Adapted from Ref. [18]

Meson i	Coupling constants: Weak PV	Weak PC	Strong	μ_i [MeV]	Λ_i [GeV]
π	$A_\pi = 1.05$	$B_\pi = -7.15$	$g_{NN\pi} = 13.3$	140.0	1.30
η	$A_\eta = 1.80$	$B_\eta = -14.3$	$g_{NN\eta} = 6.40$	548.6	1.30
ρ	$A_\rho = 1.09$	$B_\rho^V = -3.50$	$g_{NN\rho}^V = 3.16$	775.0	1.40
		$B_\rho^T = -6.11$	$g_{NN\rho}^T = 13.3$		
ω	$A_\omega = -1.33$	$B_\omega^V = -3.69$	$g_{NN\omega}^V = 10.5$	783.4	1.50
		$B_\omega^T = -8.04$	$g_{NN\omega}^T = 3.22$		

The corresponding Hamiltonians for the η and ω are completely analogous to those of the π and ρ, respectively, if one takes into consideration their isoscalar nature.

The weak couplings of the heavier mesons are theoretically inferred from those of the pion through unitary-symmetry arguments and other relationships. The strong ones are again taken from OME models for the nuclear force. We will follow the parametrization adopted in Ref. [18], where further details can be found on this matter. For definiteness, the numerical values are reproduced in Table 1.

2.3.1. One pion exchange

The local nonrelativistic one-pion-exchange transition potential is given in momentum space by

$$V_\pi^{(0)}(\boldsymbol{q}) = -\left(1+\frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \frac{g_{NN\pi}}{2M}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(A_\pi + \frac{B_\pi}{2\check{M}}\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\right)\frac{\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\boldsymbol{q}^2+\tilde{\mu}_\pi^2}, \tag{38}$$

where

$$\check{M} = \frac{M+3M_\Lambda}{3M+M_\Lambda}M. \tag{39}$$

Comparing Eq. (38) with the result that would have been obtained under approximation (31), namely [18, Eq. (24)],

$$\bar{V}_\pi^{(0)}(\boldsymbol{q}) = -G_F\mu_\pi^2 \frac{g_{NN\pi}}{2M}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(A_\pi + \frac{B_\pi}{2\bar{M}}\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\right)\frac{\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\boldsymbol{q}^2+\mu_\pi^2}, \tag{40}$$

it is possible to estimate the relative size of the effects of the more accurate treatment of the kinematics, adopted here, from the following correction factors:

$$(1+M_0/M_q)^{-1} = 0.998, \qquad \check{M}/\bar{M} = 0.996, \tag{41}$$

$$\tilde{\mu}_\pi/\mu_\pi = 0.752. \tag{42}$$

If each of these values were equal to unity, there would be no effect at all. Apparently, the situation is not much different from this, except in the last case, which will have a noticeable effect since it increases by $\sim 35\%$ the range of the corresponding contribution to the transition potential.

When changing to the coordinate representation through Eqs. (24) or (25), the shape functions

$$
\begin{aligned}
f_C(r,\mu) &= \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \frac{1}{\boldsymbol{q}^2+\mu^2} = \frac{e^{-\mu r}}{4\pi r},\\
f_V(r,\mu) &= -\frac{\partial}{\partial r} f_C(r,\mu) = \mu\left(1+\frac{1}{\mu r}\right) f_C(r,\mu),\\
f_S(r,\mu) &= \frac{1}{3}\left[\mu^2 f_C(r,\mu) - \delta(\boldsymbol{r})\right],\\
f_T(r,\mu) &= \frac{1}{3}\mu^2\left[1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right] f_C(r,\mu)
\end{aligned}
\tag{43}
$$

naturally arise, accordingly as the numerators in the Fourier transforms are, respectively, a constant, a vector, a scalar or a tensor built, at most quadratically, from $\boldsymbol{q}$. In terms of these, we get, for the potential (38) in coordinate space,

$$
\begin{aligned}
V_\pi^{(0)}(\boldsymbol{r}) &= \left(1+\frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \frac{g_{NN\pi}}{2M} \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\\
&\quad\times\left[-iA_\pi f_V(r,\tilde{\mu}_\pi)\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}} + \frac{B_\pi}{2\check{M}} f_S(r,\tilde{\mu}_\pi)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right.\\
&\qquad\left. + \frac{B_\pi}{2\check{M}} f_T(r,\tilde{\mu}_\pi) S_{12}(\hat{\boldsymbol{r}})\right],
\end{aligned}
\tag{44}
$$

where $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$ and $S_{12}(\hat{\boldsymbol{r}}) = 3(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}}) - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$. Under approximation (31), this would reduce to

$$
\begin{aligned}
\bar{V}_\pi^{(0)}(\boldsymbol{r}) &= G_F\mu_\pi^2 \frac{g_{NN\pi}}{2M} \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\\
&\quad\times\left[-iA_\pi f_V(r,\mu_\pi)\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}} + \frac{B_\pi}{2\bar{M}} f_S(r,\mu_\pi)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right.\\
&\qquad\left. + \frac{B_\pi}{2\bar{M}} f_T(r,\mu_\pi) S_{12}(\hat{\boldsymbol{r}})\right].
\end{aligned}
\tag{45}
$$

The first-order nonlocality coefficient in momentum space, appearing in Eq. (21), is given, for the pion, by

$$
\boldsymbol{V}_\pi^{(1)}(\boldsymbol{q}) = -\left(1+\frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \frac{g_{NN\pi}}{2M} \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(\frac{B_\pi}{2\grave{M}}\boldsymbol{\sigma}_1\right)\frac{\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\boldsymbol{q}^2+\tilde{\mu}_\pi^2},
\tag{46}
$$

where

$$
\frac{1}{\grave{M}} = \frac{1}{M} - \frac{1}{M_\Lambda}.
\tag{47}
$$

Changing to the coordinate representation according to Eq. (25), we get

$$
\boldsymbol{V}_\pi^{(1)}(\boldsymbol{r}) = -i\left(1+\frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \frac{g_{NN\pi}}{2M}\frac{B_\pi}{2\grave{M}} \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 f_V(r,\tilde{\mu}_\pi)(\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}})\boldsymbol{\sigma}_1
\tag{48}
$$

and introducing this into Eq. (28) yields for the first-order nonlocality correction

$$\hat{V}_\pi^{(1)}(\boldsymbol{r}) = \left(1 + \frac{M_0}{M_q}\right)^{-1} G_{\mathrm{F}} \mu_\pi^2 \frac{g_{NN\pi}}{2M} \frac{B_\pi}{2\grave{M}} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\ \times \left\{ \frac{2M_\Lambda}{3M_\Lambda + M} \left[f_S(r, \tilde{\mu}_\pi) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + f_T(r, \tilde{\mu}_\pi) S_{12}(\hat{\boldsymbol{r}}) \right] \right. \\ \left. - f_V(r, \tilde{\mu}_\pi)(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_1 \cdot \nabla) \right\}. \tag{49}$$

The mass averaging approximation (31) would set $1/\grave{M}$ to zero. Therefore, $\bar{\boldsymbol{V}}_\pi^{(1)}(\boldsymbol{q}) = 0$ and there would be no first-order nonlocality correction for the pion under this approximation, i.e.,

$$\hat{\bar{V}}_\pi^{(1)}(\boldsymbol{r}) = 0. \tag{50}$$

2.3.2. One rho exchange

The one-rho-exchange contribution to the local nonrelativistic transition potential in momentum space is

$$V_\rho^{(0)}(\boldsymbol{q}) = \left(1 + \frac{M_0}{M_q}\right)^{-1} G_{\mathrm{F}} \mu_\pi^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\ \times \left[K_\rho^1 - K_\rho^2 \boldsymbol{q}^2 - K_\rho^3 (\boldsymbol{\sigma}_1 \times \boldsymbol{q}) \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{q}) \right. \\ \left. - i K_\rho^4 (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \cdot \boldsymbol{q} + K_\rho^5 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}) \right] \frac{1}{\boldsymbol{q}^2 + \tilde{\mu}_\rho^2}, \tag{51}$$

where, for notational convenience, we have introduced the coefficients

$$K_\rho^1 = B_\rho^V g_{NN\rho}^V, \\ K_\rho^2 = B_\rho^V g_{NN\rho}^V \left[\left(\frac{1}{4M}\right)^2 + \left(\frac{1}{4\check{M}}\right)^2 + \frac{1}{2}\left(\frac{1}{M_q}\right)^2 \right] \\ + \left(\frac{B_\rho^V}{2M} \frac{g_{NN\rho}^T}{2M} + \frac{B_\rho^T}{2\bar{M}} \frac{g_{NN\rho}^V}{2\check{M}} \right) \left(1 + \frac{M_0}{M_q}\right) + \frac{B_\rho^T}{2\bar{M}} \frac{g_{NN\rho}^T}{2M} \left(\frac{M_0^2}{4M\check{M}} \right), \\ K_\rho^3 = \left[\frac{B_\rho^V}{2\check{M}} + \frac{B_\rho^T}{2\bar{M}} \left(1 + \frac{M_0}{M_q}\right) \right] \left[\frac{g_{NN\rho}^V}{2M} + \frac{g_{NN\rho}^T}{2M} \left(1 + \frac{M_0}{M_q}\right) \right], \\ K_\rho^4 = A_\rho \left[\frac{g_{NN\rho}^V}{2M} + \frac{g_{NN\rho}^T}{2M} \left(1 + \frac{M_0}{M_q}\right) \right], \\ K_\rho^5 = A_\rho \frac{g_{NN\rho}^T}{2M} \left(\frac{M_0}{2M} \right). \tag{52}$$

The corresponding potential under approximation (31), $\bar{V}_\rho^{(0)}(\boldsymbol{q})$, can be obtained from Eq. (51) through the substitutions:[5]

$$V_\rho^{(0)} \to \bar{V}_\rho^{(0)}, \qquad K_\rho^j \to \bar{K}_\rho^j \quad (j = 1\text{–}5), \qquad M_0/M_q \to 0, \qquad \tilde{\mu}_\rho \to \mu_\rho, \tag{53}$$

[5] This agrees with Eq. (34) of Ref. [18], except for a wrong sign ($A_\rho \to -A_\rho$) and an omitted term ($\propto \boldsymbol{q}^2$).

with

$$\bar{K}^1_\rho = B^V_\rho g^V_{NN\rho},$$
$$\bar{K}^2_\rho = B^V_\rho g^V_{NN\rho}\left[\left(\frac{1}{4M}\right)^2 + \left(\frac{1}{4\bar{M}}\right)^2\right] + \frac{B^V_\rho}{2M}\frac{g^T_{NN\rho}}{2M} + \frac{B^T_\rho}{2\bar{M}}\frac{g^V_{NN\rho}}{2\bar{M}},$$
$$\bar{K}^3_\rho = \left(\frac{B^V_\rho + B^T_\rho}{2\bar{M}}\right)\left(\frac{g^V_{NN\rho} + g^T_{NN\rho}}{2M}\right),$$
$$\bar{K}^4_\rho = A_\rho\left(\frac{g^V_{NN\rho} + g^T_{NN\rho}}{2M}\right),$$
$$\bar{K}^5_\rho = 0. \tag{54}$$

Let us now compare Eqs. (52) with the corresponding expressions under approximation (31), namely, Eqs. (54). Firstly, we notice that the two correction factors in Eq. (41) as well as

$$\tilde{\mu}_\rho/\mu_\rho = 0.992$$

are very close to unity. Secondly, we also notice that the relative values of the remaining correction terms can be estimated from

$$\frac{1}{2}\left(\frac{1}{M_q}\right)^2\left[\left(\frac{1}{4M}\right)^2 + \left(\frac{1}{4\check{M}}\right)^2\right]^{-1} = 1.85 \times 10^{-3},$$
$$\frac{M_0^2}{4M\check{M}} = 2.21 \times 10^{-3}, \qquad \frac{M_0}{2M} = 4.91 \times 10^{-2}$$

and are, therefore, considerably less than unity. We, thus, expect that, as far as the local contributions are concerned, only very small corrections will result from the more accurate treatment of the kinematics in the present case.

Making use of Eq. (24), we get, for the potential (51) in coordinate space,

$$V^{(0)}_\rho(\boldsymbol{r}) = \left(1 + \frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \times \big\{K^1_\rho f_C(r,\tilde{\mu}_\rho) + 3K^2_\rho f_S(r,\tilde{\mu}_\rho) + 2K^3_\rho f_S(r,\tilde{\mu}_\rho)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - K^3_\rho f_T(r,\tilde{\mu}_\rho)S_{12}(\hat{\boldsymbol{r}}) + f_V(r,\tilde{\mu}_\rho)\big[K^4_\rho(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2) + iK^5_\rho\boldsymbol{\sigma}_1\big]\cdot\hat{\boldsymbol{r}}\big\}. \tag{55}$$

Once more, the corresponding potential under approximation (31) can be obtained from the above equation by means of the substitutions (53).

For the rho meson, the coefficient of the linear term in Eq. (21) is given by

$$\boldsymbol{V}^{(1)}_\rho(\boldsymbol{q}) = -\left(1 + \frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \times \big[K^6_\rho\boldsymbol{q} - iK^7_\rho\boldsymbol{\sigma}_1\times\boldsymbol{q} - iK^8_\rho\boldsymbol{\sigma}_2\times\boldsymbol{q} - K^9_\rho(\boldsymbol{\sigma}_1\times\boldsymbol{q})\times\boldsymbol{\sigma}_2 + K^{10}_\rho(\boldsymbol{\sigma}_2\times\boldsymbol{q})\times\boldsymbol{\sigma}_1 + K^{11}_\rho\boldsymbol{\sigma}_1 - iK^{12}_\rho\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\big]\frac{1}{\boldsymbol{q}^2+\tilde{\mu}_\rho^2}, \tag{56}$$

where

$$
\begin{aligned}
K_\rho^6 &= B_\rho^V g_{NN\rho}^V \frac{1}{16\grave{M}}\left(\frac{3}{\check{M}} - \frac{1}{M}\right) + B_\rho^V \frac{g_{NN\rho}^T}{2M}\frac{M_0}{4M\acute{M}} \\
&\quad + \frac{B_\rho^T}{2\bar{M}} g_{NN\rho}^V \left[\frac{1}{2\grave{M}}\left(1 + \frac{M_0}{M_q}\right) - \frac{M_0}{2M\check{M}}\right] + \frac{B_\rho^T}{2\bar{M}}\frac{g_{NN\rho}^T}{2M}\frac{M_0^2}{4M\grave{M}}, \\
K_\rho^7 &= B_\rho^V g_{NN\rho}^V \left[\frac{1}{8\check{M}}\left(\frac{2}{M} + \frac{1}{\acute{M}}\right) + \frac{1}{8M\acute{M}}\right] + B_\rho^V \frac{g_{NN\rho}^T}{2M}\frac{M_0}{4M\grave{M}} \\
&\quad + \frac{B_\rho^T}{2\bar{M}} g_{NN\rho}^V \left[\frac{1}{2}\left(\frac{2}{M} + \frac{1}{\acute{M}}\right)\left(1 + \frac{M_0}{M_q}\right)\right] + \frac{B_\rho^T}{2\bar{M}}\frac{g_{NN\rho}^T}{2M}\frac{M_0^2}{4M\acute{M}}, \\
K_\rho^8 &= B_\rho^V g_{NN\rho}^V \frac{1}{4M}\left(\frac{1}{M} + \frac{1}{\acute{M}}\right) + B_\rho^V \frac{g_{NN\rho}^T}{2M}\left[\frac{1}{2}\left(\frac{2}{M} + \frac{1}{\acute{M}}\right)\left(1 + \frac{M_0}{M_q}\right)\right] \\
&\quad + \frac{B_\rho^T}{2\bar{M}} g_{NN\rho}^V \frac{M_0}{4M\grave{M}} + \frac{B_\rho^T}{2\bar{M}}\frac{g_{NN\rho}^T}{2M}\frac{M_0}{2}\left[\frac{1}{\grave{M}}\left(1 + \frac{M_0}{M_q}\right) + \frac{M_0}{M\check{M}}\right], \\
K_\rho^9 &= B_\rho^V \frac{g_{NN\rho}^T}{2M}\frac{M_0}{2M\check{M}} + \frac{B_\rho^T}{2\bar{M}}\frac{g_{NN\rho}^T}{2M}\frac{M_0}{M}\left(1 + \frac{M_0}{M_q}\right), \\
K_\rho^{10} &= B_\rho^V g_{NN\rho}^V \frac{1}{4M\grave{M}} + B_\rho^V \frac{g_{NN\rho}^T}{2M}\frac{1}{2\grave{M}}\left(1 + \frac{M_0}{M_q}\right) \\
&\quad + \frac{B_\rho^T}{2\bar{M}} g_{NN\rho}^V \frac{M_0}{4M\acute{M}} + \frac{B_\rho^T}{2\bar{M}}\frac{g_{NN\rho}^T}{2M}\frac{M_0}{2\acute{M}}\left(1 + \frac{M_0}{M_q}\right), \\
K_\rho^{11} &= A_\rho g_{NN\rho}^V \left[\frac{1}{2}\left(\frac{2}{M} + \frac{1}{\acute{M}}\right)\right], \\
K_\rho^{12} &= A_\rho \frac{g_{NN\rho}^T}{2M}\frac{M_0}{M},
\end{aligned}
\tag{57}
$$

with

$$
\frac{1}{\acute{M}} = \frac{1}{M} + \frac{1}{M_\Lambda}. \tag{58}
$$

To get the first-order nonlocality correction $\hat{V}_\rho^{(1)}(\boldsymbol{r})$, we need first to change (56) to the coordinate representation, according to Eq. (25). This gives

$$
\begin{aligned}
\boldsymbol{V}_\rho^{(1)}(\boldsymbol{r}) = -\left(1 + \frac{M_0}{M_q}\right)^{-1} & G_{\mathrm{F}}\mu_\pi^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
\times \Bigg\{ & \frac{f_V(r, \tilde{\mu}_\rho)}{r}\big[iK_\rho^6 \boldsymbol{r} + K_\rho^7 \boldsymbol{\sigma}_1 \times \boldsymbol{r} + K_\rho^8 \boldsymbol{\sigma}_2 \times \boldsymbol{r} \\
& \qquad - iK_\rho^9 (\boldsymbol{\sigma}_1 \times \boldsymbol{r}) \times \boldsymbol{\sigma}_2 + iK_\rho^{10}(\boldsymbol{\sigma}_2 \times \boldsymbol{r}) \times \boldsymbol{\sigma}_1\big] \\
& + f_C(r, \tilde{\mu}_\rho)\big[K_\rho^{11}\boldsymbol{\sigma}_1 - iK_\rho^{12}\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2\big]\Bigg\}.
\end{aligned}
\tag{59}
$$

Introducing (59) into Eq. (28) and noticing that $-i\boldsymbol{\sigma}\times\boldsymbol{r}\cdot\nabla=\boldsymbol{\sigma}\cdot\boldsymbol{l}$, where $\boldsymbol{l}=-i\boldsymbol{r}\times\nabla$ is the relative orbital angular momentum, we obtain, finally,

$$\begin{aligned}\hat{V}_{\rho}^{(1)}(\boldsymbol{r}) = {} & \left(1+\frac{M_0}{M_q}\right)^{-1} G_{\mathrm{F}}\mu_{\pi}^{2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\ & \times\left\{\frac{2M_{\Lambda}}{3M_{\Lambda}+M}\Big[f_S(r,\tilde{\mu}_{\rho})\big(3K_{\rho}^{6}+2\big(K_{\rho}^{10}-K_{\rho}^{9}\big)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\big)\right. \\ & \qquad -\big(K_{\rho}^{10}-K_{\rho}^{9}\big)f_T(r,\tilde{\mu}_{\rho})S_{12}(\hat{\boldsymbol{r}}) \\ & \qquad - f_V(r,\tilde{\mu}_{\rho})\big(K_{\rho}^{12}\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2+iK_{\rho}^{11}\boldsymbol{\sigma}_1\big)\cdot\hat{\boldsymbol{r}}\Big] \\ & -\frac{f_V(r,\tilde{\mu}_{\rho})}{r}\Big[K_{\rho}^{6}\boldsymbol{r}\cdot\nabla+K_{\rho}^{7}\boldsymbol{\sigma}_1\cdot\boldsymbol{l}+K_{\rho}^{8}\boldsymbol{\sigma}_2\cdot\boldsymbol{l} \\ & \qquad +\big(K_{\rho}^{10}-K_{\rho}^{9}\big)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\boldsymbol{r}\cdot\nabla \\ & \qquad +K_{\rho}^{9}\boldsymbol{\sigma}_2\cdot\boldsymbol{r}\boldsymbol{\sigma}_1\cdot\nabla-K_{\rho}^{10}\boldsymbol{\sigma}_1\cdot\boldsymbol{r}\boldsymbol{\sigma}_2\cdot\nabla\Big] \\ & \left.+f_C(r,\tilde{\mu}_{\rho})\big(K_{\rho}^{12}\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2+iK_{\rho}^{11}\boldsymbol{\sigma}_1\big)\cdot\nabla\right\}. \end{aligned} \tag{60}$$

Under approximation (31), the only surviving coefficients for the nonlocal potential would be

$$\begin{aligned}\bar{K}_{\rho}^{7} &= B_{\rho}^{V}g_{NN\rho}^{V}\frac{1}{4\bar{M}}\left(\frac{1}{\bar{M}}+\frac{2}{M}\right)+\frac{B_{\rho}^{T}}{2\bar{M}}g_{NN\rho}^{V}\left(\frac{1}{\bar{M}}+\frac{1}{M}\right), \\ \bar{K}_{\rho}^{8} &= B_{\rho}^{V}g_{NN\rho}^{V}\frac{1}{4M}\left(\frac{1}{M}+\frac{2}{\bar{M}}\right)+B_{\rho}^{V}\frac{g_{NN\rho}^{T}}{2M}\left(\frac{1}{M}+\frac{1}{\bar{M}}\right), \\ \bar{K}_{\rho}^{11} &= A_{\rho}g_{NN\rho}^{V}\left(\frac{1}{M}+\frac{1}{\bar{M}}\right),\end{aligned} \tag{61}$$

and Eq. (60) would reduce to

$$\begin{aligned}\hat{\hat{V}}_{\rho}^{(1)}(\boldsymbol{r}) = {} & -G_{\mathrm{F}}\mu_{\pi}^{2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[\frac{f_V(r,\mu_{\rho})}{r}\big(\bar{K}_{\rho}^{7}\boldsymbol{\sigma}_1\cdot\boldsymbol{l}+\bar{K}_{\rho}^{8}\boldsymbol{\sigma}_2\cdot\boldsymbol{l}\big)\right. \\ & \left.-i\bar{K}_{\rho}^{11}f_C(r,\mu_{\rho})\boldsymbol{\sigma}_1\cdot\nabla+\frac{i}{2}\bar{K}_{\rho}^{11}f_V(r,\mu_{\rho})\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}}\right].\end{aligned} \tag{62}$$

2.3.3. Extension to η and ω exchanges

If one remembers to make the replacement $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\to 1$, the results obtained above for the π and ρ mesons can be straightforwardly extended to the η and ω, respectively, and the corresponding expressions need not be reproduced here. Let us just mention that the ratios $\tilde{\mu}_{\eta}/\mu_{\eta}=0.985$ and $\tilde{\mu}_{\omega}/\mu_{\omega}=0.992$ are very close to unity and, consequently, as happened for the ρ, the effects of the reduction of the effective mass are much less important for these mesons than they are for the pion.

2.4. Contributions of strange mesons

For the strange mesons, the weak and strong vertices in Fig. 1 are interchanged with respect to those for the nonstrange ones, i.e.,

$$\begin{aligned} &a = W, \quad b = S \quad \text{(nonstrange mesons)},\\ &a = S, \quad b = W \quad \text{(strange mesons)}. \end{aligned} \tag{63}$$

For the kaon, the effective Hamiltonian for the strong coupling is [18, Eq. (28)]

$$\mathcal{H}^{S}_{\Lambda NK} = i g_{\Lambda NK} \bar{\psi}_N \gamma_5 \phi^{(K)} \psi_\Lambda, \tag{64}$$

while, for the weak one, it is [18, Eq. (29)]

$$\begin{aligned} \mathcal{H}^{W}_{NNK} = i G_{\mathrm{F}} \mu_\pi^2 \bar{\psi}_N \biggl\{ & \biggl[C_K^{\mathrm{PV}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bigl(\phi^{(K)}\bigr)^\dagger + D_K^{\mathrm{PV}} \bigl(\phi^{(K)}\bigr)^\dagger \begin{pmatrix} 0 \\ 1 \end{pmatrix} \biggr] \\ & + \gamma_5 \biggl[C_K^{\mathrm{PC}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bigl(\phi^{(K)}\bigr)^\dagger + D_K^{\mathrm{PC}} \bigl(\phi^{(K)}\bigr)^\dagger \begin{pmatrix} 0 \\ 1 \end{pmatrix} \biggr] \biggr\} \psi_N. \end{aligned} \tag{65}$$

For the K^*, we have, for the strong coupling [18, Eq. (38)],

$$\mathcal{H}^{S}_{\Lambda NK^*} = \bar{\psi}_N \biggl[\biggl(g^V_{\Lambda NK^*} \gamma^\nu + g^T_{\Lambda NK^*} \frac{\sigma^{\mu\nu} \partial_\mu}{2\bar{M}} \biggr) \phi_\nu^{(K^*)} \biggr] \psi_\Lambda, \tag{66}$$

and for the weak one [18, Eq. (39)],

$$\begin{aligned} \mathcal{H}^{W}_{NNK^*} = G_{\mathrm{F}} \mu_\pi^2 \bar{\psi}_N \biggl\{ & \gamma^\nu \biggl[C_{K^*}^{\mathrm{PC},V} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bigl(\phi_\nu^{(K^*)}\bigr)^\dagger + D_{K^*}^{\mathrm{PC},V} \bigl(\phi_\nu^{(K^*)}\bigr)^\dagger \begin{pmatrix} 0 \\ 1 \end{pmatrix} \biggr] \\ & + \frac{\sigma^{\mu\nu}\partial_\mu}{2M} \biggl[C_{K^*}^{\mathrm{PC},T} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bigl(\phi_\nu^{(K^*)}\bigr)^\dagger + D_{K^*}^{\mathrm{PC},T} \bigl(\phi_\nu^{(K^*)}\bigr)^\dagger \begin{pmatrix} 0 \\ 1 \end{pmatrix} \biggr] \\ & + \gamma^\nu \gamma_5 \biggl[C_{K^*}^{\mathrm{PV}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bigl(\phi_\nu^{(K^*)}\bigr)^\dagger + D_{K^*}^{\mathrm{PV}} \bigl(\phi_\nu^{(K^*)}\bigr)^\dagger \begin{pmatrix} 0 \\ 1 \end{pmatrix} \biggr] \biggr\} \psi_N. \end{aligned} \tag{67}$$

We again follow the parametrization adopted in Ref. [18], and collect the numerical values in Table 2 for convenience.

These mesons are isodoublets, and in terms of their different charge states we can write

$$\begin{aligned} &\phi^{(K)} \equiv \begin{pmatrix} \phi^{(K^+)} \\ \phi^{(K^0)} \end{pmatrix} = \bigl[\phi^{(K^+)} \tau_+ + \phi^{(K^0)}\bigr] \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ &\bigl(\phi^{(K)}\bigr)^\dagger \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \bigl(\phi^{(K^0)}\bigr)^\dagger, \\ &\begin{pmatrix} 0 \\ 1 \end{pmatrix} \bigl(\phi^{(K)}\bigr)^\dagger = \bigl(\phi^{(K^+)}\bigr)^\dagger \tau_- + \frac{1}{2} \bigl(\phi^{(K^0)}\bigr)^\dagger (1 - \tau_0) \end{aligned} \tag{68}$$

for the kaon, and similar equations for the K^*. As a result, when applying the Feynman rules to compute the transition potential as explained in Subsection 2.1, the isospurion will permit the introduction of isospin operators of the form

$$I = \frac{1}{2} C (1 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + D \tag{69}$$

Table 2
Coupling constants, masses (μ_i) and cutoff parameters (Λ_i) for the strange mesons. The weak couplings are in units of $G_F\mu_\pi^2$. Adapted from Ref. [18]

Meson i	Coupling constants: Weak PV	Weak PC	Strong	μ_i [MeV]	Λ_i [GeV]
K	$C_K^{\rm PV}=0.76$	$C_K^{\rm PC}=-18.9$	$g_{\Lambda NK}=-14.1$	495.8	1.20
	$D_K^{\rm PV}=2.09$	$D_K^{\rm PC}=6.63$			
K^*	$C_{K^*}^{\rm PV}=-4.48$	$C_{K^*}^{{\rm PC},V}=-3.61$	$g_{\Lambda NK^*}^V=-5.47$	892.4	2.20
		$C_{K^*}^{{\rm PC},T}=-17.9$	$g_{\Lambda NK^*}^T=-11.9$		
	$D_{K^*}^{\rm PV}=0.60$	$D_{K^*}^{{\rm PC},V}=-4.89$			
		$D_{K^*}^{{\rm PC},T}=9.30$			

for each of the different couplings in Eqs. (65) and (67). Explicitly, they are

$$I_K^{\rm PV}=\frac{1}{2}C_K^{\rm PV}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+D_K^{\rm PV},$$
$$I_K^{\rm PC}=\frac{1}{2}C_K^{\rm PC}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+D_K^{\rm PC}, \tag{70}$$

for the kaon, and

$$I_{K^*}^{{\rm PC},V}=\frac{1}{2}C_{K^*}^{{\rm PC},V}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+D_{K^*}^{{\rm PC},V},$$
$$I_{K^*}^{{\rm PC},T}=\frac{1}{2}C_{K^*}^{{\rm PC},T}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+D_{K^*}^{{\rm PC},T},$$
$$I_{K^*}^{\rm PV}=\frac{1}{2}C_{K^*}^{\rm PV}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+D_{K^*}^{\rm PV}, \tag{71}$$

for the K^*. It then becomes apparent that each such coupling will give a contribution proportional to $\frac{1}{2}C+D$ to the isoscalar potential and a similar one proportional to $\frac{1}{2}C$ to the isovector potential.

2.4.1. One K exchange

The contribution to the local nonrelativistic transition potential in momentum space due to this meson is

$$V_K^{(0)}(\boldsymbol{q})=\left(1+\frac{M_0}{M_q}\right)^{-1}G_F\mu_\pi^2\frac{g_{\Lambda NK}}{2\check{M}}\left(I_K^{\rm PV}-\frac{I_K^{\rm PC}}{2M}\boldsymbol{\sigma}_2\cdot\boldsymbol{q}\right)\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}}{\boldsymbol{q}^2+\tilde{\mu}_K^2}. \tag{72}$$

Comparing this with the result that would have been obtained under approximation (31), namely,[6]

$$\bar{V}_K^{(0)}(\boldsymbol{q})=G_F\mu_\pi^2\frac{g_{\Lambda NK}}{2\bar{M}}\left(I_K^{\rm PV}-\frac{I_K^{\rm PC}}{2M}\boldsymbol{\sigma}_2\cdot\boldsymbol{q}\right)\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}}{\boldsymbol{q}^2+\mu_K^2}, \tag{73}$$

[6] This differs from Eqs. (24) and (31) of Ref. [18] in the sign of $I_K^{\rm PV}$ and the interchange of $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$.

and noticing that the two correction factors in Eq. (41) as well as

$$\tilde{\mu}_K/\mu_K = 0.982$$

are very close to unity, one can see that, for the kaon, only very small corrections will result in the local contributions from the more accurate treatment of the kinematics. The expression for the potential (72) in coordinate space is

$$V_K^{(0)}(\boldsymbol{r}) = \left(1+\frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \frac{g_{\Lambda NK}}{2\check{M}} \times \left[i I_K^{PV} f_V(r,\tilde{\mu}_K)\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}} + \frac{I_K^{PC}}{2M} f_S(r,\tilde{\mu}_K)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \frac{I_K^{PC}}{2M} f_T(r,\tilde{\mu}_K) S_{12}(\hat{\boldsymbol{r}})\right], \tag{74}$$

and, under approximation (31), this becomes

$$\bar{V}_K^{(0)}(\boldsymbol{r}) = G_F\mu_\pi^2 \frac{g_{\Lambda NK}}{2\bar{M}} \times \left[i I_K^{PV} f_V(r,\mu_K)\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}} + \frac{I_K^{PC}}{2M} f_S(r,\mu_K)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \frac{I_K^{PC}}{2M} f_T(r,\mu_K) S_{12}(\hat{\boldsymbol{r}})\right]. \tag{75}$$

Starting with the first-order nonlocality coefficient in momentum space in Eq. (21), we have, for the kaon,

$$\boldsymbol{V}_K^{(1)}(\boldsymbol{q}) = \left(1+\frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \frac{g_{\Lambda NK}}{2\grave{M}} \left(I_K^{PV} - \frac{I_K^{PC}}{2M}\boldsymbol{\sigma}_2\cdot\boldsymbol{q}\right)\frac{\boldsymbol{\sigma}_1}{\boldsymbol{q}^2+\tilde{\mu}_K^2}, \tag{76}$$

which, in the coordinate representation, becomes

$$\boldsymbol{V}_K^{(1)}(\boldsymbol{r}) = \left(1+\frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \frac{g_{\Lambda NK}}{2\grave{M}} \times \left[I_K^{PV} f_C(r,\tilde{\mu}_K)\boldsymbol{\sigma}_1 - i\frac{I_K^{PC}}{2M} f_V(r,\tilde{\mu}_K)(\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}})\boldsymbol{\sigma}_1\right]. \tag{77}$$

Introducing this into Eq. (28) yields, for the first-order nonlocal potential,

$$\hat{V}_K^{(1)}(\boldsymbol{r}) = \left(1+\frac{M_0}{M_q}\right)^{-1} G_F\mu_\pi^2 \frac{g_{\Lambda NK}}{2\grave{M}} \times \left\{ \frac{2M_\Lambda}{3M_\Lambda+M}\left[\frac{I_K^{PC}}{2M} f_S(r,\tilde{\mu}_K)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \frac{I_K^{PC}}{2M} f_T(r,\tilde{\mu}_K) S_{12}(\hat{\boldsymbol{r}}) + i I_K^{PV} f_V(r,\tilde{\mu}_K)(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}})\right] - i I_K^{PV} f_C(r,\tilde{\mu}_K)(\boldsymbol{\sigma}_1\cdot\nabla) - \frac{I_K^{PC}}{2M} f_V(r,\tilde{\mu}_K)(\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}})(\boldsymbol{\sigma}_1\cdot\nabla)\right\}. \tag{78}$$

As already stated, the mass averaging approximation (31) would set $1/\grave{M}$, defined in Eq. (47), to zero. Therefore, there would be no first-order nonlocality correction for the kaon under this approximation, i.e.,

$$\hat{\bar{V}}^{(1)}_{K}(\boldsymbol{r}) = 0. \tag{79}$$

2.4.2. One K^* exchange

For one-K^*-exchange, the local nonrelativistic transition potential in momentum space is

$$\begin{aligned} V^{(0)}_{K^*}(\boldsymbol{q}) = {} & \left(1 + \frac{M_0}{M_q}\right)^{-1} G_{\mathrm{F}}\mu_\pi^2 \\ & \times \big[\hat{K}^1_{K^*} - \hat{K}^2_{K^*}\boldsymbol{q}^2 - \hat{K}^3_{K^*}(\boldsymbol{\sigma}_1 \times \boldsymbol{q})\cdot(\boldsymbol{\sigma}_2 \times \boldsymbol{q}) \\ & \quad - i\hat{K}^4_{K^*}(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot\boldsymbol{q} + \hat{K}^5_{K^*}(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\big]\frac{1}{\boldsymbol{q}^2 + \tilde{\mu}_{K^*}}, \end{aligned} \tag{80}$$

where we have introduced the isospin operators

$$\begin{aligned} \hat{K}^1_{K^*} = {} & g^V_{\Lambda N K^*} I^{\mathrm{PC},V}_{K^*}, \\ \hat{K}^2_{K^*} = {} & g^V_{\Lambda N K^*} I^{\mathrm{PC},V}_{K^*}\left[\left(\frac{1}{4M}\right)^2 + \left(\frac{1}{4\check{M}}\right)^2 + \frac{1}{2}\left(\frac{1}{M_q}\right)^2\right] \\ & + \left(\frac{g^T_{\Lambda N K^*}}{2\bar{M}}\frac{I^{\mathrm{PC},V}_{K^*}}{2\check{M}} + \frac{g^V_{\Lambda N K^*}}{2M}\frac{I^{\mathrm{PC},T}_{K^*}}{2M}\right)\left(1 + \frac{M_0}{M_q}\right) \\ & + \frac{g^T_{\Lambda N K^*}}{2\bar{M}}\frac{I^{\mathrm{PC},T}_{K^*}}{2M}\left(\frac{M_0^2}{4M\check{M}}\right), \\ \hat{K}^3_{K^*} = {} & \left[\frac{I^{\mathrm{PC},V}_{K^*}}{2M} + \frac{I^{\mathrm{PC},T}_{K^*}}{2M}\left(1 + \frac{M_0}{M_q}\right)\right]\left[\frac{g^V_{\Lambda N K^*}}{2\check{M}} + \frac{g^T_{\Lambda N K^*}}{2\bar{M}}\left(1 + \frac{M_0}{M_q}\right)\right], \\ \hat{K}^4_{K^*} = {} & I^{\mathrm{PV}}_{K^*}\left[\frac{g^V_{\Lambda N K^*}}{2\check{M}} + \frac{g^T_{\Lambda N K^*}}{2\bar{M}}\left(1 + \frac{M_0}{M_q}\right)\right], \\ \hat{K}^5_{K^*} = {} & I^{\mathrm{PV}}_{K^*}\frac{g^T_{\Lambda N K^*}}{2\bar{M}}\left(\frac{M_0}{2\check{M}}\right). \end{aligned} \tag{81}$$

The corresponding potential under approximation (31), $\bar{V}^{(0)}_{K^*}(\boldsymbol{q})$, can be obtained from Eq. (80) through the substitutions:

$$\begin{aligned} & V^{(0)}_{K^*} \to \bar{V}^{(0)}_{K^*}, \qquad \hat{K}^j_{K^*} \to \hat{\bar{K}}^j_{K^*} \quad (j = 1\text{–}5), \\ & M_0/M_q \to 0, \qquad \tilde{\mu}_{K^*} \to \mu_{K^*}, \end{aligned} \tag{82}$$

with

$$\begin{aligned} \hat{\bar{K}}^1_{K^*} = {} & g^V_{\Lambda N K^*} I^{\mathrm{PC},V}_{K^*}, \\ \hat{\bar{K}}^2_{K^*} = {} & g^V_{\Lambda N K^*} I^{\mathrm{PC},V}_{K^*}\left[\left(\frac{1}{4M}\right)^2 + \left(\frac{1}{4\bar{M}}\right)^2\right] + \frac{g^T_{\Lambda N K^*}}{2\bar{M}}\frac{I^{\mathrm{PC},V}_{K^*}}{2\bar{M}} + \frac{g^V_{\Lambda N K^*}}{2M}\frac{I^{\mathrm{PC},T}_{K^*}}{2M}, \end{aligned}$$

$$\hat{\hat{K}}^3_{K^*} = \left(\frac{I^{\mathrm{PC},V}_{K^*} + I^{\mathrm{PC},T}_{K^*}}{2M}\right)\left(\frac{g^V_{\Lambda N K^*} + g^T_{\Lambda N K^*}}{2\bar{M}}\right),$$
$$\hat{\hat{K}}^4_{K^*} = I^{\mathrm{PV}}_{K^*}\left(\frac{g^V_{\Lambda N K^*} + g^T_{\Lambda N K^*}}{2\bar{M}}\right),$$
$$\hat{\hat{K}}^5_{K^*} = 0. \tag{83}$$

By an analysis very similar to the one performed for the ρ meson and noticing that

$$\tilde{\mu}_{K^*}/\mu_{K^*} = 0.994$$

is also very close to unity, one concludes that, again in the present case, only very small corrections will result in the local contributions from the more accurate treatment of the kinematics.

For completeness, we give below the expression for the potential (80) in coordinate space:

$$V^{(0)}_{K^*}(\boldsymbol{r}) = \left(1 + \frac{M_0}{M_q}\right)^{-1} G_{\mathrm{F}}\mu_\pi^2 \big\{\hat{K}^1_{K^*} f_C(r, \tilde{\mu}_{K^*}) + 3\hat{K}^2_{K^*} f_S(r, \tilde{\mu}_{K^*})$$
$$+ 2\hat{K}^3_{K^*} f_S(r, \tilde{\mu}_{K^*})\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \hat{K}^3_{K^*} f_T(r, \tilde{\mu}_{K^*}) S_{12}(\hat{\boldsymbol{r}})$$
$$+ f_V(r, \tilde{\mu}_{K^*})\big[\hat{K}^4_{K^*}(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) + i\hat{K}^5_{K^*}\boldsymbol{\sigma}_2\big] \cdot \hat{\boldsymbol{r}}\big\}. \tag{84}$$

Once more, the corresponding potential under approximation (31) can be obtained from Eq. (84) by means of the substitutions (82).

The first-order nonlocality coefficient in momentum space, appearing in Eq. (21), for this meson, is

$$\boldsymbol{V}^{(1)}_{K^*}(\boldsymbol{q}) = -\left(1 + \frac{M_0}{M_q}\right)^{-1} G_{\mathrm{F}}\mu_\pi^2$$
$$\times \big[\hat{K}^6_{K^*}\boldsymbol{q} - i\hat{K}^7_{K^*}\boldsymbol{\sigma}_1 \times \boldsymbol{q} - i\hat{K}^8_{K^*}\boldsymbol{\sigma}_2 \times \boldsymbol{q}$$
$$- \hat{K}^9_{K^*}(\boldsymbol{\sigma}_1 \times \boldsymbol{q}) \times \boldsymbol{\sigma}_2 + \hat{K}^{10}_{K^*}(\boldsymbol{\sigma}_2 \times \boldsymbol{q}) \times \boldsymbol{\sigma}_1$$
$$- \hat{K}^{11}_{K^*}\boldsymbol{\sigma}_2 + i\hat{K}^{12}_{K^*}\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2\big]\frac{1}{\boldsymbol{q}^2 + \tilde{\mu}^2_{K^*}}, \tag{85}$$

where

$$\hat{K}^6_{K^*} = g^V_{\Lambda N K^*} I^{\mathrm{PC},V}_{K^*}\frac{1}{16\grave{M}}\left(\frac{3}{\check{M}} - \frac{1}{M}\right) + g^V_{\Lambda N K^*}\frac{I^{\mathrm{PC},T}_{K^*}}{2M}\frac{M_0}{4M\acute{M}}$$
$$+ \frac{g^T_{\Lambda N K^*}}{2\bar{M}} I^{\mathrm{PC},V}_{K^*}\left[\frac{1}{2\grave{M}}\left(1 + \frac{M_0}{M_q}\right) - \frac{M_0}{2M\check{M}}\right] + \frac{g^T_{\Lambda N K^*}}{2\bar{M}}\frac{I^{\mathrm{PC},T}_{K^*}}{2M}\frac{M_0^2}{4M\grave{M}},$$
$$\hat{K}^7_{K^*} = g^V_{\Lambda N K^*} I^{\mathrm{PC},V}_{K^*}\left[\frac{1}{8\check{M}}\left(\frac{2}{M} + \frac{1}{\acute{M}}\right) + \frac{1}{8M\acute{M}}\right] + g^V_{\Lambda N K^*}\frac{I^{\mathrm{PC},T}_{K^*}}{2M}\frac{M_0}{4M\grave{M}}$$
$$+ \frac{g^T_{\Lambda N K^*}}{2\bar{M}} I^{\mathrm{PC},V}_{K^*}\frac{1}{2}\left(\frac{2}{M} + \frac{1}{\acute{M}}\right)\left(1 + \frac{M_0}{M_q}\right) + \frac{g^T_{\Lambda N K^*}}{2\bar{M}}\frac{I^{\mathrm{PC},T}_{K^*}}{2M}\frac{M_0^2}{4M\acute{M}},$$

$$\hat{K}^{8}_{K^*} = g^{V}_{\Lambda NK^*} I^{\mathrm{PC},V}_{K^*} \frac{1}{4M}\left(\frac{1}{M} + \frac{1}{\acute{M}}\right) + g^{V}_{\Lambda NK^*} \frac{I^{\mathrm{PC},T}_{K^*}}{2M} \frac{1}{2}\left(\frac{2}{M} + \frac{1}{\acute{M}}\right)\left(1 + \frac{M_0}{M_q}\right)$$
$$+ \frac{g^{T}_{\Lambda NK^*}}{2\bar{M}} I^{\mathrm{PC},V}_{K^*} \frac{M_0}{4M\grave{M}} + \frac{g^{T}_{\Lambda NK^*}}{2\bar{M}} \frac{I^{\mathrm{PC},T}_{K^*}}{2M} \frac{M_0}{2}\left[\frac{1}{\grave{M}}\left(1 + \frac{M_0}{M_q}\right) + \frac{M_0}{M\check{M}}\right],$$
$$\hat{K}^{9}_{K^*} = g^{V}_{\Lambda NK^*} \frac{I^{\mathrm{PC},T}_{K^*}}{2M} \frac{M_0}{2M\check{M}} + \frac{g^{T}_{\Lambda NK^*}}{2\bar{M}} \frac{I^{\mathrm{PC},T}_{K^*}}{2M} \frac{M_0}{M}\left(1 + \frac{M_0}{M_q}\right),$$
$$\hat{K}^{10}_{K^*} = g^{V}_{\Lambda NK^*} I^{\mathrm{PC},V}_{K^*} \frac{1}{4M\grave{M}} + g^{V}_{\Lambda NK^*} \frac{I^{\mathrm{PC},T}_{K^*}}{2M} \frac{1}{2\grave{M}}\left(1 + \frac{M_0}{M_q}\right)$$
$$+ \frac{g^{T}_{\Lambda NK^*}}{2\bar{M}} I^{\mathrm{PC},V}_{K^*} \frac{M_0}{4M\acute{M}} + \frac{g^{T}_{\Lambda NK^*}}{2\bar{M}} \frac{I^{\mathrm{PC},T}_{K^*}}{2M} \frac{M_0}{2\acute{M}}\left(1 + \frac{M_0}{M_q}\right),$$
$$\hat{K}^{11}_{K^*} = g^{V}_{\Lambda NK^*} I^{\mathrm{PV}}_{K^*} \frac{1}{2}\left(\frac{2}{M} + \frac{1}{\acute{M}}\right) + \frac{g^{T}_{\Lambda NK^*}}{2\bar{M}} I^{\mathrm{PV}}_{K^*} \frac{M_0}{2\grave{M}},$$
$$\hat{K}^{12}_{K^*} = g^{V}_{\Lambda NK^*} I^{\mathrm{PV}}_{K^*} \frac{1}{2\grave{M}} + \frac{g^{T}_{\Lambda NK^*}}{2\bar{M}} I^{\mathrm{PV}}_{K^*} \frac{M_0}{2\acute{M}}, \tag{86}$$

with $1/\grave{M}$ and $1/\acute{M}$ as defined in Eqs. (47) and (58). To get the first-order nonlocality correction $\hat{V}^{(1)}_{K^*}(\boldsymbol{r})$, we need first to change (85) to the coordinate representation. This gives

$$\boldsymbol{V}^{(1)}_{K^*}(\boldsymbol{r}) = -\left(1 + \frac{M_0}{M_q}\right)^{-1} G_{\mathrm{F}}\mu_\pi^2$$
$$\times \left\{ \frac{f_V(r, \tilde{\mu}_{K^*})}{r}\left[i\hat{K}^{6}_{K^*}\boldsymbol{r} + \hat{K}^{7}_{K^*}\boldsymbol{\sigma}_1 \times \boldsymbol{r} + \hat{K}^{8}_{K^*}\boldsymbol{\sigma}_2 \times \boldsymbol{r}\right.\right.$$
$$\left. - i\hat{K}^{9}_{K^*}(\boldsymbol{\sigma}_1 \times \boldsymbol{r}) \times \boldsymbol{\sigma}_2 + i\hat{K}^{10}_{K^*}(\boldsymbol{\sigma}_2 \times \boldsymbol{r}) \times \boldsymbol{\sigma}_1\right]$$
$$\left. - f_C(r, \tilde{\mu}_{K^*})\left[\hat{K}^{11}_{K^*}\boldsymbol{\sigma}_2 - i\hat{K}^{12}_{K^*}\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2\right]\right\}. \tag{87}$$

Introducing (87) into Eq. (28), we obtain, finally,

$$\hat{V}^{(1)}_{K^*}(\boldsymbol{r}) = \left(1 + \frac{M_0}{M_q}\right)^{-1} G_{\mathrm{F}}\mu_\pi^2$$
$$\times \left\{ \frac{2M_\Lambda}{3M_\Lambda + M}\left[f_S(r, \tilde{\mu}_{K^*})\left(3\hat{K}^{6}_{K^*} + 2\left(\hat{K}^{10}_{K^*} - \hat{K}^{9}_{K^*}\right)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\right)\right.\right.$$
$$- \left(\hat{K}^{10}_{K^*} - \hat{K}^{9}_{K^*}\right) f_T(r, \tilde{\mu}_{K^*}) S_{12}(\hat{\boldsymbol{r}})$$
$$\left. + f_V(r, \tilde{\mu}_{K^*})\left(\hat{K}^{12}_{K^*}\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2 + i\hat{K}^{11}_{K^*}\boldsymbol{\sigma}_2\right) \cdot \hat{\boldsymbol{r}}\right]$$
$$- \frac{f_V(r, \tilde{\mu}_{K^*})}{r}\left[\hat{K}^{6}_{K^*}\boldsymbol{r} \cdot \nabla + \hat{K}^{7}_{K^*}\boldsymbol{\sigma}_1 \cdot \boldsymbol{l} + \hat{K}^{8}_{K^*}\boldsymbol{\sigma}_2 \cdot \boldsymbol{l}\right.$$
$$+ \left(\hat{K}^{10}_{K^*} - \hat{K}^{9}_{K^*}\right)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{r} \cdot \nabla$$
$$\left. + \hat{K}^{9}_{K^*}\boldsymbol{\sigma}_2 \cdot \boldsymbol{r}\boldsymbol{\sigma}_1 \cdot \nabla - \hat{K}^{10}_{K^*}\boldsymbol{\sigma}_1 \cdot \boldsymbol{r}\boldsymbol{\sigma}_2 \cdot \nabla\right]$$
$$\left. - f_C(r, \tilde{\mu}_{K^*})\left(\hat{K}^{12}_{K^*}\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2 + i\hat{K}^{11}_{K^*}\boldsymbol{\sigma}_2\right) \cdot \nabla\right\}. \tag{88}$$

If one assumed that the averaged-mass approximation (31) could be made, several terms in the nonlocal potential would disappear. The only remaining coefficients would be

$$\begin{aligned}
\hat{\bar{K}}^{7}_{K^*} &= g^{V}_{\Lambda NK^*} I^{\mathrm{PC},V}_{K^*} \frac{1}{4\bar{M}}\left(\frac{1}{\bar{M}}+\frac{2}{M}\right)+\frac{g^{T}_{\Lambda NK^*}}{2\bar{M}} I^{\mathrm{PC},V}_{K^*}\left(\frac{1}{M}+\frac{1}{\bar{M}}\right),\\
\hat{\bar{K}}^{8}_{K^*} &= g^{V}_{\Lambda NK^*} I^{\mathrm{PC},V}_{K^*} \frac{1}{4M}\left(\frac{1}{M}+\frac{2}{\bar{M}}\right)+g^{V}_{\Lambda NK^*}\frac{I^{\mathrm{PC},T}_{K^*}}{2M}\left(\frac{1}{\bar{M}}+\frac{1}{M}\right),\\
\hat{\bar{K}}^{11}_{K^*} &= g^{V}_{\Lambda NK^*} I^{\mathrm{PV}}_{K^*}\left(\frac{1}{M}+\frac{1}{\bar{M}}\right),
\end{aligned} \tag{89}$$

and the first order nonlocality correction would reduce to

$$\begin{aligned}
\hat{V}^{(1)}_{K^*}(\boldsymbol{r}) = -G_{\mathrm{F}}\mu_\pi^2\biggl[\ &\frac{f_V(r,\mu_{K^*})}{r}\bigl(\hat{\bar{K}}^{7}_{K^*}\boldsymbol{\sigma}_1\cdot\boldsymbol{l}+\hat{\bar{K}}^{8}_{K^*}\boldsymbol{\sigma}_2\cdot\boldsymbol{l}\bigr)\\
&+i\hat{\bar{K}}^{11}_{K^*} f_C(r,\mu_{K^*})\boldsymbol{\sigma}_2\cdot\nabla-\frac{i}{2}\hat{\bar{K}}^{11}_{K^*} f_V(r,\mu_{K^*})\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}}\biggr].
\end{aligned} \tag{90}$$

It is interesting to point out that, whereas the first-order nonlocal terms are systematically omitted in the literature on nonmesonic decay, they are routinely included in the closely related domain of strangeness-conserving, parity-violating nuclear forces. (See, for instance, Eq. (115) in Ref. [45].) We note, however, that the terms proportional to $\bar{K}^{11}$ and $\hat{\bar{K}}^{11}$, for the vector mesons, have been recently discussed in the literature [21].

2.5. Finite size effects

Before closing this section, let us mention a refinement that should always be added to the strict OME description we have been developing up to now, especially when large momentum transfers are involved, as is the case for nonmesonic hypernuclear decays. This is the effect of the finite size (FS) of the interacting baryons and mesons at each vertex.

Taking a clue from the OME models for the NN force [38,39], the FS effects are phenomenologically implemented in momentum space by inserting, at each vertex in Fig. 1, a form factor, which we choose to be of the monopole type,

$$\frac{\Lambda_i^2-\tilde{\mu}_i^2}{\boldsymbol{q}^2+\Lambda_i^2}, \tag{91}$$

where i refers to the meson involved and Λ_i are the cutoff parameters in Tables 1 and 2. This corresponds in coordinate space to replacing, in the expressions for the transition potential discussed in Subsections 2.3 and 2.4, each of the shape functions (43) as follows:

$$f_N(r,\tilde{\mu}_i)\to f_N(r,\tilde{\mu}_i)-f_N(r,\Lambda_i)+\frac{\Lambda_i^2-\tilde{\mu}_i^2}{2\Lambda_i}\frac{\partial}{\partial\Lambda_i}f_N(r,\Lambda_i), \tag{92}$$

where $N=C,V,S,T$. When the kinematical effects are ignored, Eqs. (91) and (92) should be modified by making $\tilde{\mu}_i\to\mu_i$, thus leading to agreement with Ref. [18].

In what follows, it is to be understood that these FS effects are always included.

3. Numerical results and discussion

3.1. Decay rates

We present here the numerical results for the different contributions to the nonmesonic weak decay rates of ${}^{12}_{\Lambda}$C. We consider, separately, the neutron-induced (n) and the proton-induced (p) contributions, as well as those coming from the parity-conserving (PC) and parity-violating (PV) transitions. All quantities are in units of the free Λ decay constant, $\Gamma_0 = 2.50 \times 10^{-6}$ eV. The main observables are the total nonmesonic decay constant $\Gamma_{\rm nm} = \Gamma_n + \Gamma_p$ and the ratio Γ_n/Γ_p, whose experimental estimates are in the ranges 0.89–1.14 and 0.52–1.87, respectively, with large error bars [46–50]. Most, if not all, calculations in the context of OME models give reasonable results for $\Gamma_{\rm nm}$ but fail completely for Γ_n/Γ_p. However, our main objective here is not so much to try to reproduce the experimental values for these observables, but rather to assess the relative importance of the kinematical and nonlocality effects, usually ignored, in their theoretical prediction. For simplicity, we restrict the discussion of the nonlocality corrections to those of first order.

For the explicit evaluation of the transition rates, we follow the approach of Ref. [21]. The initial and final nuclear states in Eq. (1) are described in the extreme particle–hole model (EPHM), taking as vacuum the simplest possible shell-model approximation for the ground state of ^{12}C, namely, 1s$_{1/2}$ and 1p$_{3/2}$ orbitals completely filled with neutrons and protons.[7] The Λ single-particle state has quantum numbers $j_1 = 1\mathrm{s}_{1/2}$ and the nucleon inducing the transition occupies a $j_2 = 1\mathrm{s}_{1/2}$ or $1\mathrm{p}_{3/2}$ orbital. Therefore, [21, Eqs. (4.2), (4.3)]

$$|I\rangle = \big|(j_1\Lambda)(jn)^{-1}; J_I\big\rangle, \qquad |F\rangle = \big|(jn)^{-1}(j_2N)^{-1}; J_F\big\rangle, \tag{93}$$

where $J_I = 1$, $j = 1\mathrm{p}_{3/2}$, $N = p$ or n for proton- or neutron-induced transitions, respectively, and J_F takes all the values allowed by angular momentum coupling and (when relevant) antisymmetrization. Then, changing the momentum variables in Eq. (1) to relative ($\boldsymbol{p}'$) and center-of-mass ($\boldsymbol{P}'$) momenta, making a multipole decomposition of the corresponding free waves and performing the angular integrations, one gets, for N-induced transitions, [21, Eqs. (2.4), (2.9)]

$$\begin{aligned}\Gamma_N = \frac{16M^3}{\pi} \sum_{j_2 J_F} \int_0^{\Delta_{j_2N}} d\epsilon' \sqrt{\epsilon'(\Delta_{j_2N} - \epsilon')} \\ \times \sum_{\substack{l'L'\lambda'S' \\ J'T'M'_T}} \big|\big\langle p'l'\,P'L'\,\lambda'S'J'T'M'_T, (jn)^{-1}\,(j_2N)^{-1}; J_F; J_I\big| \\ \times \hat{V}\big|(j_1\Lambda)\,(jn)^{-1}; J_I\big\rangle\big|^2,\end{aligned} \tag{94}$$

where T' is the total isospin of the two emitted nucleons and the angular momentum couplings $\boldsymbol{l}' + \boldsymbol{L}' = \boldsymbol{\lambda}'$, $\boldsymbol{\lambda}' + \boldsymbol{S}' = \boldsymbol{J}'$ and $\boldsymbol{J}' + \boldsymbol{J}_F = \boldsymbol{J}_I$ are carried out. One also has

[7] As shown in that reference, further sophistication of the nuclear structure description has little effect on the nonmesonic decay rates.

$P' = 2\sqrt{M\epsilon'}$, $p' = \sqrt{M(\Delta_{j_2N} - \epsilon')}$ and $\Delta_{j_2N} = M_\Lambda - M + \varepsilon_{j_1\Lambda} + \varepsilon_{j_2N}$, where the single-particle energies are taken from experiment, according to Table 3 of Ref. [15].

After some standard manipulations, Eq. (94) takes the form [21, Eqs. (2.13), (4.4)]

$$\Gamma_N = \frac{16M^3}{\pi} \sum_{j_2} \int_0^{\Delta_{j_2N}} d\epsilon' \sqrt{\epsilon'(\Delta_{j_2N} - \epsilon')} \times \sum_{J'} F_{J'}^{j_2N} \sum_{l'L'\lambda'S'T'} \left|\mathcal{M}(p'l'\,P'L'\,\lambda'S'J'T';\, j_1\Lambda\, j_2N)\right|^2, \tag{95}$$

which allows a nice separation between the nuclear structure aspects and those of the decay dynamics proper. The nuclear structure factor is, in second-quantized notation,

$$F_{J'}^{j_2N} = \frac{1}{2J_I + 1} \sum_{J_F} \left|\langle I \| \left(a_{j_2N}^\dagger a_{j_1\Lambda}^\dagger\right)_{J'} \| F\rangle\right|^2, \tag{96}$$

and its nonzero values, for the nuclear states in Eqs. (93), are: $F_0^{1s_{1/2}n} = F_0^{1s_{1/2}p} = 1/2$, $F_1^{1s_{1/2}n} = F_1^{1s_{1/2}p} = 3/2$, $F_1^{1p_{3/2}n} = 7/4$, $F_2^{1p_{3/2}n} = 5/4$, $F_1^{1p_{3/2}p} = 3/2$ and $F_2^{1p_{3/2}p} = 5/2$. (See Table I in Ref. [23].) On the other hand, the nuclear matrix element governing the decay is

$$\mathcal{M}(p'l'\,P'L'\,\lambda'S'J'T';\, j_1\Lambda\, j_2N) = \frac{1}{\sqrt{2}}\left[1 - (-)^{l'+S'+T'}\right](p'l'\,P'L'\,\lambda'S'J'T'M_T|\hat{V}|j_1\Lambda\, j_2N\, J'), \tag{97}$$

where $(\cdots|\hat{V}|\cdots)$ is a direct matrix element and the factor in front takes care of antisymmetrization. To compute the isospin part of this matrix element, one writes the baryon content of the ket as $|\Lambda\, N) = |\frac{1}{2}m_{t_\Lambda}\, \frac{1}{2}m_{t_N})$, where m_{t_N} takes the values $m_{t_p} = 1/2$ for protons and $m_{t_n} = -1/2$ for neutrons, while, in accordance with the isospurion stratagem, one treats the Λ as if it corresponded to $m_{t_\Lambda} = -1/2$. On the bra side, one sets $M_T = m_{t_\Lambda} + m_{t_N}$. To simplify the spatial integration, one resorts to a Moshinsky transformation [51] of the initial ΛN system. To this end, the shell-model radial wave functions are approximated by those of a harmonic oscillator with a length parameter of $b = 1.75$ fm, which is an average between the values appropriate for a Λ and for a nucleon [16]. Some useful expressions for the computation of these matrix elements are given in Appendix A.

As done in Ref. [21] and already stated above, the FS effects are taken into account as indicated in Subsection 2.5. Another important effect to include due to the relatively large momentum transfers involved in nonmesonic decays is that of short range correlations (SRC). The most satisfactory way to deal with the SRC between the Λ and the inducing nucleon in the initial state would be through a finite-nucleus G-matrix calculation [52]. However, as mentioned in Ref. [18], this can be well simulated by means of the correlation function

$$g_{\Lambda N}(r) = \left(1 - e^{-r^2/\alpha^2}\right)^2 + \beta r^2 e^{-r^2/\gamma^2}, \tag{98}$$

Table 3
Corrections due to the kinematical effects on the nonmesonic decay rates of ${}^{12}_{\Lambda}$C in several OME models, when only the *local* potential is included in the calculation. See text for detailed explanation

Model/Kinematics	Γ_n^{PC}	Γ_n^{PV}	Γ_p^{PC}	Γ_p^{PV}	Γ_{nm}	Γ_n/Γ_p
π						
Averaged	0	0	0	0	0	0
$\mu_\pi \to \tilde{\mu}_\pi$	0.0019	0.0230	0.0974	0.0601	0.1823	0.0031
Full	0.0019	0.0224	0.1000	0.0586	0.1829	0.0024
(π, η, K)						
Averaged	0	0	0	0	0	0
$\mu_\pi \to \tilde{\mu}_\pi$	0.0024	0.0292	0.0615	0.0653	0.1583	−0.0275
$\mu_i \to \tilde{\mu}_i$	0.0023	0.0359	0.0498	0.0707	0.1586	−0.0150
Full	0.0023	0.0353	0.0508	0.0694	0.1577	−0.0156
$\pi + \rho$						
Averaged	0	0	0	0	0	0
$\mu_\pi \to \tilde{\mu}_\pi$	0.0008	0.0214	0.0712	0.0658	0.1593	0.0055
$\mu_i \to \tilde{\mu}_i$	0.0008	0.0209	0.0711	0.0685	0.1614	0.0047
Full	0.0008	0.0206	0.0729	0.0652	0.1597	0.0046
$(\pi, \eta, K) + (\rho, \omega, K^*)$						
Averaged	0	0	0	0	0	0
$\mu_\pi \to \tilde{\mu}_\pi$	0.0048	0.0245	0.0710	0.0803	0.1807	−0.0103
$\mu_i \to \tilde{\mu}_i$	0.0058	0.0323	0.0605	0.0953	0.1940	−0.0032
Full	0.0058	0.0306	0.0619	0.0878	0.1862	−0.0033

with $\alpha = 0.5$ fm, $\beta = 0.25$ fm^{-2} and $\gamma = 1.28$ fm. As for the SRC between the two emitted nucleons, one might want to perform a T-matrix calculation including final state interactions along the lines of Ref. [53].[8] A simpler, if less satisfactory, way is to again appeal to a correlation function, like [54]

$$g_{NN}(r) = 1 - j_0(q_c r), \tag{99}$$

where j_0 is a spherical Bessel function and $q_c = 3.93$ fm^{-1}. For our purposes here, it is sufficient to follow Ref. [21] and opt for these phenomenological correlation functions. Thus, in the calculation of the nuclear matrix elements in Eq. (97) we simply make the replacements

$$\begin{aligned} &|j_1 \Lambda\, j_2 N\, J) \to g_{\Lambda N}(r) |j_1 \Lambda\, j_2 N\, J), \\ &\big(p'l'\, P'L'\, \lambda' S'\, J'T'M_T'\big| \to \big(p'l'\, P'L'\, \lambda' S'\, J'T'M_T'\big| g_{NN}(r). \end{aligned} \tag{100}$$

Following the discussion in Subsections 2.3 and 2.4, we initially focus our attention on the reduction of the effective meson masses, $\tilde{\mu}_i$ in Eq. (20), especially that of the pion, $\tilde{\mu}_\pi$ in Eq. (42), and show that indeed this is the main kinematical effect for the local potential, but not so for the nonlocal one. To this end we give, in Tables 3 and 4, the corrections that should be added, according to several different calculations, to the standard OME results,

[8] In fact, there are claims that this is very important for a good description of the nonmesonic decay observables [20].

Table 4
First-order nonlocality corrections for the nonmesonic decay rates of ${}^{12}_{\Lambda}$C in several OME models, and for different treatments of the kinematical effects. See text for detailed explanation

Model/Kinematics	Γ_n^{PC}	Γ_n^{PV}	Γ_p^{PC}	Γ_p^{PV}	Γ_{nm}	Γ_n/Γ_p
π						
Averaged	0	0	0	0	0	0
$\mu_\pi \to \tilde{\mu}_\pi$	0	0	0	0	0	0
Full	0.0032	0	0.0729	0	0.0761	−0.0062
(π, η, K)						
Averaged	0	0	0	0	0	0
$\mu_\pi \to \tilde{\mu}_\pi$	0	0	0	0	0	0
$\mu_i \to \tilde{\mu}_i$	0	0	0	0	0	0
Full	0.0037	−0.0332	0.0312	−0.0335	−0.0318	−0.0401
$\pi + \rho$						
Averaged	0.0001	0.0016	0.0004	−0.0102	−0.0080	0.0034
$\mu_\pi \to \tilde{\mu}_\pi$	0.0001	0.0017	0.0003	−0.0110	−0.0088	0.0032
$\mu_i \to \tilde{\mu}_i$	0.0001	0.0018	0.0003	−0.0114	−0.0091	0.0033
Full	0.0023	0.0014	0.0386	−0.0088	0.0334	−0.0001
$(\pi, \eta, K) + (\rho, \omega, K^*)$						
Averaged	0.0013	−0.0471	−0.0008	−0.0970	−0.1435	−0.0238
$\mu_\pi \to \tilde{\mu}_\pi$	0.0014	−0.0500	−0.0011	−0.1027	−0.1524	−0.0227
$\mu_i \to \tilde{\mu}_i$	0.0015	−0.0522	−0.0011	−0.1071	−0.1589	−0.0229
Full	0.0133	−0.0901	0.0404	−0.1425	−0.1790	−0.0515

i.e., those obtained when both the kinematical and the nonlocality effects are completely ignored. In Table 3, are the corrections corresponding to calculations that use only the local potential, and in Table 4, those corresponding to calculations that include also the first-order nonlocality terms. In each table, the first column indicates which mesons have been included in the exchange process and how far the kinematical effects due to the lambda-nucleon mass difference have been taken into consideration. The entry "averaged" means that the mass-averaging approximation (31) has been made and, consequently, no kinematical effects have been included, while $\mu_\pi \to \tilde{\mu}_\pi$ or $\mu_i \to \tilde{\mu}_i$ indicates that they partly have been, through these replacements made, respectively, for the pion alone or for all the mesons, in the expressions for the mass-averaged potentials $\bar{V}_i^{(0)}$ and $\hat{\bar{V}}_i^{(1)}$ in Subsections 2.3 and 2.4. (Excluding, of course, the factor $G_F\mu_\pi^2$.) Finally, the entry "full" means that the kinematical effects have been fully taken into account, by making use of the complete expressions for $V_i^{(0)}$ and $\hat{V}_i^{(1)}$ when constructing the local transition potential and (for Table 4) its first-order nonlocality correction.

Examining the first block in Table 3, one notices immediately that, when only the local potential is included in the calculation, the kinematical effects are well represented, for the pion, by the replacement $\mu_\pi \to \tilde{\mu}_\pi$ in the expression (45) for the local transition potential obtained when they are completely neglected. Thus, the further modifications caused by these effects in the local potential, which lead to the "full" expression (44), are of less importance. Comparing the last two lines in the remaining blocks of this table, one concludes that the analogous statement holds also for the other mesons. Finally, knowing

Table 5
Analysis of the different contributions to the nonmesonic decay rates of ${}^{12}_{\Lambda}$C in several OME models. All corrections are computed with "full" kinematics. See text for detailed explanation

Model/Contributions	Γ_n^{PC}	Γ_n^{PV}	Γ_p^{PC}	Γ_p^{PV}	Γ_{nm}	Γ_n/Γ_p
π						
Uncorrected value	0.0063	0.1162	0.6019	0.2887	1.0131	0.1375
Local kinem. corr.	0.0019	0.0224	0.1000	0.0586	0.1829	0.0024
1st-order nonloc. corr.	0.0032	0	0.0729	0	0.0761	−0.0062
Corrected value	0.0114	0.1386	0.7748	0.3473	1.2721	0.1337
(π, η, K)						
Uncorrected value	0.0056	0.2345	0.2124	0.3813	0.8339	0.4045
Local kinem. corr.	0.0023	0.0353	0.0508	0.0694	0.1577	−0.0156
1st-order nonloc. corr.	0.0037	−0.0332	0.0312	−0.0335	−0.0318	−0.0401
Corrected value	0.0116	0.2366	0.2944	0.4172	0.9598	0.3488
$\pi + \rho$						
Uncorrected value	0.0056	0.1004	0.5140	0.3608	0.9807	0.1212
Local kinem. corr.	0.0008	0.0206	0.0729	0.0652	0.1597	0.0046
1st-order nonloc. corr.	0.0023	0.0014	0.0386	−0.0088	0.0334	−0.0001
Corrected value	0.0087	0.1224	0.6255	0.4172	1.1738	0.1257
$(\pi, \eta, K) + (\rho, \omega, K^*)$						
Uncorrected value	0.0241	0.2218	0.2961	0.6238	1.1657	0.2672
Local kinem. corr.	0.0058	0.0306	0.0619	0.0878	0.1862	−0.0033
1st-order nonloc. corr.	0.0133	−0.0901	0.0404	−0.1425	−0.1790	−0.0515
Corrected value	0.0432	0.1623	0.3984	0.5691	1.1729	0.2124

this and comparing the second and third lines in these same blocks, one can see that the main local kinematical correction is that affecting the pion exchange. All these conclusions are in agreement with the discussion in Subsections 2.3.1, 2.3.2, 2.3.3, 2.4.1 and 2.4.2.

Going now to Table 4, one sees that the situation is quite different as regards the influence of the kinematical effects on the nonlocal potential. In fact, the first two blocks show that, in OME models containing only pseudoscalar mesons, the first-order nonlocality corrections vanish unless one takes the kinematical effects fully into account. This is just a restatement of Eqs. (50), (79) and the analogous result for the η meson. Similarly, examination of the last two blocks shows that, in OME models containing vector mesons, if one does not take the kinematical effects fully into account the first-order nonlocality corrections generally turn out very different from their actual values. The mere replacement $\mu_i \to \tilde{\mu}_i$ does not work well in this case. This is so because, as can be seen in Subsections 2.3.2 and 2.4.2, several nonlocal terms appear as a direct consequence of the kinematical effects, rather than simply being modified by them. Therefore, to be consistent, one should take the kinematical effects due to the lambda–nucleon mass difference *fully* into account when dealing with the nonlocality corrections.

To better visualize our findings, we exhibit in Table 5 an analysis of the different contributions to the nonmesonic decay rates of ${}^{12}_{\Lambda}$C in the four OME models we have been considering. In the first line of each block, we give the values that would be obtained for the transition rates in the standard OME approach, i.e., when neither the kinematical, nor the nonlocality corrections are included. On the second line, are the corrections to these values arising from the kinematical effects related to the lambda-nucleon mass difference, but still restricted to the local contributions only. On the third line, we have the first-order nonlocality corrections, and on the last one, the values of the decay rates including the two corrections. As required by consistency, according to our previous discussion, both corrections are computed with the kinematical effects fully taken into account.

Examining this table, one notices that the kinematical and the nonlocality corrections are typically of comparable sizes. Furthermore, for the partial and total decay rates, the former ones are always positive, while the latter are sometimes negative. Consequently, the two corrections should be included simultaneously, or not at all. Another point to remark is that the modifications in the uncorrected values of these decay rates when going from one OME model to another are of the same general magnitude as these corrections within each model. Therefore, it might be questionable to consider other mesons besides the pion without, at the same time, including the kinematical and nonlocality corrections.

The influence of the two effects together in the several partial decay rates varies around $\sim 80\%$ for Γ_n^{PC} and $\sim 20\%$ for the other ones, depending on the OME model. The net effect on the main decay observables is smaller: it is $\sim 15\%$ for Γ_{nm} and $\sim 10\%$ for the ratio Γ_n/Γ_p, again depending on the OME model considered. As one can see, these corrections are of no help to solve the discrepancy between the theoretical prediction and the experimental determinations for the latter quantity. They are too small for that, and usually go in the wrong direction. As a final observation, notice that the combined correction affects very differently the parity-conserving and the parity-violating transitions, especially when strange mesons are involved. For instance, in the model including the full pseudoscalar and vector meson octets (fourth block), the uncorrected value for $\Gamma^{\mathrm{PC}}/\Gamma^{\mathrm{PV}}$ is 0.379, while the corrected one is 0.604.

One might wonder how important the corrections are for each meson-exchange taken in isolation. To answer this question, we show in Table 6 the different contributions to the partial and total decay rates, as well as to the n/p ratio, coming from each meson acting alone. Of course, in actual fact the contributions of the several mesons interfere with each other, so that the numbers shown in this table do not have any direct physical meaning, but they serve as an indication of the importance of the two corrections for each meson.

The local kinematical corrections affect the pion more than any other meson, as expected due to its low mass. There, the effect on the decay rates is of the order of 20–30%, while for the other mesons it does not exceed $\sim 10\%$. For the n/p ratio, the effect is always negligible.

The first-order nonlocality corrections vary a lot, both in sign, and in magnitude. Depending on the meson and on the partial rate considered, the corrections can be as low as a few per cent, but in many cases reach the 50% level. The effect on the omega-exchange is exceptionally large, specially for the parity-violating transitions, where, in fact, more than half of the contribution comes from the nonlocal terms. However, for this meson the parity-conserving transitions are far more important and, besides this, the net nonlocal PC contribution is opposite to the PV one, such that the final effect of the nonlocal corrections on the total decay rate is of less than 5%. This is again an indication that the effects we are considering here are not exactly small, although, due to a series of compensations, they end up by not affecting the usual decay observables too much.

Table 6
Analysis of the different contributions to the nonmesonic decay rates of ${}^{12}_{\Lambda}C$ coming from each meson acting alone. The interference among the different mesons is ignored. All corrections are computed with "full" kinematics. See text for detailed explanation

Meson/Contributions	Γ_n^{PC}	Γ_n^{PV}	Γ_p^{PC}	Γ_p^{PV}	Γ_{nm}	Γ_n/Γ_p
π						
Uncorrected value	0.0063	0.1162	0.6019	0.2887	1.0131	0.1375
Local kinem. corr.	0.0019	0.0224	0.1000	0.0586	0.1829	0.0024
1st-order nonloc. corr.	0.0032	0	0.0729	0	0.0761	−0.0062
Corrected value	0.0114	0.1386	0.7748	0.3473	1.2721	0.1337
η						
Uncorrected value	0.0020	0.0031	0.0047	0.0024	0.0122	0.7200
Local kinem. corr.	0	0.0002	0.0002	0.0001	0.0006	−0.0047
1st-order nonloc. corr.	0.0003	0	0.0005	0	0.0007	−0.0130
Corrected value	0.0023	0.0033	0.0054	0.0025	0.0135	0.7023
K						
Uncorrected value	0.0058	0.0453	0.0780	0.0277	0.1569	0.4841
Local kinem. corr.	0.0002	0.0041	0.0061	0.0025	0.0129	0.0009
1st-order nonloc. corr.	−0.0002	−0.0233	0.0073	−0.0139	−0.0303	−0.1886
Corrected value	0.0058	0.0261	0.0914	0.0163	0.1395	0.2964
ρ						
Uncorrected value	0.0016	0.0017	0.1274	0.0082	0.1390	0.0246
Local kinem. corr.	0.0001	0.0002	0.0070	0.0003	0.0075	0.0004
1st-order nonloc. corr.	−0.0005	0.0007	−0.0069	−0.0013	−0.0080	0.0029
Corrected value	0.0012	0.0026	0.1275	0.0072	0.1385	0.0279
ω						
Uncorrected value	0.0073	0.0020	0.0732	0.0019	0.0843	0.1232
Local kinem. corr.	0.0007	0.0001	0.0051	0.0001	0.0062	0.0028
1st-order nonloc. corr.	0.0016	0.0037	−0.0030	0.0011	0.0034	0.0708
Corrected value	0.0096	0.0058	0.0753	0.0031	0.0939	0.1968
K^*						
Uncorrected value	0.0062	0.0145	0.0614	0.0189	0.1010	0.2579
Local kinem. corr.	0.0001	0.0011	0.0021	0.0007	0.0040	0.0060
1st-order nonloc. corr.	−0.0016	0.0086	−0.0025	−0.0004	0.0041	0.0960
Corrected value	0.0047	0.0242	0.0610	0.0192	0.1091	0.3599

3.2. Asymmetry parameter

The only remaining nonmesonic decay observable, beyond Γ_{nm} and Γ_n/Γ_p, that has been measured to date is the asymmetry parameter, a_Λ, which depends on the interference between the amplitudes for PC and PV proton-induced transitions to final states with different isospins. This parameter is a characteristic of the nonmesonic decay of a polarized Λ in the nuclear medium, having been defined so as to subdue the influence of the particular hypernucleus considered [15]. It is experimentally extracted from measurements of the asymmetry, A_y, in the angular distribution of the emitted protons in the nonmesonic decay of polarized hypernuclei [55,56]. There are large discrepancies, both experimentally and theoretically, in the determination of a_Λ [3, Section 7], specially after the newest experimental results for the decay of ${}^5_\Lambda$He [56], which give a positive value for this observable, differently from all previous measurements. In strong disagreement with that, all calculations so far find a negative value for a_Λ [3], which makes the investigation of the kinematical plus nonlocality corrections on this observable particularly relevant.

For the case of ${}^5_\Lambda$He, the following expression can be used to compute the asymmetry parameter [3]:

$$a_\Lambda = A_y\left({}^5_\Lambda\text{He}\right) = \frac{2\Re\left[\sqrt{3}\,ae^* - b(c^* - \sqrt{2}\,d^*) + \sqrt{3}\,f(\sqrt{2}\,c^* + d^*)\right]}{|a|^2 + |b|^2 + 3(|c|^2 + |d|^2 + |e|^2 + |f|^2)}, \tag{101}$$

where

$$\begin{aligned}
a &= \langle np, {}^1\text{S}_0|\hat{V}|\Lambda p, {}^1\text{S}_0\rangle \\
&= \mathcal{M}(pl = 0\ PL = 0\ \lambda = 0\ S = 0\ J = 0\ T = 1\ M_T = 0;\ \Lambda p, (1\text{s}_{1/2})^2\ J = 0),\\
b &= i\langle np, {}^3\text{P}_0|\hat{V}|\Lambda p, {}^1\text{S}_0\rangle \\
&= i\mathcal{M}(pl = 1\ PL = 0\ \lambda = 1\ S = 1\ J = 0\ T = 1\ M_T = 0;\ \Lambda p, (1\text{s}_{1/2})^2\ J = 0),\\
c &= \langle np, {}^3\text{S}_1|\hat{V}|\Lambda p, {}^3\text{S}_1\rangle \\
&= -\mathcal{M}(pl = 0\ PL = 0\ \lambda = 0\ S = 1\ J = 1\ T = 0\ M_T = 0;\ \Lambda p, (1\text{s}_{1/2})^2\ J = 1),\\
d &= -\langle np, {}^3\text{D}_1|\hat{V}|\Lambda p, {}^3\text{S}_1\rangle \\
&= \mathcal{M}(pl = 2\ PL = 0\ \lambda = 2\ S = 1\ J = 1\ T = 0\ M_T = 0;\ \Lambda p, (1\text{s}_{1/2})^2\ J = 1),\\
e &= i\langle np, {}^1\text{P}_1|\hat{V}|\Lambda p, {}^3\text{S}_1\rangle \\
&= -i\mathcal{M}(pl = 1\ PL = 0\ \lambda = 1\ S = 0\ J = 1\ T = 0\ M_T = 0;\ \Lambda p, (1\text{s}_{1/2})^2\ J = 1),\\
f &= -i\langle np, {}^3\text{P}_1|\hat{V}|\Lambda p, {}^3\text{S}_1\rangle \\
&= -i\mathcal{M}(pl = 1\ PL = 0\ \lambda = 1\ S = 1\ J = 1\ T = 1\ M_T = 0;\ \Lambda p, (1\text{s}_{1/2})^2\ J = 1).
\end{aligned} \tag{102}$$

The extra factors in the transition amplitudes are due to differences in phase conventions, as explained in Appendix B, and we have rewritten them in terms of the nuclear matrix elements defined in Eq. (97).

It is important to realize that the formula (101) is not of general validity, and is, in fact, merely an approximation taken over from the result valid for a free space process [57] and adapted somehow to the hypernuclear decay situation. In particular, there is no definitive prescription on how to divide the energy liberated in the decay, Δ_F, between the relative and CM motions of the emitted nucleons. It seems that, based on the expectation that the final result be insensitive to this point, some authors merely take $P = 0$ in Eq. (102), while others integrate over phase space, both the numerator, and the denominator in Eq. (101). We have checked that the two prescriptions indeed give similar results for ${}^5_\Lambda$He, and opted to tabulate only those corresponding to the first one. A more rigorous calculation of a_Λ is planned for the near future.

We give, in Table 7, the results we obtained for the decay rates and asymmetry parameter of ${}^5_\Lambda$He. The calculation goes along similar lines to those explained in Subsection 3.1, except for the obvious changes. The (hyper)nuclear model-space is restricted to the $1\text{s}_{1/2}$

Table 7
Analysis of the different contributions to the nonmesonic decay rates and asymmetry parameter of ${}^{5}_{\Lambda}$He in several OME models. All corrections are computed with “full” kinematics. See text for detailed explanation

Model/Contributions	$\Gamma_n^{\rm PC}$	$\Gamma_n^{\rm PV}$	$\Gamma_p^{\rm PC}$	$\Gamma_p^{\rm PV}$	$\Gamma^{\rm PC}/\Gamma^{\rm PV}$	a_Λ
π						
Uncorrected value	0.0004	0.0739	0.3889	0.1479	1.7553	−0.4351
Local kinem. corr.	0.0005	0.0130	0.0599	0.0258	−0.0295	−0.0084
1st-order nonloc. corr.	0.0011	0	0.0461	0	0.1810	−0.0021
Corrected value	0.0020	0.0869	0.4949	0.1737	1.9068	−0.4456
(π, η, K)						
Uncorrected value	0.0013	0.1734	0.1261	0.2077	0.3342	−0.5852
Local kinem. corr.	0.0008	0.0234	0.0280	0.0327	0.0229	−0.0106
1st-order nonloc. corr.	0.0013	−0.0285	0.0176	−0.0215	0.0952	−0.0406
Corrected value	0.0034	0.1683	0.1717	0.2189	0.4523	−0.6364
$\pi + \rho$						
Uncorrected value	0.0001	0.0648	0.3135	0.1939	1.2118	−0.2665
Local kinem. corr.	0.0001	0.0121	0.0433	0.0299	−0.0251	−0.0217
1st-order nonloc. corr.	0.0004	0.0016	0.0245	−0.0056	0.1003	−0.0273
Corrected value	0.0006	0.0785	0.3813	0.2182	1.2870	−0.3155
$(\pi, \eta, K) + (\rho, \omega, K^*)$						
Uncorrected value	0.0112	0.1672	0.1778	0.3642	0.3558	−0.5131
Local kinem. corr.	0.0026	0.0206	0.0344	0.0442	0.0233	−0.0090
1st-order nonloc. corr.	0.0054	−0.0715	0.0237	−0.0897	0.2073	−0.0167
Corrected value	0.0192	0.1163	0.2359	0.3187	0.5864	−0.5388

orbital; the relevant nuclear structure factors, Eq. (96), are [23] $F_0^{1s_{1/2}n} = F_0^{1s_{1/2}p} = 1/2$, $F_1^{1s_{1/2}n} = F_1^{1s_{1/2}p} = 3/2$; we used 1.62 fm for the oscillator length parameter; and took $\Delta_F = 153.83$ MeV. It is clear from this table that the comments made above about the decay rates of ${}^{12}_{\Lambda}$C remain qualitatively valid in the present case. As to the asymmetry parameter, the effect of the two corrections reaches $\sim 18\%$ in the $\pi + \rho$ model, but is of only $\sim 5\%$ in the complete model. On the average, it varies around $\sim 10\%$. It is interesting to observe that the two corrections always go in the direction of making a_Λ even more negative, thus confirming the sign of the existing theoretical predictions for this observable in contraposition to its most recent experimental determination.

4. Summary and conclusions

We have proposed an approach that naturally establishes a hierarchy for the different levels of approximation in the extraction of the nonrelativistic transition potential in OME models in general. The central result is Eq. (21). The first term corresponds to the local approximation, usually adopted in the literature on nonmesonic decay. The second one, to the first-order nonlocality correction, which we have included in our calculations here. And the last one, to the second-order nonlocality correction, which we have neglected. We have also given a detailed and general account on how to deal accurately with the kinematical effects that result when one has different baryon masses on the four legs in the OME

Feynman amplitude in Fig. 1. All this was particularized to Λ hypernuclear nonmesonic decay and detailed expressions for all contributions to the transition potential coming from the exchange of the complete pseudoscalar and vector meson octets were given.

Using this formalism, we have investigated the relative importance of two effects sistematically ignored in OME models for the nonmesonic weak decay of Λ-hypernuclei. First, that of an accurate treatment of the kinematics, i.e., of taking into account the difference in mass between the hyperon and the nucleon, when determining the OME transition potential. Secondly, we considered the influence of the first-order nonlocality-correction terms. Surprisingly, in view of the nonnegligible value of the mass-asymmetry ratio in Eq. (32), we came to the conclusion that the kinematical effect on the local potential is small, except for the reduction of the effective mass of the pion, Eq. (42), which implies an increase of $\sim 35\%$ in the range of the corresponding transition potential. However its indirect influence is important, since it activates several nonlocal terms in the transition potential.

Our conclusion is that the influence of the two effects together on the partial decay rates is sizeable, the full amount depending on which mesons are included. It can be very large for Γ_n^{PC}, often exceeding 100%, while it typically stays in the 20–30% range for the other partial rates. The effects are somewhat washed out in the main decay observables, averaging to $\sim 15\%$ for the total nonmesonic decay rate, $\Gamma_{\mathrm{nm}} = \Gamma_n + \Gamma_p$, and to $\sim 10\%$ for, both the neutron- to proton-induced ratio, Γ_n/Γ_p, and the asymmetry parameter, a_Λ. In particular, they do not in any way account for the well-known discrepancy between the standard OME predictions for the latter two observables and their measured values [3]. To summarize, although the kinematical and nonlocality effects can be very important for particular transitions, they end up by not affecting the main decay observables too much. One can partly understand this from the following two facts:

1. The most affected transitions are by far the neutron-induced, parity-conserving ones. However those contribute very little to Γ_{nm} and to Γ_n/Γ_p, and not at all to a_Λ, which is a property of proton-induced transitions only.
2. The largest relative effect of the nonlocality corrections is on the PV transitions coming from the one-omega-exchange process. However, for this meson, the PC transitions are much more important and they are affected in the opposite direction. As a result the two effects are largely neutralized in any of the main decay observables, which depend on, either the sum of the intensities, or the product of the amplitudes for these two types of transitions.

We have also shown that the parameter $\Gamma^{\mathrm{PC}}/\Gamma^{\mathrm{PV}}$ is strongly affected in OME models that include strange mesons. The relative correction is of $\sim 30\%$ in the $(\pi + \eta + K)$ model and of $\sim 60\%$ in the complete model. However, none of the three nonmesonic decay observables that have been measured up to now is sensitive to this ratio.

Let us finalize by saying that, although the effects we have studied here can be considered small in view of the imprecision of the available measurements and the degree of uncertainty presently existing in the parameters of OME models for nonmesonic decay (particularly coupling constants), they are not altogether negligible. In fact, in many cases they appear to influence the theoretical predictions by roughly as much as the inclusion of other mesons beyond the pion in the exchange process. It seems, therefore, that they have a part to play in OME models for nonmesonic hypernuclear decays, specially if one takes into consideration that more detailed and accurate experimental data on this issue is forthcoming.

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Appendix A. Calculation of nuclear matrix elements

We collect in this appendix some useful expressions for the calculation of the several matrix elements contributing to Eq. (97). As mentioned below that equation, the first step is to factor out the baryon content of the initial state,

$$|j_1 \Lambda\, j_2 N\, J) = |j_1 j_2 J)\,|\Lambda N) \equiv |n_1 l_1 j_1\, n_2 l_2 j_2 J)\,|\Lambda N), \tag{A.1}$$

and perform a Moshinsky transformation [51] for its space-spin part,

$$\begin{aligned} |n_1 l_1 j_1\, n_2 l_2 j_2\, J) &= \sqrt{(2j_1+1)(2j_2+1)} \sum_{\lambda S} \sqrt{(2\lambda+1)(2S+1)} \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ \lambda & S & J \end{Bmatrix} \\ &\quad \times \sum_{nlNL} |nl\, NL\, \lambda S J)\,(nl\, NL\, \lambda | n_1 l_1\, n_2 l_2\, \lambda). \end{aligned} \tag{A.2}$$

A look at Subsections 2.3 and 2.4 shows that the operators involved are always of the general form $v(r)\Omega(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \hat{\boldsymbol{r}}, \nabla)$. All matrix elements are diagonal in the quantum numbers L and J, and the needed results are listed below. Some of them have already been given in Ref. [21] but are repeated here for completeness.[9]

$$(p'l'\, P'L\, \lambda' S' J | v(r) | nl\, NL\, \lambda S J) = \delta_{l'l}\delta_{\lambda'\lambda}\delta_{S'S}(P'L|NL)(p'l'|v(r)|nl), \tag{A.3}$$

$$\begin{aligned} &(p'l'\, P'L\, \lambda' S' J | v(r)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 | nl\, NL\, \lambda S J) \\ &\quad = \delta_{l'l}\delta_{\lambda'\lambda}\delta_{S'S}\big[2S(S+1)-3\big](P'L|NL)(p'l'|v(r)|nl), \end{aligned} \tag{A.4}$$

$$\begin{aligned} &(p'l'\, P'L\, \lambda' S' J | v(r) S_{12}(\hat{\boldsymbol{r}}) | nl\, NL\, \lambda S J) \\ &\quad = (-)^{L+J+1}\delta_{S1}\delta_{S'1}\sqrt{120(2l'+1)(2l+1)(2\lambda'+1)(2\lambda+1)} \end{aligned}$$

[9] There are some misprints in the formulas for the matrix elements in the published version of Ref. [21]. These misprints, however, do not occur in its preprint version, nucl-th/0011092.

$$\times \begin{pmatrix} l' & 2 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \lambda' & 2 & \lambda \\ l & L & l' \end{Bmatrix} \begin{Bmatrix} 1 & 2 & 1 \\ \lambda & J & \lambda' \end{Bmatrix} (P'L|NL)(p'l'|v(r)|nl), \tag{A.5}$$

$$\begin{aligned}
&(-)^{S'+S}(p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}}|nl\,NL\,\lambda SJ) \\
&\quad = (p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}}|nl\,NL\,\lambda SJ) \\
&\quad = (-)^{L+J+1}\sqrt{6(2l'+1)(2l+1)(2\lambda'+1)(2\lambda+1)(2S'+1)(2S+1)} \\
&\qquad \times \begin{pmatrix} l' & 1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \lambda' & 1 & \lambda \\ l & L & l' \end{Bmatrix} \begin{Bmatrix} S' & 1 & S \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} S' & 1 & S \\ \lambda & J & \lambda' \end{Bmatrix} \\
&\qquad \times (P'L|NL)(p'l'|v(r)|nl),
\end{aligned} \tag{A.6}$$

$$\begin{aligned}
&(p'l'\,P'L\,\lambda'S'J|iv(r)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\hat{\boldsymbol{r}}|nl\,NL\,\lambda SJ) \\
&\quad = (-)^{L+J+S}(\delta_{S'0}\delta_{S1}+\delta_{S'1}\delta_{S0})\sqrt{12(2l'+1)(2l+1)(2\lambda'+1)(2\lambda+1)} \\
&\qquad \times \begin{pmatrix} l' & 1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \lambda' & 1 & \lambda \\ l & L & l' \end{Bmatrix} \begin{Bmatrix} S' & 1 & S \\ \lambda & J & \lambda' \end{Bmatrix} (P'L|NL)(p'l'|v(r)|nl),
\end{aligned} \tag{A.7}$$

$$\begin{aligned}
&(-)^{S'+S}(p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}|nl\,NL\,\lambda SJ) \\
&\quad = (p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla}|nl\,NL\,\lambda SJ) \\
&\quad = (-)^{L+J+1}\sqrt{6(2l'+1)(2l+1)(2\lambda'+1)(2\lambda+1)(2S'+1)(2S+1)} \\
&\qquad \times \begin{pmatrix} l' & 1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \lambda' & 1 & \lambda \\ l & L & l' \end{Bmatrix} \begin{Bmatrix} S' & 1 & S \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} S' & 1 & S \\ \lambda & J & \lambda' \end{Bmatrix} \\
&\qquad \times (P'L|NL)(p'l'|v(r)\hat{d}_V(l',l;r)|nl),
\end{aligned} \tag{A.8}$$

$$\begin{aligned}
&(p'l'\,P'L\,\lambda'S'J|iv(r)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\boldsymbol{\nabla}|nl\,NL\,\lambda SJ) \\
&\quad = (-)^{L+J+S}(\delta_{S'0}\delta_{S1}+\delta_{S'1}\delta_{S0})\sqrt{12(2l'+1)(2l+1)(2\lambda'+1)(2\lambda+1)} \\
&\qquad \times \begin{pmatrix} l' & 1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \lambda' & 1 & \lambda \\ l & L & l' \end{Bmatrix} \begin{Bmatrix} S' & 1 & S \\ \lambda & J & \lambda' \end{Bmatrix} \\
&\qquad \times (P'L|NL)(p'l'|v(r)\hat{d}_V(l',l;r)|nl),
\end{aligned} \tag{A.9}$$

$$\begin{aligned}
&(-)^{S'+S}(p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{l}|nl\,NL\,\lambda SJ) \\
&\quad = (p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_2\cdot\boldsymbol{l}|nl\,NL\,\lambda SJ) \\
&\qquad \times (-)^{l+L+J+1}\delta_{l'l}\sqrt{6l(l+1)(2l+1)(2\lambda'+1)(2\lambda+1)(2S'+1)(2S+1)} \\
&\qquad \times \begin{Bmatrix} \lambda' & 1 & \lambda \\ l & L & l' \end{Bmatrix} \begin{Bmatrix} S' & 1 & S \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} S' & 1 & S \\ \lambda & J & \lambda' \end{Bmatrix} \\
&\qquad \times (P'L|NL)(p'l'|v(r)|nl),
\end{aligned} \tag{A.10}$$

$$\begin{aligned}
&(p'l'\,P'L\,\lambda'S'J|iv(r)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\boldsymbol{l}|nl\,NL\,\lambda SJ) \\
&\quad = (-)^{l+L+J+S}\delta_{l'l}(\delta_{S'0}\delta_{S1}+\delta_{S'1}\delta_{S0})\sqrt{12l(l+1)(2l+1)(2\lambda'+1)(2\lambda+1)} \\
&\qquad \times \begin{Bmatrix} \lambda' & 1 & \lambda \\ l & L & l' \end{Bmatrix} \begin{Bmatrix} S' & 1 & S \\ \lambda & J & \lambda' \end{Bmatrix} (P'L|NL)(p'l'|v(r)|nl),
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
&(p'l'\,P'L\,\lambda'S'J|v(r)\hat{\boldsymbol{r}}\cdot\boldsymbol{\nabla}|nl\,NL\,\lambda SJ) \\
&\quad = (p'l'\,P'L\,\lambda'S'J|v(r)\hat{d}_S(r)|nl\,NL\,\lambda SJ) \\
&\quad = \delta_{l'l}\delta_{\lambda'\lambda}\delta_{S'S}(P'L|NL)(p'l'|v(r)\hat{d}_S(r)|nl),
\end{aligned} \tag{A.12}$$

$$
\begin{aligned}
&(p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\hat{\boldsymbol{r}}\cdot\nabla|nl\,NL\,\lambda SJ)\\
&\quad=(p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\hat{d}_S(r)|nl\,NL\,\lambda SJ), \qquad\text{(A.13)}\\
&(p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}}\boldsymbol{\sigma}_2\cdot\nabla|nl\,NL\,\lambda SJ)\\
&\quad=\frac{1}{3}(p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\hat{d}_S(r)|nl\,NL\,\lambda SJ)\\
&\qquad+\frac{1}{3}(p'l'\,P'L\,\lambda'S'J|v(r)S_{12}(\hat{\boldsymbol{r}})\hat{d}_T(l',l;r)|nl\,NL\,\lambda SJ)\\
&\qquad+\frac{1}{2}(p'l'\,P'L\,\lambda'S'J|i\frac{v(r)}{r}(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\boldsymbol{l}|nl\,NL\,\lambda SJ), \qquad\text{(A.14)}\\
&(p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}}\boldsymbol{\sigma}_1\cdot\nabla|nl\,NL\,\lambda SJ)\\
&\quad=\frac{1}{3}(p'l'\,P'L\,\lambda'S'J|v(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\hat{d}_S(r)|nl\,NL\,\lambda SJ)\\
&\qquad+\frac{1}{3}(p'l'\,P'L\,\lambda'S'J|v(r)S_{12}(\hat{\boldsymbol{r}})\hat{d}_T(l',l;r)|nl\,NL\,\lambda SJ)\\
&\qquad-\frac{1}{2}(p'l'\,P'L\,\lambda'S'J|i\frac{v(r)}{r}(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\boldsymbol{l}|nl\,NL\,\lambda SJ). \qquad\text{(A.15)}
\end{aligned}
$$

We have introduced the following effective radial differential operators:

$$
\begin{aligned}
&\hat{d}_S(r)=\frac{\partial}{\partial r},\\
&\hat{d}_V(l',l;r)=\frac{\partial}{\partial r}+\frac{l(l+1)-l'(l'+1)+2}{2r},\\
&\hat{d}_T(l',l;r)=\frac{\partial}{\partial r}+\frac{l(l+1)-l'(l'+1)+6}{4r}. \qquad\text{(A.16)}
\end{aligned}
$$

When the short range correlations are implemented as indicated in Eqs. (100), one should, accordingly, make the following replacements in the relative radial matrix elements in Eqs. (A.3)–(A.15):

$$
\begin{aligned}
&|nl)\rightarrow g_{\Lambda N}(r)|nl),\\
&(p'l'|\rightarrow(p'l'|g_{NN}(r). \qquad\text{(A.17)}
\end{aligned}
$$

Clearly, the correlation function $g_{\Lambda N}(r)$ is also subject to the action of the differential operators in these equations. Explicitly, the center-of-mass radial overlap and the relative radial matrix elements are given by

$$
(P'L|NL)=\int R^2\,dR\,j_L(P'R)\mathcal{R}_{NL}\big(b/\sqrt{2},R\big) \qquad\text{(A.18)}
$$

and

$$
(p'l'|v(r)\hat{d}(r)|nl)=\int r^2\,dr\,j_{l'}(p'r)g_{NN}(r)v(r)\hat{d}(r)\big[g_{\Lambda N}(r)\mathcal{R}_{nl}\big(\sqrt{2}b,r\big)\big], \qquad\text{(A.19)}
$$

where j_L and $j_{l'}$ are spherical Bessel functions, $\mathcal{R}_{NL}$ and $\mathcal{R}_{nl}$ are harmonic oscillator radial wave-functions with the first arguments giving the respective length parameters, and $\hat{d}(r)$ is either unity or one of the differential operators (A.16).

Finally, let us remark that the convention for the relative coordinate adopted here, Eq. (14), has the opposite sign to those of Refs. [21] and [51], for instance. This introduces an extra phase factor of $(-1)^l$ and $(-1)^{l'}$, respectively, in our kets and bras involving the relative motion. As a result, the transformation brackets in Eq. (A.2) differ by a factor of $(-1)^l$ from those originally defined in Ref. [51]. To avoid this adjustment, one can simply shift to the opposite convention for the relative coordinate by making the transcriptions $\boldsymbol{r} \to -\boldsymbol{r}$, $\hat{\boldsymbol{r}} \to -\hat{\boldsymbol{r}}$ and $\nabla \to -\nabla$ in the expressions for the transition potential in coordinate space given in Subsections 2.3 and 2.4. In either case, the equations in Subsection 3.1 and here in Appendix A remain formally unaltered.

Appendix B. Phase conventions for *a*, *b*, ..., *f*

With the phase conventions of Nabetani et al. (N.O.S.K.), in Ref. [57], from which Eq. (101) is obtained, the transition amplitudes are defined as follows:

$$\begin{aligned} a &= \big(\langle pn, {}^1\mathrm{S}_0|\hat{V}|p\Lambda, {}^1\mathrm{S}_0\rangle\big)_{\mathrm{N.O.S.K.}}, & d &= \big(\langle pn, {}^3\mathrm{D}_1|\hat{V}|p\Lambda, {}^3\mathrm{S}_1\rangle\big)_{\mathrm{N.O.S.K.}}, \\ b &= \big(\langle pn, {}^3\mathrm{P}_0|\hat{V}|p\Lambda, {}^1\mathrm{S}_0\rangle\big)_{\mathrm{N.O.S.K.}}, & e &= \big(\langle pn, {}^1\mathrm{P}_1|\hat{V}|p\Lambda, {}^3\mathrm{S}_1\rangle\big)_{\mathrm{N.O.S.K.}}, \\ c &= \big(\langle pn, {}^3\mathrm{S}_1|\hat{V}|p\Lambda, {}^3\mathrm{S}_1\rangle\big)_{\mathrm{N.O.S.K.}}, & f &= \big(\langle pn, {}^3\mathrm{P}_1|\hat{V}|p\Lambda, {}^3\mathrm{S}_1\rangle\big)_{\mathrm{N.O.S.K.}}. \end{aligned} \tag{B.1}$$

The relationship with our phase conventions is

$$\begin{aligned} &\big(\langle pn, {}^{2S'+1}l'_J|\hat{V}|p\Lambda, {}^{2S+1}l_J\rangle\big)_{\mathrm{N.O.S.K.}} \\ &\quad = (-)^{S'+S} i^{-l'} \langle np, {}^{2S'+1}l'_J|\hat{V}|\Lambda p, {}^{2S+1}l_J\rangle, \end{aligned} \tag{B.2}$$

where the first correction is due to the change in ordering in the Clebsch–Gordan couplings for the spins, and the second one, to the fact that we do not include the phase $i^{l'}$ in the final partial-wave radial function (cf. Eq. (A.19)). This explains the extra phase factors appearing in Eq. (102).

Notice that there is no factor $(-)^{l'+l}$ in Eq. (B.2) as might be expected due to the change in ordering of the two particles in both the initial and final states, the reason being that this is compensated by the fact that we use the opposite convention for the relative coordinates.

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