MACROECONOMIC EFFECTS OF DELAYED CAPITAL LIQUIDATION

Wei Cui
University College London

Abstract
This paper studies the macroeconomic effects of capital reallocation with financial shocks. I develop a model in which firms face borrowing constraints, idiosyncratic productivity shocks, and idiosyncratic liquidation costs. The idiosyncratic risks of productivity and liquidation costs generate an option value of staying in business and a liquidation delay for unproductive firms. A new feature arises from the delay. Unproductive firms that are not liquidated increase their leverage over time, pushing them to hit the borrowing limit. I show that adverse financial shocks that tighten borrowing constraints can raise the option value, and equilibrium effects can further delay capital liquidation and reallocation. Capital is thus persistently misallocated, leading to long-lasting economic contractions. (JEL: E22; E32; E44; G11.)

1 Introduction

The market for reallocating used capital is sizable in the United States. In 2018, the capital reallocation recorded by publicly listed non-financial firms was \$0.81 trillion, about 32% of total capital expenditures that include both new investment and reallocation. Based on the productivity differences across firms within narrow industries, previous studies have found that reallocation in general improves the productivity of capital.[1] At the same time, the process of reallocating capital is costly and may be subject to financial frictions.[2] Capital reallocation under resale and financial frictions can thus have important aggregate implications.

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E-mail: w.cui@ucl.ac.uk

1. Maksimovic and Phillips (2001, 2002), Bloom (2009), İmrohoroğlu and Tüzel (2014), and Kehrig (2015) document the heterogeneity of total factor productivity (TFP) in U.S. corporate firms and plants.

2. See, e.g., Ramey and Shapiro (2001) and Shleifer and Vishny (1992).

I document two new stylized facts about cyclical financial conditions and reallocation. First, in the aggregate, (non-financial) business debt, new investment, and capital reallocation are all procyclical. In recessions, firms can typically borrow less, invest less, and reallocate less. The fall in reallocation, however, exceeds the fall in capital expenditures, generating a *procyclical* reallocation-to-expenditures (R-E) ratio. Second, the reallocation process can either be in the form of full liquidation through selling a whole firm or in the form of partial liquidation through sales of property, plant, and equipment. The share of partial liquidation to reallocation (P share) is *countercyclical*. In other words, when reallocation falls in recessions, firms substitute full liquidation with partial liquidation.

The goal of the paper is to provide a tractable macroeconomic model that accounts for these facts. This model features costly capital reallocation with financial shocks besides conventional aggregate total factor productivity (TFP) shocks. Financial shocks are modeled as perturbations that originate from the financial sector and that directly affect firms' debt limits. The shocks' impact on employment and new investment during the business cycle is well known (e.g., Jermann and Quadrini 2012, Buera and Moll 2015, and Del Negro et al. 2017). Less known is the impact on the two types of capital reallocation discussed above.

My theory integrates resale option-value analysis into a heterogenous-firms macroeconomic model with financial frictions. The core of the theory is the interaction between the financial and resale frictions, which pushes unproductive firms to become financially constrained. This interaction generates a financially-constrained option value of staying in business, reflecting the option to delay full liquidation for future smaller liquidation costs, future higher productivity, and/or future better financial conditions. This value also depends on the equilibrium interest rate and wage rate.

I show that adverse financial shocks can generate a recession by reducing the investment of productive firms, reducing debt for staying unproductive firms, and depressing equilibrium interest and wage rates. The recessionary financial shocks also raise the option value of staying for unproductive firms, thus delaying full liquidation but encouraging partial liquidation.

The option-value channel of financial frictions is fundamentally different from but complements existing work on financial frictions (e.g., Buera et al. 2011 and Khan and Thomas 2013). In previous work, financial frictions prevent the expansion of constrained high-productivity firms. Equilibrium effects usually help the expansion of unconstrained low-productivity firms. In this paper, however, the unproductive firms are financially constrained.[3] Some unproductive firms that would have been liquidated without adverse financial shocks are thus kept running after the shocks, and they benefit from the equilibrium effects.

To illustrate the above idea, I start with a simplified decision problem without equilibrium effects. Consider abandoning a permanently unproductive firm subject to

3. Unproductive firms thus may have higher leverage ratios, which is consistent with the empirical findings of İmrohoroğlu and Tüzel (2014) and online Appendix D.

borrowing constraints. The entrepreneur or manager of the firm has a preference for smoothing dividends. Assume that the resale constraint restricts partially liquidating any capital. Also assume that the firm, if liquidated in full, faces random liquidation costs drawn identically and independently from a fixed distribution each period.[4]

Two features of this decision problem emerge. First, the entrepreneur sells the unproductive firm if the liquidation cost is smaller than a *threshold*, which is an *increasing function* of its debt level for a given level of capital stock. That is, the firm is more likely to be liquidated if its debt level is higher. Second, this cash-poor firm, if it is staying in business because of high liquidation costs, may borrow more over time to smooth dividends since the resale constraint prevents it from partially selling capital for funding.[5] As a result, the resale constraint can drive the *staying* unproductive firm to hit the borrowing limit.

Now consider the impact of financial shocks by permanently tightening the borrowing constraint in each period. Firstly, on impact, the unproductive firm that is not liquidated has to cut down its debt if the initial debt level is high, which limits its ability to smooth dividends. The liquidation option thus becomes more attractive, so the entrepreneur is more likely to liquidate the firm on impact of the shocks. Secondly, if the firm is not liquidated initially because of a high liquidation-cost draw, future liquidation incentives can fall. Since future firm debt will be lower than in the scenario without the credit tightening, the staying cash-poor firm will have more financial flexibility or lower debt servicing costs. This implies that running the business can be easier after the initial painful adjustment caused by the credit tightening. Hence, future liquidation incentives can be weaker beyond the initial period, as the liquidation threshold can fall because of a lower debt level. In other words, liquidation can be delayed beyond the initial credit tightening.

Then I study the equilibrium version of the model with primary and secondary capital markets, a credit market, and a labor market. In this environment, a continuum of firms face idiosyncratic productivity shocks besides idiosyncratic liquidation costs. Both idiosyncratic risks contribute to the option value. I also relax the resale constraint so that staying firms can partially liquidate capital at a discount to fund expenses. In the quantitative exercise, staying unproductive firms are financially constrained while productive ones are not. I show that persistent adverse financial shocks can again delay full capital liquidation and, thus, reallocation in the secondary market. Compared to the decision problem, the equilibrium effects can even delay the reallocation on impact. To see the equilibrium effects, notice that tightened borrowing constraints lead firms to cut borrowing, capital expenditures, and employment. The drop in credit and labor demand after the financial shocks implies a lower interest rate and a lower wage rate in equilibrium.[6] The lower interest and wage rates improve the attractiveness of

4. The modeling captures, for example, integration costs because of capital specificity. See Cabrales, Calvó-Armengol, and Pavoni (2008) for an application of a related "mobility" cost in labor markets.

5. Other non-flexible running costs can have the same effect.

6. Other financial shocks can simultaneously reduce credit and enlarge the spread between the lending rate and the deposit rate, see e.g., Gilchrist and Zakrajšek (2012) and Cui and Kaas (2020); then the debt

running a business. As a result, such equilibrium effects further delay the liquidation of unproductive firms, worsen misallocation, and reduce aggregate output, including at the initial period.

In other words, a credit tightening can encourage more full liquidations on impact according to the result of the decision problem, but equilibrium effects can overturn this result. Therefore, persistent recessionary financial shocks can generate a persistent delay in full liquidation. More unproductive firms decide to stay in business in response to the shocks. Though not liquidated in full, these firms partially liquidate capital so that the P share goes up. In the aggregate, reallocation still falls significantly, as some capital that should have been reallocated is not available for reallocation in response to the shocks. That is why financial shocks can generate a *procyclical R-E ratio* and a *countercyclical P share* as in the stylized facts. Exogenous TFP shocks, however, lead to a countercyclical R-E ratio. For example, an exogenous fall in aggregate TFP reduces the attractiveness of running a business, leading to a drop in output but a rise in liquidation and reallocation of firms.

In fact, only with the equilibrium effects (especially the interest rate effect) can the general equilibrium model with financial shocks explain the positive co-movements among output, investment, and capital reallocation. Without the equilibrium effects, adverse financial shocks generate counterfactual dynamics as well. In particular, instead of falling initially, reallocation rises initially in response to adverse financial shocks, as demonstrated by the decision problem. Another outcome in the absence of equilibrium effects is that new investment falls and does not positively co-move with output. However, in the data, new investment and output positively co-move. The reason for this counterfactual outcome is that in the model more secondary market transactions imply less need to accumulate capital via the primary market.

What the option-value channel reveals is a new effect of financial frictions on capital misallocation. This effect is related to zombie firms, i.e., firms that are unproductive but kept afloat by current market conditions, such as the low interest rate in this paper. The research on zombie firms is vast,[7] but the result that tightened borrowing also reduces capital liquidations is, to my knowledge, new. The conventional wisdom is that zombies might be kept alive inefficiently by over-borrowing easy credit, which implies that tightened credit should encourage reallocation. This paper, however, shows that credit tightening, while on impact it can indeed encourage liquidations of unproductive firms (at least in the decision problem), might in fact delay liquidations and lead to more zombies in the medium term. Additionally, the equilibrium effects can be strong enough to delay reallocation even on impact.

servicing cost could go up even if firms deleverage, which reduces the value of staying. However, the deposit rate still falls in this case, so the interest rate channel may still delay liquidation after adverse financial shocks. I thank the editor for pointing out this possibility.

7. For example, Caballero et al. (2008) document Japanese zombie firms in detail, and they find useful creative destructions are not sufficiently frequent in the aggregate. This is because banks supply low-interest loans to incumbents; some are actually productive, but many are zombies.

Related Literature. The literature on financial frictions and their impact on capital allocation efficiency is vast. It includes Kiyotaki and Moore (1997), Bernanke et al. (1999), Mendoza (2010), and Brunnermeier and Sannikov (2014). Adverse financial shocks that tighten financing constraints and reduce productivity and output can be the consequence of the collapse of bubbles (e.g., Miao and Wang 2012, 2018). However, the interaction with capital liquidation is relatively unexplored. The proposed theory of option value with financial frictions is closest to the literature on entrepreneurship with financial development, e.g., Buera and Shin (2013) and Buera et al. (2015). There, agents can choose between being an entrepreneur or a worker, and the degree of financial frictions affects the choice. In this paper, entrepreneurs/managers that run firms can choose between running their firms and saving the proceeds from liquidation in risk-free assets, and the choice depends on the degree of financial frictions.

The option value links liquidation and financial frictions, which complements the literature on the option value of delaying investment under uncertainty.[8] This paper focuses on how financing constraints directly affect liquidation. But if one considers that at least a part of credit tightening comes from increased uncertainty (e.g., Alfaro et al. 2018) or similar beliefs perceived by lenders (e.g., Cui and Kaas 2020), then the equilibrium effect is amplifying the initial uncertainty shock, while in previous work such as Bloom et al. (2018), it can be a dampening force.

The literature on capital reallocation starts at least from Ramey and Shapiro (1998) and Eisfeldt and Rampini (2006).[9] Eisfeldt and Rampini (2008) and Fuchs et al. (2016) use information frictions to endogenize liquidity/price and study procyclical reallocation. There is another growing line of research that uses search-and-matching to endogenize liquidity and generate procyclical reallocation, including Cao and Shi (2016), Cui and Radde (2019), Ottonello (2017), Dong et al. (2018), and Wright et al. (2018). Additionally, Lanteri (2018) focuses on the price of used capital within a heterogenous-firms model, calibrating an exogenous function that determines the substitutability between new and old capital. In the industry-equilibrium model of Caggese (2007), financially constrained firms respond to idiosyncratic shocks by adjusting variable investment first and fixed capital later.

My contribution is to analyze full and partial liquidation with option value, which allows for a better assessment of the importance of financial shocks and productivity shocks. The new notion of misallocation discussed before is also crucial. Treating full and partial liquidation differently is similar to the "bundled and disassembled selling" idea in Jovanovic and Rousseau (2002), who do not consider financial frictions. The analytical results of this paper illustrate how financial shocks affect different types of capital reallocation, which previous papers are silent on. Acquisitions (or full

8. Seminal papers include Dixit and Pindyck (1994), Abel and Eberly (1996), and Bloom (2009).

9. Eisfeldt and Rampini (2006) show that aggregate capital reallocation is procyclical, while the potential benefits (the dispersion of Tobin's Q) to reallocation are countercyclical. Eberly and Wang (2009) look at the relation between reallocation and growth in a two-sector growth model. Lee (2016) identifies a "reallocation" shock, which correlates well with investment-specific technology shocks. A recent survey by Eisfeldt and Shi (2018) contains other useful references on this topic.

liquidations in the model) are often accompanied by borrowing by buyers, for example, through leveraged buyouts (see, e.g., Opler and Titman 1993), while partial sales are not. Using data on capital expenditures, acquisitions, and partial sales, we can then assess the contribution of real and financial shocks to the business cycle. To the best of my knowledge this paper is the first to do so with these data.

Finally, the technical aspect might be of independent interest. I extend firm problems with portfolio choices, such as in Angeletos (2007), Moll (2014), and Liu and Wang (2014), and I add random liquidation costs but still maintain tractability. The tractable model generates a threshold liquidation cost below which entrepreneurs liquidate, leading to straightforward aggregation. This feature also allows me to use the standard perturbation method for equilibrium dynamic analysis.

The remainder of this paper is structured as follows. Section 2 shows the cyclical properties of debt, output, investment, and reallocation, motivating why I study the two types of capital reallocation with financial shocks. In Section 3, I analyze a firm decision problem that highlights the option-value channel of financial frictions. I show that permanently tightened borrowing constraints can delay capital liquidation beyond the initial period. Section 4 presents the general equilibrium framework. In Section 5's quantitative analysis with general equilibrium, I show that financial shocks are necessary to capture those cyclical properties. This section also highlights that recessionary financial shocks can delay reallocation through interest and wage rate channels. Section 6 concludes.

2 Some Stylized Facts

In this section, I present some stylized facts concerning the U.S. economy from 1971 to 2018. Both business debt and capital reallocation (relative to capital expenditures) are procyclical. Partial reallocation (relative to total reallocation) is countercyclical. All data details are in online Appendix B.

Business debt includes only the liabilities of *non-financial* businesses, and it is directly related to credit market transactions or bank loans (excluding, e.g., relative short-term debt to fund wage bills).[10] I normalize the data to 2012 dollars by using the business value-added price deflator. The corresponding aggregate output is real business value-added, and investment is real non-residential fixed investment.[11] The debt-to-output ratio has a mean of 65.5% for the period 1971-2018.

The firm-level data of reallocating capital (excluding financial institutions) starting from 1971 is obtained from COMPUSTAT (following Eisfeldt and Rampini (2006)). Reallocation measures the changes of firm-level ownership of capital so that new

10. The model introduced later has a frictionless labor market, and the analysis here is thus consistent. Notice that the stylized facts presented below are most similar even if we include short-term debt.

11. One can restrict output to total value-added of COMPUSTAT firms (see online Appendix D) with capital reallocation used below. The cyclical statistics are similar.

productivity should apply.[12] This reallocation has two components. One is labeled *full liquidation*, i.e., full buyouts or acquisitions; the other is *partial liquidation*, i.e., partial sales or sales of property, plant, and equipment (SPP&E). I also obtain capital expenditures of each firm. For each year, I then aggregate the firm-level data and obtain aggregate full liquidation, aggregate partial partial liquidation, and aggregate capital expenditures. Capital reallocation is defined as the sum of aggregate full liquidation and aggregate partial liquidation.

To proceed, I compute the *R-E ratio*, the ratio of aggregate capital reallocation to aggregate capital expenditures (spending on both new and old capital), and the *P share*, i.e., aggregate partial liquidation divided by aggregate capital reallocation. Compared to previous studies using the level of capital reallocation or turnover rates of reallocation relative to assets, the measures I propose contain more information about the importance of old capital relative to new capital; these measures could also attenuate the effect of variation in capital prices, since the prices in primary and secondary markets are likely to co-move positively (although not perfectly). The R-E ratio was about 0.38 in 2018; the 1971-2018 sample mean is 0.28. The P share was about 0.16 in 2018; the 1971-2018 sample mean is 0.30.

The top two panels of Figure 1 display the cyclical components of debt, investment, output, and the R-E ratio. Two stylized facts are visible. First, debt falls in recessions and rises in booms, especially since 1984 (likely a result of financial liberalization). This fits the usual findings that recessions push firms to restructure their financial positions by cutting debt. Second, since both the R-E ratio and new investment fall in recessions, the reduction in spending on used capital has to be deeper than the reduction in new capital to generate a fall of R-E ratio, if we recall that

$$\text{R-E ratio} = \frac{\text{Reallocation}}{\text{Reallocation} + \text{New Investment}}.$$

That is, capital reallocation falls in recessions, and the fall is deeper than that of new investment.

Which type of reallocation further contributes to the drop of the R-E ratio in recessions? In Figure 1, the bottom left panel shows the cyclical components (with the same filtering) of the P share, which is clearly countercyclical. That is, reallocation in the form of full liquidation is less frequent in recessions than in booms. This aspect (together with the procyclical R-E ratio) is a new finding. To consider a concept similar to the R-E ratio, we can define the *P-E ratio,* which is the ratio of aggregate partial liquidation to aggregate capital expenditures. Although the overall countercyclicality becomes less obvious, the P-E ratio generally rises in recessions.[13]

12. This process can include organizational capital of a firm, namely its culture, customer base, and organizational learning, which can be a source of competitive advantage. Carlin et al. (2012) show that the most efficient mergers/acquisitions are between firms with substantial organization capital and firms with little organization capital.

13. This attenuation in cyclicality is possibly due to distinctive price variations in the primary market, the secondary market with full buyouts, and the secondary market with partial sales.

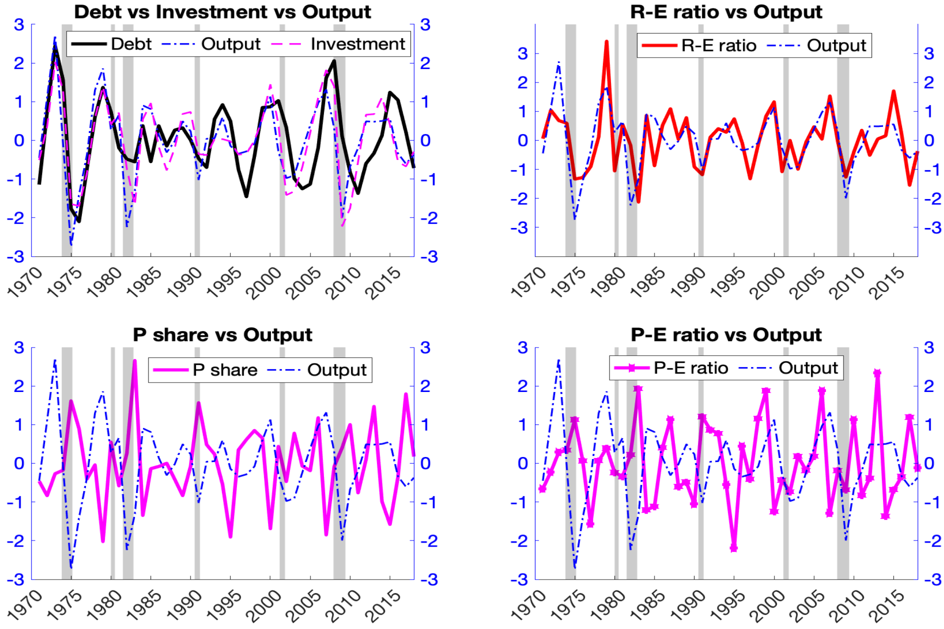

FIGURE 1. Financial and reallocation cycles. The series plotted are cyclical components of non-financial business debt, non-residential fixed investment, the R-E ratio (capital reallocation divided by capital expenditures), the P share (partial capital sales divided by capital reallocation), and the P-E ratio (partial capital sales divided by capital expenditures). I use the default band-pass filter approximation by Christiano and Fitzgerald (2003) with 2-9 years frequencies to calculate the cyclical components. Alternative specifications and other filters (e.g. the Hodrick-Prescott filter) generate similar results. All series are normalized by their standard deviations. Shaded regions denote NBER recessions, and the dash-dotted line in each graph is the cyclical component of business output.

As a summary, Table 1 presents the cyclical statistics. Debt, investment, and the R-E ratio are all procyclical (i.e., positively co-move with output), while the P share is countercyclical, and the P-E ratio is mildly countercyclical or close to acyclical. Although debt is already 19% more volatile than output, the volatilities of the R-E ratio, the P-E ratio, and the P share, are 5.79, 5.59, and 8.68 times that of output, respectively. Interestingly, *all* the correlations are strengthened if we only look at recession dates. For example, debt and output move more closely in recessions (with a positive correlation of 0.79), and the mildly countercyclical P-E ratio also has a much larger negative correlation (-0.39) with output. The takeaway is simple: the observed reallocation process is quite volatile and sensitive to business cycle conditions, especially during business downturns.

Although reallocation is costly, the process is important as it moves capital from less productive firms to more productive ones in general, as documented for example in Maksimovic and Phillips (2001, 2002). The above observed financial and reallocation

TABLE 1. Summary statistics.

Correlation	Debt	R-E ratio	P-E ratio	P share	Output
Debt	1	0.52 (0.60)	-0.15 (-0.38)	-0.45 (-0.62)	0.59 (0.79)
R-E ratio	-	1	-0.16 (-0.30)	-0.77 (-0.86)	0.64 (0.66)
P-E ratio	-	-	1	0.75 (0.75)	-0.17 (-0.39)
P share	-	-	-	1	-0.53 (-0.66)
Output	-	-	-	-	1
Relative Standard Deviations	1.19	5.79	5.59	8.68	1

Note: Numbers in brackets are the corresponding correlations for NBER recessions. The correlations of investment with output, the R-E ratio, the P share are 0.85, 0.57, and -0.49, respectively.

cycles thus motivate me to study the impact of external financial conditions on capital liquidation, aggregate productivity, and output.

3 A Model of Liquidation

This section presents the basic environment to illustrate how financing constraints affect capital liquidation decisions. The key is the financially-constrained option value of staying in business.

3.1 An Entrepreneur's Problem

Time is discrete and infinite. Consider entrepreneurs who can either run firms using a linear production technology with capital, or else they can liquidate existing firms and save in risk-free bonds.

Preferences and Technology. A typical entrepreneur has a per-period utility from consumption (or dividends) c represented by

$$u(c) = \log(c).$$

It may at first seem that the entrepreneur owns the firm, but the entrepreneur could also be the firm's manager; in the latter case, c should be interpreted as dividend payout, and the curvature in $u(.)$ captures dividend smoothing. In addition, increasing c can be interpreted as equity share repurchases while reducing c can be thought of as sales of new shares. Lintner (1956) first showed that managers take into account dividend smoothing over time, a fact further confirmed by subsequent studies. Putting curvature in $u(.)$ is thus a simple way of modeling the speed with which firms can vary the funding source when financial conditions change. The log utility further allows closed-form solution.[14]

14. Alternatively, one can follow Jermann and Quadrini (2012) with dividend adjustment costs, but there is no closed-form solution in that environment.

The gross return on capital is $R^k = r + 1 - \delta$, where $r \geq 0$ is profit/income per unit of capital, and $\delta \in [0, 1]$ is the depreciation rate. The gross return on risk-free bonds is R. The two returns R^k and R are, for now, exogenous. R will be endogenized later in this section, and R^k will be endogenized in the macroeconomic model. Importantly, adjusting capital is subject to both resale frictions and financial frictions.

Resale Frictions. The entrepreneur can invest in capital. But if the entrepreneur decides to sell capital, the whole firm has to be sold (note: this assumption will be relaxed in Section 4), i.e.,

$$k_{t+1} \in \{0\} \cup [(1 - \delta)k_t, +\infty).$$

If selling the firm, the entrepreneur incurs an i.i.d. *stochastic liquidation cost* $\zeta \in [\underline{\zeta}, \bar{\zeta}]$ across time with a cumulative distribution function $F(.)$. The time-varying ζ drives the entrepreneur to liquidate the firm when it is low and to stay in business when it is high. It is modeled as a utility cost for tractability.[15]

One interpretation of liquidation costs could be that capital is specific to certain production needs and the entrepreneur has to spend time and resources in searching for and negotiating with potential acquirers. These costs can also come from adviser fees (mostly through investment banks), legal fees, and post-acquisition integration/reorganization costs (e.g., HR costs and re-branding costs). In theory, one can model these costs through fees paid to intermediaries addressing search frictions (see, e.g., Cao and Shi 2016 and Cui and Radde 2019) and/or asymmetric information (see, e.g., Eisfeldt and Rampini 2008 and Kurlat 2013). Since the aim is to examine the impact of financial shocks on liquidation decisions, I abstract from the microfoundation of liquidation costs. Finally, notice that while in practice the cost is likely incurred by both buyers and sellers, the cost in the model is only paid by sellers, which makes the solution simple. The results are not sensitive to this modeling choice, because buyers (especially in the full macroeconomic model examined later) are indifferent between new capital and used capital.[16]

Financial Frictions. There is no insurance market for idiosyncratic risks, including shocks to liquidation cost and productivity to be introduced in the macroeconomic model. In this section, only liquidation-cost shock is considered for exposition simplicity. The only financial market is the credit market, where the entrepreneur can save and borrow. If the entrepreneur decides to borrow, debt must be collateralized due to limited commitment issues. That is, the entrepreneur's external borrowing is bounded, because of collateral constraints similar to those in Kiyotaki and Moore

15. This modeling strategy admits closed-form solutions, even when both option values and portfolio choices are present, as in this paper. Such simplification is not crucial. As an alternative, one can assume that ζ represents consumption goods. As long as it is proportional to the entrepreneur's net worth, we have identical analytical results according to the scale-invariant property in Proposition 1.

16. If we also ask buyers to pay a cost, the sellers have to sell assets at a discount relative to the price of new assets (which is one unit of consumption goods).

(1997) and Hart and Moore (1994). In case of default, the lender should be able to seize a fraction $\theta \in [0, 1-\delta]$ of the capital, where $1-\delta$ indicates that the seizure is after depreciation and reallocation. As a result of no "run away" default, the entrepreneur can only borrow up to the fraction θ of the residual capital at time $t+1$. Let b_t be the level of bonds held by the entrepreneur at the beginning of t. If $b_{t+1} < 0$, the entrepreneur borrows, and the collateral constraint implies a *lower* bound on b_{t+1}

$$Rb_{t+1} \geq -\theta k_{t+1}.$$

The Entrepreneur's Problem. Let $0 < \beta < 1$ be a subjective discount factor, $\mathbb{E}$ be a mathematical expectation operator taken over the random liquidation-cost draw, and $\mathbb{1}$ be an indicator function. The entrepreneur maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t) - \mathbb{1}_{\{\text{liquidating a firm}\}} \zeta_t\}.$$

Notice that ζ_t is drawn every period, but it only matters when the entrepreneur has a firm and may consider liquidating the firm. It is simpler to write the entrepreneur's problem in a recursive form. I omit subscript t and use x_{+1} to denote x_{t+1}. Let V be the optimal value of the entrepreneur with state (k, b, ζ) at the beginning of t. When the entrepreneur has a firm (i.e., $k > 0$), the problem consists of two actions: either to liquidate the firm and get a value of $V^0(k,b) - \zeta$; or to stay in business and get a value of $V^1(k,b)$. The entrepreneur chooses the higher of these two:

$$V(k,b,\zeta) = \max_{\text{not running business / running business}} \{V^0(k,b) - \mathbb{1}_{\{k>0\}}\zeta, \quad V^1(k,b)\}.$$

When the entrepreneur does not own a firm (i.e., $k = 0$), V^0 and V^1 respectively correspond to the values of continuing without a firm and starting a new firm.

As for V^0, we have the Bellman equation as

$$V^0(k,b) = \max_{c, b_{+1}} \{u(c) + \beta \mathbb{E}[V(0, b_{+1}, \zeta_{+1})]\} \tag{1}$$

$$\text{subj. to:} \quad c + b_{+1} = R^k k + Rb; \tag{2}$$

$$b_{+1} \geq 0, \tag{3}$$

in which the entrepreneur chooses consumption c and savings b_{+1} in bonds. (2) is the resource constraint with returns on capital $R^k k$ and on bonds Rb on the right-hand side. Notice that in the next period, $k_{+1} = 0$ and the liquidation-cost draw ζ_{+1} will not matter (but still written to be consistent with V's definition). Therefore, in the next period, the entrepreneur does not have collateral, and b_{+1} can only be non-negative as shown in (3). As for V^1, we have the Bellman equation as

$$V^1(k,b) = \max_{c, k_{+1}, b_{+1}} \{u(c) + \beta \mathbb{E}[V(k_{+1}, b_{+1}, \zeta_{+1})]\} \tag{4}$$

$$\text{subj. to:} \quad c + b_{+1} + k_{+1} = R^k k + Rb; \tag{5}$$

$$Rb_{+1} \geq -\theta k_{+1}; \tag{6}$$

$$k_{+1} - (1-\delta)k \geq 0, \tag{7}$$

in which the entrepreneur chooses optimal consumption and a portfolio of capital stock and bonds. In so doing, the entrepreneur needs to respect both the financing constraint (6) and the resale constraint (7). Note that (5) is the resource constraint and that $k_{+1} \geq 0$, which distinguishes (2) from (5).

3.2 The Entrepreneur's Decision Rules

I first derive some useful properties of the entrepreneur's value functions and decision rules. It is convenient to work with the *leverage ratio*, defined as

$$\lambda \equiv \frac{k}{k+b}.$$

Notice that $0 \leq \lambda \leq \bar{\lambda}$, where the lower bound implies that the entrepreneur does not have a firm, and the upper bound $\bar{\lambda} = \bar{\lambda}(\theta) \equiv (1-\theta/R)^{-1}$ is the highest leverage.

PROPOSITION 1. *The value functions have the following properties:*

$$V^0(k,b) = J^0 + \frac{\log N^0(k,b)}{1-\beta} \text{ and } V^1(k,b) = J^1\left(\frac{k}{k+b}\right) + \frac{\log N^1(k,b)}{1-\beta}, \tag{8}$$

where $N^0(k,b) \equiv rk + (1-\delta)k + Rb$ *and* $N^1(k,b) \equiv rk + q(k/k+b)(1-\delta)k + Rb$ *are net worths, where* J^0 *is a constant, and where* $J^1(k/k+b)$ *and* $q(k/k+b) \leq 1$ *do not depend on k or b separately and are functions of the leverage* $\lambda = k/(k+b)$*. Further,* $q < 1$ *means that the resale constraint is strictly binding. The consumption, capital, and bond policy functions have the following algebraic forms:*

$$c = \begin{cases} (1-\beta)N^0 & \textit{liquidating} \\ (1-\beta)N^1 & \textit{staying} \end{cases}; \quad k_{+1} = \begin{cases} 0 & \textit{liquidating} \\ \frac{\lambda_{+1}\beta N^1}{1+(q-1)\lambda_{+1}} & \textit{staying} \end{cases};$$

$$b_{+1} = \begin{cases} \beta N^0 & \textit{liquidating} \\ \frac{(1-\lambda_{+1})\beta N^1}{1+(q-1)\lambda_{+1}} & \textit{staying} \end{cases}.$$

Proof. See Appendix A. □

In words, the entrepreneur pays a $(1-\beta)$ fraction of the "economic" net worth N^0 or net worth N^1 as dividends and saves the other β fraction. N^1 evaluates capital at a shadow price $q \leq 1$, reflecting the tightness of the resale constraint. Further, (8) also implies a "scale-invariant" property. For any $\rho > 0$, the following is true

$$V(\rho k, \rho b, \zeta) = V(k,b,\zeta) + \frac{\log \rho}{1-\beta}. \tag{9}$$

That is, if the entrepreneur has a new firm that is twice as large as the previous one (assuming the two firms have the same productivity), all choices are scaled up exactly by two (though the value increases by $\log 2/(1-\beta)$). Thanks to this scale invariant property, when characterizing liquidation decisions, I only need to consider an entrepreneur with $k = 1$, leverage λ (so that $b = \lambda^{-1} - 1$), and liquidation cost ζ.

3.3 When to Liquidate An Unproductive Firm?

In order to focus on the liquidation decision, I assume that the firm is not productive compared to the return on bonds:

$$R^k \equiv r + 1 - \delta < R. \tag{A1}$$

Therefore, it is equivalent to think of the entrepreneur's problem as a real option problem of abandoning a permanently unproductive firm (e.g., a firm that receives a permanent productivity drop). With A1, the entrepreneur wants to sell the existing firm, but the liquidation cost may be too high. Waiting for a low liquidation cost in the future generates an *option value of staying in business*. Through its effect on the option value, the tightness of financing constraints determines when (or, with what liquidation cost) the entrepreneur should liquidate the firm. In Section 4, firms will receive idiosyncratic productivity, so A1 will be relaxed, and the option value also takes into account a future productivity rebound.

Understanding Financing and Resale Constraints. The resale constraint (7) may interact with the financing constraint (6). To see this, suppose the resale constraint (7) binds for an entrepreneur staying in business, and then the resource constraint becomes

$$c + b_{+1} = rk + Rb.$$

That is, this entrepreneur uses "cash on hand" $rk + Rb$ to finance the dividend payout c and the savings b_{+1} in bonds. Under A1, $r < R - (1 - \delta)$ so that the "cash on hand" is not sufficient and thus the dividend payout c needs to be cut. But it might not be easy to do so (or correspondingly, to issue new equity shares), because of the curvature in adjusting the dividend represented by $u(.)$. The entrepreneur thus needs to borrow to smooth dividends. As a result, the resale and financing constraints can simultaneously bind, since the capital return can be so low that b_{+1} from the above resource constraint has to reach the lower bound implied by (6).

Then, there are two possibilities. First, the entrepreneur is able to stay, with both the financing constraint (6) and the resale constraint (7) binding. This can happen if the productivity/profit of the firm measured by r is not too small. Second, the entrepreneur is forced to liquidate the whole firm when r is rather small, because the dividend payout could be non-positive even if the entrepreneur chooses to stay and borrow to the limit. But non-positive dividends are impossible and the firm will have to be liquidated in this case, since the marginal value of dividends is negative infinity when $c \to 0$.

For any leverage limit $\bar{\lambda}$, the highest beginning-of-period leverage without forced liquidation is denoted as $\Psi(\bar{\lambda})$. With this leverage, a firm with unit capital and bonds $1/\Psi(\bar{\lambda}) - 1$ still has to borrow to the credit limit even if no dividend is paid, i.e.,

$$\bar{\lambda}\left[r + 1 - \delta + R\left(\frac{1}{\Psi(\bar{\lambda})} - 1\right)\right] = 1 - \delta,$$

where the left-hand side says no resource is spent on c and $1-\delta$ comes from the fact that the firm is not liquidated. Then, we can write

$$\Psi(\bar{\lambda}) \equiv \frac{R\bar{\lambda}}{1-\delta+\left(R-R^k\right)\bar{\lambda}}.$$

I focus on $\lambda < \Psi(\bar{\lambda})$ from now on to rule out forced liquidation in order to highlight the interaction between the resale constraint and the financing constraint.[17]

It is worth mentioning that the general idea is not restrictive to a curvature in $u(.)$ in the context of financial and resale frictions; these are assumed here for tractability reasons. One can alternatively assume that firms have certain fixed overhead costs that need to be paid as long as the firm is in operation, without restricting the non-negative dividend/consumption smoothing incentives (under log utility). The main message in this alternative case will be similar: the financial flexibility is low when there are certain non-flexible costs in operation. Then, those unproductive firms, if staying in business, are likely to borrow to the limit to finance these costs. For this reason, the main results shown below still hold, even if we allow for a different dividend-smoothing elasticity or allow for the possibility of a zero dividend.

Liquidation Threshold. Proposition 1 allows me to normalize $k = 1$ for those with a firm, and I denote $\omega^0(\lambda) \equiv V^0(1, \lambda^{-1} - 1)$ as the (before-liquidation-cost) value to the entrepreneur if *the firm were sold,* and I denote $\omega^1(\lambda) \equiv V^1(1, \lambda^{-1} - 1)$ as the value to the entrepreneur if *the firm were kept in place*. The decision to liquidate the firm is irreversible; in the period after the liquidation, the entrepreneur cannot liquidate the firm again even if drawing a smaller liquidation cost. The decision to delay liquidation, however, is reversible. This asymmetry leads to a simple liquidation rule: liquidate the firm only if the liquidation cost is small enough. It will thus be only optimal to sell the firm if $\omega^0(\lambda)$ exceeds $\omega^1(\lambda)$ by a positive amount. That is, the *net gain from liquidation* today $\omega^0(\lambda) - \omega^1(\lambda)$ should be larger than a threshold to encourage liquidation. In the following, I show the liquidation threshold, which turns out to be recursive because $\omega^1(\lambda)$ includes future liquidation thresholds.[18]

17. This is not saying that forced liquidation never happens in practice, but implies that it is not the main reason for acquisition. If forced liquidation accounts for most acquisitions, then we should observe an increase in acquisitions when firm debt falls (arising from a tighter financing constraint in my interpretation). However, the data seems to suggest the opposite. In the macroeconomic model, partial sales by firms can partly take into account forced liquidation.

18. Calculating the threshold $\tilde{\xi}$ is more useful for studying capital reallocation than calculating the exact option value. The option value at a particular time t is the expected value of $\omega^0(\lambda_\tau) - \xi_\tau - \omega^1(\lambda_\tau)$ when first exercising the option at a random time τ. This logic follows McDonald and Siegel (1986).

Using the Bellman equation (1) and the property of $V^0(k,b)$ in (8), I can write

$$\omega^0(\lambda) = \log\left((1-\beta)n^0(\lambda)\right) + \beta\left[J^0 + \frac{\log\left(\beta n^0(\lambda)R\right)}{1-\beta}\right], \quad (10)$$

$$\text{where } n^0(\lambda) \equiv N^0\left(1, \frac{1}{\lambda}-1\right) = R^k + R\left(\frac{1}{\lambda}-1\right) = R^k - R + \frac{R}{\lambda}.$$

To understand (10), first notice that the entrepreneur consumes $c^0 = (1-\beta)n^0$, which yields the utility of consumption. Second, βn^0 is saved in risk-free bonds today, earning the gross return R. That is why next period we have $\log\left(\beta n^0 R\right)/(1-\beta)$, in which the denominator $(1-\beta)$ comes from the form of the value function in (8). Finally, to interpret the net worth n^0, notice that the total amount of assets is $1+b = 1/\lambda$, earning the basic return R; the one unit of capital also earns an excess return $R^k - R$; the two parts thus generate n^0.

Using the Bellman equation (4) and the property of $V^1(k,b)$ in (8), I can write

$$\omega^1(\lambda) = \log\left(n^0(\lambda) - \frac{1-\delta}{\lambda_{+1}}\right) + \frac{\beta\log(1-\delta)}{1-\beta} + \beta\int_{\underline{\zeta}}^{\tilde{\zeta}(\lambda_{+1})}\left[\omega^0(\lambda_{+1}) - x\right]dF(x) + \beta\int_{\tilde{\zeta}(\lambda_{+1})}^{\bar{\zeta}}\omega^1(\lambda_{+1})dF(x), \quad (11)$$

where the future threshold $\tilde{\zeta}(\lambda_{+1})$ appears. When the entrepreneur keeps the firm today, capital becomes $(1-\delta)$, which explains the future value $\beta\log(1-\delta)/(1-\beta)$ after I use the scale property (9). This means that we can normalize tomorrow's capital to be unity again and use $\omega^0(.)$ and $\omega^1(.)$, which are in the integrals of (11). To see this, notice that the target sum of capital $(1-\delta)$ and bonds $(1-\delta)\left(1/\lambda_{+1}-1\right)$ amounts to $(1-\delta)/\lambda_{+1}$, and the dividend/consumption in this case is the difference between the net worth and the target sum, i.e., $c^1 = n^0 - (1-\delta)\lambda_{+1}^{-1}$. Tomorrow, the value will either be $\omega^0(\lambda_{+1}) - \zeta$ when $\zeta < \tilde{\zeta}(\lambda_{+1})$ and the firm is liquidated; or $\omega^1(\lambda_{+1})$ when $\zeta \geq \tilde{\zeta}(\lambda_{+1})$ and the firm is kept.

Importantly, $c^1 < c^0$, which can be seen from the consumption policy function. That is, $c^0 = (1-\beta)\left(r+1-\delta-R+R/\lambda\right)$ if the firm is sold, and $c^1 = (1-\beta)[r+(1-\delta)q - R + R/\lambda]$ if the firm stays in business as well as $q<1$ since the resale constraint is binding. The consumption c^1 has to be smaller because the productivity of the firm is low relative to the interest rate (recall A1), and the firm needs to service its debt. Staying in business is thus painful, but the liquidation cost may force the firm to stay. To calculate the liquidation cost threshold, I subtract (11) from (10).

PROPOSITION 2. *Suppose A1 holds. For any $\lambda \in [0, \Psi(\bar{\lambda}))$, there exists a unique threshold value $\tilde{\zeta}(\lambda) = \min\{\max\{\omega^0(\lambda) - \omega^1(\lambda), \underline{\zeta}\}, \bar{\zeta}\}$ such that the entrepreneur liquidates the entire firm when drawing $\zeta < \tilde{\zeta}(\lambda)$. If $\omega^0(\lambda) - \omega^1(\lambda) \in \left(\underline{\zeta}, \bar{\zeta}\right)$, then:*

$$\tilde{\zeta}(\lambda) = \log\left(\frac{(1-\beta)n^0(\lambda)}{n^0(\lambda) - (1-\delta)/\lambda_{+1}}\right) + \frac{\beta}{1-\beta}\log\left(\frac{R}{n^0(\lambda_{+1})}\right) + \frac{\beta}{1-\beta}\log\left(\frac{\beta n^0(\lambda)}{1-\delta}\right) + \beta\left[\tilde{\zeta}(\lambda_{+1}) - \int_{\underline{\zeta}}^{\tilde{\zeta}(\lambda_{+1})} F(x)dx\right]. \quad (12)$$

Proof. See Appendix A. □

(12) links the leverage with liquidation threshold. I use that to understand the effect of financial shocks on the liquidation strategy. The first term on the RHS of (12) is the utility difference from consumption between the two choices, liquidating and staying. The second term represents the relative continuation value from net worth tomorrow. The third term comes from the fact that the relative continuation value needs to be adjusted by different levels of wealth. That is, in the case of liquidation, we have $\beta n^0(\lambda)$; in the case of staying, the capital becomes $1-\delta$. Finally, the last term on the RHS comes from future liquidation costs: $\tilde{\zeta}(\lambda_{+1})$ only applies when $\zeta_{+1} \geq \tilde{\zeta}(\lambda_{+1})$, and that is why $\int_{\underline{\zeta}}^{\tilde{\zeta}(\lambda_{+1})} F(x)dx$ is subtracted.

(12) implies a trade-off from holding on to an unproductive firm. On the one hand, the entrepreneur needs to borrow to smooth dividend payouts today since the firm is unproductive, increasing the debt burden tomorrow. On the other hand, the entrepreneur does not have to pay the liquidation cost today. Considering the possibility that the liquidation cost in the future will become smaller, the entrepreneur derives an option value of staying. The entrepreneur finds gambling for tomorrow's low liquidation cost not worthwhile when the cost of choosing the safer option (i.e. liquidation) is smaller than a threshold shown in (12).

We will see that the staying unproductive firm's leverage goes up, which affects the liquidation threshold. Intuitively, the higher the leverage to begin with, the tougher the situation is if the firm stays; therefore, the liquidation incentive increases with leverage λ, or the threshold $\tilde{\zeta}$ goes up with leverage. In other words, a higher leverage λ generally implies a higher likelihood of liquidation (see the numerical example in Figure 2 below and those in Appendix A).

3.4 The "Dually Constrained" Outcome and Adverse Financial Shocks

Using (12), I can study the impact of financial shocks (to borrowing constraints) on liquidation strategy. The following discussion shows that adverse financial shocks encourage liquidation on impact but delay liquidation later on.

Consider adverse financial shocks, which may come from the spillover from the collapse of bubbles in another sector (e.g., the financial sector). Depending on the liquidation cost and leverage, the firm may find it optimal to liquidate; or the firm may find it optimal to stay, deleveraging or accumulating leverage more slowly. On impact of the shocks, if the firm (which has been financially constrained) chooses to stay and bear the painful cut in c, in the future it will have a lower leverage and more financial flexibility so that the future liquidation threshold can drop. In other words, the probability of liquidation can fall later.[19]

To begin with, let us make the financing constraint tight enough:

$$\bar{\lambda} < \lambda^* \equiv \frac{R-(1-\delta)}{R-R^k} = \Psi(\lambda^*), \tag{A2}$$

where λ^* is solved from $\lambda^* = \Psi(\lambda^*)$. Notice that under A1 and A2 the threshold $\bar{\lambda} < \Psi(\bar{\lambda})$. There will not be forced liquidation if we focus on the meaningful range of leverage $\lambda \in [0, \bar{\lambda}]$.[20]

The Dually Constrained Outcome. As briefly discussed before, if the firm is sufficiently unproductive, it will not have much financial flexibility when staying in business, and it will have to borrow and could thus be financially constrained. The following proposition shows that the entrepreneur's leverage has to go up if the firm is not liquidated; when the beginning-of-period leverage λ is sufficiently high, the entrepreneur is financially constrained today, i.e., $\lambda_{+1}(\lambda) = \bar{\lambda}$.

PROPOSITION 3. *Suppose A1 and A2 hold, and $\beta R < 1-\delta$. Suppose the entrepreneur draws a high liquidation cost $\zeta \geq \tilde{\zeta}(\lambda)$ so that the firm stays in operation (i.e., the resale constraint binds). For any $\lambda \in [0, \bar{\lambda})$, the end-of-period leverage must be higher $\lambda_{+1}(\lambda) > \lambda$. Finally, for $\lambda \in [\psi(\bar{\lambda}), \bar{\lambda}]$, the entrepreneur's financing constraint binds today, i.e., $\lambda_{+1}(\lambda) = \bar{\lambda}$, where $\psi(\bar{\lambda})$ is defined as*

$$\psi(\bar{\lambda}) \equiv \frac{\bar{\lambda}}{\frac{1-\delta}{\beta R} - \left(\frac{1-\delta}{\beta R} - 1\right)\left[\frac{1-\delta}{R} + \frac{\bar{\lambda}}{\lambda^*}\left(1 - \frac{1-\delta}{R}\right)\right]} < \bar{\lambda}.$$

Proof. See Appendix A. □

Leverage increases if the cash-poor firm stays in business. As implied by Proposition 3, the entrepreneur indeed may immediately hit the borrowing limit $\bar{\lambda}$ today, as long as the firm starts with a high enough leverage $\lambda \geq \psi(\bar{\lambda})$ and the entrepreneur is relatively impatient ($\beta R < 1-\delta$). A smaller β means that the entrepreneur needs to finance more dividends (recall $(1-\beta)$ fraction) out of net worth. Thus the firm has to borrow more to pay back previous debt and smooth (or, in this case, front-load) dividends as much as possible. Of course, the entrepreneur can sell the firm immediately to reduce the debt burden today, but the resale cost may prevent him/her from doing so. Immediately or after staying in operation for a few periods, the firm will be financially constrained. If the firm is less productive (i.e., R^k decreases), λ^* falls;

19. Notice that, in the macroeconomic model introduced later, such declining incentives to liquidate after adverse financial shocks determine the persistent drop in reallocation and productivity.

20. In the macro model, an entrepreneur can act in a precautionary way and reduce leverage when the firm is productive, preventing forced liquidation in the future.

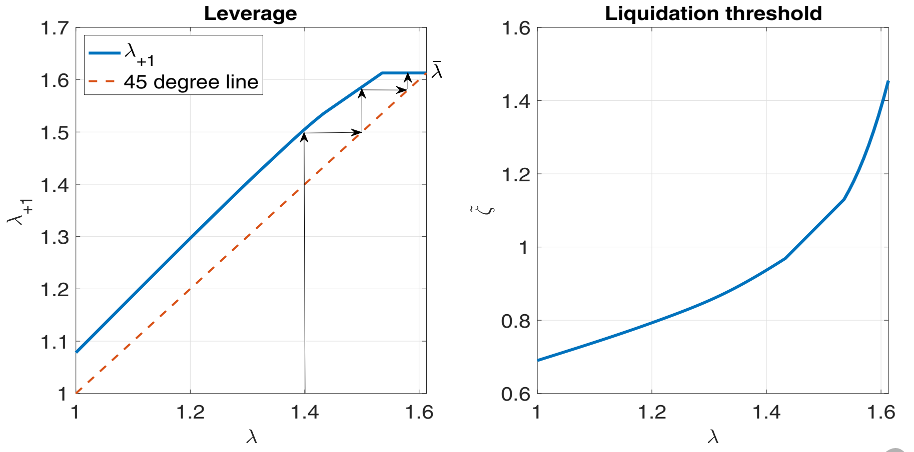

FIGURE 2. An example leverage and liquidation policy. In this example, $\delta = 0.10$, $\theta = 0.38$, $\beta = 0.85$, $R = 1.00$, and $R^k = R/1.03 \approx 0.97$. ζ is drawn from a uniform distribution with support $[0, 2]$. The arrows show a path of leverage dynamics if the entrepreneur is not immediately financially constrained and also chooses to keep the firm in each period.

$\psi(\bar{\lambda})$ becomes smaller and there is a wider range of today's leverage $\lambda \in [\psi(\bar{\lambda}), \bar{\lambda}]$ that makes the entrepreneur dually constrained, simply because the firm is even more cash-poor and has to rely more on borrowing.

Notice that $\psi(\bar{\lambda})$ is a convenient lower bound.[21] The exact leverage lower bound with which an entrepreneurs is just financially constrained could be smaller. The complete characterization of whether an entrepreneur is financially constrained for arbitrary leverage λ requires solving $\lambda_{+1}(\lambda)$, $q(\lambda)$, and $\tilde{\zeta}(\lambda)$ simultaneously. To illustrate, Figure 2 shows a numerical example of leverage dynamics: the leverage upper bound is $\bar{\lambda} = 1.613$ with the parameterization shown in the figure's note; and $\psi(\bar{\lambda}) = 1.608$. However, with leverage $\lambda = 1.535 < \psi(\bar{\lambda})$, the staying entrepreneur is already financially constrained at the end of the period. If not constrained immediately, the entrepreneur builds up leverage as indicated by the arrows. When leverage is higher, the liquidation threshold is higher as well.

Adverse Financial Shocks. Consider a tightening of the financing constraint, i.e., a fall in θ from θ^h to θ^l that changes the highest leverage $\bar{\lambda} = \bar{\lambda}(\theta) = [1 - \theta/R]^{-1}$. Suppose the exogenous credit limit $\bar{\lambda}$ suddenly drops permanently from a high level λ^h to a low level λ^l from $t = 0$. The left panel of Figure 3 shows the time path of $\bar{\lambda}$. Suppose λ^l is not too far from λ^h.[22]

21. $\psi(\bar{\lambda})$ is derived such that the shadow capital return next period will be the same as the interest rate R if the firm stays in business again.

22. That is, I again rule out forced liquidation, or $\lambda^h < \Psi(\lambda^l)$ so that the "old" leverage λ^h is still below the threshold leverage of forced liquidation implied by λ^l.

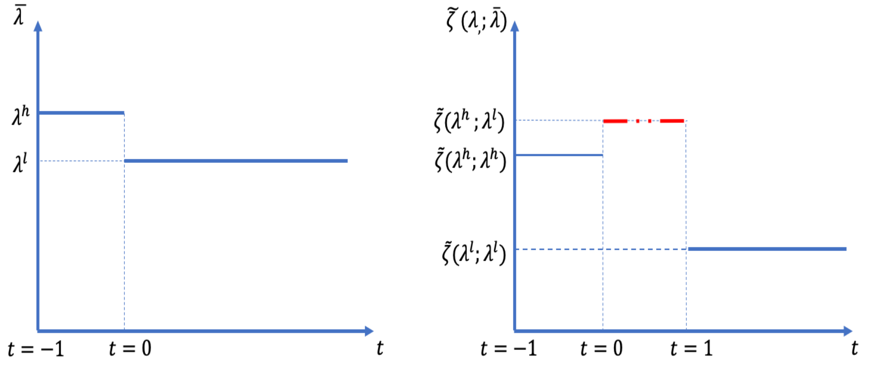

FIGURE 3. A permanent tightening of credit with a fixed interest rate. The unproductive firm (with $R^k < R$) has a leverage constraint measured by $\bar{\lambda} = \lambda^h$ before time $t = 0$. From $t = 0$ onward, the credit constraint is permanently tightened so that $\bar{\lambda} = \lambda^l$. The second panel shows the liquidation threshold if the firm is not liquidated.

I examine the time path of the liquidation threshold $\tilde{\zeta}$. Because of the variation in $\bar{\lambda}$, the value function $\omega^1(\lambda)$ now depends on the "state" of the credit constraint implied by $\bar{\lambda}$, as will $\tilde{\zeta}(\lambda)$. Similar to an aggregate state that an individual entrepreneur cannot influence, I use $\bar{\lambda}$ after a semi-colon in functions to indicate the exogenous credit limit. That is, the notations $\omega^1(\lambda;\bar{\lambda})$ and $\tilde{\zeta}(\lambda;\bar{\lambda})$ will be used.

For simplicity, let us look at sufficiently high λ, i.e., $\lambda > \psi(\bar{\lambda})$, so that at the end of the period the firm hits the leverage upper bound, i.e., $\lambda_{+1} = \bar{\lambda}$. We can solve for the liquidation threshold $\zeta\left(\lambda;\bar{\lambda}\right)$ through the following strategy.

First, start with $\lambda = \bar{\lambda}$ and compute $\tilde{\zeta}(\bar{\lambda};\bar{\lambda}) = \omega^0(\bar{\lambda}) - \omega^1(\bar{\lambda};\bar{\lambda})$. Why do we solve for the threshold $\tilde{\zeta}(\bar{\lambda};\bar{\lambda})$ first? With a high enough beginning-of-period leverage today, the firm hits $\bar{\lambda}$ at the end of today; then the firm is going to start with leverage $\bar{\lambda}$ tomorrow, hitting $\bar{\lambda}$ again if it is not going to be liquidated tomorrow. Therefore, the threshold $\tilde{\zeta}(\bar{\lambda};\bar{\lambda})$ is the liquidation threshold from tomorrow onward, and it is the "steady-state" threshold that should be derived first. After simplifying (12), we know that $\tilde{\zeta}(\bar{\lambda};\bar{\lambda})$ solves $\tilde{\zeta}$ in the following condition:

$$(1-\beta)\tilde{\zeta} + \beta\int_{\underline{\zeta}}^{\tilde{\zeta}} F(x)dx = \log\left(\frac{(1-\beta)\left[\bar{\lambda}\left(R^k - R\right) + R\right]}{\bar{\lambda}\left(R^k - R\right) + R - (1-\delta)}\right) + \frac{\beta\log\left(\frac{\beta R}{1-\delta}\right)}{1-\beta}. \tag{13}$$

The LHS, a function of the liquidation threshold $\tilde{\zeta}$, collects the steady-state liquidation threshold. The RHS, a function of $\bar{\lambda}$, includes both the gain from higher consumption today and the gain from savings if the firm is liquidated. Second, after deriving $\tilde{\zeta}(\bar{\lambda};\bar{\lambda})$, I solve for the liquidation threshold $\tilde{\zeta}(\lambda;\bar{\lambda})$ for leverage $\lambda \in [\psi(\bar{\lambda}), \bar{\lambda}]$ today. One can go back to (12), and today's liquidation threshold $\tilde{\zeta}(\lambda;\bar{\lambda})$ is obtained by setting $\lambda_{+1} = \bar{\lambda}$.

We can now characterize the behavior of the liquidation threshold shown in the right panel of Figure 3, thanks to the following proposition.

PROPOSITION 4. *Suppose A1 and A2 hold, and $\beta R < (1-\delta)$. We have $\partial\tilde{\xi}(\lambda;\bar{\lambda})/\partial\bar{\lambda} < 0$ for any given $\lambda \in [\psi(\bar{\lambda}),\bar{\lambda}]$, and $\partial\tilde{\xi}(\bar{\lambda};\bar{\lambda})/\partial\bar{\lambda} > 0$ holds.*

Proof. See Appendix A. □

Using the result of Proposition 4, let us examine the experiment in which the financial constraint is permanently tighter. The economy starts with the old "steady-state" liquidation threshold $\tilde{\xi}(\lambda^h;\lambda^h)$. At $t=0$ shown in Figure 3, the threshold $\tilde{\xi}$ goes up when $\bar{\lambda}$ drops on impact, i.e., $\tilde{\xi}(\lambda;\lambda^l) > \tilde{\xi}(\lambda;\lambda^h)$, since $\partial\tilde{\xi}(\lambda;\bar{\lambda})/\partial\bar{\lambda}$ is negative from the proposition. It is more likely that the firm is liquidated on impact of the adverse financial shock. Intuitively, for a given initial leverage λ, tighter credit conditions make staying in business more painful as the firm will have to cut the dividend payout today. Unambiguously, the tightened leverage constraint raises the net gain from liquidation $\omega^0(\lambda)-\omega^1(\lambda;\bar{\lambda})$, for any leverage $\lambda \in [\psi(\bar{\lambda}),\bar{\lambda}]$. To understand further, notice that $\lambda^l < \lambda^h$ was always in the choice set when $\bar{\lambda}=\lambda^h$. The reason that λ^l is not chosen when $\bar{\lambda}=\lambda^h$ must be that $\omega^1(\lambda^l;\lambda^h) < \omega^1(\lambda^h;\lambda^h)$ as the dividend payout is lower by choosing λ^l when a higher leverage λ^h is available.

Except in the initial period of the shock, the entrepreneur actually has a stronger incentive to keep the firm running. At $t=1$, the firm has a beginning-of-period leverage $\lambda_1=\lambda^l$ if it was not liquidated at $t=0$. If the entrepreneur also chooses to stay in period $t=1$, then the end-of-period leverage λ_2 will hit the credit limit λ^l again. Therefore, the liquidation threshold at $t=1,2,3,...$ becomes $\tilde{\xi}(\lambda^l;\lambda^l)$. Proposition 4 shows that $\tilde{\xi}(\lambda^l;\lambda^l) < \tilde{\xi}(\lambda^h;\lambda^h)$. The tighter credit conditions have two effects on the "steady-state" threshold in (13). On the one hand, a smaller $\bar{\lambda}$ implies a tighter borrowing limit, reflected by the second argument in $\tilde{\xi}(\bar{\lambda};\bar{\lambda})$. The lower borrowing limit worsens the dividend payout in each period, pushing down the value of staying relative to the value of liquidating (i.e., increasing the RHS in (13)). On the other hand, if the entrepreneur does not liquidate, a lower $\bar{\lambda}$ also implies deleveraging at $t=0$ and a lower debt servicing cost every period later, reflected by the first argument of $\tilde{\xi}(\bar{\lambda};\bar{\lambda})$. For any given liquidation cost, the staying option is less painful once the leverage has adjusted after the initial financial shock. The proposition implies that the second effect dominates. The reason is that the firm's capital return is low relative to the risk-free rate (according to A1); when a firm decides to delay liquidation, the entrepreneur mainly worries about the debt servicing cost before liquidation. After the adjustment, a lower leverage to start with every period generates a lower debt servicing cost every period, which can give more room for dividend smoothing even if the credit constraint is permanently tighter.

To summarize, on impact at $t=0$, the firm starts with leverage $\lambda_0=\lambda^h$, and the tighter financing constraint makes it more likely to be liquidated, i.e., the threshold $\tilde{\xi}_0=\tilde{\xi}(\lambda^h;\lambda^l)$ is above both the old steady-state threshold $\tilde{\xi}_{-1}=\tilde{\xi}(\lambda^h;\lambda^h)$ and the new threshold $\tilde{\xi}(\lambda^l;\lambda^l)$. For $t \geq 1$, a firm staying in business will start with the leverage

λ^l and also end up at λ^l; the new "steady-state" liquidation threshold is below the old one, i.e., $\tilde{\zeta}_t = \tilde{\zeta}(\lambda^l; \lambda^l) < \tilde{\zeta}(\lambda^h; \lambda^h) = \tilde{\zeta}_{-1}$.

3.5 Introducing Credit Market Equilibrium

To close this section, I study financial shocks with credit market equilibrium that endogenizes the interest rate. Unlike the previous discussion, the liquidation threshold can also fall on impact of the adverse financial shocks. This can happen when the interest rate is depressed enough by the shocks, reducing the benefit of liquidation and further pushing up the option value of staying. An unproductive firm is thus less likely to be liquidated in all periods following the shocks.

Suppose there is a continuum of entrepreneurs with firms, and each faces the same problem discussed before. All these firms have total capital stock K every period. That is, in each period t, entrepreneurs that liquidate their firms in period $t-1$ are replaced by entrepreneurs with the same amount of capital liquidated so that the total amount of capital stock stays the same every period. Additionally, these new firms have the same leverage as other existing firms. Suppose further that there is an exogenous supply of credit B each period. The exact level of B does not matter for our result. These assumptions simplify the discussion, while still featuring the impact of credit demand on the interest rate. To see this, the market clearing condition for bonds is

$$(\lambda_{+1}^{-1} - 1)K(1-\delta)\left[1 - F(\tilde{\zeta})\right] + \beta(R^k - R + R\lambda^{-1})KF(\tilde{\zeta}) + B = 0, \qquad (14)$$

where the first term on the LHS is the demand for credit from staying firms (note: there is a fraction $1 - F(\tilde{\zeta})$ of capital from them), the second term is the supply of credit from those entrepreneurs who liquidate this period, and the last term B is the exogenous supply of credit. This market clearing condition, as will be clear soon, is a simplified version of the one in the macroeconomic model. Unlike the macroeconomic model, K and B are *not* endogenous state variables yet; in the macroeconomic model, B includes supply of credit from entrepreneurs who sold their firms in the past.

As before, we look at the dually constrained economy so that the financing constraint binds for those entrepreneurs who do not liquidate (and therefore are against their resale constraint). According to (14), the endogenous interest rate R_{+1} is a function of the threshold $\tilde{\zeta}$ by setting $\lambda_{+1} = \bar{\lambda}$. We then replace R_{+1} in the liquidation threshold of (12) and obtain a single recursive equation for the threshold $\tilde{\zeta}$.

The steps to solve the threshold are again straightforward. (i). We look at $\theta = \theta^h$ and solve for the steady-state liquidation threshold, which is also the threshold $\tilde{\zeta}_t$ for $t \leq -1$. (ii). We then look at $\theta = \theta^l$ and again solve for the steady-state liquidation threshold, which is the threshold $\tilde{\zeta}_t$ for $t \geq 1$. (iii). Finally, we compute the threshold $\tilde{\zeta}_0$ at time $t = 0$, when the staying entrepreneurs deleverage because of adverse financial conditions. The result can be summarized in the following proposition.

PROPOSITION 5. *Suppose A1 and A2 hold, the credit market is in equilibrium captured by (14), and the economy features a dually constrained outcome with $\theta = \theta^h$ before*

$t = 0$. *From* $t = 0$, θ *permanently falls from* θ^h *to* θ^l *in a neighborhood of* θ^h. *Then,* $\tilde{\zeta}_t < \tilde{\zeta}_{-1}$ *for any* $t \geq 0$, *as well as* $R_1 < R_0$.

Proof. See Appendix A. □

As before, the negative financial shocks push down the liquidation threshold for periods $t \geq 1$, that is, $\tilde{\zeta}_t < \tilde{\zeta}_{-1}$. The new result is that on impact, the liquidation threshold $\tilde{\zeta}_0$ is also below $\tilde{\zeta}_{-1}$. This result comes from the fact that a negative financial shock also reduces credit demand, pushing down the interest rate R_1 relative to R_0. A lower interest rate implies a lower return on savings from liquidation and a lower debt servicing cost; both of these make the staying option more attractive than the liquidating option. Therefore, negative financial shocks can also push down the liquidation threshold at $t = 0$.

Both the liquidation threshold and the interest rate fall on impact at $t = 0$. But also notice that when entrepreneurs are less willing to liquidate capital, there is less supply of credit as well. The supply effect may or may not dominate the effect of credit demand from financial shocks. Therefore, the relationship between $\tilde{\zeta}_0$ and $\tilde{\zeta}_t$ for $t \geq 1$ is ambiguous; this also applies to the relationship between R_1 and the new "steady-state" interest rate R_{t+1} for $t \geq 1$.[23]

To close this section, note that other types of financial shocks can simultaneously reduce credit and enlarge the spread between the borrowing interest rate and the savings interest rate (e.g., Gilchrist and Zakrajšek (2012) and Cui and Kaas (2020)). But a higher borrowing rate, putting upward pressure on debt servicing costs, does not immediately imply that unproductive firms are more willing to liquidate. First, leverage still falls, so that the debt servicing costs for firms can still be lower in the future if they choose to stay in business today. Second, more importantly, besides the borrowing rate, what also matters for the option value is the benefit from saving in risk-free assets after liquidation. A lower savings interest rate may still reduce the attractiveness of the liquidation option. The ambiguity requires future research.

4 General Equilibrium with Aggregate Shocks

Now, I extend the simple partial-equilibrium model into a more realistic general-equilibrium model with aggregate shocks. Capital liquidated from one firm becomes productive capital in another firm. The model can thus be used to study the dynamics of capital reallocation, total factor productivity (TFP), and aggregate output.

23. For example, the interest rate can fall initially and then fall further: $R_0 > R_1 > R_2 = R_3 = R_4 \ldots$. Since entrepreneurs are less willing to liquidate, there is less supply of credit. If $\tilde{\zeta}_0 < \tilde{\zeta}_1$ occurs, the supply effect in period 0 is stronger than that in period 1, and the interest rate R_1 can be above R_2. Expecting a further lower interest rate in period 1 onward, entrepreneurs in period 0 indeed are less likely to liquidate their firms than in period 1, justifying $\tilde{\zeta}_0 < \tilde{\zeta}_1$.

4.1 The Extended Environment

The economy is now populated by a continuum of entrepreneurs and a continuum of households, and they have population measure m and measure one, respectively. The households do not face any idiosyncratic risk, so I can focus on a representative household. Let X be the aggregate state variable (with details discussed later) whose law of motion is taken as given by all agents.

The Entrepreneur's Problem. If an entrepreneur i has a firm at the beginning of t, the firm uses capital stock $k(i)$ installed at $t-1$ and hires labor hours $\ell(i)$ to produce general consumption goods at the beginning of t, according to a constant-return-to-scale (CRS) production technology

$$y(i) = [z(i)k(i)]^{\alpha} [A\ell(i)]^{1-\alpha},$$

where $\alpha \in (0,1)$ is the capital share in the production function, A is the level of aggregate labor augmenting productivity, and $z(i) \in \{z^l, z^h\}$ follows a two-state Markov process. The idiosyncratic productivity risk features an extra reason for staying in the business for unproductive firms with z^l, as productivity of the firm might improve in the future. For notation consistency, powers are always used together with brackets.[24] Aggregate and idiosyncratic productivities are common knowledge.

All entrepreneurs take the competitive wage rate $w = w(X)$ as given, and entrepreneur i's profits after production are represented by

$$\Pi(z(i), k(i); w) = [z(i)k(i)]^{\alpha} [A\ell(i)]^{1-\alpha} - w\ell(i).$$

Online Appendix C shows that an individual entrepreneur's profits are still linear in $k(i)$, i.e., $\Pi(z(i), k(i); w) = z(i)\pi(w)k(i)$, where the associated labor demand and profit rate are given by

$$\ell^*(i) = \frac{z(i)k(i)}{A}\left[\frac{(1-\alpha)A}{w}\right]^{\frac{1}{\alpha}} \text{ and } \pi = \alpha\left(\frac{(1-\alpha)A}{w}\right)^{\frac{1-\alpha}{\alpha}}. \tag{15}$$

The output produced by firm i is thus $y(i) = \pi z(i)k(i)/\alpha$, of which $\pi z(i)k(i)$ is claimed by firm i. In contrast to Section 3, the return $r = z\pi$ is now endogenous, and it is firm specific.

The simple model of Section 3 abstracts from forced liquidation and/or fire sales.[25] Partial sales can reflect these aspects. Entrepreneurs can now choose partial liquidation up to a fraction $\varphi \in [0,1]$ of their capital. This means that if firms delay full liquidation, they can use partial sales to partly fund their expenses. In addition, I also consider a resale discount d with partial liquidation, which reflects the fact that forced liquidation

24. For example, z^l does *not* mean z raised to the power of l, but it is used to indicate the low productivity.

25. This does not mean that firms are never forced to be liquidated (e.g., a fire sale) in practice. Those firms in the productivity left tail are, of course, more likely to be liquidated after adverse financial shocks.

may put sellers in an unfavorable bargaining position as they need to sell assets in a short period of time. That is, the capital is sold in pieces that do not reflect the capital's full productivity; additionally, the capital may be purchased by a firm in a different industry that cannot make the best use of it (see, e.g., Shleifer and Vishny 1992). I do not consider d to be time-varying as previous findings suggest that reallocation costs as a cause can explain a limited part of endogenous productivity dispersion and fluctuations (e.g., Cooper and Schott 2013 and David and Venkateswaran 2019).

Let V be the optimal value of an entrepreneur with individual state (k, b, z, ζ), given the aggregate state X. The value function $V(k, b, z, \zeta; X)$ satisfies

$$V(k,b,z,\zeta;X) = \max_{\text{not running a firm / running a firm}} \{V^0(k,b,z;X) - \mathbb{1}_{\{k>0\}}\zeta, \quad V^1(k,b,z;X)\}.$$

V^0 represents the value of liquidating the existing firm (excluding the liquidation cost) when $k > 0$, and it can also represent the value of not running a firm when $k = 0$:

$$\begin{aligned} V^0(k,b,z;X) = \max_{c,b_{+1}} \quad & \{u(c) + \beta\mathbb{E}[V(0,b_{+1},z_{+1},\zeta_{+1};X_{+1})|z,X]\} \\ \text{subj. to:} \quad & c + b_{+1} = z\pi k + (1-\delta)k + Rb; \\ & b_{+1} \geq 0. \end{aligned}$$

V^1 takes into account the financing constraint and resale constraint:

$$\begin{aligned} V^1(k,b,z;X) = \max_{c,k_{+1},b_{+1}} \quad & \{u(c) + \beta\mathbb{E}[V(k_{+1},b_{+1},z_{+1},\zeta_{+1};X_{+1})|z,X] \\ \text{subj. to:} \quad & \end{aligned}$$

$$\begin{cases} c + b_{+1} + k_{+1} - (1-\delta)k = z\pi k + Rb & \text{if } k_{+1} > (1-\delta)k; \\ c + b_{+1} + (1-d)\left[k_{+1} - (1-\delta)k\right] = z\pi k + Rb & \text{if } k_{+1} \leq (1-\delta)k; \end{cases} \tag{16}$$

$$R_{+1}b_{+1} \geq -\theta(z)(1-d)k_{+1}; \tag{17}$$

$$k_{+1} \geq (1-\varphi)(1-\delta)k. \tag{18}$$

There are two changes to the financing constraint (17) compared to that in the simple model. First, it is now tighter because lenders may have to liquidate capital at a discount. Second the parameter θ that controls the tightness of the constraint is allowed to depend on productivity z. This reflects the fact that in practice borrowing is not only collateralized by physical capital but can also be related to the performance of the firm. The resale constraint (18) takes into account the partial selling represented by φ. When $k_{+1} \leq (1-\delta)k$, the entrepreneur obtains $(1-d)\left[(1-\delta)k - k_{+1}\right]$ from selling part of the capital stock, as shown in the budget constraint (16). When an entrepreneur invests or restarts a new firm (because of a new z^h draw), $k_{+1} > (1-\delta)k$ and the resale constraint does not matter.

A Representative Household. A representative household with a discount factor $\beta^H > \beta$ solves the following Bellman equation

$$W(B^H; X) = \max_{C^H, L^H, B^H_{+1}} \left\{ U\left(C^H, L^H\right) + \beta^H \mathbb{E}\left[W(B^H_{+1}; X_{+1})|X\right]\right\}$$
$$\text{subj. to:} \quad C^H + B^H_{+1} = wL^H + RB^H, \tag{19}$$

where $U(C^H, L^H)$ is a concave utility function of consumption and hours of work. (19) is the resource constraint: the household uses labor income wL^H, where $L^H \in [0, 1]$, and return on savings RB^H to finance consumption C^H and new savings B^H_{+1}.

4.2 Recursive Competitive Equilibrium

I restrict the analysis to the equilibrium of interest. When drawing z^h, an entrepreneur invests in the firm's technology. When entrepreneurs draw z^l, it is better to save in risk-free bonds, but the liquidation costs could prevent them from doing so. To have these two features, the expected return on production needs to be higher than the return on bonds when z^h is drawn, but lower when z^l is drawn.[26]

4.3 Characterization

Since $\beta^H > \beta$, households save and some entrepreneurs borrow. The savings choice $B^H_{+1} = B^H_{+1}(B^H; X)$ and the labor supply decision $L^H = L^H(B^H; X)$ satisfy the following first-order conditions:

$$\mathbb{E}_X \left[\frac{\beta^H U_C\left(C^H_{+1}, L^H_{+1}\right)}{U_C\left(C^H, L^H\right)} R_{+1} \right] = 1; \tag{20}$$

$$U_C(C^H, L^H)w + U_L(C^H, L^H) = 0. \tag{21}$$

The consumption $C^H = C^H(B^H; X)$ can be solved from the resource constraint (19).

As in the simple model, I focus on the type of equilibrium in which unproductive firms today (whether they were previously unproductive or productive) are both financially and resale constrained if they stay in business (and this will be checked in the quantitative analysis). These firms sell all the φ fraction of their existing capital to smooth dividends. Therefore, I only need to keep track of two leverage ratios.

Proposition 3 implies that a low β can generate such a dually constrained outcome. Notice that a high β can generate multiple "cohorts" of unproductive firms, which I abstract. In that case, we will have unconstrained unproductive firms with different ages since they were productive *before* they become constrained; we also need to keep

26. First, z^h should be high enough so that entrepreneurs invest when they draw it. Otherwise, no entrepreneur would run a firm and there would not be any production, which cannot hold in equilibrium. Second, when drawing z^l, entrepreneurs liquidate their firms only if the liquidation cost is sufficiently low, similarly to that in the simple model. If no one liquidates a firm, we cannot study the full liquidation.

track of endogenous leverage ratios of all the staying entrepreneurs captured by the arrow lines in Figure 2 before they hit the financing constraint. But to the extent that a tightened credit condition can have an effect on borrowing today or in the future, which feeds back to today's leverage decisions, the main channel still holds. Therefore, to simplify, I focus on the dually constrained outcome (and the quantitative exercises later will check this.).

Since the time-varying interest rate R is predetermined in each period, it is easier to work with a slightly modified definition of leverage that includes debt repayment:[27]

$$\tilde{\lambda} \equiv \frac{k}{k + Rb} \quad \text{and} \quad \lambda = \frac{\tilde{\lambda}}{\tilde{\lambda} + (1 - \tilde{\lambda})/R}. \tag{22}$$

Let $\tilde{\lambda}^j$, where $j \in \{h, l\}$, be the time t beginning-of-period leverage of an entrepreneur who had z^j at $t-1$. Then, $\tilde{\lambda}^l_{+1}$ hits the borrowing limit and satisfies $\tilde{\lambda}^l_{+1} = \left[1 - \theta\left(z^l\right)(1-d)\right]^{-1}$. For an unproductive entrepreneur with state $\tilde{\lambda}^j$, let $\tilde{\zeta}(\tilde{\lambda}^j; X)$ be the threshold liquidation cost and $q(\tilde{\lambda}^j; X)$ be the shadow value of capital. The characterization of $\{\tilde{\lambda}^h_{+1}(\tilde{\lambda}^j; X), \tilde{\lambda}^l_{+1}(\tilde{\lambda}^j; X), \tilde{\zeta}(\tilde{\lambda}^j; X), q(\tilde{\lambda}^j; X)\}$ generalizes the results of the simple model. Online Appendix C contains the details.

For the aggregate economy, the state can be written as $X = (\Gamma(k, b, z), \tilde{B}^H, Z)$, where $\Gamma(k, b, z)$ measures the distribution of firms (captured by the individual firm's capital stock, bonds, and productivity) and where the level of bonds $\tilde{B}^H = RB^H$ is accumulated by households. The exogenous state $Z \equiv (A, \theta(z))$ has exogenous aggregate productivity and collateralizability parameters. It turns out that we do not need to keep track of the full distribution thanks to the policy functions.

Let K^h and K^l be the levels of aggregate capital held by productive and unproductive firms at the beginning of t, respectively. Let $\tilde{B} = RB$ be the corresponding level of aggregate bonds held by entrepreneurs who did not run firms. Because of the linearity of the entrepreneurs' policy functions according to Proposition 1, they save a common β fraction of their net worth. Capital stock and bonds thus move proportionally with the mass of entrepreneurs, so that the distribution of firms can be summarized by leverage and aggregate wealth. As a result, the aggregate state can thus be simplified to $X = (\tilde{\lambda}^h, \tilde{\lambda}^l, K^h, K^l, \tilde{B}, \tilde{B}^H, Z)$. The wealth dynamics can be represented by the backward-looking equations after we use Proposition 1 and the

27. One does not need to keep track of the pre-determined interest rate using this new definition. In essence, we can reinterpret bonds as discounted bonds; that is, $\tilde{b}_{+1}/R_{+1}$ should be regarded as investment in bonds, and $\tilde{b}_{+1}$ denotes the repayment where $\tilde{b}_{+1} = R_{+1}b_{+1}$. I thank an anonymous referee for suggesting this simplification.

relationship between λ and $\tilde{\lambda}$ in (22):

$$K_{+1}^{h} = \frac{\tilde{\lambda}_{+1}^{h}}{\tilde{\lambda}_{+1}^{h} + (1-\tilde{\lambda}_{+1}^{h})/R_{+1}} \beta \left[\sum_j \left(z^h \pi + 1 - \delta - 1 + \frac{1}{\tilde{\lambda}^j} \right) p^{jh} K^j + p^{lh} \tilde{B} \right]; \tag{23}$$

$$K_{+1}^{l} = (1-\varphi)(1-\delta) \sum_j \left[1 - F(\tilde{\xi}^j) \right] p^{jl} K^j; \tag{24}$$

$$\tilde{B}_{+1}/R_{+1} = \beta \sum_j F(\tilde{\xi}^j) \left(z^l \pi + 1 - \delta - 1 + \frac{1}{\tilde{\lambda}^j} \right) p^{jl} K^j + \beta p^{ll} \tilde{B}, \tag{25}$$

(23) describes the law of motion of capital held by productive entrepreneurs. LHS is the end-of-period capital stock, while the RHS is the savings in capital stock from those who are productive today. (24) describes the law of motion of capital held by unproductive entrepreneurs. Since only a fraction φ of capital is salable, those who were previously productive contribute $(1-\varphi)(1-\delta)[1-F(\tilde{\xi}^h)]p^{hl}K^h$, while those who were previously unproductive contribute $(1-\varphi)(1-\delta)[1-F(\tilde{\xi}^l)]p^{ll}K^l$. (25) represents savings in bonds from entrepreneurs who do not run firms; $\tilde{B}_{+1}/R_{+1}$ includes the savings from z^l entrepreneurs who decide to liquidate today, as well as the savings from those who did not run firms previously (i.e., $\beta p^{ll}\tilde{B}$).

Finally, two market clearing conditions determine the interest rate and the profit rate of firms (or equivalently the wage rate). Since an individual entrepreneur's bond position is $b_{+1} = (1/\lambda_{+1} - 1)k_{+1}$, with k_{+1} as the target capital level, the credit clearing condition is

$$\sum_j \left(\frac{1}{\tilde{\lambda}_{+1}^j} - 1 \right) K_{+1}^j + \tilde{B}_{+1} + \tilde{B}_{+1}^H = 0, \tag{26}$$

which generalizes the partial-equilibrium clearing condition (14). Aggregating individual labor demand (15), we write the market clearing condition for labor as

$$A^{-1} \left(\frac{\pi}{\alpha} \right)^{\frac{1}{1-\alpha}} \sum_j \left(p^{jh} z^h + p^{jl} z^l \right) K^j = L^H, \tag{27}$$

where the LHS is the demand for households' hours and the RHS represents hours supplied by households.

Definition 1. Let $\mathbb{L} \in \mathbb{R}_+^2$ be the compact set containing all leverage ratios $\{\tilde{\lambda}^h, \tilde{\lambda}^l\}$ and let $\mathbb{X}$ be the compact set containing all possible values of X. A *recursive competitive equilibrium with capital reallocation* is a mapping $\mathbb{X} \to \mathbb{X}$ with pricing functions (w, π, R_{+1}): $\mathbb{X} \to \mathbb{R}_+^3$, the household's policy functions $(C^H, L^H, \tilde{B}_{+1}^H)$: $[0, +\infty) \times \mathbb{X} \to \mathbb{R}_+^3$, and low-productivity firms' liquidation strategies $(\tilde{\xi}, q)$: $\mathbb{L} \times \mathbb{X} \to [\underline{\xi}, \bar{\xi}] \times [0,1]$ and leverage ratios $\tilde{\lambda}_{+1}^j$ (for $j = h, l$): $\mathbb{L} \times \mathbb{X} \to \mathbb{L}$, such that

(i) the household's choice $(C^H, L^H, \tilde{B}_{+1}^H)$ solves (19), (20), and (21);

(ii) the portfolio choice $\{\tilde{\lambda}^j_{+1}\}$, liquidation strategy $\tilde{\xi}$, and the shadow price q solve entrepreneurs' problems (see online Appendix C);

(iii) $(K^h_{+1}, K^l_{+1}, \tilde{B}_{+1})$ satisfies the wealth dynamics in (23)-(25);

(iv) the wage rate w solves (15), and R_{+1} and π are determined by (26) and (27).

Aggregate Variables. For subsequent quantitative analysis, I define a few aggregate variables. First, aggregate capital reallocated L can be expressed as

$$L \equiv \underbrace{\varphi(1-d)(1-\delta)\left[\sum_j \left[1 - F\left(\tilde{\xi}^j\right)\right] p^{jl} K^j\right]}_{L^P} + \underbrace{(1-\delta)\left[\sum_j F(\tilde{\xi}^j) p^{jl} K^j\right]}_{L^F},$$

where the first part is the aggregate partial liquidation (L^P) and the second part is the aggregate full liquidation (L^F).

Second, aggregate investment here includes the increment of capital of productive firms and the adjustment costs from full liquidation net of capital reallocation:

$$I \equiv K^h_{+1} - (1-\delta)\sum_j p^{jh} K^j + \Phi - L,$$

where Φ represents aggregate adjustment costs. To calculate Φ, I transform the utility adjustment costs into goods costs. Notice that we can rewrite the utility cost as $\zeta = \underline{\zeta} + (\bar{\zeta} - \underline{\zeta})(\zeta - \underline{\zeta})/(\bar{\zeta} - \underline{\zeta})$. Then, the cost can be thought of as entrepreneurs' additional labor input $(\zeta - \underline{\zeta})/(\bar{\zeta} - \underline{\zeta}) \in [0, 1]$ for producing a specialized service (e.g., intermediation costs/integration costs) to facilitate the transaction. Otherwise, the capital acquired will be less useful or even useless, and buyers are better off purchasing new capital (note: buyers are indifferent between new and used capital). That is, this type of service production is combined within the entrepreneurs' sector. I choose this simplification in order to focus on the liquidation decision and its aggregate effect. This service can be measured through a production function with $1 - \alpha$ as the labor share $A\xi\left[(\zeta - \underline{\zeta})/(\bar{\zeta} - \underline{\zeta})\right]^{1-\alpha}$, where ξ is scaling parameter capturing fixed inputs that are not modeled. The total adjustment costs can be calculated as

$$\Phi = A\xi m \sum_j s^j p^{jl} \int_{\underline{\zeta}}^{\tilde{\xi}^j} \left(\frac{\zeta - \underline{\zeta}}{\bar{\zeta} - \underline{\zeta}}\right)^{1-\alpha} dF(\zeta),$$

where s^j is the population share of entrepreneurs in the $j \in \{h, l\}$ group. Online Appendix C shows how to compute s^j explicitly.

Finally, aggregate output is simply $Y = C + I$, where $C = \int c(i)di + C^H$ is the sum of the consumption of entrepreneurs and workers. Online Appendix C contains an alternative way of computing output and the corresponding TFP definition for producing firms' output $\int y(i)di$. There, $Y = C + I$ is satisfied because of Walras' law.

5 Quantitative Analysis

I now assess the quantitative effects of financial and productivity shocks. Compared to the analysis above, I assess the general-equilibrium effect of financial shocks on liquidation decisions, besides the direct channel highlighted in the firm problem.

5.1 *Parameterization*

The steady-state aggregate productivity A is normalized to 1. The rest of the parameters are calibrated to annual frequencies, as the earliest capital reallocation recorded in the COMPUSTAT dataset was for 1971 explained in Section 2.

Steady-state Calibration. The calibration matches the model steady state to several U.S. long-run statistics (Table 2).

For the idiosyncratic productivity transition matrix and the levels of idiosyncratic productivity, I follow the method of Tauchen (1986). I set the transition probability as $p^{hh} = p^{ll}$ and the two levels of idiosyncratic productivity as

$$\log(\tilde{z}^h) = \sigma \text{ and } \log(\tilde{z}^l) = -\sigma.$$

According to a micro-level study by Ábrahám and White (2006), the idiosyncratic productivity fits an AR(1) process well. The yearly persistence is around 0.69, while the standard deviation of shocks is about 0.18.[28] I equate the variance and the serial correlation of the two-point Markov chain in the model and the AR(1) process, i.e.,

$$\sigma = \sqrt{\frac{0.18^2}{1-0.69^2}} = 0.2487 \text{ and } p^{hh} = \frac{1+0.69}{2} = 0.845.$$

This calculation suggests that a productive firm is about 64% (note: $\tilde{z}^h/\tilde{z}^l = \exp\{2\sigma\} \approx 1.64$) more productive than an unproductive one.

β^H is the household's discount factor that targets a 2% annual risk-free interest rate. To leave out the wealth effect on the household's labor supply, I assume that the household has a non-separable GHH utility function

$$U(C^H, L^H) = \frac{\left[C^H - \frac{\mu}{1+\nu}\left(L^H\right)^{1+\nu}\right]^{1-\varepsilon} - 1}{1-\varepsilon}.$$

I set $\varepsilon = 1$ to allow the same risk-aversion as the entrepreneurs (with log utility). I set $\nu = 1/1.5$ so that the household's labor supply elasticity is 1.5, in line with many macroeconomic calibrations. Finally, $\mu = 2.34$ is calibrated so that total hours

28. If I only use COMPUSTAT, then the persistence and the standard deviation increase to 0.70 and 0.27, respectively, as documented by İmrohoroğlu and Tüzel (2014). Doing so will increase the power of financial shocks. In order to capture a wider extent of firms, I choose to be conservative in the calibration.

TABLE 2. Parameters.

	Value	Explanation/Target		Value	Explanation/Target
β^H	0.98	Annualized risk-free rate 2%	φ	3.96%	Share of partial sales: 28%
ε	1	Household risk aversion	δ	8.88%	Effective depreciation rate: 10%
ν	1/1.5	Inverse labor supply elasticity	$\bar{\zeta}$	10.5434	See the discussion in text.
μ	2.3443	Hours worked 1/3	$\underline{\zeta}$	3.5587	R-E ratio: 30%
p^{hh}	0.8450	Productivity persistence 0.69	ξ	34.4930	Acquisition costs: 1.68% of output
p^{ll}	0.8450	$p^{ll} = p^{hh}$	θ	0.4203	Debt-to-output: 65.5%
$\tilde{z}^h$	1.2824	Productivity st.dev. 0.18	m	0.10	exogenous
$\tilde{z}^l$	0.7798	$\log(\tilde{z}_l) = -\log(\tilde{z}_h)$	ρ_A	0.83	exogenous
α	0.30	Capital share	ρ_θ	0.83	exogenous
d	0.10	10% cost of partial sells	σ_A	0.52%	Output volatility 1.92%
β	0.9002	Investment-to-output: 18.1%	σ_θ	2.09%	Relative R-E ratio volatility 5.79

used for work are 1/3 of all agents' time. Notice that if one increases either ε or $1/\nu$, the household's saving/labor supply decisions become more sensitive to changes in the interest rate and the wage rate, and the effect of financial shocks will be further amplified.

The capital share α in production is set to 0.3. The discount in partial liquidation d is set to 0.1 to be conservative. Previous empirical findings suggest that the discount in partial liquidation can easily be above 20% and sometimes more than 50% (see, e.g., Ramey and Shapiro 1998, 2001). In order to obtain a tractable solution, $F(.)$ is set to the CDF of a uniform distribution.[29]

I set $\theta\left(z^h\right) = \gamma\theta(z^l)$, where $\gamma - 1 \geq 0$ measures the extra borrowing capacity of productive firms. Let $\gamma = 1.2$ (i.e., 20% more capacity), but the exact value of γ is not crucial because productive firms turn out to have a lower leverage than unproductive firms in equilibrium. That is, even when $\gamma = 1$, productive firms never hit the borrowing limit in the calibration. Therefore, compared to previous studies cited in the introduction, financial shocks have smaller effects on productive firms.[30] For this reason, we can just focus on $\theta(z^l)$, which is denoted simply as θ from now on.

Then, I use seven targets to jointly calibrate seven parameters: the depreciation rate δ (note: the effective depreciation depends on the resale cost d), the entrepreneurs' discount factor β, the tightness of the financing constraint θ, partial salability φ, the lower and upper bounds $\underline{\zeta}$ and $\bar{\zeta}$ for liquidation costs, and finally the parameter ξ affecting the total reallocation costs. The seven targets are the effective depreciation rate (0.10), the investment-to-output ratio (0.181), the aggregate debt-to-output ratio (0.655), the P share (0.30), the R-E ratio (0.28), $F(\tilde{\zeta}^h)/F(\tilde{\zeta}^l)$ the relative likelihood of

29. Different distribution types and different parameters for these distributions generate similar quantitative results. This is due to the fact that the dominating force is the option value of staying in business, instead of the exact distribution of ζ.

30. Setting $\gamma = 1.2$, instead of $\gamma = 1$, allows enough borrowing capacity so that productive firms are not constrained even when adverse financial shocks hit. Importantly, this fact limits the power of financial shocks as otherwise investment and capital reallocation from productive firms become more procyclical.

being acquired (0.60), and finally the size of reallocation costs explained below. Notice that the population measure of entrepreneurs (m) only affects ξ, and I set $m = 0.1$. The last six targets are briefly explained below.

The R-E ratio, the P share, investment, and output have been explained in Section 2. The costs from acquisitions, unlike the loss of productive capital captured by d in partial liquidation, are paid to services that facilitate acquisitions; the costs also affect new investment and are thus related to investment adjustment costs more broadly. In the data, financial service costs for reallocation and investment purposes can be associated with investment banking and related security activities - it is possible to measure trading fees and commissions, securities underwriting fees, and management fees for financial market and clearing products. The relevant value-added under "securities, commodity contracts, and investments" published by Bureau of Economic Analysis (BEA) is about 1.81% of business value-added between 1997 and 2018. Value-added from other sub financial sectors may also be relevant for capital reallocation, but to be conservative, I use 1.81%.[31] For z^l firms, I set the liquidation probability of those previously productive to be 60% of the liquidation probability of those previously unproductive, i.e., $F(\tilde{\zeta}^h)/F(\tilde{\zeta}^l) = 0.6$. This calibration is in line with previous findings that firms with better past performance is less likely to be acquired.[32] Also, as long as $F(\tilde{\zeta}^h)/F(\tilde{\zeta}^l) < 1$, changing the target seems to have neglibile effect on the calibration. Finally, $\theta = 0.42$ so that the average debt-to-output ratio is 65.5% across time as in the sample period. Note that firm debt in the model is funded by savings from both the households and the entrepreneurs who choose not to run firms. The latter can be classified as corporate savings.

REMARK. *Productive firms borrow and invest because capital offers a return higher than the real interest rate, but they do not borrow to the credit limit for precautionary reasons. Unproductive firms, on the contrary, do not invest; some of them are constrained because the value of keeping the firm is high, and they need to borrow to smooth dividends as their resources are limited. Therefore, a firm in the economy has* occasionally binding (financing and resale) constraints. *In Online Appendix* D, *I look at the relationship between firm-level productivity and leverage. If I sort firms along the productivity dimension into deciles, more productive firms seem to have lower*

31. There are other kinds of costs. The post-integration costs (e.g., for employee-related, consultant and reconfiguration, IT system changes, etc.) are around 3% of a typical deal. See a report by Ernst & Young that reviewed more than 70 deals from 2010 to 2016 valued at over one billion US dollars each with publicly listed buyer companies, https://www.ey.com/en_gl/transactions/four-current-trends-estimating-mergers-acquisitions-integration-costs. Legal and accounting fees are said to be comparable but unfortunately not well documented.

32. See Morck et al. (1988), Hirshleifer and Thakor (1992), and Comment and Schwert (1995). In the calibrated economy, one percetnage-point fall in firm return on equity increases the probability of being acquired by about 1.7 percentage points, in line with empirical findings such as by Schwert (2000).

leverage.[33] *The leverage ratios $\lambda^l = 1.59$ and $\lambda^h = 1.56$ are within the observed range of firm-level leverage ratios, which supports the calibration.*

Shocks. For shocks to productivity A_t and the external financial condition θ_t, I follow the tradition in quantitative models by specifying AR(1) processes:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A; \quad \log \theta_t = \rho_\theta \log \theta_{t-1} + (1-\rho_\theta)\log\theta + \varepsilon_t^\theta,$$

where ρ_A and ρ_θ are persistent parameters, and $\varepsilon_t^A \sim N(0,\sigma_A^2)$ and $\varepsilon_t^\theta \sim N(0,\sigma_\theta^2)$ are i.i.d. normal random "shocks," with variance σ_A^2 and σ_θ^2.

I solve the system dynamics around the deterministic steady state using log-linear approximations. The persistent parameters are set such that $\rho_A = \rho_\theta = 0.83$, a reasonable number for a yearly model in the business cycle literature.[34] To discipline the sizes of the shocks, I set $\sigma_A = 0.52\%$ and $\sigma_\theta = 2.09\%$ so that the model generates the same standard deviations of the output and the R-E ratio as in the data.

5.2 Equilibrium Dynamics

I now assess the effects of varying external financial conditions. To highlight the effects of financial shocks, I start with equilibrium responses to textbook-style exogenous aggregate productivity shocks.

(Exogenous) Productivity Shocks. As in a standard business-cycle model, a negative one-standard-deviation productivity shock generates a persistent fall in consumption, investment, and output (blue continuous lines in Figure 4). The size of the initial TFP drop is $(1-\alpha)0.52\% = 0.36\%$; the fall of output on impact (i.e., in year 0) is larger (0.60%), since the model features a financial accelerator through the borrowing constraint, i.e., when entrepreneurs' wealth falls, they can borrow and invest less, reducing the ability to accumulate capital and bringing down their wealth further. However, the decline in TFP also generates a "cleansing" effect or "creative destruction," which improves resource allocation after the initial shock.

To see this cleansing effect, notice that falling aggregate labor-augmenting productivity A_t reduces the wage rate, and households start to save less to make up for the fall in consumption (about 0.44% on impact). As a result, both the profit rate π_t and the credit supply decline. Falling net worth following the lower profit rate π_t, together with the reduction in credit supply, pushes productive firms to borrow less for capital expenditures. New investment falls by about 1.32% on impact. Nevertheless, spending on used capital rises by 0.49% on impact. Such a rise in reallocation is because of the fact that z^l firms have less incentive to produce and stay in business in response to a

33. Firm entry with different productivities may also contribute to this pattern. That is, new entrants may need to borrow to start their business, and a less productive firm may also have higher leverage than a more productive firm. But this pattern shown in Appendix D is robust even if we control for firm age.

34. An early version of paper estimates the stochastic processes along with more shocks to fit output, investment, the R-E ratio, and the P share. The results are mostly similar (available upon request).

falling profit rate π_t; fewer z^l entrepreneurs keep operating their firms, thus increasing full liquidation and creating a cleansing effect. Although capital expenditures from productive firms fall, more available used capital implies that the R-E ratio rises on impact and stays persistently above the steady-state level.

The above discussion can be seen from the adjusted TFP, i.e., the TFP after adjusting the impact of *exogenous* productivity A_t (see online Appendix C). This measure looks at the capital held by productive firms relative to that held by unproductive firms:

$$TFP_t^{adj} = \left(\bar{z}^h - \frac{\bar{z}^h - \bar{z}^l}{K^h / K^l + 1} \right)^{\alpha},$$

where $\bar{z}^h \equiv \sum_j p^{hj} z^j$ as average productivity in this period for a previously productive firm, and similarly $\bar{z}^l \equiv \sum_j p^{lj} z^j$ for a previously unproductive firm. Figure 4 shows that the adjusted TFP rises slightly on impact after aggregate productivity shocks, indicating that capital is better allocated, since the only reason for adjusted TFP to rise is that K^h / K^l is higher. In other words, adverse productivity shocks push more unproductive firms to be sold; the liquidated funds are then saved in bonds borrowed by productive firms, and capital is thus more efficiently allocated.

As a result, debt does not fall dramatically, and the change in the interest rate is almost negligible. The winners of the cleansing effect, of course, are the productive firms. They also expand and hire more labor a few years after the initial shock (although aggregate hours fall), which explains why the profit rate π_t overshoots about 6 years after the initial shock.

Financial Shocks. Adverse financial shocks, in contrast, do *not* generate the cleansing effect (red solid lines with markers in Figure 4). The positive co-movement among reallocation, investment, and output is similar to what the data shows. The R-E ratio falls by 6.9% on impact and stays persistently below the steady-state level; the P share, the share of partial liquidation in reallocation, increases by about 11.3% and 10.7% in the first two periods, and it also stays persistently above its steady-state level.

The option value of staying in business increases when financing constraints become tighter. As explained in the firm problem, this will be the case except for the period of impact, when firms' leverage cannot be adjusted and unproductive firms are more likely to be liquidated. However, a tighter constraint reduces credit demand from all firms, pushing down the interest rate. With the falling interest rate, unproductive firms are more likely kept operating even on impact of the adverse financial shocks. Additionally, although staying unproductive firms are hoarding more labor, falling acquisition activities and the weaker expansion from productive firms still creates a fall in labor demand. Less competition in the labor market then reduces the wage rate and pushes up π_t from year 1 onward. The benefit of staying in business, in the view of z^l firms, therefore becomes even greater. Capital reallocation is thus persistently delayed, with 10% less on impact, and still 2% less after 10 years.

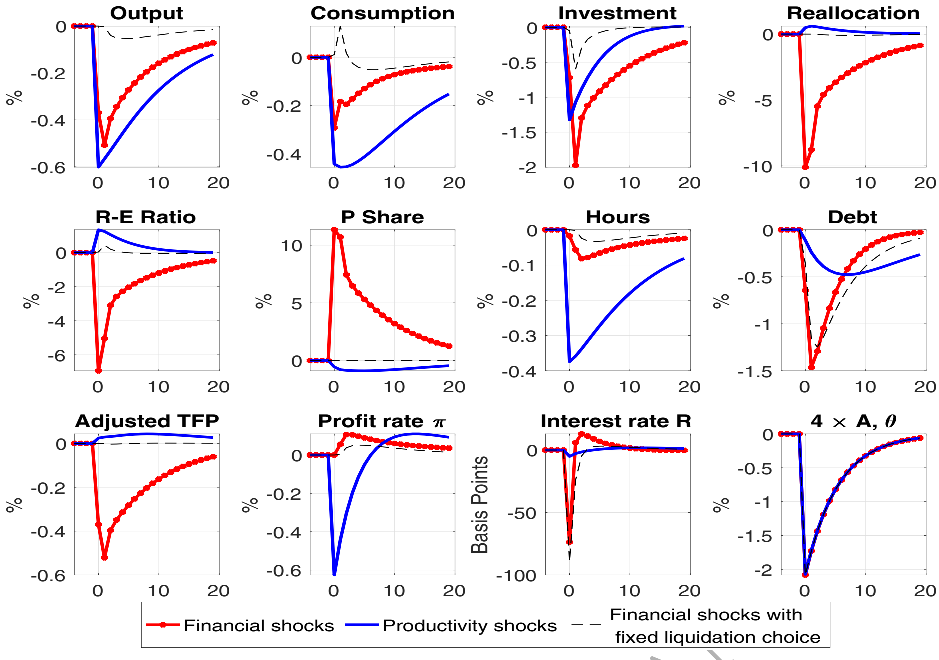

FIGURE 4. Impulse responses. All series plotted are changes from their steady-state levels in response to one standard deviation innovations to the exogenous processes.

To see the equilibrium effects that lead to a long-lasting recession, first note that the initial negative shock brings down the interest rate by about 74 basis points, as the demand for credit falls significantly. A lower real rate reduces the net gain from liquidation, since the benefit of liquidating firms and saving the proceeds in risk-free bonds fall. As a result, z^l entrepreneurs are more willing to stay. Notice that the interest-rate drop is significant on impact compared to that under shocks to exogenous productivity, but the interest rate quickly adjusts to a level slightly above the steady-state level. The reason is that credit supply is related to capital stock, and on impact it is less flexible than credit demand controlled by the borrowing constraint. A sudden fall of credit demand thus reduces the interest rate significantly. However, a delayed liquidation reduces credit supply in the medium term, which explains why interest rate can overshoot. It then has a negative impact on productive firms in the credit market. Additionally, adverse financial shocks cause investment from productive firms to contract, and less capital means less need for hiring labor as well.

The magnitude of the fall of reallocation is *not* mainly caused by the lower demand from productive firms. To understand this claim, notice the R-E ratio can be written as

$$\text{R-E ratio} = \frac{L^F + L^P}{L^F + L^P + I},$$

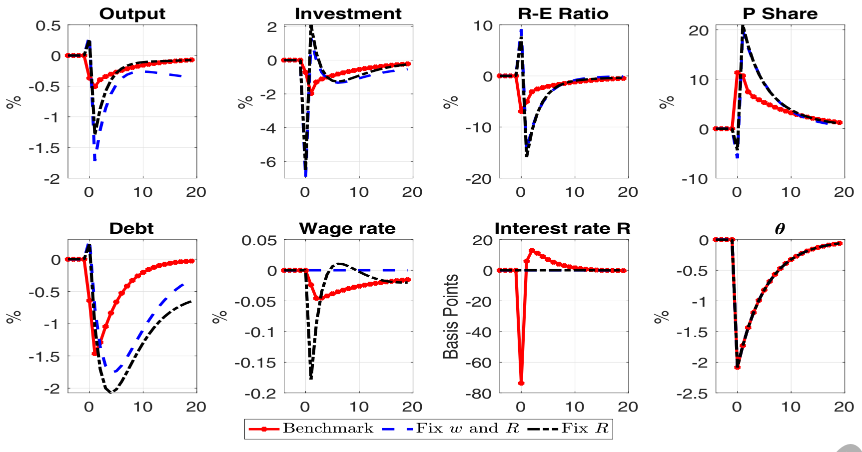

FIGURE 5. Effects of financial shocks. All series plotted are changes from their steady-state levels in response to a negative one standard-deviation innovations to the log θ process.

where L^F is full liquidation, L^P is partial liquidation, and I is new investment. To generate a falling R-E ratio, capital reallocation $(L^F + L^P)$ must fall by more than the fall of new investment (I). In other words, the lower supply of liquidated capital has to be the main driving force behind the fall in reallocation. If, instead, the demand from productive firms is the main factor, $L^F + L^P$ and I would drop at the same rate, leaving the R-E ratio unchanged.

A related aspect is that if full liquidation falls, then partial liquidation jumps up (see again the rise of the P share in Figure 4). This can be interpreted as a rise in fire sales, as units of efficient capital disappear because of the physical cost d. The magnitude of the fall in full liquidation (L^F) is larger than the rise of partial liquidation (L^P). This is because for one unit of capital that should have been liquidated with full liquidation, only $\varphi = 3.96\%$ units are sold.

Notice that the consequence of financial shocks on real activities is significant. We have an initial 0.29% drop in consumption. Investment and output exhibit hump-shaped dynamics, as investment falls almost by 0.72% and 1.97% in year 0 and year 1, respectively, while output drops by 0.37% and 0.51% in the first 2 years. Compared to aggregate productivity shocks, financial shocks generate more persistent dynamics of macro aggregates. Output recovers less than half of the initial drop even after 10 years. This is because the debt falls by 0.64% on impact and reaches the lowest level (1.47% lower) 1 year after; it takes about 8 to 9 years to have half of the recovery of the initial fall. As a comparison, negative aggregate productivity shocks generate a much milder response in debt. One reason for the persistence in the response to financial shocks is the stronger financial accelerator, and another comes from long periods of capital misallocation. These reasons may shed light on the long-lasting recovery experience since the 2007-2009 global financial crisis.

Due to less supply of credit as mentioned before, from period 1 onward the interest rate is higher than the steady-state value, but still we observe a persistent drop in the R-E ratio and a persistent increase in the P share. To highlight the direct and equilibrium effects, Figure 5 compares the benchmark responses to financial shocks with two other counterfactuals. One counterfactual has an economy with all the wage rate and the interest rate fixed at the steady-state levels, the other has the constant steady-state interest rate while the labor market is in equilibrium in every period.

When the interest rate is fixed in the two counterfactuals, on impact the R-E ratio increases by about 7.7%-9.1% while the P share falls by about 4.4%-6.0%, which confirms the finding in the firm-decision problem: when the adverse financial shocks hit, firms do not have time yet to adjust their leverage, and unproductive firms' managers find it more attractive to liquidate. Reallocation of capital rises on impact, leading to better allocation of capital and thus more output. Given the substitution between new and used capital, investment falls on impact. Investment and reallocation also move in opposite directions in the first 6 periods. However, these features are at odds with the fact that investment and reallocation measures are procyclical.

When the credit market is in equilibrium, the initial falling interest rate contributes to the fall of the R-E ratio and the rise of the P share. The labor market equilibrium strengthens the effect; but since the lower wage rate is short-lived, it has a limited impact if the credit market equilibrium is absent. In the benchmark economy, the rise of the interest rate seen after the initial drop limits the fall of reallocation, but it does not lead to a reallocation boom because firms have adjusted their leverage after the initial financial tightening. After the initial tightening, both the persistent low wage rate and the improving financial conditions encourage the unproductive firms to stay in business longer so that reallocation remains below the steady-state level persistently.

5.3 Model Performance

I examine the model's performance against other parts of the data.

First, I back out the shocks that generate the observed cyclical output and R-E ratio, which is done by reformulating the model in a state-space form. The estimated shocks are calculated through the standard Kalman smoother approach after Kalman filtering (see online Appendix C). These estimated shocks are fed into the model, and then I assess the model's performance. The estimated productivity and financial shocks (at their mean levels) are plotted in Figure 6, normalized by their own standard deviations for straightforward comparison.

As illustrated in the impulse response analysis, financial shocks are quantitatively important for the business cycle, since they can move leverage and significantly affect capital reallocation, productivity, and aggregate activities. Through the lens of the model, aggregate productivity shocks are much more important before 1984, while financial shocks are also present. The 2007-2009 recession (the "Great Recession") seems to be the only recession that has a combination of significant adverse shocks to aggregate productivity and worsening of external financial conditions for at least 2 years. The 2007-2009 recession featured a large drop in the liquidity and pledgeability

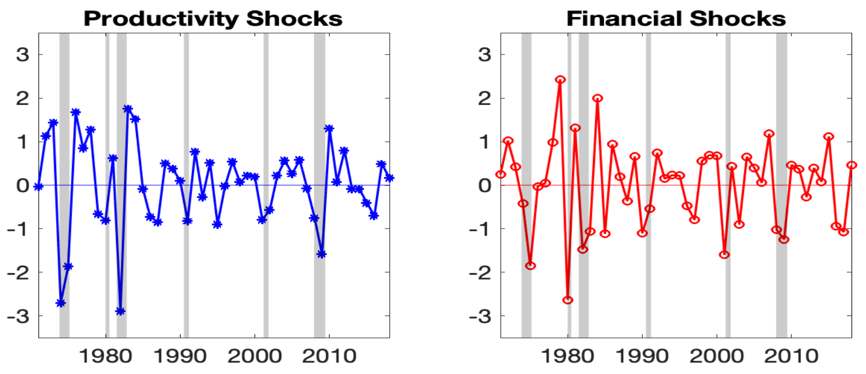

FIGURE 6. Estimated innovations (shocks) to productivity and financial conditions. Shocks are smoothed results from Kalman Smoother and are normalized by their respective standard deviations. Shaded areas indicate NBER dated recessions.

of financial assets, which is captured in the model by negative innovations to θ, the measure of collateralizability. Note that the data and the model exclude debt and reallocation in the financial sector as well as household debt, so financial shocks in practice could be even larger in the Great Recession. Finally, note the frequent positive shocks to θ in the years prior to the Great Recession, which may reflect the real-estate boom and the surge of collateral assets in this period.

Second, I present simulated data after feeding the estimated shocks into the model. Table 3 reports the corresponding model statistics compared to the stylized facts in Table 1. It shows that the model does a reasonable job in matching the volatility and cyclical patterns of debt, investment, and reallocation. Statistics related to P-E ratio and P share are more cyclical in the model, likely because the model does not capture special secondary-market price features of partial sales in practice.

TABLE 3. Business cycle statistics: Data vs model (with smoothed shocks).

Correlation	Debt	R-E ratio	P-E ratio	P share	Output
Debt	1	0.52 (0.54)	-0.15 (-0.77)	-0.45 (-0.67)	0.59 (0.51)
R-E ratio	-	1	-0.16 (-0.87)	-0.77 (-0.97)	0.64 (0.64)
P-E ratio	-	-	1	0.75 (0.96)	-0.17 (-0.87)
P share	-	-	-	1	-0.53 (-0.78)
Output	-	-	-	-	1
Relative Std. Dev	1.19 (1.29)	5.79 (5.79)	5.59 (5.00)	8.68 (10.43)	1

Note: Numbers in brackets are results from the model after I feed the smoothed shocks into the model. The correlations of investment with output, the R-E ratio, and the P share are 0.85 (0.95), 0.57 (0.56), -0.49 (-0.74), respectively.

Figure 7 further presents simulated results of variables not directly targeted. Hours are less volatile in the model than in the data because of the frictionless labor market. TFP is mostly close to the data, which supports the view that financial shocks can bring endogenous fluctuations in aggregate productivity, whose impact on investment

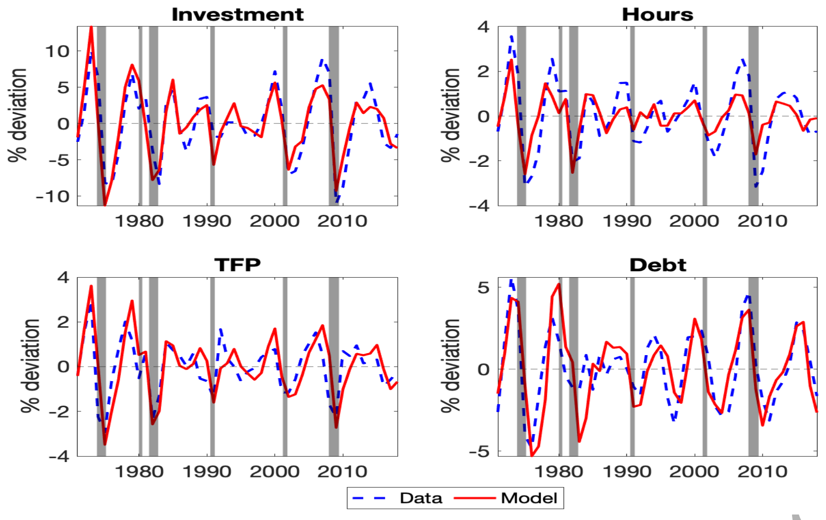

FIGURE 7. Other variables not targeted All series plotted are changes from their steady-state levels. The red solid lines are the model outcomes, while the blue dashed lines represent the data. Shaded areas indicate NBER dated recessions.

is similar to that of exogenous aggregate productivity shocks. For debt dynamics, the model outcome tracks the data most of the time, with some exceptions between 1975 and 1983. This period is characterized by high inflation, strong monetary policy measures, and financial liberalization, and the model does not touch on these aspects.

Finally, If I do not use estimated shocks, Table 4 shows the comparison between two simulated versions of the model. One is with both aggregate productivity shocks and financial shocks using the parameters in Table 2, and the other uses only aggregate productivity shocks (recalibrated to hit the exact same output volatility).

Though the persistence parameters of the shocks are exogenous and I only calibrate the sizes of the shocks to hit the volatilities of output and the R-E ratio, the key message is still clear. Compared to the data, aggregate productivity shocks generate the opposite reallocation dynamics; the volatility of reallocation is much smaller. Financial shocks generate a *procyclical R-E ratio* and a *countercyclical P share*. Overall, financial shocks improve the model performance in terms of correlations with output and most of the relative standard deviations. Debt volatility seems higher with financial shocks than that in the data, suggesting that financial and productivity shocks could be correlated (not implemented in the exercises). Hours are less volatile with financial shocks, since hiring is not directly linked to borrowing constraints as in Jermann and Quadrini (2012), who emphasize the variations in employment and production *directly* from financial shocks. This paper emphasizes the allocation of capital directly from financial shocks and then analyzes variations in employment, investment, and output arising *indirectly* from financial shocks.

TABLE 4. Simulated business cycle statistics.

Relative Std. Dev.	Consumption	Investment	Hours	TFP	Debt	R-E ratio	P share
with only A shocks	0.96	1.42	0.64	0.35	1.15	1.64	2.18
with both shocks	0.84	2.21	0.55	0.64	1.69	5.79	12.09
Data	0.85	2.58	0.79	0.62	1.19	5.79	8.68

Correlation	Consumption	Investment	Hours	TFP	Debt	R-E ratio	P share
with only A shocks	0.99	0.87	0.99	0.80	0.86	-0.91	0.93
with both shocks	0.95	0.85	0.92	0.80	0.84	0.33	-0.43
Data	0.96	0.85	0.89	0.79	0.59	0.64	-0.53

5.4 Counterfactuals

I use two counterfactuals to illustrate the importance of costly capital liquidation.

First, I discuss the impact of financial shocks on productive firms and unproductive firms separately. Notice that productive firms are not immediately financially constrained, but future binding financing constraints still have an impact on their investment decisions. However, it is not straightforward to separate these two groups of firms, because they take into account productivity transitions when making decisions, and capital from primary investment and the secondary market are linked. I look at the extreme case when liquidation thresholds are fixed at their steady-state levels. That is, financial shocks are not affecting the cyclical liquidation decisions, and the same fractions of unproductive firms will be liquidated every period.

The dashed lines in Figure 4 show that the investment decline is smaller (0.05% in period 0 and 0.60% in period 1) and much less persistent than under the baseline. Reallocation is almost unchanged, and it is hard to observe worsening capital misallocation, as the ratio of capital held by the two groups remains roughly constant over time. Adverse financial shocks, in this case, act as a cut of water inflow into one pond (representing capital stock of productive firms); this pond is connected to another pond (representing capital stock of unproductive firms), and both ponds have constant fractions of water outflow so that the outflow is also lessened following the cut. The ratio of the amount of water between the two ponds will stay roughly the same. Therefore, adjusted TFP is almost unaffected, and the fall of output mainly reflects the reduction of capital stock arising from falling investment.

Second, modeling partial liquidation improves the model performance. I compare the log-likelihoods after using output and reallocation data across two versions of the model. (1) The baseline which generates Figure 6; (2) A version of the model with $\varphi = 0$ so that no partial liquidation is allowed. The version with $\varphi = 0$ is recalibrated with the same targets (including the output volatility and the R-E ratio volatility) except that I set P share to be 0. In fact, setting $\varphi = 0$ and removing partial liquidation reduce the model's log-likelihood by more than 50. This comparison implies that having both types of capital reallocation is useful for our understanding of recessions driven by financial and productivity shocks.

6 Final Remarks

This paper highlights the impact of financing constraints on capital reallocation. Tighter financing constraints can increase the unproductive firms' option value of staying in business in the medium term, if they can survive. In general equilibrium, productive firms cut capital spending and production when the economy enters a financial recession; following adverse financial shocks, wages and interest rates fall because of input misallocation, further raising the option value. This effect reduces the unproductive firms' probability of exercising the liquidation option, worsening the allocation of capital/labor and further deepening the recession.

The model may have implications for low interest rate environments through the lens of capital misallocation. It can also shed some light on interest rate policies. For example, while an interest rate cut helps productive firms to borrow and buy capital, it also reduces the incentive for unproductive firms to reallocate capital, as the falling interest rate reduces their debt servicing cost and the benefit of liquidation. Therefore, "zombies" can exist for a long time. The past decade of low interest rates in developed economies seems to have contributed significantly to the rise of zombie firms.[35]

Finally, in developing economies, the liquidity of secondary capital markets is heterogeneous and could have large impacts on the saving decisions of entrepreneurs. This may shed light on why financial development across countries has different impacts on capital allocation, productivity, and output (see, e.g., Buera et al. 2011 and Midrigan and Xu 2012). These further explorations are left for future research.

Appendix: Proofs

A.1. Proof of Proposition 1

The proof proceeds with a guess-and-verify strategy. Guess that the value functions $V^0(k,b)$ and $V^1(k,b)$ have the properties

$$V^0(k,b) = J^0 + \frac{\log N^0(k,b)}{1-\beta} \quad \text{and} \quad V^1(k,b) = J^1\left(\frac{k}{k+b}\right) + \frac{\log N^1(k,b)}{1-\beta},$$

where $N^0(k,b) \equiv rk + (1-\delta)k + Rb$ and $N^1(k,b) \equiv (rk+q)(k/k+b)(1-\delta)k + Rb$ are net worths, J^0 is a constant, and $J^1(k/k+b)$ and $q = q(k/k+b) \le 1$ are functions of the leverage $\lambda = k/(k+b)$. I will verify these properties later. Using the guessed value functional forms, we know that the "scale-invariant" property is satisfied. That is, for any $\rho > 0$,

$$V(\rho k, \rho b, \zeta) = V(k, b, \zeta) + \frac{\log \rho}{1-\beta}. \tag{A.1}$$

35. See Banerjee and Hofmann (2018) on zombie firms from 14 advanced economies.

Now, I verify these properties in two separate scenarios: one with the policy $k_{+1} = 0$ and the other with the policy $k_{+1} > 0$. The first scenario says that an entrepreneur with existing firm liquidates the firm, or an entrepreneur who does not have a firm continues without a firm. The second scenario says that an entrepreneur runs a firm.

Scenario 1: $k_{+1} = 0$. In this scenario, the entrepreneur either chooses liquidating a firm or not running a firm. That is, $\lambda_{+1} = k_{+1}/(k_{+1} + b_{+1}) = 0$. The value of doing so is $V(k, b, \zeta) = V^0(k, b) - \mathbb{1}_{\{k>0\}}\zeta$, where

$$\begin{aligned} V^0(k,b) &= \max_{b_{+1}} \{\log(rk + (1-\delta)k + Rb - b_{+1}) + \beta\mathbb{E}[V(0, b_{+1}, \zeta_{+1})]\} \\ &= \max_{b_{+1}} \left\{\log(rk + (1-\delta)k + Rb - b_{+1}) + \beta\mathbb{E}[V(0, 1, \zeta_{+1})] + \frac{\beta\log(b_{+1})}{1-\beta}\right\}, \end{aligned}$$

after using the scale invariant property (A.1). The expression implies the optimal saving rule $b_{+1} = \beta[rk + (1-\delta)k + Rb] = \beta N^0(k, b)$, which also implies that the consumption function is $c = (1-\beta)N^0(k, b)$. We can thus rewrite $V^0(k, b)$ as

$$V^0(k,b) = \underbrace{\log(1-\beta) + \frac{\beta\log\beta}{1-\beta} + \beta\mathbb{E}[V(0,1,\zeta_{+1})]}_{\equiv J^0} + \frac{\log N^0(k,b)}{1-\beta}, \qquad \text{(A.2)}$$

where we obtain an expression for J^0. Therefore, the value function $V^0(k, b)$ and the policy functions $c = (1-\beta)N^0$, $k_{+1} = 0$, and $b_{+1} = \beta N^0$ are verified.

Scenario 2: $k_{+1} > 0$. In this scenario, the entrepreneur either chooses keeping the existing firm or starting a new firm. That is, $\lambda_{+1} > 0$ and $b_{+1} > 0$. The value of doing so is $V(k, b, \zeta) = V^1(k, b)$, where

$$V^1(k,b) = \max_{k_{+1}, b_{+1}} \{\log(c) + \beta\mathbb{E}[V(k_{+1}, b_{+1}, \zeta_{+1})]\}, \text{ s.t. } (5) - (7).$$

The budget constraint (5) is repeated here for convenience:

$$c + k_{+1} + b_{+1} = rk + (1-\delta)k + Rb. \qquad \text{(A.3)}$$

Since the current ζ does not affect the value function $V^1(k, b)$, we know that λ_{+1} and q are independent of ζ. Additionally, from the definition of λ, we have

$$b_{+1} = k_{+1}\left(\frac{1}{\lambda_{+1}} - 1\right) \quad \text{and} \quad k_{+1} + b_{+1} = \frac{k_{+1}}{\lambda_{+1}},$$

and thus $c = N^0(k,b) - k_{+1}/\lambda_{+1}$. We then use (A.1) to rewrite $V^1(k,b)$ as

$$V^1(k,b) = \max_{\lambda_{+1}>0, k_{+1}} \left\{ \log\left(N^0(k,b) - \frac{k_{+1}}{\lambda_{+1}}\right) + \frac{\beta \log\left(\frac{k_{+1}}{\lambda_{+1}}\right)}{1-\beta} + \beta \mathbb{E}\left[V\left(\lambda_{+1}, 1-\lambda_{+1}, \zeta_{+1}\right)\right] \right\}, \tag{A.4}$$

$$\text{subj. to:} \quad \lambda_{+1} \leq \bar{\lambda} = \left(1 - \frac{\theta}{R}\right)^{-1}; \tag{A.5}$$

$$k_{+1} \geq (1-\delta)k. \tag{A.6}$$

Notice again that $\bar{\lambda}$ denotes the upper bound of the leverage ratio. The optimal choice becomes now (k_{+1}, λ_{+1}), and I break the rest of the verification into two steps.

Step 1. Verifying the policy functional forms. First, let $\mu_k/k \geq 0$ be the Lagrange multiplier attached to the resale constraint (A.6). The first-order condition for k_{+1} is

$$\frac{-1/\lambda_{+1}}{rk + (1-\delta)k + Rb - k_{+1}/\lambda_{+1}} + \frac{\beta}{(1-\beta)\,k_{+1}} + \frac{\mu_k}{k} = 0,$$

with the complementary slackness condition $[k_{+1} - (1-\delta)k]\,\mu_k/k = 0$. Rearranging the above condition, we have

$$\frac{k_{+1}}{\lambda_{+1}} = \frac{\beta + \mu_k(1-\beta)k_{+1}/k}{1 + \mu_k(1-\beta)k_{+1}/k}\left[rk + (1-\delta)k + Rb\right].$$

The above result of k_{+1}/λ_{+1} is then used in the budget constraint (A.3) (after using again that $k_{+1} + b_{+1} = k_{+1}/\lambda_{+1}$) to rewrite consumption $c = N^0(k,b) - k_{+1}/\lambda_{+1}$ as

$$c = \frac{(1-\beta)N^0(k,b)}{1 + \mu_k(1-\beta)k_{+1}/k} = \frac{(1-\beta)\left[r + (1-\delta) + R(\lambda^{-1} - 1)\right]k}{1 + \mu_k(1-\beta)(1-\delta)}, \tag{A.7}$$

where the last equality uses the complementary slackness condition.

Second, instead of working with the multiplier μ_k/k, I work with the economically meaningful shadow value q of capital, defined by using the following relationship:

$$\frac{r + (1-\delta)q + R(\lambda^{-1} - 1)}{r + (1-\delta) + R(\lambda^{-1} - 1)} = \frac{1}{1 + \mu_k(1-\beta)(1-\delta)}. \tag{A.8}$$

Intuitively, q depends on the leverage λ, and importantly $q \leq 1$ because $\mu_k \geq 0$. Also, $q = 1$ when the resale constraint is slack (i.e., $\mu_k = 0$); this could happen when capital return r is high and the entrepreneur invests. q will be jointly determined with the portfolio choice λ_{+1} to be shown later. Now, replacing μ_k in (A.7) by using the definition (A.8), we verify the consumption function:

$$c = (1-\beta)\left[rk + (1-\delta)qk + Rb\right] = (1-\beta)N^1(k,b). \tag{A.9}$$

Finally, adding the identity $(q-1)k_{+1} = (q-1)(1-\delta)k$ (which is true either because $q = 1$ or because $k_{+1} = (1-\delta)k$ when the resale constraint is binding) to both sides of the budget constraint (A.3), we have

$$c + qk_{+1} + b_{+1} = rk + (1-\delta)qk + Rb = N^1(k,b),$$

which, together with (A.9) and $k_{+1} + b_{+1} = k_{+1}/\lambda_{+1}$, verifies

$$k_{+1} = \frac{\lambda_{+1}}{1+(q-1)\lambda_{+1}}\beta N^1(k,b) \quad \text{and} \quad b_{+1} = \frac{1-\lambda_{+1}}{1+(q-1)\lambda_{+1}}\beta N^1(k,b). \tag{A.10}$$

Step 2. Verifying the property of V^1. Using the definition $\lambda_{+1} = k_{+1}/(k_{+1}+b_{+1})$, (A.9), and (A.10) to replace c , k_{+1}, and b_{+1}, we can rewrite $V^1(k,b)$ in (A.4) as

$$V^1(k,b) = J^1\left(\frac{k}{k+b}\right) + \frac{\log N^1(k,b)}{1-\beta} \tag{A.11}$$

$$\text{where } J^1\left(\frac{k}{k+b}\right) \equiv \frac{(1-\beta)\log(1-\beta)+\beta\log\beta}{1-\beta}$$
$$+\beta \max_{0<\lambda_{+1}\le\bar{\lambda}}\left\{-\frac{\log\left(1+\left[q\left(\frac{k}{k+b}\right)-1\right]\lambda_{+1}\right)}{1-\beta} + \mathbb{E}\left[V\left(\lambda_{+1}, 1-\lambda_{+1}, \zeta_{+1}\right)\right]\right\}. \tag{A.12}$$

J^1 depends only on the leverage λ. This is because q depends on λ, not on ζ. Therefore, I verify the property of $V^1(k,b)$ and the proof is completed.[36] □

A.1.1. Choosing λ_{+1}. Although not immediately used in the proposition, the optimal choice of λ_{+1} is derived here, which will be used in the characterization later. I start from the maximization in (A.12).

$$\max_{0<\lambda_{+1}\le\bar{\lambda}}\left\{-\frac{\log\left(1+(q-1)\lambda_{+1}\right)}{1-\beta} + \mathbb{E}\left[V\left(\lambda_{+1}, 1-\lambda_{+1}, \zeta_{+1}\right)\right]\right\}.$$

36. The sufficient conditions of the verification theorem 4.14 in Stokey et al. (1989) with unbounded returns (unbounded utility here) are satisfied. Therefore, the supremum of the value functions (or the economic meaningful one) has the same properties in the proof. To see this, we need well-defined upper bound functions. When r is small, $V^0(k,b)$ is naturally the candidate value function upper bound in the theorem, and has closed-form solution (see numerical examples below). When r is large (not in the simple model), the entrepreneur always invests; then $V^1(k,b)$ is the candidate value function upper bound with a binding financing constraint (so that there is a single value $J^1(k/(k+b))$ to be determined because of the single leverage $k/(k+b)$) and a slack resale constraint ($q = 1$). This case also has closed-form solution.

From the envelope conditions,[37] we know that

$$V_k^0(k,b) = \frac{r+1-\delta}{(1-\beta)\,[rk+(1-\delta)k+Rb]} \;;\; V_b^0(k,b) = \frac{R}{(1-\beta)\,[rk+(1-\delta)k+Rb]};$$
$$V_k^1(k,b) = \frac{r+(1-\delta)q}{(1-\beta)\,[rk+(1-\delta)qk+Rb]} \;;\; V_b^1(k,b) = \frac{R}{(1-\beta)\,[rk+(1-\delta)qk+Rb]}.$$

which can be verified by using (A.2), (A.10), and (A.11). Let the probability of liquidation be $P = P(\lambda_{+1}) = F(\tilde{\xi}(\lambda_{+1}))$ where $\tilde{\xi}$ is the liquidation threshold shown later. After using the envelop conditions above, I derive the first-order condition for λ_{+1}[38]

$$P\left[\frac{r+1-\delta-R}{(r+1-\delta)\,\lambda_{+1}+R(1-\lambda_{+1})}\right] + (1-P)\left[\frac{r+(1-\delta)q_{+1}-R}{[r+(1-\delta)q_{+1}]\,\lambda_{+1}+R(1-\lambda_{+1})}\right] = \mu_\lambda + \frac{q-1}{1+(q-1)\lambda_{+1}},$$

where $\mu_\lambda/(1-\beta) \geq 0$ is the Lagrange multiplier attached to the borrowing constraint $\lambda_{+1} \leq \bar{\lambda}$. After multiplying both sides by $1+(q-1)\lambda_{+1}$, we obtain

$$P\left[\frac{qR_{+1}^{k,L}-R}{\frac{q\lambda_{+1}}{1+(q-1)\lambda_{+1}}\left(R_{+1}^{k,L}-R\right)+R}\right] + (1-P)\left[\frac{qR_{+1}^{k,S}-R}{\frac{q\lambda_{+1}}{1+(q-1)\lambda_{+1}}\left(R_{+1}^{k,S}-R\right)+R}\right] = [1+(q-1)\lambda_{+1}]\,\mu_\lambda + q - 1,$$

where $R_{+1}^{k,L}$ and $R_{+1}^{k,S}$ are capital (shadow) returns if the firm will be liquidated or will be staying next period, respectively:

$$R_{+1}^{k,L} = \frac{r+1-\delta}{q} \text{ and } R_{+1}^{k,S} = \frac{r+(1-\delta)q_{+1}}{q}. \tag{A.13}$$

Finally, I subtract $q-1$ on both sides of the first-order condition before (A.13) and rearrange it to obtain

$$P\left[\frac{R_{+1}^{k,L}-R}{\frac{q\lambda_{+1}}{1+(q-1)\lambda_{+1}}\left(R_{+1}^{k,L}-R\right)+R}\right] + (1-P)\left[\frac{R_{+1}^{k,S}-R}{\frac{q\lambda_{+1}}{1+(q-1)\lambda_{+1}}\left(R_{+1}^{k,S}-R\right)+R}\right] = \frac{\mu_\lambda\,[1+(q-1)\lambda_{+1}]^2}{q} \geq 0, \tag{A.14}$$

37. The envelope condition requires differentiability of $V(k,b,\xi)$. When $k_{+1} > (1-\delta)k$, this relies on the differentiability of standard dynamic programming problem as proved by Benveniste and Scheinkman (1979) (or see again Stokey et al. (1989)). When $k_{+1} = (1-\delta)k$, I apply the method from Clausen and Strub (2012) in Banach space (the space of k and b) to the dynamic programming problem of my model.

38. Note that the choice of λ_{+1} also affects the liquidation threshold $\tilde{\xi}(\lambda_{+1})$ and the probability of liquidation next period. However, the sum of the terms that have $\partial\tilde{\xi}(\lambda_{+1})/\partial\lambda_{+1}$ is zero after I use the liquidation threshold shown in Proposition 2 (not requiring the knowledge of this first-order condition).

with inequality ">" if $\lambda_{+1} = \bar{\lambda}$. This is similar to a standard Euler equation (in terms of the excess return on capital), or an optimal portfolio choice between capital and bonds, taking into account potential borrowing constraints. The difference is that it has the shadow price of capital depending on the entrepreneur's choice. Again from the definition of q in condition (A.8), we know $q = 1$ if the entrepreneur invests in capital (so that the resale constraint is slack); $q < 1$ if the entrepreneur is not investing but still decides to stay in business (so that the resale constraint is binding).

A.1.2. Numerical Examples. I show examples to visualize value functions. Notice that J^0 can be derived explicitly under Assumption A1, because entrepreneurs who liquidate will not start a new firm with low productivity. As a result, $V(0, 1, \zeta_{+1}) = V^0(0, 1) = J^0 + \log R/(1-\beta)$, which can be plugged into (A.2); we obtain

$$J^0 = (1-\beta)^{-1}\left[\log(1-\beta) + \beta\frac{\log \beta R}{1-\beta}\right].$$

The parameterizations are $\delta = 0.10$, $\beta = 0.85$, $R = 1.00$, and $R^k = R/1.03 \approx 0.97$ (so that $r = R^k - (1-\delta) = 0.07$). These are sufficient to show $V^0(k, b)$. To calculate $V^1(k, b)$, I specify that ζ is drawn from a uniform distribution with a support $[0, 2]$. θ is set to be 0.38 so that the highest leverage is $\bar{\lambda} = 1.613$.

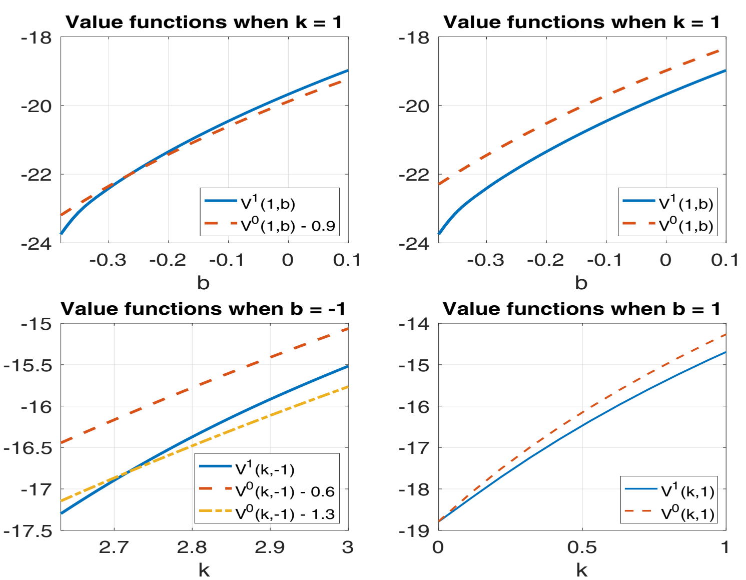

FIGURE A.1. Example value functions.

The top row panels in Figure A.1 show the value functions when $k = 1$. One has a liquidation cost $\zeta = 0.9$, and the other has no liquidation cost. The bottom row panels show value functions by fixing b. The left-bottom panel has $b = -1$, so that the firm has debt and capital k cannot go below a certain positive level (otherwise, consumption is negative). The right-bottom panel has $b = 1$, and in this case the entrepreneur can either run a firm (i.e., $k > 0$) or no firm (i.e., $k = 0$).

As expected, because the return from running the firm R^k is smaller than the interest rate R in this parameterization, it is always better to enjoy V^0 in the absence of liquidation cost (the second column). Positive costs of liquidation (shown in the first column) can prevent the entrepreneur exercising this option. Therefore, for $k > 0$, the value function $V(k, b, \zeta) = \max\{V^0(k, b) - \zeta, V^1(k, b)\}$ will be the upper envelope of $V^0(k, b) - \zeta$ and $V^1(k, b)$ in each panel.

A.2. Proof of Proposition 2 with Derivation of $\omega^0(\lambda)$ and $\omega^1(\lambda)$

I consider an entrepreneur with $k = 1$ and leverage λ, and I use $\omega^0(\lambda) \equiv V^0(1, \lambda^{-1} - 1)$ and $\omega^1(\lambda) \equiv V^1\left(1, \lambda^{-1} - 1\right)$ as in the main text. Also, one can define $\omega(\lambda, \zeta) = \max\{\omega^0(\lambda) - \zeta, \omega^1(\lambda)\}$ as the value if the entrepreneur draws liquidation cost ζ.

There are three cases. (1). If $\omega^0(\lambda) - \bar{\zeta} \geq \omega^1(\lambda)$, the liquidation option is always preferred (i.e., even if paying the largest liquidation cost $\bar{\zeta}$, the value $\omega^0(\lambda) - \bar{\zeta}$ weakly dominates); in this case, $\tilde{\zeta}(\lambda) = \bar{\zeta}$. (2). If $\omega^0(\lambda) - \underline{\zeta} \leq \omega^1(\lambda)$, the staying option is always preferred; in this case, $\tilde{\zeta}(\lambda) = \underline{\zeta}$. (3). If $\omega^0(\lambda) - \omega^1(\lambda) \in \left(\underline{\zeta}, \bar{\zeta}\right)$, then we have $\tilde{\zeta} = \omega^0(\lambda) - \omega^1(\lambda)$; in this case, the entrepreneur liquidates the entire firm iff ζ is smaller than the the threshold $\tilde{\zeta}$, since $\omega^0(\lambda) - \zeta \geq \omega^1(\lambda)$ for $\zeta \in [\underline{\zeta}, \tilde{\zeta})$ and $\omega^0(\lambda) - \zeta \leq \omega^1(\lambda)$ for $\zeta \in [\tilde{\zeta}, \bar{\zeta}]$. Combining the three cases, we have $\tilde{\zeta}(\lambda) = \min\{\max\{\omega^0(\lambda) - \omega^1(\lambda), \underline{\zeta}\}, \bar{\zeta}\}$, and the following shows $\omega^0(\lambda) - \omega^1(\lambda)$.

First, for an entrepreneur who liquidates the entire firm, his value $\omega^0(\lambda) = V^0(1, \lambda^{-1} - 1)$ satisfies

$$
\begin{aligned}
\omega^0(\lambda) &= \log\left((1-\beta)\underbrace{n^0(\lambda)}_{r+1-\delta-R+R\lambda^{-1}}\right) + \beta\left[J^0 + \frac{\log \beta n^0(\lambda) R}{1-\beta}\right] \\
&= \log(1-\beta) + \log\left(n^0(\lambda)\right) + \frac{\beta}{1-\beta}\log(\beta R) + \beta\omega^0(\lambda), \qquad \text{(A.15)}
\end{aligned}
$$

where I use (A.2) for $V^0(1, \lambda^{-1} - 1)$, the consumption function $(1-\beta)n^0(\lambda)$, and net worth at the beginning of next period $\beta n^0(\lambda) R$ according to Proposition 1. Second, for an entrepreneur who does not liquidate, the level of capital stock is $(1-\delta)$. The budget constraint (A.3) implies that consumption satisfies $c^1 = n^0(\lambda) - (1-\delta)/\lambda_{+1}$, where we use $k = 1$, $b = \lambda^{-1} - 1$, $k_{+1} = (1-\delta)$, and $b_{+1} = (1-\delta)\left(\lambda_{+1}^{-1} - 1\right)$ according to Proposition 1. Then, $\omega^1(\lambda) = V^1(1, \lambda^{-1} - 1)$ satisfies

$$
\begin{aligned}
\omega^1(\lambda) = \log&\left(n^0(\lambda) - \frac{1-\delta}{\lambda_{+1}}\right) + \frac{\beta \log(1-\delta)}{1-\beta} \\
&+ \beta\int_{\underline{\zeta}}^{\tilde{\zeta}(\lambda_{+1})}\left[\omega^0(\lambda_{+1}) - x\right]dF(x) + \beta\int_{\tilde{\zeta}(\lambda_{+1})}^{\bar{\zeta}}\omega^1(\lambda_{+1})dF(x), \qquad \text{(A.16)}
\end{aligned}
$$

where I use (A.4) for $V^1(1, \lambda^{-1} - 1)$. The RHS includes the utility from consumption, together with three possible continuation values next period. Finally, assuming an interior solution $\tilde{\zeta}(\lambda) \in (\underline{\zeta}, \bar{\zeta})$, I subtract (A.15) from (A.16). Noticing that $\tilde{\zeta}(\lambda_{+1}) = \omega^0(\lambda_{+1}) - \omega^1(\lambda_{+1})$, I obtain the recursion for $\tilde{\zeta} = \tilde{\zeta}(\lambda)$ that proves (12)

$$\tilde{\zeta} = \log\left(\frac{(1-\beta)(R^k - R + R\lambda^{-1})}{R^k - R + R\lambda^{-1} - (1-\delta)/\lambda_{+1}}\right) + \frac{\beta}{1-\beta}\log\left(\frac{R}{R^k - R + \frac{R}{\lambda_{+1}}}\right)$$
$$+ \frac{\beta}{1-\beta}\log\left(\frac{\beta\left(R^k - R + \frac{R}{\lambda}\right)}{1-\delta}\right) + \beta\left[\tilde{\zeta}_{+1} - \int_{\underline{\zeta}}^{\tilde{\zeta}_{+1}} F(\zeta) d\zeta\right]. \qquad \text{(A.17)}$$

□

REMARK. *Now we can solve $\tilde{\zeta}$, q, and λ_{+1} jointly. We have two equations (A.14) and (A.17) already, and the last one is*

$$\underbrace{r + 1 - \delta}_{R^k} + R\left(\frac{1}{\lambda} - 1\right) - \frac{1-\delta}{\lambda_{+1}} = (1-\beta)\left[r + (1-\delta)q + R\left(\frac{1}{\lambda} - 1\right)\right], \qquad \text{(A.18)}$$

in which LHS is consumption from the budget constraint (A.3) and RHS is from the consumption policy function (A.9) in Proposition 1. Figure A.2 shows numerical examples of the liquidation threshold (and leverage policy) using the same parameters for Figure A.1. In the comparative-static analysis, $R^k = R/1.05$ instead of $R^k = R/1.03$. Notice that the liquidation threshold increases with leverage λ. This is because the dividend payment c falls with leverage if the entrepreneur chooses to stay as the debt servicing cost goes up. Therefore, staying in business is more painful with a higher leverage λ, and it is more likely (i.e., a rising threshold) that the entrepreneur chooses to liquidate the firm. Finally, when R^k falls relative to the interest rate R, the firm is even less productive. It is intuitive that for any given λ, the liquidation threshold goes up. The leverage dynamics will be discussed in the next proposition.

A.3. Proof of Proposition 3

The proof is outlined as follows. First, the proof shows that leverage grows, i.e., $\lambda_{+1} > \lambda$, if the entrepreneur does not liquidate the firm and if the firm is not financially constrained. Second, the proof looks at a range of λ that implies the immediate financially constrained outcome (i.e., $\lambda_{+1}(\lambda) = \bar{\lambda}$).

I use (A.18) to understand leverage dynamics. As before, I normalize $k = 1$. (A.18) implies that the shadow price satisfies

$$q(\lambda) = 1 - \frac{1}{(1-\beta)\lambda_{+1}(\lambda)} + \frac{\frac{\beta R}{1-\delta}}{1-\beta}\frac{1}{\lambda} - \frac{\beta\left(R - R^k\right)}{(1-\beta)(1-\delta)}, \qquad \text{(A.19)}$$

or, after rearrangement:

$$\frac{1}{\lambda_{+1}} = \frac{\beta R}{1-\delta}\frac{1}{\lambda} + (1-\beta)\left[1 - q(\lambda)\right] - \frac{\beta\left(R - R^k\right)}{1-\delta}, \qquad \text{(A.20)}$$

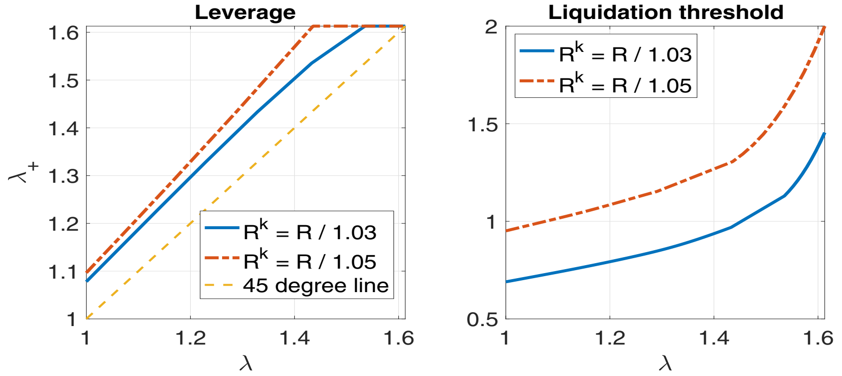

FIGURE A.2. Example leverage policy and liquidation threshold $\tilde{\xi}$.

both of which will be used later.

Step 1. We focus on $\lambda_{+1} < \bar{\lambda}$, i.e., the financing constraint is not binding today for an entrepreneur who decides to stay. We need some information about the shadow price for the leverage dynamics. From (A.13), tomorrow the rates of return on capital when the firm will be liquidated and when the firm will stay are

$$R^{k,L} = \frac{r + 1 - \delta}{q} \quad \text{and} \quad R^{k,S} = \frac{r + (1-\delta)q_{+1}}{q}.$$

Since $q_{+1} < 1$, $R^{k,L} > R^{k,S}$. We must have $R^{k,S} < R$, otherwise $R^{k,L} > R^{k,S} \geq R$, and the entrepreneur has to be financially constrained today according to the portfolio choice (A.14), a contradiction to non-binding financing constraint. Also, we must have $R^{k,L} > R$, otherwise $R^{k,S} < R^{k,L} \leq R$, and the entrepreneur should choose $\lambda_{+1} = 0$ or $k_{+1} = 0$, which is a contradiction to the fact that the entrepreneur stays in business.

With $R^{k,S} < R < R^{k,L}$, we know

$$q(\lambda) > \frac{r + (1-\delta)q(\lambda_{+1})}{R}, \tag{A.21}$$

and $q_{+1} \leq q$ (otherwise $R^{k,L} \leq R^{k,S}$). Let the lower bound of the shadow price when the entrepreneur is not financially constrained be $\underline{q}$, and (A.21) implies that

$$q(\lambda) \geq \underline{q} > \frac{r}{R - (1-\delta)}.$$

With the knowledge of q, we analyze two possibilities of leverage dynamics.

First, if tomorrow the entrepreneur is not financially constrained in the case of staying, i.e., $\lambda_{+1}(\lambda_{+1}(\lambda)) < \bar{\lambda}$, then $q_{+1} \geq \underline{q}$, which together with (A.20) implies the following relationship between $1/\lambda_{+1}$ (note: $\lambda_{+1} < \bar{\lambda}$) and $1/\lambda$:

$$\begin{aligned}\frac{1}{\lambda_{+1}} &\leq \frac{\beta R}{1-\delta}\frac{1}{\lambda} + (1-\beta)\left(1-\underline{q}\right) - \frac{\beta\left(R-R^k\right)}{1-\delta} \\ &< \frac{\beta R}{1-\delta}\frac{1}{\lambda} + (1-\beta)\left[\frac{R-R^k}{R-(1-\delta)}\right] - \frac{\beta(R-R^k)}{1-\delta} \\ &= \frac{\beta R}{1-\delta}\frac{1}{\lambda} + \left(1-\frac{\beta R}{1-\delta}\right)\frac{1}{\lambda^*}.\end{aligned}$$

Second, if tomorrow the entrepreneur is financially constrained in the case of staying, i.e., $\lambda_{+1}(\lambda_{+1}(\lambda)) = \bar{\lambda}$, then (A.19) and $\lambda_{+1} < \bar{\lambda}$ lead to

$$q(\lambda_{+1}) =$$

$$1 - \frac{1}{(1-\beta)\bar{\lambda}} + \frac{\frac{\beta R}{1-\delta}}{1-\beta}\frac{1}{\lambda_{+1}} - \frac{\beta\left(R-R^k\right)}{(1-\beta)(1-\delta)} > 1 - \frac{1-\frac{\beta R}{1-\delta}}{1-\beta}\frac{1}{\bar{\lambda}} - \frac{\beta\left(R-R^k\right)}{(1-\beta)(1-\delta)},$$

which, together with (A.20), implies the following relationship between $1/\lambda_{+1}$ (note: $\lambda_{+1} < \bar{\lambda}$) and $1/\lambda$:

$$\begin{aligned}\frac{1}{\lambda_{+1}} &< \frac{\beta R}{1-\delta}\frac{1}{\lambda} + (1-\beta)\left[1-\frac{r+(1-\delta)q(\lambda_{+1})}{R}\right] - \frac{\beta\left(R-R^k\right)}{1-\delta} \\ &< \frac{\beta R}{1-\delta}\frac{1}{\lambda} + (1-\beta)\left(\frac{R-R^k}{R}\right) + \frac{1-\delta}{R}\left(1-\frac{\beta R}{1-\delta}\right)\frac{1}{\bar{\lambda}} \\ &\quad + \frac{\beta\left(R-R^k\right)}{R} - \frac{\beta(R-R^k)}{1-\delta} \\ &= \frac{\beta R}{1-\delta}\frac{1}{\lambda} + \frac{1-\delta}{R}\left(1-\frac{\beta R}{1-\delta}\right)\frac{1}{\bar{\lambda}} + \frac{R-(1-\delta)}{R\lambda^*}\left(1-\frac{\beta R}{1-\delta}\right) \\ &< \frac{\beta R}{1-\delta}\frac{1}{\lambda} + \left(1-\frac{\beta R}{1-\delta}\right)\frac{1}{\bar{\lambda}},\end{aligned}$$

where the first "<" uses (A.21), the "=" uses the fact that $\lambda^* = [R-(1-\delta)]/(R-R^k)$, and the last "<" uses the fact that $\bar{\lambda} < \lambda^*$ and $\beta R < 1-\delta$.

Therefore, combining the two cases above, we have

$$\lambda_{+1} > \frac{1}{\frac{\beta R}{1-\delta} + \left(1-\frac{\beta R}{1-\delta}\right)\lambda/\bar{\lambda}}\lambda > \lambda, \tag{A.22}$$

since $\beta R < 1-\delta$ and $\lambda < \bar{\lambda}$. That is, λ_{+1} grows before reaching the upper bound $\bar{\lambda}$.

Step 2. I derive a (sufficient but not necessary) lower bound $\lambda^{**} = \psi\left(\bar{\lambda}\right)$ above which the associated entrepreneur is financially constrained immediately. Let us look at a specific leverage λ^{**} such that

$$R^{k,S} = \frac{r+(1-\delta)q(\bar{\lambda})}{q(\lambda^{**})} = R. \tag{A.23}$$

Since $R^{k,L} > R^{k,S} = R$, (A.14) features strict inequality ">", and thus the staying entrepreneur with $\lambda = \lambda^{**}$ is financially constrained. In fact, with some $\lambda < \lambda^{**}$ the entrepreneur may also be financially constrained, as (A.14) can still holds as inequality; but I pick λ^{**} so that $q(\lambda^{**})$ can be computed from (A.23) without knowing the liquidation threshold. To obtain λ^{**} as a function of $\bar{\lambda}$, I solve $q(\bar{\lambda})$ with $\lambda_{+1}(\bar{\lambda}) = \bar{\lambda}$ from (A.19)

$$q(\bar{\lambda}) = 1 + \frac{\beta R - (1-\delta)}{(1-\beta)(1-\delta)} \frac{1}{\bar{\lambda}} - \frac{\beta \left(R - R^k\right)}{(1-\beta)(1-\delta)}, \tag{A.24}$$

I obtain $q(\lambda^{**}) = [r + (1-\delta) q(\bar{\lambda})]/R$ from (A.23), and I finally set $\lambda_{+1}(\lambda^{**}) = \bar{\lambda}$ in (A.19) to obtain an equation that links λ^{**} and $\bar{\lambda}$:

$$\lambda^{**} = \frac{\bar{\lambda}}{\frac{1-\delta}{\beta R} - \left(\frac{1-\delta}{\beta R} - 1\right)\left[\frac{1-\delta}{R} + \frac{\bar{\lambda}}{\lambda^*}\left(1 - \frac{1-\delta}{R}\right)\right]} \equiv \psi(\bar{\lambda}).$$

Finally, we can verify that $\psi(\bar{\lambda}) < \bar{\lambda}$ by using $\beta R < 1-\delta$ and $R > 1-\delta$. □

REMARK. *As an illustration, the second panel of Figure A.2 also shows the leverage dynamics. When $R^k = R/1.03$, $\lambda^{**} = 1.608$ and we have immediate financially constrained outcomes $\lambda_{+1}(\lambda) = \bar{\lambda}$ for any $\lambda^{**} \leq \lambda \leq \bar{\lambda}$ (see the flat part of the blue-solid line). Notice that an entrepreneur is already financially constrained when the beginning-of-period leverage λ is roughly 1.534 (i.e., smaller than λ^{**}). When R^k falls relative to the interest rate R, leverage goes up more quickly, and there is a larger range of λ that puts a staying entrepreneur financially constrained.*

A.4. Proof of Proposition 4

The proof is outlined as follows. First, I will show $\partial\tilde{\zeta}(\bar{\lambda};\bar{\lambda})/\partial\bar{\lambda} > 0$, which completes the second statement of the proof; Second, I will show the expression for $\tilde{\zeta}(\lambda;\bar{\lambda})$; Third, I will show $\partial\tilde{\zeta}(\lambda;\bar{\lambda})/\partial\bar{\lambda} < 0$, which completes the first statement of the proof. To simplify, define the LHS of (13) as $G(\tilde{\zeta})$, and define the RHS of (13) as $H(\bar{\lambda})$.

Step 1. Notice that $\tilde{\zeta}(\bar{\lambda};\bar{\lambda})$ solves $G(\tilde{\zeta}) = H(\bar{\lambda})$. Additionally, $G(\tilde{\zeta})$ is an increasing function of $\tilde{\zeta}$ since $G'(\tilde{\zeta}) = 1 - \beta + \beta F(\tilde{\zeta}) > 0$; $H(\bar{\lambda})$ is also an increasing function of $\bar{\lambda}$ because

$$H'(\bar{\lambda}) = \frac{-R/\bar{\lambda}^2}{R^k - R + R/\bar{\lambda}} + \frac{[R-(1-\delta)]/\bar{\lambda}^2}{R^k - R + R/\bar{\lambda} - (1-\delta)/\bar{\lambda}} > 0 \Longleftrightarrow R^k - R < 0,$$

which is true under Assumption A1. According to the implicit function theorem, we thus have the second statement of the proposition:

$$\frac{\partial\tilde{\zeta}(\bar{\lambda};\bar{\lambda})}{\partial\bar{\lambda}} = \frac{H'(\bar{\lambda})}{G'(\tilde{\zeta})} = \frac{\frac{-R/\bar{\lambda}^2}{R^k - R + R/\bar{\lambda}} + \frac{[R-(1-\delta)]/\bar{\lambda}^2}{R^k - R + R/\bar{\lambda} - (1-\delta)/\bar{\lambda}}}{1 - \beta + \beta F(\tilde{\zeta}(\bar{\lambda};\bar{\lambda}))} > 0.$$

Step 2. Using (A.17) and setting $\lambda_{+1} = \bar{\lambda}$, we obtain the expression $\tilde{\xi}(\lambda;\bar{\lambda})$ for any given $\lambda \in [\psi(\bar{\lambda}), \bar{\lambda}]$

$$\tilde{\xi}(\lambda;\bar{\lambda}) = \log\left(\frac{(1-\beta)(R^k - R + R/\lambda)}{R^k - R + R/\lambda - (1-\delta)/\bar{\lambda}}\right) + \beta\left[\tilde{\xi}(\bar{\lambda};\bar{\lambda}) - \int_{\underline{\xi}}^{\tilde{\xi}(\bar{\lambda})} F(x)dx\right] + \frac{\beta}{1-\beta}\log\left(\frac{R}{R^k - R + R/\bar{\lambda}}\right) + \frac{\beta}{1-\beta}\log\left(\frac{\beta\left(R^k - R + R/\lambda\right)}{1-\delta}\right).$$

Now, make the first argument in $\tilde{\xi}(\lambda;\bar{\lambda})$ to be $\bar{\lambda}$ and obtain $\tilde{\xi}(\bar{\lambda};\bar{\lambda})$ so that I can relate $\tilde{\xi}(\lambda;\bar{\lambda})$ and $\tilde{\xi}(\bar{\lambda};\bar{\lambda})$:

$$\tilde{\xi}(\lambda;\bar{\lambda}) = \frac{\log\left(\frac{R^k - R + R/\lambda}{R^k - R + R/\bar{\lambda}}\right)}{1-\beta} + \log\left(\frac{R^k - R + R/\bar{\lambda} - (1-\delta)/\bar{\lambda}}{R^k - R + R/\lambda - (1-\delta)/\bar{\lambda}}\right) + \tilde{\xi}(\bar{\lambda};\bar{\lambda}).$$

Step 3. Now, taking the first order derivative of the threshold $\tilde{\xi}(\lambda;\bar{\lambda})$ w.r.t. $\bar{\lambda}$:

$$\begin{aligned}\frac{\partial\tilde{\xi}(\lambda;\bar{\lambda})}{\partial\bar{\lambda}} &= \frac{R/\bar{\lambda}^2}{(1-\beta)\left(R^k - R + R/\bar{\lambda}\right)} - \frac{\left[R-(1-\delta)\right]/\bar{\lambda}^2}{R^k - R + R/\bar{\lambda} - (1-\delta)/\bar{\lambda}} \\ &\quad - \frac{(1-\delta)/\bar{\lambda}^2}{R^k - R + R/\lambda - (1-\delta)/\bar{\lambda}} + \frac{\frac{-R/\bar{\lambda}^2}{R^k - R + R/\bar{\lambda}} + \frac{[R-(1-\delta)]/\bar{\lambda}^2}{R^k - R + R/\bar{\lambda} - (1-\delta)/\bar{\lambda}}}{1-\beta+\beta F(\tilde{\xi}(\bar{\lambda};\bar{\lambda}))} \\ &\le \frac{1}{\bar{\lambda}^2}\left[\frac{\frac{\beta}{1-\beta}\left[R-(1-\delta)\right]}{R^k - R + \frac{R}{\bar{\lambda}} - \frac{1-\delta}{\bar{\lambda}}} - \frac{1-\delta}{R^k - R + \frac{R}{\lambda} - \frac{1-\delta}{\bar{\lambda}}}\right],\end{aligned} \tag{A.25}$$

where the expression after "≤" uses the fact that $F(\tilde{\xi}(\bar{\lambda};\bar{\lambda})) \geq 0$. The rest of the proof shows the whole term in the bracket after "≤" of (A.25) is negative. Or, equivalently,

$$\frac{R^k - R + R/\lambda - (1-\delta)/\bar{\lambda}}{R^k - R + R/\bar{\lambda} - (1-\delta)/\bar{\lambda}} < \frac{\frac{1-\beta}{\beta}(1-\delta)}{R-(1-\delta)}. \tag{A.26}$$

To see (A.26), first notice that an entrepreneur with a leverage $\lambda \in [\psi(\bar{\lambda}), \bar{\lambda}]$ is financially constrained. Since $k = 1$, $k_{+1} = 1-\delta$, and $\lambda_{+1} = \bar{\lambda}$, then

$$R^k - R + R/\lambda - (1-\delta)/\bar{\lambda} = c = \frac{1-\beta}{\beta}(1-\delta)\left[q(\lambda) - 1 + 1/\bar{\lambda}\right],$$

where the first equality uses (A.18) that has different ways of expressing consumption on both sides and where the second equality uses the policy function (A.10). This means that the LHS of (A.26) satisfies

$$\begin{aligned}\frac{R^k - R + R/\lambda - (1-\delta)/\bar{\lambda}}{R^k - R + R/\bar{\lambda} - (1-\delta)/\bar{\lambda}} &= \frac{\frac{1-\beta}{\beta}(1-\delta)\left[1+\bar{\lambda}\left(q(\lambda)-1\right)\right]}{\bar{\lambda}\left(R^k - R\right) + R - (1-\delta)} \\ &\le \frac{\frac{1-\beta}{\beta}(1-\delta)\left[1+\bar{\lambda}\left(q(\psi(\bar{\lambda}))-1\right)\right]}{\bar{\lambda}\left(R^k - R\right) + R - (1-\delta)} = \frac{1-\beta}{\beta}(1-\delta)\frac{1+\frac{\bar{\lambda}}{R}\left(r + q(\bar{\lambda})(1-\delta) - R\right)}{\bar{\lambda}\left(R^k - R\right) + R - (1-\delta)},\end{aligned}$$

where the inequality uses the fact that $\lambda_{+1}(\lambda) = \bar{\lambda}$ for all $\lambda \in \left[\psi(\bar{\lambda}), \bar{\lambda}\right]$ so that $q(\lambda) \le q(\psi(\bar{\lambda})) = q(\lambda^{**})$ according to (A.19), and where the second equality uses (A.23) that links λ^{**} and $\bar{\lambda}$. Finally, to show (A.26), we now only need to show

$$\frac{1 + \frac{\bar{\lambda}}{R}\left(r + q(\bar{\lambda})(1-\delta) - R\right)}{\bar{\lambda}\left(R^k - R\right) + R - (1-\delta)} < \frac{1}{R - (1-\delta)}.$$

It is equivalent to

$$r + q(\bar{\lambda})(1-\delta) - R < R\left[\frac{R^k - R}{R - (1-\delta)}\right],$$

or (after using the expression (A.24) for $q(\bar{\lambda})$ and $R^k = r + 1 - \delta$)

$$\frac{\beta R - (1-\delta)}{1-\beta}\frac{1}{\bar{\lambda}} < \frac{\beta R - (1-\delta)}{1-\beta}\frac{(R - R^k)}{R - (1-\delta)} \Leftrightarrow \bar{\lambda} < \frac{R - (1-\delta)}{R - R^k} = \lambda^*,$$

which is true under $\beta R < 1 - \delta$ and Assumption A2. That is, (A.26) is verified, and the term after "$\le$" of (A.25) is negative so that $\partial\tilde{\xi}(\lambda;\bar{\lambda})/\partial\bar{\lambda} < 0$, verifying the first statement of the proposition. □

A.5. Proof of Proposition 5

The proof is organized as follows. First, I will show the expression of interest rate to be used to substitute out the interest rate in the liquidation threshold. I then look at the impact of θ on the liquidation threshold. Second, I show the "steady-state" liquidation threshold $\tilde{\xi}_1$ in period 1 *after* the initial shock (hitting period 0) compared to the "steady-state" liquidation threshold $\tilde{\xi}_{-1}$ *before* the shock. Third, knowing the properties of $\tilde{\xi}_1$, I show the liquidation threshold $\tilde{\xi}_0$ *on impact of* the financial shock compared to the "steady-state" liquidation threshold $\tilde{\xi}_{-1}$. Finally, as a by-product, I show interest rate R_1 falls below R_0.

Step 1. The version of (12) that has time-varying interest rates is

$$\tilde{\xi}(\lambda;\bar{\lambda}_{+1}) = \log\left(\frac{(1-\beta)\left(R^k - R + \frac{R}{\lambda}\right)}{R^k - R + \frac{R}{\lambda} - \frac{1-\delta}{\bar{\lambda}_{+1}}}\right) + \frac{\beta}{1-\beta}\log\left(\frac{R_{+1}}{R^k - R_{+1} + \frac{R_{+1}}{\bar{\lambda}_{+1}}}\right)$$
$$+ \frac{\beta}{1-\beta}\log\left(\frac{\beta\left(R^k - R + \frac{R}{\lambda}\right)}{1-\delta}\right) + \beta\left[\tilde{\xi}(\bar{\lambda}_{+1};\bar{\lambda}_{+2}) - \int_{\underline{\xi}}^{\tilde{\xi}(\bar{\lambda}_{+1};\bar{\lambda}_{+2})} F(x)dx\right]. \quad \text{(A.27)}$$

Next using the leverage upper bound $\bar{\lambda}_{+1} \equiv (1 - \theta/R_{+1})^{-1}$, $\lambda_{+1} = \bar{\lambda}_{+1}$, and the market clearing for bonds (14), we have

$$\left(-\frac{\theta}{R_{+1}}\right)(1-\delta)\left[1-F(\tilde{\xi})\right]+\beta\left(R^k-\theta_{-1}\right)F(\tilde{\xi})+\frac{B}{K}=0,$$

where $R^k = r + 1 - \delta > \theta$ since $\theta \in [0, 1-\delta]$. Therefore, the interest rate R_{+1} is a function of θ and $\tilde{\xi}$:

$$R_{+1}=\frac{\theta(1-\delta)\left[1-F(\tilde{\xi})\right]}{B/K+\beta\left(R^k-\theta_{-1}\right)F(\tilde{\xi})}. \tag{A.28}$$

Further notice that $R_{+1} > \theta$, otherwise $\bar{\lambda}_{+1} < 1$ which means that the entrepreneur is saving in bonds, a contradiction to the fact that the entrepreneur is borrowing constrained. Since $R^k \geq 1-\delta > \theta_{-1}$ and $R_{+1} > \theta$, (A.28) implies that $1-\delta > (B/K/1-F(\tilde{\xi})$, which will be used later. Notice the condition is trivial when $B \leq 0$.

Now, for $\lambda = \bar{\lambda} = (1-\theta_{-1}/R)^{-1}$ and $\lambda_{+1} = \bar{\lambda}_{+1} = (1-\theta/R_{+1})^{-1}$, the expression for R_{+1} in (A.28) can be used to express the liquidation threshold (A.27) as

$$\tilde{\xi}-\beta\tilde{\xi}_{+1}+\beta\int_0^{\tilde{\xi}_{+1}}F(x)dx=\log\left(\frac{(1-\beta)\left(R^k-\theta_{-1}\right)}{R^k-\theta_{-1}-(1-\delta)+\frac{B/K+\beta\left(R^k-\theta_{-1}\right)F(\tilde{\xi})}{1-F(\tilde{\xi})}}\right)$$
$$+\frac{\beta}{1-\beta}\log\left(\frac{R^k-\theta_{-1}}{R^k-\theta}\right)+\frac{\beta}{1-\beta}\log\left(\frac{\beta\theta\left[1-F(\tilde{\xi})\right]}{B/K+\beta\left(R^k-\theta_{-1}\right)F(\tilde{\xi})}\right). \tag{A.29}$$

It is convenient to define the left-hand side of (A.29) as $G(\tilde{\xi}, \tilde{\xi}_{+1})$ where

$$G(x,y)\equiv x-\beta y+\beta\int_0^y F(s)ds.$$

Importantly, $G_x(x,y) > 0$ and $G_y(x,y) = -\beta(1-F(y)) \leq 0$. Finally, $dG(x,x)/dx > 0$.

Step 2. I prove that $\tilde{\xi}_1 < \tilde{\xi}_{-1}$. For periods $t = -1$ and $t = 1$, the thresholds are at different steady states. At $t = -1$, $\theta_{-1} = \theta = \theta^h$; at $t = 1$, $\theta_{-1} = \theta = \theta^l$. Writing $G(\tilde{\xi}_1, \tilde{\xi}_1)$ and $G(\tilde{\xi}_{-1}, \tilde{\xi}_{-1})$ according to (A.29) and subtracting $G(\tilde{\xi}_1, \tilde{\xi}_1)$ from $G(\tilde{\xi}_{-1}, \tilde{\xi}_{-1})$, we reach

$$G(\tilde{\xi}_{-1},\tilde{\xi}_{-1})-G(\tilde{\xi}_1,\tilde{\xi}_1)=\log\left(\frac{1+\frac{\frac{B/K}{1-F(\tilde{\xi}_1)}-(1-\delta)}{R^k-\theta^l}+\frac{\beta F(\tilde{\xi}_1)}{1-F(\tilde{\xi}_1)}}{1+\frac{\frac{B/K}{1-F(\tilde{\xi}_{-1})}-(1-\delta)}{R^k-\theta^h}+\frac{\beta F(\tilde{\xi}_{-1})}{1-F(\tilde{\xi}_{-1})}}\right)$$
$$+\frac{\beta}{1-\beta}\log\left(\frac{\theta^h}{\theta^l}\right)+\frac{\beta}{1-\beta}\log\left(\frac{\frac{B/K}{1-F(\tilde{\xi}_1)}+\frac{\beta\left(R^k-\theta^l\right)F(\tilde{\xi}_1)}{1-F(\tilde{\xi}_1)}}{\frac{B/K}{1-F(\tilde{\xi}_{-1})}+\frac{\beta(R^k-\theta^h)F(\tilde{\xi}_{-1})}{1-F(\tilde{\xi}_{-1})}}\right). \tag{A.30}$$

Suppose, instead, we assume $\tilde{\zeta}_1 \geq \tilde{\zeta}_{-1}$ and thus $F(\tilde{\zeta}_1) \geq F(\tilde{\zeta}_{-1})$. It is straightforward to verify that on the RHS the first term is positive by using $1-\delta > (B/K/1-F(\tilde{\zeta}))$ discussed before. The last two terms are both positive because $\theta^h > \theta^l$ and $F(\tilde{\zeta}_1) \geq F(\tilde{\zeta}_{-1})$. So, $G(\tilde{\zeta}_{-1}, \tilde{\zeta}_{-1}) > G(\tilde{\zeta}_1, \tilde{\zeta}_1)$ according to (A.30) if we assume $\tilde{\zeta}_1 \geq \tilde{\zeta}_{-1}$. However, since $G_x(x,x) > 0$, then $\tilde{\zeta}_1 \geq \tilde{\zeta}_{-1}$ implies that $G(\tilde{\zeta}_{-1}, \tilde{\zeta}_{-1}) \leq G(\tilde{\zeta}_1, \tilde{\zeta}_1)$ must be true, a contradiction. Therefore, $\tilde{\zeta}_1 \geq \tilde{\zeta}_{-1}$ cannot hold, and we have $\tilde{\zeta}_1 < \tilde{\zeta}_{-1}$.

Further notice that, $\tilde{\zeta}_t = \tilde{\zeta}_1$ for $t \geq 1$ (because the entrepreneurs adjust to the new "steady-state" leverage upper bound), we thus know that $\tilde{\zeta}_t < \tilde{\zeta}_{-1}$ for any $t \geq 1$.

Step 3. I prove that $\tilde{\zeta}_0 < \tilde{\zeta}_{-1}$. At time $t=0$, $\theta_{-1} = \theta^h$ and $\theta = \theta^l$. Writing $G(\tilde{\zeta}_0, \tilde{\zeta}_1)$ according to (A.29) and subtracting $G(\tilde{\zeta}_0, \tilde{\zeta}_1)$ from $G(\tilde{\zeta}_{-1}, \tilde{\zeta}_{-1})$ (obtained in Step 2), we reach:

$$G(\tilde{\zeta}_{-1}, \tilde{\zeta}_{-1}) - G(\tilde{\zeta}_0, \tilde{\zeta}_1) = \log\left(\frac{1 + \frac{\frac{B/K}{1-F(\tilde{\zeta}_0)} - (1-\delta)}{R^k - \theta^h} + \frac{\beta F(\tilde{\zeta}_0)}{1-F(\tilde{\zeta}_0)}}{1 + \frac{\frac{B/K}{1-F(\tilde{\zeta}_{-1})} - (1-\delta)}{R^k - \theta^h} + \frac{\beta F(\tilde{\zeta}_{-1})}{1-F(\tilde{\zeta}_{-1})}}\right)$$
$$+ \frac{\beta}{1-\beta}\log\left(\frac{R^k - \theta^l}{R^k - \theta^h}\right)$$
$$+ \frac{\beta}{1-\beta}\left[\log\left(\frac{\theta^h}{\theta^l}\right) + \log\left(\frac{\frac{B/K}{1-F(\tilde{\zeta}_0)} + \frac{\beta(R^k - \theta^h)F(\tilde{\zeta}_0)}{1-F(\tilde{\zeta}_0)}}{\frac{B/K}{1-F(\tilde{\zeta}_{-1})} + \frac{\beta(R^k - \theta^h)F(\tilde{\zeta}_{-1})}{1-F(\tilde{\zeta}_{-1})}}\right)\right]. \tag{A.31}$$

Suppose, instead, we assume $\tilde{\zeta}_0 \geq \tilde{\zeta}_{-1}$ and thus $F(\tilde{\zeta}_0) \geq F(\tilde{\zeta}_{-1})$. Then, following a similar check as in Step 2, we can verify that $G(\tilde{\zeta}_{-1}, \tilde{\zeta}_{-1}) > G(\tilde{\zeta}_0, \tilde{\zeta}_1)$ according to (A.31). However, since we already proved that $\tilde{\zeta}_{-1} > \tilde{\zeta}_1$ and we assume that $\tilde{\zeta}_0 \geq \tilde{\zeta}_{-1}$, $G(\tilde{\zeta}_{-1}, \tilde{\zeta}_{-1}) \leq G(\tilde{\zeta}_0, \tilde{\zeta}_{-1}) \leq G(\tilde{\zeta}_0, \tilde{\zeta}_1)$ must be true because $G_x(x,y) > 0$ and $G_y(x,y) \leq 0$, a contradiction. Therefore, $\tilde{\zeta}_0 \geq \tilde{\zeta}_{-1}$ cannot hold, and we have $\tilde{\zeta}_0 < \tilde{\zeta}_{-1}$.

Step 2 and Step 3 together show that $\tilde{\zeta}_t < \tilde{\zeta}_{-1}$ for $t \geq 0$.

Step 4. As a by-product of Step 3, the interest rate R_1 has to fall below R_0, i.e. $R_1 < R_0$. Otherwise, first suppose instead $R_1 = R_0$; then on impact of the adverse financial shock $t = 0$, the end-of-period leverage (i.e., $(1 - \theta^l/R_1)^{-1}$) is lower than that (i.e., $(1-\theta^h/R_0)^{-1}$) of period $t=-1$. But notice that the beginning-of-period leverages in periods $t=-1$ and $t=0$ are the same, Proposition 4 tells us that the value of the liquidation option is higher, and thus $\tilde{\zeta}_0 > \tilde{\zeta}_{-1}$ should be true, a contradiction. Second, suppose instead $R_1 > R_0$, then $\tilde{\zeta}_0$ goes up even further because a higher interest rate further raises the value of liquidation, another contradiction. □

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