

Adaptive Water Resource Planning Using Decision-Rules^{*}

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Abstract

Dealing with uncertainty in water resource planning is problematic because insufficient or underused infrastructure can have social and environmental costs. Multistage stochastic optimisation provides a mechanism to deal with this challenge in water supply capacity expansion planning. However, for real systems it can be mathematically hard and computationally expensive. The ‘Decision-rule’ formulation represents an attempt to remedy this by approximating the multistage problem where decisions at each stage are a function of the uncertainty and the state of the system. We introduce a family of rules to show how they approximate the multistage problem and investigate the implications of the approximation for adaptive water resources planning.

1. Introduction and background

A core challenge of water supply infrastructure planning is deciding the capacity of infrastructure interventions given unknown future supply and demand. New asset selections should be efficient with respect to key performance metrics and flexible such that they can adapt to a range of future conditions. Flexibility is necessary to capitalise on upside conditions while avoiding the risk of the downside situation. Motivated by desire for flexible strategies in engineering design such as to expand, delay and or replace an option until more information is available (Trigeorgis, 1996; Dixit and Pindyck, 1994; De Neufville and Scholtes, 2011), the water planning literature has proposed different methods to solve the capacity expansion decision-making problem (Watkins Jr and McKinney, 1997; Lempert et al., 2006; Kang and Lansey, 2012; Hall et al., 2012; Ray et al., 2012; Matrosov et al., 2013; Haasnoot et al., 2013; Mortazavi-Naeini et al., 2014; Beh et al., 2014; Jeuland and Whittington, 2014; Kwakkel et al., 2015; Ray and Brown, 2015; Huskova et al., 2016; Erfani et al., 2018, 2020).

Among those, multistage stochastic programming is a classical method that has been successfully adapted to many fields of application such as transport,

energy, and water and flood system (one can refer to for example (Ukkusuri and Patil, 2009; Woodward et al., 2014; Ceseña et al., 2015; Erfani et al., 2018)). However, the multistage problem can be mathematically hard to solve, and it has been criticised for being computationally expensive due to the large dimensionality of real systems and the difficulty of implementation beyond a few stages (Shapiro and Nemirovski, 2005; Kuhn et al., 2008). To mitigate these problems, several strategies have been proposed with different degrees of generality, tractability and performance guarantee. ‘Decision rules’ investigated in-depth by Ben-Tal et al. (2004) enable scalability and help model the sequential decisions of multistage stochastic problems. In decision-rule-based optimisation models, decisions are explicit functions of uncertainty and of the current state of the system. That is, the sequentially optimised decisions adapt to the state of the system as uncertainty is progressively resolved over time. This is similar to the earlier conceptual work of Charnes et al. (1958) on relating the decision variables to heating oil stochastic demand and later adopted by Young (1967); Revelle et al. (1969); Loucks et al. (1981) for reservoir rule operation using dynamic programming. In this paper we examine an extended family of decision-rules applied to the sequential water resource capacity expansion problem; we show how they approximate multistage stochastic programming and investigate implications for water resource infrastructure planning.

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2. Problem description and formulation

To formalise the problem statement, consider the vector of uncertain parameters $\xi \in \Xi$. A sequential decision $x_t \in X$ at stages $t = 1, \dots, T$ is to be made before the value of ξ^t is known. That is, at time t a decision maker only knows the realisation of the uncertainty up until t , while future realisations at $t + 1, \dots, T$ are still unknown. The goal is to find a sequence of decisions x_t for which a vector of objective functions is optimised given a state function of the system, i.e., gap between supply and demand, vulnerability of system, failure frequency of an asset, etc.

2.1. Multistage stochastic formulation

To model this sequential problem, we use scenario generation methods in which a finite number of scenarios $w \in \Omega$ (with Ω the discretisation set of Ξ) representing what may happen to ξ in the future, are generated. The system is simulated using those scenarios while planning decisions are being optimised using (multi-objective) optimisation search algorithms. Consider a water resource capacity expansion problem with a total expected cost objective function. For the sake of simplicity of explanation, we assume the planning decisions are solely related to the state of the system and the gap between the existing supply and the demand. The decision maker at each stage t decides the new capacity intervention (dS) and its supply use (S) considering the available information on ξ and the gap between the water resource supply and demand. Given $x = \{S, dS\}$, $\xi = \{Supply\}$, and uncertain resource supply represented by an ensemble of scenarios, the above problem can be presented using the following multistage decision problem:

$$\min_{S, dS} obj = \sum_{w \in \Omega, t \in T, i \in I} \frac{P_w}{(1+r)^t} [cC_i \times (dS_{t,i}^w - dS_{t-1,i}^w) + fC_i \times dS_{t,i}^w + vC_i \times S_{t,i}^w], \quad (1a)$$

s.t.

$$\sum_{i \in I} S_{t,i}^w + eS_t^w \geq D_t, \quad \forall w \in \Omega, t \in T, \quad (1b)$$

$$S_{t+\lambda_i,i}^w \leq dS_{t,i}^w \times cS_i^w, \quad \forall w \in \Omega, t \in T, i \in I, \quad (1c)$$

$$dS_{t+1,i}^w \leq dS_{t,i}^w, \quad \forall w \in \Omega, t \in T, i \in I, \quad (1d)$$

$$dS_{t,i}^w = dS_{t,i}^v, \quad \forall w, v \in \Omega, i \in I, v \neq w, \quad (1e)$$

where w is a scenario with probability of occurrence of p_w , t denotes planning decision time (stages), i is an intervention decision, r is the discount rate, cC_i ,

fC_i and vC_i are respectively the undiscounted capital, fixed, and variable operational costs of intervention i . The optimisation model minimises the total expected cost of interventions discounted back to the present; the expected value can readily be replaced by other metrics (Kasprzyk et al., 2009; Ray et al., 2014; Herman et al., 2015; Giuliani and Castelletti, 2016; McPhail et al., 2018). Constraint 1b, by investing in intervention i , allows the existing supplies eS_t^w to be augmented to meet the water demand D_t in time t . Constraint 1c allows intervention i to be used up to its maximum capacity (cS_i^w) considering its construction period λ_i . Constraint 1d forces an irreversible decision that once activated needs to remain active until the end of the planning horizon. Constraint 1e ensures that the decision on stage t on each scenario is based on the information available up until time t .

Next, we present special cases of the above problem augmented with a family of decision-rule constraints.

3. Decision-rule based method

To allow adaptive planning, decision variables in the multistage problem 1 above are different for each scenario w . The idea behind the decision-rule method is to still allow for this adaptability by approximating future sequential decisions as a function of a vector of uncertain parameters ξ . That is, at each planning stage, the decision maker only needs the observation of the vector of uncertain parameters (e.g. supply value at t), and the water resource capacity expansion decisions will be provided using the approximated function. To accommodate this, in the multistage problem above, $S_{t,i}$ is replaced by the following equation:

$$S_{t,i} = \Gamma(\xi^t, s) \quad t \in T, i \in I, \quad (2)$$

where $\Gamma(\cdot)$ is a family of decision-rule functions related to uncertainty (ξ) and the current state of the system (s) that is revealed up to time t . Below we present a family of rules for water resource planning.

3.1. Linear rule

If planning decisions and the uncertainty are linearly dependant, $\Gamma(\cdot)$ in Equation 2 reduces to the following linear function of uncertainty ξ (Ben-Tal et al., 2004; Kuhn et al., 2008; Chen et al., 2008):

$$S_{t,i} = \alpha_0^{t,i} + \sum_l \alpha_l^{t,i} \times \xi_l^t \quad t \in T, i \in I. \quad (3)$$

where l is the dimension of the vector of uncertainties and α are the coefficients of the linear function. In this decision rule, decision variables are linearly adaptive to uncertainty.

3.2. Piecewise linear rule

To increase the accuracy of the linear rule, equation 3 can be replaced by a piecewise linear function of uncertainty (with one breakpoint $\hat{\xi}$) as (Georghiou et al., 2015):

$$S_{t,i} = \alpha_0^{t,i} + \sum_l [\alpha_{l,1}^{t,i} \times \{\min(\xi_l^t, \hat{\xi}) - \underline{\xi}\} + \alpha_{l,2}^{t,i} \times \{\hat{\xi} - \max(\xi_l^t, \underline{\xi})\}], \quad t \in T, i \in I, \quad (4)$$

where $\underline{\xi}$ and $\hat{\xi}$ are minimum and maximum of the support of the uncertainty set Ξ .

3.3. Polynomial based rule

In order to model the case for which the decision variables are nonlinearly related to the uncertainty, we extend Equation 3 to a *polynomial* function. This results to the following equation (Bertsimas et al., 2011):

$$S_{t,i} = \alpha_0^{t,i} + \sum_{l,p} \alpha_{l,p}^{t,i} \times \xi_l^{t,p} \quad t \in T, i \in I. \quad (5)$$

where p is the degree of the polynomial function.

3.4. Conditional if-then based rule

In this family of decision-rule, function $\Gamma(\cdot)$ is based on the state of the system. This human-interpretable rule employs a simple if-then condition that reads as ‘if a condition is met then an action should follow’. The if-then based rule is similar to dynamic adaptive policy pathways (Kwakkel et al., 2015, 2016) for adaptive decision making. Within our context, it can be formulated as:

$$S_{t,i} > 0 \quad \text{if} \quad \Theta(\xi_l^t, s). \quad (6)$$

In Equation 6, $\Theta(\cdot)$ defines a trigger for the planning intervention i based on the uncertainty (ξ) and the state of the system (s). In the equation, $\Theta(\cdot)$ and S are both decision variables.

Next, we explain how the optimality of the original multistage problem 1 is affected by introducing decision-rules into the formulation.

4. Optimality of decision-rule formulation

The above family of rules are the functions of uncertainty space. That is, the decision-rule formulations introduced above map realisation ξ to decision variables. This reduces the multistage problem in 1 to a simpler problem by augmenting the constraint sets. To explain, in the original multistage problem 1, the scenarios $w \in \Omega$ are the discretisation of the uncertainty set Ξ and for each scenario w a decision variable is chosen. In the augmentation forced by the decision-rule method, decision variables are restricted to a function of uncertainty and hence this reduces the feasible space (Bertsimas et al., 2010; Bampou and Kuhn, 2011; Wiesemann et al., 2012). This reduction in feasible space, although it simplifies the problem formulation and reduces the computation time, comes at the price of producing solutions which are suboptimal (Georghiou et al., 2019).

To conceptualise this, we define the problem \mathcal{P} as a compact general type of uncertain optimisation problem $\mathcal{P} = \{\min_x \{f(x, \xi) \mid x \in \mathbf{K}\} : \xi \in \Xi\}$ where f is the objective function with the constraint set \mathbf{K} . We also define a decision-rule based problem by $\mathcal{D} = \{\min_x \{f(x, \xi) \mid x \in \mathbf{H}\} : \xi \in \Xi\}$, where $\mathbf{H} = \mathbf{K} \cap \mathbf{R}$ and \mathbf{R} is the additional set of decision-rule constraints added to the original set \mathbf{K} . This leads to the following theorem.

Theorem 4.1. *If Z^* and \hat{Z} are the objective value solution to \mathcal{P} and \mathcal{D} , respectively, $Z^* \leq \hat{Z}$.*

Proof. Let $Z^* = f(x^*, \xi)$ for some $x \in \mathbf{K}$, and $\hat{Z} = f(\hat{x}, \xi)$ for some $x \in \mathbf{H}$. Given that $\hat{x} \in \mathbf{H}$ and that $\mathbf{H} \subseteq \mathbf{K}$, therefore, $\hat{x} \in \mathbf{K}$. However, the optimiser to the f in problem \mathcal{P} with constraint set \mathbf{K} is x^* . This implies that $f(x^*) \leq f(\hat{x})$. That is, $Z^* \leq \hat{Z}$. \square

To demonstrate the above theory, as an example, consider the following simple sequential decision making problem introduced in Chen and Zhang (2009); Bertsimas et al. (2011), with uncertainty set $\Xi = \{w \in \mathbf{R}^N : \|w\|_2 \leq 1\}$. x is the intervention decision that should be taken before the value of uncertain parameter w is revealed, and y are the next stage decisions (the usage of the capacity installed) that depend on the value of uncertain parameter w . The following multistage

optimisation problem:

$$\min_{x,y(w)} obj = x \quad (7a)$$

s.t.

$$x \geq \sum_{i=1}^N y_i, \quad (7b)$$

$$y_i \geq w_i^2, \quad \forall w \in \Xi, \quad (7c)$$

has the optimal solution $obj = 1$ while under linear decision-rule for example, the optimal solution is $obj = N$ (refer to appendix in [Bertsimas et al. \(2011\)](#) for proof). This simple example along with the above theory demonstrates that the optimal solution as a result of employing decision-rules may approximate the original multistage problem poorly (in this case N times larger).

Next, we show how the decision-rule formulations introduced earlier are employed in a water resource planning problem and discuss their optimality implications.

5. Application to a water resource planning problem

We illustrate an application of a decision-rule optimisation model formulation in a sequential water resource capacity expansion planning as it is conceptualised in England and Wales ([Padula et al., 2013](#); [von Lany et al., 2013](#)). Every 5 years, the economic regulator requires water utilities produce a plan demonstrating that the supply-demand balance is met at least cost throughout their operating area over a 25-year planning period. To accommodate this, we consider five planning stages ($t_1 \dots t_5$) in which each stage is a 5-year period. A plan is an optimal combination of five ($o_1 \dots o_5$) new supply and demand management interventions, scheduled to meet estimated water supply zone demand at least expected discounted total cost including the capital, operational and fixed costs. Supply uncertainty is represented by deployable output or the ‘safe’ yield of each source, typically estimated as the lowest volume a supply source was able to provide in the historical period, and remains constant during each 5-year planning decision periods. The upper and lower bound of supply uncertainty along with the demand growth estimation are shown in Table 1. Existing supply uncertainty is represented using 100 uniformly distributed scenarios given the uncertainty bounds in Table 1. We consider five different supply and demand management interventions with specifications listed in Table 2. The alternative interventions have

unique capital, operational and fixed usage costs and capacities (firm yield water supply volume they add to the system). To solve the multistage problem 1, we make a scenario tree out of the 100 scenarios following [Erfani et al. \(2018\)](#) with 1 percent information loss. For linear, piecewise, polynomial (of order 4) we use the formulation 8, 9 and 10, respectively,

$$S_{t,i}^w = \alpha_0^{t,i} + \alpha_1^{t,i} \times eS_t^w \quad w \in \Omega, t \in T, i \in I, \quad (8)$$

$$S_{t,i}^w = \alpha_0^{t,i} + \alpha_1^{t,i} \times \{\min(eS_t^w, \hat{\xi}) - \underline{\xi}\} + \alpha_2^{t,i} \times \{\bar{\xi} - \max(eS_t^w, \hat{\xi})\}, \quad w \in \Omega, t \in T, i \in I, \quad (9)$$

$$S_{t,i}^w = \alpha_0^{t,i} + \sum_p \alpha_p^{t,i} \times (eS_t^w)^p \quad w \in \Omega, t \in T, i \in I. \quad (10)$$

For the conditional if-then based formulation we use the following rule: ‘if the existing supply capacity in the last time period drops below α_0 Ml/d , the capacity should be expanded by α_1 Ml/D , otherwise nothing should be done’. This is formalised as below for each intervention decision i :

$$S_{t,i}^w = \begin{cases} \alpha_1^{t,i} & \text{if } eS_t^w \leq \alpha_0^{t,i} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

This is easy to implement in the case of simulation-based optimisation where simulation models emulate the if-then rule condition. In a mathematical programming context, one needs to incorporate this via mixed integer ‘big-M’ formulation as described in [Williams \(2013\)](#).

To compare the results, we also find the worst-case solution for which the worst value of supply is used to make decisions at each time stage. Next, we discuss the results.

6. Results

The optimal activation of options and their usage are reported in Table 3 for the second time step t_2 for brevity. The multistage stochastic optimisation problem 1 and the conditional if-then based decision-rule model results are illustrated in Figure 1 and 2. To explain the decision-rule results in Table 3 for t_2 , assume that the existing supply $eS_2 = 1900Ml/d$. With demand of $2080Ml/d$ from Table 1 at t_2 , the shortfall of

Table 1: Existing supply uncertainty bound and demand growth projection

	t_1	t_2	t_3	t_4	t_5
Demand (Ml/d)	2050	2080	2200	2300	2350
Max Supply (Ml/d)	2068	2247	2334	2354	2360
Min Supply (Ml/d)	2068	1825	1711	1684	1659

Table 2: Intervention options' cost, capacity and construction time

Name	ID	Capacity		Cost (GBP)		Construction time
		Ml/d	Capital million	Fixed thousand	Operational thousand/Ml/d	5-yearly
Option 1	o_1	150	4	276	28	-
Option 2	o_2	180	52	2,217	63	1
Option 3	o_3	200	23	1,302	50	1
Option 4	o_4	250	18	1,017	76	2
Option 5	o_5	300	435	7,732	186	3

180Ml/d should be compensated using a combination of intervention options.

Applying the worst-case formulation, the optimal planning suggests an overinvestment decision: that options o_1 and o_3 should be used.

The linear decision-rule suggests o_1 and o_3 to be used by 114.7 and 94.8Ml/d, respectively when $eS_2 = 1900\text{Ml/d}$. The usage levels are derived by replacing the eS_2 value in the corresponding functions in Table 3. The same can be shown for polynomial and piecewise rules.

As for the conditional if-then based rule, the results states that o_1 is used by 150Ml/d and o_3 by 104Ml/d. Compared to the worst-case solution, the if-then rule avoids overinvestment by triggering options' activation and use and monitoring the state of the existing supply; it only activates the options if the existing supply eS_2 falls below the optimal threshold value shown in Figure 2.a.

The multistage model formulation results in a set of rules that depend on the scenario tree, i.e., the capacity options that should be activated and used at each time step and for each level of supply-demand gap. In our example, given that $eS_2 = 1900\text{Ml/d}$ is less than 1963Ml/d, from Table 3 and Figure 1.b, o_1 should be used by 116Ml/d. It is also noted in Figure 1.a that in the same time period that o_1 is used, o_3 and o_4 are also activated to be used in later time periods considering their construction time.

7. Discussion

Multistage stochastic formulation 1 provides a mechanism to allow decision variables to adjust themselves to the actual values of the uncertain data when they are progressively revealed in the planning horizon. This is done by making 'here-and-now' decisions at the first time period ($t = 1$) and employing 'wait-and-see' decisions for each scenario of the uncertainty set Ξ for the other periods ($t = 2, \dots, N$). 'Wait-and-see' decisions are used to correct/adjust the 'here-and-now' decisions as new information becomes available. This allows decision-making to adjust later on in the planning horizon to capitalise on eventual favourable conditions while avoiding the risk of critical situations. For example, from Table 3, in time period 2, the use of o_1 is adjusted based on different levels of supply conditions (from a critical usage of 116 Ml/d to a more favourable zero use). At the same time (from Figure 1), if a drier condition with a lower supply level is realised (the bottom branch of scenarios on the tree in Figure 1), contingency investment plan o_3 is made. This way of structuring the sequential decision problem allows asset managers to review plans periodically (every 5 years in our test case) and respond to water needs by selecting additional interventions or expanding existing ones to achieve security of supply over the entire planning horizon (next 25-years). However, the use of multistage stochastic formulations can be prohibitive in practice due to scalability of the real world system, because it cannot always be solved to optimality (NP-hardness (Shapiro and Nemirovski,

Table 3: Decision-rules and multistage stochastic results ranked based on their total expected cost from the cheapest (top of the table) to the most expensive one. Only time period 2 (t_2) results are shown for brevity. The percentage in the last column are the percentage distance from the solution of multistage model known as optimality gap.

	Decision-rules (values are in MI/d unit)	Cost (million unit)
Multistage stochastic	If existing supply eS_2 is more than 2025, do nothing, if this is between 2025 and 1963 use o_1 by 54, and, if it drops below 1963, use o_1 by 116	39.54
Piecewise linear	$S_{2,o_1} = -54 + 0.36[2400 - \max(eS_2, 1800)]$, $S_{2,o_3} = -38 + 0.25[2400 - \max(eS_2, 1800)]$	58.11 (47%)
Polynomial (4)	$S_{2,o_1} = 1261.2 - 5.9eS_2^2 + 0.7eS_2^4$, $S_{2,o_3} = 63.7 + 0.3eS_2^2 - 0.07eS_2^4$	100.75 (155%)
Conditional if-then	If existing supply eS_2 is below 2079, use o_1 by 150 and o_3 by 104	111.53 (182%)
Linear	$S_{2,o_1} = 742.9 - 0.33eS_2$, $S_{2,o_3} = 614.2 - 0.27eS_2$	111.58 (182%)
Worst Case	Use o_1 by 150 and o_3 by 104	112.26 (184%)

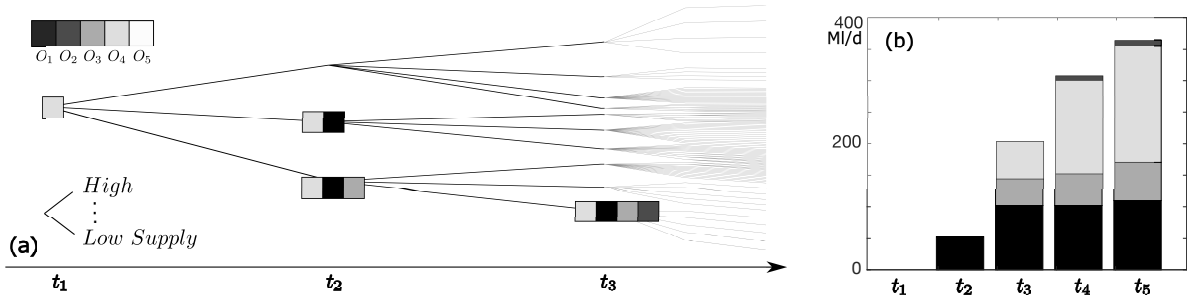


Figure 1: (a) Intervention decisions for the first three time period, (b) Average usage plan of capacity options

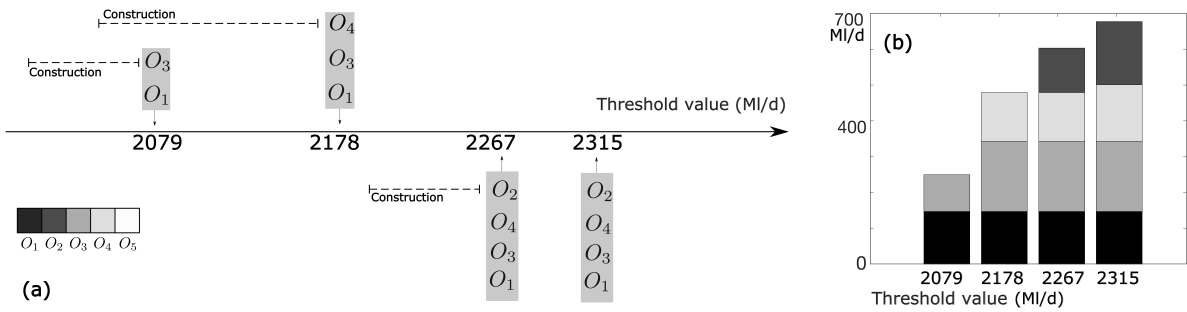


Figure 2: (a) Intervention decisions and usage of different options if existing supply drops below the trigger values on the threshold line, (b) usage plan of capacity options

2005)) or because its implementation beyond a few stages might be unsolvable. Examples in [Georghiou et al. \(2019\)](#) show how multistage formulations for some practical problems become intractable beyond the first few planning stages. This encourages the use of decision-rules. Indeed, the rationale for using decision-rules is their tractability, simplicity of use and their implementability for all decision stages. That is, they can be employed to derive planning decisions as a function of the water resource system's uncertain conditions. As shown in [Table 3](#), all derived rules provide adaptability as they are all series of functions of the state of the system and the uncertainty. That is, the intervention decision and the use of each option are adaptive to the conditions of the system and they change as new information about uncertain parameters becomes available. Unlike a multistage stochastic approach that explicitly models the 'wait-and-see' decisions as a tool for adaptability, decision-rule formulations adapt by relating the decision variables with the uncertain parameters in the form of a functional relationship. In the linear case, for example, in time period 2, the decision on o_1 use is in the form of a linear function $S_{2,o_1} = 742.9 - 0.33eS_2$ ([Table 3](#)). In this equation, as the existing supply, eS_2 , changes, different optimal uses of option o_1 are suggested. This is replaced by the optimal threshold value in the case of the if-then formulation ([Figure 2](#)) for which the threshold works as a trigger in optimally activating and using different interventions. We note however that, depending on the interaction between decision variables and uncertain parameters, the choice of different functional components within each rule may result in different solutions. This implies the optimal solutions provided by the decision rules are dependent on the structure of the rule. This is not the case in the multistage stochastic formulation where the solutions are not bounded by a pre-set functional relationship between decision variables and uncertain parameters.

As demonstrated by [Theorem 4.1](#), decision-rules reduce the efficiency of multistage stochastic problem [1](#). This is empirically shown in [Table 3](#) using the proposed test problem. As can be seen, multistage stochastic formulation [1](#) outperforms the decision-rule formulations. The efficiency of the linear rule, initially performing poorly when applied to this test case, is improved by the polynomial of degree 4 and piecewise formulations by almost 20 and 100 percent, respectively. The accuracy of approximation may be improved further by higher order polynomials or adding more break points to the piecewise linear approximation. However, this is not always the case.

As can be seen in [Table 3](#), a piecewise linear function with one break point outperforms the high order polynomial approximation in this case. The accuracy of approximation depends on several underlying factors including the interval of approximation and underlying structure of data and their interaction with decision variables. For example, for a data set that is linearly dependant, polynomial approximation performs worse than the linear one. In addition, there is a chance that the polynomial approximation bends away from the optimal point whereas the linear one coincides with it or gets closer to it. This was empirically observed by [Young \(1967\)](#) in relevant water resources work where it was stated that the linear approximation provides as good or better a fit to the data than more complicated ones, e.g., quadratic or cubic. Nevertheless, given the difficulties discussed above about the multistage stochastic formulation and that the decision-rules may be the only feasible approach, the approximation accuracy and the potential for suboptimality should be considered and investigated when using decision-rule formulations in capacity expansion problems.

Moreover, although for simplicity of explaining the results we employed a single objective function to discuss the decision rule application, the results in this paper also apply to the case of multiobjective optimisation as demonstrated in [Theorem 4.1](#). This is because for each solution of the multiobjective optimisation, there exists a scalar single objective optimisation problem ([Jahn, 2009](#); [Erfani and Utyuzhnikov, 2011](#)), and hence the vectorised objective function does not reduce the generality of the results. In addition, we hypothesise that [Theorem 4.1](#) also applies to the case of simulation-based optimisation for which the decision rules can be implemented as a set of constraints embedded in a simulation engine. The demonstration of this is left for future work.

8. Conclusion

This paper proposes and examines the use of decision-rules for multistage adaptive water resources infrastructure planning optimisation models. When using decision-rules, given a realisation of uncertainty, the current state of the water infrastructure system and the unknown future, the activation and use of future options are functions of the uncertainties. We introduce a family of decision-rules, namely, linear, polynomial, piecewise and conditional if-then based rules, and show how these formulations approximate the multistage stochastic programming formulation. We investigate their performance on a synthetic water

supply capacity expansion example problem with a small number of intervention options and compare their results with those of the multistage stochastic problem. Results demonstrate that even though the decision-rule formulations simplified the multistage stochastic program, in the example provided, they have more than 50 percent optimality gap with the solution of original multistage problem in most cases. The piecewise and polynomial work better than linear rules. This potential for suboptimal problem approximation when using decision-rules in adaptive optimisation planning models should be considered by analysts. Nevertheless, given that multistage program is hard to solve for large scale systems, and that the decision-rules such as conditional if-then based rules are simple to interpret and implement, they may still be appropriate for adaptive water resource planning. Future research could focus on verifying and improving their efficiency.

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