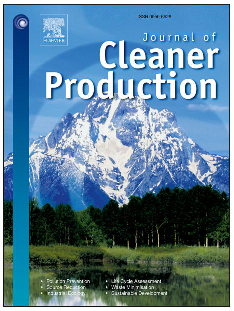

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A Study of Degrowth Paths based on the von Neumann Equilibrium Model

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Abstract

This paper revisits the von Neumann equilibrium model from a degrowth perspective. Degrowth can be either planned or autonomous based on voluntary simplicity (VS), characterized by voluntary constraints on the private consumptions. Any share of the working population is allowed to limit their private consumptions (or ecological footprint). Degrowth means a lower economic growth rate as well as reductions in the usage of materials and fossil energy; however, when the economy cannot fit within the biophysical boundary despite such reductions, degrowth can also mean a deliberate transition towards lesser and cleaner production of a smaller number of goods. Taking ecological boundaries into consideration may thus switch the direction of an economy from growth to degrowth. Degrowth is consistent with economic equilibrium in the dynamic input-output model. Degrowth equilibrium results when a critical mass of the population prioritizes diminishing the ecological deficit over increasing the output levels. Green growth can take place during degrowth. While the economy gradually shrinks, the interest (profit) rate in macroeconomic equilibrium is negative due to the conservation target (excessive consumption has negative value to VS-type agents). Degrowing private consumptions implies less work, more leisure and better quality of the environment, improving social welfare. Degrowth can be advanced by adopting a more cooperative production system or by imposing high ecological taxes. The equilibrium model yields an argument for basic income as a means of supporting welfare during degrowth and of compensating for unpaid work. The open economy version of the model shows reduced scope for intra-industry trade between similar countries.

Keywords: degrowth, von Neumann model, dynamic equilibrium, input-output models, ecological economics, voluntary simplicity, re-localization

1. Introduction

The environmental crisis is severe and worsening. According to the recent second warning to humanity, signed by more than 15000 scientists, "we are jeopardizing our future by not reining in our intense but geographically and demographically uneven material consumption" (Ripple et al., 2017). Economic growth is a major driver of the crisis (Pachero et al., 2018; Ripple et al., 2018). Historically, economic growth has been largely based on cheap fossil energy. Continued growth along the historical trend cannot safely be assumed (Ayres et al., 2013). Empirical research suggests that attempts to curb the carbon footprint have been largely negated by the rebound effect or by leakage due to "off-shoring" of carbon-intensive industry (Druckman and Jackson, 2009). The global ecological footprint, accounting for the carbon footprint, already is 1.7 (see (Global Footprint Network)). In rich countries the resource consumption exceeds the sustainable level by factors between 5 and 10, which explains the enormous losses in biodiversity, declining at unprecedented rates.

The main alternatives to continued growth, recently discussed by Raworth (2017), are green growth and degrowth. The proponents of green growth, including IMF, OECD and mainstream economists, argue that is both necessary and possible to decouple GDP growth from its environmental consequences. In the absence of evidence of the feasibility of the level of absolute decoupling required for fitting the economic system within the planetary boundaries (see (Hickel and Kallis, 2019)), Latouche (2009); Trainer (2012); Kallis (2011, 2013) defend degrowth: a deliberate downscaling of the economy to shrink towards the ecological boundaries. Likewise, Meadows et al. (2004) argue that the global material flows are too large to be sustainable without modifying the consumptions even when accounting for green growth. A survey of literature on degrowth has been presented in (Weiss and Cattaneo, 2017) (see also (Asara et al., 2015)). An assessment of different ecological macroeconomic models for the study of post-growth economies has been presented by Hardt and O'Neill (2017), recommending a combination of input-output analysis with stock-flow consistent modelling (Godley and Lavoie, 2012).

The first equilibrium growth model was the dynamic input-output model by von Neumann (1937)[1], widely studied in various applications; however, it

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[1] Despite his work on game theory, this model was the only paper by John von Neumann that was directly concerned with economics. The work was published in German in 1937; The English version, titled "A Model of General Equilibrium", was published only in 1945.

has not been previously discussed from a degrowth perspective. This work generalized Brouwers fixed-point theorem and applied it for the first time in the proof of the existence of a competitive economic equilibrium. The balanced growth path solution of the equilibrium model allows for different outcomes: degrowth, growth or a steady state[2]. Within the equilibrium model it turns out that degrowth and green growth can be seen as complementary. This paper makes the thought experiment of applying the first growth model to study degrowth paths in an economy that has grown too large in terms of material consumptions, so that despite additional investments in green sectors, fitting the economic system within the biophysical boundaries is infeasible unless the economy also deliberately degrows in size.

Under severe ecological deficit, the conservation of the environment, not economic growth, ought to be the driving force of development (Pachero et al., 2018). A degrowth society is conserving and frugal, embracing the idea to produce and consume only as much as is needed for a high quality of life (Trainer, 2012). The first growth model is based on a similar idea: the role of consumption is to support labour as input of production (von Neumann, 1937). A related notion in deep ecological thinking is that humans do not have the right to reduce the diversity of life except to satisfy the necessities of life (Naess, 1973). In practise, the set of goods considered as necessities can be broadly defined. Voluntary constraints on the private consumptions characterize deep ecological thinking and voluntary simplicity (VS) (Alexander and Ussher, 2012; Elgin and Mitchell, 1977). This paper introduces ecological boundaries into equilibrium growth theory by allowing for any share of the working population to choose VS and limit their private consumptions/ecological footprint. The analysis of the ecological-economic equilibrium model yields the following main results:

- Introducing ecological constraints to equilibrium growth theory implies a breaking point for growth at the ecological boundary; as the share of VS-type agents increases, the equilibrium growth rate falls;
- The direction of the economy can switch from growth to degrowth when the number of VS-type agents rises above a critical mass;
- A shrinking economy is consistent with economic equilibrium while capable of supporting a high quality of life;

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[2] The so-called turnpike theorems in economics show that optimal growth paths will run near to or on the von Neumann-path most of the time, provided that the time horizon is long enough, see (Koopmans, 1964).

- The equilibrium model yields an argument for basic income as a means of supporting welfare during degrowth and of compensating for unpaid work;
- Green growth can take place during degrowth; it is possible to increase environmental work as well as investments in green sectors even if the overall level of production falls;
- Short-term interest rate is negative in macroeconomic equilibrium during degrowth. Degrowth challenges capitalism and motivates different forms of centralized control.

The climate crisis is largely due to overconsumption and calls for new ways of thinking in economics. Mainstream economics omits the biophysical boundaries; consumerist agents are assumed to improve their utilities by maximizing their individual consumptions, omitting environmental externality effects, while firms re-invest profits from production to make more money. Georgescu-Roegen (1971) argued for the necessity of relating economics to the biosphere; however, economic theory has ignored the basic laws of biology and physics (Latouche, 2009; Cochet, 2005). This paper demonstrates that traditional economic thinking can become meaningless under a severe ecological overshoot; in the von Neumann model of a multi-sector economy, the interest (profit) rate is negative in macroeconomic equilibrium during degrowth; on macroeconomic scale, capital endowments cannot yield profits in degrowth equilibrium where excessive consumption has negative value. In general, it is possible that in macroeconomic equilibrium during degrowth some firms are able to maintain positive profits while others would disappear. The theoretical equilibrium model abstracts from asymmetries between firms; assuming the firms are symmetric, all profits really are negative in a degrowth equilibrium. Even if the endowments yield negative short-term profits during degrowth transition, the socially responsible firms or cooperatives would nevertheless continue production for otherwise the labour would cease to exist in the absence of the production of the goods considered necessities[3]. A negative short-term profit rate in degrowth equilibrium necessitates a redefinition of the role of the firms. In a degrowth society firms must care about the ecological impacts of their activities as

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[3] In theory, it is possible that the capitalists would continue to recruit only a small part of the labour force to maintain the production level they consider necessary; the focus in this paper is on the case with full employment where all agents obtain at least subsistence level income.

well as about the welfare of the workers; economic profit cannot be the only motivation for production in a post-growth society.

An equilibrium framework, where the production side is presented in terms of an aggregate production function, was presented in (Heikkinen, 2018), addressing degrowth with heterogeneous agents; therein, a capitalist economy based on endogenous growth coexists with a subsystem based on VS, converging to a steady state; in equilibrium, perfect competition makes long-term profits zero. Degrowth in terms of an increase of the share of VS-type agents diminishes the growth rate but does not disturb the economy in the equilibrium model (ibid.). This paper observes an analogous stability result in the physical multi-sector model of an economy based on (von Neumann, 1937), abstracting from endogenous growth based on technical progress. As the share of VS-type agents increases, the growth rate in the multi-sector model falls; however, in the equilibrium model, degrowth does not disrupt the economic system, as often claimed by the opponents of the degrowth proposal. Degrowth equilibrium in theory means a deliberate, smooth and bounded reduction of material consumptions, while the activities related to environmental work and leisure increase. During degrowth, green investments based on existing techniques is allowed, including additional investments in renewables and organic agriculture. However, gradual degrowth whereby the economy actually shrinks can take place only if sufficiently many agents agree that this is necessary for enforcing the economy to fit within the ecological boundaries. In a democratic system an oversized economy would start shrinking only if a critical mass of the population adopts deep ecological values. Accounting for ecological constraints thus challenges capitalism as well as democracy. To fit an oversized economy within the planetary boundaries in practise is likely to require different forms of centralized control such as ecological taxes and direct constraints. The von Neumann model is general and can be interpreted either as a planned economy or as a decentralized system; planned contraction means sector-specific constraints on production.

Autonomous degrowth can be advanced via self-organizing cooperatives. In a degrowth society, the workers can be better compensated for their personal investments, cf. (Gorz, 1989; Morishima, 1974). The society can support degrowth by introducing a basic income by taxing profits; furthermore, ecological taxes would discourage excessive consumptions. In a steady state equilibrium with given technology and population, the profits are zero; the motivation for production cannot then be wealth accumulation (only perfect competition implies zero long-term profits). In a post-growth society, the purpose of production is to provide the necessary consumption goods, not to maximize the level of economic activities. Thus, degrowth moves the society towards a Keynesian utopia where the economic problem would become of secondary importance, as envisioned already by (Keynes, 1932). In the equilibrium model degrowth does not imply unemployment but less work and better welfare. Likewise, Sekulova et al. (2013) suggest that climate policy can have a positive impact on well-being in spite of reductions in carbon-intensive consumptions, income and working hours. This can be partly due to a downward adjustment of reference consumption and income levels (ibid.). In a post-growth society, social comparisons based on income and expenditures would perhaps become irrelevant.

The paper is organized as follows. The von Neumann model of economic equilibrium and its extension due to (Morishima, 1964, 1974) are presented in Section 2. The model can be applied to determine whether green growth enables the economy to fit within the planetary boundary. Under severe ecological deficit this target can be impossible to achieve within a limited time frame, motivating the study of different degrowth paths in Section 3. Degrowth may result from high ecological taxes or from additional leisure or both. Green growth can take place during degrowth. Degrowth transition is facilitated by participatory resource management based on cooperation (Ostrom, 1990). Section 3 demonstrates the existence of welfare-improving degrowth paths towards lesser production of a smaller number of goods, as well as the consistency of the paths with a solid equilibrium notion. Degrowth in an open economy setting is also addressed. Section 4 presents concluding comments.

2. The von Neumann model of dynamic economic equilibrium

Given a set of products $G_i,\ i = 1, .., n$ that can be produced by production processes $P_j,\ j = 1, ..., m$, von Neumann (1937) asks what processes will be used and what prices will emerge in equilibrium where the intensities x_j grow (or degrow) at a constant rate α. The study contains the first explicit statement of what has subsequently been called the activity-analysis model (Koopmans, 1964). This model assumes a finite number of production processes, characterized by constant ratios of inputs to outputs (constant returns to scale). Formally, the model is based on an input matrix $\mathbf{A}$ and an output matrix $\mathbf{B}$; the ijth element in matrix $\mathbf{A}$, $\mathbf{A}_{ij} = a_{ij}$ denotes the quantity of good i used up in production process j while the ijth element in matrix $\mathbf{B}$, $\mathbf{B}_{ij} = b_{ij}$ denotes the output of good i produced by process j at unit intensity. The inputs have to exist in the beginning of each time period of production. To produce n different goods as outputs requires the necessary activities to be in operation for one time period. For simplicity, time indices will be omitted in what follows. The linear production model abstracts from economies of scale but simplifies the analysis of the role of ecological boundaries in economic theory.

2.1. Equilibrium solution

The unknowns of the von Neumann problem are: (i) the intensities $x_j,\ j = 1, ..., m$ of the processes $P_j,\ j = 1, ..., m$; (ii) the coefficient of expansion of the whole economy α; (iii) the prices $y_i,\ i = 1, ..., n$ of goods $G_i,\ i = 1, .., n$; (iv) the interest factor β, corresponding to a uniform rate of profit. The model allows the growth rate of the economy $\alpha - 1$ to be positive, negative or zero. Assuming $x_j \geq 0,\ j = 1, .., m$, $y_i \geq 0,\ i = 1, ..., n$, $\sum_j x_j > 0$ and $\sum_i y_i > 0$, the model is summarized by the inequalities:

$$\alpha \mathbf{A}\mathbf{x} \leq \mathbf{B}\mathbf{x}, \tag{1}$$

and

$$\beta \mathbf{y}'\mathbf{A} \geq \mathbf{y}'\mathbf{B}. \tag{2}$$

The factor $\alpha > 0$ in (1) is the technological expansion factor while $\beta > 0$ in (2) is the economic expansion factor (Bapat and Raghavan, 1996). According to (1), it is impossible to consume more of a good G_i than is being produced. If, however, there is excess production, G_i becomes a free good and its price $y_i = 0$. This condition corresponds to the so called complementary slackness condition in the theory of linear programming (Bazaraa et al., 1990). According to (2), in equilibrium no profit can be made on any process. However, if P_j is unprofitable, then $x_j = 0$.

Let $a_{ij} \geq 0,\ b_{ij} \geq 0\ \forall i, j$. The key result of the model is the existence and uniqueness of the common equilibrium growth (or degrowth) factor $\alpha = \beta$, assuming

$$a_{ij} + b_{ij} > 0. \tag{3}$$

The equilibrium $\alpha > 0$ can be characterized as

$$\alpha = \max_{\mathbf{x}} \min_{\mathbf{y}} \frac{\mathbf{y}'\mathbf{B}\mathbf{x}}{\mathbf{y}'\mathbf{A}\mathbf{x}}. \tag{4}$$

von Neumann (1937) notes that ”one would expect $\alpha > 1$, but $\alpha < 1$ cannot be excluded in view of the generality of our formulation”; in terms of short-term profit, the processes P_j may really be unproductive from purely economic perspective, omitting environmental values. Equilibrium is a state where the ratios of the intensities $\{x_j\}$ remain unchanged. In equilibrium, the x_js are multiplied by a common factor α a per unit of time. The production and the use of various goods thus change at the same constant rate $\alpha - 1$. Depending on the sign of $\alpha - 1$, the economy grows, stays at steady state or degrows.

Since the a_{ij}, b_{ij} may be arbitrarily small, the assumption in (3) is "not very far-reaching" (von Neumann, 1937). This assumption ensures that the economic system is irreducible (Bapat and Raghavan, 1996), i.e. the economic system is not decomposable into isolated independent subsystems:

Definition 1. *For the von Neumann model, let* $H \subset \{1, ..., n\}$ *be a set of goods that can be produced using only goods in* H*. Such set of goods is defined as independent set. Given* H*, consider a set of activities* $V \subset \{1, ..., m\}$ *such that* $a_{ij} = 0$ *if* $j \in V$*,* $i \notin H$*, and* $\forall i \in H$*,* $b_{ij} > 0$ *for some* $j \in V$*. The model is irreducible (indecomposable) if* V *admits no proper independent subset (Bapat and Raghavan, 1996).*

Thus, the system defined by matrices $(\mathbf{A}, \mathbf{B})$ is indecomposable, if there is no subset of goods that can be produced without using at least one input not in the subset. This condition is necessary to guarantee the existence of an equilibrium solution for the system of linear inequalities. An important special case of the linear von Neumann model is the Leontief model (see Appendix A).

2.2. Consumption in the von Neumann-Morishima model

In (von Neumann, 1937), consumption consists of necessities while all income in excess of necessities of life will be reinvested. Even labour is produced as input (as "consumption of goods takes place only through the processes of production"). Accordingly, consumption can be explicitly introduced in the model by decomposing the input coefficients a_{ij} into uses in production (r_{ij}) and in consumption (c_{ij}) so that $a_{ij} = r_{ij} + c_{ij}$. Morishima (1964, 1974) presents a more concrete specification of the consumption coefficients as $c_{ij} = c_i \cdot m_j$, where c_i is the amount of good i required for the reproduction of one hour labour and m_j is the number of hours employed in activity j at unit level of intensity. Define the consumption matrix $\mathbf{C}$ as the tensor product

$$\mathbf{C} = \mathbf{c} \otimes \mathbf{m}. \tag{5}$$

Denoting by $\mathbf{R}$ as the matrix of material input coefficients with elements r_{ij}, the Morishima-von Neumann model can be summarized as (Zalai, 2003):

$$\alpha(\mathbf{R} + \mathbf{C})\mathbf{x} \leq \mathbf{B}\mathbf{x} \tag{6}$$

$$\beta \mathbf{y}'(\mathbf{R}+\mathbf{C}) \geq \mathbf{y}'\mathbf{B}, \tag{7}$$

$$\mathbf{m}'\mathbf{x} > 0 \tag{8}$$

and

$$\mathbf{y}'\mathbf{B}\mathbf{x} > 0, \tag{9}$$

where by (8), production requires the use of some positive amount of labour and by (9), the value of the total output is positive. The cost of consumption equals the unit wage rate: $\mathbf{y}'\mathbf{c} = \mathbf{w}$, where the vector $\mathbf{w}$ denotes the vector of wages. Let $L = \mathbf{m}'\mathbf{x}$ denote the total time in hours spent on different activities. The total wage cost in the equilibrium model then is

$$wL = (\mathbf{y}'\mathbf{c})\mathbf{m}'\mathbf{c} = \mathbf{y}'\mathbf{c} \otimes \mathbf{m}'\mathbf{x} = \sum_{i,j} y_i c_{ij} x_j, \tag{10}$$

corresponding to the total expenditure on consumption.

Due to its generality, the von Neumann model is applicable to the study of both planned and capitalist economies. Applying the von Neumann framework, (Morishima, 1974) studies a capitalist system, consisting of a given set $\eta = \{1, ..., N\}$ agents. Assuming private ownership, let ω_v denote the initial endowment of commodities owned by agent $v \in \eta$ and let $w = \mathbf{y}'\mathbf{c}$ denote the wage rate. The agents possessing the initial endowments are capitalists, maximizing their profit in terms of interest rate[4]. Typically, workers are assumed to be symmetric and have only labour power. When $\alpha > 1$ in equilibrium, the endowments at t, $\{\omega_v(t)\}$ then equal $\mathbf{x}(t) = \alpha^t \mathbf{x}(0)$.

Definition 2. *Given an economy* $\{\mathbf{A}, \mathbf{B}, \mathbf{m}\}$*,* $\eta, \{\omega_v\}$*, the labor value of a consumption bundle* $\mathbf{c}$ *is the solution,* $\mathbf{m}\mathbf{x}^*$*, of the following constrained optimization program (Morishima, 1974):*

$$\min\ \mathbf{m}'\mathbf{x}\ \ s.t.\ \mathbf{B}\mathbf{x} - \mathbf{R}\mathbf{x} \geq \mathbf{c}.$$

Denote by $\mathbf{x}^*$ the solution vector to (6)-(9). During one time period ($\Delta t = 1$), labour obtains the share $\mathbf{m}'\mathbf{x}^* \leq 1$ of the consumption bundle $\mathbf{c} \in R^n_+$. The solution implies *labour exploitation* whenever $\mathbf{m}'\mathbf{x}^* < 1$ (Morishima, 1974). Labour exploitation means that labour is not fully compensated when yields from production are being distributed to accumulate the wealth of the capital owners (cf. Gorz (1989)). Using this definition,

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[4] Note that the consumptions of the capitalists can be assumed to be incorporated in the consumption matrix $\mathbf{C}$, modifying the consumptions of the labour force in proportion to the share of the capitalists in the society.

Morishima (1974) presents the *Generalized Fundamental Marxian Theorem* as follows (see (Yoshihara, 2014)):

Remark 1. *Given a von Neumann capitalist economy, Morishima (1974) shows that, under the balanced-growth equilibrium where* $\alpha = \beta$*, labor exploitation exists if and only if the growth rate (profit rate) is strictly positive:*

$$\mathbf{m}'\mathbf{x}^* < 1 <=> \alpha - 1 > 0.$$

The interest (profit) rate $\alpha - 1$ *is zero whenever* $\mathbf{m}'\mathbf{x} = 1$.

Assuming a steadily growing population and constant per capita consumption level, the model determines the necessary rate of growth (Zalai, 2003). Before the worldwide collapse of socialism, the von Neumann model was applied to production planning in socialist countries. Participatory economics (parecon), advocated by Ostrom (1990), is an alterative to both capitalism and socialism. This paper studies the von Neumann model from the new viewpoint of participatory economics based on cooperative management of common resources (commodity endowments).

2.3. Consistency of growth equilibrium with ecological boundary

The physical input-output model can be applied to determine whether green growth in terms of capital investments in green sectors[5] make it feasible for the economy to fit within the planetary boundaries. By allowing for the activities to have joint outputs, the von Neumann model is well adapted to include capital goods that can be used in the production of each other while also accounting for the depreciation in use (Lancaster, 1968); the model thus embraces capital formation in green sectors including renewables and organic agriculture. Given the level of technology, there is a given set of inputs and processes that can be used to produce green capital goods, including solar panels and wind turbines. Denote by $g > 1$ a green growth parameter, modifying the subset of input parameters and corresponding output parameters in connection with green capital formation. The green growth parameter can be chosen so as to satisfy a given target reduction in the consumptions of fossil energy and materials. The perturbation of the relevant subsystem of the input-output model, reflecting an increase in the use of resources in green sectors, affects all variables in the system; for given matrices $\mathbf{A}$ and $\mathbf{B}$ in the linear equilibrium model, perturbation analysis

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[5] For different definitions of green growth, see (Hickel and Kallis, 2019).

(Trefethen, 1997) can be applied to determine whether positive equilibrium growth with $\alpha = \alpha(g) > 1$ is feasible with some parameter $g > 1$ that would enable fitting the economy within the ecological boundary.

Absolute decoupling of GDP from resource use is physically impossible to maintain in the longer term (Hickel and Kallis, 2019). Despite green growth, sustainability can thus be impossible to achieve within a limited time without degrowing excessive consumptions. For example, Victor (2012) has estimated that even if global carbon emissions were reduced by 40 % by 2035 compared to 1980 then this would allow an average global sustainable level of GDP/capita of less than \$4000, or one tenth of Canada's GDP/capita in 2010. In what follows, the main focus will be on the case where the economy cannot fit within the ecological boundaries unless the scale of the economy becomes smaller, despite additional investments in green sectors including renewables and organic agriculture.

3. Degrowth paths in the equilibrium model

Economics builds on the insatiability axiom, explaining the quest for infinite economic growth. In contrast, deep ecological thinking (Naess, 1973) leads to voluntary simplicity (VS), characterized by downscaled lifestyles (Elgin and Mitchell, 1977; Alexander and Ussher, 2012; Kallis, 2011). Voluntary simplicity does not necessarily mean asceticism but in practise can take many different shapes; transition to VS means searching for a point where the individual feels that there are enough resources, not just for necessities but also some luxuries, sufficient for a good life. VS embraces sharing and collaborative consumption and enables a reduction in working hours. A number of studies have found that shorter average working time is associated with lower greenhouse gas emissions[6]. A shorter average working time translates into degrowth when the population size is fixed: a reduction in labour supply means lower aggregate production.

Degrowth as a deep ecological transformation is addressed Section 3.1. Degrowth scenarios based on ecological taxes and VS are discussed in 3.2. Degrowth via green growth in terms of additional investments in renewables is discussed in 3.3. Section 3.4 discusses the welfare implications of incorporating key elements of degrowth (frugality, conservation, cooperation, sharing) into equilibrium growth theory while giving up short-term profit considerations. Degrowth in multi-country setting is addressed in 3.5.

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[6] For example, Rosnick (2013) estimates that reducing the average annual hours by just 0.5 percent per year would mitigate 1/4-1/2 of any warming which is not yet locked-in.

3.1. Degrowth as a deep ecological transformation

The von Neumann model captures the key idea of capitalism in that all excess income from production is assumed to be reinvested to grow the economy. The model is consistent with (reproducible as) a system where capitalists maximize profits while the labour obtains the wage to cover the expenditure on necessities (broadly defined), see (Romer, 1989; Yoshihara, 2014). However, the original von Neumann model omits the utility functions of the agents (workers). A utility model for the VS-type workers, incorporating constraints on consumption, is introduced below. In the equilibrium model, any share of the working population is allowed to choose VS. Adding a formal utility model of the VS-type agents, embracing ecological constraints, complements the original von Neumann model and shows a breaking point for growth; for agents with deep ecological values, continued growth beyond the ecological boundary makes no sense.

Consider a society where the population has stopped growing. A positive initial growth rate then leads to steadily increasing per capita consumptions and an ecological overshoot. To alleviate the ecological deficit, VS-type agents set constraints on their private consumptions. Formally, denote the sustainable target consumptions of the VS-type agents by $c_{i,\max},\ i \in S$; consumptions beyond this level do not increase the utility and have thus no value (or may even have negative value, see below). Sharing and collaborative consumption reduce the target consumption level that is sufficient. A shift in the structure of the consumptions to products with lower environmental impacts also plays an important role in transition to post-growth society (Hardt and O'Neill, 2017). Accordingly, it is assumed that the agents complying with deep ecological values demand only a given subset of goods S, sufficient to guarantee a high quality of life:

$$S = \{1, ..., n_{\max}\} \subseteq \{1, ..., n\}. \tag{11}$$

Under severe ecological deficit, increasing the activity levels at constant rate $\alpha - 1 > 0$ only worsens the deficit. Traditional economic thinking guiding to continued growth makes no sense when excessive consumption has zero or negative value due to ecological deficit.

In order to comply with the ecological target, the VS-type agents would moderate their consumptions, enabling an increase in leisure. The essential characteristic of leisure is that it serves no economic purpose. Additional leisure can be introduced in the model by allowing the households to spend more time on unproductive activities; however, even activities that are not directly used to increase production, such as household work, require the use of some consumption goods. In what follows, it is assumed (unless otherwise stated) that all activities by households, whether in connection of production or used for other purposes, require the same amount of consumption goods per unit time. The households may thus consume goods to maintain both labour and leisure. To formally introduce leisure in the model, let $l(0) = 1$ denote the pre-transition leisure-parameter scaled at one, incorporated in $L(0) = \sum_j m_j x_j(0)$. Denote by $l(T) > 1$ the post-transition leisure parameter, corresponding to the factor by which the average time spent for unproductive purposes increases at time T. Thus, the post-transition parameters measuring the time allocations on the consumptions are $m_j(T) = l(T)m_j,\ j = 1, ..., m$ (as the consumptions per unit time are the same for both work and leisure). Denoting by $p \in [0, 1]$ the share of agents adopting deep ecological values, the post-transition average leisure parameter is

$$l(T, p) = (1 - p) + pl_v(T),$$

where $l_v(T) > 1$ is the leisure parameter of the VS-type agents. In what follows, denote by $\mathbf{C}_u$ the submatrix of $\mathbf{C}$:

$$\mathbf{C}_u = \mathbf{C}(i \in S, j \in \{1, ..., m\}), \tag{12}$$

corresponding to the set S of necessities and let $\mathbf{x}_u$ and $\mathbf{y}_u$ denote the corresponding subvectors of $\mathbf{x}$ and $\mathbf{y}$, respectively. The total consumption expenditure at t as function of p is:

$$pl_v(T)\mathbf{y}'_u\mathbf{C}_u\mathbf{x}_u(t) + (1 - p)\mathbf{y}'\mathbf{C}\mathbf{x}(t),\ t \geq T.$$

Allowing for the workers also to posses capital endowments, autonomous degrowth can take place via self-organizing cooperatives, enabling additional leisure by lowering the profit target. In the equilibrium model the society can support degrowth by introducing a *basic income* paid out by taxing profits (or production)[7]. Standard-type agents prefer a purely capitalist system to increase the level of the private consumptions; however, VS-type agents would use their basic income to support additional leisure via $l_v > 1$, which requires an additional income $I(T)$:

$$I(t) = (l_v(T) - 1)\mathbf{y}'_u(t)\mathbf{C}_u x_u(t),\ t \geq T. \tag{13}$$

The additional income $I(t)$ can be interpreted as a basic income that can be used to support additional leisure, household work or self-employment. The

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[7] The degrowth literature suggests taxing wealth and fossil fuels as potential concrete sources of funding BI in reality.

additional income $I(t)$ is directly proportional to labour income. A similar idea has been earlier presented by Gorz (1989), advocating basic income independent of labour as a means of compensating for unpaid work carried out outside the economic system as well as for the personal investments from the workers which are not paid back (however, Gorz (1989) did not present a formal model of this idea). Ecological profit taxes support an ecological basic income (cf. (Andersson, 2009)). When the society cannot agree on taxing profits, a more cooperative system can nevertheless agree to distribute the corresponding additional income in the amount of $I(t)$ to its members.

The dynamic equilibrium model yields exact conditions for degrowth equilibrium where the economy shrinks:

Proposition 1. *A degrowth equilibrium where* $\alpha < 1$ *results when the share of VS-type agents* p *rises above threshold* $\bar{p}$ *so that*

$$p > \bar{p} \equiv \frac{\mathbf{y}'(\mathbf{B}-\mathbf{A})\mathbf{x}}{l_v(T)\mathbf{y}_u'\mathbf{C}_u\mathbf{x}_u - \mathbf{y}'\mathbf{C}\mathbf{x}}, \tag{14}$$

where $\mathbf{y} = \mathbf{y}(\bar{p})$ *and* $\mathbf{x} = \mathbf{x}(\bar{p})$ *denote the solution vectors corresponding to* $\alpha(\bar{p}) = 1$ *based on* α *in (4). In order to* $\bar{p} > 0$*, the leisure parameter* $l_v = l_v(T) > 1$ *at time of transition* T *must exceed threshold* $\bar{l}_v(T)$*:*

$$l_v(T) > \bar{l}_v(T) \equiv \frac{\mathbf{y}'\mathbf{C}\mathbf{x}}{\mathbf{y}_u'\mathbf{C}_u'\mathbf{x}_u}. \tag{15}$$

Proof: Given $l_v(T)$ satisfying (15), $\alpha(l(T))$ based on (4) is strictly decreasing in $l(T,p)$ while $l(T,p)$ is monotonically increasing in p; hence, there exists a unique threshold value $\bar{p}$ in (14), so that $\forall p > \bar{p}$, $\alpha(p) < 1$ where the threshold $\bar{p} > 0$ is defined so that $\alpha(\bar{p}) = 1$. $\square$

If only a small fraction $p < \bar{p}$ of the agents chooses VS, allocating more time on leisure or on other activities outside the economic system, the equilibrium growth rate decreases but still remains positive[8]. In order for the scale of the economy to start decreasing, it is necessary that the number of VS-type agents exceeds the critical mass $\bar{p}P$ where P is the population size at the time of transition and $\bar{p}$ is the threshold in (14). In a democratic capitalist system based on rewarding excessive consumptions, it can be difficult to find such critical mass.

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[8] Latouche (2009) suggests that reducing growth means slowing down and defines degrowth as "the diametrical opposite of the goal of the technocrats", focused on short-term profitability.

Given constant population and technology, sustainability is possible in a steady-state economy whose scale is sufficiently small (Daly, 1996). Define the parameter $\gamma_{\max}$ as the level of target consumption in a sustainable steady state where the population size is constant P. Assuming the structure of the consumptions is constant, let $\mathbf{s} = \{s_i\},\ i \in S$ denote the coefficients of the amounts consumed at unit level of $\gamma_{\max}$. For a given post-transition consumption structure $\mathbf{s}(T) = \mathbf{s}$, there exists $\gamma_{\max} > 0$ at which

$$\gamma_{\max} s_i = c_{i,\max},\ \forall i \in S, \tag{16}$$

where the target parameter $\gamma_{\max}$ can be determined using estimates on remaining carbon budget and technical progress; even if technical progress enables a higher value of the target parameter, the ongoing depletion of the remaining carbon budget constantly diminishes it. For simplicity, in what follows the parameter is assumed to be constant. The aggregate target consumption $\gamma_{\max} \sum_i s_i$ then corresponds to an estimated ecological share, including individual carbon budget, of a household. Consider the case where at time $T > 0$ the population is constant and where the consumption level exceeds the target level $\gamma_{\max}$:

$$\alpha(0)^T (\mathbf{C}\mathbf{x}(0))_i > \gamma_{\max} \mathbf{s}_i,\ \forall i \in S. \tag{17}$$

The utility of the VS-type agents is assumed to be unaffected by the necessary labour hours required to cover the consumption expenditure. Denote by $\tilde{\mathbf{c}}(t)$ the vector of consumptions at t, with elements $\tilde{c}_i(t) = (\mathbf{C}\mathbf{x}(t))_i,\ i \in S$. Denoting by d the average discrepancy between level of excessive and sustainable level of consumption, $d = \sum_i s_i(\tilde{\mathbf{c}}_i - c_{i,\max}) / \sum_i s_i$. Let $\sigma = \sigma(d)$ designate a parameter measuring environmental quality (see 3.2 below), which decreases in d. The utility of the VS-type households can then be formalized for $t \geq T$ as:

$$u(\tilde{\mathbf{c}}, \tilde{l}, \sigma) = \begin{cases} u(\tilde{\mathbf{c}}, \tilde{l}, \sigma) & \text{if } \tilde{c}_i \leq c_{i,\max},\ i \in S, \\ u(\tilde{c}_{-i}, \tilde{c}_{i,\max}, \tilde{l}, \sigma), & \text{whenever } \tilde{c}_i > c_{i,max},\ i \in S, \end{cases} \tag{18}$$

where $\tilde{c}_{-i} = [\tilde{c}_1, ..., \tilde{c}_{i-1}, \tilde{c}_{i+1}, .., \tilde{c}_{n_{\max}}]$ and the additional leisure time in hours for $t \geq T$ is

$$\tilde{l}(t) = (l_v(T) - l(0))L_u(t),\ l(0) = 1,\ t \geq T,$$

where $L_u = \mathbf{m}_u' \mathbf{x}_u$ is the working time of the VS-type agents allocated to production. Assuming the marginal utility of additional leisure and environmental quality is positive, let $du/d\tilde{l} > 0$ and $du/d\sigma > 0$.

When the economy has grown too large, the equilibrium model can be applied to define an equilibrium degrowth path[9] where $\alpha(T) < 1$ until the target consumption is reached.

Proposition 2. *Assuming (14)-(15) and (17)-(18), consider a degrowth transition in a multisector economy at* $T > 0$ *whereby the share of VS-type agents rises above* $\bar{p}$ *in (14). An equilibrium degrowth path, defined by its duration* $\hat{T} - T \geq 0$ *and intensity* $l(T, l_v) > 1$*, can then be determined using steady state conditions based on zero profit and the target level* $\gamma_{\max}$.

Proof: see Appendix B

While $\alpha = \alpha(T) < 1$ during the transition, the overall consumption level falls. The target consumption determines the equilibrium labour requirement in steady state. During transition, the total time spent in connection with different activities consuming resources, $L(t)$, decreases towards one. While the level of per capita consumptions falls, the equilibrium output falls. However, degrowth is welfare-improving in terms of the utility model in (18), when $\tilde{l}(T) > 0$ or $\Delta\sigma(d) > 0$ as d (discrepancy from target consumption) decreases. While $\alpha(l(T)) < 1$, the equilibrium profit (interest) rate is negative due to ecological deficit; on macroeconomic scale, the endowments cannot yield economic profit when excessive consumption has negative value via the environmental quality parameter σ. Due to the higher utility value from a better environment, the outcome where social welfare is high (or even increasing) when profits are shrinking, can be grounded in the degrowth literature (D'Alisa et al., 2015).

Growth equilibrium is a noncooperative equilibrium that changes only if the underlying game changes. The game profoundly changes when the share of agents adopting deep ecological values exceeds the threshold in (14); growth equilibrium switches to a degrowth equilibrium then. Degrowth is advanced by a system level change from a non-cooperative competitive system towards a cooperative system based on more cooperative management of local resources (cf. (Ostrom, 1990)). A cooperative system can distribute the additional income $I(t)$ in (13) to its members and redistribute endowments, supporting welfare-improving degrowth. Even if the endowments remain in the possession of firms, deep ecological values in the society, valuing environmental quality σ, can still guide the economy towards the ecological boundary (see 3.2 below). A post-transition steady state outcome is consistent with a post-capitalist system as outlined by Trainer (2012), consisting

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[9] To obtain a unique solution for the degrowth path, it is necessary to consider the block-symmetric version of the model (see Remark 2 in Appendix B).

of micro-firms, farms and cooperatives. In equilibrium with zero profits, the labour obtains its full share of the production (see Remark 1)[10].

The value of excessive consumption is negative when the agents prioritize consuming at the ecological target level and prefer leisure to consumption; thus, the equilibrium profit rate is negative during degrowth. In degrowth equilibrium, ecological deficit makes production in all sectors (on average) economically unprofitable until the economy fits within the ecological boundaries. However, all agents in a degrowth society can be entitled to a basic income. Degrowth transition can take place either planned or autonomously. Assuming planned economic contraction, the degrowth path can be defined so that it leads to a target level of consumption. Leisure-related consumptions e.g. in connection with permaculture can be environmentally friendly; accordingly, the target consumption level in (18) can be relaxed so as to allow for additional leisure-related consumptions. Denoting by $\mu \in [1, l(T)]$ the parameter by which the target consumption in (18) is allowed to be modified, let

$$\gamma_{\max} = \mu\gamma \in [\gamma, \gamma l(T)], \tag{19}$$

where γ is the target level at $l(T) = 1$. The total consumption budget is fixed when $\mu = 1$; if $\mu = l(T)$, all leisure-related consumptions are allowed to extend the budget.

In summary, degrowth can in theory result from abandoning the expansionist growth target. By curbing production, restrictions on consumption, whether voluntary or mandatory/planned, contribute to conservation. A transition towards participatory resource management (cf. (Ostrom, 1990)), whereby the agents obtain additional leisure even if the total working time falls, resembles a transition towards the Keynesian utopia (Keynes, 1930). For the overconsuming economies such welfare-improving transition would have been possible for decades ago; however, such transition can take place only if a critical mass of agents agrees on rescaling the economy. In practise, without any support from the prevalent expansionist capitalist system, degrowth via a large scale system level change is not likely to take place autonomously; thus, shrinking an oversized economy is likely to require some direct constraints or ecological taxes (see below).

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[10] Perfect competition between firms implies an equilibrium outcome that is similar to the outcome achieved by cooperatives, minimizing their cost of production. For growth potential in a non-capitalist economy, see (Washida, 1988).

3.2. Degrowth via ecological taxes

Environmental impacts due to production, including emissions, can be partly compensated by introducing ecological taxes. For example, to remove gigatons of CO2 from the atmosphere, the firms can pay labour for planting large amounts of trees or for restoring marshlands as compensation. The compensation cost is an ecological tax that can be modelled as follows. Extending the previous analysis, define the cost parameter $z_j > 1$ as the factor by which the time spent in connection with activity j, m_j, $j = 1, ..., m$, is modified upwards, to account for the compensation cost at unit activity level. Normalizing the initial value of each z_j at one, let $z_j(0) = 1\ \forall j$; thus, setting $z_j(T) \geq 1$ at $T > 0$, the additional working time due to the ecological compensation is $\sum_j (z_j - 1) m_j(T) x_j(T)$. This time can be allocated to carry out environmental work directly by labour, either in connection with existing processes (e.g. via better waste management) or outside the economic system. High ecological taxes can trigger degrowth similarly as in Proposition 1 when

$$\sum_j (z_j - 1) m_j(T) x_j(T) > \bar{p}(\bar{l}_v(T) - 1),$$

using $\bar{p}$ and $\bar{l}_v(T)$ in (14)-(15). Ecological taxes signal the true cost of ecological deficit, making average economic profit rate negative in macroeconomic equilibrium during degrowth.

Consider a transition at $T > 0$, translating into a uniform increase in the time spent on each activity from m_j to $q_j m_j$, $j = 1, ..., m$ where $q_j > 1$ while $q_j(0) = 1\ \forall j$ and $l(0) = 1$. Leisure can also be incorporated in the parameters m_j, as in the preceding subsection. For simplicity, let $q_j = l z_j$, where $l \geq 1$ reflects the factor by which the average leisure increases relative to $l(0) = 1$. Within one working day at T, normalized to unity, $L(T, q) = \sum_j q_j m_j x_j(T)$ then denotes the total active time in hours, including labour and leisure hours as well as time spent on environmental work. If degrowth is driven by VS and sharing, $q = l > 1$ means an increase in l relative to $l(0) = 1$. If degrowth reflects ecological taxes, $l = 1$ remains constant whereas environmental work increases, with a positive impact on the environmental quality σ. In general, degrowth whereby $q_j(T) > 1\ \forall j$, may reflect any combination of $\{z_j\}$ and l, depending on the preferences and the distribution of the endowments. The quality of the environment $\sigma = \sigma(d, \mathbf{z})$ in (18) is a positive function of the parameters $\{z_j\}$. Similarly as $l(T) > 1$ in (19), the parameters $\{z_j\}$ can be allowed to modify the target level, reducing the discrepancy $d = d(\mathbf{z})$ between excessive and target consumptions.

Table 1: A degrowth scenario when $\alpha(0) > 1$

	Transition stage
0	At time zero $q_j(0) = 1\ \forall j$, $\alpha(0) > 1$;
1	At $T > 0$ when $(\mathbf{Cx}(T))_i > c_{i,\max}\ \forall i \in S$, set $q_j(T) > 1\ \forall j$ s.t. $\alpha(T) < 1$;
2	The economy then degrows at rate $\alpha(T) - 1 < 0$ until target consumption is reached at steady state with zero profit;
3	In steady state consumption stays at target level while $\alpha = 1$.

Let $\alpha(q(0))$ denote α at $t = 0$. Similarly as in Proposition 2 where $l(T)$ and $\hat{T}$ can be determined by the steady state conditions, for given set $\{z_j\}$, it is possible to determine $l(T)$ and $\hat{T} \geq T$, defining a feasible degrowth path. Accordingly, either there exists a gradual degrowth path with $q_j(T) > 1\ \forall j$ and $\hat{T} > T$, or otherwise degrowth to sustainable level of consumption takes place instantaneously at $\hat{T} = T$. Table 1 outlines a degrowth scenario, assuming gradual transition. In Table 1, the growth factor $\alpha(\mathbf{q})$ is negatively affected by the cost parameters $q_j = z_j l,\ j = 1, ..., m$.

The degrowth scenario in Table 1 is an outline of an evolution towards a sustainable steady state equilibrium. The exact origin of the degrowth transition is left unspecified, as this will depend on the chosen degrowth path in terms of $\{z_j(T)\}$ and $l(T)$. The main idea is to show the feasibility of such path as well as their consistency with a solid equilibrium concept. In the equilibrium model degrowth does not imply unemployment but less work (disequilibria are not allowed). In steady state at $\hat{T}$, $\alpha(\hat{T}) = 1$ (see Remark 1). Steady state allows for the coexistence of cooperatives and firms; however, in steady state the long term profits are zero like under perfect competition. Assuming all agents in a degrowth society have similar preferences in terms of the utility model in (18), the post-transition values l^* and $\{z_j^*\}$ can be determined by utility maximization. The ecological taxes in terms of $\{z_j(T)\}$ imposed on the different processes may differ. In macroeconomic equilibrium during degrowth, all economic processes are equally unprofitable, accounting for the true cost of ecological deficit when the size of the economy is too large.

3.3. Degrowth via green growth in terms of investments in renewables

Green growth via additional investments in renewables or organic agriculture can be incorporated in the equilibrium model as described in section

2.3. When the growth factor $\alpha = \alpha(g)$ drops below one as the result from $\Delta g > 0$, green growth in terms of additional capital formation in green sectors results in degrowth, shrinking the size of the economy. Additional manufacturing of capital goods improving input efficiency can be incorporated in the model in a similar way: the input efficiency of some processes then improves by some constant factor and the use of other inputs increases while new capital is being produced.

Above in section 3.2, green growth reflected additional environmental work carried out by labour. The degrowth scenario in Table 1 is based on $g = 1$, abstracting from additional capital formation in the green sectors. Allowing for $g \geq 1$, the scenario can be generalized by allowing for additional green investments.

3.4. Degrowth and welfare

Consider a social welfare function $W = W(\beta, u)$ as function of the profit rate $\beta - 1 = \alpha - 1$ and utility level u in (18). Assuming an additional form for the welfare function, the post-transition social welfare for $t \geq T$ can be defined as

$$W(l, g, t) = u(\tilde{l}, \sigma(\mathbf{z}, \mathbf{g}), t) + \zeta(\alpha^*(T) - 1),\ t \geq T, \tag{20}$$

where $\zeta \geq 0$ is the utility parameter associated with profit and $\alpha^*(T) = \beta^*(T)$ is the equilibrium growth factor. The cost of consumption equals the unit wage rate in the equilibrium model: $\mathbf{y}'\mathbf{c} = \mathbf{w}$; moreover, as noted above, the utility model assumes that there is no disutility associated with providing the necessary labour.

Consider a degrowth scenario in a multi-sector economy as outlined in Table 1, based on the set of parameters $\{m_j(T)\} = \{q_j(T)m_j\}$ where $q_j(T) = l(T)z_j(T) > 1\ \forall j$. The direct effect of degrowth via $q_j(T) > 1\ \forall j$ on social welfare is positive via increased leisure and/or better environment. However, during degrowth, the equilibrium profit (interest) rate $\beta - 1$ is negative, reflecting the negative value of excessive consumptions and also the cost of additional environmental work and green investments during transition. On macroeconomic scale, the capital endowments cannot yield profits during degrowth. In equilibrium theory where all firms are symmetric, the same is true on microeconomic scale so all profits really are negative during degrowth transition. Even if the endowments yield negative short-term profits, the socially responsible firms or cooperatives would nevertheless continue production during transition for otherwise the labour would cease to exist in the absence of the production of the necessities required to maintain life. Idle capital has no value to the society, nor to the capital owners when all production requires labour, whereas using capital in production to create social welfare has. A severe ecological deficit requires major rethinking in economic theory.

The social welfare in (20) is positive during degrowth when the parameter $\zeta \geq 0$ measuring the value of profit (interest) rate in (20) is small. Seeking growth equilibrium driven by green growth can be motivated provided the growing economy can fit within the ecological boundaries. However, for high income countries where the consumption levels already exceed the sustainable level by large factors of 10 or more, such green growth does not seem realistic in near future.

Average leisure can increase autonomously or through policies such as the basic income scheme outlined above. Abandoning the expansionist targets of maximizing consumptions and profits yields potential for welfare-improving degrowth; any growing economy eventually then reaches the size at which degrowth to a lower level of consumption would improve the social welfare (Heikkinen, 2018). This argument continues to hold in the equilibrium model of a multi-sector economy, provided the society can tolerate a negative equilibrium interest rate during the transition; cooperation in production, collaborative consumption and/or basic income support welfare during degrowth (only zero profits under perfect competition would be irrelevant). High preference for short-term profits guides towards continued growth. An ecological deficit forces to reconsider the motivations for growth as well as the meaningfulness of purely profit-motivated technology-improvements.

Assuming the conservation of the environment replaces growth as the target of the society, degrowth may take place as a deliberate transition towards lesser, greener, more cooperative and more labour-intensive production. Welfare-improving degrowth is a transition towards a "Keynesian utopia" (Keynes, 1930), with lesser work and production. Ecologists have noted that such transition would have been both necessary and possible in high income countries for decades ago (see e.g. (Linkola, 2009)). The equilibrium model shows that a major system level change is necessary in order for a society to even start moving towards such utopia; as long as the growth target, wealth distribution and the preferences remain unaltered, the economies continue expanding above the ecological boundary. It is a mechanism design problem to encourage frugality, resource-efficiency and sharing. Ecological taxes on consumptions can signal the true cost of an ecological deficit.

3.5. Degrowth and re-localization in a multi-country setting

Re-localization means production at local basis and is a key element in the degrowth program (Latouche, 2009). Consider the open economy version of the von Neumann model due to Morgenstern and Thompson (1969), (see Appendix C). Open economy models widely apply the CES-utility model which takes the form (Dixit and Stiglitz, 1977)

$$u_{ces}(\mathbf{c}, n) = (\sum_{i=1}^{n} c_i^{\frac{\nu-1}{\nu}})^{\frac{\nu}{\nu-1}},\ \nu < 1, \tag{21}$$

where ν is the constat elasticity of substitution parameter. Each product variety enters symmetrically in the utility, increasing in the number of varieties n. Assuming similar high income countries with similar technologies, the CES-utility model explains intra-industry trade (IIT) between similar countries even in the absence of a comparative advantage (Greenanaway and Milner, 1986). This is trade in goods within the same industry, constituting the vast majority of trade between high income countries. Assuming deep ecological values, there would be a limited preference for variety, recognizing the high resource cost associated with the international transport[11]. Consider the utility function $U(\tilde{\mathbf{c}}, n, \tilde{l})$, by which the VS-type agents demand $n_{\max}$ product varieties:

$$U(\tilde{\mathbf{c}}, n, \tilde{l}) = \begin{cases} U(u_{ces}(\tilde{\mathbf{c}}, n_{\max}), \tilde{l}) & \text{if } \ \tilde{c}_i \leq c_{i,\max}\ \forall i \in \{1, .., n_{\max}\}, \\ U(u_{ces}(\tilde{c}_{-i}, \tilde{c}_{\max}, n_{\max}), \tilde{l}) & \text{if } \ \tilde{c}_i > c_{max},\ i \in \{1, .., n_{\max}\}. \end{cases}$$

In a post-growth world consisting of similar economies, trade takes place in fewer varieties when the demand for variety drops. Trade based on "horizontal differentiation" would drop to minimum. Accordingly, degrowth increases the share of local production by reducing the volume of trade. Moreover, local production is advanced when degrowth in associated with additional leisure, self-employment and permacultural production/organic agriculture.

Applying a multi-country model, accounting for damages from global carbon emissions, Larch et al. (2018) study degrowth in term of an exogenous reduction in factor supplies. In this study degrowth via a reduction

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[11] For example, Cristea et al. (2013) estimate that international transport is responsible for 1/3 of world-wide trade-related emissions, and over 3/4 of emissions for major manufacturing categories.

in all national production factors (rather than only the energy input) reduces leakage, i.e. the (partial) offset of the intended emission reduction by increases in emissions in other countries. However, the study observes no relation between degrowth and the share of local production. This result is primarily due to the application of the CES-type utility model, underlying the demand for variety. Degrowth via VS would reduce the leakage, not only via a fall in real income but also directly, by affecting the demand structure. In a multi-country setting, degrowth means a transition towards a system based on more self-sufficient transition towns. For example, the Oberlin project in Ohio, USA, has the ambitious target of increasing the share of locally produced agricultural products to 70 % relative to the local demand (Raworth, 2017).

4. Conclusion

Global consumption uses the equivalent of 1.7 earths to provide the resources and absorb the waste. There is an ongoing discussion on different possible transition paths towards sustainable levels of consumption and production. This paper has applied the first dynamic model of economic equilibrium (von Neumann, 1937) to study degrowth paths in a multi-sector economy based on a given technology. In the equilibrium model, any share of the working population may limit their private consumptions. The main focus has been on the case where despite additional green investments, it is not possible for the economy to fit within the biophysical boundaries unless the scale of economy becomes smaller. Degrowth via system level change, including a redefinition of the role of the firms, is then necessary. In the theoretical equilibrium model, degrowth paths to a sustainable level of consumption exist assuming indecomposability of the economic system. On macroeconomic scale, economic processes are on average unprofitable in a degrowth equilibrium where excessive consumption has negative value. This means that capitalism needs to be redefined so as to temporarily allow for a negative short term interest rate. Degrowth can take place either as planned contraction or autonomously. In order for the scale of the economy to start shrinking autonomously, a critical mass of the agents must prioritize diminishing the ecological deficit over increasing the output level. Accounting for the biophysical boundaries thus challenges traditional economic thinking and motivates the introduction of different forms of centralized control. A major challenge in practise is, how to create the pre-conditions and mechanisms (such as binding ecological taxes) under which the global economic system would support and encourage saving and frugality instead of excessive consumption.

Constraints on consumption, whether voluntary or mandatory, account for personal carbon budget and provide a direct channel for degrowth, while reducing the scope for rebounds and leakage of emissions. A deliberate moderation of excessive consumption levels can improve the quality of life, enhance the social welfare and reduce the pressure on the environment. Degrowth originates at micro-level from a change in the distribution of the agents, accompanied by a change in the production system, reflecting a transition towards cleaner, lesser, more cooperative and more labour-intensive production. Green growth in terms additional environmental work and investments in renewables and in organic agriculture can take place during degrowth. Reinterpreting the open economy version of the equilibrium model from degrowth perspective has implications for trade: in a post-growth society, there is less scope for intra-industry trade and more scope for local production.

The overconsuming societies seem to be paradoxically close to the "Keynesian utopia", without really being able to move towards it. The equilibrium model makes such utopia more transparent, by showing the existence of feasible degrowth-paths supporting a high quality of life with additional leisure and green investments despite falling production levels. Degrowth via a system level change requires a profound change in the valuations. In a post-growth society the firms must care about the ecological constraints as well as about the welfare of the workers. As the global consumption levels continue to increase and all economic activities consume some resources, some deliberate degrowth will eventually be necessary for shrinking the world economy to fit within the planetary boundaries.

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Appendix A

Definition 3. *The von Neumann-Leontief economy* $\{\mathbf{A}, \mathbf{B}, \mathbf{m}\}$ *is the special case of the von Neumann model where each productive activity has a single output (no joint products) whereas there may be many activities producing the same output (Lancaster, 1968).*

The von Neumann-Leontief model has a unique expansion factor α^* and unique equilibrium vectors $\mathbf{x}^*$ and $\mathbf{y}^*$. The outputs grow or degrow according to the relation:

$$\alpha^* \mathbf{A}^* \mathbf{x}^* = \mathbf{x}^*.$$

For maximal α^*, $\mathbf{A}^*$ is the submatrix of $\mathbf{A}$ with the least dominant characteristic root $\lambda^* = 1/\alpha^*$ (Lancaster, 1968). In general, the submatrix $\mathbf{A}^*$ can be defined using multiple criteria. To ensure a high quality of life at any level of activity, it is necessary to include in $\mathbf{A}^*$ the consumptions of goods in set S in (11).

Appendix B

Proof of Proposition 2

Accounting for the post-transition values of $p > \bar{p}$ and $l_v(T) > 1$ in (15), let $\alpha(T) = \alpha(l(T)) < 1$ denote the equilibrium rate of degrowth as function of $l(T, l_v) = pl_v + (1-p)$. The end-point of the transition $\hat{T}$ is the time at which the consumption level of the VS-type agents reaches the target:

$$\alpha(l(T))^{\hat{T}-T} (\mathbf{C}_u(l_v(T))\mathbf{x}_u(l_v(T))_i = \gamma_{\max}\mathbf{s}_i, \ \forall i \in S, \tag{22}$$

where $\mathbf{C}_u$ is the matrix in (12) and $\mathbf{x}_u(l_v(T))$ is the corresponding post-transition activity vector (see below). Given the pre-transition vector of parameters $\mathbf{m}$ based on $l(0) = 1$, let $\mathbf{m}_u(l_v(T))$ denote the subvector of $\mathbf{m}$, corresponding to $\mathbf{x}_u(l_v(T))$. At steady state at $\hat{T} > T$, the labour consumes its full share of the production (see Remark 1),

$$\alpha(\hat{T}) = 1 \ \ <=> \ \ L(\hat{T}) = 1.$$

The steady state condition requiring $\alpha(\hat{T}) = 1$ at $\hat{T}$ can be stated as:

$$L(\hat{T}) = [pl_v(T)\mathbf{m}'_u\mathbf{x}_u(l_v(T)) + (1-p)\mathbf{m}'\mathbf{x}]\alpha(l(T))^{\hat{T}-T} = 1. \tag{23}$$

Denote by $\mathbf{x}_u(T, l_v(T))$ a feasible solution to the post-transition equilibrium condition defined at T:

$$\alpha(T)\mathbf{A}\mathbf{x}_u(T, l_v(T)) = \mathbf{B}\mathbf{x}_u(T, l_v(T)).$$

Accounting for the pre-transition growth rate $\alpha(0) - 1 > 1$ applicable for $0 \le t \le T$, modifies the level of intensities $\mathbf{x}_u(T, l_v(T))$ at time T by $\alpha(0)^T$; thus, $\mathbf{x}_u(l_v(T))$ in (22)-(23) is defined as:

$$\mathbf{x}_u(l_v(T)) = \alpha(0)^T \mathbf{x}_u(T, l_v(T)) \text{ at } t = T. \tag{24}$$

Regarding the solution pair $\{l(T), \hat{T}\}$, there are then two cases:

1. Either $\exists\ l(T, l_v) = pl_v + 1 - p > 1$ and $\hat{T} > T$ that solve (22)-(23), taking $\gamma_{\max}$ and $T > 0$ as given. During transition $\alpha(l(T)) < 1$ by (14).
2. Otherwise, a steady state with $\alpha(T) = 1$ can be obtained instantaneously by setting $\hat{T} = T$ and

$$l_v(\hat{T}) = \arg\max_{l_v(\hat{T})} \text{ s.t. } \mathbf{A}(l_v(\hat{T}))\mathbf{x}(l_v(\hat{T})) = \mathbf{B}\mathbf{x}(l_v(\hat{T})),\ \mathbf{C}_u\mathbf{x}_u(l_v(\hat{T})) = \gamma_{\max}\mathbf{s}.$$

The growth rate then immediately drops to zero while the consumption level falls to the target level.

$\square$

As $\gamma_{\max}$ increases, $\hat{T} \to T$ by (22) if $\alpha(l(T)) < 1$; thus for large values of $\gamma_{\max}$, the steady state solution at $T = \hat{T}$ (Case 2 above) is the only feasible solution.

Remark 2. *Proposition 2 is based on the general von Neumann model, allowing for multiple solutions in terms of the equilibrium intensities and prices. To obtain a unique solution, it is necessary to assume the absence of joint products (see Appendix A above).*

Example 1. *Consider a symmetric case of the model. Denote by* $\mathbf{E}$ *the* $n \times n$ *matrix of ones. Let* $\mathbf{s} = \mathbf{1}$, $m = n = n_{\max}$ *and* $\mathbf{A} = \mathbf{C} = (1.05n)^{-1}\mathbf{E}$*. Then* $\mathbf{A}^* = \mathbf{C}$*, and* $\mathbf{x}_u(l(T))$ *in (23) equals* $\mathbf{x}_u(0)\alpha(0)^T$ *where* $\mathbf{x_u} = \mathbf{x} = \mathbf{e}$*. At time zero,* $\mathbf{Cx} = (1/1.05)\mathbf{e}$ *and* $\alpha(0) = 1.05$*. Using* $\mu = l(T)$ *and* $\gamma = 0.8$ *in (19), let* $\gamma_{\max}(l(T)) = 0.8l(T)$*. In post-transition equilibrium with* $l(T) > 1$, $\alpha^* = \alpha(T) = \alpha(0)/l(T) < 1$*. Consider gradual degrowth at* $T = 12$ *towards the target level* $\gamma_{\max} = 0.8l(T)$*. Solving the system (22)-(23) yields the unique solution with* $l(T) = 1.19$ *and* $\hat{T} = 18$*. The total consumption falls at the rate* $\alpha(T) - 1 = \alpha(0)/l(T) - 1 = -0.12$ *until* $\mathbf{Cx}(\hat{T}) = \gamma_{\max}\mathbf{e}$ *at* $\hat{T} = 18$*. The transition reduces the level of consumption by 44 %.*

Example 2. *Modifying Example 1 so that the target is fixed at* $\gamma_{\max} = 1$ *(*$\mu = 1$ *in (19)), no solution with* $\hat{T} > T$ *exists to (22)-(23). In this case, steady state is obtained at* $T = \hat{T}$ *by setting* $l(T) = \alpha(0) = 1.05$ *while instantaneously dropping consumption to the target.*

Appendix C

Given export and import prices, the open economy extension of the von Neumann model can be summarized based on (Morgenstern and Thompson,

1969) as follows. Denote by $\mathbf{e}$ the vector of exports and by $\mathbf{g}$ the vector of imports. Let $\mathbf{h}_p$ and $\mathbf{h}_l$ denote the vectors of profits and losses. Using the previous notations regarding $\alpha, \mathbf{x}, \mathbf{y}, \mathbf{B}$ and $\mathbf{A}$, the conditions (1)-(2) can be stated in an open economy context as

$$\mathbf{B}\mathbf{x} - \alpha\mathbf{A}\mathbf{x} = \mathbf{e} - \mathbf{g},\ \mathbf{y}'\mathbf{B} - \alpha\mathbf{y}'\mathbf{A} = \mathbf{h}_p - \mathbf{h}_l,$$

where the first condition guarantees the equality between total production and total demand accounting for net exports while the second condition ensures the equality between the value of outputs accounting for the losses and the value of imports plus profits. The value of the total output must be positive, as in (9). Balance of payments condition requires $\mathbf{e}'\mathbf{y}_e = \mathbf{g}'\mathbf{p}_g$.

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Highlights

- This paper studies degrowth paths applying the von Neumann equilibrium model.
- Any share of the working population can choose voluntary simplicity (VS) and constrain their private consumptions.
- Ecological boundaries imply a breaking point for growth; the direction of an economy can switch from growth to degrowth when the number of VS-type agents rises above a critical mass.
- Degrowth means a lower growth rate as well as reductions in the usage of materials and fossil energy. Degrowth can also mean a transition towards lesser and cleaner production of a smaller number of goods.
- Degrowth is consistent with equilibrium in a multi-sector economy while supporting a high quality of life.
- Green growth can take place during degrowth.
- In degrowth equilibrium, ecological deficit can make economic activities economically unprofitable.
- The equilibrium model yields an argument for basic income for supporting welfare during degrowth.
- Degrowth challenges capitalism and motivates different forms of centralized control.

Declaration of interest statement

This work has been carried out as independent research without any financial and personal relationships with other people or organizations.