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# Finite-time active fuzzy sliding mode approach for deep surge control in nonlinear disturbed compressor system with uncertainty in charactrisitic curve

Xiuwei Fu[a], Li Fu[a] and Hamid Malekizade[b]

[a]College of Information & Control Engineering, Jilin Institute of Chemical Technology, Jilin, People's Republic of China; [b]Department of Electrical Engineering, Imam Khomeini University of Maritime Sciences, Noshahr, Mazandaran, Iran

**ABSTRACT**

In this paper, a novel active control approach is designed for surge instability in the compressor system using the finite-time fuzzy sliding mode scheme. The primary novelty of this study lies in the development of a finite-time fuzzy sliding mode control for the surge instability in a compressor system in the presence of disturbance and uncertainty in the characteristic curve of the compressor and also throttle valve. To ensure the stability of the closed-loop system in Lyapunov's concept, a finite time active control method is proposed based on fuzzy estimation method and robust adaptive and sliding mode methods. Achieving finite time stability and rapid elimination of deep surge instability occurs through a fast sliding mode design, while fuzzy and adaptive techniques are used to estimate uncertainty and nonlinear terms, as well as to obtain optimal estimation weights. The simulation results in MATLAB environment and comparison show that the suggested method provides better quality control in terms of surge suppression, robustness, and overcoming uncertainty and disturbance effects.

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Centrifugal compressor; deep surge; finite-time active control; fast sliding mode; disturbance; uncertainty; noise

## 1. Introduction

Compressors are broadly used in various industries such as oil and gas, petrochemicals, and used to increase the pressure or transfer of compressible liquids. In many of these applications, compressor failure can cause upstream and downstream equipment failure, so much effort is being put into their stable and optimal performance [1–5]. There are two main categories of axial and centrifugal compressors; the former is preferred for flow and higher speeds and the latter for greater compression at lower flows [6].

Surge, stall and choke, or stone wall are instabilities that affect the performance of the compressor, and among them, surge is the most common and destructive instability. During surge instability, as the flow decreases to a critical value so-called the surge point, the flow reverses and forms an unstable pattern. Surge affects the entire compressor system, and its deep type leads to severe fluctuations in flow and pressure, as well as severe mechanical damage to the compressor.

So far, numerous efforts have been made to prevent and control surge instability [7–11]. Passive and active control are the two main categories of surge management, the former of which sacrifices optimal performance for reliable and stable performance, while the latter focuses on optimal performance though ensuring stability [12,13]. The active controller has the ability to move the compressor operating point close to the surge line and improve the operating range of the compressor system while suppresses surge instability online. Although the compressor characteristic curve is assumed to be known in many surge controller design studies, but due to the effect of different parts of the compressor system such as the pipe and the parts connected to the compressor, this characteristic curve changes and this uncertainty is one of the main reasons for the inefficiency of the active surge controller [14]. Considering the nonlinear dynamics, the effect of disturbance with unknown upper bound as well as the uncertainty of the throttle valve are other important influential elements on the performance of the compressor system. Therefore, the focus and innovation of this study are in the design of active controller taking into account the mentioned bugs.

So far, various actuators such as inlet guide vanes, throttle control valve, close-coupled valve (CCV), etc have been used in different active surge control algorithms [15–19]. Due to the efficiency as well as the range of penetration and the presence of CCV in most industrial compressor systems, this actuator is the most common means for active control of surge, and in this study, it is considered to apply a control signal.

Using the CCV actuator, various techniques have been designed to control the instability of the surge,

**CONTACT** Hamid Malekizade malekizadehhamid@gmail.com

especially its deep type. The second-order sliding mode [20], the feedback control method [21], the positive feedback method [22], the backstepping control method [23], and a variety of model predictive and robust approaches in Ref. [24–27] have been employed to control the surge instability, but all of these methods require accurate knowledge of the compressor characteristic curve as well as the upper limit of disturbance. Also, despite the destructive effect of surge instability on the whole compressor system, the control methods designed to date are not able to guarantee the stability of the closed-loop system in a limited time. Another problem in this regard is the uncertainty in other parts of the dynamics of the compressor system and the need for simultaneous coverage of these effects.

Accordingly, due to the widespread use of robust control methods in various applications [28–33], this paper presents a finite-time robust nonlinear method for controlling the surge instability of a compressor system using a CCV actuator. The capabilities and advantages of the proposed method are as follows: 1 – Compressor operating range is increased by CCV actuator which has high response speed and good reliability. 2 – Estimation of uncertainties on the throttle valve and compressor characteristic curve, estimation of nonlinear effects as well as upper limits of uncertainty and disturbance are done through fuzzy estimator, which gives a favorable estimate of the mentioned effects by using the optimal gains obtained from the adaptive method. 3 – Sliding mode method as a robust control approach provides the ability to control the instability of the surge and eliminate the effects of disturbance and uncertainty. 4 – Implementing the finite time approach also adds advantages such as higher convergence speed, more accurate performance tracking, faster transient response and limited time stability of the closed-loop system to the proposed control method.

Therefore, the body of the paper is in this way: the second part presents the model for the compressor system. In the third part, the proposed finite-time active fuzzy sliding mode control method is presented. In the fourth part, the simulations are carried out, and finally, in the fifth part, the conclusion of the paper is given.

## 2. Model of the compressor system

In most studies, the compression system proposed by Immons is used to provide a one-dimensional model [34], but in 1976, Greitzer improved it by presenting a nonlinear model [35,36]. The main drawback of the previous models was that they were linear around the work point and limited to small disturbances. The Greitzer (G-Model) was a major model for axial compressors. However, Hansen et al. (37) showed that the Greitzer model could also be used for centrifugal compressors [37]. The Greitzer model represented the surge cycle well, but did not include the rotating stall that caused the pressure drop. To model rotating stall, it seems that a two-dimensional model should be used. This was first done by Greitzer and Moore in 1986 for such turbomachines [38].

Despite the introduction of numerous innovative models for compression systems, the Greitzer model is the main choice for active surge control in centrifugal compressors due to its low order equations and simple construction. This model can prepare a qualitative explanation of the relevant phenomena, while its simplicity enables the physical interpretation of the model parameters and their influence on the overall dynamics. In addition, a set of ordinary differential equations (ODE) makes real-time computations applicable and implementable. The reader is referred to [39] for comprehensive information on compressor system models and their use in surge control design.

Figure 1 shows the schematic of the compression system with a closed coupled valve. Considering CCV as a control actuator, the dynamical equations of the compressor system in the presence of disturbance are given by:

$$\begin{aligned}\dot{\psi} &= \frac{1}{4B^2 l_c}(\phi - \phi_T(\psi) + d_\phi(\xi)) \\ \dot{\phi} &= \frac{1}{l_c}(\psi_c(\phi) - \psi - \psi_V(\phi) + d_\psi(\xi))\end{aligned} \tag{1}$$

where $\psi$ and $\phi$ are the coefficients of the compressor's pressure rise and mass flow, respectively. $\xi$ stands for the dimensionless time. Also, $\phi_T(\psi)$ and $\psi_c(\varphi)$ signify the characteristics of the throttle valve and the compressor, respectively. $l_c$ indicates the length of the ducts and compressors. $d_\phi(\xi)$ and $d_\psi(\xi)$ are the disturbances in the flow and pressure of the compressor. Also, $\phi_T(\psi)$ and $\psi_c(\varphi)$ denote the characteristics of the throttle valve and the compressor, respectively. According to Moor and Greitzer [40], the $\psi_c(\varphi)$ cube characteristic of the compressor is described as:

$$\begin{aligned}\psi_c(\phi) = \psi_{c0} + H\Bigg(1 + \frac{3}{2}\left(\frac{\phi}{W} - 1\right) \\ - \frac{1}{2}\left(\frac{\phi}{W} - 1\right)^3\Bigg)\end{aligned} \tag{2}$$

where $\psi_{c0}$ indicates the value of the characteristic curve in zero dB. $H$ is the half-height and $W$ the half-width of the characteristic curve.

Also, according to [19], the throttle valve characteristic is given by:

$$\phi_T(\psi) = \gamma_T\sqrt{\psi} \tag{3}$$

where $\gamma_T$ specifies the valve's yield.

$\psi_V(\phi)$ indicates the input for the control system. $B$ is the Greitzer parameter obtained from the following

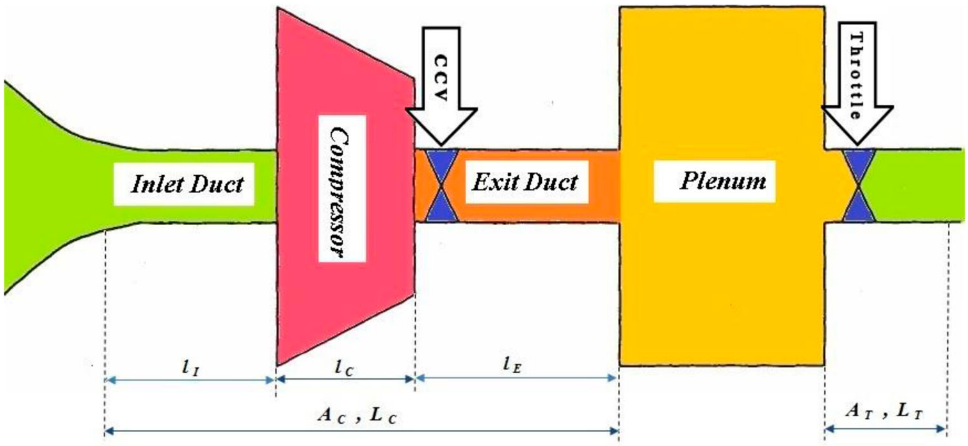


**Figure 1.** The compressor system with CCV.

relation:

$$B = \frac{U}{2a_s}\sqrt{\frac{V_p}{A_c l_c}} \tag{4}$$

In this equation $U$ is the constant of compressor's tangent speed. $a_s$ is the sound speed, $V_p$ the plenum volume, and $A_c$ is the cross-section of the compressor.

Taking $x_1 = \psi$ and $x_2 = \phi$, the equations of compressor's state space will be:

$$\begin{aligned} \dot{x}_1 &= \frac{1}{4B^2 l_c}(x_2 - \phi_T(x_1) + d_\phi(\xi)) \\ \dot{x}_2 &= \frac{1}{l_c}(\psi_c(x_2) - x_1 - u + d_\psi(\xi)) \end{aligned} \tag{5}$$

The governing equations of the system exhibit the unmatched characteristic for the flow disturbance, whereas the disturbance in its second state is as a matched type.

## 3. Finite-time active fuzzy sliding mode control for surge instability

The most important innovation of this paper is to present a finite time fuzzy sliding mode control method with the assumption that the opening percentage of the throttle valve is unknown and also there is an uncertainty on the compressor characteristic curve, while the disturbance enters both states of the system, and of course the actuator used in the compressor system is CCV. A schematic of the robust finite time active fuzzy sliding mode control is shown in Figure 2. First, the fuzzy estimator used in this paper is described, and then the controller design steps are presented.

### 3.1. Fuzzy estimator

A Fuzzy system is specified as a system that provides an outline from the input vector to output vector: $x \to y$, where $x = [x_1, \ldots, x_n] \in X_1 \times \ldots \times X_n \subseteq R^n, y \in R$. The fuzzy system has an integrated fuzzification, Gauss membership function, product inference, and principal mean defuzzification, the $i_{th}$ rule of the fuzzy logic system is as follows:

Rule $i$ : *if* $x_i$ *is* $F_{i1}$, $x_n$ *is* $F_{in}$ *then* $y = w_i$, where $i = 1, \ldots, m$, $m$ indicates how many fuzzy logic rules there are, $w_i$ shows the ith fuzzy law, $F_{ij}(j = 1, \ldots, n)$ indicates a fuzzy collection in the world of discourse $X_i$. Gauss function is as membership function

$$\mu F_{ij}(x_j) = e^{-\left(\frac{x_j - a_{ij}}{b_{ij}}\right)^2} \tag{6}$$

where $a_{ij}$ and $b_{ij}$ are design components.

To realize the fuzzy system, one should combine the fuzzy rules one after the other, i.e.

$$y(x) = \frac{\sum_{i=1}^{m} w_i \left(\prod_{j=1}^{n} \mu F_{ij}(x_j)\right)}{\sum_{i=1}^{m}\left(\prod_{j=1}^{n} \mu F_{ij}(x_j)\right)} \tag{7}$$

$$W^T = [w_1, \ldots, w_m] \tag{8}$$

$$P(x) = [p_1(x), p_2(x), \ldots, p_m(x)]^T \tag{9}$$

$$p_i(x) = \frac{\prod_{j=1}^{n} \mu F_{ij}(x_j)}{\sum_{i=1}^{m}\left(\prod_{j=1}^{n} \mu F_{ij}(x_j)\right)} \tag{10}$$

By determining a constant value for membership function (i.e. $a_{ij}$ and $b_{ij}$ constant), and describing the fuzzy rule $w_i$ as a variable component, we have

$$y(x) = W^T P(x) \tag{11}$$

where $P(x)$ is a fuzzy basis function vector and $W$ is a component vector.

There exists a fuzzy system $y^*(x)$ for each real continuous function $y(x)$ in the set $X \subseteq R^n$ and also for each real number $\varepsilon > 0$, that satisfies sup $|y^*(x) - y(x)| < \varepsilon$, so using the fuzzy system to estimate a continuous function $y(x)$ is offered as follows

$$f(x) = W^{*T} P(x) + \Delta f(x) \tag{12}$$

where $\Delta f(x)$ contents $\Delta f(x) < \varepsilon$ [40].

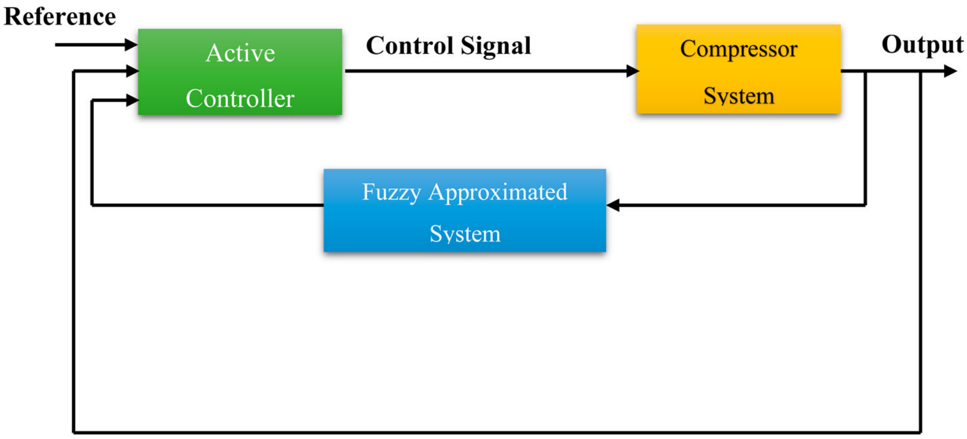


**Figure 2.** Schematic of robust finite time active fuzzy sliding mode control

### 3.2. Active controller design

Now the steps of designing the controller are given below. First, the error variables are defined as follows:

$$e_1 = x_1 - x_{1d}$$
$$e_2 = x_2 - \alpha \tag{13}$$

By differentiating the Equation (13) and according to (5), the systems error equations are gotten as follow:

$$\dot{e}_1 = \frac{1}{4B^2 l_c}(e_2 + \alpha - \phi_T(x_1) + d_\phi(\xi)) - \dot{x}_{1d}$$
$$\dot{e}_2 = \frac{1}{l_c}(\psi_c(x_2) - e_1 - x_{1d} - u + d_\psi(\xi)) - \dot{\alpha} \tag{14}$$

The nonlinear functions $\Psi_1$ and $\Psi_2$ include the uncertainties and disturbances of the system and are described as:

$$\Psi_1(.) = -\phi_T(x_1) + d_\phi(\xi) - 4B^2 l_c \dot{x}_{1d}$$
$$\Psi_2(.) = \psi_c(x_2) - x_{1d} + d_\psi(\xi) - l_c\dot{\alpha} \tag{15}$$

Therefore, the system error equations are reduced to the following:

$$\dot{e}_1 = \frac{1}{4B^2 l_c}(e_2 + \alpha + \Psi_1(.))$$
$$\dot{e}_2 = \frac{1}{l_c}(-e_1 - u + \Psi_2(.)) \tag{16}$$

Now using the fuzzy approximator, (16) can be rewritten as follows.

$$\dot{e}_1 = \frac{1}{4B^2 l_c}(e_2 + \alpha + W_1^T P_1 + \varepsilon_1)$$

$$\dot{e}_2 = \frac{1}{l_c}(-e_1 - u + W_2^T P_2 + \varepsilon_2) \tag{17}$$

The Lyapunov candidate is defined as:

$$V = 2B^2 l_c e_1^2 + \frac{l_c}{2}e_2^2 + \frac{1}{2\lambda_1}\tilde{W}_1^2 + \frac{1}{2\lambda_2}\tilde{W}_2^2 \tag{18}$$

where:

$$\tilde{W}_1 = W_1 - \hat{W}_1$$
$$\tilde{W}_2 = W_2 - \hat{W}_2 \tag{19}$$

Where $\hat{W}_1, \hat{W}_2$ are estimates of the weights of the fuzzy system for $W_1, W_2$.

By derivation of V, it is obtained

$$\dot{V} = e_1(e_2 + \alpha - \phi_T(x_1) + W_1^T P_1 + \varepsilon_1)$$
$$+ e_2(-e_1 - u + W_2^T P_2 + \varepsilon_2) - \frac{1}{\lambda_1}\tilde{W}_1\dot{\hat{W}}_1$$
$$- \frac{1}{\lambda_2}\tilde{W}_2\dot{\hat{W}}_2 \tag{20}$$

The virtual control input $\alpha$, and the control input $u$ in this equation are described as:

$$\alpha = -k_1 e_1 - k_2 e_1^v - \hat{W}_1 P_1 sgn(e_1)$$
$$u = k_3 e_2 + k_4 e_2^v + \hat{W}_2 P_2 sgn(e_2) \tag{21}$$

where $v = p/q$ and $q > p > 0$. $k_1$, $k_2$, $k_3$, and $k_4$ are the system gains, and $sgn(.)$ specifies the sign function.

From Equations (20) and (21), it is obtained

$$\dot{V} = -k_1 e_1^2 - k_2 e_1 e_1^v + e_1 P_1(W_1 - \hat{W}_1 sgn(e_1))$$
$$+ e_1\varepsilon_1 - k_3 e_2^2 - k_4 e_2 e_2^v + e_2 P_2(W_2 - \hat{W}_2 sgn(e_2))$$
$$+ e_2\varepsilon_2 - \frac{1}{\lambda_1}\tilde{W}_1\dot{\hat{W}}_1 - \frac{1}{\lambda_2}\tilde{W}_2\dot{\hat{W}}_2 \tag{22}$$

The late equation is simplified to yield:

$$\dot{V} \le -k_1 e_1^2 - k_2 e_1 e_1^v - k_3 e_2^2 - k_4 e_2 e_2^v + e_1\varepsilon_1 + e_2\varepsilon_2$$
$$+ |e_1| P_1 \tilde{W}_1 + |e_2| P_2 \tilde{W}_2 - \frac{1}{\lambda_1}\tilde{W}_1\dot{\hat{W}}_1$$
$$- \frac{1}{\lambda_2}\tilde{W}_2\dot{\hat{W}}_2 \tag{23}$$

By defining the adaptive laws as follow:

$$\dot{\hat{W}}_1 = \lambda_1 |e_1| P_1$$
$$\dot{\hat{W}}_2 = \lambda_2 |e_2| P_2 \tag{24}$$

which give the following inequality:

$$\dot{V} \le -k_1 e_1^2 - k_2 e_1 e_1^v - k_3 e_2^2 - k_4 e_2 e_2^v + e_1\varepsilon_1 + e_2\varepsilon_2 \tag{25}$$

By simplifying the mathematical formula (25), it is obtained.

$$\dot{V} \le -\left(k_1 - \frac{1}{2}\right) e_1^2 - k_2 e_1 e_1^v - \left(k_3 - \frac{1}{2}\right) e_2^2 - k_4 e_2 e_2^v + \frac{1}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_2^2 \tag{26}$$

By describing $\sigma = \frac{1}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_2^2$ and $k = \min((k_1 - \frac{1}{2}), k_2, (k_3 - \frac{1}{2}), k_4)$, it is obtained

$$\dot{V} \le -k(e_1^2(t) + e_2^2(t) + e_1(t)e_1^v(t) + e_2(t)e_2^v(t)) + \sigma \tag{27}$$

Integrating the above equation on the region $\xi \in [0, T]$ gives:

$$V(T) - V(0) \le -k\int_0^T (e_1^2(\xi) + e_2^2(\xi) + e_1(\xi)e_1^v(\xi) + e_2(\xi)e_2^v(\xi)))d\xi + \int_0^T \sigma\, d\xi \tag{28}$$

Considering $V(T) \ge 0$, it can be obtained that:

$$\int_0^T (e_1^2(\xi) + e_2^2(\xi) + e_1(\xi)e_1^v(\xi) + e_2(\xi)e_2^v(\xi)))d\xi \le \frac{1}{k}V(0) + \frac{1}{k}\int_0^T \sigma\, d\xi \tag{29}$$

From the above equation, it is determined that the closed-loop system is stable and ultimately bounded, which is in-line with tracking error. Therefore, a fuzzy sliding mode control method has been developed for the compressor system that acts in a desired manner.

**Lemma 1** ([41]). *Suppose there exists a continuous positive definite function $V(t)$ satisfying the following differential inequality:*

$$\dot{V} \le -\rho_1 V - \rho_2 V^{\varrho}, \quad \forall t > t_0 \tag{30}$$

*where $\rho_1 > 0, \rho_2 > 0$, and $0 < \varrho < 1$. Then $V(t)$ converges to the equilibrium point in the finite-time $t_f$ defined by:*

$$t_f \le t_0 + \frac{1}{\rho_1(1-\varrho)} \ln \frac{\rho_1 V^{1-\varrho}(t_0) + \rho_2}{\rho_2} \tag{31}$$

**Lemma 2** ([41]). *Suppose that $\tau_1, \tau_2, \ldots, \tau_n$ and $0 < h < 1$ are positive constants, then the following inequality holds:*

$$(\tau_1^2 + \tau_2^2 + \ldots + \tau_n^2)^{2h} \le (\tau_1^{2h} + \tau_2^{2h} + \ldots + \tau_n^{2h})^2 \tag{32}$$

*Let the Lyapunov function candidate is supposed:*

$$V = 2B^2 l_c e_1^2 + \frac{l_c}{2} e_2^2 \tag{33}$$

Taking the time derivative of the $V$ and from the Equation (21), the following inequality is obtained:

$$\dot{V} \le -k_1 e_1^2 - k_2 e_1 e_1^v - k_3 e_2^2 - k_4 e_2 e_2^v + |e_1|P_1\tilde{W}_1 + |e_2|P_2\tilde{W}_2 + e_1\varepsilon_1 + e_2\varepsilon_2 \tag{34}$$

By considering the different parts of the above expression, the following inequalities hold:

$$|e_1|P_1\tilde{W}_1 \le c_1 e_1^2 + \frac{(P_1\tilde{W}_1)^2}{4c_1} \tag{35}$$

$$|e_2|P_2\tilde{W}_2 \le c_2 e_2^2 + \frac{(P_2\tilde{W}_2)^2}{4c_2} \tag{36}$$

$$|e_1|\varepsilon_1 \le c_3 e_1^2 + \frac{\varepsilon_1^2}{4c_3} \tag{37}$$

$$|e_2|\varepsilon_2 \le c_4 e_2^2 + \frac{\varepsilon_2^2}{4c_4} \tag{38}$$

$$\begin{aligned} -k_1 e_1^2 - k_3 e_2^2 + |e_1|P_1\tilde{W}_1 + |e_2|P_2\tilde{W}_2 + e_1\varepsilon_1 + e_2\varepsilon_2 \\ = -\left(\frac{k_1 - c_1 - c_3}{2B^2 l_c}\right)(2B^2 l_c e_1^2) \\ - \left(\frac{2(k_3 - c_2 - c_4)}{l_c}\right)\left(\frac{l_c}{2}e_2^2\right) + \frac{(P_1\tilde{W}_1)^2}{4c_1} \\ + \frac{(P_2\tilde{W}_2)^2}{4c_2} + \frac{\varepsilon_1^2}{4c_3} + \frac{\varepsilon_2^2}{4c_4} \\ \le -\gamma_1\left(2B^2 l_c e_1^2 + \frac{l_c}{2}e_2^2\right) + c_5 \end{aligned} \tag{39}$$

where

$$\gamma_1 = min\left(\frac{k_1 - c_1 - c_3}{2B^2 l_c}, \frac{2(k_3 - c_2 - c_4)}{l_c}\right)$$
$$c_5 = \frac{(P_1\tilde{W}_1)^2}{4c_1} + \frac{(P_2\tilde{W}_2)^2}{4c_2} + \frac{\varepsilon_1^2}{4c_3} + \frac{\varepsilon_2^2}{4c_4} \tag{40}$$

Again, from Equation (34) and Lemma 2, the following inequality is satisfied:

$$\begin{aligned} k_2 e_1^{v+1} + k_4 e_2^{v+1} &\le min(k_2, k_4)(e_1^{v+1} + e_2^{v+1}) \\ &\le min(k_2, k_4)(e_1^2 + e_2^2)^{\frac{v+1}{2}} \\ &\le min(k_2, k_4) min\left(\frac{1}{2B^2 l_c}, \frac{2}{l_c}\right) \\ &\quad \times \left(2B^2 l_c e_1^2 + \frac{l_c}{2}e_2^2\right)^{\frac{v+1}{2}} \\ &\le \gamma_2 V^{\frac{v+1}{2}} \end{aligned} \tag{41}$$

where $\gamma_2 = min(k_2, k_4) min\left(\frac{1}{2B^2 l_c}, \frac{2}{l_c}\right)$ is a positive constant. By using Equations (39–41), the Equation (34) is rewritten as

$$\dot{V} \leq -\gamma_1 V - \gamma_2 V^{\frac{v+1}{2}} + c_5 \tag{42}$$

Then, Equation (32) can be provided in the following forms:

$$\dot{V} \leq -\left(\gamma_1 - \frac{c_5}{V}\right) V - \gamma_2 V^{(v+1)/2} \tag{43}$$

$$\dot{V} \leq -\gamma_1 V - (\gamma_2 - \frac{c_5}{V^{(v+1)/2}}) V^{(v+1)/2} \tag{44}$$

From Equation (43), if $\left(\gamma_1 - \frac{c_5}{V}\right) > 0$, then the finite-time stability will still be guaranteed using Lemma 1, which implies that $V$ converges to the region $V \leq c_5/\gamma_1$in the finite-time. Hence, in the finite-time the sliding manifold $s$ will converge to the region:

$$s \leq \sqrt{\frac{c_5}{min\left(\frac{k_1-c_1-c_3}{2B^2 l_c}, \frac{2(k_3-c_2-c_4)}{l_c}\right)}} \tag{45}$$

From (44), if $(\gamma_2 - \frac{c_5}{V^{(v+1)/2}}) > 0$, then the finite-time stability will still be guaranteed by using Lemma 2, which implies that $V$ converges to the region $V \leq (c_5/\gamma_2)^{2/(v+1)}$ in the finite-time. Therefore, the sliding manifold $s$ will converge to the region:

$$s \leq \sqrt{\left(\frac{c_5}{min(k_2, k_4) min\left(\frac{1}{2B^2 l_c}, \frac{2}{l_c}\right)}\right)^{2/(v+1)}} \tag{46}$$

Finally, the finite-time convergence region $s \leq \Delta$ for the sliding manifold would be:

$$s \leq \Delta = min\left\{\sqrt{\frac{c_5}{min\left(\frac{k_1-c_1-c_3}{2B^2 l_c}, \frac{2(k_3-c_2-c_4)}{l_c}\right)}}, \sqrt{\left(\frac{c_5}{min(k_2, k_4) min\left(\frac{1}{2B^2 l_c}, \frac{2}{l_c}\right)}\right)^{2/(v+1)}}\right\} \tag{47}$$

## 4. Simulation results

In this section, to show the robustness and efficiency of the proposed method, the Greitzer model is simulated in MATLAB environment, and the results are compared with robust adaptive fuzzy backstepping (RAFB) and feedback control methods [42,43]. For this purpose, three different scenarios have been considered for the simulation. The compressor simulation parameters are exactly in accordance with the reference [19].

$$B = 1.8, \quad l_c = 3, \quad H = 0.18, \quad W = 0.25, \quad \psi_{c0} = 0.3 \tag{48}$$

The initial points in the process were selected as $(x_1(0), x_2(0)) = (0.15, 0.4)$. The controller parameters are taken as:

$$k_1 = 8, \quad k_2 = 1, \quad k_3 = 5, \quad k_4 = 1, \quad \lambda_1 = 0.01, \quad \lambda_2 = 0.01, p = 3, q = 5 \tag{49}$$

The fuzzy system is used to estimate nonlinear functions $\Psi_1$ and $\Psi_2$. To do this, 9 membership functions with the following equations are used

$$p_i(x) = e^{-(x-0.1i)^2}, \quad i = 1, 2, \ldots, 9 \tag{50}$$

Due to the fact that the control input is applied through the CCV actuator, the effects of disturbance at the input appear in the second state of the compressor system, i.e. on $\psi$. Therefore, in the first simulation scenario, the successive disturbance effect at the input is considered as follows:

$$\begin{aligned} d_\phi(\xi) &= 0 \\ d_\psi(\xi) &= 0.02\sin(0.1\xi) + 0.02\cos(0.4\xi) \end{aligned} \tag{51}$$

The results of this simulation are shown in Figures 3–6. Figure 3 shows the compressor pressure under the three controllers. As can be seen from Figure 3, the proposed method, in addition to high compression, is able to quickly eliminate disturbance effects. A stable operating point of the system is obtained under the proposed controller while the robust adaptive fuzzy backstepping method does not provide proper compression. Of course, the effect of disturbance has not been removed by the feedback control method. Figure 4 also shows the flow of the compressor system. The amount of flow is stabilized in the two methods of active fuzzy sliding mode and RAFB and the effects of disturbance on the flow are removed, but the effects of disturbance on the flow are still clear in the control feedback method, and the mentioned method is not able to eliminate these effects. Of course, we must also note that in the RAFB method, the flow experiences a terrible overshoot. Attention to Figure 5 which shows the control signals obtained from the three methods, it can be seen that the higher compression obtained from the control feedback method is due to the non-operational and infeasible control signal obtained from this method because the operating range of the CCV actuator is not observed by this method. Low fluctuations and observation of functional limitation by the CCV actuator are the advantages of the proposed active fuzzy sliding mode method. The phase portrait shown in Figure 6 also confirms that the optimum operating point of the compressor is obtained under the proposed method, while under the RAFB method, the operating point of the compressor is far from the optimal operating point, and of course, under the feedback control method, the compressor system experiences surge conditions.

In the second scenario, the effects of disturbance on the flow are considered. Flow disturbance enters the

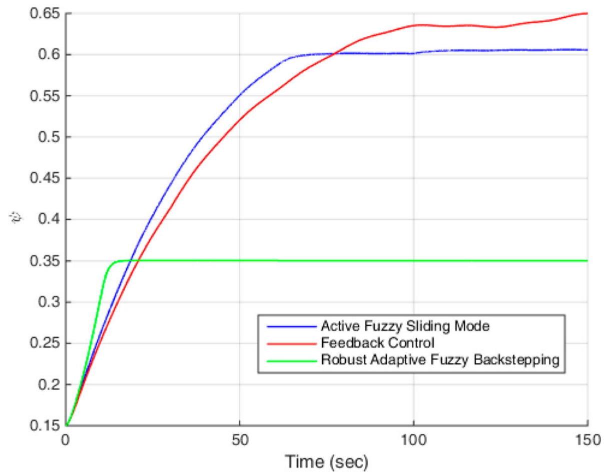


**Figure 3.** The compressor pressure rise in the first scenario.

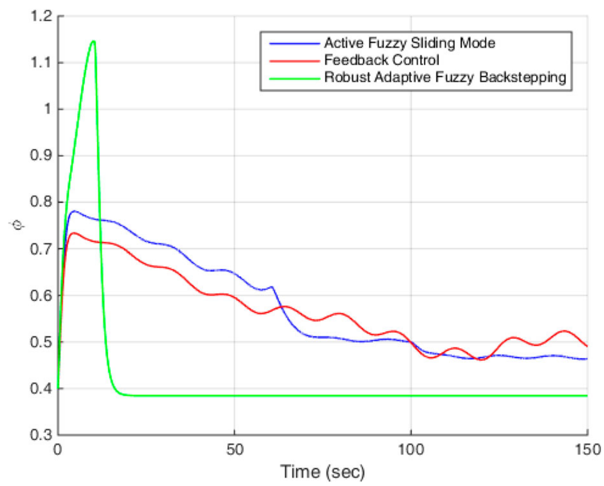


**Figure 4.** The compressor flow rate in the first scenario.

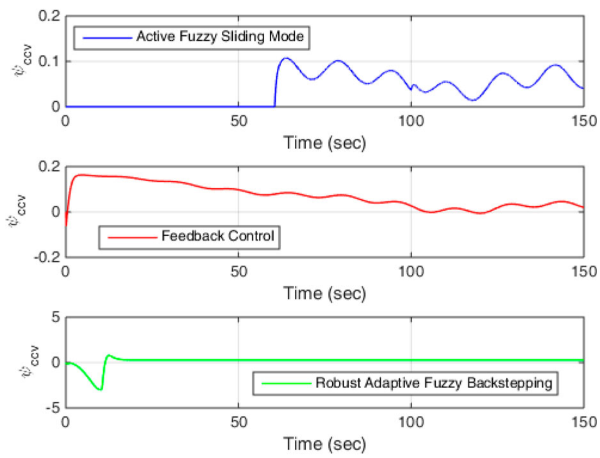


**Figure 5.** The control signals in the first scenario.

compressor system with the following equation:

$$\begin{aligned} d_\phi(\xi) &= 0.05e^{-0.015\xi}\cos(0.2\xi) \\ d_\psi(\xi) &= 0 \end{aligned} \tag{52}$$

The simulation results are shown in Figures 7–10. As can be seen from Figures 7 and 8, the optimal compression with the stable flow is achieved through the proposed method, while the control feedback method

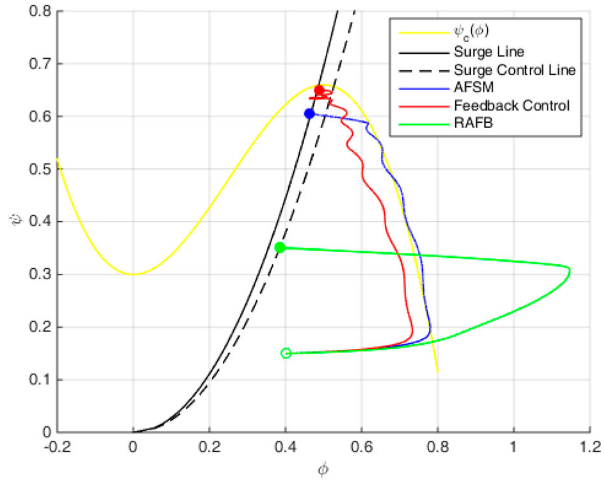


**Figure 6.** Compression system trajectories in the first scenario.

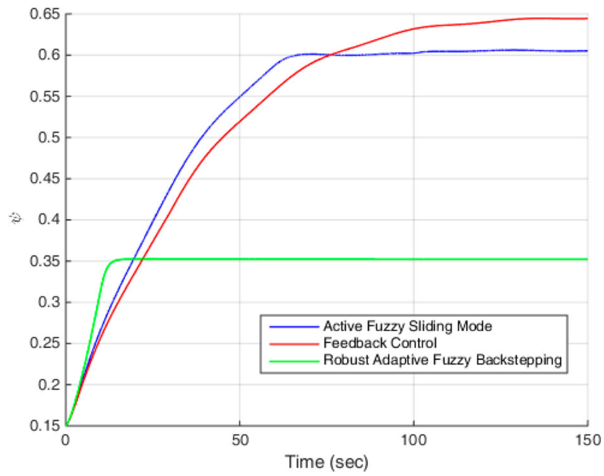


**Figure 7.** The compressor pressure rise in the second scenario.

is not able to completely eliminate the effects of disturbance and does not provide a stabilized operating point. Also, in the RAFB method, in addition to poor compression, a heavy overshoot occurs in the flow. Figure 9 shows the control signals obtained from the three methods. As it is known, the control signal obtained from the proposed method observes the operating range of the CCV operator, while the control signal obtained from the other two methods is non-operational and impractical. Figure 10 shows the phase portrait under three control methods. The RAFB method gives a non-optimal working point, while the operating point obtained under the control feedback method is optimal, but due to the impractical control signals obtained from these methods, none of them can be achieved.

In the third scenario, the effects of noise on the performance of control methods are investigated. A Gaussian white noise is considered as follows:

$$\begin{aligned} d_\phi(\xi) &= 0 \\ d_\psi(\xi) &= 0 \end{aligned}, y_1 = \psi + 0.01n(t), y_2 = \phi + 0.01n(t) \tag{53}$$

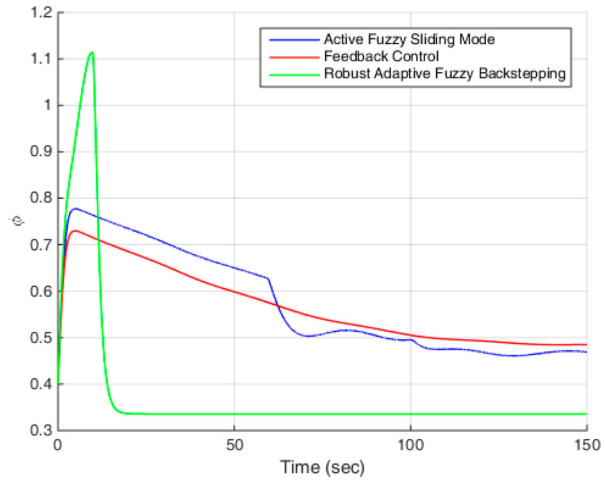


**Figure 8.** The compressor flow rate in the second scenario.

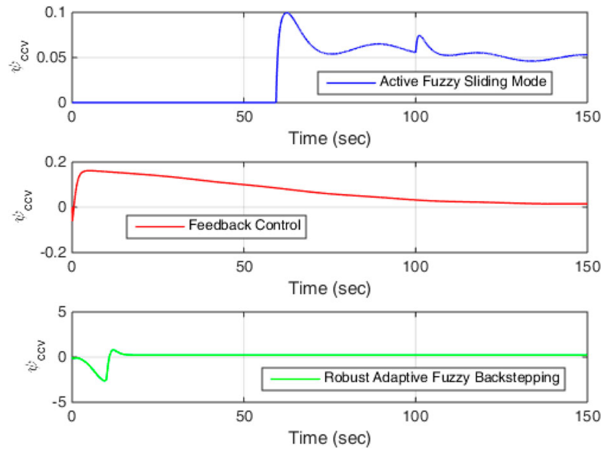


**Figure 9.** The control signals in the second scenario.

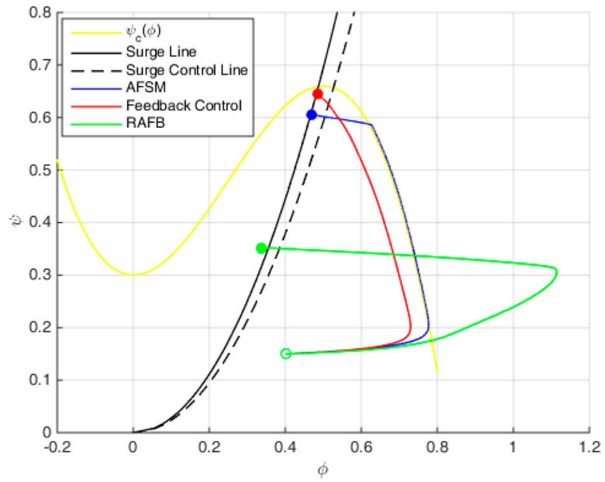


**Figure 10.** Compression system trajectories in the second scenario.

$n(t)$ is the unit white Gaussian noise. The pressure diagram is shown in Figure 11 displays that the system is completely unstable under the RAFB method, while the proposed method and the control feedback method provide satisfactory compression and the effects of

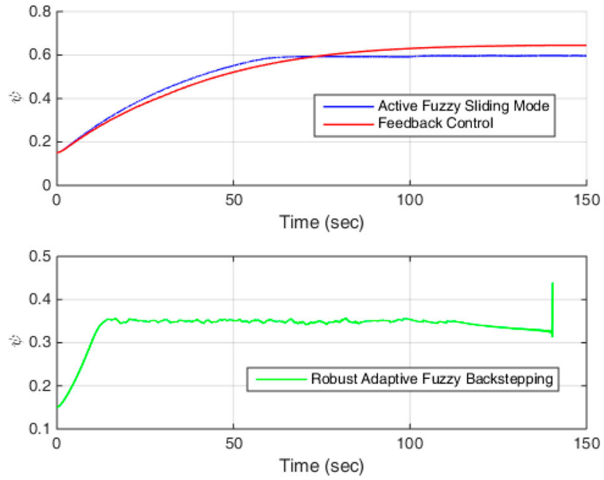


**Figure 11.** The compressor pressure rise in the third scenario.

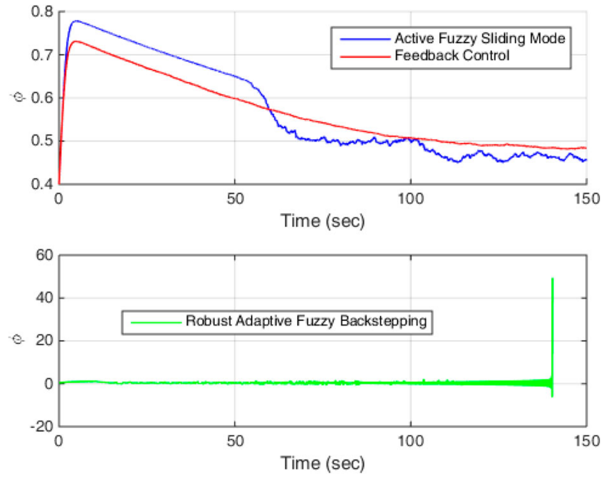


**Figure 12.** The compressor flow rate in the third scenario.

noise on the compressor system pressure are eliminated under these two methods. Figure 12 shows the compressor flow. As can be seen, under the RAFB method and with the entry of noise, the compressor system becomes unstable, while the two proposed and feedback control methods provide good flows. Figure 13 also shows the control signals obtained from the three methods. The instability of the control signal obtained from RAFB is quite obvious. Also, the control signal obtained from the feedback control method adopts negative values which indicates this signal is non-operational and non-physical. Due to the high frequency of white noise, drastic changes are observed in the control signal obtained from the proposed method. However, due to the high speed of the CCV actuator and the fact that the signal obtained is within the operating range of the actuator, the elimination of noise effects is possible and feasible under this controller. Figure 14 shows the pressure-flow diagram of the compressor under the proposed method which shows the achievement of the optimal operating point despite of the destructive effects of noise.

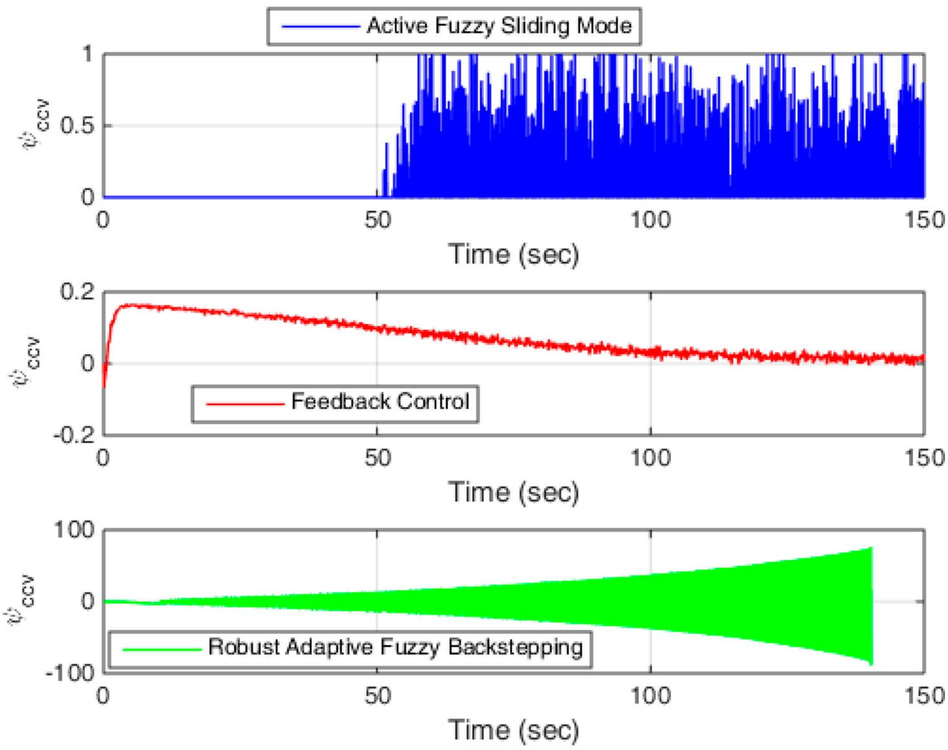


**Figure 13.** The control signals in the third scenario.

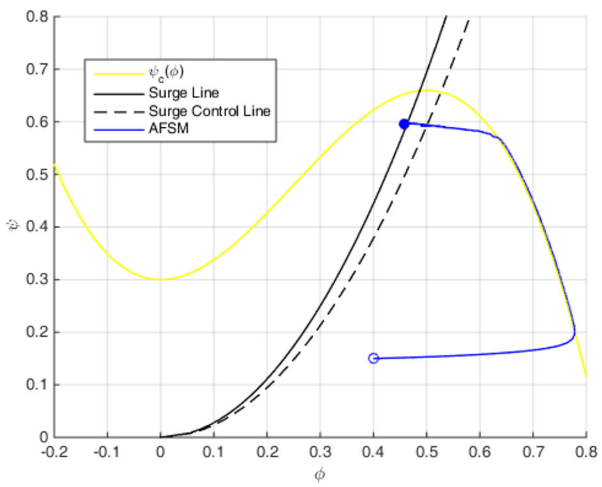


**Figure 14.** Compression system trajectories in the third scenario.

To better evaluate the proposed controller in terms of convergence rate and tracking performance, the error values based on the four criteria ISE, ITSE, IAE, and ITAE for the three simulation scenarios are given in Tables 1–3. As can be seen from the perspective of these criteria, the active fuzzy sliding mode method provides lower error values than the adaptive backstepping method. In addition, although the errors obtained in the feedback control method are less than the proposed method in some cases, but it should be noted that the signal obtained from the feedback control method is an unrealizable signal and naturally, it will not be an effective way to control surge instability.

**Table 1.** Performance index comparison and time-domain specifications.

| | Scenario 1 | | | |
|---|---|---|---|---|
| Controllers | ISE | ITSE | IAE | ITAE |
| Robust adaptive fuzzy backstepping | 15.4578 | 1.0831e3 | 47.8545 | 3.4896e3 |
| Feedback control | 5.2748 | 97.7517 | 19.3393 | 662.3055 |
| Active fuzzy sliding mode | 4.8671 | 19.8333 | 19.8333 | 844.2985 |

**Table 2.** Performance index comparison and time-domain specifications.

| | Scenario 2 | | | |
|---|---|---|---|---|
| Controllers | ISE | ITSE | IAE | ITAE |
| Robust adaptive fuzzy backstepping | 15.2411 | 1.0697e3 | 47.5232 | 3.4680e3 |
| Feedback control | 5.2767 | 97.8637 | 19.2794 | 652.1210 |
| Active fuzzy sliding mode | 4.8705 | 98.5984 | 19.8529 | 845.2736 |

**Table 3.** Performance index comparison and time-domain specifications.

| | Scenario 3 | | | |
|---|---|---|---|---|
| Controllers | ISE | ITSE | IAE | ITAE |
| Robust adaptive fuzzy backstepping | unstable | unstable | unstable | unstable |
| Feedback control | 5.2995 | 97.6467 | 19.2869 | 650.7936 |
| Active fuzzy sliding mode | 4.9854 | 108.0924 | 20.6049 | 923.8855 |

In this paper, uncertainty on the compressor characteristic curve, uncertainty on the throttle valve characteristic, and disturbance with an unknown upper bound were considered simultaneously. The simulation results showed that compared to the feedback control and adaptive fuzzy backstepping methods, the proposed controller shows superior performance in preventing surge occurrence and is robust to uncertainties imposed on the system. In addition, the proposed controller generates a positive control signal during the simulations, which is consistent with the physical limitations of the system. Although feedback control and adaptive fuzzy backstepping methods are able to ensure an increase in pressure rise and low flow performance, respectively, they did not meet the limitation that the signals from the control actuator must be non-negative, as a result, the output control signals from these controllers are imaginary signals that are impractical for real-world applications.

## 5. Conclusion

In this paper, a new robust method is presented for controlling surge instability of a compressor system. First, by considering the nonlinear equations related to pressure and flow, the complete dynamics of the compressor system is described. A fuzzy estimator is used to study the uncertain effects on the compressor characteristic curve as well as the throttle valve, and a robust sliding mode control method is planned for the active suppression of surge instability. Using the finite time approach, the proposed method was able to ensure the stability of the closed loop with faster convergence and also did not require knowledge of the upper limit of disturbance and uncertainty. The simulation results using the proposed active method showed that in addition to observing the physical constraints related to CCV actuator, flow and pressure disturbances as well as model uncertainties were completely covered and the finite-time control of deep surge instability was performed. For future studies, simultaneous control of surge and stall instabilities by covering pipe effects is recommended.

## Disclosure statement

No potential conflict of interest was reported by the author(s).

## References


[1] Zhang C, Dong X, Liu X, et al. A method to select loss correlations for centrifugal compressor performance prediction. Aerosp Sci Technol. 2019;93:105335.

[2] Ekradi K, Madadi A. Performance improvement of a transonic centrifugal compressor impeller with splitter blade by three-dimensional optimization. Energy. 2020;201:117582.

[3] Mojaddam M, Pullen KR. Optimization of a centrifugal compressor using the design of experiment technique. Appl Sci. 2019;9(2):291.

[4] Liang D, Yang H, Xu C, et al. The recent progresses in industrial centrifugal compressor designs. Int J Fluid Mech Therm Sci. 2020;6(2):61.

[5] Xu C, Muller M. (2017, June). The development of high performance centrifugal compressor using CFD and other considerations. In *ASME Power Conference* (Vol. 57618, p. V002T13A013). American Society of Mechanical Engineers.

[6] Standard API. (2004). Axial and centrifugal compressors and expander-compressors for petroleum, chemical and gas industry services.

[7] Uddin N, Gravdahl JT. Introducing back-up to active compressor surge control system. IFAC Proc Vol. 2012;45(8):263–268.

[8] Uddin N, Gravdahl JT. Active compressor surge control system by using piston actuation: implementation and experimental results. IFAC-PapersOnLine. 2016;49(7):347–352.

[9] Pak SJ, Yoon JW, Kim S, et al. Case study of surge avoidance design for centrifugal compressor systems during emergency shut-down (ESD). Korean J Chem Eng. 2016;33(3):764–774.

[10] Jokar A, Zomorodian R, Ghofrani MB, et al. Active control of surge in centrifugal compressors using a brain emotional learning-based intelligent controller. Proc Inst Mech Eng J Mechan Eng Sci. 2016;230(16):2828–2839.

[11] Torrisi G, Grammatico S, Cortinovis A, et al. Model predictive approaches for active surge control in centrifugal compressors. IEEE Trans Control Syst Technol. 2017;25(6):1947–1960.

[12] Willems F, De Jager B. Modeling and control of compressor flow instabilities. IEEE Control Systems. 1999;19(5):8–18.

[13] Epstein AH, Williams JF, Greitzer EM. Active suppression of aerodynamic instabilities in turbomachines. J Propul Power. 1989;5(2):204–211.

[14] Tamaki H. Effect of piping systems on surge in centrifugal compressors. J Mech Sci Technol. 2008;22(10): 1857–1863.

[15] Liu M, Tan L, Cao S. Influence of geometry of inlet guide vanes on pressure fluctuations of a centrifugal pump. J Fluids Eng. 2018;140:9.

[16] Imani H, Malekizade H, Asadi Bagal H, et al. Surge explicit nonlinear model predictive control using extended Greitzer model for a CCV supported compressor. Automatika: časopis za automatiku, mjerenje, elektroniku, računarstvo i komunikacije. 2018;59(1): 43–50.

[17] Ma X, Zheng S, Wang K. Active surge control for magnetically suspended centrifugal compressors using a variable equilibrium point approach. IEEE Trans Ind Electron. 2019;66(12):9383–9393.

[18] Shehata RS, Abdullah HA, Areed FF. Variable structure surge control for constant speed centrifugal compressors. Control Eng Pract. 2009;17(7):815–833.

[19] Ziabari MT, Jahed-Motlagh MR, Salahshoor K, et al. Robust adaptive control of surge instability in constant speed centrifugal compressors using tube-MPC. Cogent Eng. 2017;4(1):1339335.

[20] Bartolini G, Muntoni A, Pisano A, et al. Compressor surge suppression by second-order sliding mode control technique. IFAC Proc Vol. 2008;41(2):6238–6244.

[21] Shiriaev AS, Johansson R, Robertsson A, et al. (2009). Criteria for global stability of coupled systems with application to robust output feedback design for active surge control. In 2009 *IEEE Control Applications,(CCA) & Intelligent Control,(ISIC)* (pp. 1021–1026). IEEE.

[22] Willems F, Heemels WPMH, De Jager B, et al. Positive feedback stabilization of centrifugal compressor surge. Automatica (Oxf). 2002;38(2):311–318.
[23] Ghanavati M, Salahshoor K, Motlagh MRJ, et al. A novel combined approach for gas compressors surge suppression based on robust adaptive control and backstepping. J Mech Sci Technol. 2018;32(2), 823–833.
[24] Imani H, Jahed-Motlagh MR, Salahshoor K, et al. Constrained nonlinear model predictive control for centrifugal compressor system surge including piping acoustic using closed coupled valve. Syst Sci Control Eng. 2017;5(1):342–349.
[25] Huofa L, Imani Marrani H. A new robust LMI-based model predictive control for continuous-time uncertain nonlinear systems. Automatika. 2020;61(4):670–681.
[26] Marrani HI, Fazeli S, Malekizade H, et al. Tube model predictive control for a class of nonlinear discrete-time systems. Cogent Eng. 2019;6(1):1629055.
[27] Fazeli S, Abdollahi N, Imani Marrani H, et al. A new robust adaptive decentralized tube model predictive control of continuous time uncertain nonlinear large-scale systems. Cogent Eng. 2019;6(1):1680093.
[28] Wang J, Zhu P, He B, et al. An adaptive neural sliding mode control with ESO for uncertain nonlinear systems. Int J Contr Automat Syst. 2021;19(2):687–697.
[29] Zhang X, Wang Y, Wang C, et al. Adaptive estimated inverse output-feedback quantized control for piezoelectric positioning stage. IEEE Trans Cybern. 2019;49(6):2106–2118.
[30] Luo J, Li M, Liu X, et al. Stabilization analysis for fuzzy systems with a switched sampled-data control. J Franklin Inst. 2020;357(1):39–58.
[31] Xie W, Zhang R, Zeng D, et al. Strictly dissipative stabilization of multiple-memory Markov jump systems with general transition rates: A novel event-triggered control strategy. Int J Robust Nonlinear Control. 2020;30(5):1956–1978.
[32] Wang J, Luo W, Liu J, et al. Adaptive type-2 FNN-based dynamic sliding mode control of DC-DC boost converters. IEEE Trans Syst Man Cybernet Syst. 2019;30(5).
[33] Wang J, Gao Y, Qiu J, et al. Sliding mode control for non-linear systems by Takagi–Sugeno fuzzy model and delta operator approaches. IET Control Theor Appl. 2017;11(8):1205–1213.
[34] Emmons HW. Compressor surge and stall propagation. Trans ASME. 1955;77(4):455–467.
[35] Greitzer EM. (1976). Surge and rotating stall in axial flow compressors – Part I: Theoretical compression system model.
[36] Greitzer EM. (1976). Surge and rotating stall in axial flow compressors – Part II: experimental results and comparison with theory.
[37] Hansen KE, Jorgensen P, Larsen PS. (1981). Experimental and theoretical study of surge in a small centrifugal compressor.
[38] Moore FK, Greitzer EM. (1986). A theory of post-stall transients in axial compression systems: Part I – development of equations.
[39] Uddin N, Gravdahl JT. (2012). A compressor surge control system: combination active surge control system and surge avoidance system. *ISUAAAT Scientific Committee with JSASS Publication*.
[40] Wang LX, Ying H. Adaptive fuzzy systems and control: design and stability analysis. J Intell Fuzzy Syst Appl Eng Technol. 1995;3(2):187–188.
[41] Yu S, Yu X, Shirinzadeh B, et al. Continuous finite-time control for robotic manipulators with terminal sliding mode. Automatica (Oxf). 2005;41(11):1957–1964.
[42] Sheng H, Huang W, Zhang T, et al. Robust adaptive fuzzy control of compressor surge using backstepping. Arab J Sci Eng. 2014;39(12):9301–9308.
[43] Shafieian M, Zavar M, Rahmanian M. Simulation and control of surge phenomenon in centrifugal compressors. Trait Signal. 2019;36(3):259–264.