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A NEW SLOPE INDEX FOR SOLVING NxM FLOW SHOP SEQUENCING PROBLEMS WITH MINIMUM MAKESPAN

Summary

A flow shop sequencing problem is one of the classical problems in the production scheduling. In a flow shop, a particular case of manufacturing process follows a fixed linear structure. The purpose of this paper is to find the minimum total processing time (makespan) of sequencing ‘n’ jobs on ‘m’ machines for a flow shop problem in a static workshop. The proposed approach is based on the slope of each job on its journey from the first to the last machine. This approach is compared with five well-known heuristics (Palmer, Gupta, CDS, Dannenbring, Hundal) and one more recent technique that is based on the harmonic triangle. The results obtained from this study for different sizes of ‘n’x’m’ flow shop sequencing problems ranging from 4x4 to 50x20 indicate that the proposed approach is efficient with an encouraging percentage of improvements compared with all other six heuristic techniques.

***Key words**:* *scheduling, flow shop, sequencing, makespan, Gantt chart*

1. Introduction

Fundamentally, the flow shop scheduling consists of a fixed linear structure of a particular case of a job shop. The process involves establishing a step-by-step procedure to be adhered to strictly and all job operations are to be performed in the exact order. Such scheduling is important in designing the layout of a new and current industrial production facilities due to its ability to reduce and eliminate wasteful idle machine time during operations[1].

Sequencing is a key in determining or selecting a particular order in which a limited number of machines can complete different jobs in the shortest possible time and the least expensive manner. Flow shop problems require sequencing of ‘n’ jobs by ‘m’ machines to reduce the overall completion time, known as makespan, for all jobs. The savings in time and production costs drive the search for improved sequencing in workshop scheduling, with the goal being increased productivity. This paper proposes a new job sequencing technique for the static flow shop problem based on the slope equation of each job on its journey from the first to the last machine.

This paper is organized as follows: Section 2 provides a brief historical overview of six job sequencing techniques and their associated equations. Section 3 presents a proposed technique for reducing the makespan time. A numerical example of the proposed method is presented in Section 4. Section 5 compares the proposed technique to other six alternative techniques in a tabular form. Discussion and conclusions comprise Section 6.

2. Six efficient heuristic procedures

Over the past several decades, issues relating to the flow shop scheduling and job sequencing have generated extraordinary interest in operations research with a nearly inexhaustible development of new methods and improvement of the old ones. Increased automation in nearly every industry and nearly every facet of each organization's operations in a highly competitive global environment demands maximum optimization of all available resources. The only constant resource in all scenarios is time. Thus, the demand for the most efficient and effective way to optimize a project from beginning to end by applying advanced scheduling and sequencing techniques is paramount importance.

Johnson [2] first studied the flow shop problem for 'n' jobs to be processed on two machines. Completion of the job in its entirety represented the total completion time as an objective function. Johnson's solution, known as Johnson's rule, states that job i precedes job j in an optimal sequence if and only if $\min\{t_{i1}, t_{j2}\} \leq \min\{t_{i2}, t_{j1}\}$, where t is the time interval that one machine has to devote to the job in order to complete it. The potential complexity of the flow shop scheduling is evident when 'm' is greater than 2 and the problem, in this case, becomes non-deterministic polynomial-time hard (NP-hard).

Palmer [3] provided methods for reaching lower bounds, which the makespans must exceed or equal. The Palmer heuristic calculates a slope index S_i for each job and then schedules the jobs in descending order given by these indices [4]. Formally, the slope index is:

$$S_i = \sum_{j=1}^{M} (M + 2j + 1) t_{ij} \qquad for\ i = 1,2, \dots, N \tag{1}$$

where t_{ij} is the processing time of job i on machine j.

Campbell, Dudek, and Smith [5], who developed the CDS heuristic, followed Johnson's algorithm. This heuristic involves solving M-1 two-machine problems by dividing M machines into a set of two groups of the flow shop problem, then the authors applied Johnson's two-machine rule to find M-1 schedules in which the best schedule is selected. The processing times P_{ig}^{k} of the i-th job on the g-th machine group for the k-th reduced problem, g=1 or 2 and k=1,…, M-1, are:

$$P_{i1}^{k} = \sum_{j}^{k} t_{ij} \quad \text{and} \quad P_{i2}^{k} = \sum_{j}^{k} t_{i,m-j+1} \tag{2}$$

Gupta [6] designed a heuristic to solve a flow shop problem with a given number of 'n' jobs on a given number of 'm' machines where the workflow is unidirectional. Such a scenario requires the technological order of all jobs and all machines be identical. Since the numbering of machines is random, the machine numbers can reflect the end goal that the jobs are processed first on machine 1, then passed on to machine 2, 3...., all the way to machine 'm' as the last machine in a consecutive manner [7]. Gupta assigned an index f_i to each job i; then sorted the jobs in ascending order given by these indices. He generalized the index as follows:

$$f_i = \frac{A}{min_{1 \leq m \leq M-1}(t_{im} + t_{im+1})}, \tag{3}$$

where $A = \begin{cases} 1 & if\ t_{im} < t_{im+1} \\ -1 & Otherwise \end{cases}$.

Dannenbring [8] introduced a Rapid Access (RA) heuristic method. Dannenbring attempted to utilize the Campbell, Dudek, and Smith (CDS) heuristic methods and Palmer's slope index. Dannenbring constructed an artificial two-machine problem with the processing times reflecting the same behaviors as Palmer's slope index, then applied Johnson's algorithm. The constructed processing times were:

$$P_{i1} = \sum_{j=1}^{M}(M-j+1)t_{ij}\ , \qquad P_{i2} = \sum_{j=1}^{M}(j)t_{ij} \quad \text{for i=1, 2,…, } N \tag{4}$$

The RA heuristic method was proposed to solve problems as quickly as possible. It effectively resolves permutations of flow shop problems and the overall time of completion (makespan). It only uses Johnson's rule to solve idle time for two auxiliary machines [9].

Hundal and Rajgopal [10] extended Palmer's heuristic by computing two other sets of slope indices. Consequently, two more schedules are produced, and the best one is selected. The two sets of slope indices are:

$$S_i = \sum_{j=1}^{M}(M-2)t_{ij} \quad \text{and} \quad S_i = \sum_{j=1}^{M}(M-2j+2)t_{ij} \quad \text{for i=1, 2,…, } N \tag{5}$$

Dhanasakkaravarthi and Krishnamoorthy [11] used a harmonic triangle that is similar to Pascal's triangle involving binomial coefficients. The harmonic triangle for the r_{th} entry in the n_{th} row is given mathematically by:

$$H(n,r) = \frac{(r-1)!}{n(n-1)(n-2)\ldots(n-r+1)} = \frac{(r-1)!(n-r)!}{n!} \tag{6}$$

Dhanasakkaravarthi and Krishnamoorthy used the harmonic triangle form to solve permutation flow shop sequencing problems by reducing 'n' jobs, 'm' machines to 'n' jobs, '2' machines, then the optimum makespan can be determined using Johnson's rule.

3. Illustrative example of the proposed flow shop sequencing technique

To illustrate the proposed heuristic technique for the static flow shop, the following assumptions are made:

- There are 'n' number of jobs (J) and 'm' number of machines (M).
- The order of sequence of operations of 'n' jobs on all 'm' machines is the same.
- The setup time is not considered for calculating the total processing time (makespan).

In Table 1, the data from Ajay and Rajan [12] are used and an eight-job and three-machine problem is presented. Each cell in the table represents the processing time t_{ij} of an operation job *i* on machine *m*.

Table 1 A numerical example for 8-job and 3-machine problem

J\M	Machine 1	Machine 2	Machine 3
Job 1	6	5	1
Job 2	3	9	5
Job 3	8	1	6
Job 4	4	6	3
Job 5	9	8	2
Job 6	3	2	4
Job 7	5	9	4
Job 8	2	8	9

Step #1: Due to the fluctuation of the processing time for job i from one machine to another, calculate the slope of the trend line T_i of the job i starting its journey from the first machine to the last machine as follows:

$$T_i = \frac{t_{im} - t_{i1}}{m-1} \tag{7}$$

For example, the slope of the trend line T_i of job 1, which starts on machine 1, which requires a processing time of 6 minutes and ends in machine 3 with a processing time of 1 minute, can be calculated as follows:

$$T_1 = \frac{1-6}{3-1} = -2.5 \quad (8)$$

Step #2: Assign slope index S_i of job i by dividing its trend T_i by its total processing times in 'm' machines multiplied by 100 to clear the differences among the jobs. The slope index S_i can be calculated as follows:

$$S_i = \left(\frac{T_i}{\sum_{j=1}^{m} t_{ij}}\right) * 100 \quad (9)$$

For example, the value of T_1 from the previous step is -2.5, and then the slope index S_i of job 1 can be calculated as follows:

$$S_i = \left(\frac{-2.5}{6+5+1}\right) * 100 = -20.83 \quad (10)$$

Step #3: Rank the jobs in descending order by their indices and calculate the total processing time. The rationale of using descending order is to delay the execution of the jobs that have low processing times on the production line and this correspondingly reduces the total idle times of the machines.

Table 2 shows all results obtained for all jobs, while Figure 1 shows the trend lines of the eight jobs. After ranking the job sequence in descending order the final job sequence result is 8-2-6-7-4-3-5-1, and the makespan is 51. Figure 2 shows the flow of each job on each machine.

Table 2 Results (slope of the trend lines and slope indices) of eight jobs

Jobs	J1	J2	J3	J4	J5	J6	J7	J8
T_i	-2.5	1	-1	-0.5	-3.5	0.5	-0.5	3.5
S_i	-20.83	5.88	-6.67	-3.85	-18.42	5.56	-2.78	18.42
Rank	8	2	6	5	7	3	4	1

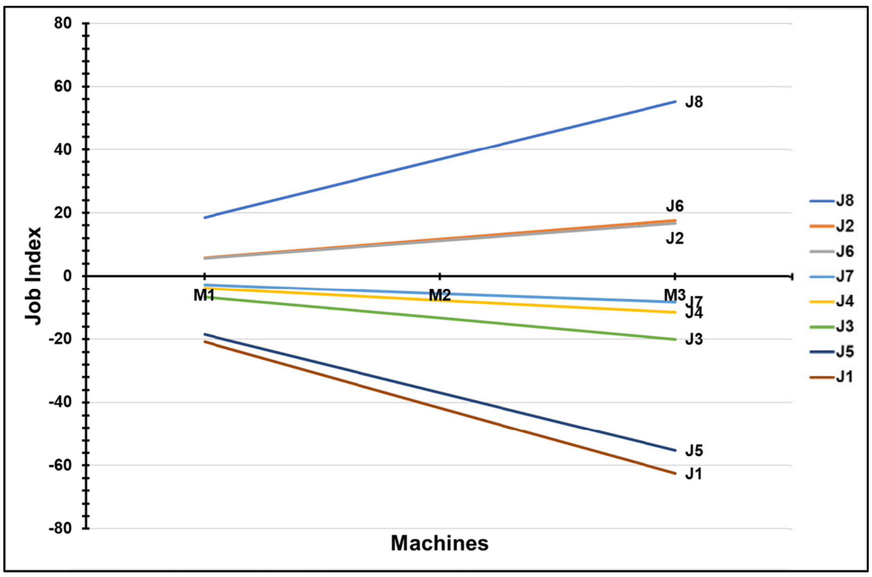

Fig. 1 Trend lines of the original problem of eight jobs and three machines

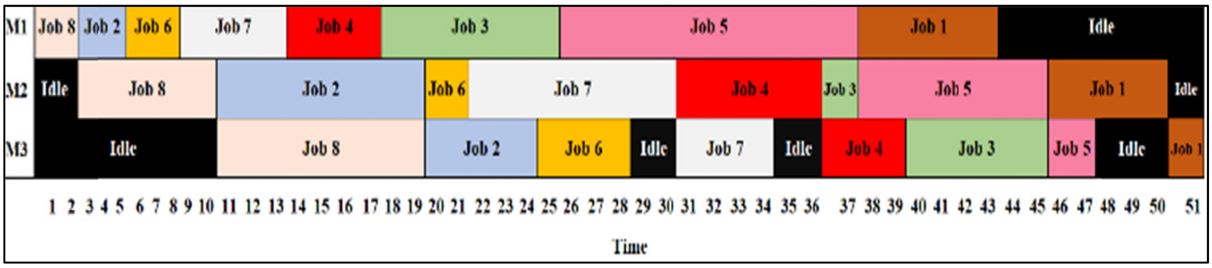

Fig. 2 Flow of each job on each machine

4. Comparison of the proposed heuristic with six benchmark algorithms

To study the effectiveness of the proposed heuristic technique, the six algorithms mentioned in Section 2 were compared with the proposed heuristic for minimum makespan. For this purpose, ten flow shop problems were selected from the literature and reproduced in Figure 3 (a-j). All ten problem sizes range from 4x4 to 50x10 to cover both small and large cases. The benchmarks of Taillard [13] and Vallade et al. [14] provided the parameters for the 20x10, 20x20, and 50x10 problems. The results (job sequence and corresponding makespan) obtained with the six benchmark methods are demonstrated in Figure 4.

Case 1 (4x4): Dhanasakkaravarthi (2019)				
J\M	M1	M2	M3	M4
J1	4	3	7	8
J2	3	7	2	5
J3	1	2	4	7
J4	3	4	3	2

(a)

Case 2 (5x5):Yunior (2017)					
J\M	M1	M2	M3	M4	M5
J1	10	11	6	8	11
J2	15	9	14	10	14
J3	12	11	9	10	6
J4	8	4	8	9	12
J5	6	6	8	6	3

(b)

Case 3 (5x6): Mostafa Khatami (2019)						
J\M	M1	M2	M3	M4	M5	M6
J1	2	3	15	3	8	8
J2	3	4	21	7	13	15
J3	9	12	30	15	19	21
J4	13	14	34	19	24	26
J5	15	16	37	24	28	35

(c)

Case 4 (7x7): Vladimír (2010)							
J\M	M1	M2	M3	M4	M5	M6	M7
J1	3	2	4	5	1	3	5
J2	5	5	8	7	2	5	2
J3	7	8	1	6	8	4	8
J4	1	1	6	1	4	6	4
J5	6	6	7	8	6	8	6
J6	9	7	9	4	7	1	3
J7	4	9	1	3	4	2	2

(d)

Case 5 (8x3): Illustrative Example			
J\M	M1	M2	M3
J1	6	5	1
J2	3	9	5
J3	8	1	6
J4	4	6	3
J5	9	8	2
J6	3	2	4
J7	5	9	4
J8	2	8	9

(e)

Case 6 (10x8): Ajay Kumar Agarwal (2013)								
J\M	M1	M2	M3	M4	M5	M6	M7	M8
J1	6	5	1	7	9	3	4	2
J2	3	9	5	7	2	5	6	1
J3	8	1	6	4	3	9	5	9
J4	4	6	3	1	5	6	7	7
J5	9	8	2	9	2	5	9	6
J6	3	2	4	3	7	2	3	5
J7	5	9	4	2	4	8	6	6
J8	2	8	9	1	6	3	4	8
J9	1	4	6	2	5	4	3	9
J10	6	3	5	5	2	7	1	9

(f)

Case 7 (10x10): Dhanasakkaravarthi (2019)										
J\M	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
J1	5	2	3	5	7	9	7	8	2	7
J2	2	6	4	2	6	2	5	2	6	1
J3	1	2	2	1	3	7	2	5	4	4
J4	7	5	6	3	2	3	2	4	2	2
J5	6	6	1	8	6	4	3	9	6	4
J6	3	7	5	2	2	1	5	3	2	6
J7	7	2	4	6	5	5	1	2	5	2
J8	5	1	7	1	7	3	6	6	2	2
J9	7	8	6	9	1	8	2	1	6	6
J10	4	3	5	8	3	1	3	8	3	7

(g)

Case 8 (20x10): Taillard (1993)										
J\M	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
J1	74	28	89	60	54	92	9	4	25	15
J2	21	3	52	88	66	11	8	18	15	84
J3	58	27	56	26	12	54	88	25	91	8
J4	4	61	13	58	57	97	72	28	49	30
J5	21	34	7	76	70	57	27	95	56	95
J6	28	76	32	98	82	53	22	51	10	79
J7	58	64	32	29	99	65	50	84	62	9
J8	83	87	98	47	84	77	2	18	70	91
J9	31	54	46	79	16	51	49	6	76	76
J10	61	98	60	26	41	36	82	90	99	26
J11	94	76	23	19	23	53	93	69	58	42
J12	44	41	87	48	11	19	96	61	83	66
J13	97	70	7	95	68	54	43	57	84	70
J14	94	43	36	78	58	86	13	5	64	91
J15	66	42	26	77	30	40	60	75	74	67
J16	6	79	85	90	5	56	11	4	14	3
J17	37	88	7	24	5	79	37	38	18	98
J18	22	15	34	10	39	74	91	28	48	4
J19	99	49	36	85	58	24	84	4	96	71
J20	83	72	48	55	31	3	67	80	86	62

(h)

Case 9 (20x20): Taillard (1993)																				
J\M	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13	M14	M15	M16	M17	M18	M19	M20
J1	50	90	39	34	66	81	27	48	46	68	48	92	78	84	93	39	43	1	65	87
J2	78	56	9	43	84	73	66	38	83	57	97	52	77	13	12	2	65	93	39	1
J3	36	43	10	19	55	48	85	70	82	39	91	82	85	17	6	54	87	85	4	72
J4	85	88	60	98	4	99	53	21	33	53	63	18	45	29	43	41	80	4	31	19
J5	9	92	98	44	51	8	31	15	47	31	80	83	20	84	69	49	93	39	13	88
J6	75	64	96	95	22	41	26	33	68	9	81	28	61	69	37	57	36	80	96	74
J7	46	94	6	19	20	51	85	92	43	75	70	70	36	31	76	63	89	46	25	88
J8	73	3	56	73	80	82	36	98	90	46	10	46	65	83	75	47	61	28	59	22
J9	71	49	36	87	8	25	76	73	80	6	6	33	79	10	93	65	26	73	42	18
J10	7	40	33	64	5	25	89	95	58	83	28	35	74	5	6	9	3	2	35	41
J11	49	49	15	18	65	55	1	79	10	37	77	80	79	84	93	21	85	64	46	35
J12	3	53	59	7	65	58	24	55	26	40	89	94	51	74	54	86	22	83	19	44
J13	60	88	15	26	11	16	55	59	81	53	92	23	55	79	13	89	2	17	97	41
J14	12	47	46	17	43	16	91	94	73	89	12	58	25	24	55	1	67	3	1	71
J15	75	19	60	87	27	48	72	88	48	59	74	86	49	94	15	95	41	94	15	71

Case 9 (20x20): Taillard (1993)																				
J16	31	61	47	32	34	69	32	1	1	80	19	57	98	37	31	51	66	38	62	72
J17	70	78	41	9	47	94	26	65	17	42	59	80	7	75	63	96	7	10	47	38
J18	20	78	38	26	64	62	11	38	68	37	74	9	65	16	38	85	50	62	39	97
J19	88	30	34	33	21	7	94	10	73	85	82	62	99	67	61	10	4	70	31	49
J20	9	41	22	34	83	55	3	8	75	30	57	65	89	60	90	84	74	17	2	19

(i)

Case 10 (50x10): Taillard (1993)										
J\M	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
J1	46	61	3	51	37	79	83	22	27	24
J2	52	87	1	24	16	93	87	29	92	47
J3	79	51	58	21	42	68	38	99	75	39
J4	45	25	85	57	47	75	38	25	94	66
J5	97	73	33	69	94	37	86	98	18	41
J6	10	93	71	51	14	44	67	55	41	46
J7	44	28	58	50	94	34	23	80	37	24
J8	24	90	56	51	34	39	19	82	58	23
J9	85	94	64	21	72	76	97	33	56	68
J10	75	59	43	19	36	62	78	68	20	50
J11	66	64	48	63	88	74	66	47	2	93
J12	49	2	69	91	51	28	67	74	39	22
J13	95	16	96	11	41	78	7	26	91	64
J14	61	35	35	6	71	43	23	61	81	81
J15	19	53	82	31	94	98	67	95	33	94
J16	47	40	53	63	99	83	8	55	14	97
J17	84	81	64	36	11	91	77	11	88	54
J18	13	26	11	39	97	27	71	42	22	82
J19	11	85	61	57	44	6	85	72	36	11
J20	19	4	36	47	77	82	29	14	65	91
J21	98	4	53	56	69	60	49	8	79	23
J22	2	10	87	65	91	44	3	98	23	32
J23	85	63	88	59	38	43	94	90	66	26
J24	44	96	10	4	25	76	76	36	5	22
J25	7	55	32	10	87	99	95	75	15	12
J26	73	71	38	12	7	66	48	69	51	23
J27	19	66	25	62	66	11	4	26	2	34
J28	69	94	24	43	54	35	37	24	81	87
J29	12	7	90	49	86	52	82	55	12	59
J30	73	15	7	54	49	8	57	98	40	2
J31	85	11	11	87	3	40	61	86	59	38
J32	23	99	49	29	48	62	6	30	32	84
J33	53	37	2	2	44	25	97	92	16	62
J34	16	50	76	18	93	24	5	94	87	10
J35	88	56	17	75	37	30	27	66	78	11
J36	8	69	32	39	82	1	95	47	41	93
J37	26	22	39	77	31	73	46	3	43	57
J38	42	56	9	69	59	27	92	41	94	81
J39	58	67	83	15	78	16	46	41	1	10
J40	63	63	69	78	33	91	52	47	93	40
J41	7	96	67	68	36	33	8	89	22	62
J42	2	74	28	37	3	11	11	28	93	49
J43	44	4	88	22	58	99	7	39	62	90
J44	38	42	23	41	10	2	54	80	53	34
J45	24	40	91	92	98	60	72	47	30	11

Case 10 (50x10): Taillard (1993)										
J46	76	30	71	67	6	90	57	57	34	81
J47	85	93	3	24	44	36	85	74	27	51
J48	61	36	26	87	62	62	22	38	30	21
J49	32	25	41	91	24	15	87	59	54	39
J50	90	87	96	31	94	3	65	5	77	27

(j)

Fig. 3 Input data set for testing the efficiency of the proposed heuristic technique (a – j)

Case (#) J x M	Method	Sequence	Makespan	Case (#) J x M	Method	Sequence	Makespan
(1) 4 x 4	Palmer Slop Index	3-1-2-4	30	(5) 8 x 3	Palmer Slop Index	8-2-6-4-7-3-1-5	52
	Gupta's Heuristics	3-1-2-4	30		Gupta's Heuristics	6-8-2-7-5-4-3-1	55
	CDS	3-1-2-4	30		CDS	8-2-6-3-7-4-5-1	51
	Dannenbring	3-2-1-4	31		Dannenbring	6-8-2-7-5-3-4-1	52
	Hundal	3-1-2-4	30		Hundal	8-2-7-6-3-4-5-1	51
	Harmonic Triangle	3-2-1-4	31		Harmonic Triangle	8-2-7-5-4-1-6-3	59
	This Paper	**3-1-2-4**	**30**		**This Paper**	**8-2-6-7-4-3-5-1**	**51**
(2) 5 x 5	Palmer Slop Index	4-2-1-5-3	90	(6) 10 x 8	Palmer Slop Index	9-3-4-6-10-8-7-5-1-2	92
	Gupta's Heuristics	4-1-2-3-5	89		Gupta's Heuristics	4-6-9-7-10-8-3-5-2-1	96
	CDS	4-1-2-3-5	89		CDS	9-4-6-10-3-7-5-1-8-2	93
	Dannenbring	3-1-2-5-4	93		Dannenbring	6-9-4-10-8-3-7-5-2-1	92
	Hundal	1-2-3-4-5	89		Hundal	9-4-3-6-10-8-7-1-5-2	90
	Harmonic Triangle	3-1-5-4-2	90		Harmonic Triangle	9-6-8-10-2-4-7-5-3-1	94
	This Paper	**3-4-1-2-5**	**89**		**This Paper**	**9-4-3-6-10-8-7-5-1-2**	**92**
(3) 5 x 6	Palmer Slop Index	5-4-2-3-1	225	(7) 10 x 10	Palmer Slop Index	3-1-10-5-6-2-8-7-9-4	99
	Gupta's Heuristics	1-2-3-4-5	229		Gupta's Heuristics	3-6-10-1-5-9-8-2-7-4	103
	CDS	2-3-4-5-1	224		CDS	3-6-10-1-9-5-8-2-7-4	102
	Dannenbring	1-2-3-4-5	229		Dannenbring	3-10-1-5-9-8-6-2-7-4	97
	Hundal	2-1-5-3-4	214		Hundal	3-1-10-6-5-2-8-7-4-9	100
	Harmonic Triangle	1-2-3-4-5	229		Harmonic Triangle	3-6-1-10-9-5-4-2-7-8	96
	This Paper	**2-1-5-3-4**	**214**		**This Paper**	**3-1-10-5-6-8-2-9-7-4**	**97**
(4) 7 x 7	Palmer Slop Index	4-1-5-3-2-7-6	70	(8) 20 x 10	Palmer Slop Index	5-12-15-17-18-4-9-2-20-3-10-13-11-19-6-7-14-8-16-1	1790
	Gupta's Heuristics	4-1-3-5-2-7-6	70		Gupta's Heuristics	2-8-17-12-5-9-6-4-15-19-13-10-7-11-3-18-20-14-16-1	2027
	CDS	4-1-3-5-6-2-7	68		CDS	2-17-18-5-9-15-12-20-4-6-7-13-19-10-14-8-11-3-1-16	1757
	Dannenbring	4-1-3-5-6-2-7	68		Dannenbring	18-2-17-5-3-4-9-12-15-20-10-13-8-19-14-11-7-6-1-11	1771
	Hundal	4-5-3-1-2-7-6	69		Hundal	5-12-18-17-15-4-2-9-3-20-10-11-6-13-7-19-14-8-16-1	1741
	Harmonic Triangle	1-7-3-5-6-2-4	78		Harmonic Triangle	16-2-6-4-17-5-9-8-13-20-14-10-19-12-11-15-7-1-3-18	2016
	This Paper	**4-1-5-3-2-6-7**	**66**		**This Paper**	**5-12-18-15-17-4-9-2-3-20-10-13-19-11-7-6-14-8-1-16**	**1758**
(9) 20 x 20	Palmer Slop Index	11-16-12-18-3-7-20-5-13-15-1-6-19-9-14-8-17-10-2-4					2466
	Gupta's Heuristics	16-14-10-20-3-7-5-1-18-12-15-6-8-4-11-13-17-19-2-9					2555
	CDS	20-3-12-14-11-18-7-16-2-6-5-15-1-8-9-13-19-4-17-10					2493
	Dannenbring	16-20-18-12-11-18-5-3-19-7-6-1-15-8-2-9-17-14-4-10					2568
	Hundal	11-12-16-18-7-3-5-20-15-1-6-13-19-9-14-8-17-2-10-4					2536
	Harmonic Triangle	20-14-12-5-10-3-18-7-16-1-6-13-17-4-15-11-19-9-2-8					2580
	This Paper	**11-16-18-12-3-20-7-5-13-1-15-6-19-9-14-8-17-2-10-4**					**2492**
(10) 50 x 10	Palmer Slop Index	20-38-18-36-15-14-33-43-42-2-44-29-4-49-31-25-37-34-22-16-3-46-28-30-6-12-32-13-41-40-7-10-8-11-1-47-19-26-17-9-23-21-24-35-45-48-27-5-39-50					3461
	Gupta's Heuristics	33-22-42-27-29-20-25-32-18-49-14-41-43-37-11-6-28-16-4-38-36-15-46-40-9-23-50-3-5-8-7-21-35-10-48-12-1-17-19-45-13-34-47-2-31-30-26-24-44-39					3672
	CDS	18-20-37-33-25-30-49-22-44-42-31-38-29-36-12-7-14-43-2-16-34-15-4-41-28-3-23-13-40-46-8-9-5-35-17-47-32-26-11-6-10-19-21-50-48-45-1-24-27-39					3421
	Dannenbring	42-44-18-33-20-37-49-14-22-36-34-31-43-25-29-2-38-4-28-16-3-46-15-9-40-23-5-11-17-45-13-47-10-6-12-41-50-21-7-8-32-19-35-26-1-30-48-24-39-27					3510
	Hundal	20-18-38-36-33-14-42-43-15-44-2-49-29-4-37-31-25-22-34-30-16-3-46-28-32-6-12-41-13-7-1-8-10-19-26-40-47-11-24-27-17-21-35-48-9-23-45-5-39-50					3469
	Harmonic Triangle	27-29-22-25-18-41-36-24-19-45-32-6-37-39-16-15-1-11-33-20-8-42-3-10-46-47-38-9-28-17-2-50-14-40-5-4-43-23-26-13-44-34-35-49-7-48-31-21-12-30					3856
	This Paper	**20-18-33-42-36-38-14-43-15-44-2-49-29-4-31-25-37-22-34-16-3-46-28-6-30-12-13-32-41-40-7-11-10-8-47-1-17-9-19-26-23-21-5-45-35-24-48-27-50-39**					**3415**

Fig. 4 Comparative results between the proposed technique and six benchmark techniques

It is clear from the table presented in Figure 4 that the proposed technique is more effective than the other heuristic techniques in all of the ten tested flow shop problems. Figure 4 shows the makespan for each of the ten problems for all six methods including the method proposed in this study. Table 3 shows that the proposed technique gives a better result than the harmonic triangle, Dannenbring, Palmer, Gupta, CDS, and Hundal methods by 90%, 80%, 70%, 70%, 60%, and 40%, respectively.

Table 3 A comparison between the proposed method and six other methods

Other methods	Number of times when the solution by the proposed method is better	Number of times when the solution by the proposed method is equal	Number of times when the solution achieved by other methods is better	Percentage of proposed method when it is better than other method
Palmer Slope Index	7	2	1	70.00%
Gupta's Heuristics	7	3	---	70.00%
CDS	6	3	1	60.00%
Dannenbring	8	2	---	80.00%
Hundal	4	4	2	40.00%
Harmonic Triangle	8	---	1	88.89%

As shown in Table 3, the proposed method has a better result in seven out of ten problems compared to the Palmer Slope Index, in seven problems compared to Gupta's, in six problems compared to CDS, in eight problems compared to Dannenbring, in four problems compared to Hundal, and in eight problems compared to the harmonic triangle. Table 4 shows the percentage of improvement provided by the proposed method corresponding to the problem size with respect to each of the other methods. Moreover, Table 4 indicates that the average improvement of the proposed technique over the other six methods is notable. The proposed technique has improved the results on average by 6.86%, 4.81%, 2.53%, 1.77%, 1.35%, and 0.74% over the harmonic triangle, Gupta, Dannenbring, Palmer, CDS, and Hundal, respectively.

Table 4 Percentage of improvement achieved by proposed method compared to other methods

Problem size (N x M)	Palmer Slope Index	Gupta's Heuristics	CDS	Dannenbring	Hundal	Harmonic Triangle
4 x 4	0.00%	0.00%	0.00%	**3.23%**	0.00%	**3.23%**
5 x 5	**1.11%**	0.00%	0.00%	**4.30%**	0.00%	**1.11%**
5 x 6	**4.89%**	**6.55%**	**4.46%**	**6.55%**	0.00%	**6.55%**
7 x 7	**5.71%**	**5.71%**	**2.94%**	**2.94%**	**4.35%**	**15.38%**
8 x 3	**1.92%**	**7.27%**	0.00%	**1.92%**	0.00%	**13.56%**
10 x 8	0.00%	0.00%	**1.08%**	0.00%	-2.22%	**2.13%**
10 x 10	**2.02%**	**5.83%**	**4.90%**	0.00%	**3.00%**	-1.04%
20 x 10	**1.79%**	**13.27%**	-0.06%	**0.73%**	-0.98%	**12.80%**
20 x 20	-1.05%	**2.47%**	**0.04%**	**2.96%**	**1.74%**	**3.41%**
50 x 10	**1.35%**	**7.00%**	**0.18%**	**2.71%**	**1.56%**	**11.44%**
Average improvement	1.77%	4.81%	1.35%	2.53%	0.74%	6.86%

5. Conclusion

The primary purpose of the paper was to develop and demonstrate a new heuristic technique to solve ‘n’ jobs on ‘m’ machines for static flow shop sequencing problems with a minimum makespan time as an objective function. The proposed technique assigns a slope index to each job based on its trend journey from the first to the last machine. After that, the indices are ranked in descending order to determine the job sequencing and corresponding makespan time. The technique is simple and quick to produce one sequence at a time. Based on ten tested flow shop problems with sizes ranging from 4x4 to 50x10, a comparative analysis is made to find the effectiveness of the proposed technique with respect to the other five well-known heuristics (Palmer, Gupta, CDS, Dannenbring, and Hundal) as well as a recent one (harmonic triangle). The results showed that the proposed technique successfully improved (on average) the makespan time over the other six heuristics. As an overall observation, the proposed technique is more efficient than the harmonic triangle methodology, while the Hundal technique is the closest competitor to the proposed technique. In future work, examining the proposed technique on a job shop, or extending the proposed technique to a dynamic job shop might be an area of study.

REFERENCES

[1] Y. MarfoMissah, “Business Innovation with Enterprise Architecture,” *Int. J. Comput. Appl.*, vol. 120, no. 9, pp. 12–15, 2015, https://doi.org/10.5120/21254-4097

[2] S. M. Johnson, “Optimal two- and three-stage production schedules with setup times included,” *Nav. Res. Logist. Q.*, vol. 1, no. 1, pp. 61–68, Mar. 1954, https://doi.org/10.1002/NAV.3800010110

[3] D. Palmer, “Sequencing Jobs Through a Multi-Stage Process in the Minimum Total Time - A Quick Method of Obtaining a Near Optimum,” *J. Oper. Res. Soc.*, vol. 16, pp. 101–107, 1965, https://doi.org/10.1057/jors.1965.8

[4] T. Meng and Q.-K. Pan, “A distributed heterogeneous permutation flowshop scheduling problem with lot-streaming and carryover sequence-dependent setup time,” *Swarm Evol. Comput.*, vol. 60, 2021, https://doi.org/10.1016/j.swevo.2020.100804

[5] H. G. Campbell, R. A. Dudek, and M. L. Smith, “ A Heuristic Algorithm for the n Job, m Machine Sequencing Problem ,” *Manage. Sci.*, vol. 16, no. 10, p. B-630-B-637, Jun. 1970, https://doi.org/10.1287/MNSC.16.10.B630

[6] J. N. D. Gupta, “A heuristic algorithm for the flowshop scheduling problem,” *RAIRO - Oper. Res. - Rech. Opérationnelle*, vol. 10, no. V2, pp. 63–73, 1976, Accessed: Aug. 03, 2021. [Online]. Available: http://www.numdam.org/item?id=RO_1976__10_2_63_0. https://doi.org/10.1051/ro/197610V200631

[7] I. Ribas, R. Companys, and X. Tort-Martorell, “Efficient heuristics for the parallel blocking flow shop scheduling problem,” *Expert Syst. Appl.*, vol. 74, pp. 41–54, 2017, https://doi.org/10.1016/j.eswa.2017.01.006

[8] D. G. Dannenbring, “An Evaluation of Flow Shop Sequencing Heuristics,” *Manage. Sci.*, vol. 23, no. 11, 1977, https://doi.org/10.1287/mnsc.23.11.1174

[9] A. Garay-Sianca and S. G. Nurre Pinkley, “Interdependent integrate d network design and sche duling problems with movement of machines,” *European Journal of Operational Research,* Volume 289, Issue 1, pp. 297-327, 2021, https://doi.org/10.1016/j.ejor.2020.07.013

[10] T. S. HUNDAL and J. RAJGOPAL, “An extension of Palmer’s heuristic for the flow shop scheduling problem,” *International Journal of Production Research*, vol. 26, no. 6, pp. 1119–1124, 1988, https://doi.org/10.1080/00207548808947922

[11] B. Dhanasakkaravarthi and A. Krishnamoorthy, “A NEW HEURISTIC ALGORITHM TO DETERMINE MORE THAN ONE SEQUENCE IN PERMUTATION FLOW SHOP SCHEDULING BY USING HARMONIC TRIANGLE,” *Int. J. Mech. Eng. Technol.*, vol. 10, no. 03, pp. 284–289, 2019.

[12] A. K. Agarwal and R. Garg, “Advanced Modelistic Approach of Flowshop Scheduling Problem for 10-Jobs , 10-Machines By Heuristics Models Using Makespan Criterion,” *Int. J. Eng. trend Technol.*, vol. 4, no. 5, pp. 1742–1748, 2013.

[13] E. Taillard, "Benchmarks for basic scheduling problems," *Eur. J. Oper. Res.*, vol. 64, no. 2, pp. 278–285, 1993, https://doi.org/10.1016/0377-2217(93)90182-M

[14] E. Vallade, R. Ruiz, and J. M. Framinan, "New hard benchmark for flowshop scheduling problems minimising makespan," *Eur. J. Oper. Res.*, vol. 240, no. 3, pp. 666–677, 2015, https://doi.org/10.1016/j.ejor.2014.07.033

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